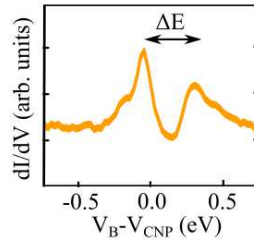


# How heterostrain controls the flat bands of twisted bilayers of graphene ?

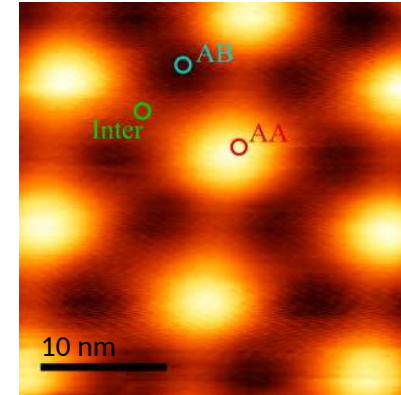


## Scanning Tunneling Spectroscopy



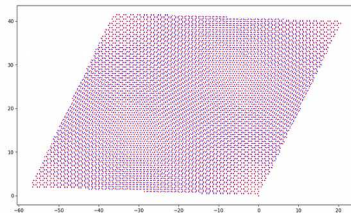
—► Information on the flat bands

## Scanning Tunneling Microscopy



—► Information on heterostrain & stacking configuration

## Link with Continuum and Tight binding calculations



# The most important



**Hartree-Fock  
Continuum calculations**  
Tommaso Cea  
Paco Guinea  
IMDEA Nanociencia

instituto  
**imdea**  
nanociencia

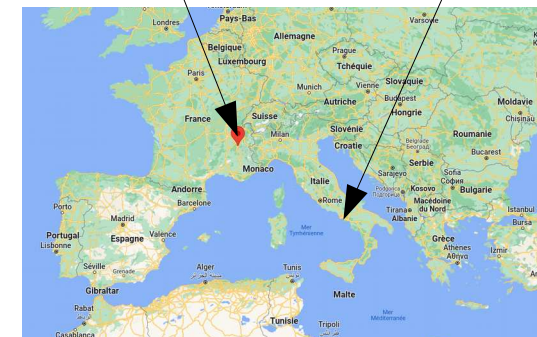


**Tight binding calculations**  
Ahmed Missaoui  
Guy Trambly de Laissardière  
LPTM Cergy Pontoise

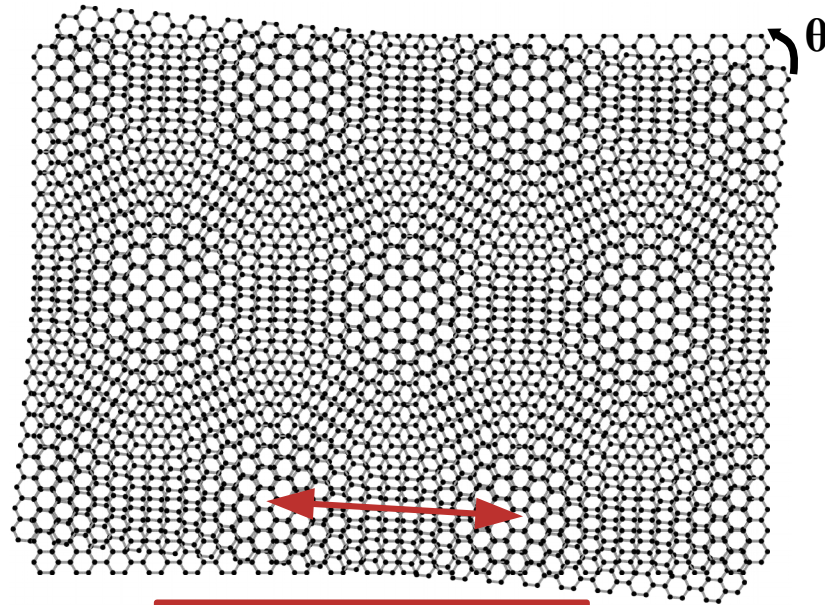
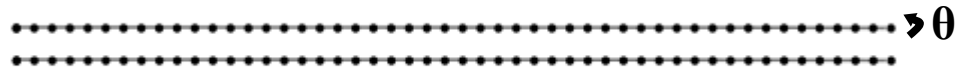
 UNIVERSITÉ  
de Cergy-Pontoise



**STM team**  
Loic Huder  
Vincent T. Renard  
Claude Chapelier  
IRIG PHELIQS



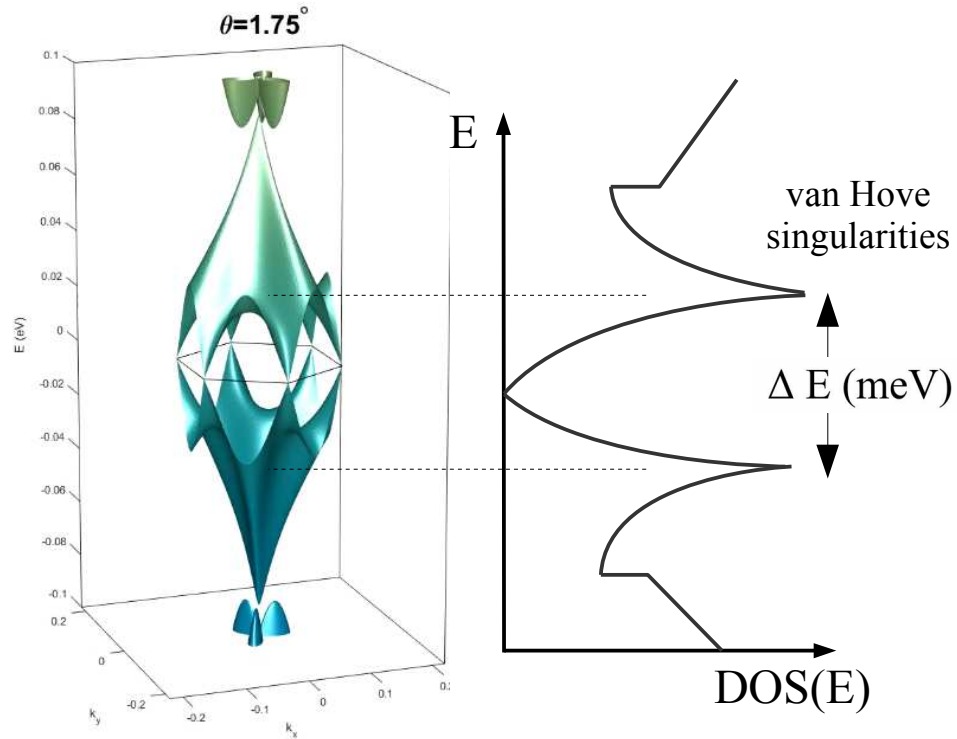
# Twisting two layers of graphene



$$a_M = \frac{a_{Gr}}{2\sin(\theta/2)}$$

# Flat-bands in low angle twisted bilayers of graphene

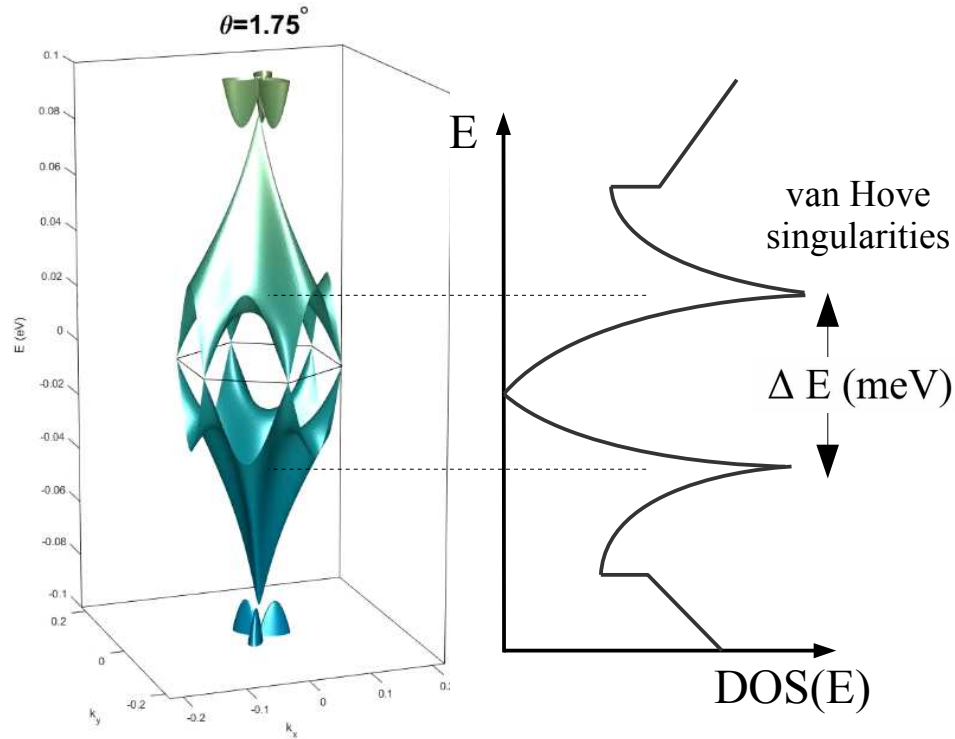
Highly tunable system



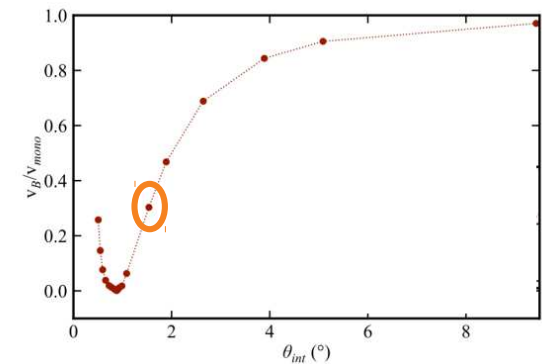
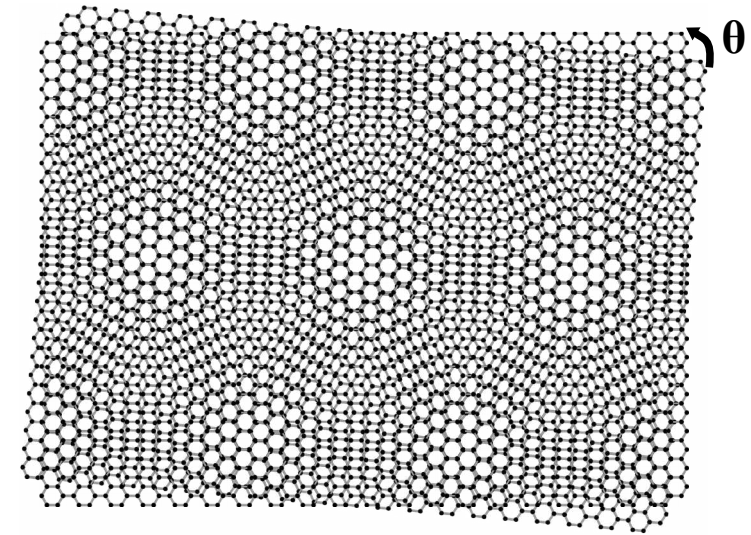
Y. Cao et al. Nature 556, 80 (2019)

# Flat-bands in low angle twisted bilayers of graphene

Highly tunable system



Y. Cao et al. Nature 556, 80 (2019)



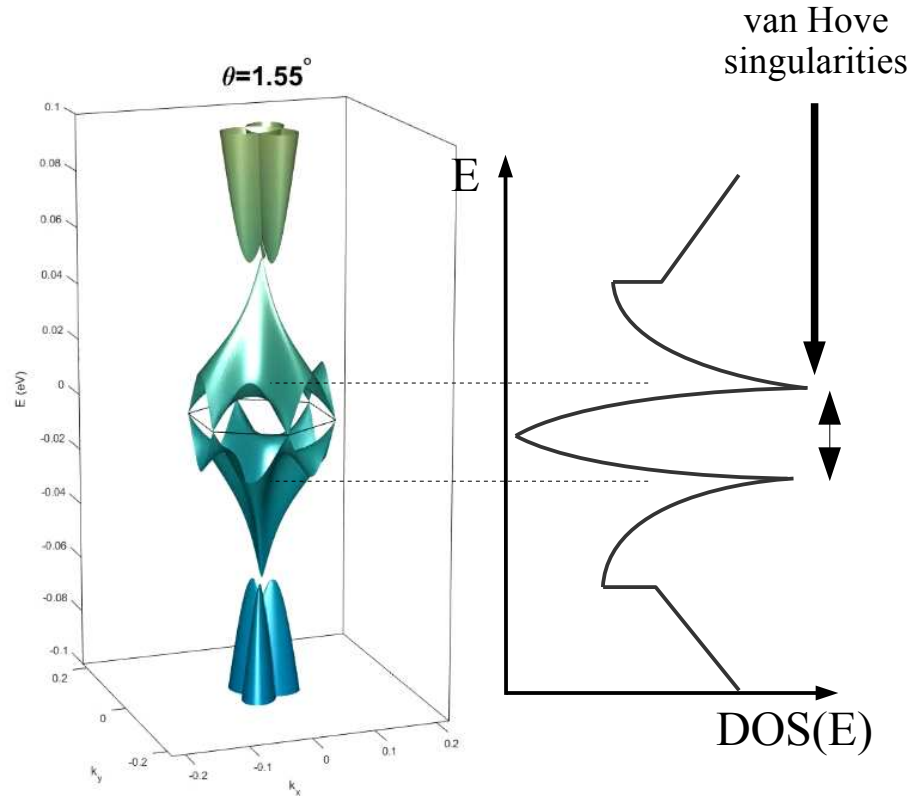
G. T. de Laissardière et. al ; Nano Lett. 10, 804–808 (2010)

R. Bistritzer and A. H. MacDonald. PNAS 108(30), (2011)

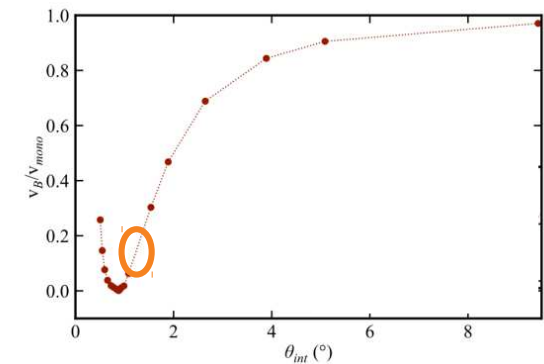
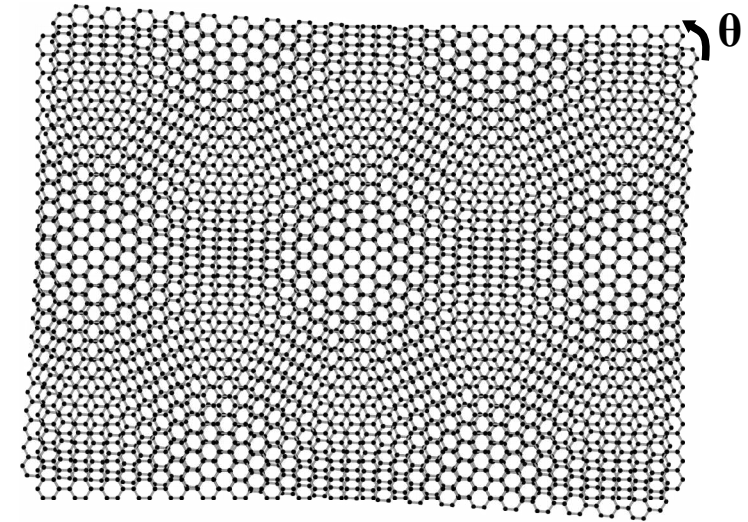
J. M. B. Lopes dos Santos et al P.R.B 86, 155449, (2012)

