Capri Summer School 2022

Theory of Twisted Bilayer Graphene

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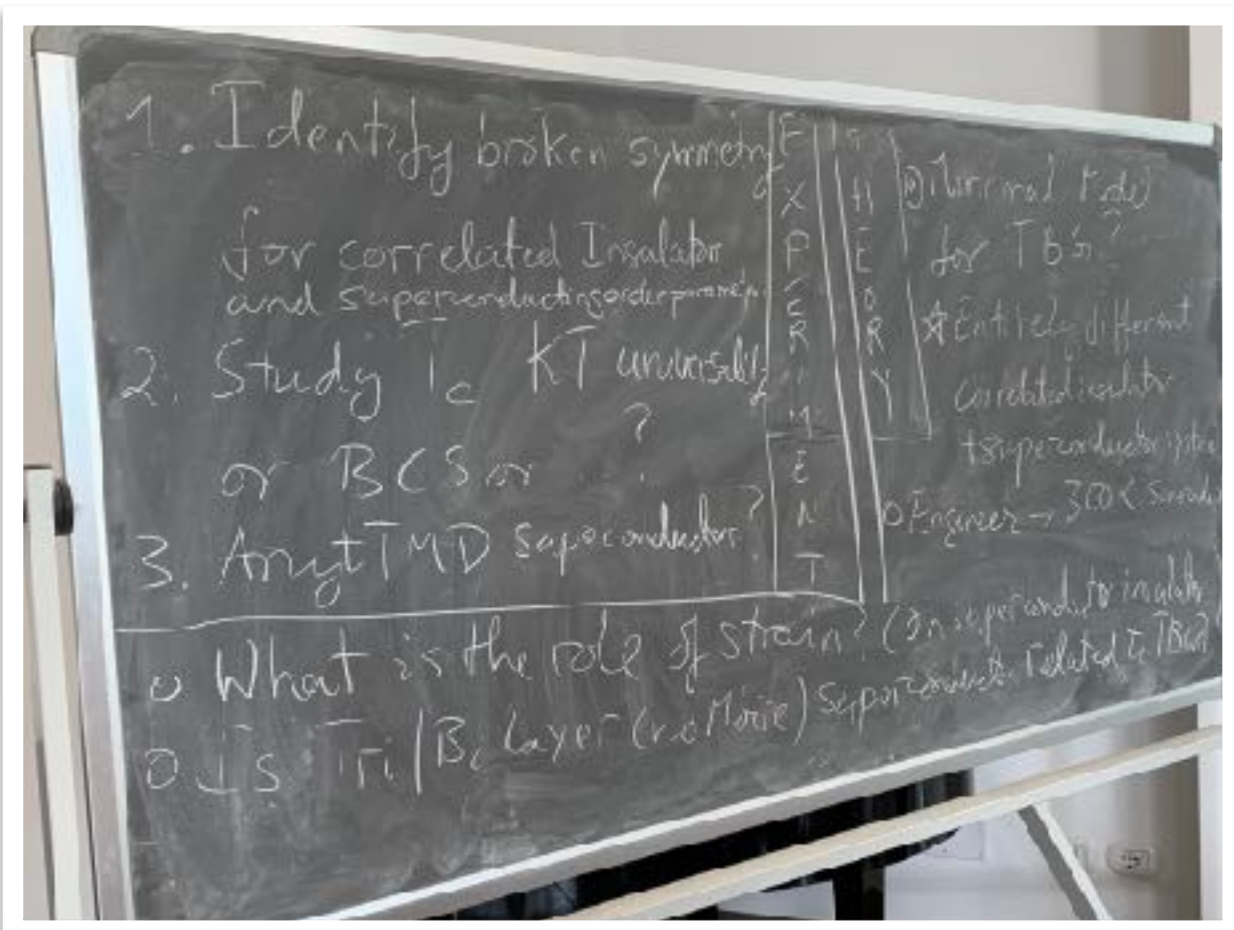
Harvard University

Lecture Notes for More Technical Details:
Strong Coupling Theory of Magic-Angle Graphene: A Pedagogical Introduction

Ledwith, Khalaf AV

(P.W. Anderson Special Issue). https://arxiv.org/pdf/2105.08858.pdf

OPEN QUESTIONS -Crowdsourced



1. Experimentally, identify the broken symmetry of the correlated insulators and the pairing symmetry of the superconductor.
2. What is the nature of the transition into the superconductor KT/BCS..?
3. What is the role of strain and disorder in magic angle graphene?
4. Are there any TMD moire' superconductors? Is there an entirely different setting for correlated insulators+superconductors in moire materials different from the twisted bilayer and alternating multilayers.
5. Theoretically, what is the minimal model for TBG?
6. Can we engineer a 300k superconductor using the lessons from moire' materials?

OUTLINE

- Lecture 1 - Preliminaries, the chiral model, *wave functions,* from bilayer to n=3,4,5..
- Lecture 2 - Correlated Insulators - exact solutions, Hartree Fock, topology and σ model.
- Lecture 3 - Superconductivity - disordered σ model.
- Lecture 4 - Fractional Chern insulators in magic angle graphene

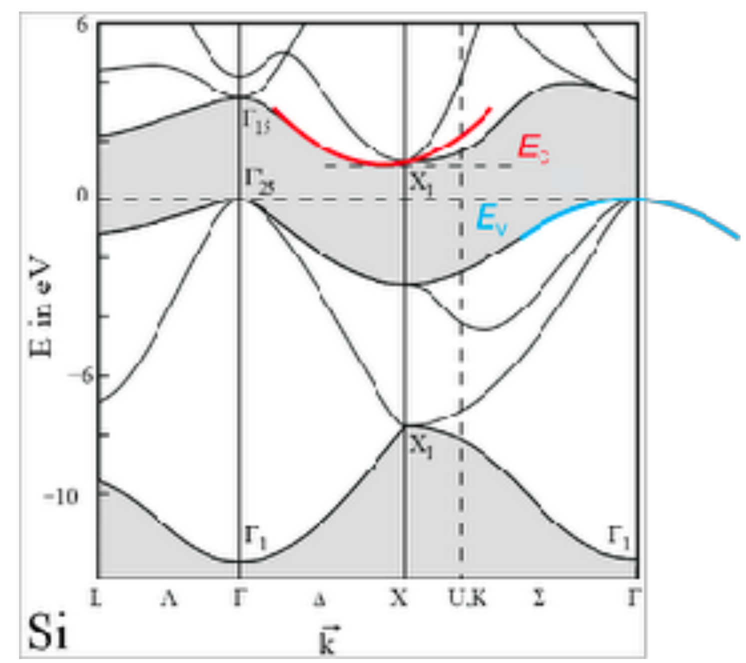
Crystals - Artificial Vacuum for Electrons

Vacuum Tubes <1960s

Transistor

"A sudden gasp filled the room when he flicked on an oscillator circuit, and it emitted a shrill tone <u>instantaneously</u>, with no <u>warmup delay</u> whatsoever."

Demonstration of the Transistor 1948
From - Crystal Fire

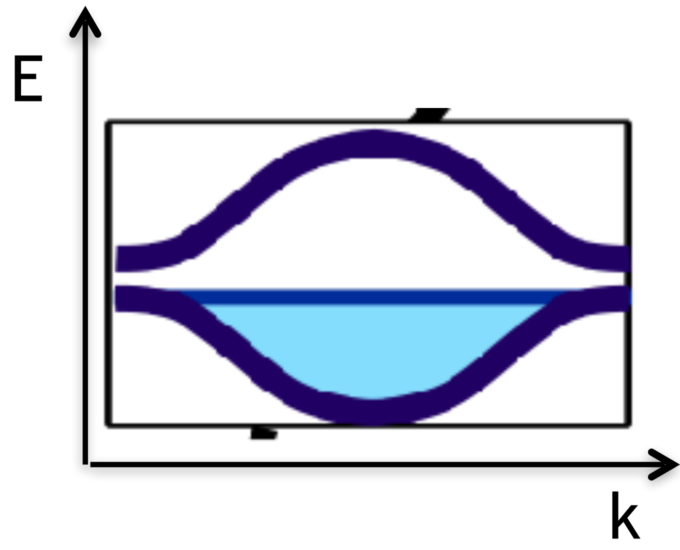


Modify properties of the electron
Effective mass, electrons+holes etc.

Qualitatively new effects?

- Semi-classical theory of electrons in a crystal

$$\dot{x} = \nabla_k \mathcal{E}(k) \quad + \dot{k} \times \tilde{B}$$
$$\dot{k} = -\nabla_r \mathcal{V}(r) + e\dot{x} \times B$$



- Symmetry restored in a crystal – Berry Flux $\tilde{B}$ leads to an anomalous velocity.

- Berry Flux is related to the Berry's phase acquired by states in the band. "Quantum Geometry" of bands.

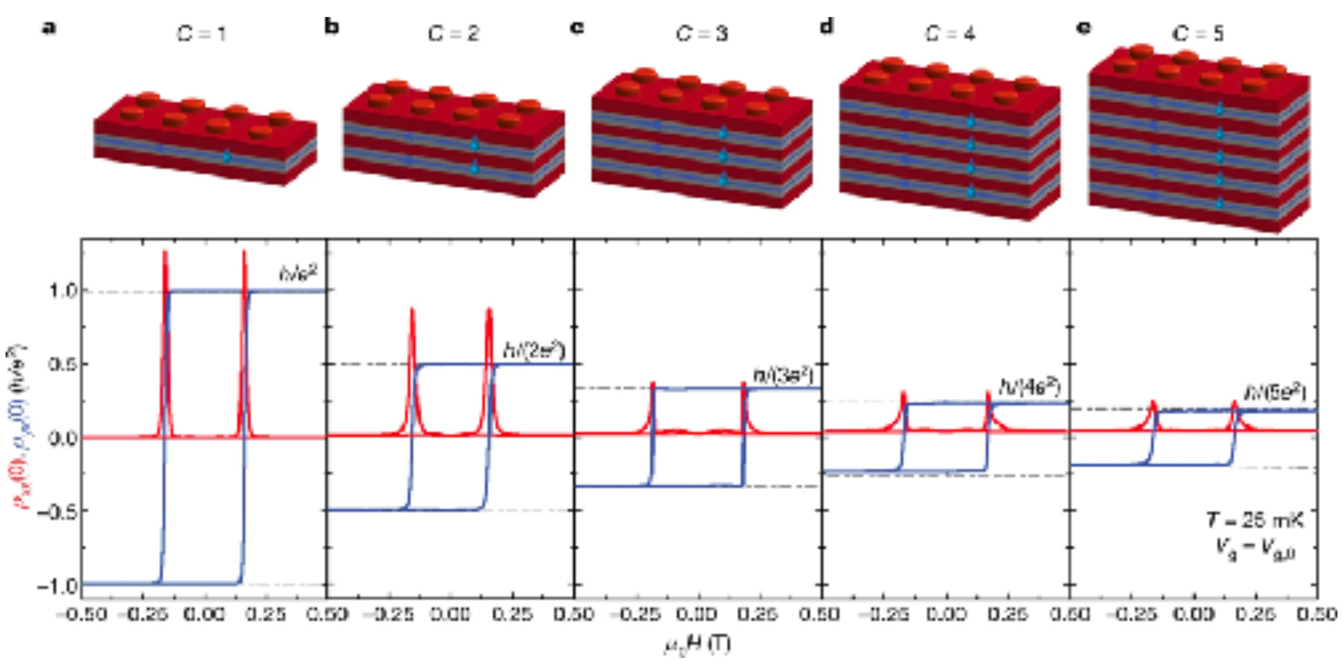
Tuning the Topology and Geometry of Bands

Quantized Hall Conductance in a Two-Dimensional Periodic Potential

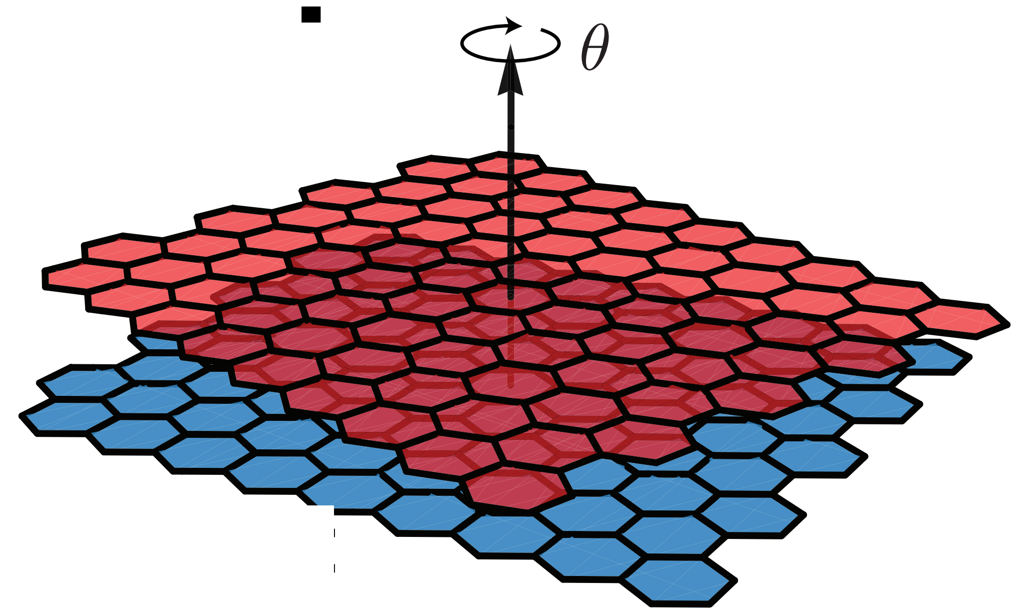
D. J. Thouless, M. Kohmoto,[a] M. P. Nightingale, and M. den Nijs

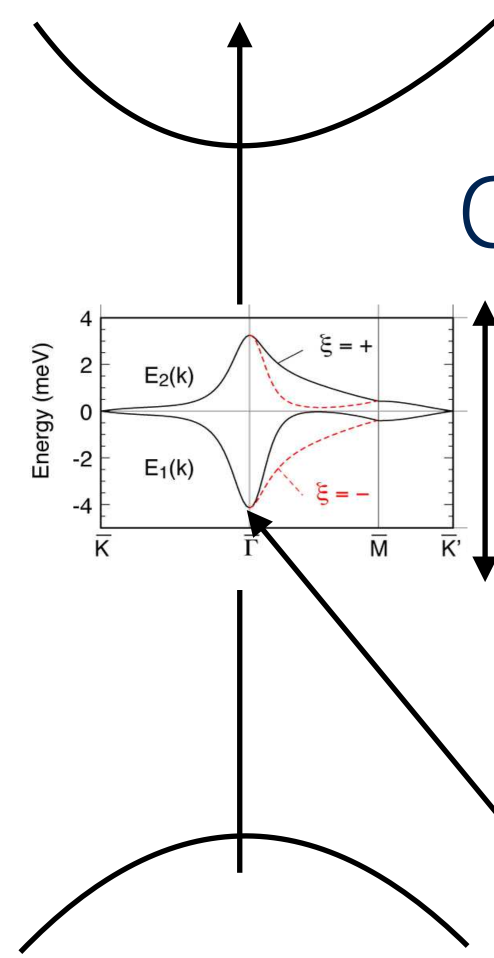
Department of Physics, University of Washington, Seattle, Washington 98195

(Received 30 April 1982)



$(Bi,Sb)_{2-x}Cr_xTe_3$ Zhao et al. Nature 2020





Gap ~ 30 meV

W ~ 8 meV

Koshino et al

Wavefunctions!

0. Recall Some Properties of Graphene

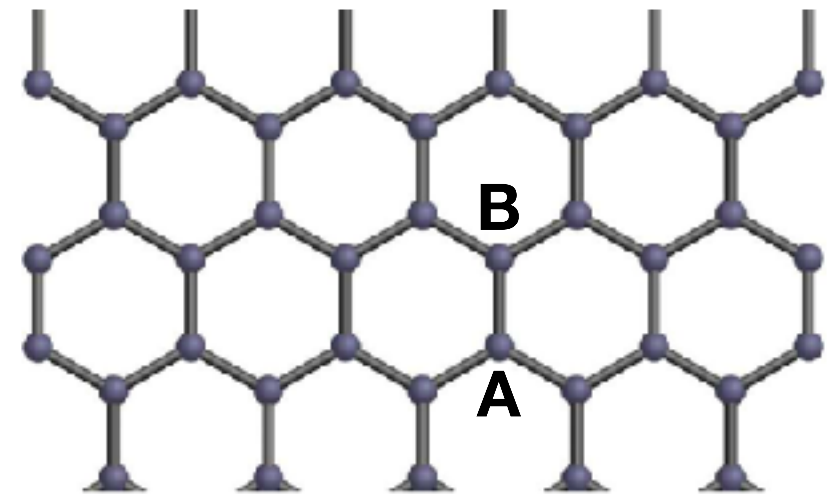
Graphene and Dirac Points

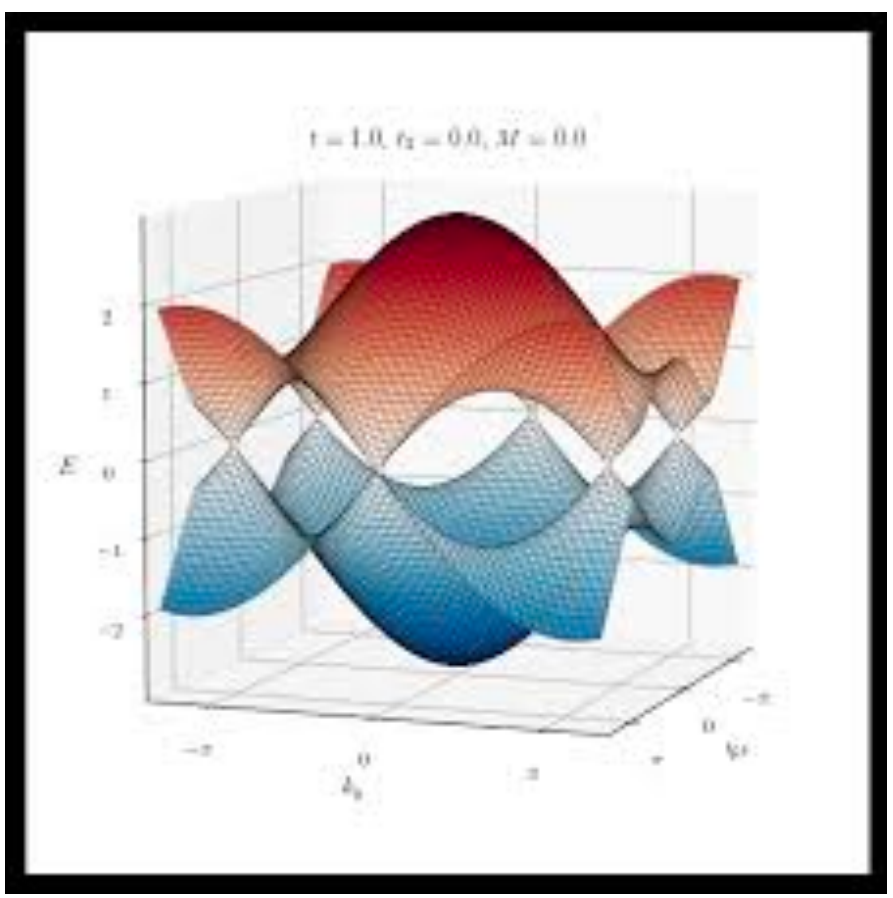
$$H_k = \vec{d}_k \cdot \vec{\sigma}$$

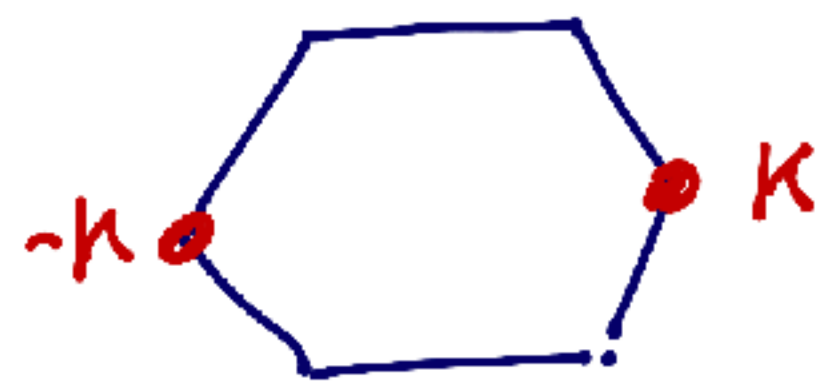
$$d_x = -t\left(\cos k \cdot a_1 + \cos k \cdot a_2 + \cos k \cdot a_3\right)$$

$$d_y = -t\left(\sin k \cdot a_1 + \sin k \cdot a_2 + \sin k \cdot a_3\right)$$

$d_z = 0$ **"Chiral Symmetry"** $\left\{\sigma_z, H_k\right\} = 0$

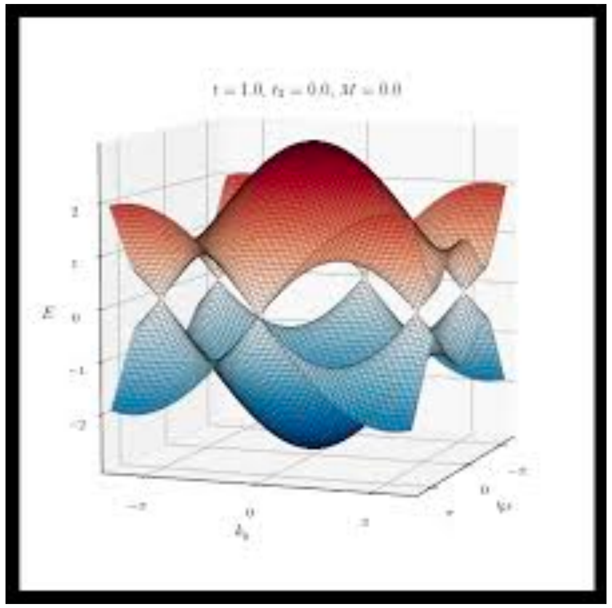




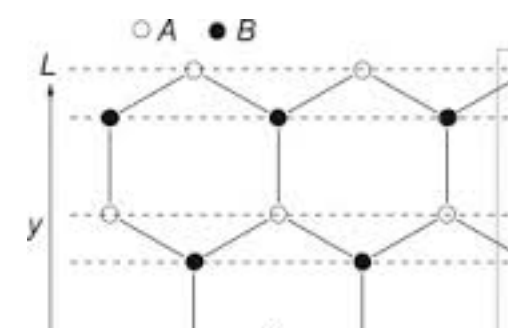


Dirac Points - vortices in d vector

Graphene and Dirac Points

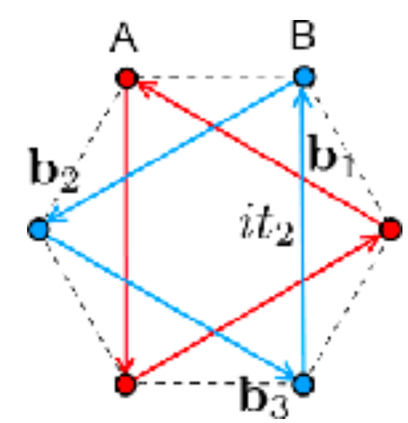


Three ways to remove Dirac Cones:



1. Break P (r—> -r) symmetry - staggered potential (B-N)

$$\Delta H = m\sigma_z$$

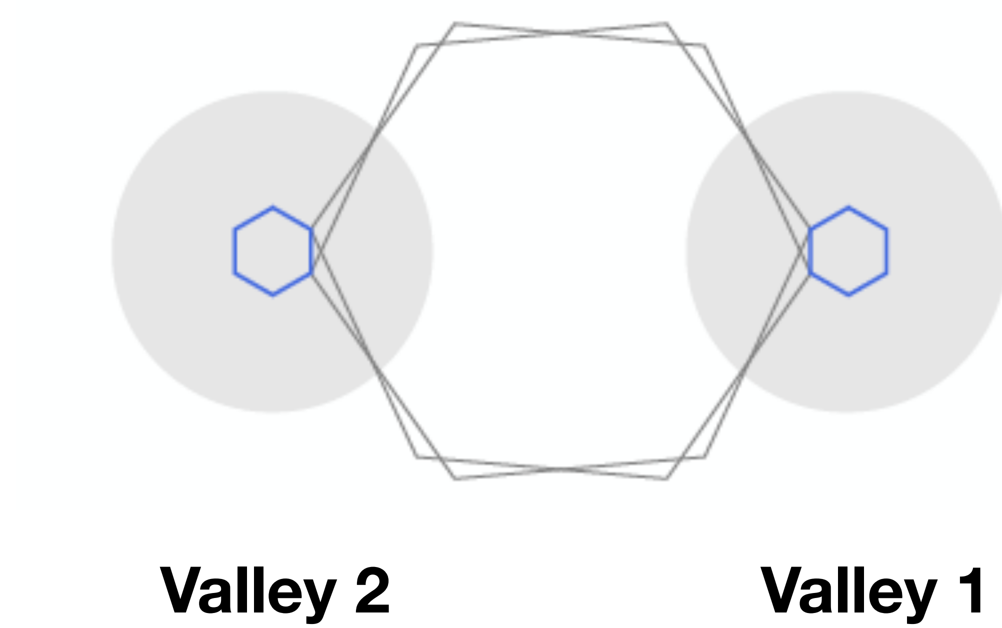


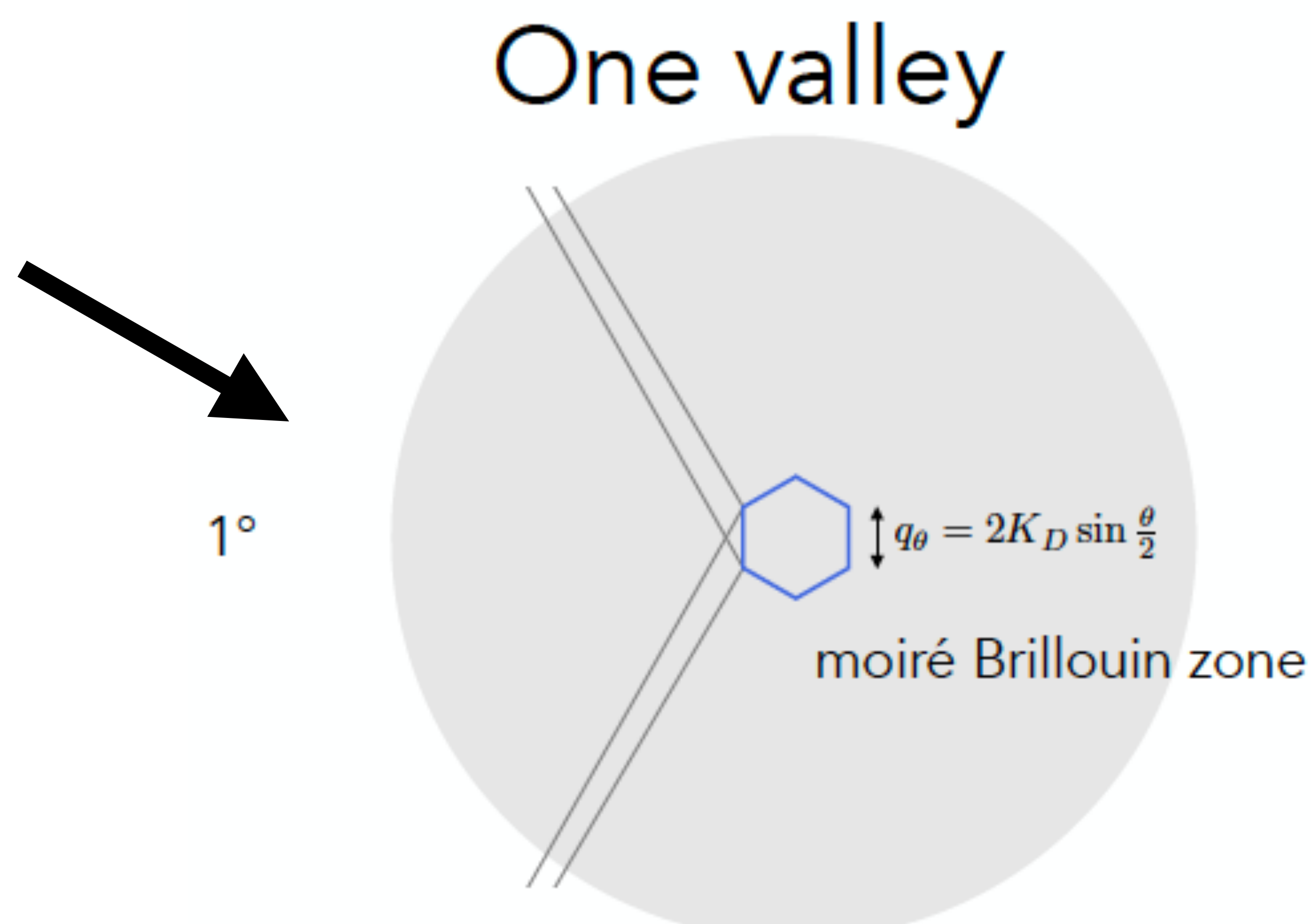
2. Break T symmetry - Haldane term

3. Break C3 rotation and annihilate
Dirac points (needs finite strength)

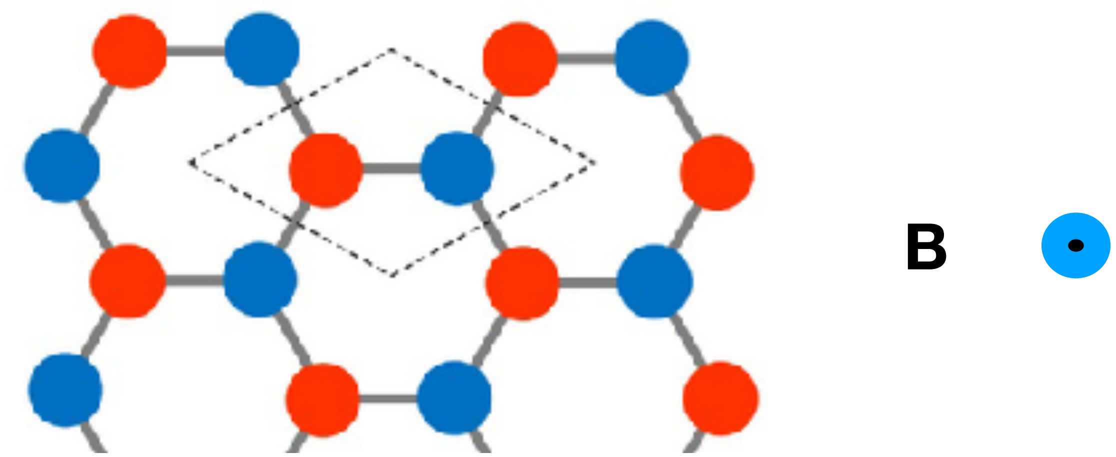
Twisted Bilayer Graphene

Continuum Approximation - each layer Dirac points.





Dirac Landau Levels



$$H_{\pm} = (p_x - eA_x)\sigma_x \pm (p_y - eA_y)\sigma_y$$

(+K, -K) valleys

$$H_{-} = \hbar\omega_c \begin{pmatrix} 0 & a \\ a^{\dagger} & 0 \end{pmatrix}$$

Zeroth Landau Level
is a `zero mode'.
Sublattice polarized

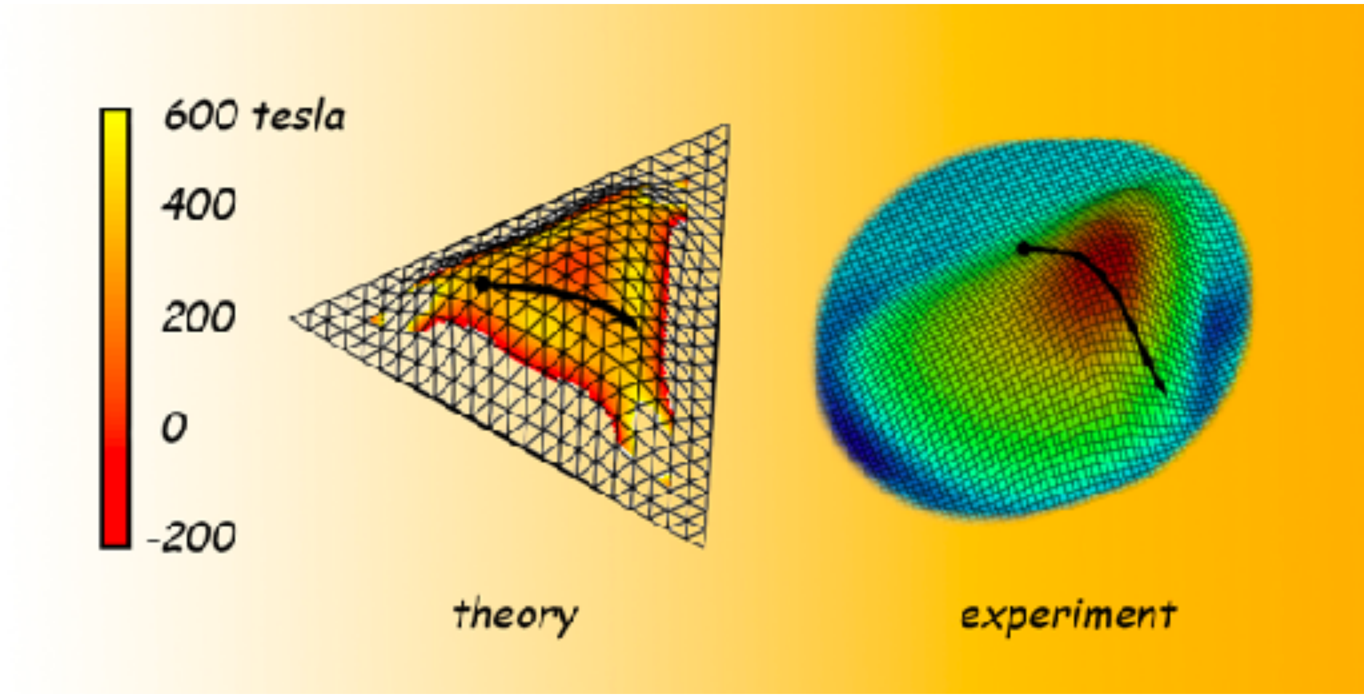
$$H \begin{pmatrix} 0 \\ |0\rangle \end{pmatrix} = 0$$

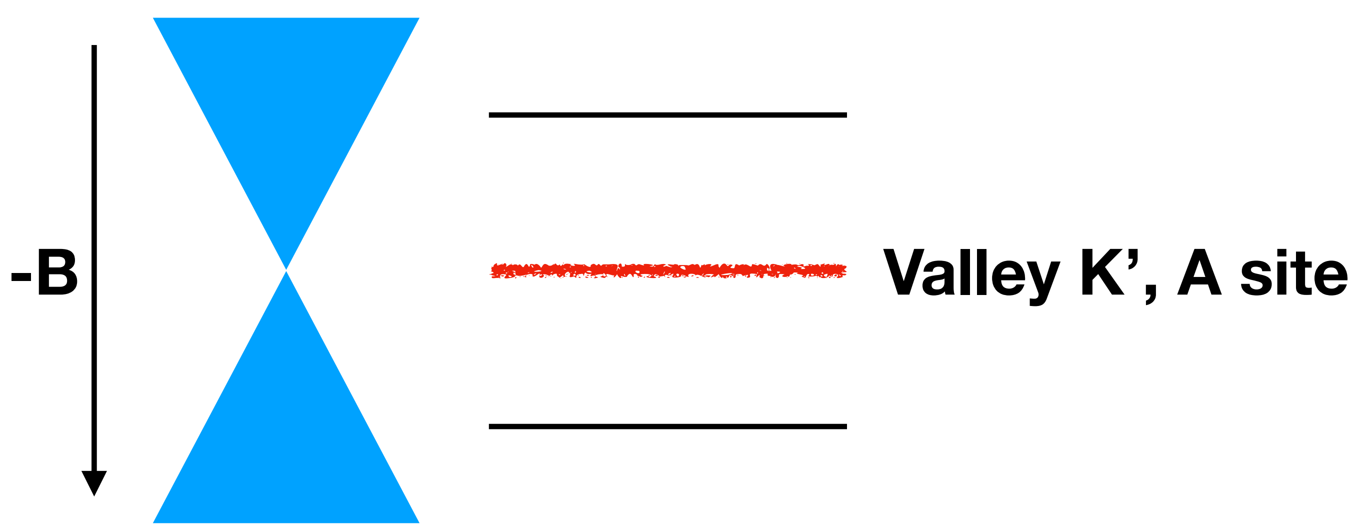
$$A = \frac{B}{2}(-y, x)$$

$$a = -i\left(2\frac{\partial}{\partial \bar{z}} + \frac{1}{2}z\right)$$

$$\Psi_0(z = x + iy) = f(z)e^{-\frac{1}{4}|z|^2}$$

Strained Graphene - Dirac Landau Levels





- Strained graphene Landau levels [Guinea, Crommie, Castro-Neto, 2010]

PRL 108, 266801 (2012) PHYSICAL REVIEW LETTERS week ending 29 JUNE 2012

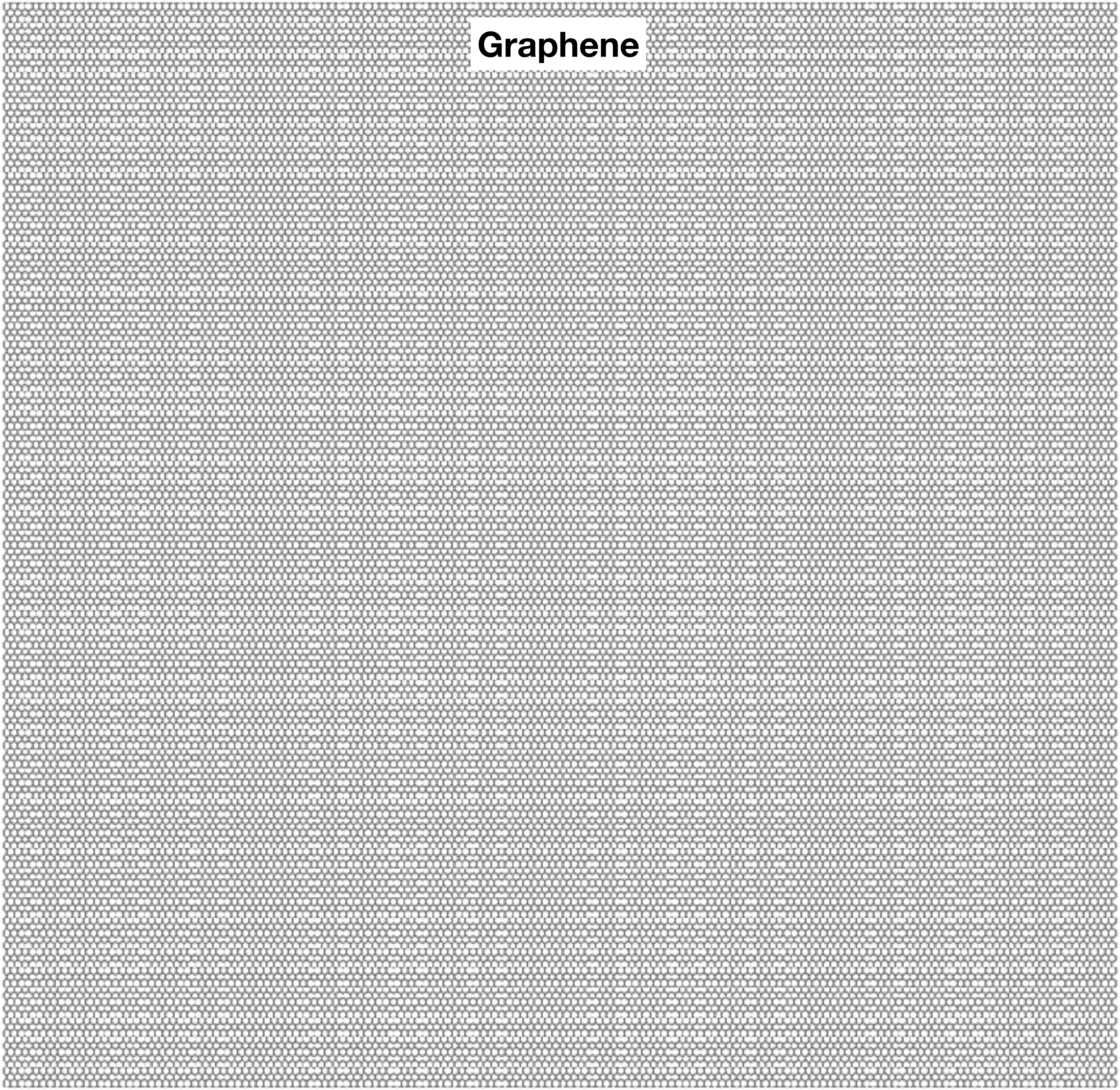
Fractional Topological Phases and Broken Time-Reversal Symmetry in Strained Graphene

Pouyan Ghaemi,[1,2,3,*] Jérôme Cayssol,[2,4,5] D. N. Sheng,[6] and Ashvin Vishwanath[2,3]

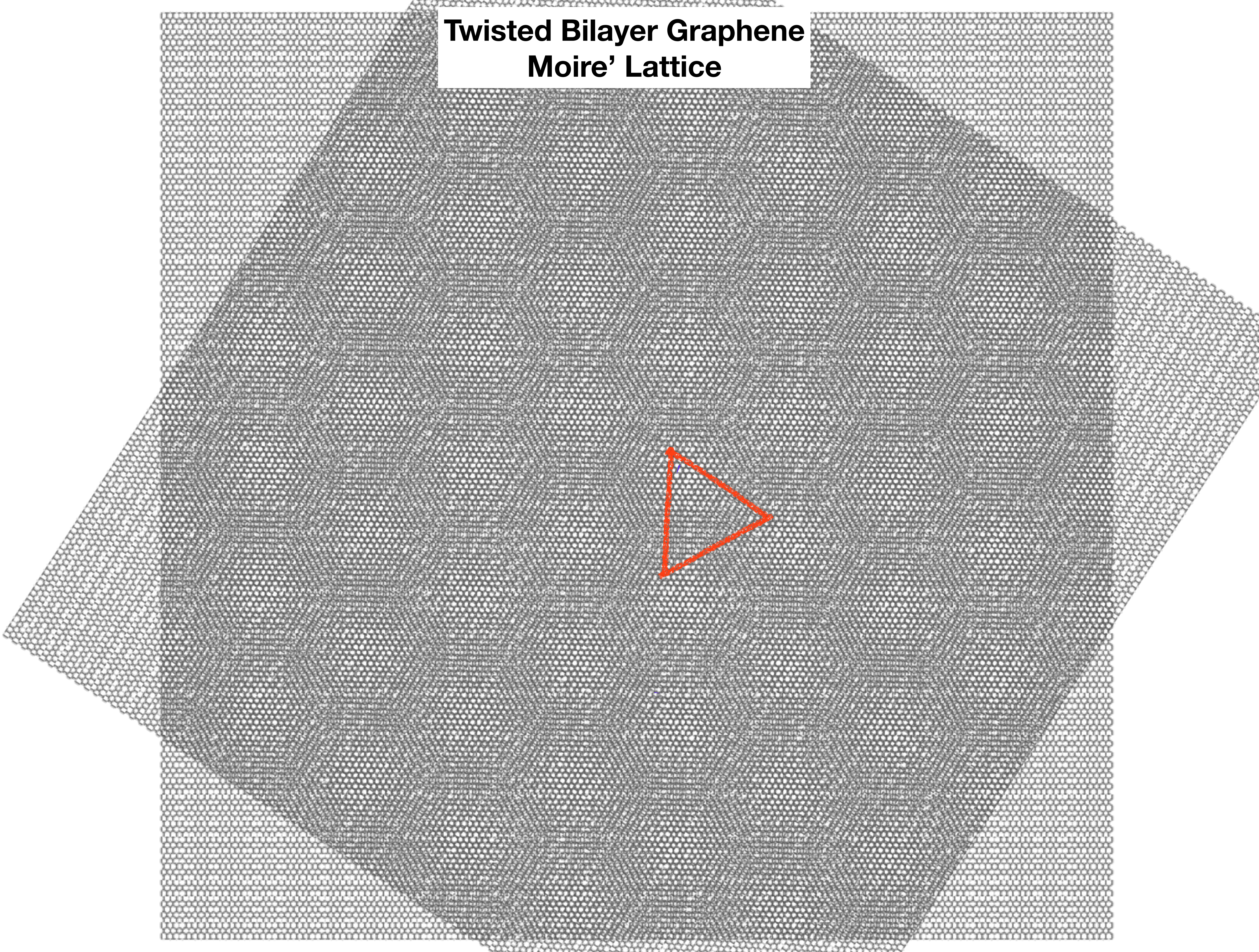
- Here - “2-copies” of strained graphene Landau levels