# Flat-bands in low angle twisted bilayers of graphene

Highly tunable system



Y. Cao et al. Nature 556, 80 (2019)



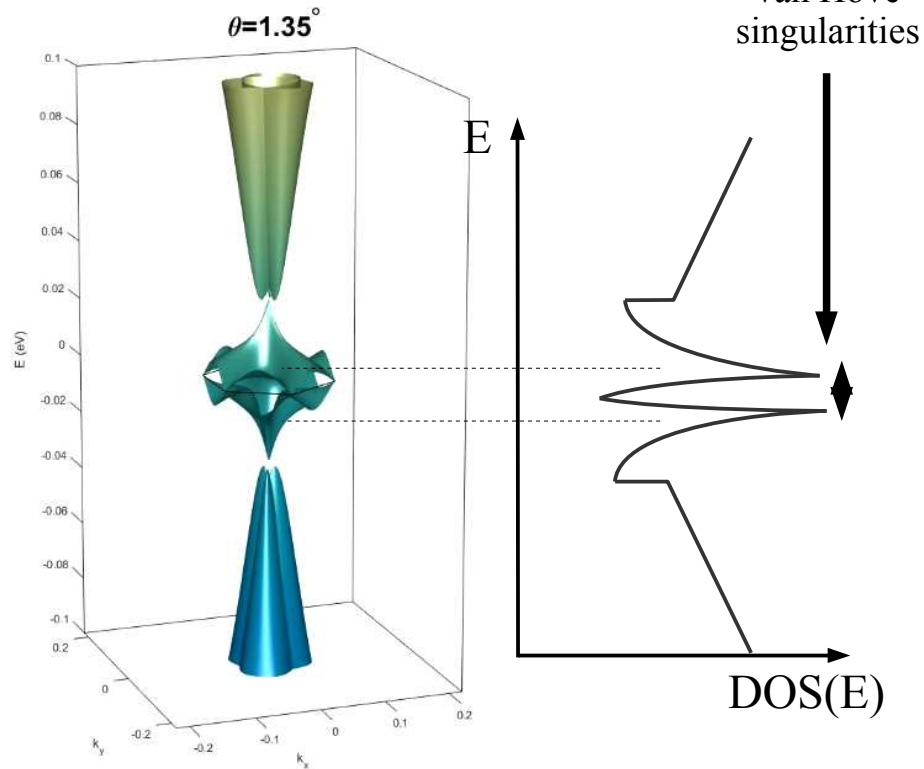
G. T. de Laissardière et. al ; Nano Lett. 10, 804–808 (2010)

R. Bistritzer and A. H. MacDonald. PNAS 108(30), (2011)

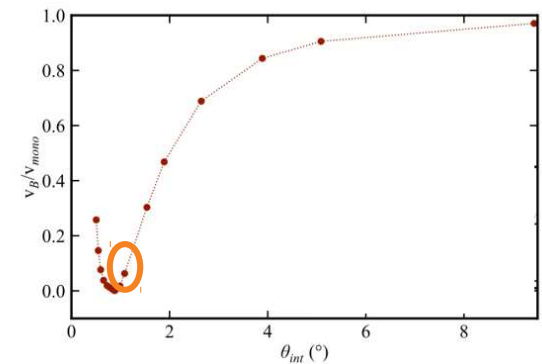
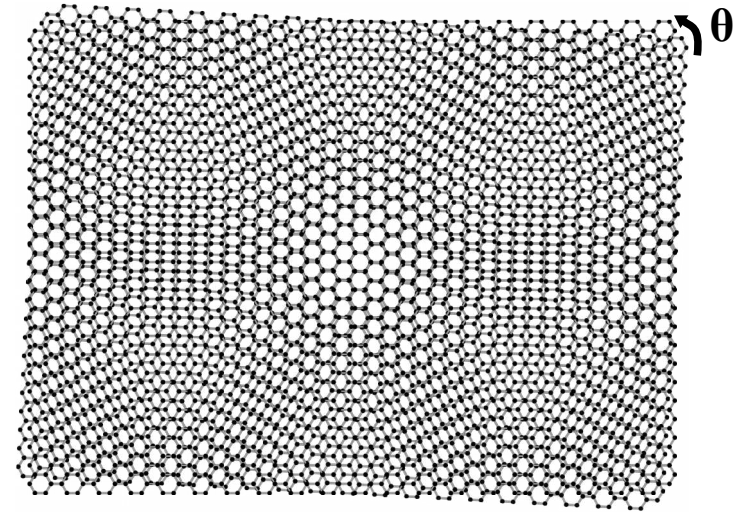
J. M. B. Lopes dos Santos et al P.R.B 86, 155449, (2012)

# Flat-bands in low angle twisted bilayers of graphene

Highly tunable system



Y. Cao et al. Nature 556, 80 (2019)



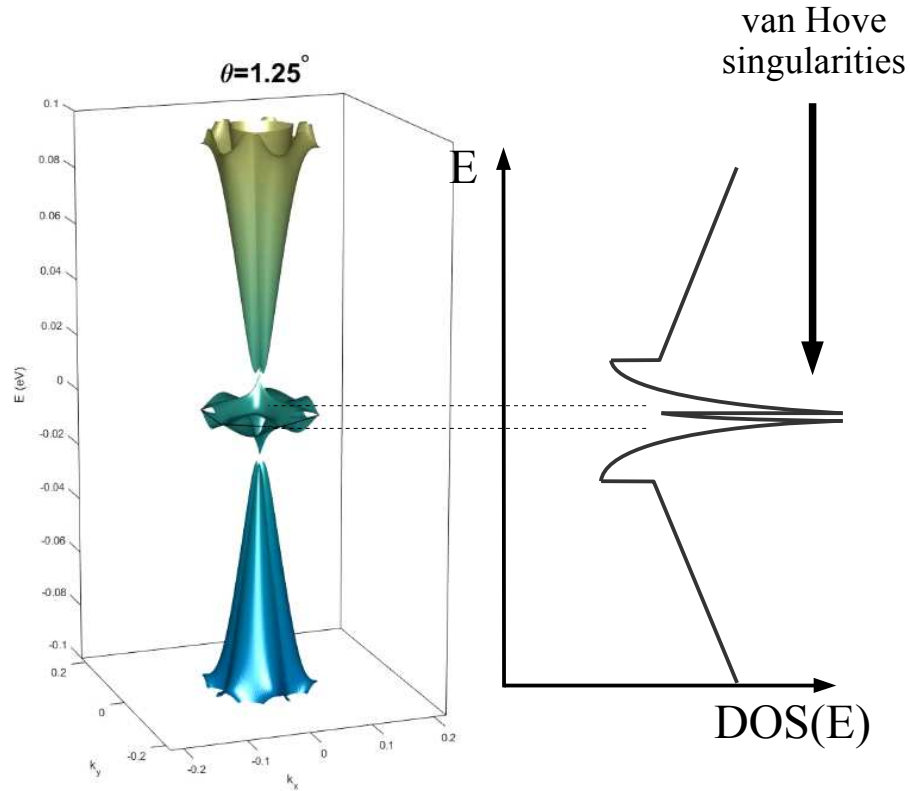
G. T. de Laissardière et. al ; Nano Lett. 10, 804–808 (2010)

R. Bistritzer and A. H. MacDonald. PNAS 108(30), (2011)

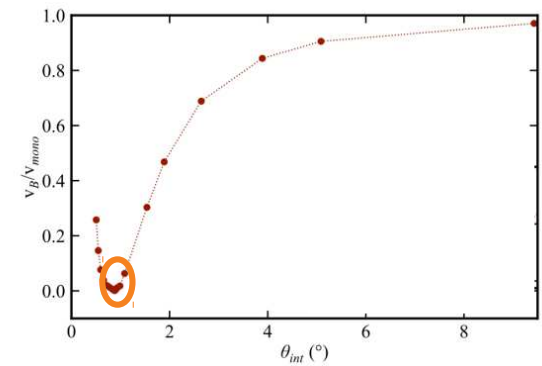
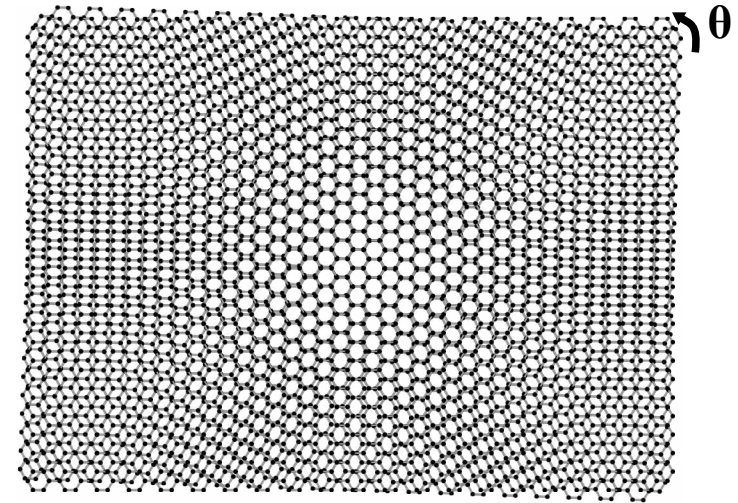
J. M. B. Lopes dos Santos et al P.R.B 86, 155449, (2012)

# Flat-bands in low angle twisted bilayers of graphene

Highly tunable system



Y. Cao et al. Nature 556, 80 (2019)



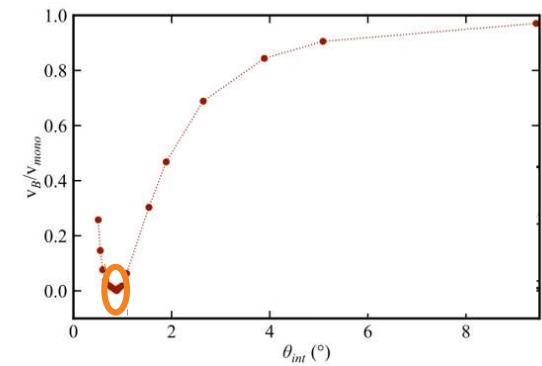
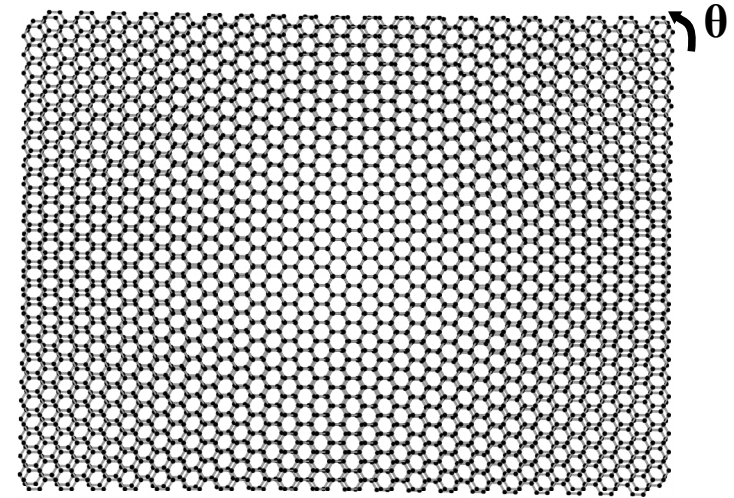
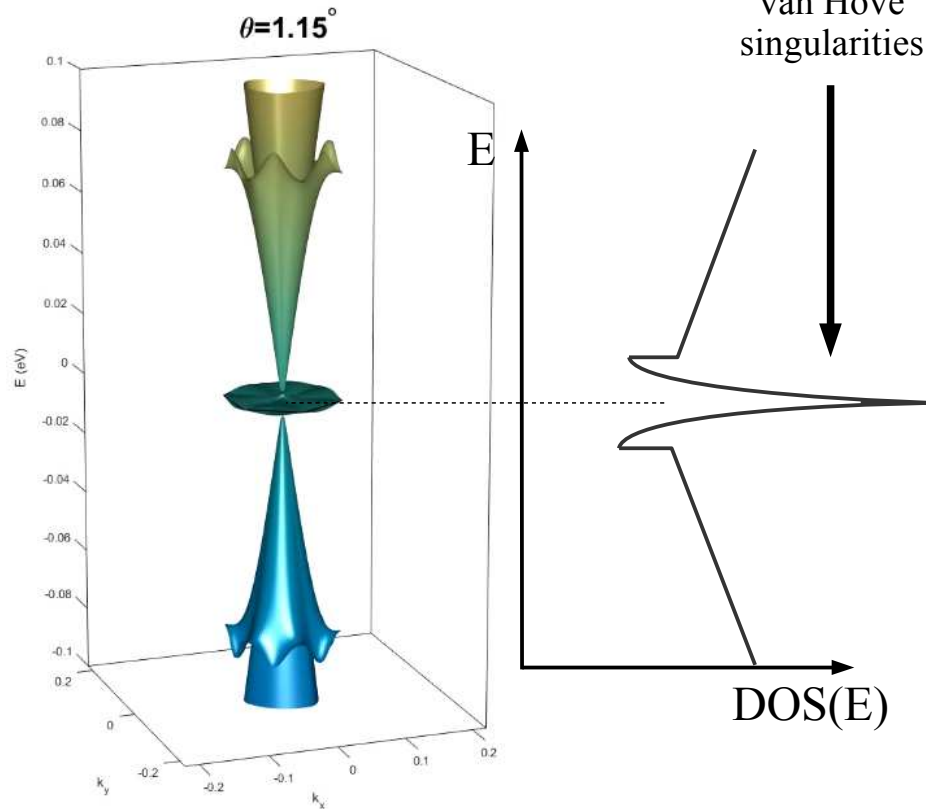
G. T. de Laissardière et. al ; Nano Lett. 10, 804–808 (2010)

R. Bistritzer and A. H. MacDonald. PNAS 108(30), (2011)

J. M. B. Lopes dos Santos et al P.R.B 86, 155449, (2012)

# Flat-bands in low angle twisted bilayers of graphene

Highly tunable system



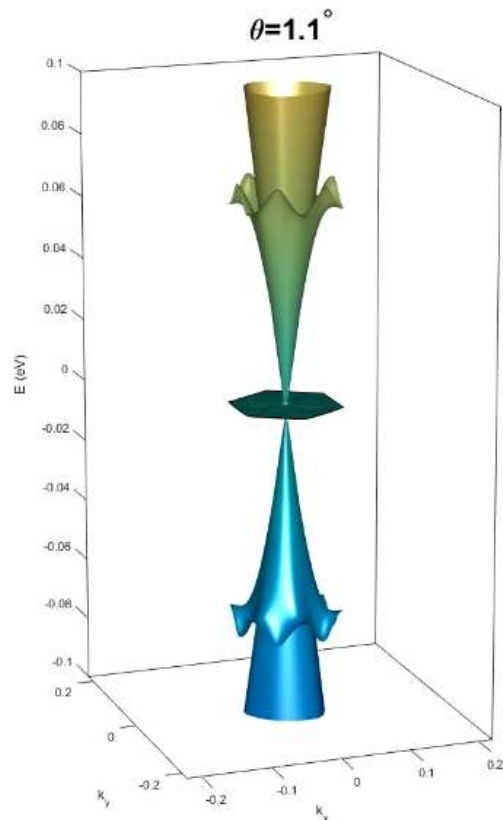
G. T. de Laissardi re et. al ; Nano Lett. 10, 804–808 (2010)

R. Bistritzer and A. H. MacDonald. PNAS 108(30), (2011)

J. M. B. Lopes dos Santos et al P.R.B 86, 155449, (2012)

# Flat-bands in low angle twisted bilayers of graphene

Highly tunable system



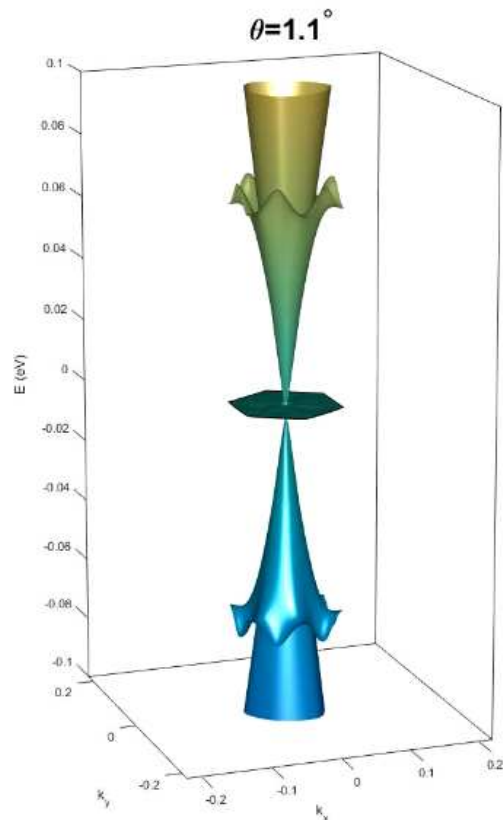
Y. Cao et al. Nature 556, 80 (2019)

$$\frac{E_C}{E_k} \rightarrow \infty$$

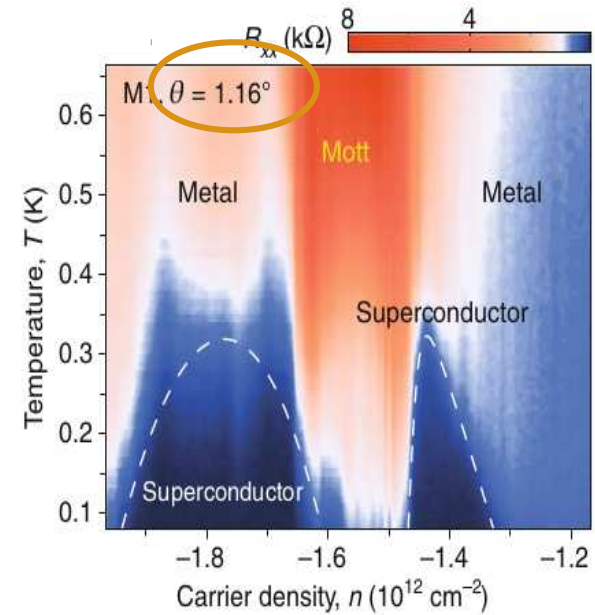
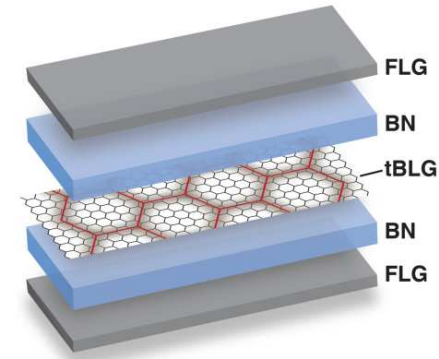


boosted interactions

## Highly tunable system



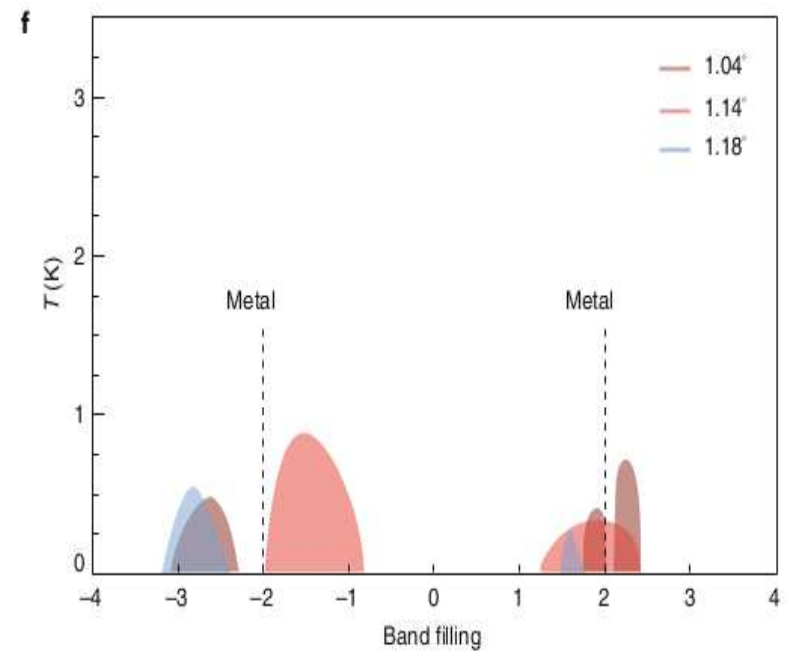
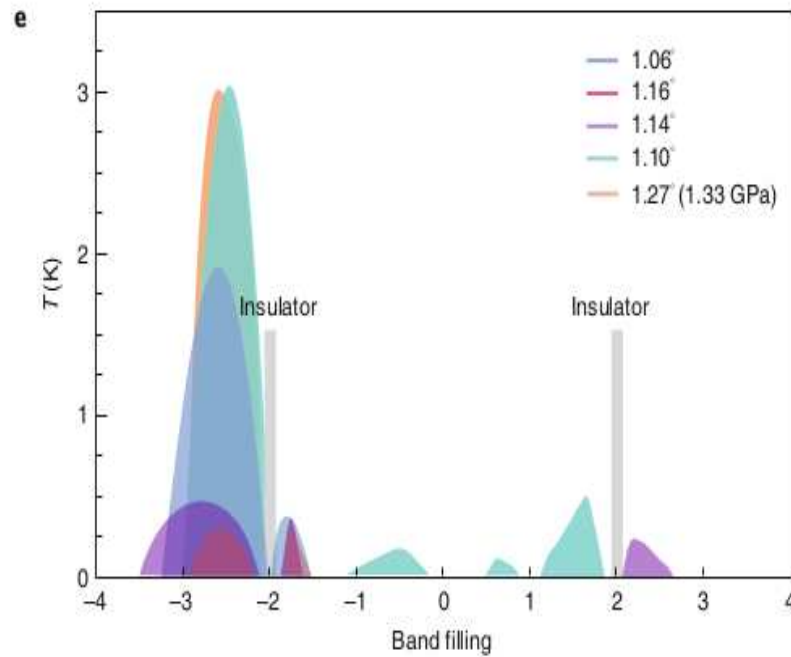
Y. Cao et al. Nature 556, 80 (2019)

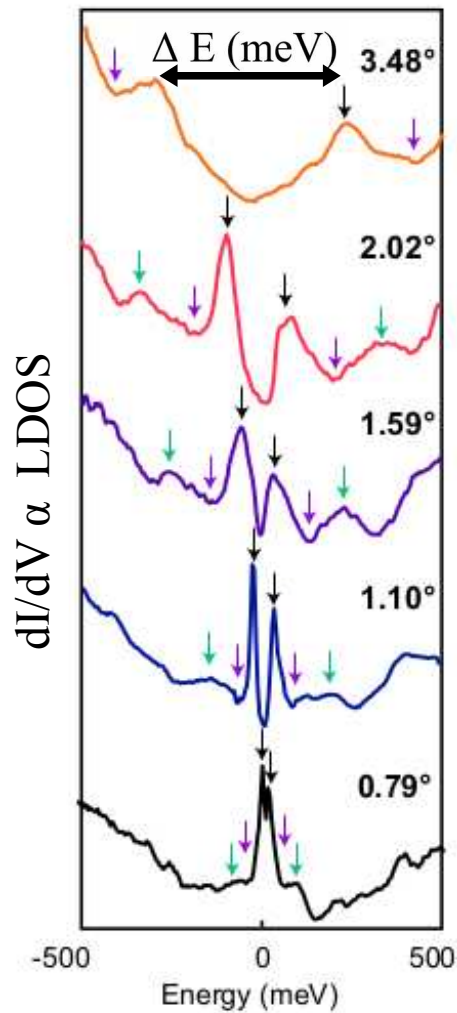


Y. Cao et al. Nature 55, 43-50, (2019)

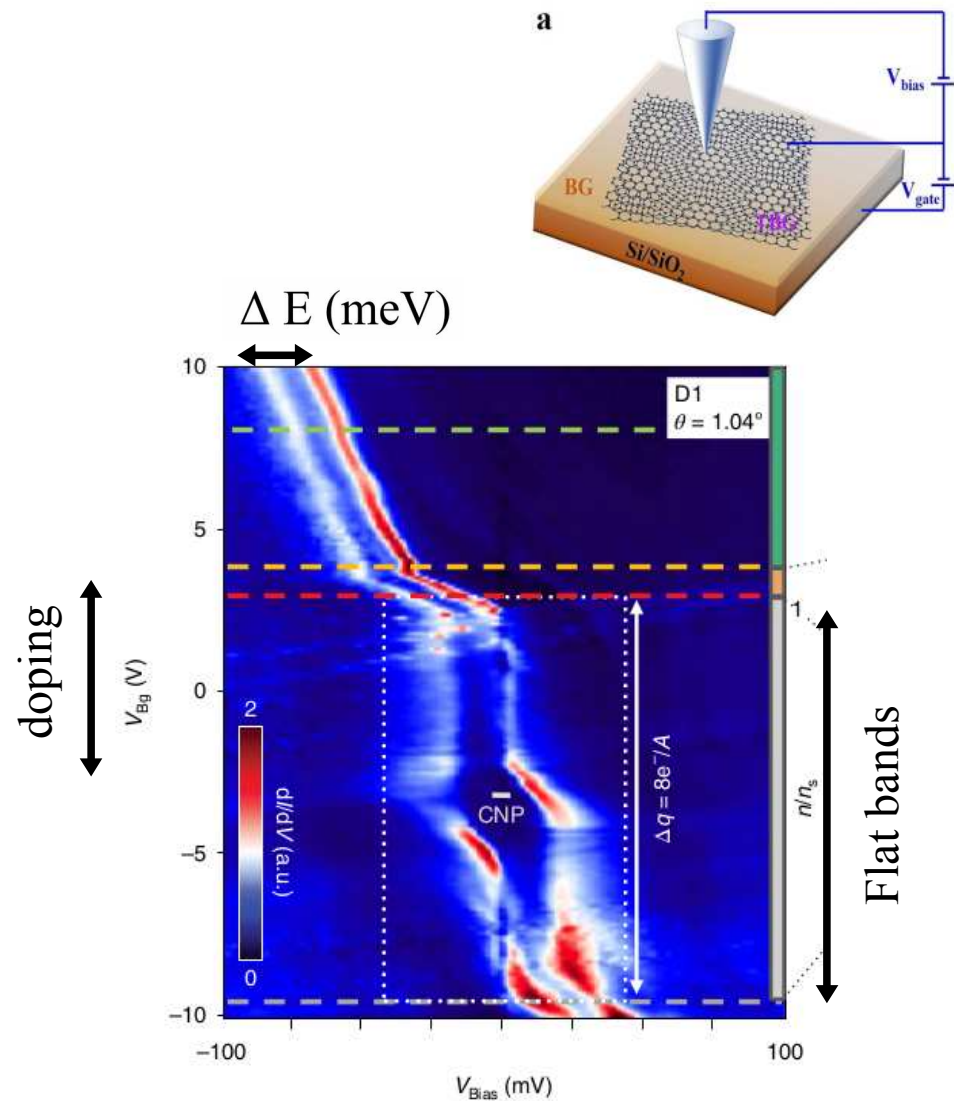
# A variety of Twisted bilayers of graphene

L. Balents, C. R. Dean, D. K. Efetov, and A. F. Young Nature Physics 16(7), 725–733 (2020)



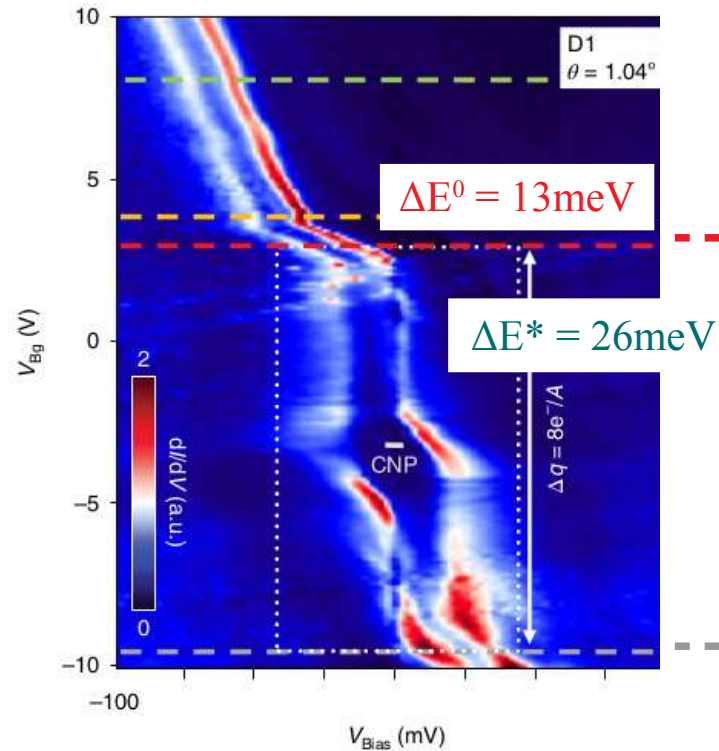


A. Kerelsky et al. Nature 572, 95-100 (2019)



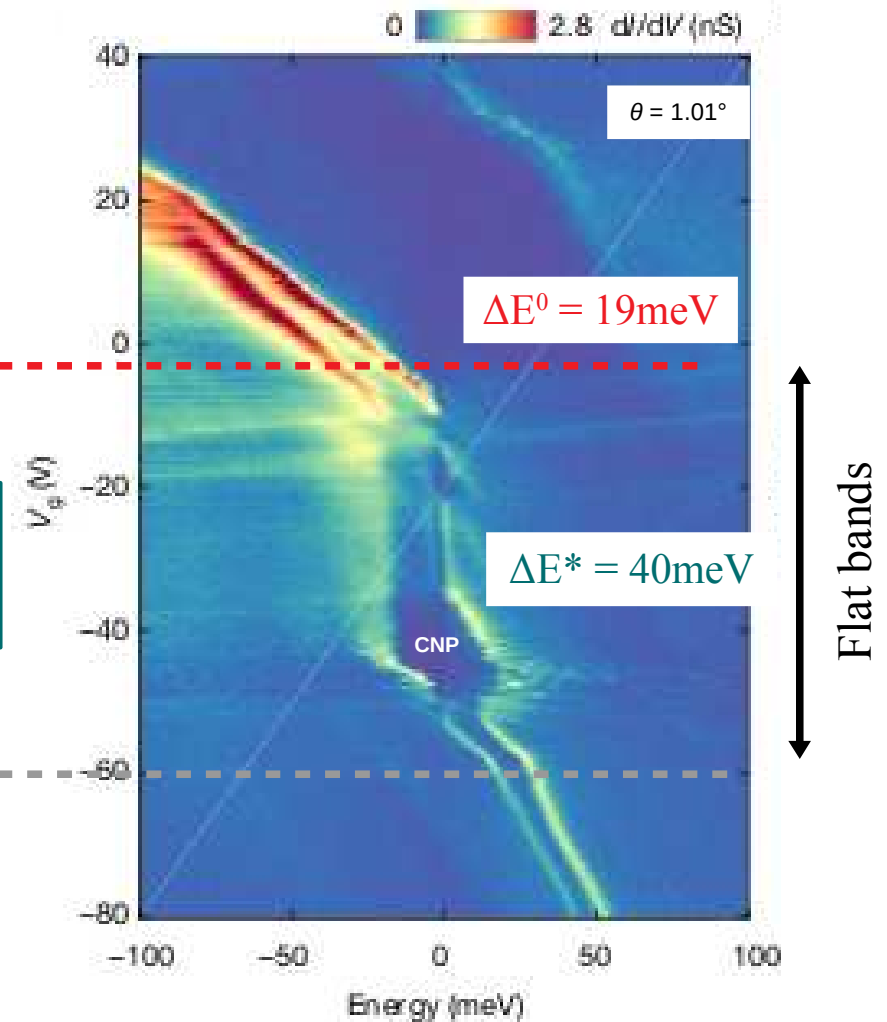
Y. Choi et al. Nature Physics 15, 1174-1180 (2019)

# A variety of Twisted bilayers of graphene



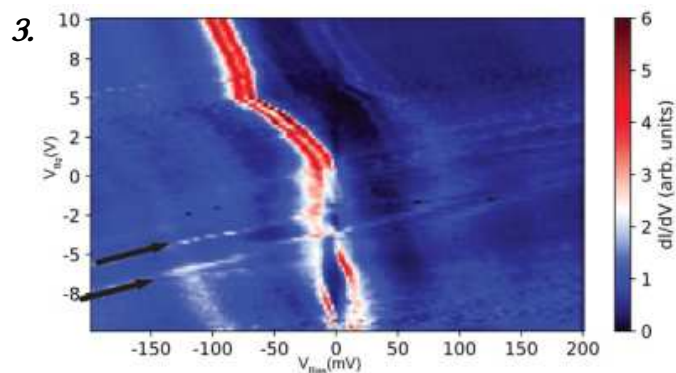
Y. Choi et al. Nature Physics 15, 1174-1180 (2019)

$$\frac{E_C}{E_k} \rightarrow \infty$$

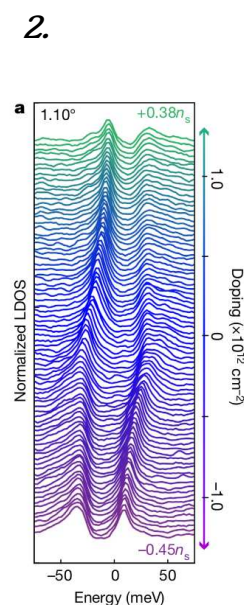
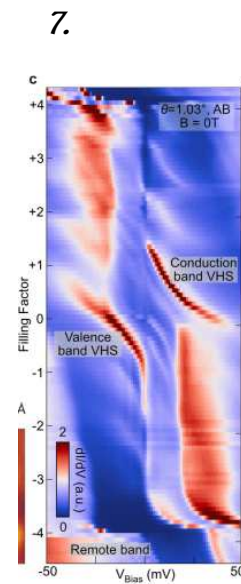
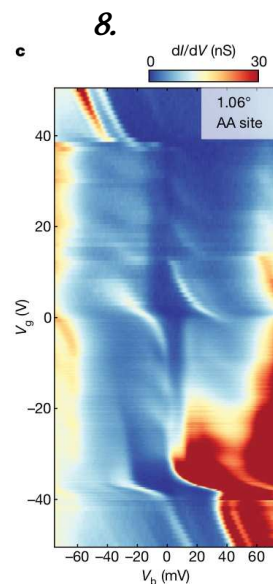
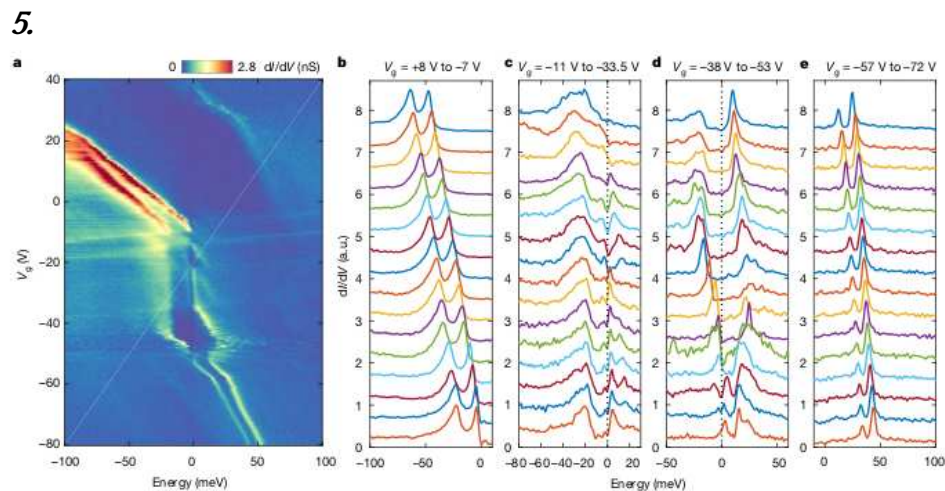