1. The Wavefunctions of Magic Angle Flat Bands

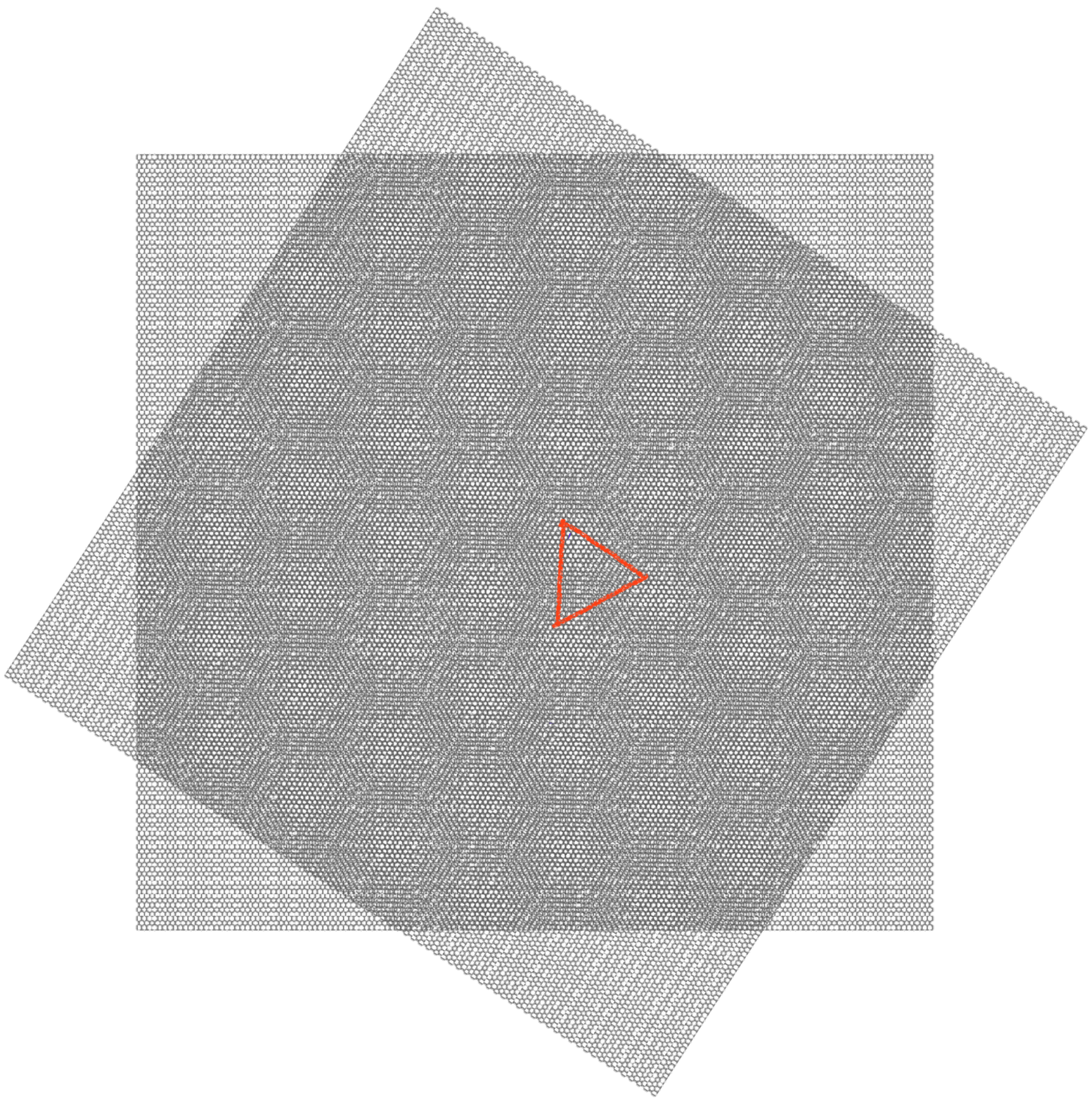
Graphene



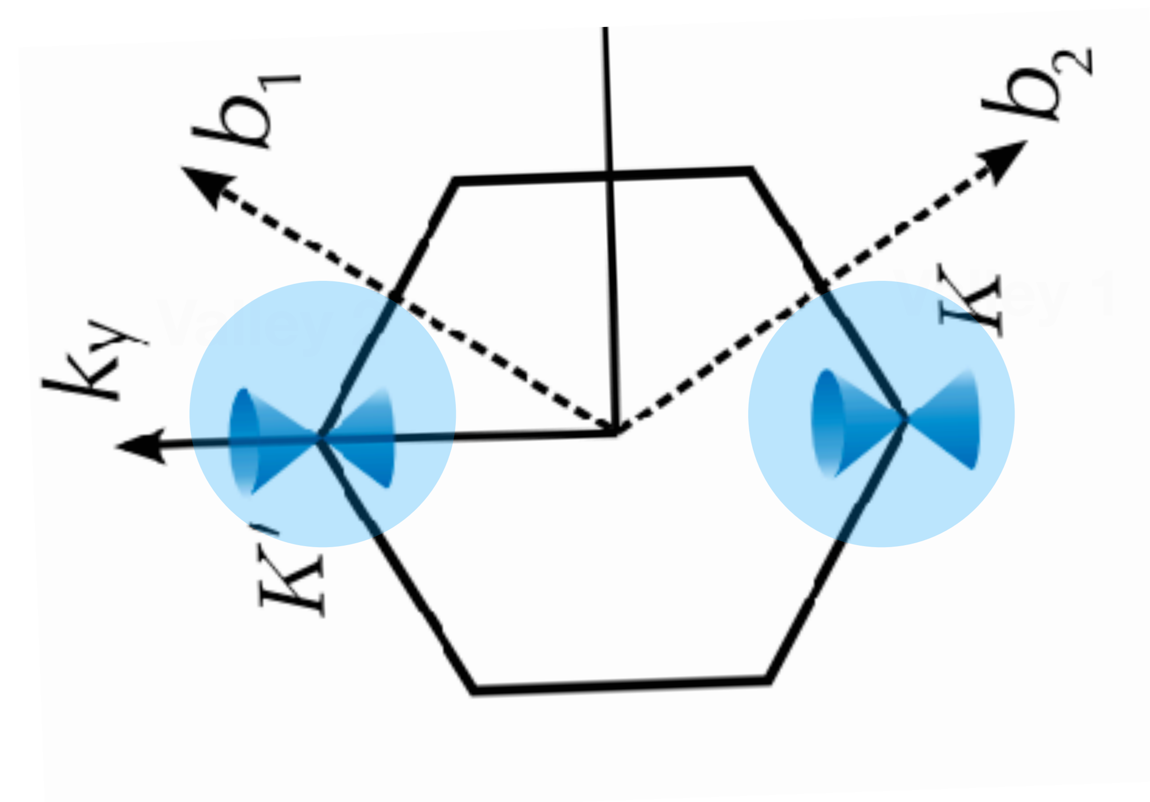
Twisted Bilayer Graphene Moire' Lattice



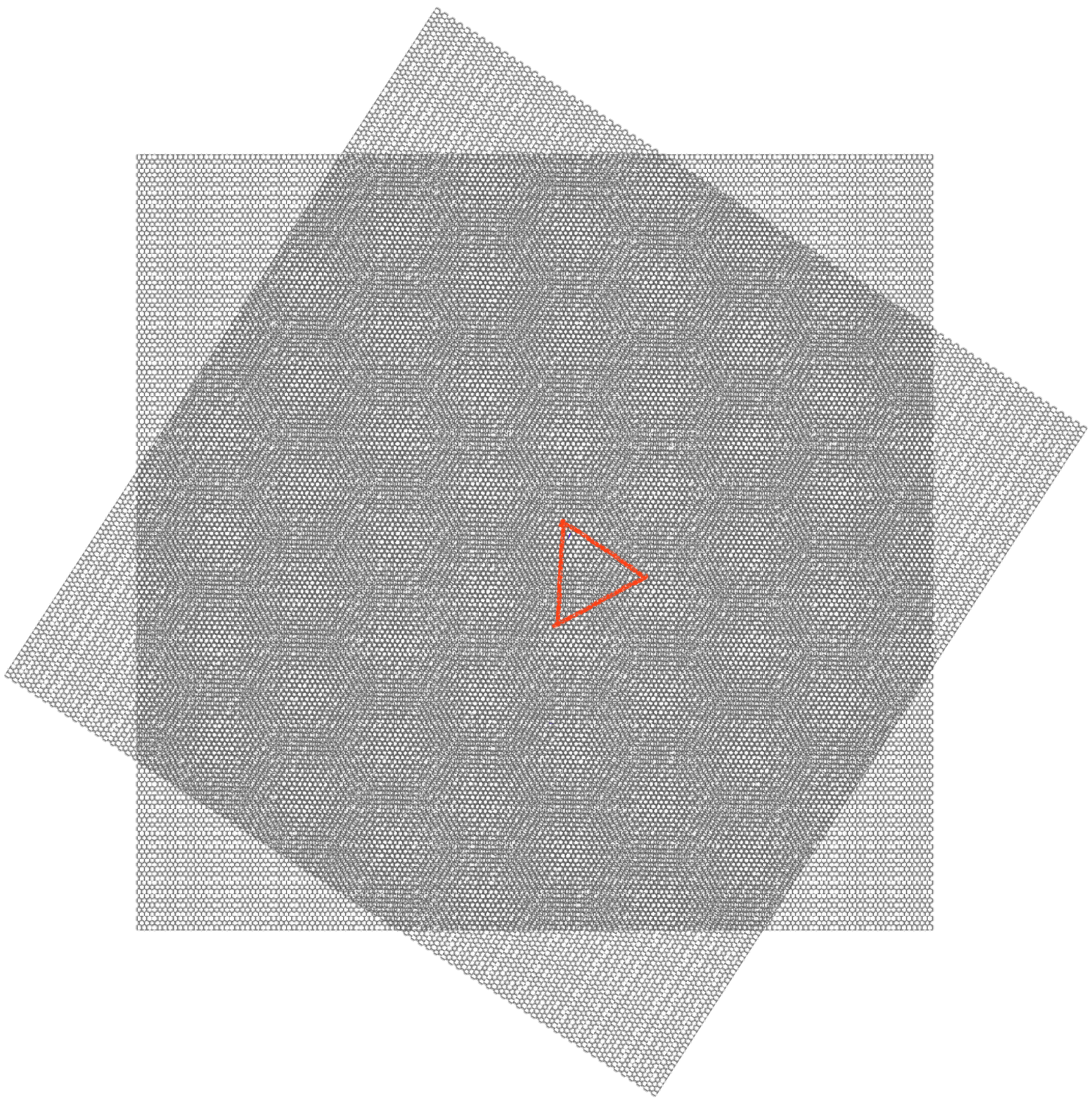
Moire Unit Cell and Brillouin Zone

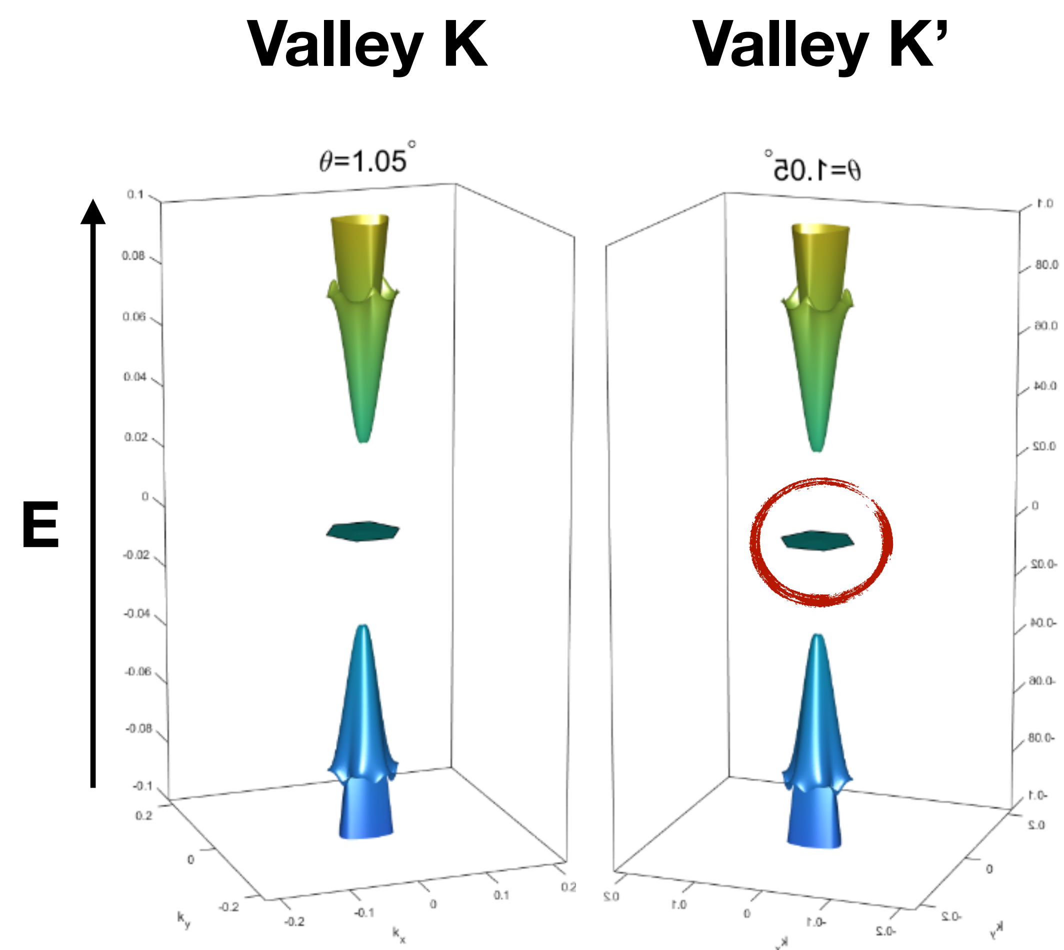


Graphene BZ

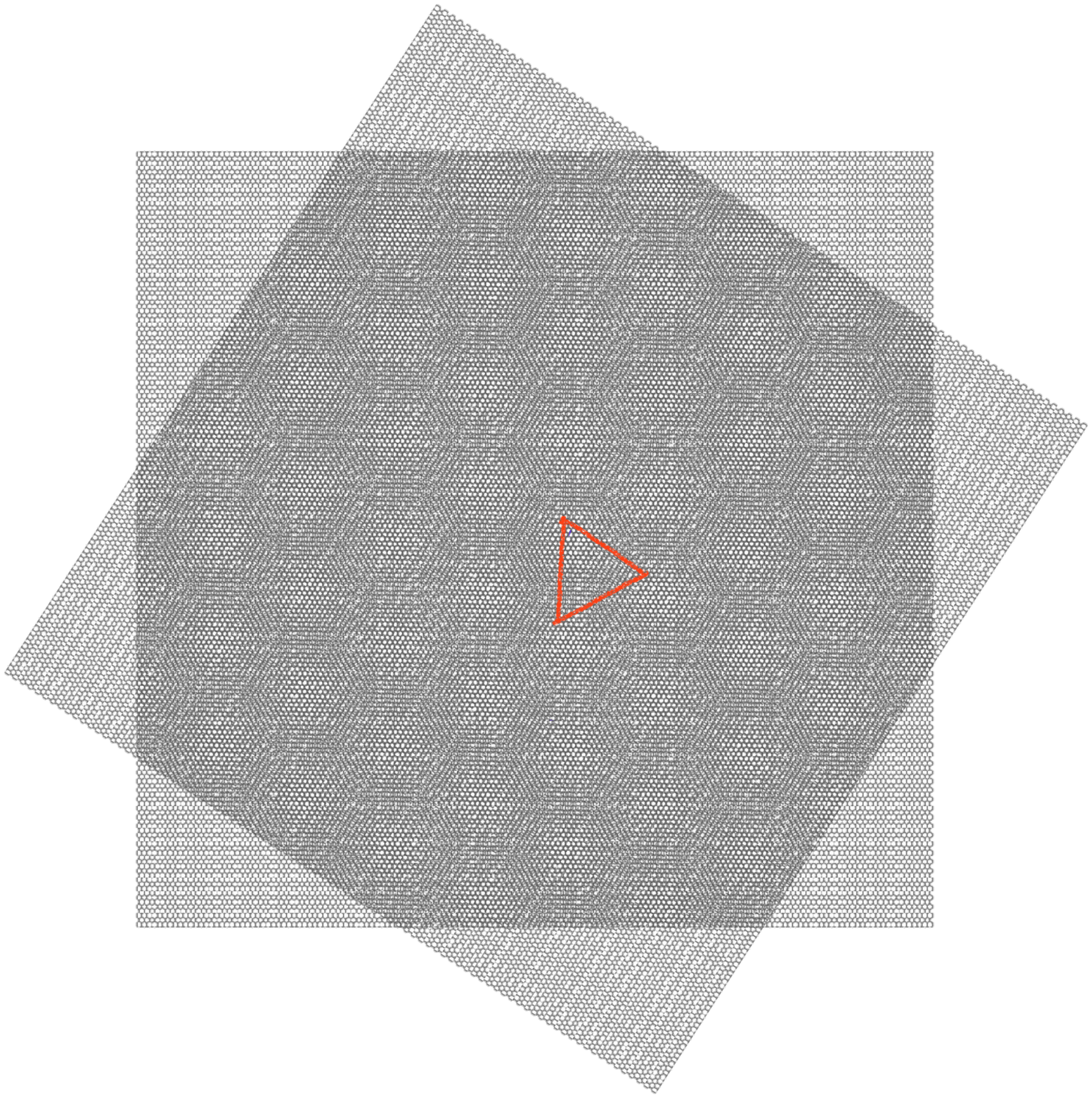


Moire Unit Cell and Brillouin Zone

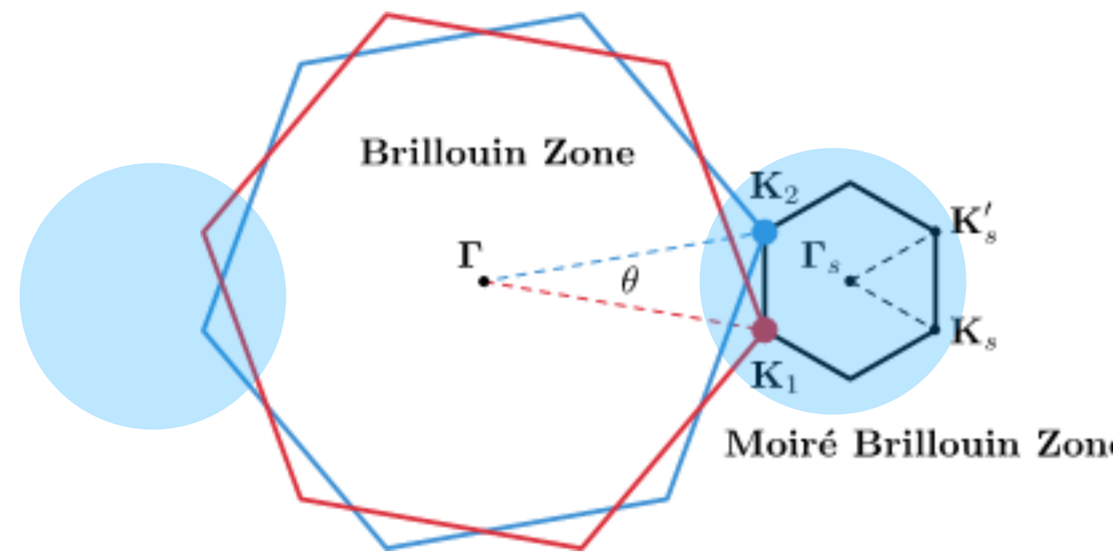




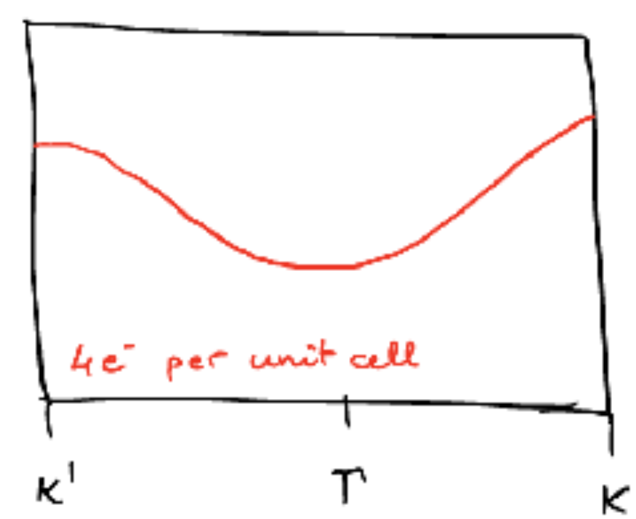
Moire Unit Cell and Brillouin Zone



Graphene BZ



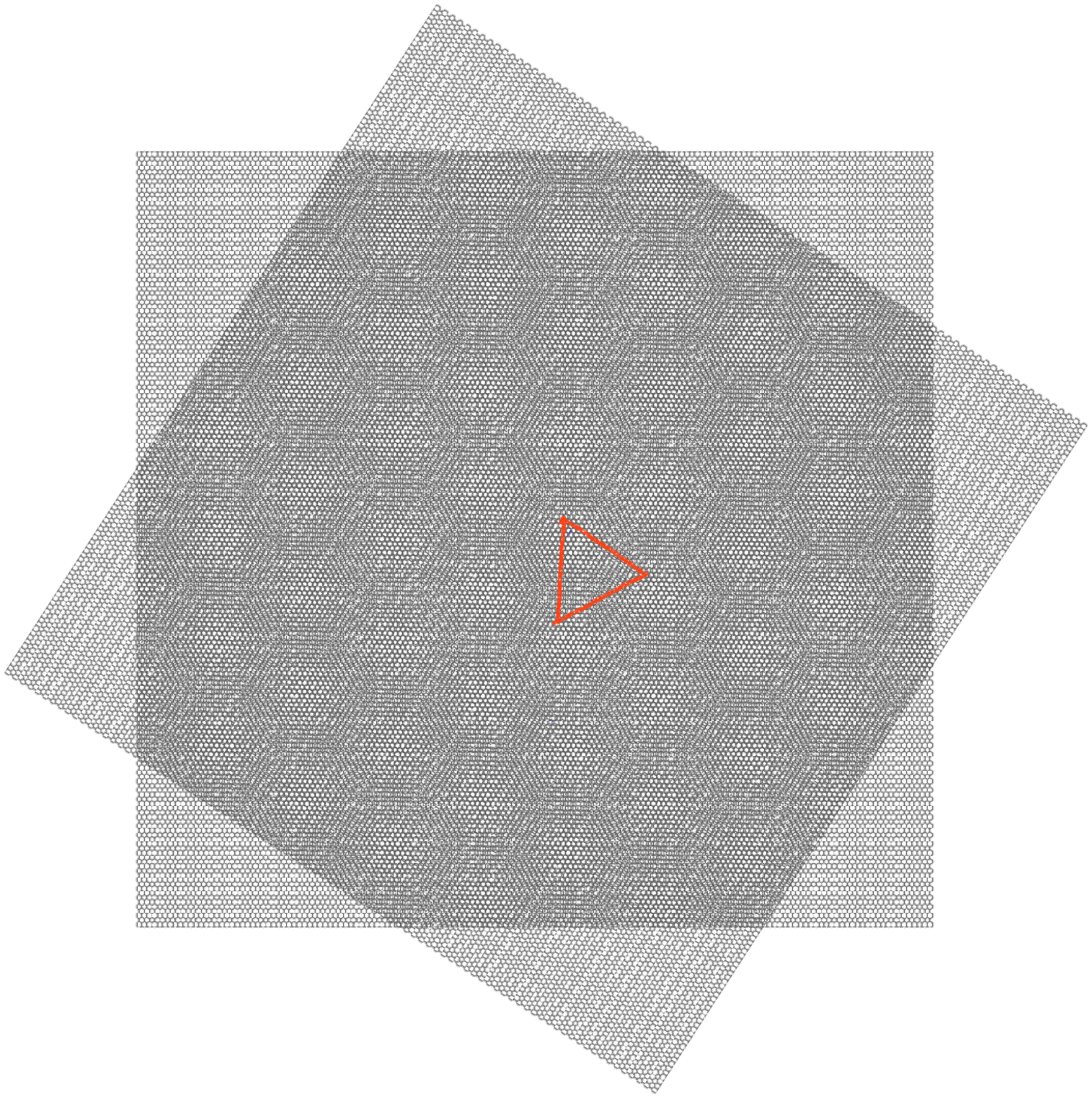
Valley 2 **Valley 1**



Moire Brillouin Zone

4e to fill a unit cell (spin and valley)?

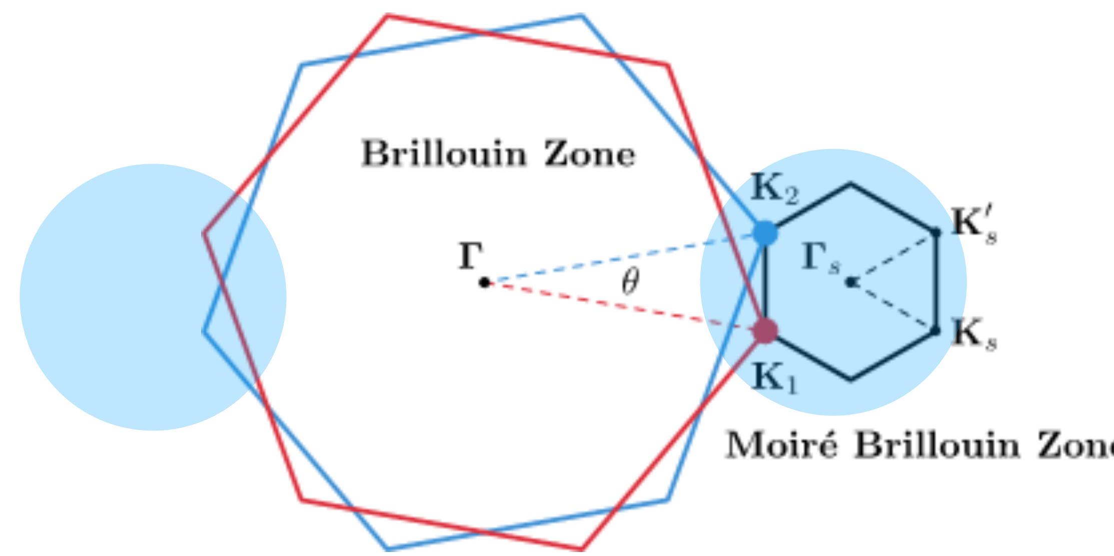
Moire Unit Cell and Brillouin Zone



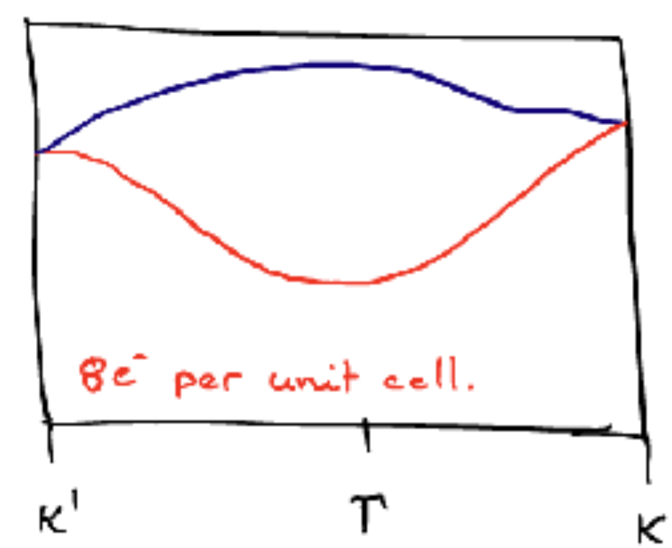
Actually, 8e to fill a unit cell

New degree of freedom: spin + valley + band

Graphene BZ



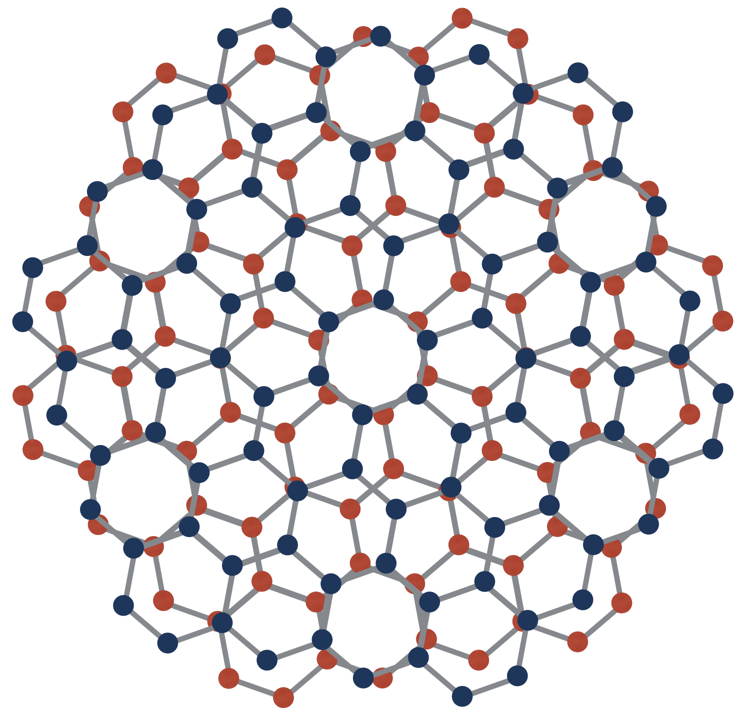
Valley 2 **Valley 1**

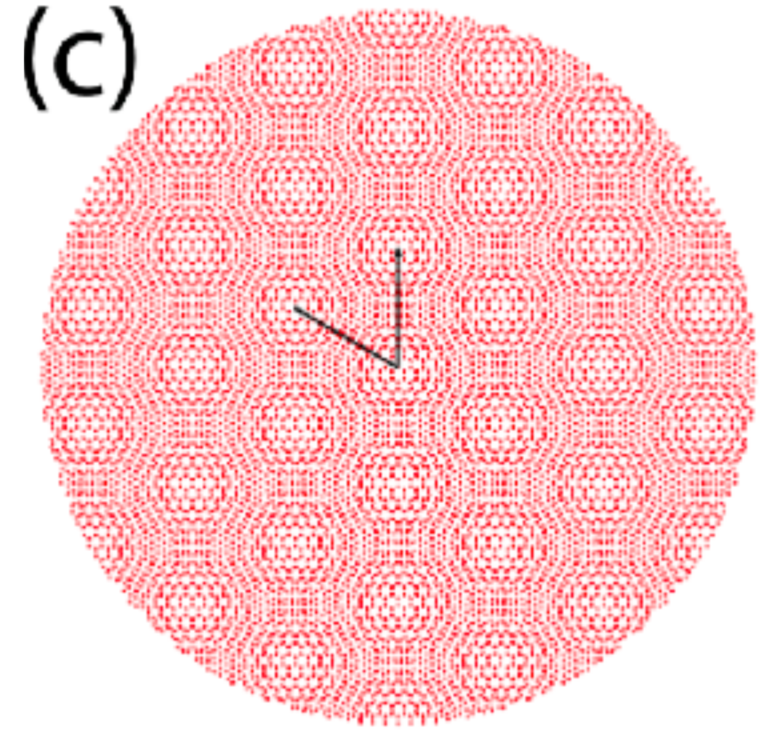


Moire Brillouin Zone

Symmetries of Twisted Bilayer Graphene

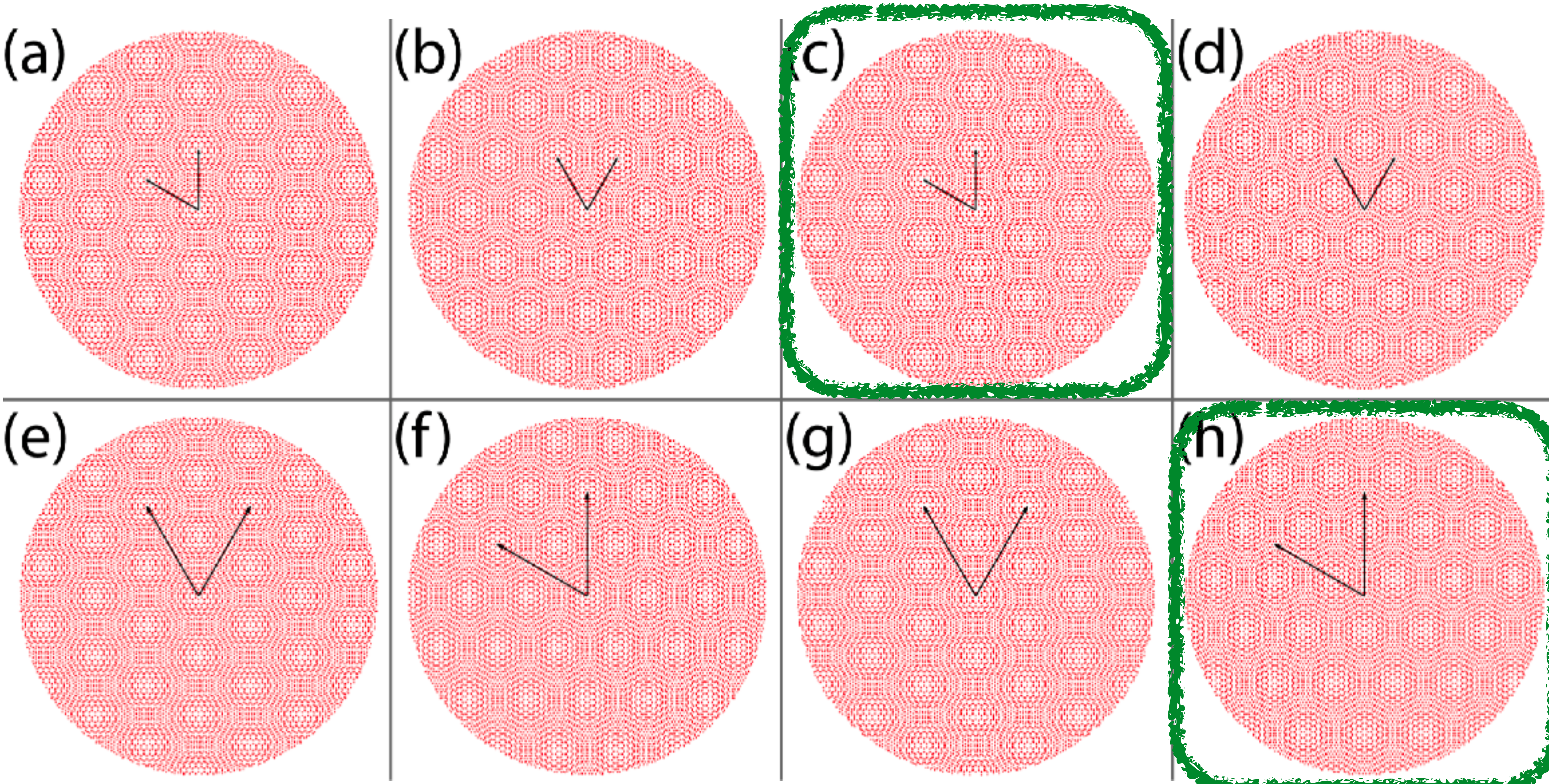
$\theta \simeq 6^\circ$





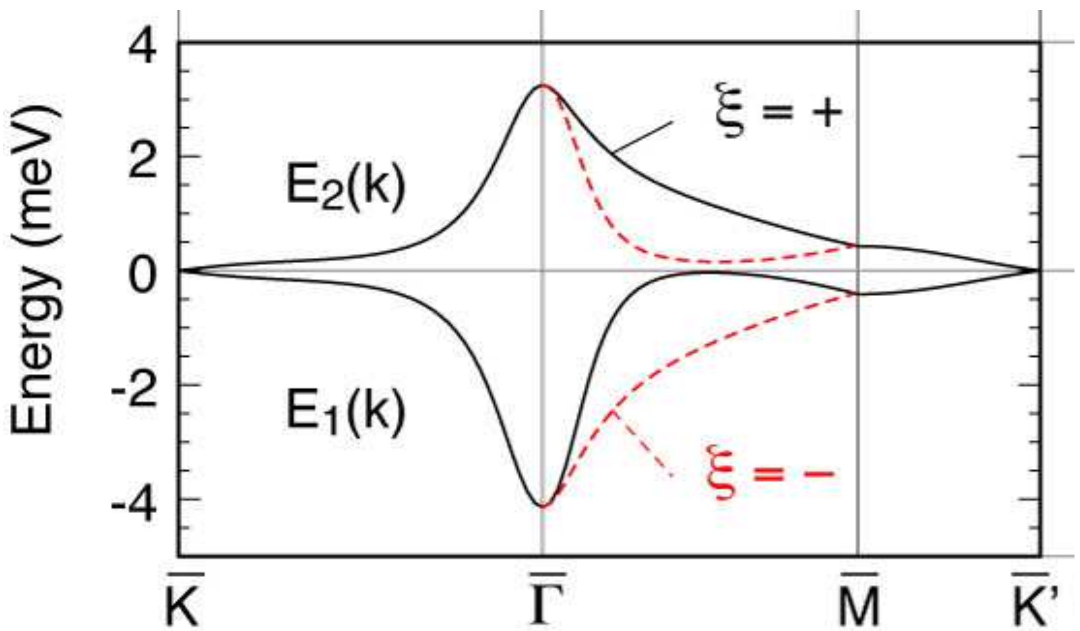
- At *small angles* - emergent C_6 symmetry. Commensurate vs incommensurate is irrelevant.
- Valley conservation -> $U(1)_v$ symmetry.
- The **C_2** part of the symmetry is crucial and specific to twisted bilayer graphene.
- Flat band topology - no tight binding model of just flat bands that preserves C_2T & $U(1)v$ symmetries. [Po, Zou, Senthil, *AV* - PRX, PRB 2018]

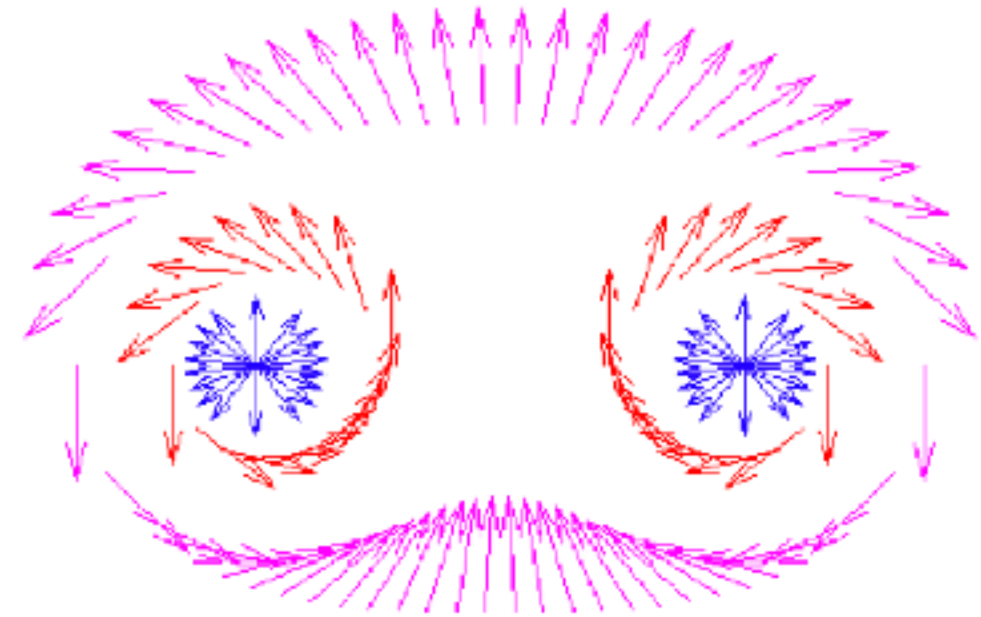
Symmetries: C6 vs C3?



Which one(s) **doesn't** have six-fold rotation symmetry about the bright spots?

Topology and Obstruction to 2-band model





[Castro Neto et al., PRB 2011, Goerbig & Montambaux, 2017]

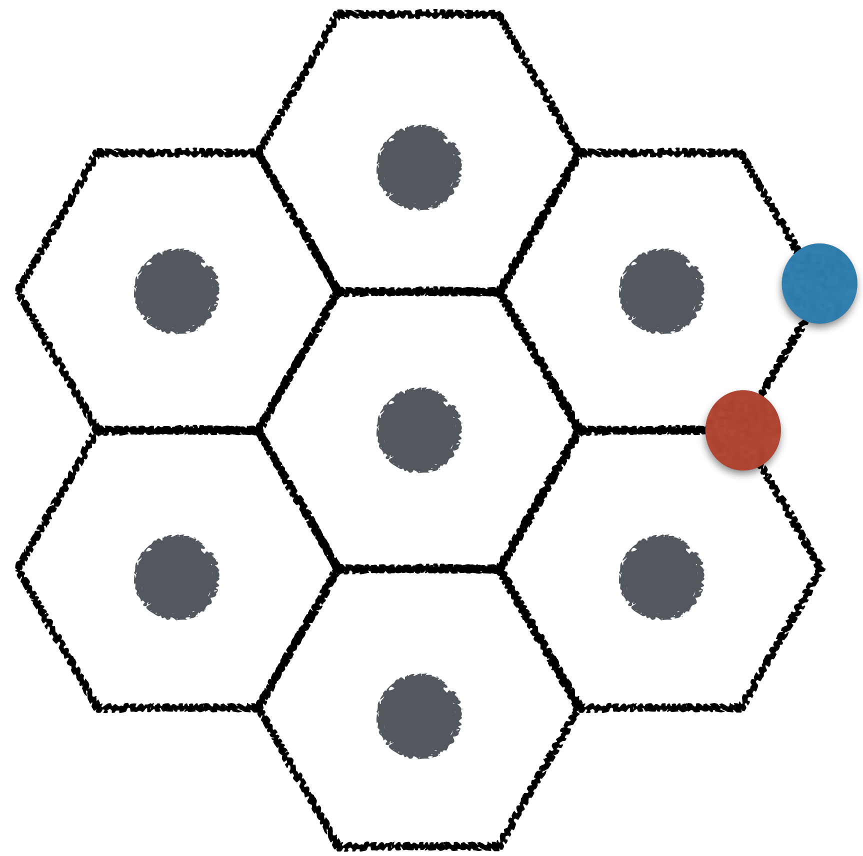
The Dirac points in each valley have the **same** chirality.

<u>Origin:</u> the graphene Dirac dispersion.

Prohibits a symmetric 2-band tight binding model. [Po, Zou, Senthil, *AV* - PRX, PRB 2018]

See Also: Ahn, Park and Yang; Liu, Liu, Dai; Song, Wang, Shi, Li, Fang, Bernevig;

No-go argument- Flipped Haldane Model



- With *same* chirality - onsite staggered potential will lead to Dirac gap and Chern number +1, -1.

- Large onsite potential - atomic insulator, incompatible with Chern number

Work in the Continuum like for Quantum Hall

Continuum model

- Larger unit cell→ smaller BZone
- Bistrizer-Macdonald (BM) model (2011)

$$\mathcal{H}_K = \begin{pmatrix} -iv_F \boldsymbol{\sigma}_{\theta/2}\cdot\nabla & T(\boldsymbol{r}) \\ T^\dagger(\boldsymbol{r}) & -iv_F\boldsymbol{\sigma}_{-\theta/2}\cdot\nabla \end{pmatrix}_{12},$$

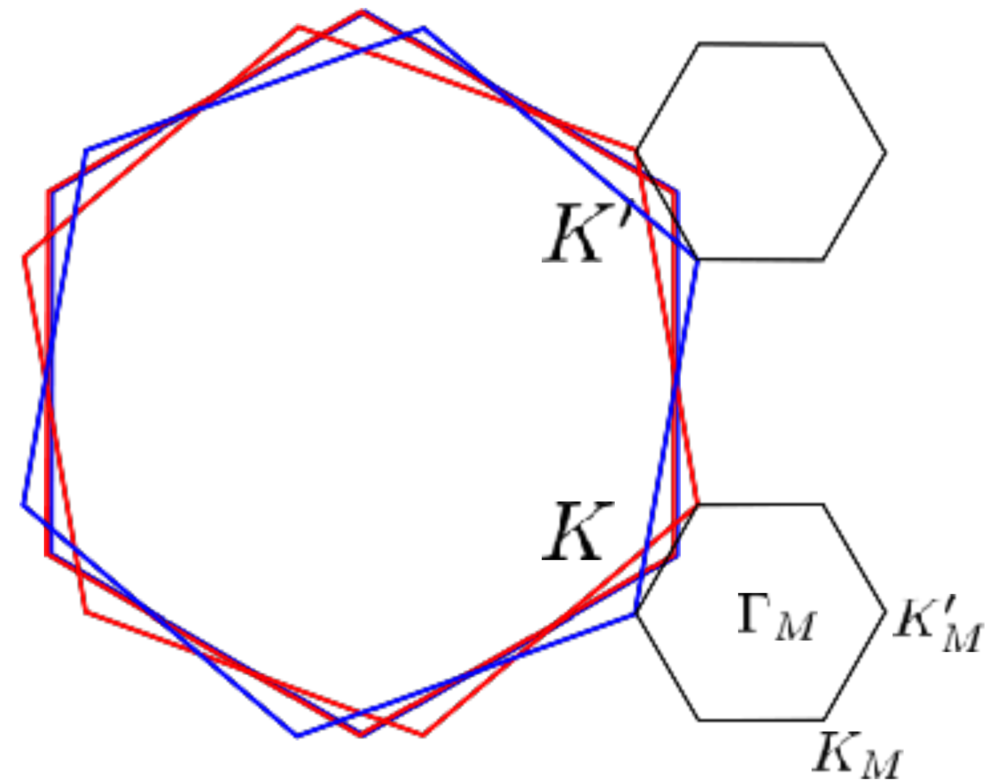
- Moire “potential”

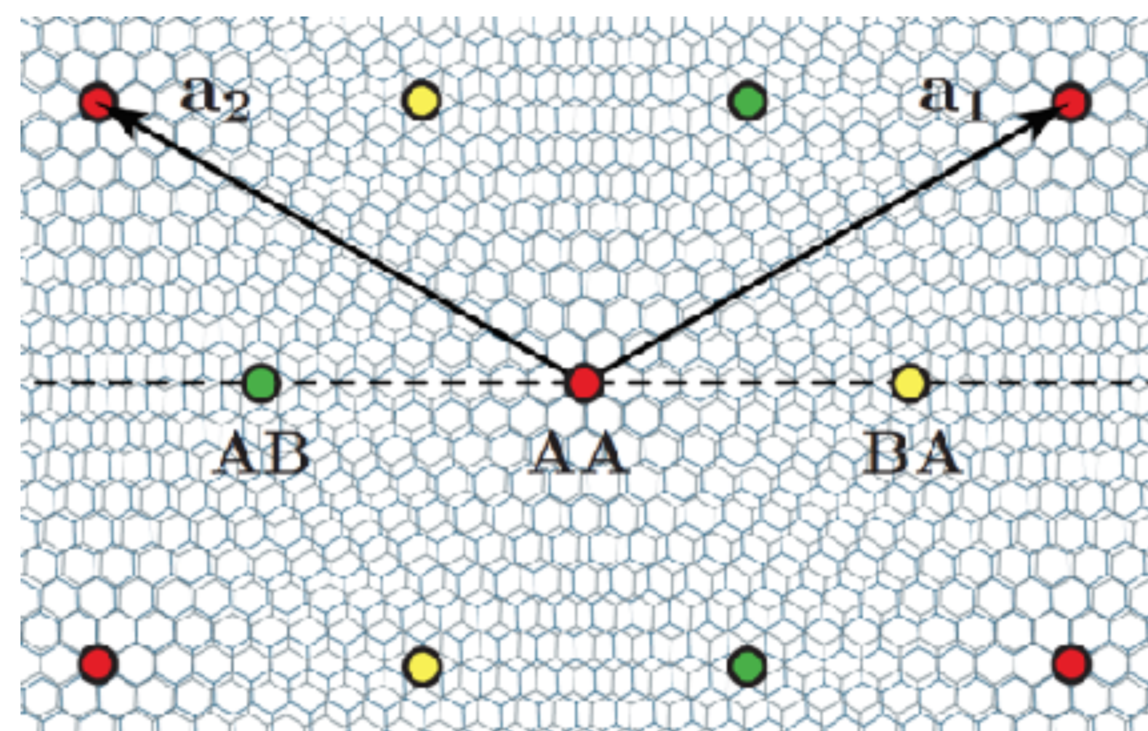
$$T(\boldsymbol{r}) = \begin{pmatrix} w_0 U_0(\boldsymbol{r}) & w_1 U(\boldsymbol{r}) \\ w_1 U^*(-\boldsymbol{r}) & w_0 U_0(\boldsymbol{r}) \end{pmatrix}_{AB}$$

- Lattice relaxation: AB stacking favored to AA stacking (Carr *et al*. 2019, Nam, Koshino 2017)

$$\Longrightarrow \quad w_0/w_1 \approx 0.7$$

$$\frac{w_0}{w_1} = \approx 0.55 - 0.8$$ **(AA vs AB/BA)**





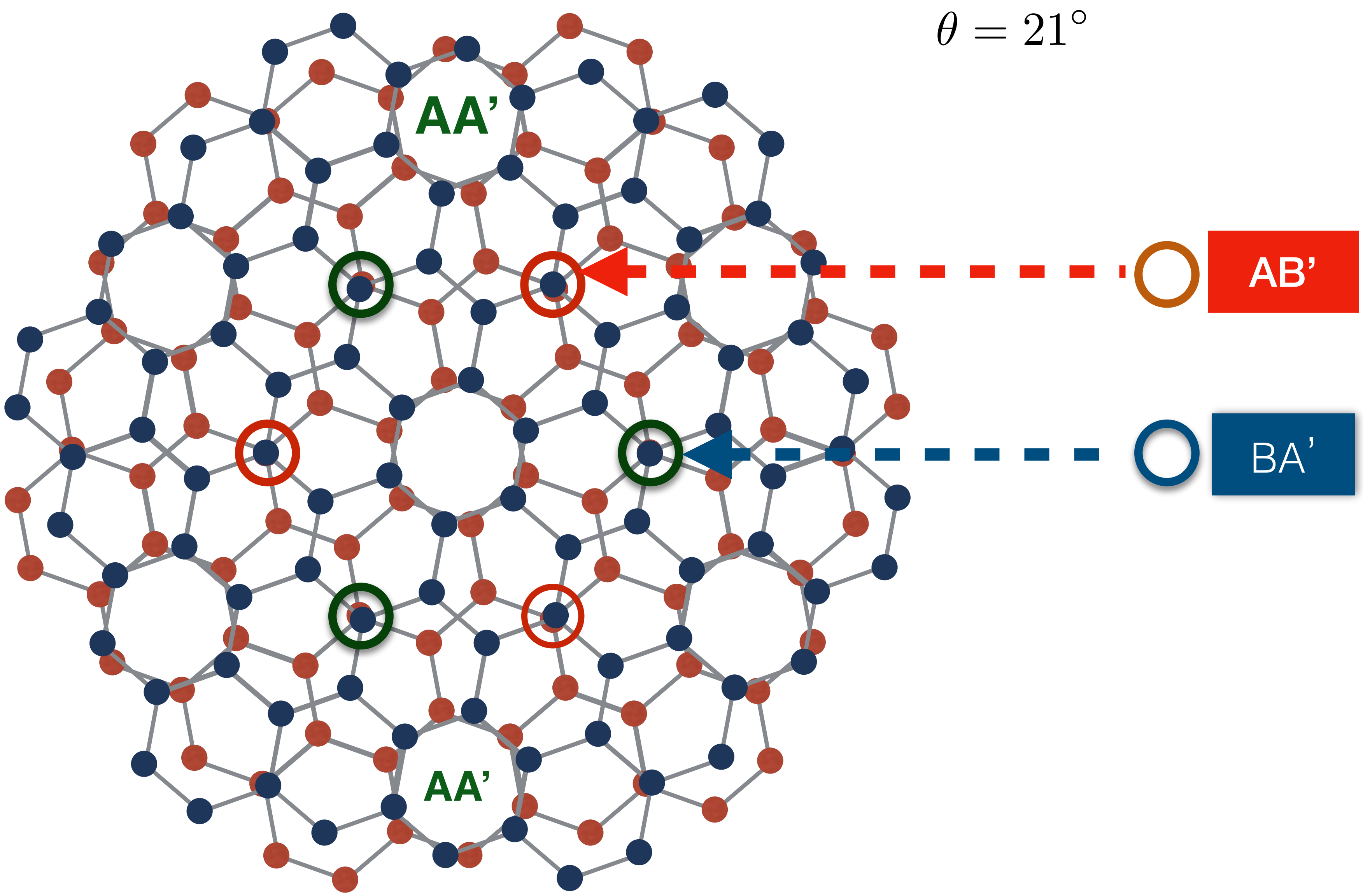
Chiral Model

Tarnopolski, Kruchkov, AV PRL 2019

Switch off AA coupling. Only AB coupling

$$T(\boldsymbol{r}) = \begin{pmatrix} \cancel{w_0 U_0(\boldsymbol{r})} & w_1 U(\boldsymbol{r}) \\ w_1 U^*(-\boldsymbol{r}) & \cancel{w_0 U_0(\boldsymbol{r})} \end{pmatrix}$$