Y. Xie et al. Nature 572, 101-105 (2019)

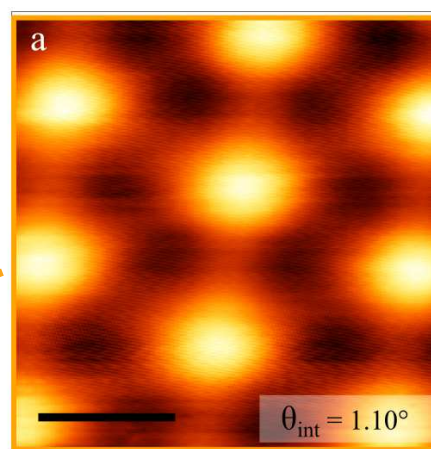
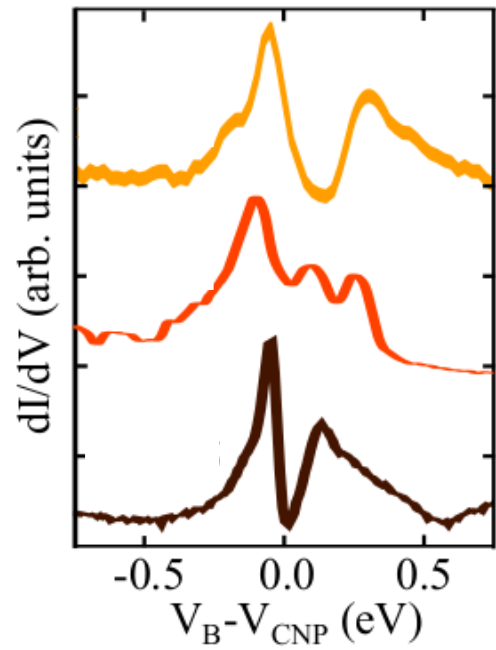
# A variety of Twisted bilayers of graphene



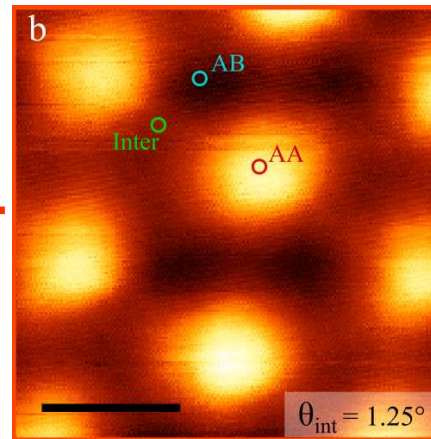
1. L. Huder et al. Phys. Rev. Lett. 120, 156405 (2018)
2. A. Kerelsky et al. Nature 572, 95-100 (2019)
3. Y. Choi et al. Nature Physics 15, 1174-1180 (2019)
4. Y. Jiang et al. Nature 573, 91-95 (2019)
5. Y. Xie et al. Nature 572, 101-105 (2019)
6. Z. Zhang et al. Phys. Rev. Research 2, 033181 (2020)
7. D. Wong et al. Nature 582(7811), 198-202 (2020)
8. Y. Choi et al. arXiv 2008.11746 (2020)
9. K. P. Nuckolls et al. arXiv 2007.03810 (2020)



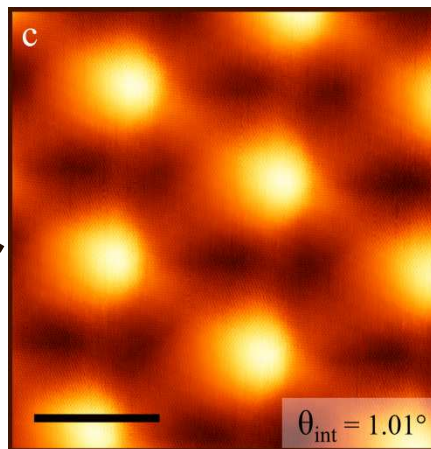
# A sensitive system



A. Kerelsky et al. Nature 572, 95-100 (2019)



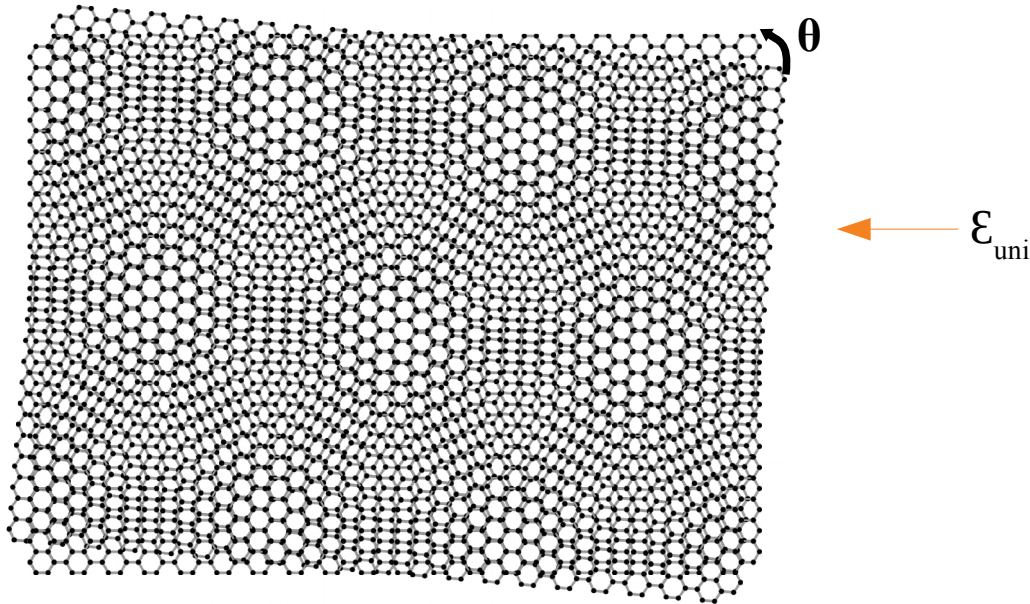
L. Huder et al. Phys. Rev. Lett. 120, 156405 (2018)



Y. Choi et al. Nature Physics 15, 1174-1180 (2019)

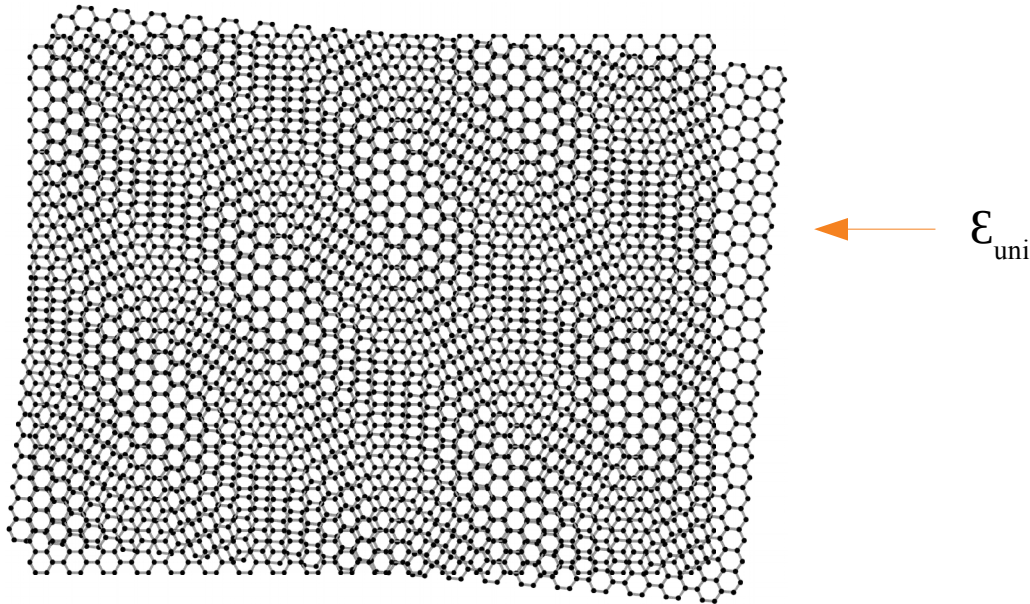
# Heterostrain : schematic view

- $\theta$
- $\varepsilon_{uni}$



$$\begin{pmatrix} (\varepsilon_{uni} + 1) & 0 \\ 0 & 1 \end{pmatrix}$$

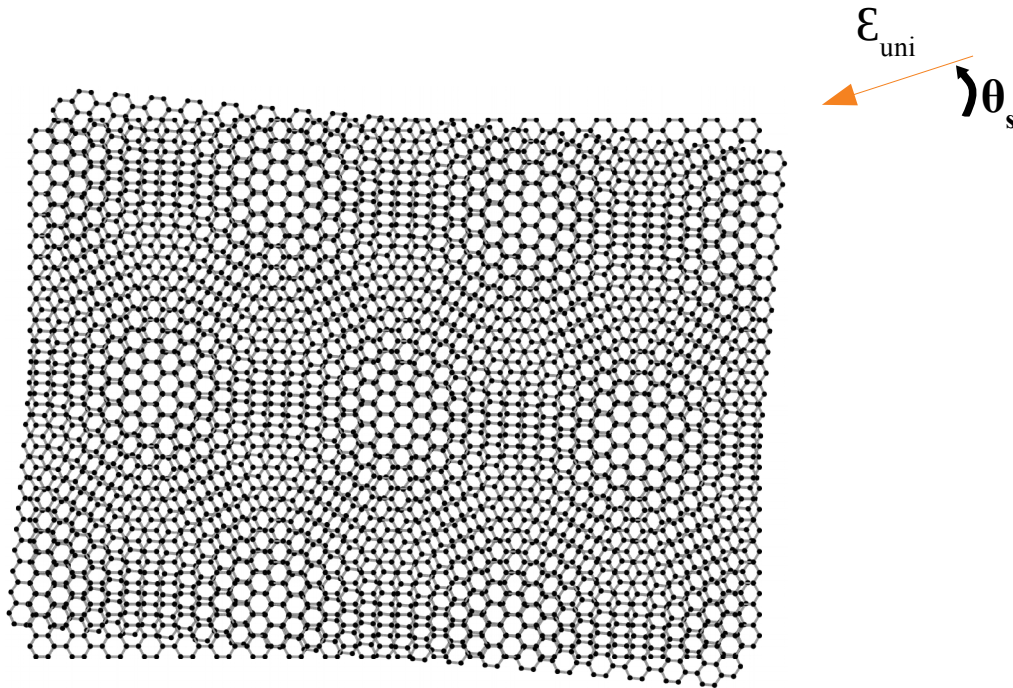
- $\theta$
- $\varepsilon_{\text{uni}}$



$$\begin{pmatrix} (\varepsilon_{\text{uni}} + 1) & 0 \\ 0 & 1 \end{pmatrix}$$

# Heterostrain : schematic view

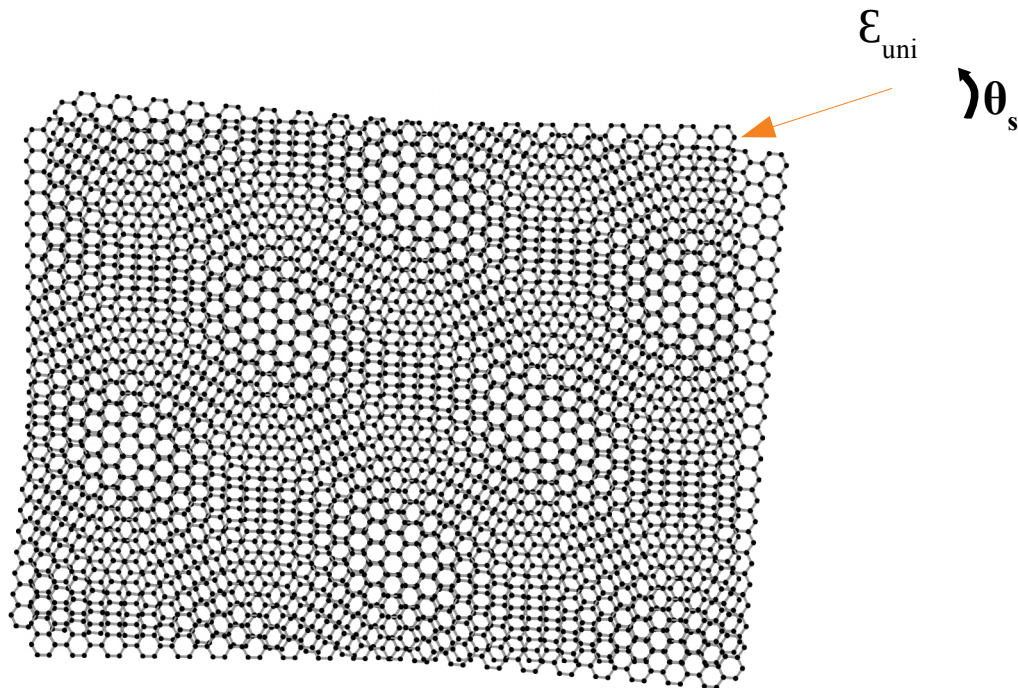
- $\theta$
- $\varepsilon_{\text{uni}}$
- $\theta_s$



$$\begin{pmatrix} \cos\theta_2 & -\sin\theta_2 \\ \sin\theta_2 & \cos\theta_2 \end{pmatrix} \begin{pmatrix} (\varepsilon_{\text{uni}} + 1) & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta_1 & -\sin\theta_1 \\ \sin\theta_1 & \cos\theta_1 \end{pmatrix}$$

# Heterostrain : schematic view

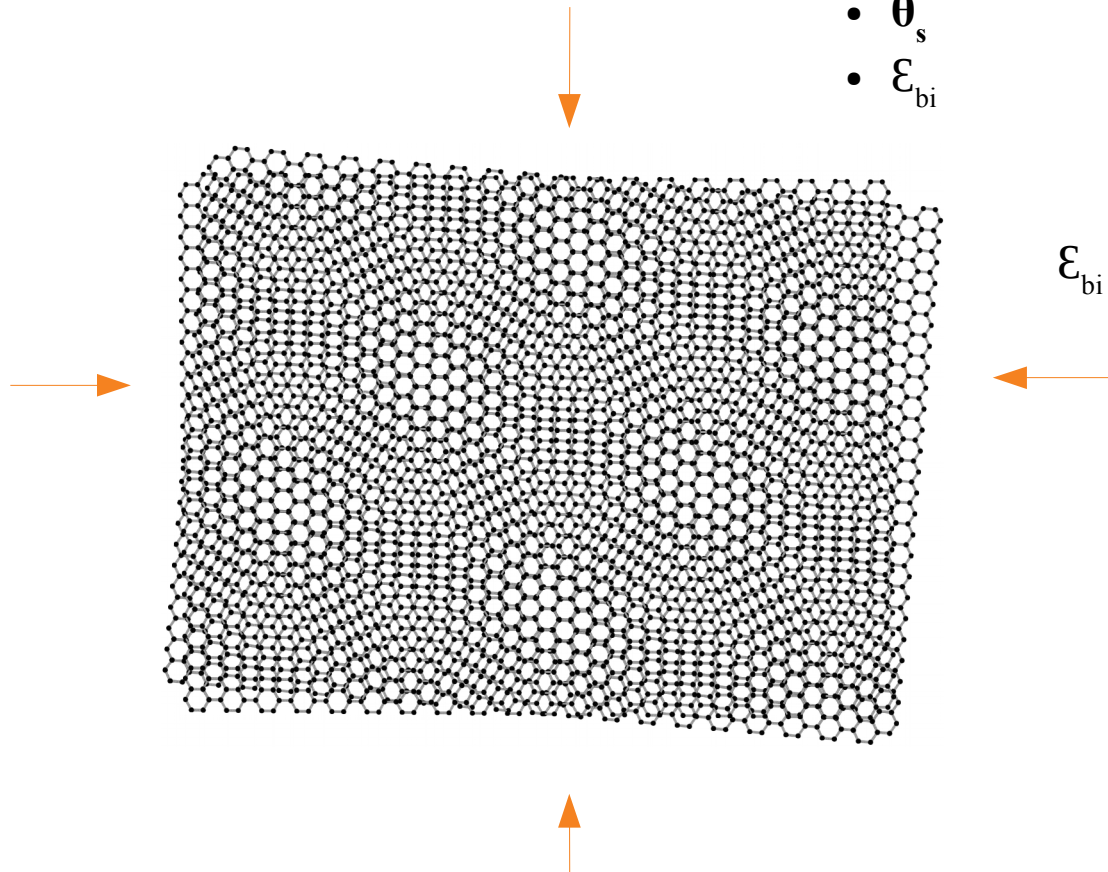
- $\theta$
- $\varepsilon_{uni}$
- $\theta_s$



$$\begin{pmatrix} \cos\theta_2 & -\sin\theta_2 \\ \sin\theta_2 & \cos\theta_2 \end{pmatrix} \begin{pmatrix} (\varepsilon_{uni} + 1) & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta_1 & -\sin\theta_1 \\ \sin\theta_1 & \cos\theta_1 \end{pmatrix}$$