$\theta = 21^\circ$



Grisha Tarnopolski
+ Alex Kruchkov
Harvard

Chiral Symmetry

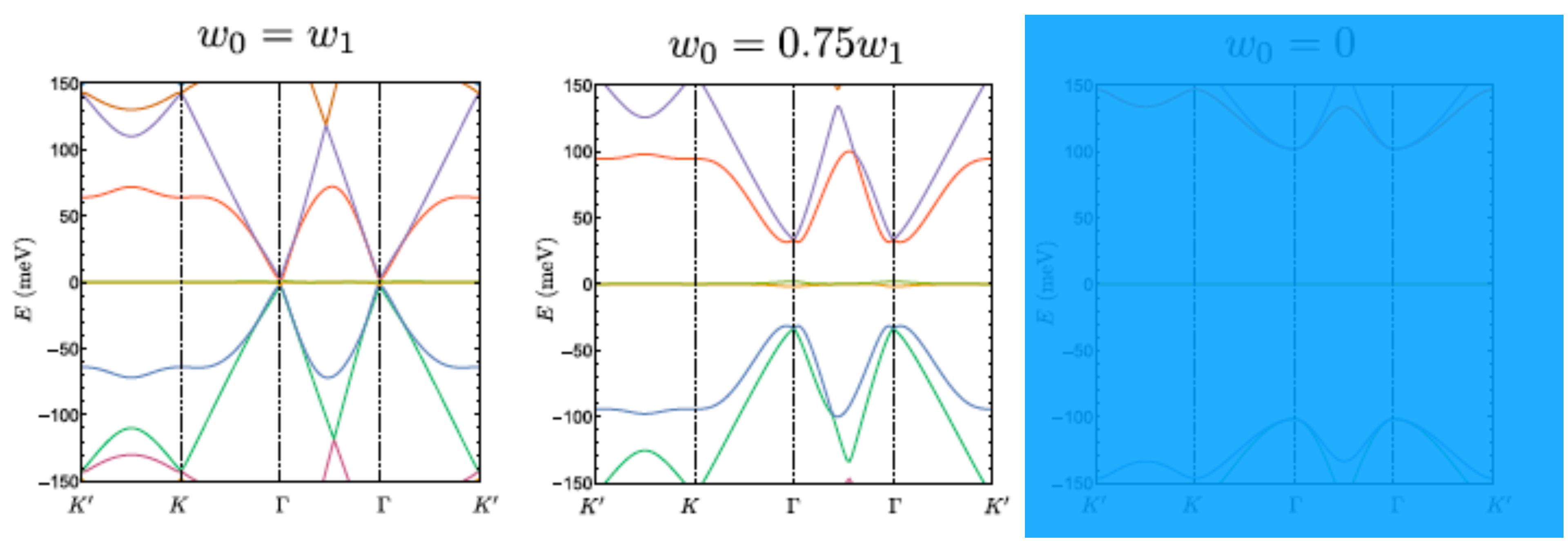
$$\{\sigma_z \otimes 1, \mathcal{H}\} = 0$$

Chiral Model

Tarnopolski, Kruchkov, AV
PRL 2019

Switch off AA coupling. Only AB coupling

$$T(\boldsymbol{r}) = \begin{pmatrix} \cancel{w_0 U_0(\boldsymbol{r})} & w_1 U(\boldsymbol{r}) \\ w_1 U^*(-\boldsymbol{r}) & \cancel{w_0 U_0(\boldsymbol{r})} \end{pmatrix}$$



Chiral Symmetry

$\{\sigma_z \otimes 1, \mathcal{H}\} = 0$

sublattice

$$\alpha = \frac{w_1}{2 v_F k_D \sin \theta/2}$$

Large angle: $\alpha \approx 0$

Magic angle: $\alpha \approx 0.6$

Chiral Model

Tarnopolski, Kruchkov, AV PRL 2019

Switch off AA coupling. Only AB coupling

$$\mathcal{H} = \begin{array}{c} \begin{array}{cc} \mathrm{A} & \mathrm{B} \end{array} \\ \begin{pmatrix} 0 & \mathcal{D}^*(-\mathbf{r}) \\ \mathcal{D}(\mathbf{r}) & 0 \end{pmatrix} \end{array}, \quad \mathcal{D}(\mathbf{r}) = \begin{array}{cc} \begin{array}{cc} \mathrm{U} & \mathrm{D} \end{array} & \\ \begin{pmatrix} -2i\bar{\partial} & \alpha U(\mathbf{r}) \\ \alpha U(-\mathbf{r}) & -2i\bar{\partial} \end{pmatrix} & \begin{array}{c} \mathrm{U} \\ \mathrm{D} \end{array} \end{array}$$

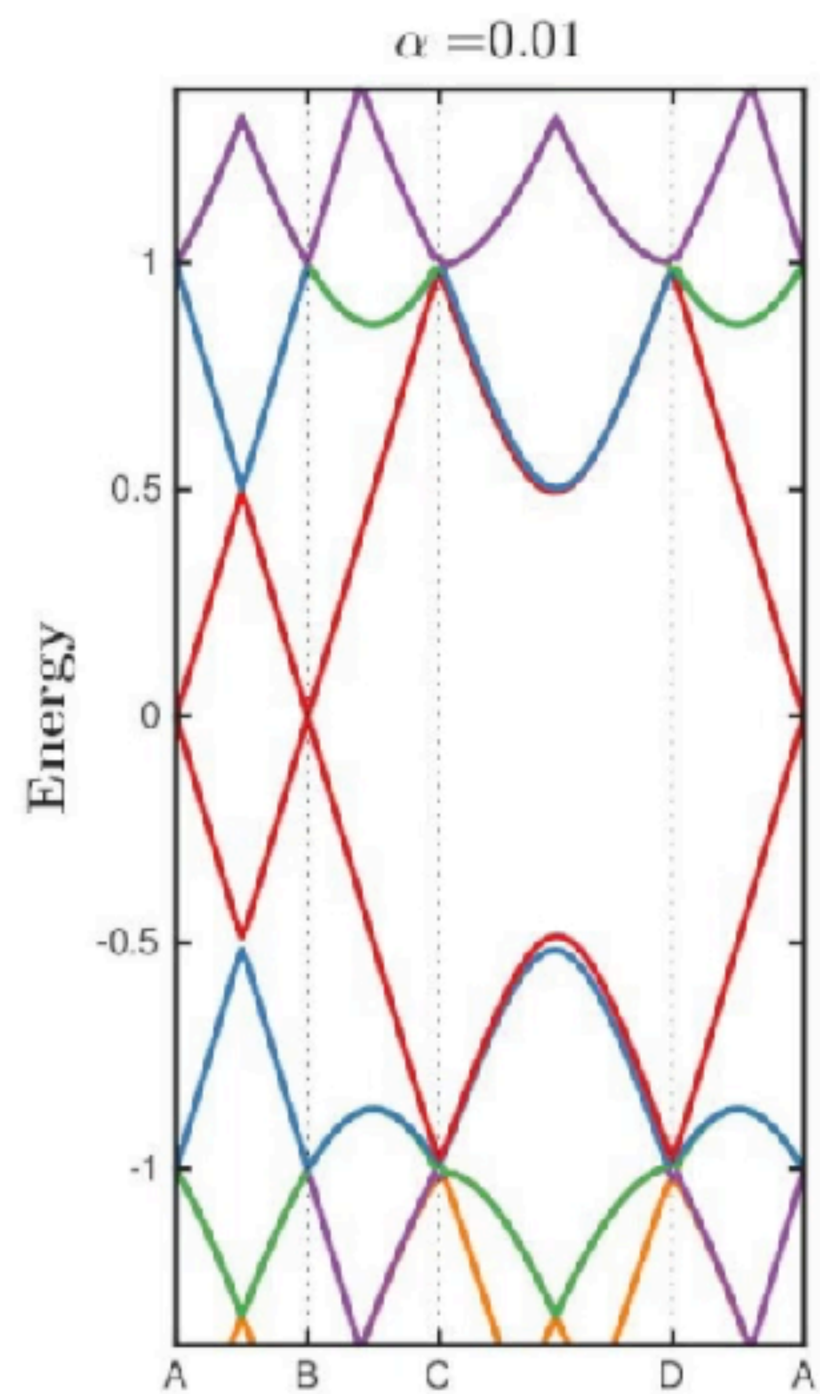
$$\bar{\partial} = \tfrac{1}{2}(\partial_x + i\partial_y)$$

Can be viewed as Dirac fermions in a non-abelian SU(2) gauge field *P. San-Jose, J. Gonz´alez, and F. Guinea*

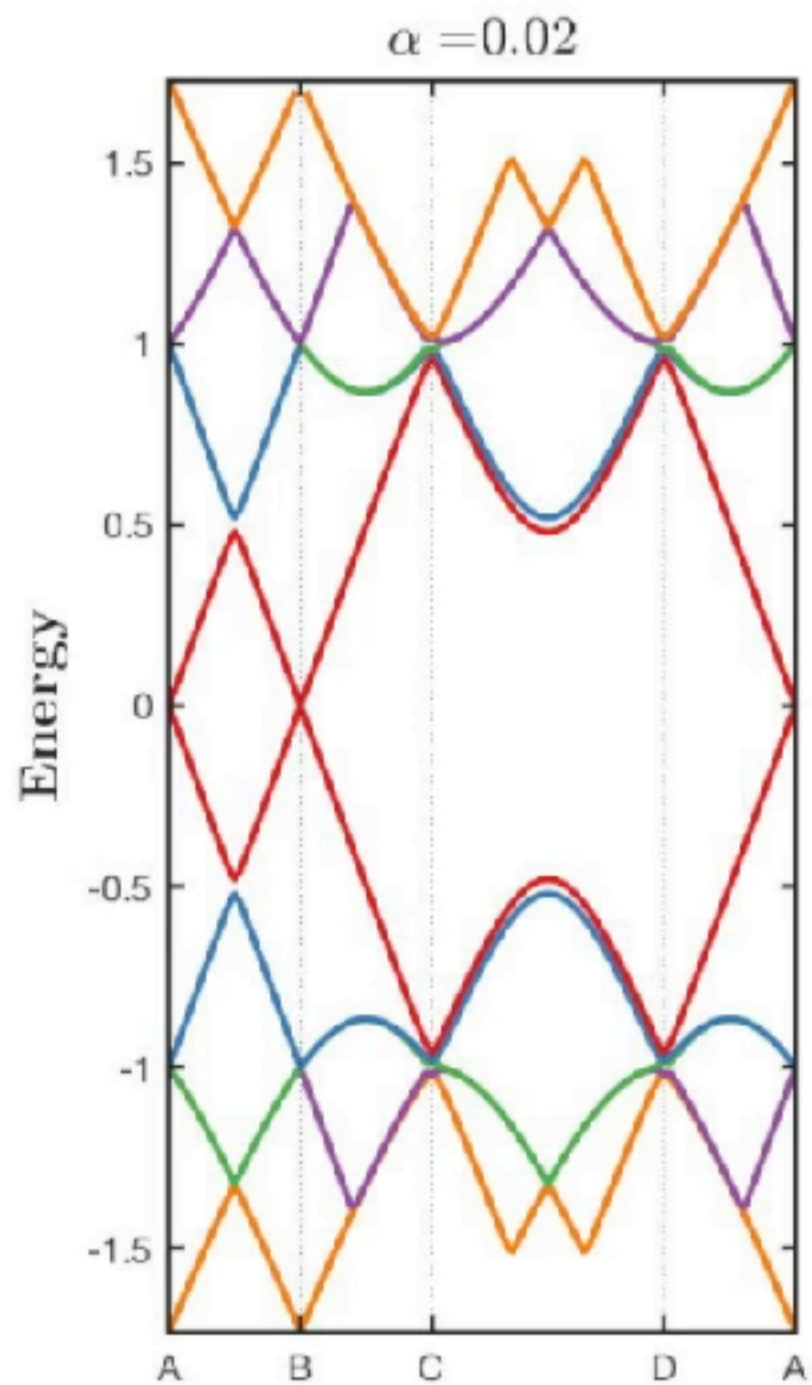
Non-Abelian vector potential:

$$\bar{A} = \frac{U(r) + U(-r)}{2}\tau^x + i\frac{U(r) - U(-r)}{2}\tau^y$$

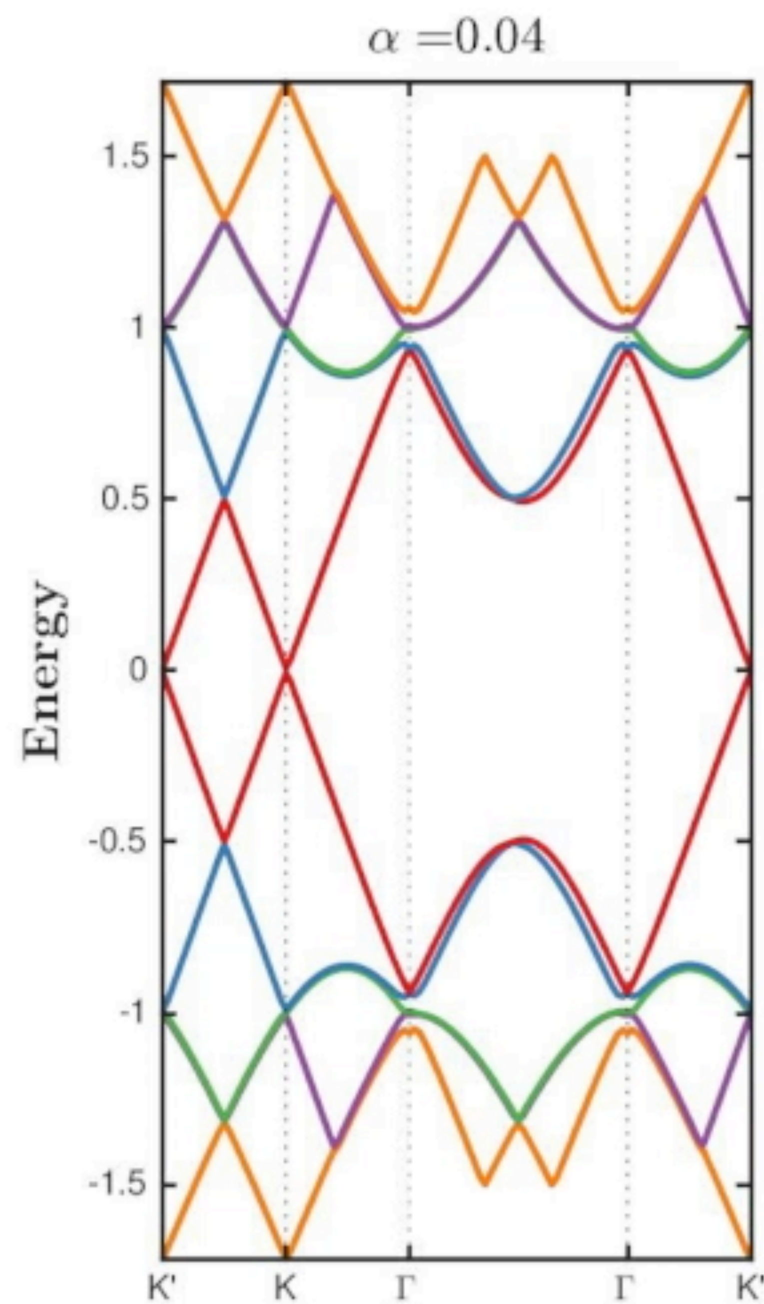
Bistritzer-MacDonald Model



Perfectly Flat Bands in the Chiral Model



Absence of Flat Bands in anti-Chiral Model

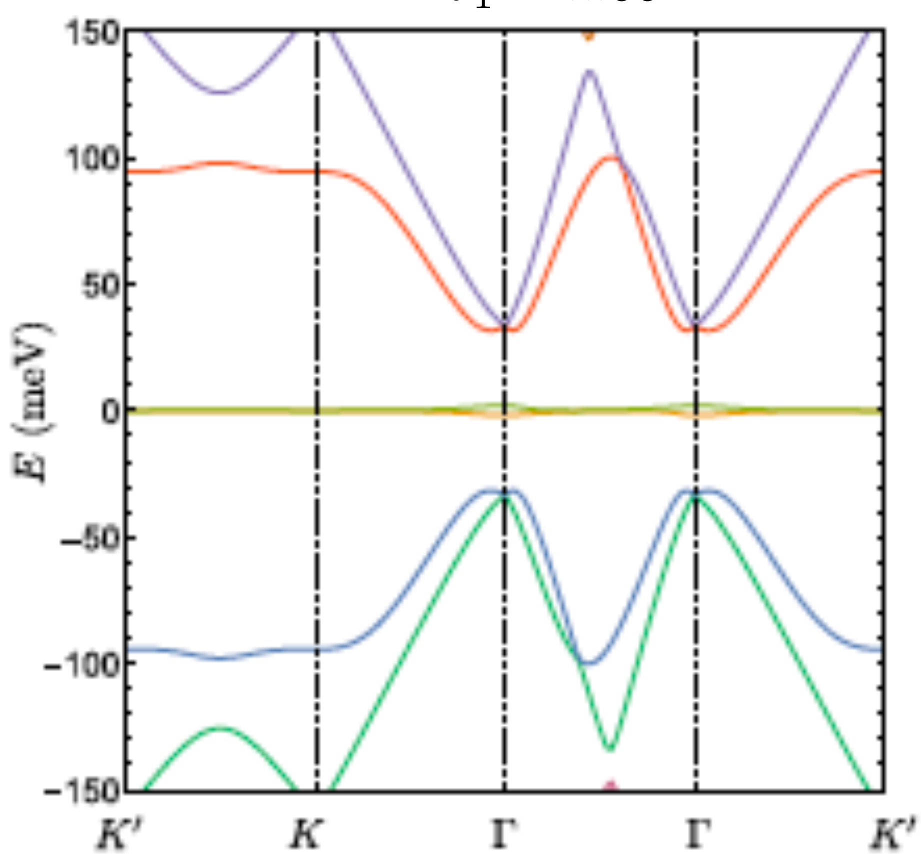


- Switch off A-B hopping (retain only A-A)

From Flat*tish* to Perfectly Flat Bands in the Chiral Limit

$w_0 = 0.75 w_1$

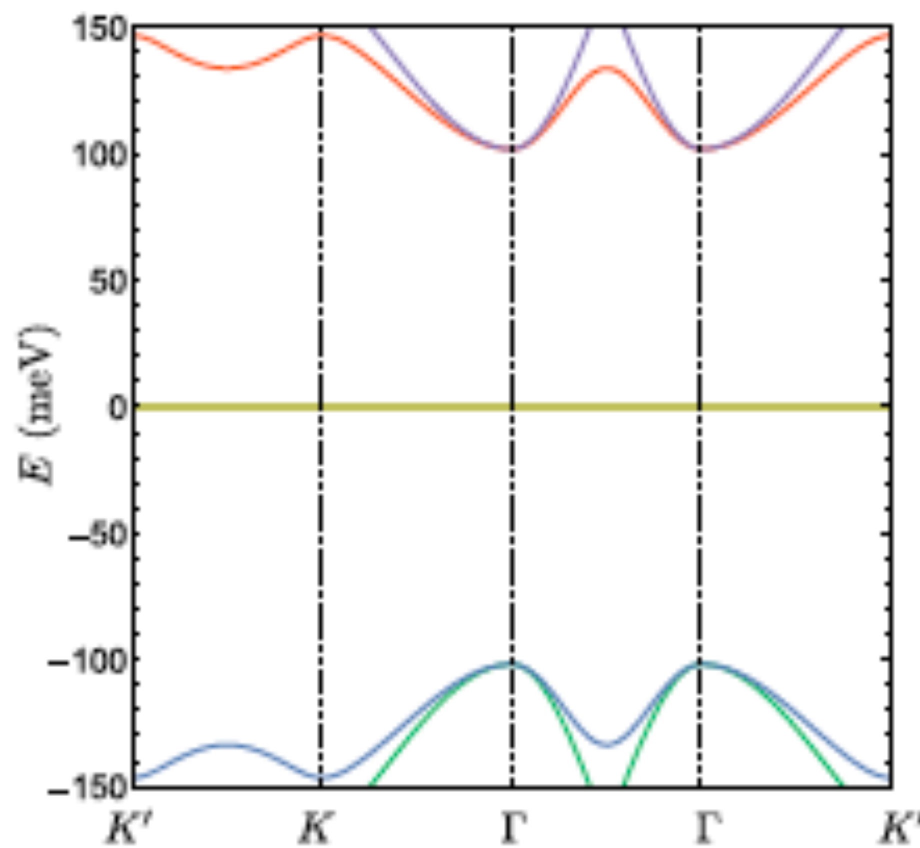
$\theta_1 \approx 1.05^\circ$



Exactly flat bands

$w_0 = 0$

$\alpha = 0.586$



$$\alpha = \frac{w_1}{2 v_0 k_D sin(\theta/2)}$$

Chiral Symmetry

sublattice $\longrightarrow \{\sigma_z \otimes 1, \mathcal{H}\} = 0$

$\alpha = 0.58566355838955$

Becker, Embree, Wittsten, Zworski `20

Tarnopolski, Kruchkov, AV;

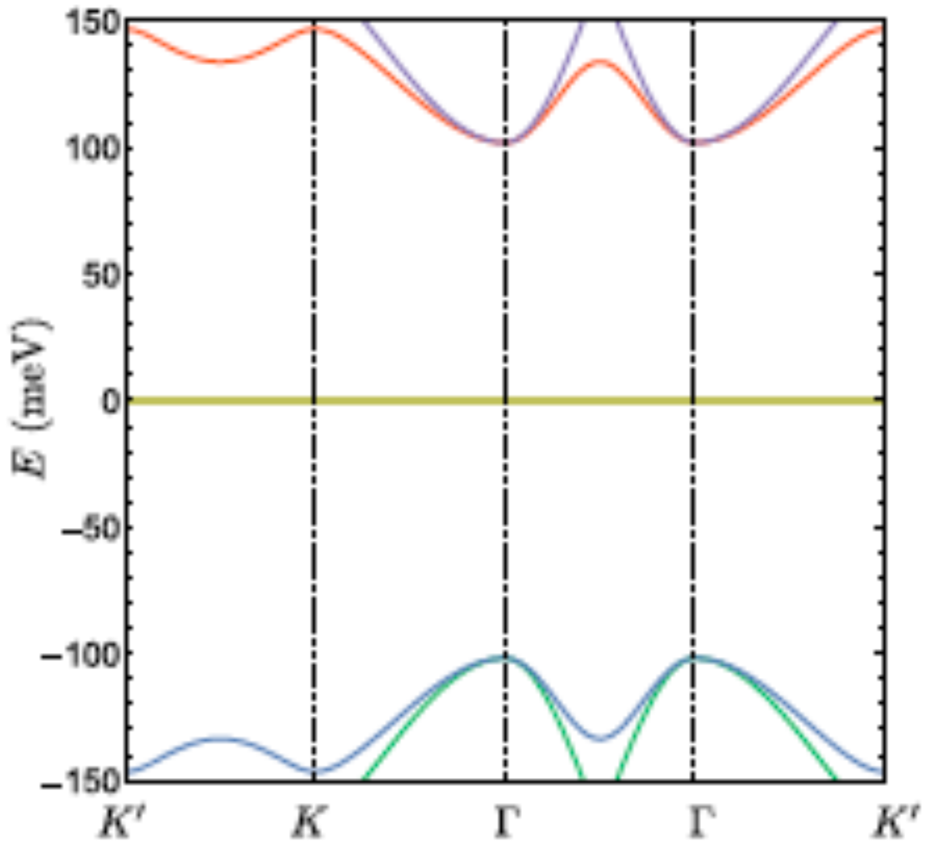
Chiral Limit - Attractive fixed point (Kang and Vafek)

From Flat*tish* to Perfectly Flat Bands in the Chiral Limit

Tarnopolski, Kruchkov, AV
P. San-Jose, Gonz´alez, and Guinea

Exactly flat bands

$\alpha = 0.586$



Wavefunctions:

$$\frac{\Psi_{\mathrm{k}}(\mathrm{z})}{\Psi_{\Gamma}(\mathrm{z})} = e^{-\frac{i}{2}\mathrm{k}\bar{\mathrm{z}}}\frac{\sigma(\mathrm{z}+i\ell^2\mathrm{k})}{\sigma(\mathrm{z})}$$

$\sigma(z)$ Weierstrass-Haldane sigma function.
Or, ratio of Theta fns.
(Symmetric vs. Landau gauge)

1. There is a zero at the origin for Gamma point wfn.

2. Has Chern number =1
Can be seen from $u_{k+G} = e^{i\phi_k(G)}u_k$

$$\sigma(z) = -\sigma(-z) \qquad ,$$

$$\sigma(z+a_{1,2}) = -\exp\left(\frac{1}{2\ell^2}a^*_{1,2}(z+\frac{a_{1,2}}{2})\right)\sigma(z)$$

Tarnopolski, Kruchkov, AV;
P. Ledwith, Tarnopolsky, Khalaf, AV;
Jie Wang, Cano, Millis, Liu, Yang.
Haldane.

Exactly Flat Bands

Look for *exactly* zero energy states:

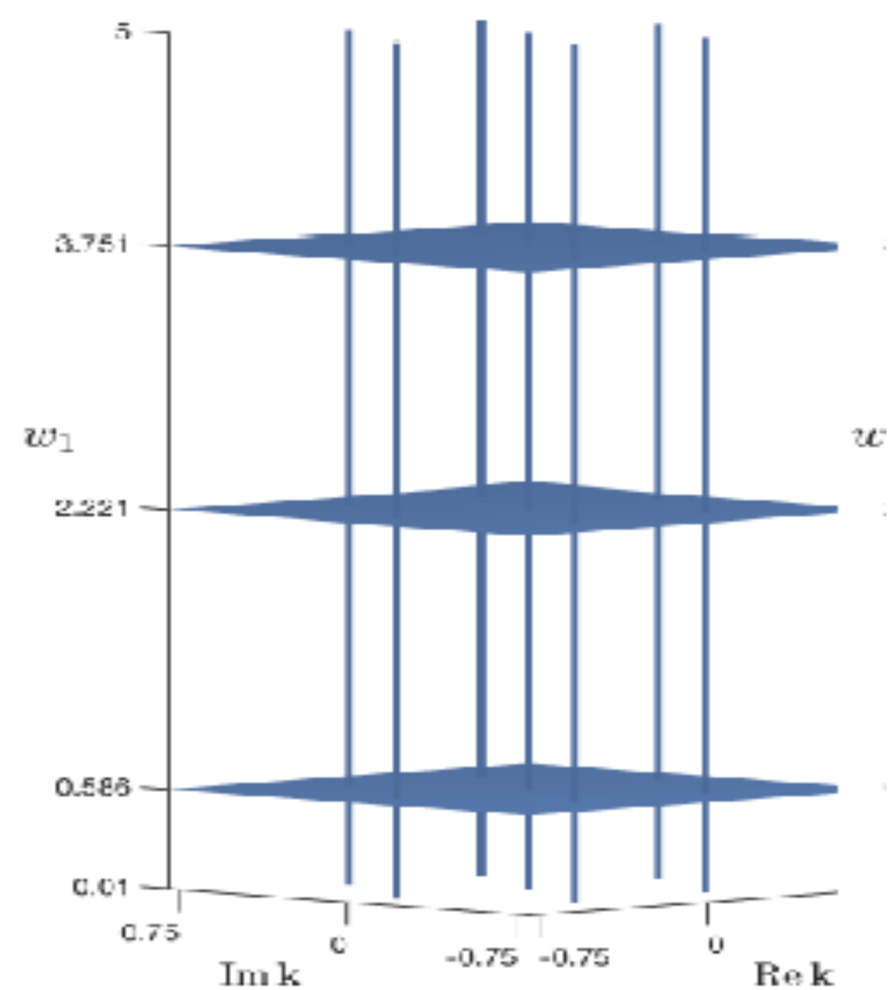
$$\mathscr{D}(r)\,\psi(r) = 0$$

Usually: $H\,\psi_k = E(k)\,\psi_k$. BUT HERE $D\,\psi_k = 0$

- Simpler `zero mode' equation -

$$D\,u_{\mathbf{k}} = (k_x + ik_y)\,u_{\mathbf{k}}$$

- Always has solution at Dirac point, but flat bands only at magic angles.



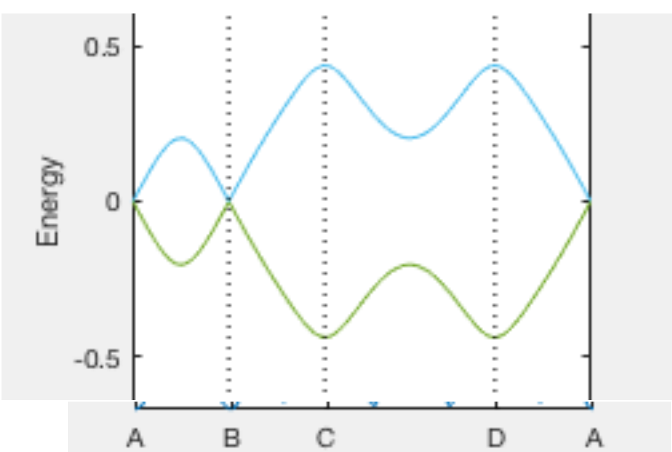
Becker, Embree, Wittsten, Zworski `20

Exactly Flat Bands

Look for *exactly* zero energy states:

$$\mathscr{D}(r)\,\psi(r) = 0$$

For all angles there exists a zero-mode solution at K point $\mathcal{D}(\mathbf{r})\psi_K(\mathbf{r}) = 0$



$$\begin{pmatrix} -2i\bar{\partial} & \alpha U(\mathbf{r}) \\ \alpha U(-\mathbf{r}) & -2i\bar{\partial} \end{pmatrix} \begin{pmatrix} \psi_{K1} \\ \psi_{K2} \end{pmatrix} = 0$$

Generate new zero modes
- flat band?

?? $\begin{pmatrix} \psi_{k1} \\ \psi_{k2} \end{pmatrix} = f(z) \begin{pmatrix} \psi_{K1} \\ \psi_{K2} \end{pmatrix}$ **??**

Theory of Exactly Flat Band

$$\begin{pmatrix}\psi_{k1}\\ \psi_{k2}\end{pmatrix} = f(z)\begin{pmatrix}\psi_{K1}\\ \psi_{K2}\end{pmatrix}$$

another Zero mode? -
but needs correct periodicity

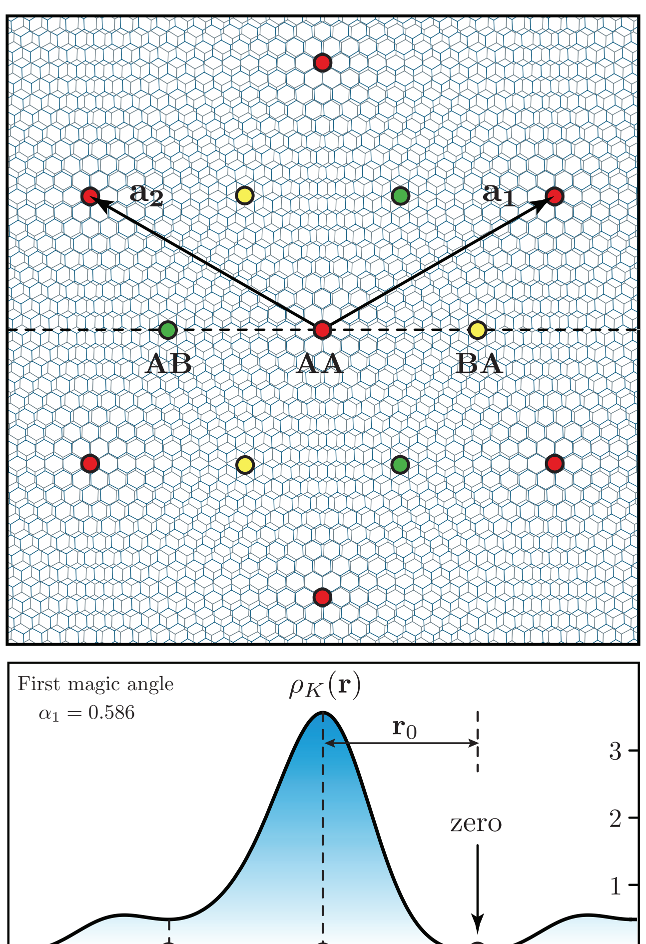
Theta functions

$$f(z) \sim \frac{\theta_{a',b'}(z|\tau)}{\theta_{a,b}(z|\tau)}$$

BUT requires wave-function zero
to cancel pole in denominator

At special (magic) angles,
the spinor wfn.
vanishes at **points** in the unit cell

$$\psi_{\boldsymbol{k}}(\boldsymbol{r}) = e^{2\pi i k_1 z'}\frac{\vartheta\left(z' - k_2 + \omega k_1|\omega\right)}{\vartheta\left(z'|\omega\right)}\psi_K(\boldsymbol{r})$$



Tarnopolski, Kruchkov, AV 1808.05250

Chiral Model

$$\mathcal{H} = \begin{array}{c} \begin{matrix} \quad A \quad & \quad B \quad \end{matrix} \\ \begin{pmatrix} 0 & \mathcal{D}^*(-\mathbf{r}) \\ \mathcal{D}(\mathbf{r}) & 0 \end{pmatrix} \end{array}, \quad \mathcal{D}(\mathbf{r}) = \begin{array}{cc} \begin{matrix} \quad U \quad & \quad D \quad \end{matrix} & \\ \begin{pmatrix} -2i\bar{\partial} & \alpha U(\mathbf{r}) \\ \alpha U(-\mathbf{r}) & -2i\bar{\partial} \end{pmatrix} & \begin{matrix} U \\ D \end{matrix} \end{array}$$

$$\bar{\partial} = \frac{1}{2}(\partial_x + i\partial_y)$$

$$U(\boldsymbol{r}) = e^{-i\boldsymbol{q}_1\boldsymbol{r}} + e^{i2\pi/3}e^{-i\boldsymbol{q}_2\boldsymbol{r}} + e^{-i2\pi/3}e^{-i\boldsymbol{q}_3\boldsymbol{r}}$$

$$\begin{pmatrix} 0 & \mathcal{D}^*(-r) \\ \mathcal{D}(r) & 0 \end{pmatrix} \begin{pmatrix} 0 \\ \psi_B(r) \end{pmatrix} = 0$$

Gives:
perfectly flat bands at a series of magic angles.

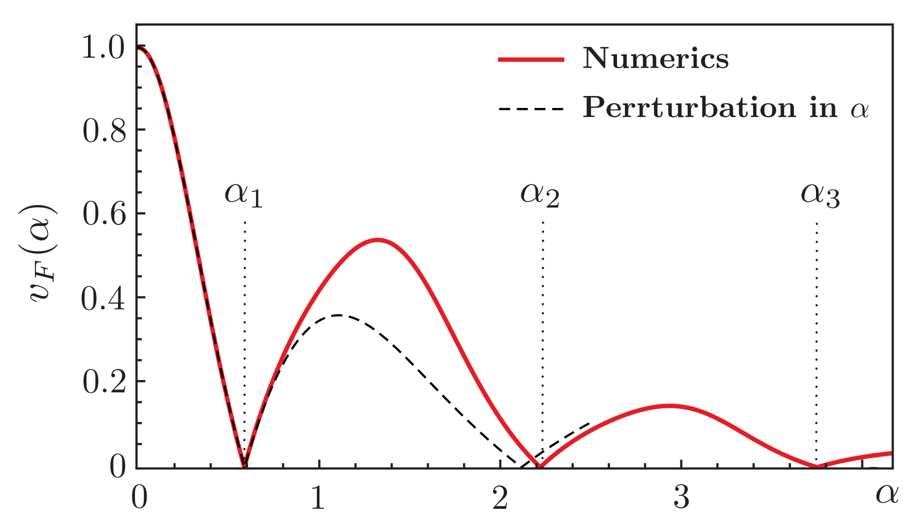
Perturbation Theory:

$$\psi_K(\mathbf{r}) = \begin{pmatrix} \psi_{K,1} \\ \psi_{K,2} \end{pmatrix} = \begin{pmatrix} 1 + \alpha^2 u_2 + \alpha^4 u_4 + \dots \\ \alpha u_1 + \alpha^3 u_3 + \dots \end{pmatrix}$$

$$u_1(r_0) = 0;$$

$$u_2(r_0) = |u_1(r_0)|^2 - 3$$

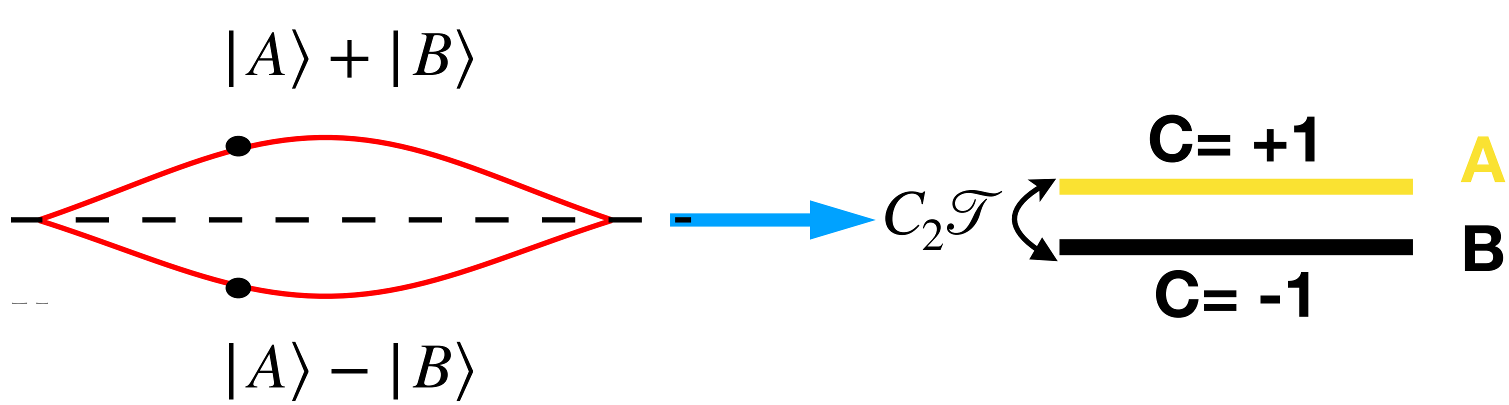
$$v_F(\alpha) = \frac{1 - 3\alpha^2 + \alpha^4 - \frac{111\alpha^6}{49} + \frac{143\alpha^8}{294} + \dots}{1 + 3\alpha^2 + 2\alpha^4 + \frac{6\alpha^6}{7} + \frac{107\alpha^8}{98} + \dots}$$



Tarnopolski, Kruchkov, AV PRL 2019

Chiral Model and Quantum Hall

- Exactly flat bands are eigenstates of chirality (live on a. Single sub lattice A or B)
- They have opposite Chern number
- Staggered Potential + SPONTANEOUS spin & valley polarization=> Chern Insulator

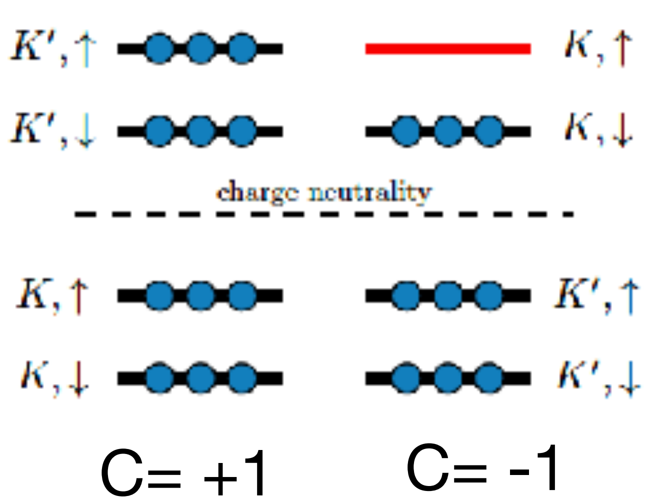


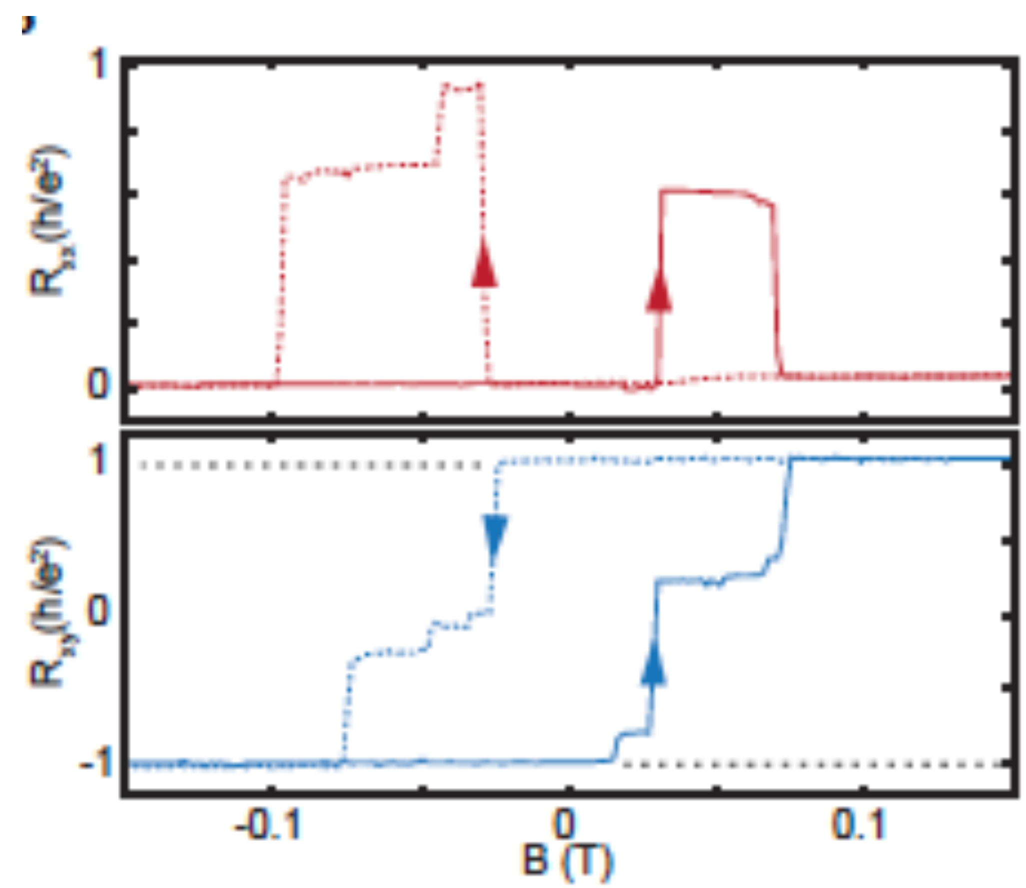
Valley K
Linear combination of bands

Chern =±1;
Sublattice polarized

Intrinsic quantized anomalous Hall effect in a moiré heterostructure

M. Serlin,[1,*] C. L. Tschirhart,[1,*] H. Polshyn,[1,*] Y. Zhang,[1] J. Zhu,[1] K. Watanabe,[2] T. Taniguchi,[2] L. Balents,[3] and A. F. Young[1,†]



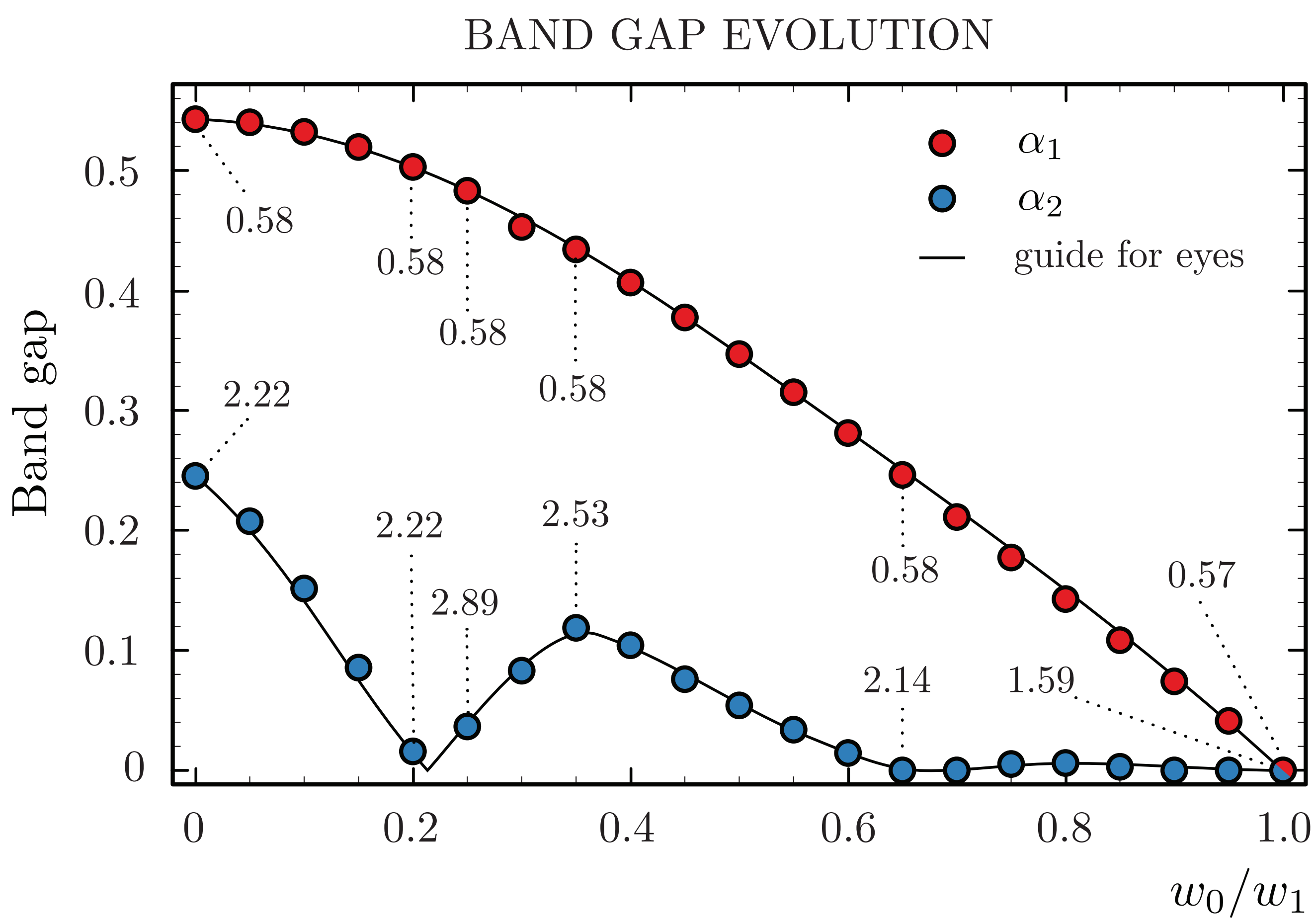


$\nu = 3$

Integer Chern Insulator

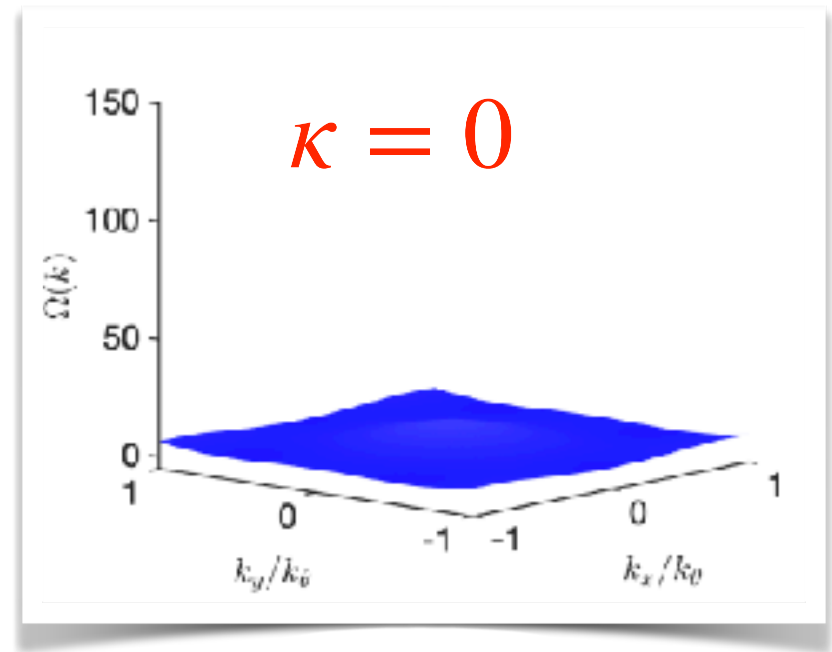
Real Magic Angle Graphene

CHIRAL: 0 < w0/w1 < 1 BM

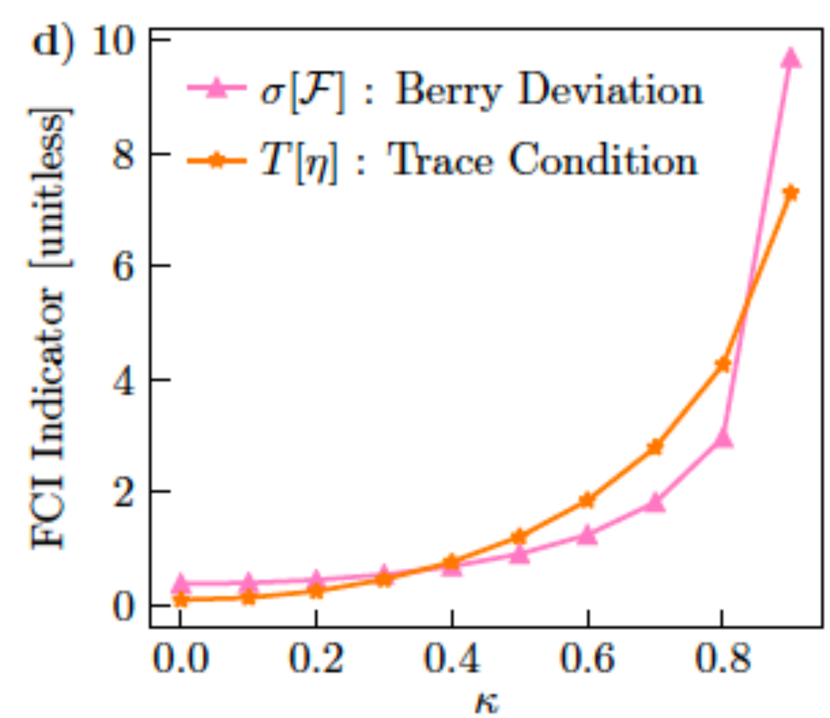


First magic angle
Connected to chiral limit.