# Heterostrain : schematic view

- $\theta$
- $\epsilon_{uni}$
- $\theta_s$
- $\epsilon_{bi}$

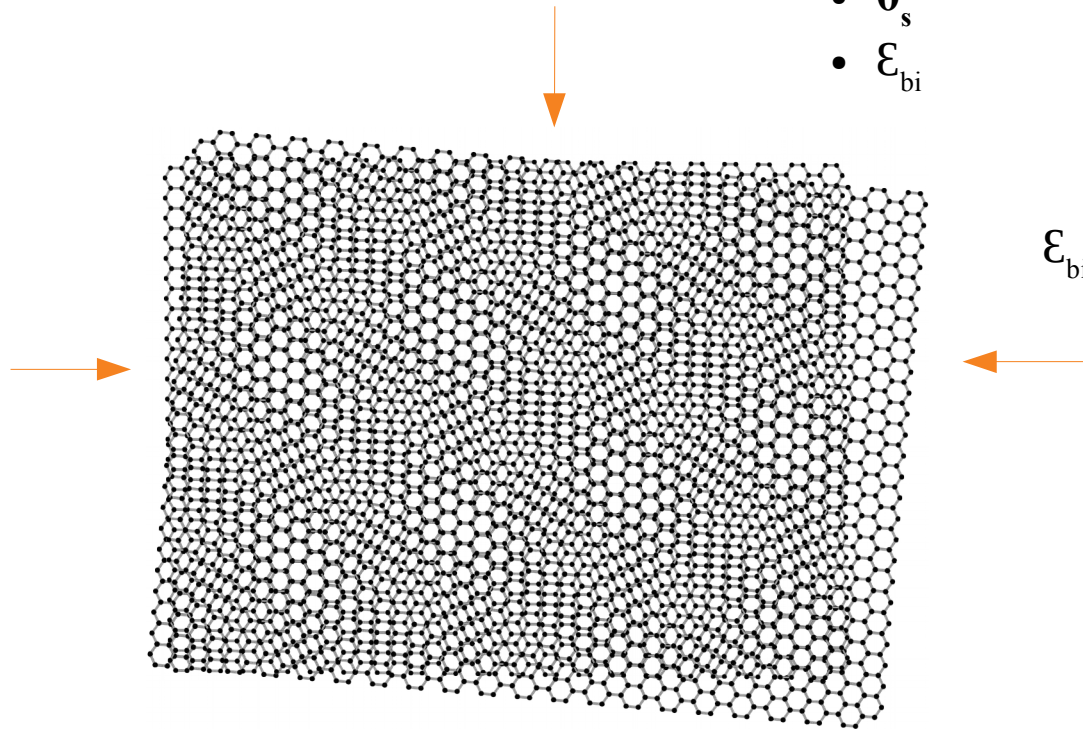


$$\begin{pmatrix} \cos\theta_2 & -\sin\theta_2 \\ \sin\theta_2 & \cos\theta_2 \end{pmatrix} \begin{pmatrix} (\epsilon_{uni} + 1) & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta_1 & -\sin\theta_1 \\ \sin\theta_1 & \cos\theta_1 \end{pmatrix} \begin{pmatrix} (\epsilon_{bi} + 1) & 0 \\ 0 & (\epsilon_{bi} + 1) \end{pmatrix}$$

# Heterostrain : schematic view

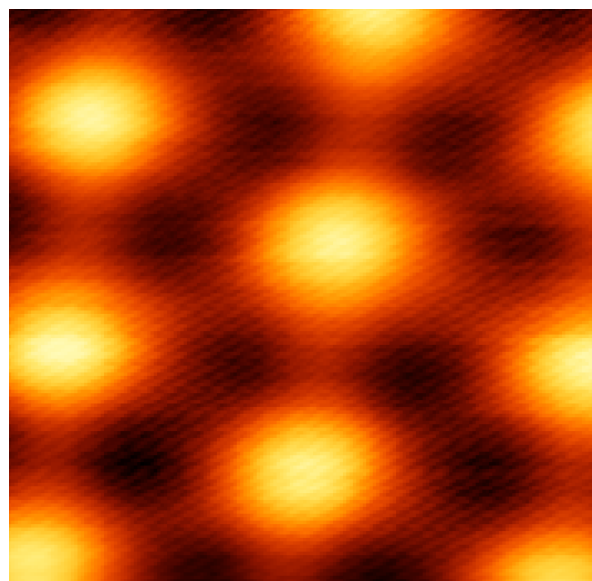
- $\theta$
- $\epsilon_{uni}$
- $\theta_s$
- $\epsilon_{bi}$

A. Artaud et al. Scientific Reports 6, 25670 (2016)

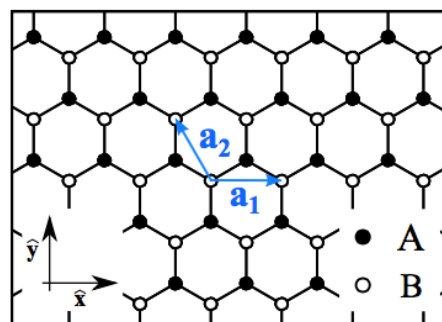
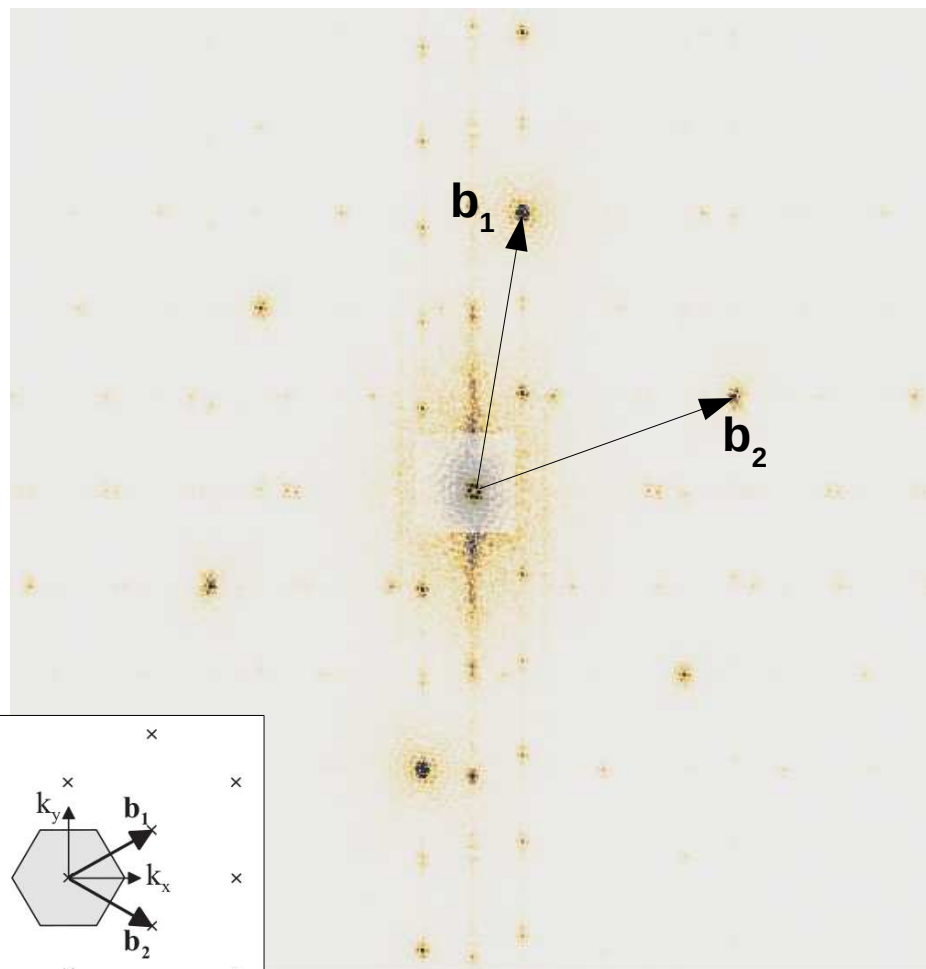


$$\begin{pmatrix} a_{t1} \\ a_{t2} \end{pmatrix} = \begin{pmatrix} \cos\theta_2 & -\sin\theta_2 \\ \sin\theta_2 & \cos\theta_2 \end{pmatrix} \begin{pmatrix} (\epsilon_{uni} + 1) & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta_1 & -\sin\theta_1 \\ \sin\theta_1 & \cos\theta_1 \end{pmatrix} \begin{pmatrix} (\epsilon_{bi} + 1) & 0 \\ 0 & (\epsilon_{bi} + 1) \end{pmatrix} \begin{pmatrix} a_{b1} \\ a_{b2} \end{pmatrix}$$

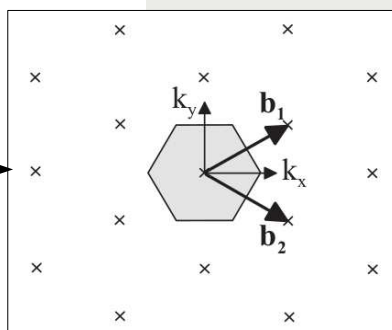
# Heterostrain in experiments



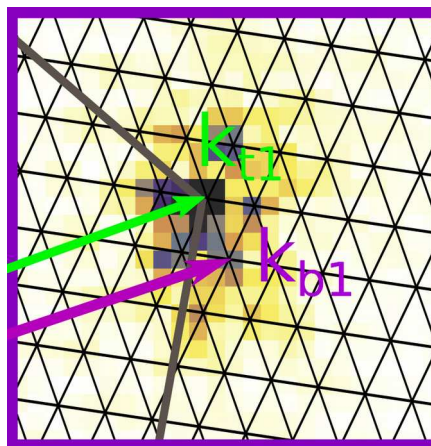
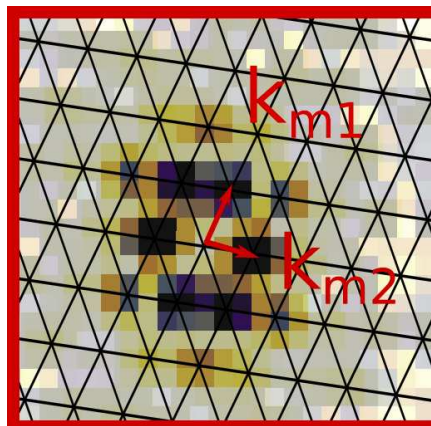
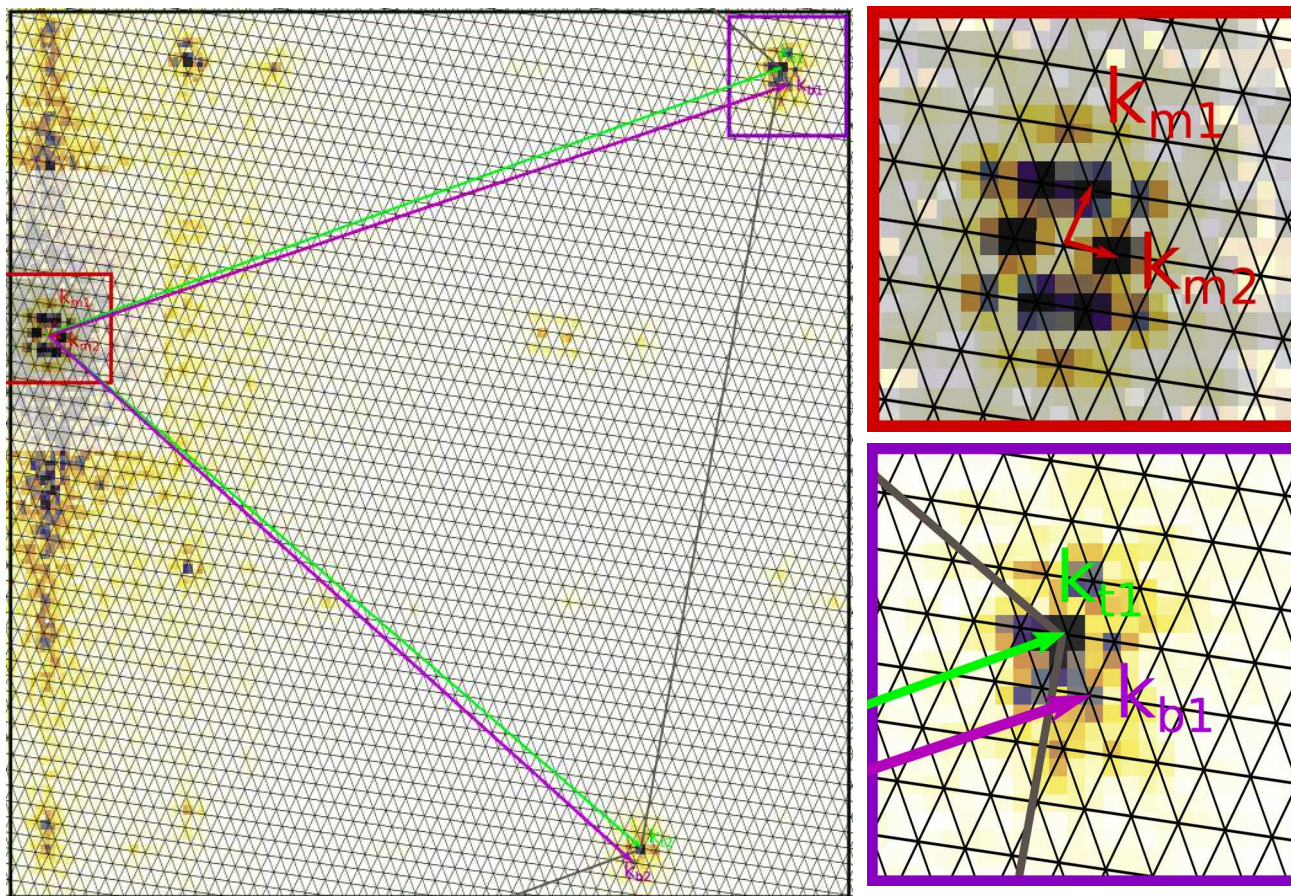
FFT



FT



# Heterostrain in experiments



$$\begin{pmatrix} k_{b1} \\ k_{b2} \end{pmatrix} = \begin{pmatrix} a & c \\ b & d \end{pmatrix} \begin{pmatrix} k_{t1} \\ k_{t2} \end{pmatrix}$$



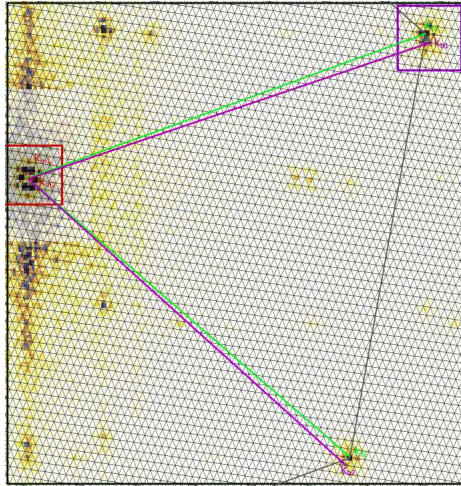
No calibration artefacts

$$\begin{pmatrix} k_{t1} \\ k_{t2} \end{pmatrix} = \begin{pmatrix} i & k \\ j & l \end{pmatrix} \begin{pmatrix} k_{m1} \\ k_{m2} \end{pmatrix} = \begin{pmatrix} 29 & 34 \\ -23 & 66 \end{pmatrix} \begin{pmatrix} k_{m1} \\ k_{m2} \end{pmatrix}$$

Commensurate cell :

$$\begin{pmatrix} k_{b1} \\ k_{b2} \end{pmatrix} = \begin{pmatrix} m & q \\ n & r \end{pmatrix} \begin{pmatrix} k_{m1} \\ k_{m2} \end{pmatrix} = \begin{pmatrix} 28 & 35 \\ -24 & 66 \end{pmatrix} \begin{pmatrix} k_{m1} \\ k_{m2} \end{pmatrix}$$

# Heterostrain in experiments



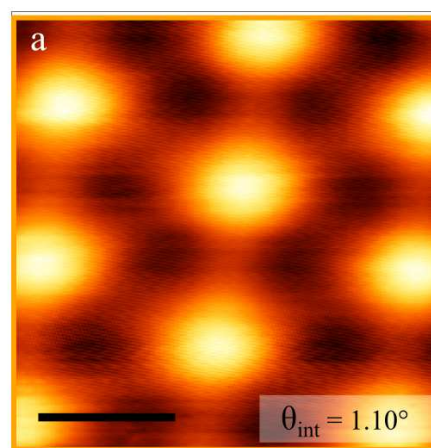
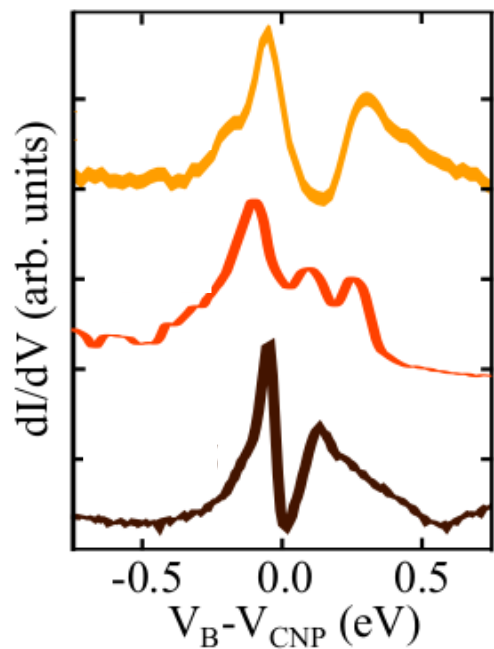
$$\begin{pmatrix} k_{b1} \\ k_{b2} \end{pmatrix} = \begin{pmatrix} a & c \\ b & d \end{pmatrix} \begin{pmatrix} k_{t1} \\ k_{t2} \end{pmatrix}$$

$$\begin{pmatrix} a_{t1} \\ a_{t2} \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} a_{b1} \\ a_{b2} \end{pmatrix}$$

$$\begin{pmatrix} a_{t1} \\ a_{t2} \end{pmatrix} = \begin{pmatrix} \cos\theta_2 & -\sin\theta_2 \\ \sin\theta_2 & \cos\theta_2 \end{pmatrix} \begin{pmatrix} (\varepsilon_{uni} + 1) & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta_1 & -\sin\theta_1 \\ \sin\theta_1 & \cos\theta_1 \end{pmatrix} \begin{pmatrix} (\varepsilon_{bi} + 1) & 0 \\ 0 & (\varepsilon_{bi} + 1) \end{pmatrix} \begin{pmatrix} a_{b1} \\ a_{b2} \end{pmatrix}$$

$$\longrightarrow (\theta, \varepsilon_{uni}, \theta_s, \varepsilon_{bi})$$

# Heterostrain in experiments

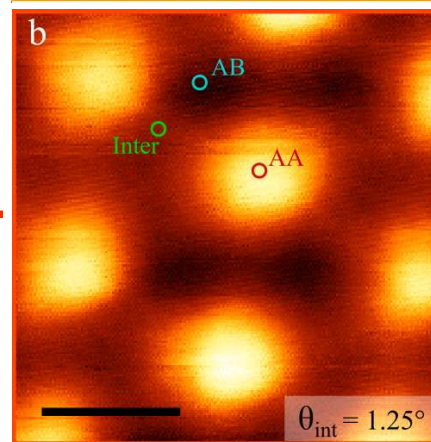


A. Kerelsky et al. Nature 572, 95-100 (2019)

$$\epsilon_{\text{bi}} = 0.13\%$$

$$\epsilon_{\text{uni}} = -0.55\%$$

$$\theta_s = 34^\circ$$

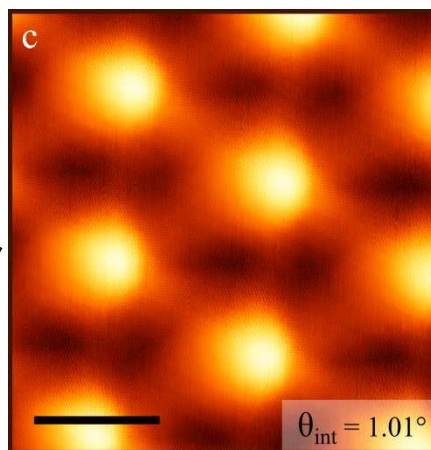


L. Huder et al. Phys. Rev. Lett. 120, 156405 (2018)

$$\epsilon_{\text{bi}} = -0.06\%$$

$$\epsilon_{\text{uni}} = 0.35\%$$

$$\theta_s = 10.6^\circ$$



Y. Choi et al. Nature Physics 15, 1174-1180 (2019)

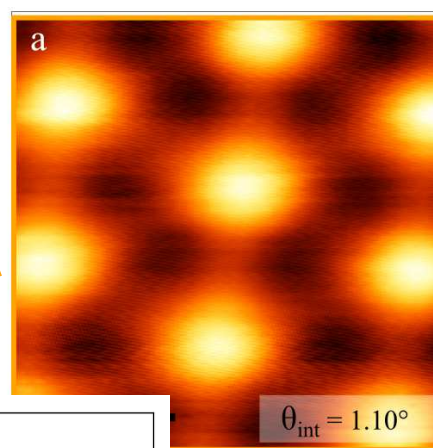
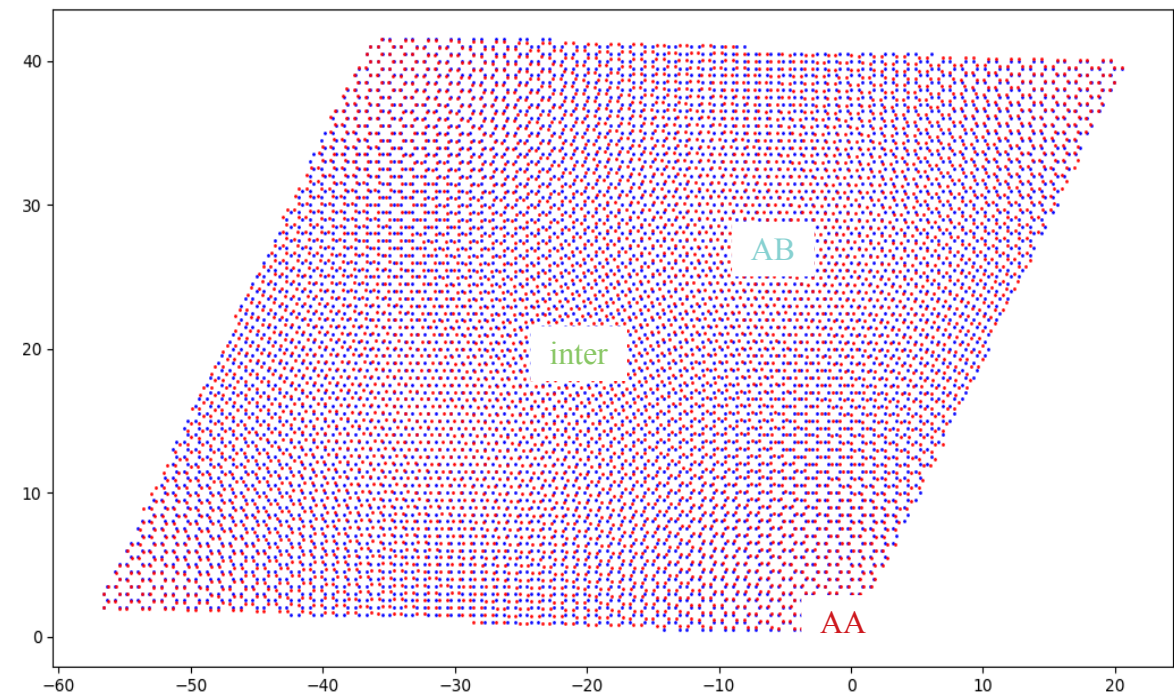
$$\epsilon_{\text{bi}} = -0.0\%$$

$$\epsilon_{\text{uni}} = 0.16\%$$

$$\theta_s = -26^\circ$$

# Heterostrain in experiments

## Periodic cell including heterostrain



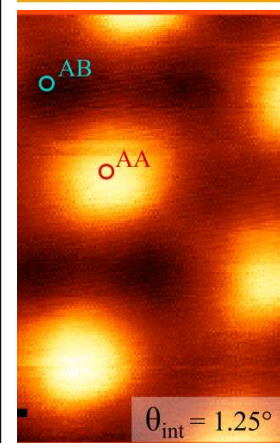
A. Kerelsky et al. Nature 572, 95-100 (2019)

$$\epsilon_{bi} = 0.13\%$$

$$\epsilon_{uni} = -0.55\%$$

$$\theta_s = 34^\circ$$

$$\theta_{int} = 1.10^\circ$$



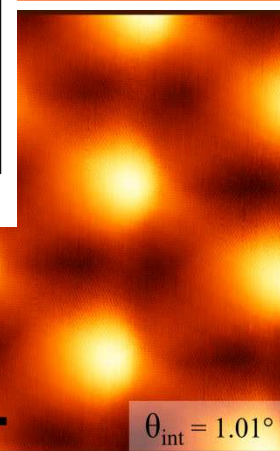
L. Huder et al. Phys. Rev. Lett. 120, 156405 (2018)

$$\epsilon_{bi} = -0.06\%$$

$$\epsilon_{uni} = 0.35\%$$

$$\theta_s = 10.6^\circ$$

$$\theta_{int} = 1.25^\circ$$



Y. Choi et al. Nature Physics 15, 1174-1180 (2019)

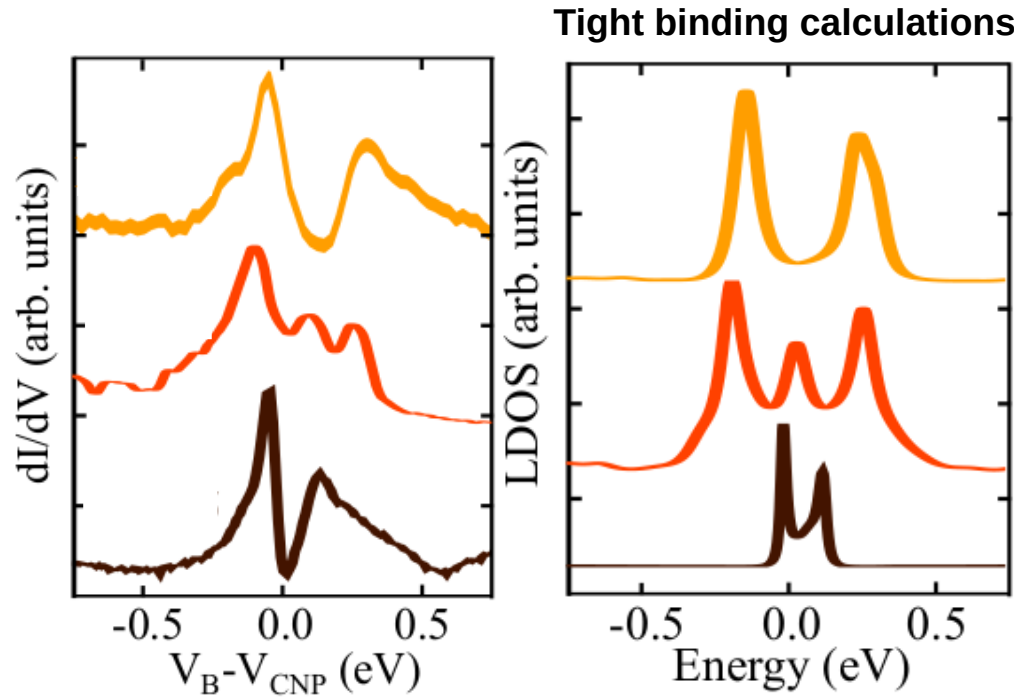
$$\epsilon_{bi} = -0.0\%$$

$$\epsilon_{uni} = 0.16\%$$

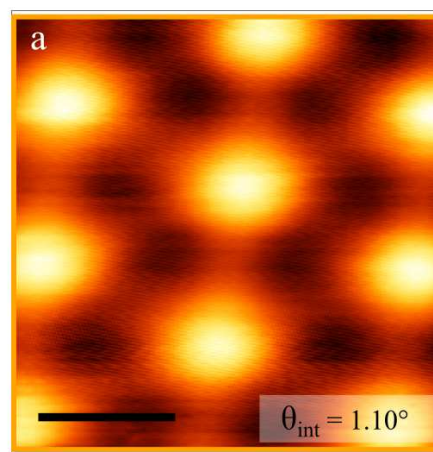
$$\theta_s = -26^\circ$$

$$\theta_{int} = 1.01^\circ$$

# Heterostrain in TB calculations



- By including heterostrain, we describe the physics !