Nearly uniform Berry curvature



Geometry

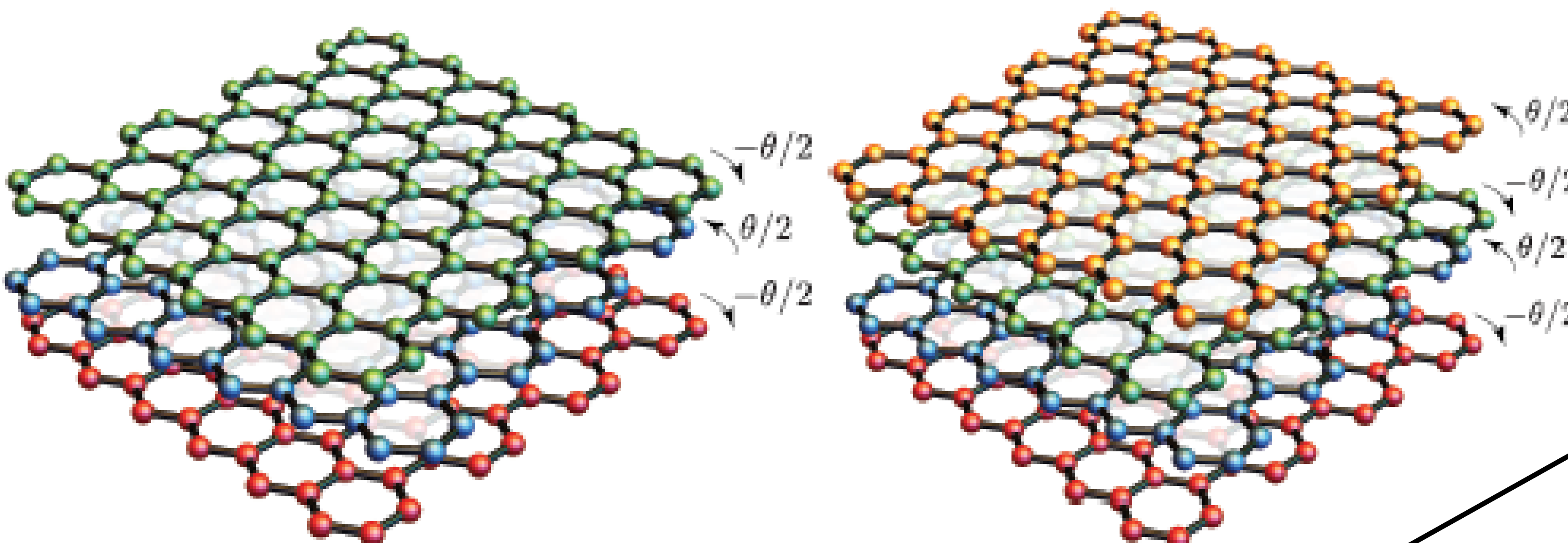


Tarnopolski, Kruchkov, AV 1808.05250

Alternating Twist Multilayers

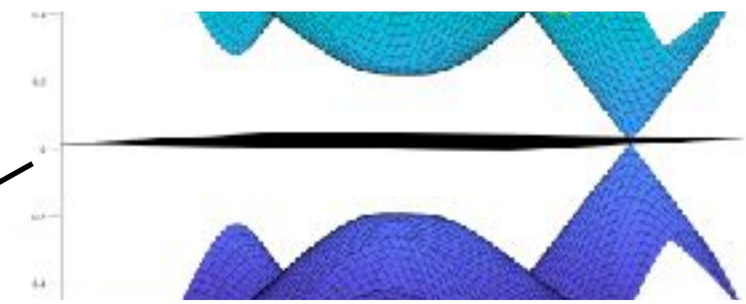
- Other systems with C2 symmetry

Alternating twist sandwich



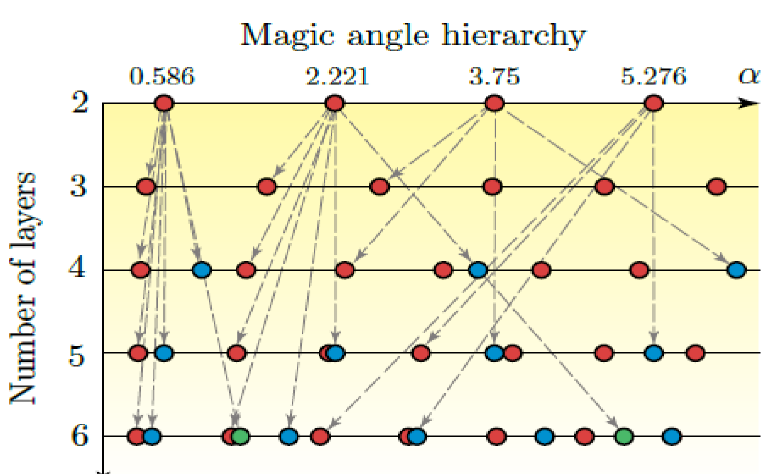
Eslam Khalaf

Magic angle = $\sqrt{2}\, 1.1^o \sim 1.55^o$



Flat band+Dirac

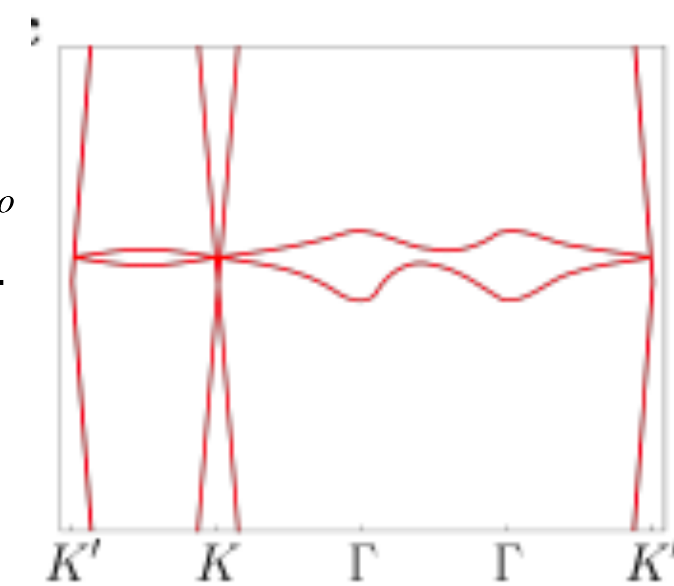
Carr et al. stability `19



$\sqrt{2}$

$$\varphi = \frac{1+\sqrt{5}}{2}$$

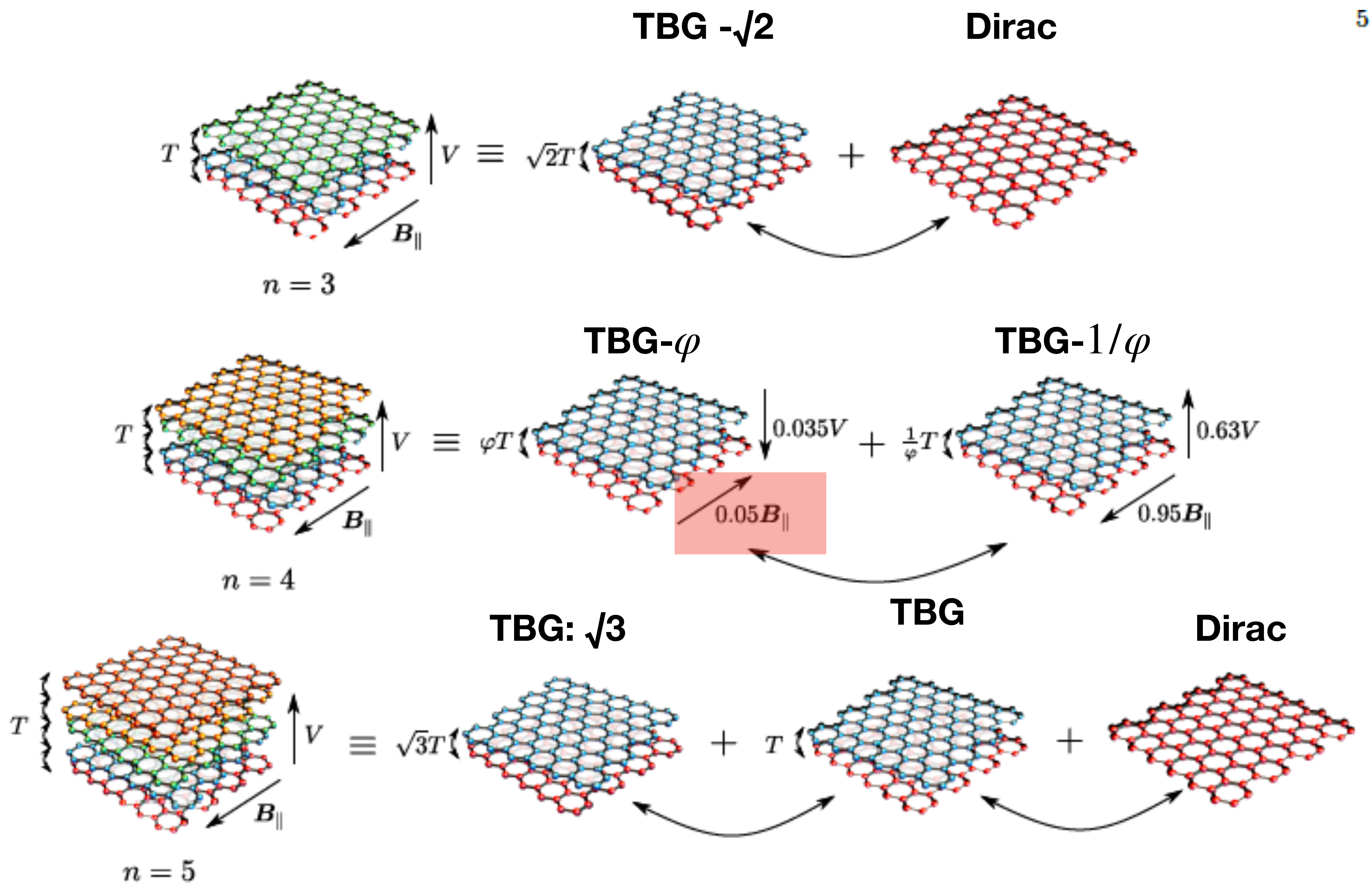
Magic angle = $\dfrac{1+\sqrt{5}}{2}\, 1.1^o \sim 1.78^o$



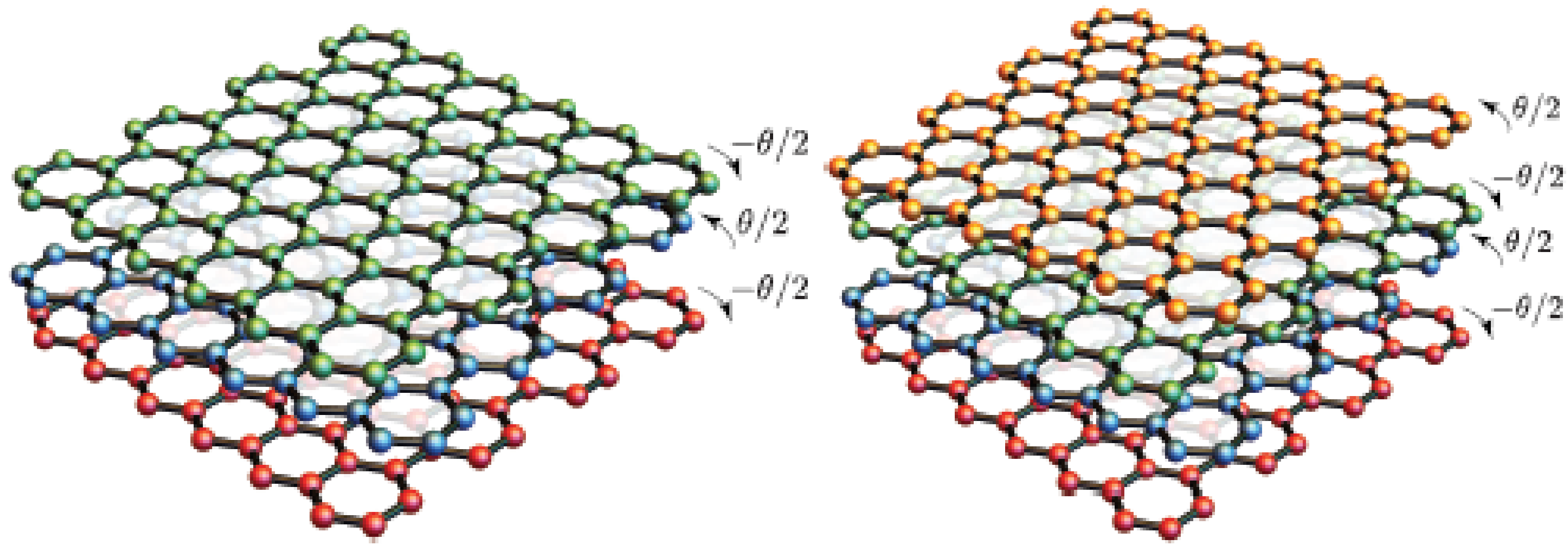
Khalaf, Kruchkov, Tarnopolsky, AV, 1901.10485

Larger magic angles!

Deconstructing n=3, 4, 5

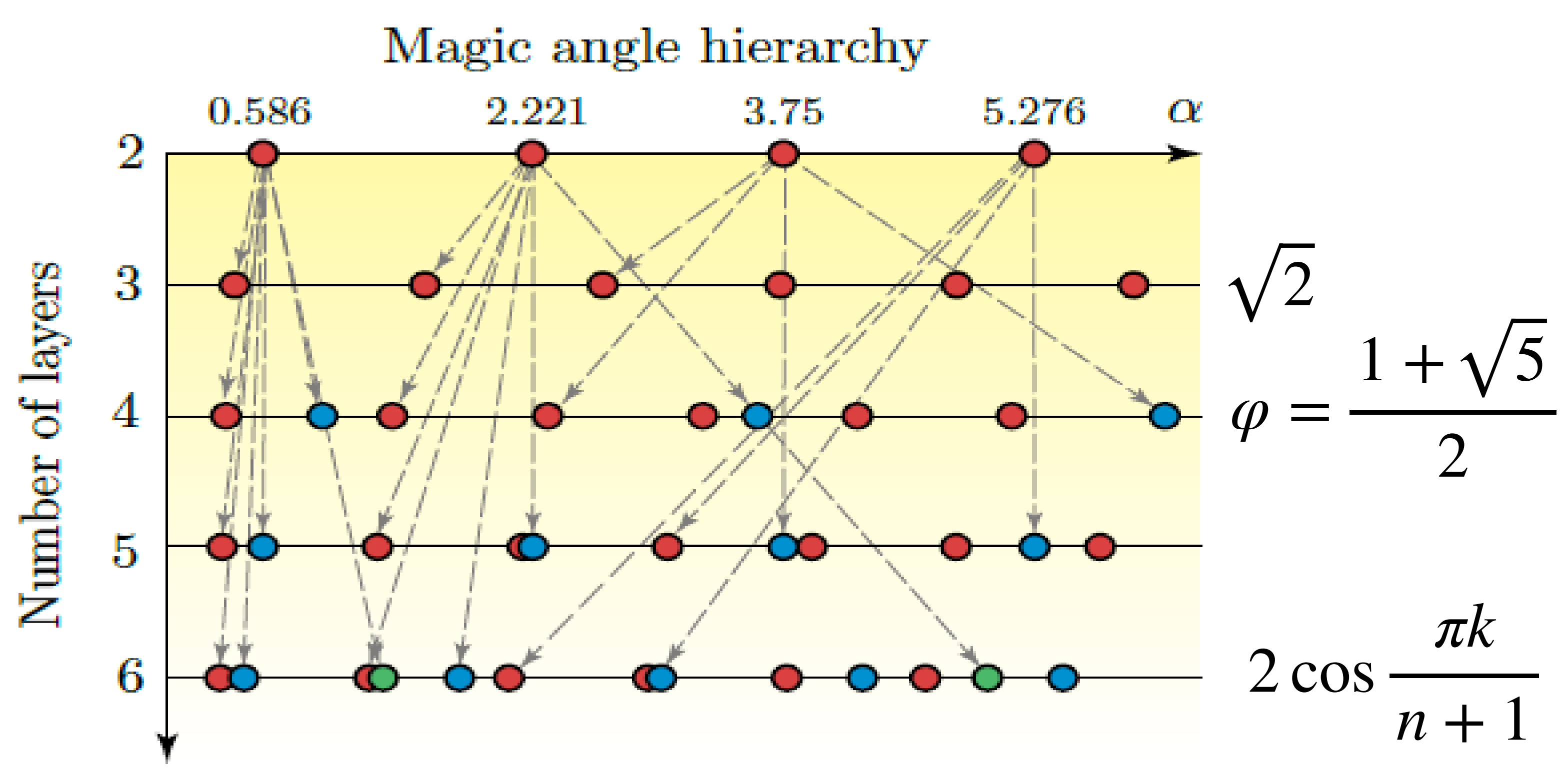


ASIDE: Alternating-twist multilayer graphene



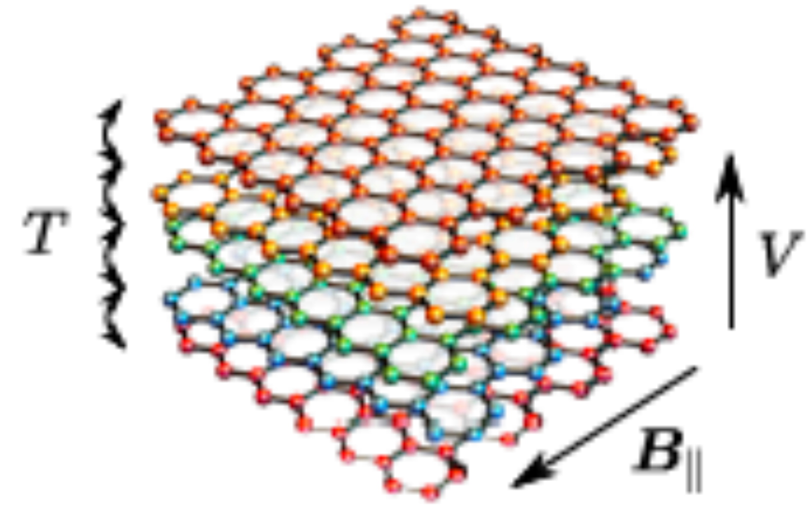
Eslam Khalaf

- Alternating twist:
  - Magic angles *simply* related to the bilayer case.



Khalaf, Kruchkov, Tarnopolsky, AV, 1901.10485

Displacement Field Effect



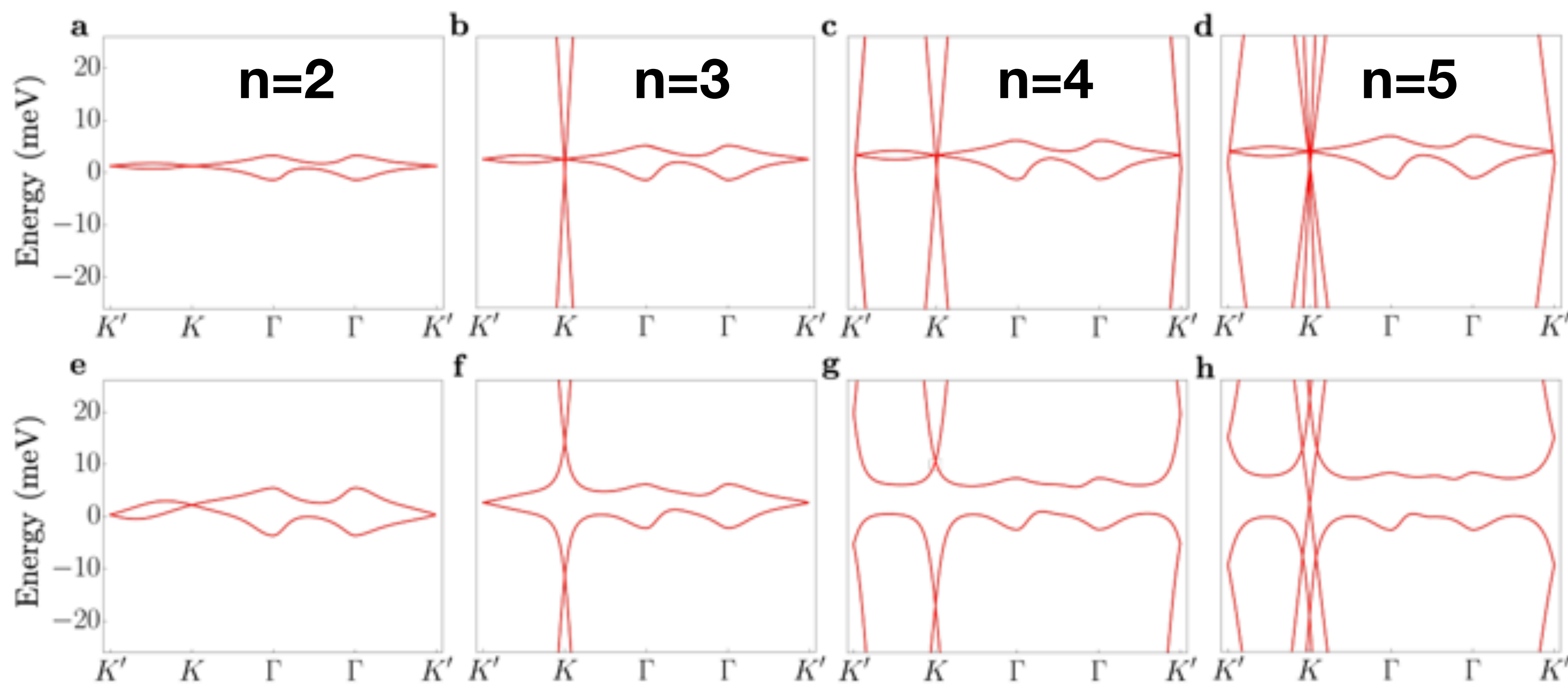
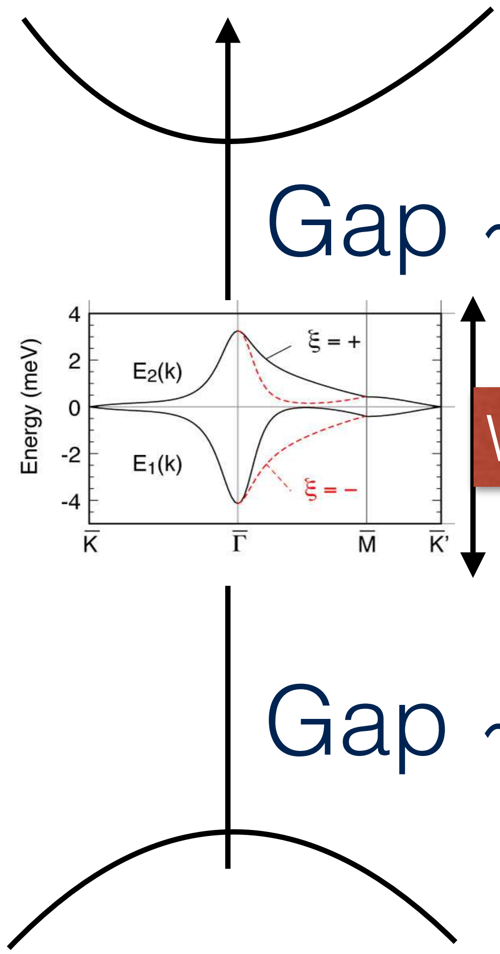


FIG. 2: ATMG band structures with and without displacement field: Panels **a-d** have no displacement field and correspond to $n = 2, 3, 4, 5$ respectively. Panels **e-h** have $V = 50$ meV and also correspond to $n = 2, 3, 4, 5$ respectively. We used the parameters $\kappa = 0.58$ and $\theta = \lambda_1 1.05°$ for $\lambda_1 = 1, \sqrt{2}, \varphi, \sqrt{3}$ for $n = 2, 3, 4, 5$ respectively. Note, $n = 2$ which corresponds to ordinary MATBG shows little change with displacement field (**a** vs. **e**). In contrast, a gap opens at $n = 3, 4, 5$ on applying moderate V (**b, c, d** vs. **f, g, h**).