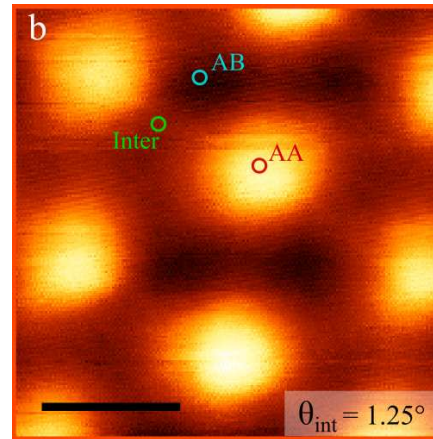


A. Kerelsky et al. Nature 572, 95-100 (2019)

$$\epsilon_{bi} = 0.13\%$$

$$\epsilon_{uni} = -0.55\%$$

$$\theta_s = 34^\circ$$

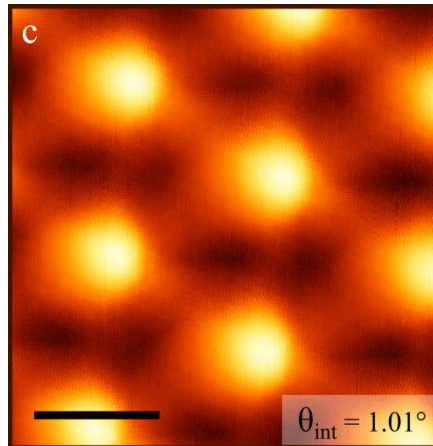


L. Huder et al. Phys. Rev. Lett. 120, 156405 (2018)

$$\epsilon_{bi} = -0.06\%$$

$$\epsilon_{uni} = 0.35\%$$

$$\theta_s = 10.6^\circ$$



Y. Choi et al. Nature Physics 15, 1174-1180 (2019)

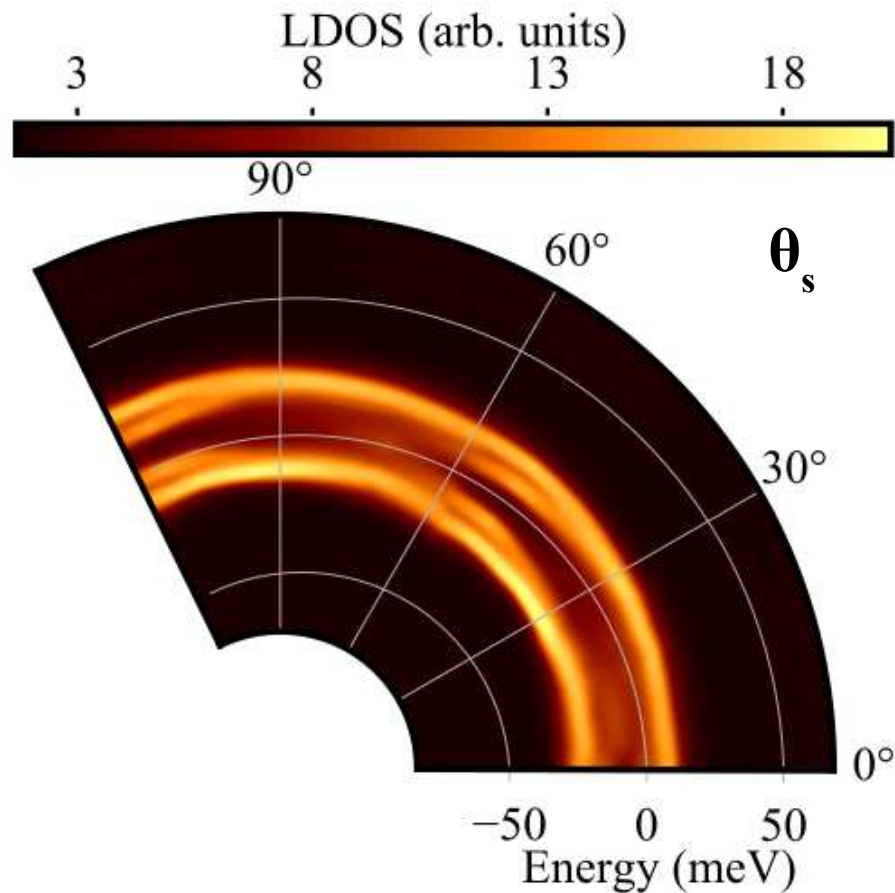
$$\epsilon_{bi} = -0.0\%$$

$$\epsilon_{uni} = 0.16\%$$

$$\theta_s = -26^\circ$$

$$\theta = 1.1^\circ$$

$$\epsilon_{\text{uni}} = 0.4 \%$$

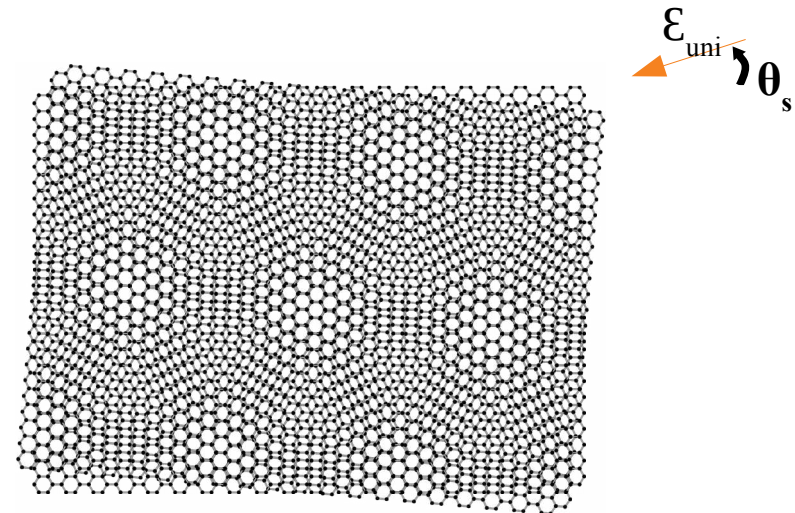


## Tight binding calculations

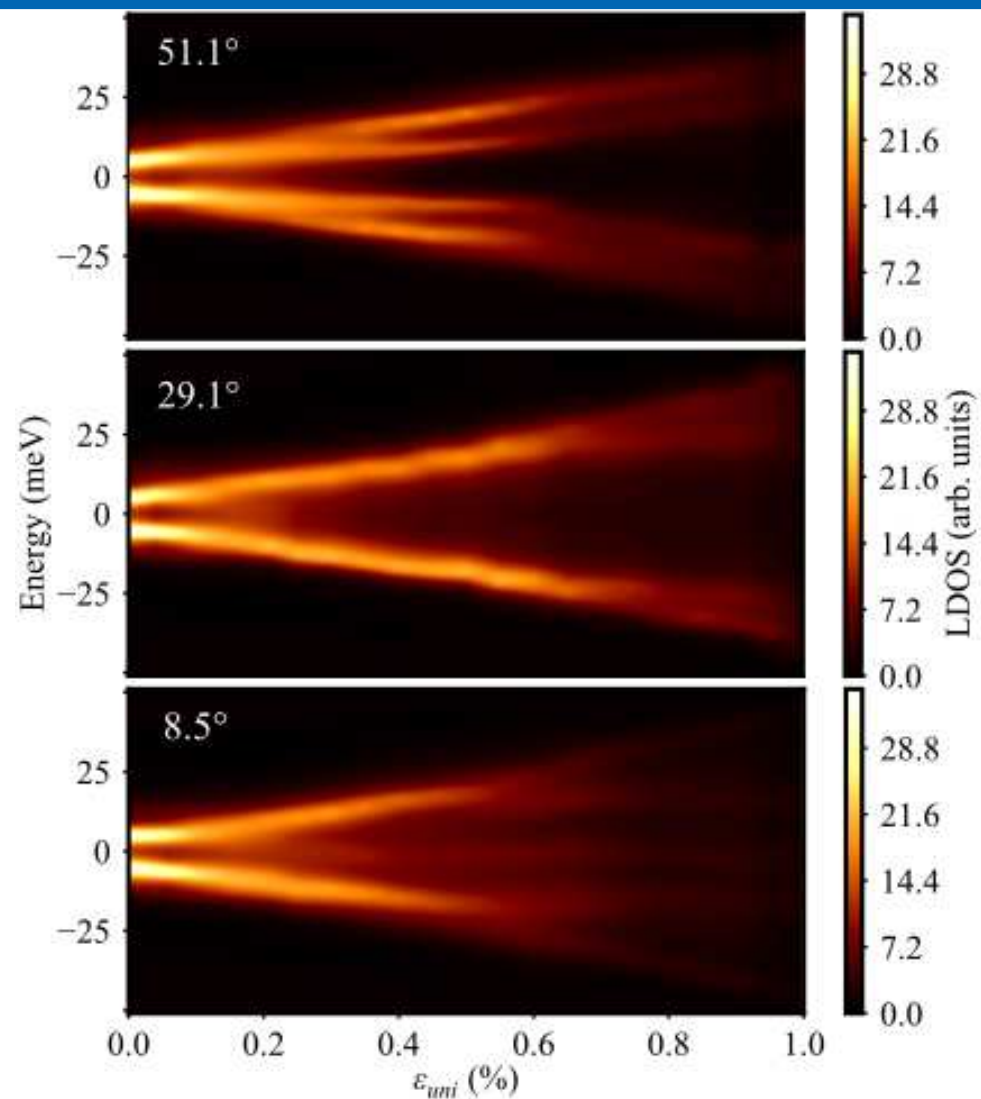
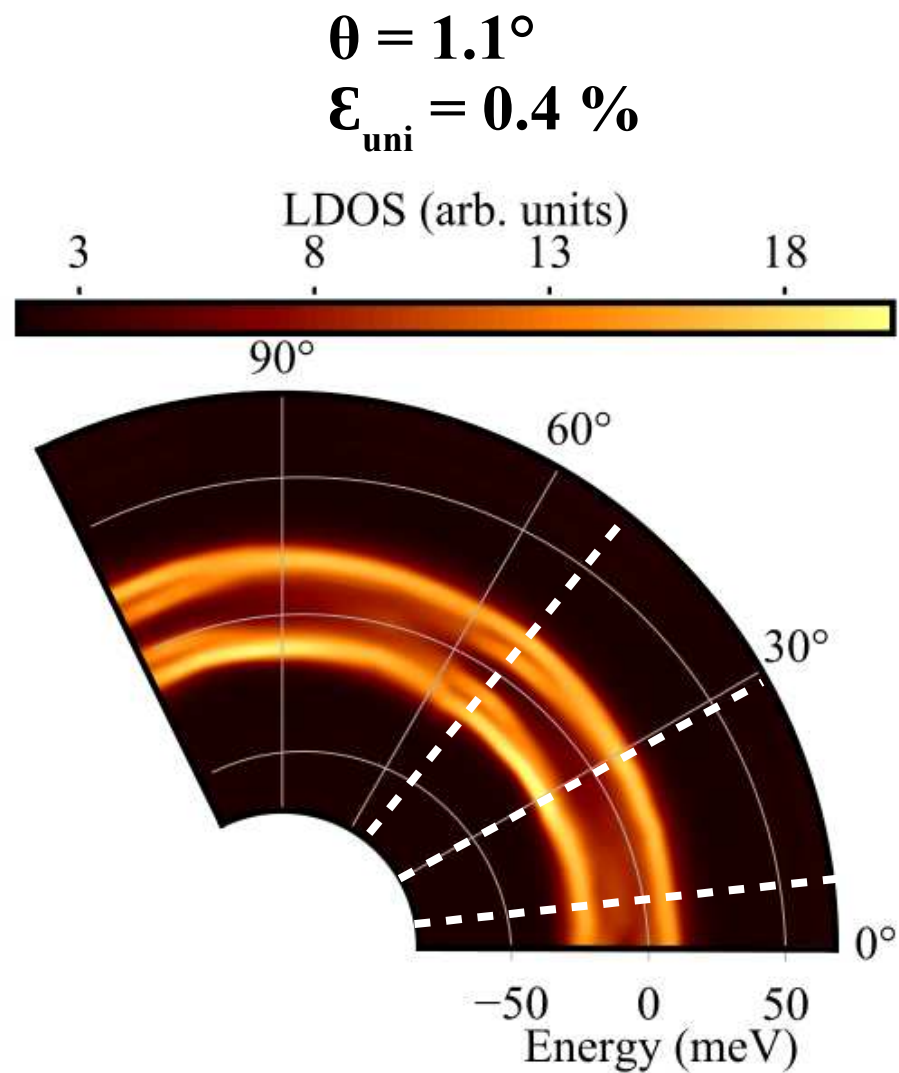
Ahmed Missaoui

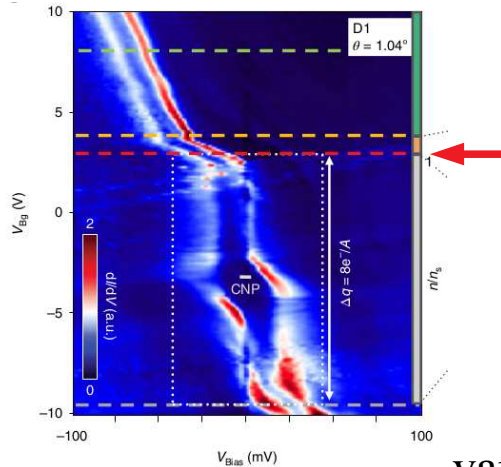
Guy Trambly de Laissardière

LPTM Cergy Pontoise



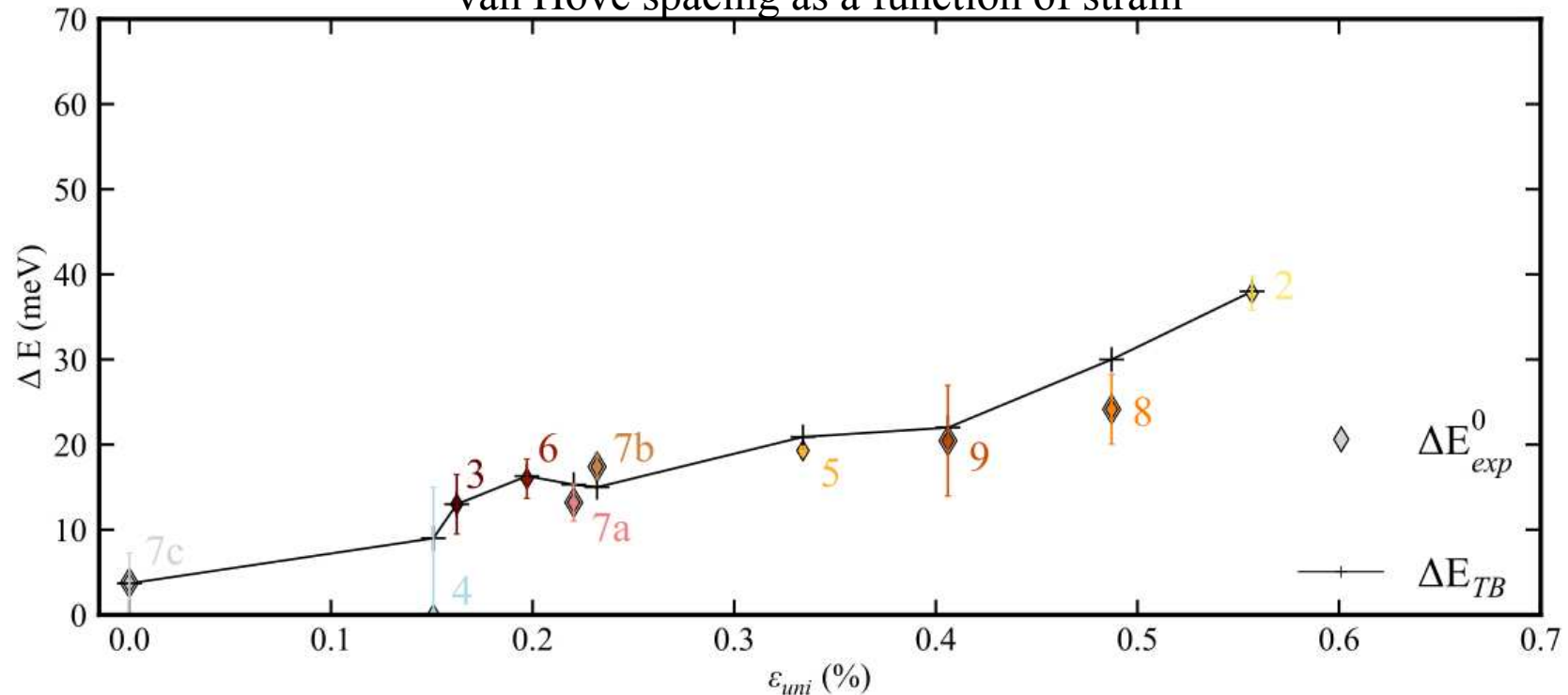
# Heterostrain in TB calculations

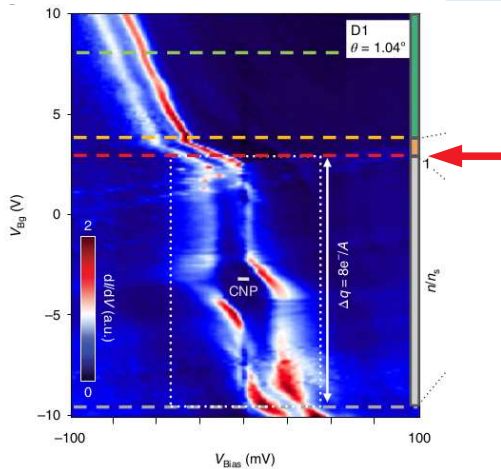




1. L. Huder et al. Phys. Rev. Lett. 120, 156405 (2018)
2. A. Kerelsky et al. Nature 572, 95-100 (2019)
3. Y. Choi et al. Nature Physics 15, 1174-1180 (2019)
4. Y. Jiang et al. Nature 573, 91-95 (2019)
5. Y. Xie et al. Nature 572, 101-105 (2019)
6. Z. Zhang et al. Phys. Rev. Research 2, 033181 (2020)
7. D. Wong et al. Nature 582(7811), 198-202 (2020)
8. Y. Choi et al. arXiv 2008.11746 (2020)
9. K. P. Nuckolls et al. arXiv 2007.03810 (2020)

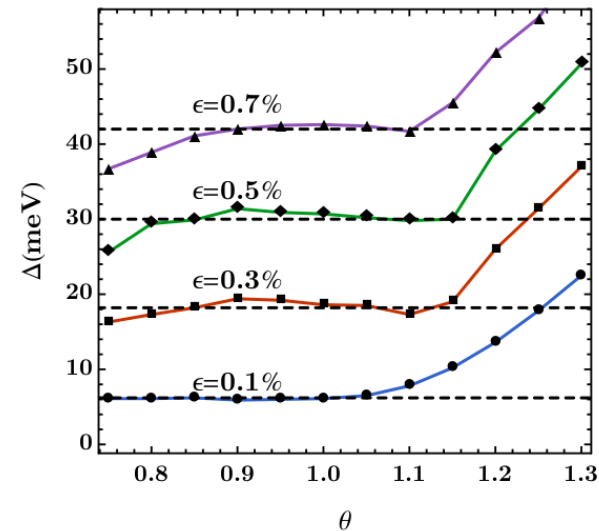
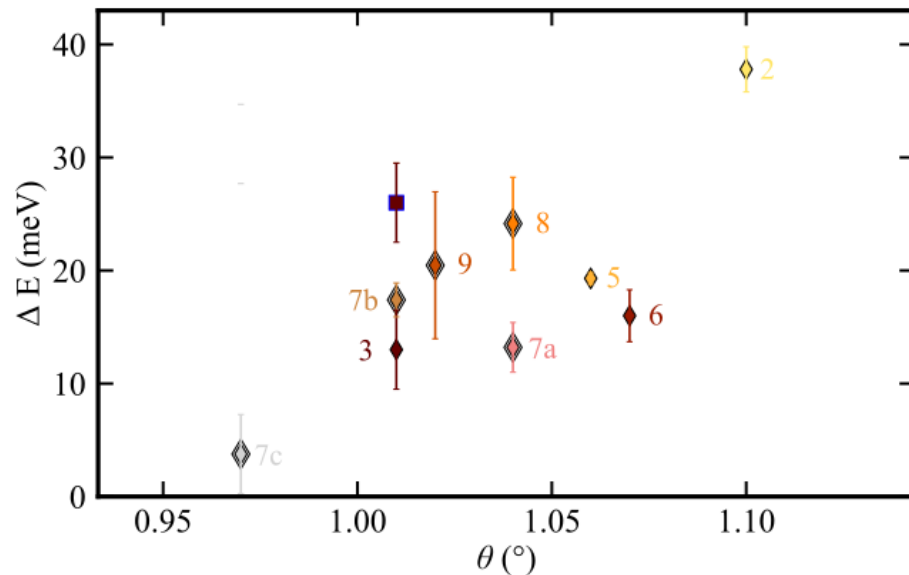
van Hove spacing as a function of strain



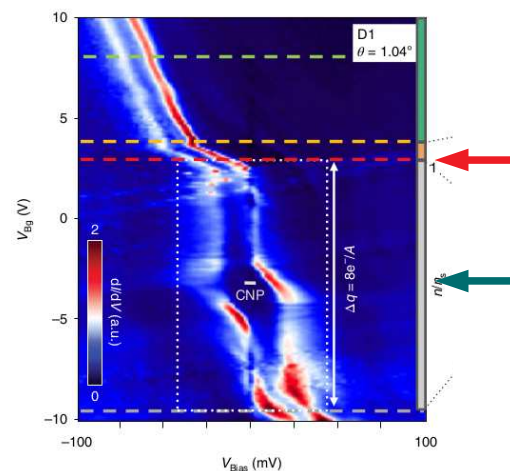


1. L. Huder et al. Phys. Rev. Lett. 120, 156405 (2018)
2. A. Kerelsky et al. Nature 572, 95-100 (2019)
3. Y. Choi et al. Nature Physics 15, 1174-1180 (2019)
4. Y. Jiang et al. Nature 573, 91-95 (2019)
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6. Z. Zhang et al. Phys. Rev. Research 2, 033181 (2020)
7. D. Wong et al. Nature 582(7811), 198-202 (2020)
8. Y. Choi et al. arXiv 2008.11746 (2020)
9. K. P. Nuckolls et al. arXiv 2007.03810 (2020)

van Hove spacing as a function of twist angle

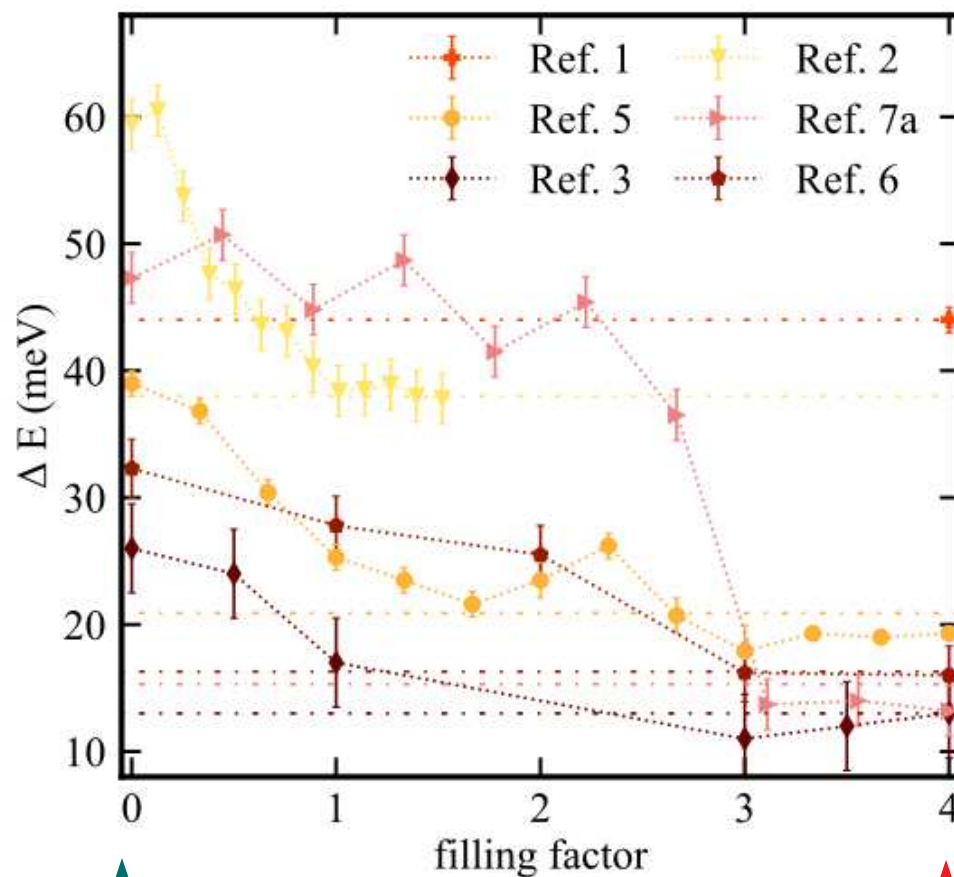


Z. Bi, N. F. Q. Yuan, and L. Fu. Phys. Rev. B 100, 035448 (2019)



$$\frac{E_C}{E_k} \rightarrow \infty$$

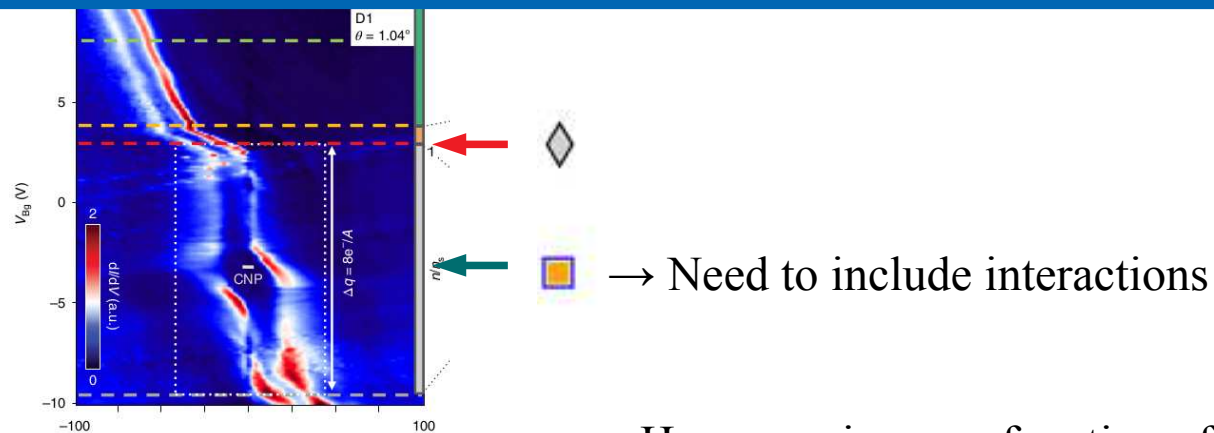
van Hove spacing as a function of doping



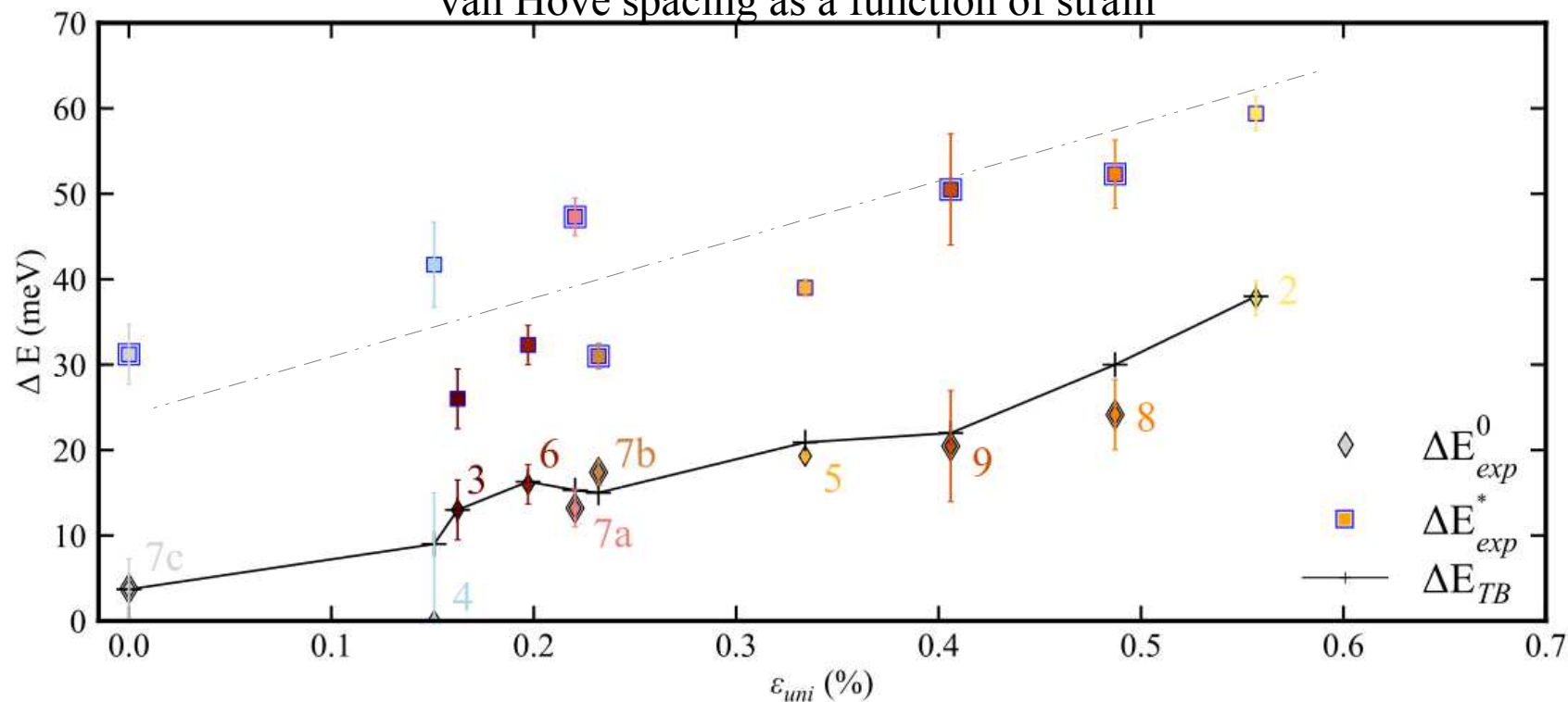
--- TB predictions

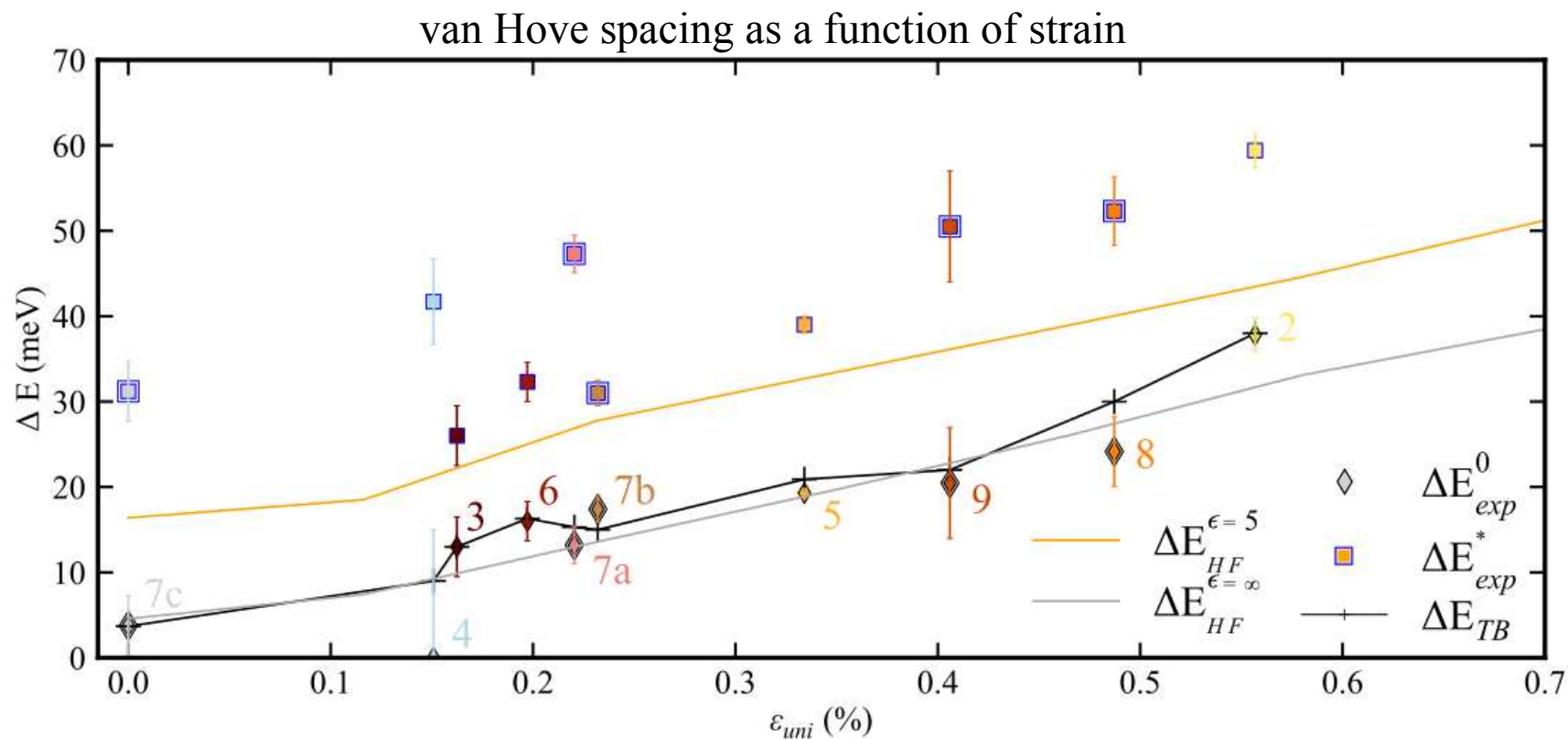
$\Delta E^*$

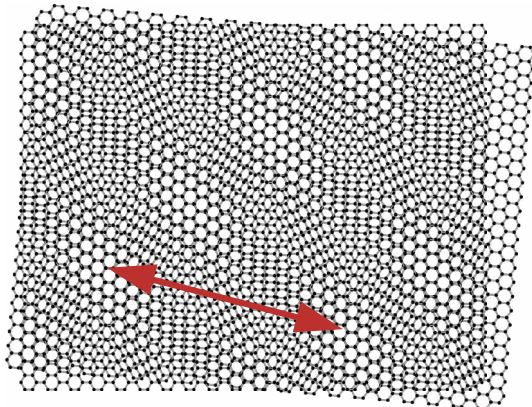
$\Delta E^0$



van Hove spacing as a function of strain

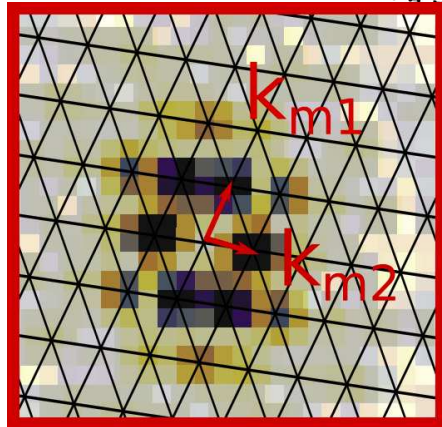




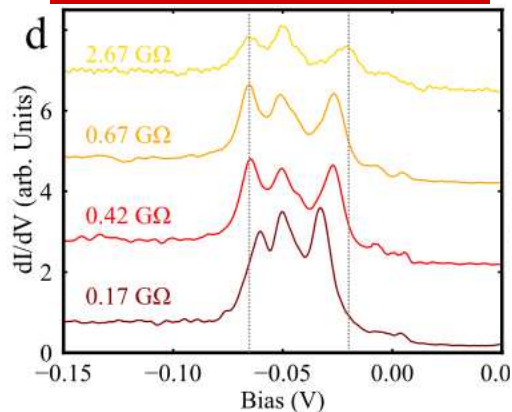


→ The physics of TBLG near magic angle are controlled by strain, not by interlayer angle.

**N. Nakatsuji, M. Koshino, arXiv (2022)**



→ Efficient method to extract « average » strain from STM images



→ Motivation to further investigate the complex parameter space that controls the physics at play (*tip induced strain, inhomogeneous strain, electrostatic environment, local tip doping, ...*)