OUTLINE

- Lecture 1 - Preliminaries, the chiral model, *wave functions,* from bilayer to n=3,4,5..
- Lecture 2 - Correlated Insulators - exact solutions, Hartree Fock, topology and σ model.
- Lecture 3 - Superconductivity - disordered σ model.
- Lecture 4 - Fractional Chern insulators in magic angle graphene

Correlation Effects in Twisted Bilayer Graphene

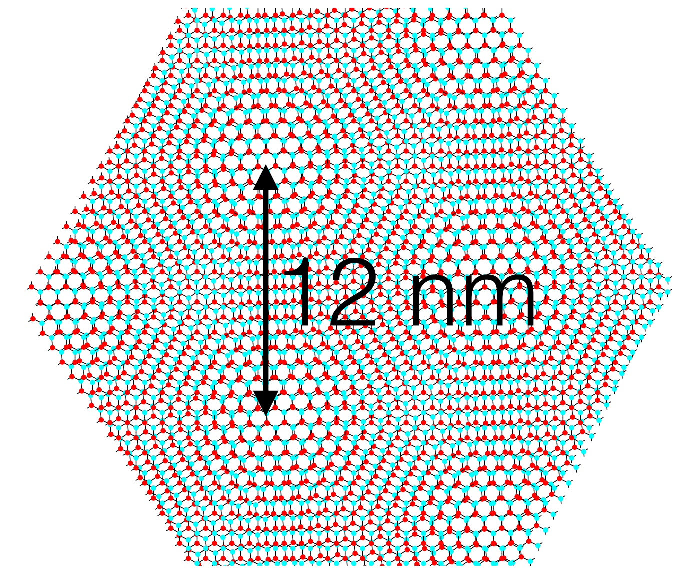


Gap ~ 30 meV

W_{bare} ~ 8 meV

Koshino et al

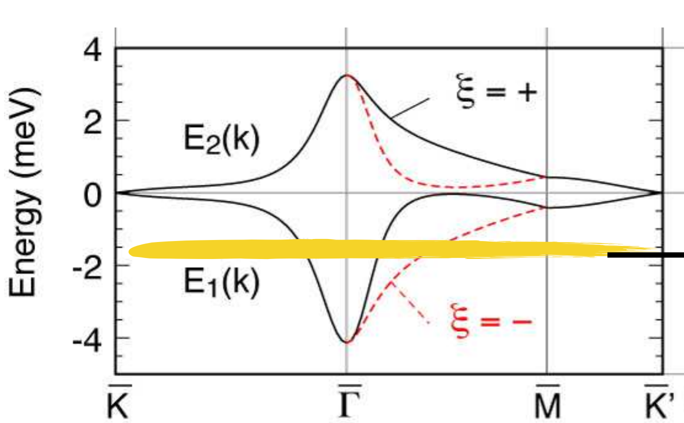
Gap ~ 30 meV



V~30meV

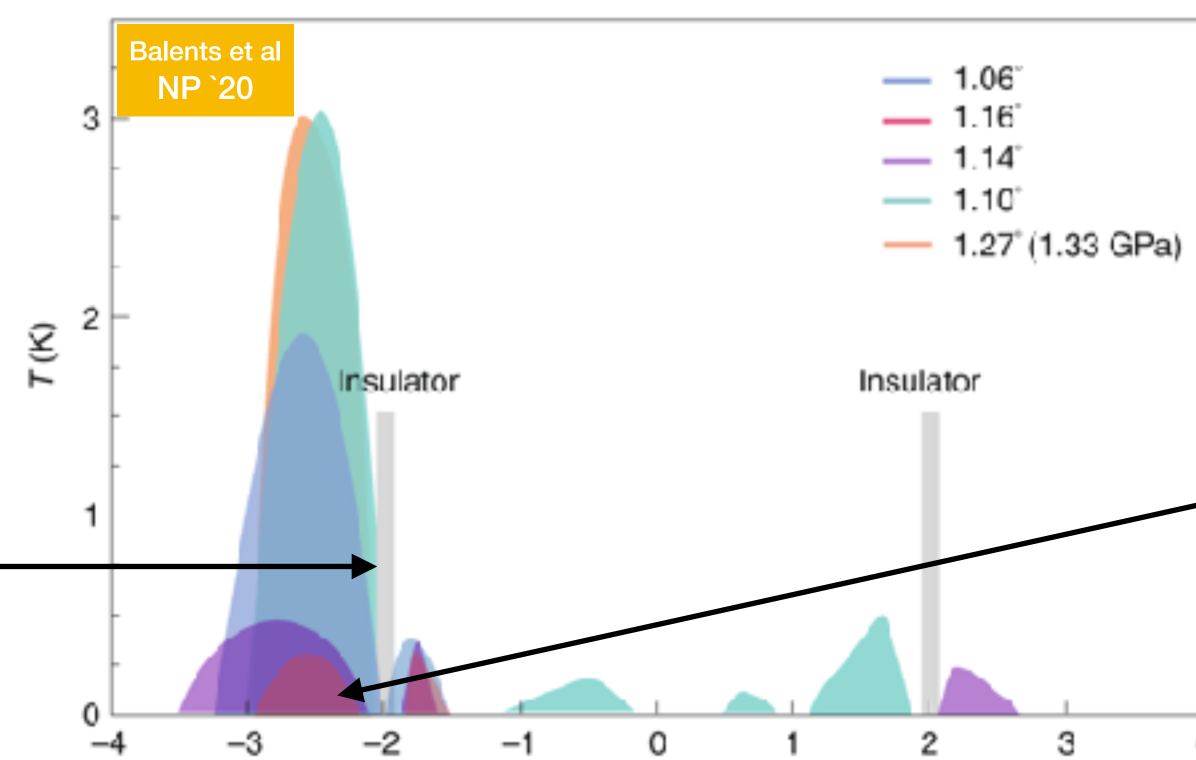
$\theta \sim 1/60$ radians

Electron Count

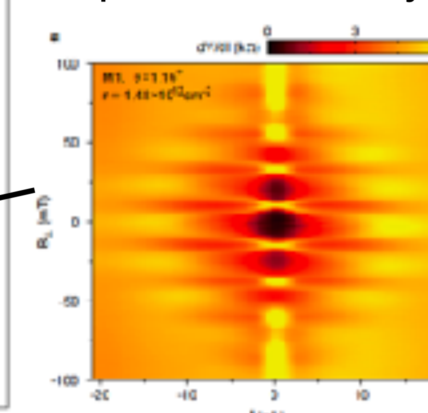


$\nu = +4$

$\nu = -4$



Superconductivity



Chiral Limit Wavefunctions

Many special properties:

(i) Chern number+ sub-lattice polarized

$$\psi_A(x,y) = \frac{\psi_K(x,y)}{\theta_1(\frac{z-z_0}{a_1}\,|\,\omega)} f(x+iy)$$

Apply C_2T symmetry - *same* valley *opposite* Chern number, *opposite* sublattice.

C= +1 A

$$H = H_0 + \sum V(x-x')\rho(x)\rho(x')$$

Prefers band basis

Prefers polarizing electrons, *sublattice basis*

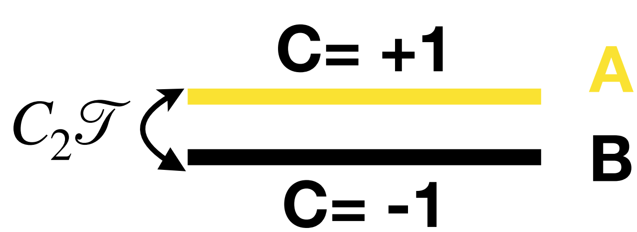
Chiral Limit Wavefunctions

Many special properties:

(i) Chern number+ sub-lattice polarized

$$\psi_A(x,y) = \frac{\psi_K(x,y)}{\theta_1(\frac{z-z_0}{a_1}\,|\,\omega)} f(x+iy)$$

Apply C_2T symmetry - *same* valley *opposite* Chern number, *opposite* sublattice.



Valley K

Chern =±1;

Sublattice polarized

Valley K

$$\rho(x) = \sum_{a=A,B} c_a^\dagger(x) c_a(x)$$

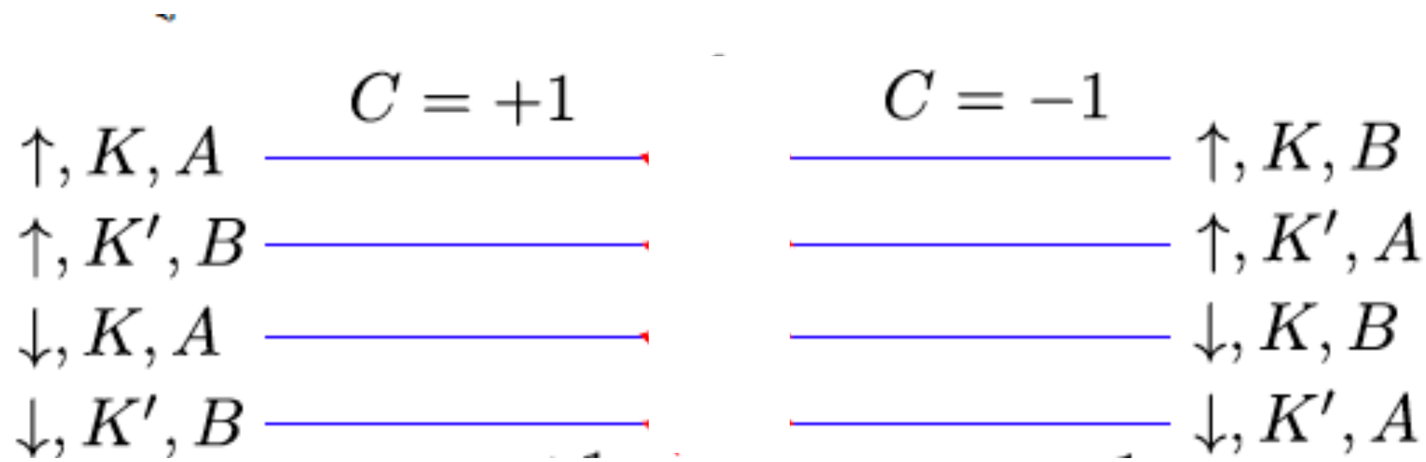
Wfns.

$$c_a^\dagger(x) = u_a^{(+)}(x) c_+^\dagger + u_a^{(-)}(x) c_-^\dagger$$

Sublattice polarized:

$$\rho = \rho_{C=+1} + \rho_{C=-1}$$

Interactions



$$\mathcal{H} = \sum_{\boldsymbol{k}} c^{\dagger}_{\boldsymbol{k}} h_{\boldsymbol{k}} c_{\boldsymbol{k}} + \frac{1}{2A} \sum_{\boldsymbol{q}} V_{\boldsymbol{q}} \delta\rho_{\boldsymbol{q}} \delta\rho_{-\boldsymbol{q}}, \quad \delta\rho_{\boldsymbol{q}} = \rho_{\boldsymbol{q}} - \bar{\rho}_{\boldsymbol{q}},$$

$$\rho(q) = \sum_{k \in BZ} c^{\dagger}_{k} \Lambda_{q}(k) c_{k+q}$$

Screened Coulomb

Form factor - wave functions of flat bands -plays a key role

PT

Kang, Vafek PRL 2019: SU(4)

Bultnick, Khalaf et al. arXiv:1911.02045

Flavor Ordered Insulator

Ground State and Hidden Symmetry of Magic Angle Graphene at Even Integer Filling

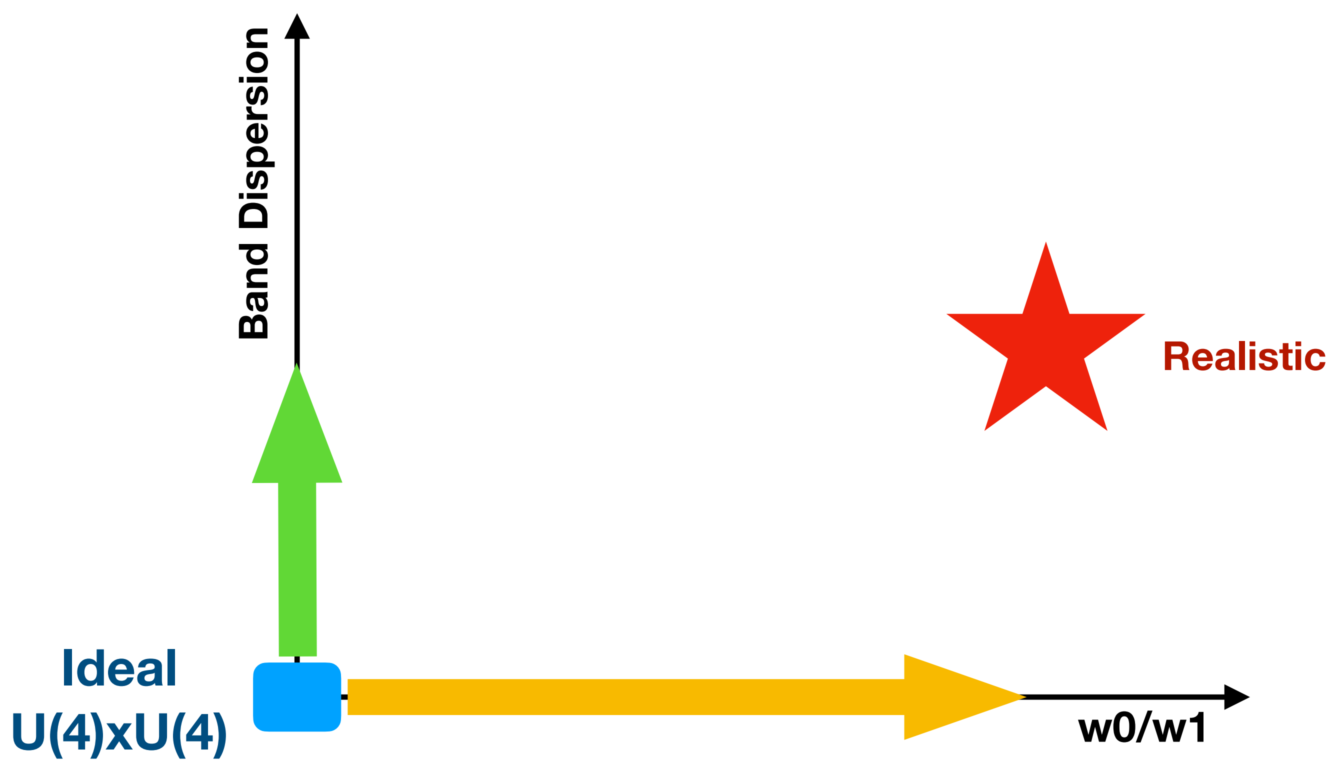
Nick Bultinck,[1,*] Eslam Khalaf,[2,*] Shang Liu,[2] Shubhayu Chatterjee,[1] Ashvin Vishwanath,[2] and Michael P. Zaletel[1,†]

[1]*Department of Physics, University of California, Berkeley, CA 94720, USA*

[2]*Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA*

arXiv:1911.02045 (PRX 2020)

Correlated Insulators - Ideal Limit



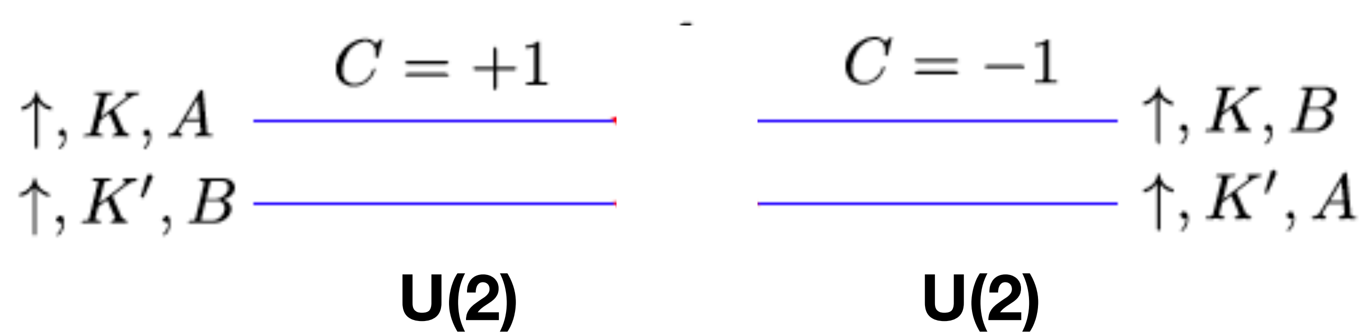
Term	Sublattice-diagonal Int.	Dispersion	Sublattice-off-diagonal Int.
Energy	E_{Coulomb}	$0.2-0.3E_{\text{Coulomb}}$	$0.2-0.3E_{\text{Coulomb}}$
Symmetry	$\mathrm{U}(4)\times\mathrm{U}(4)$	$\mathrm{U}_R(4)$	$\mathrm{U}(4)_R$

Density:

$$\rho \approx \rho_{C=+1} + \rho_{C=-1}$$

Simplified Model - Spinless TBG

$$\mathcal{H}_{\text{int}} = \frac{1}{2A} \sum_{q} V_{q} \delta\rho_{q} \delta\rho_{-q},$$



No Dispersion & Chiral limit:

Density: $\rho \approx \rho_{C=+1} + \rho_{C=-1}$

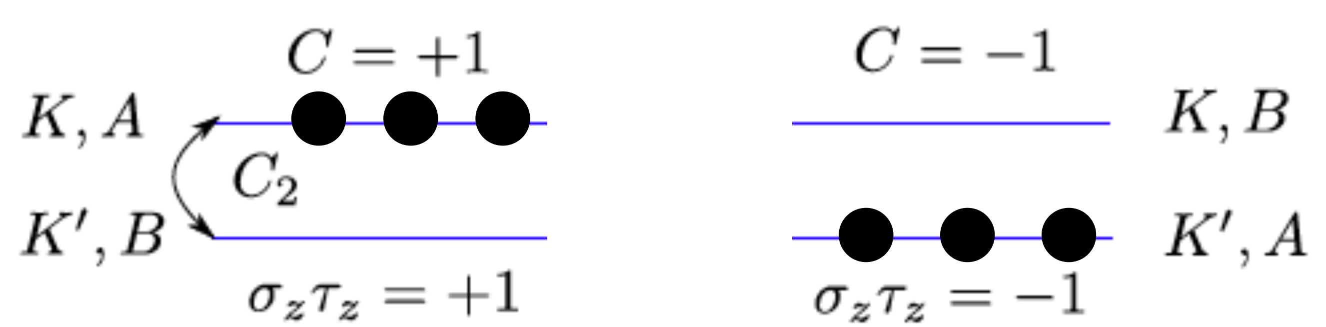
(Spinless Model) **Or** $\nu = \pm 2$

- Family of exact ground states - generalized ferromagnets. Fill Chern Bands.

Argument:

$V_q \geq 0$ and

$\delta\rho_q |\Psi\rangle = 0$



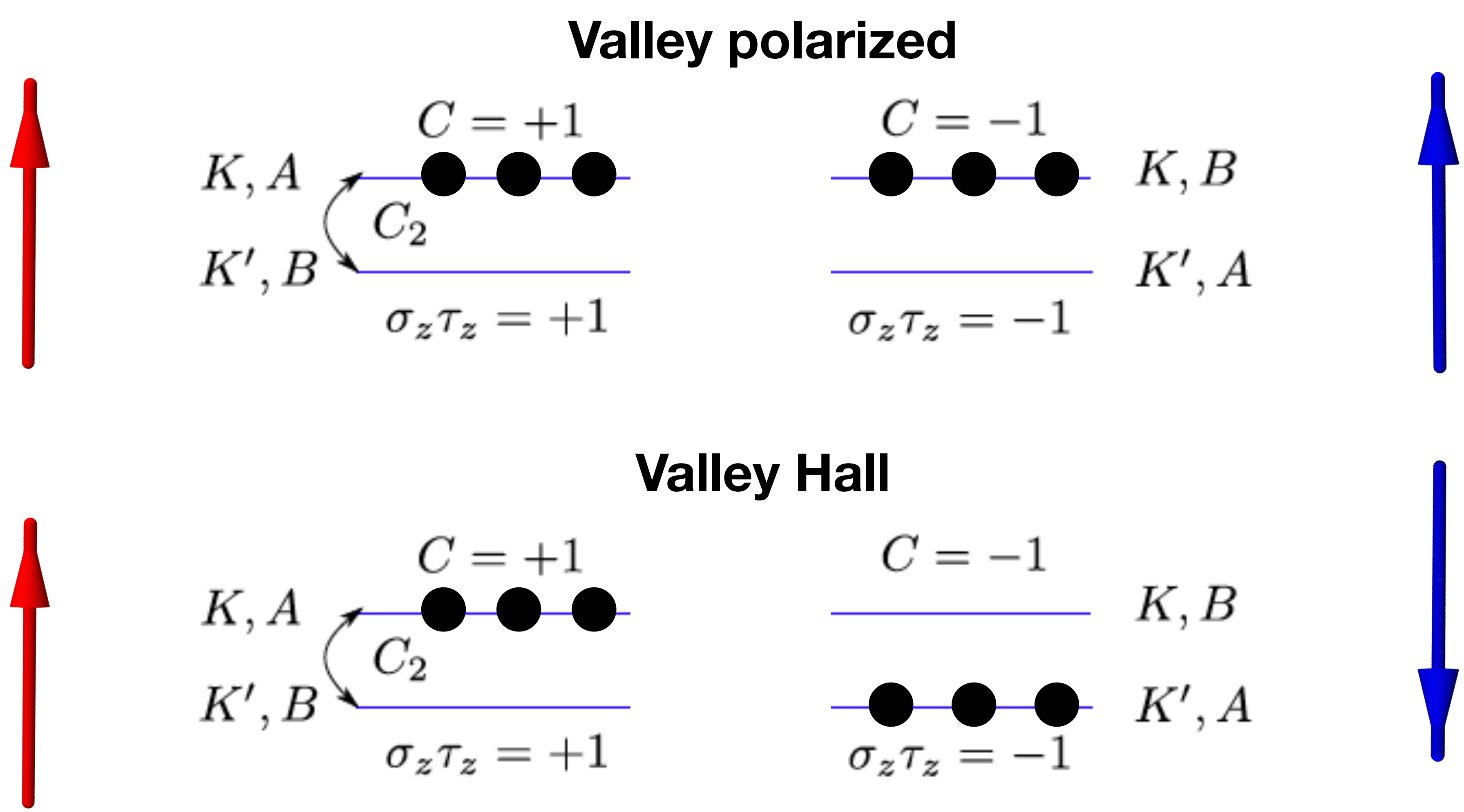
Ground States of Ideal Model

$$Q_+ = 1 - |z_+\rangle\langle z_+| = \sigma \cdot \hat{n}_+$$

$$Q_- = 1 - |z_-\rangle\langle z_-| = \sigma \cdot \hat{n}_-$$

Psuedo-Spin

Psuedo-Spin



$$\hat{n}_+ = \hat{n}_- = -\hat{z}$$

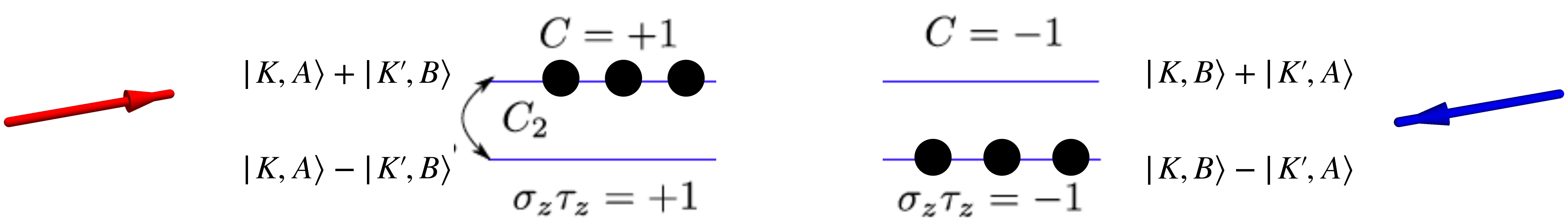
$$\hat{n}_+ = -\hat{n}_- = -\hat{z}$$

Ground States of Ideal Model

- Intervalley coherent (IVC) states break valley U(1): translation symmetry at graphene scale.

Psuedo-Spin

Psuedo-Spin



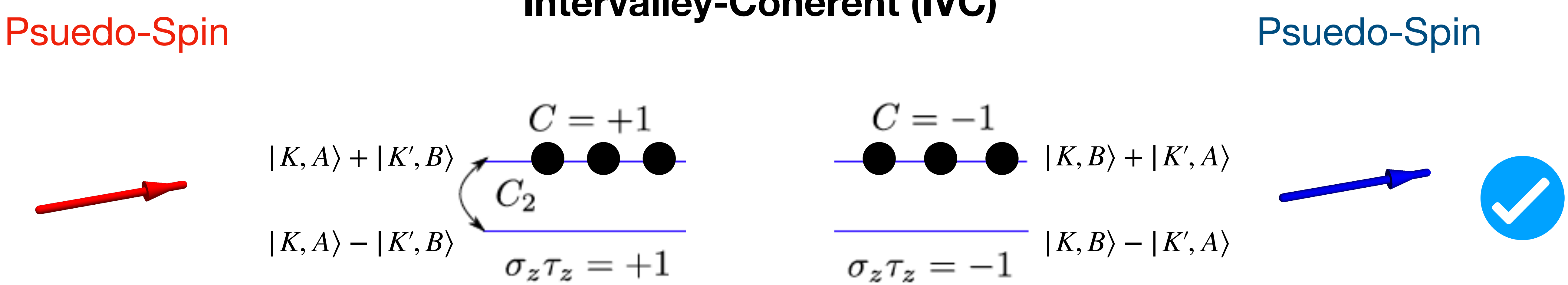
$$\hat{n}_+ = \hat{n}_- = -\hat{x}$$

K-IVC

Ground State of Chiral Model-Generalized Ferromagnet

- Family of exact ground states - Intervalley coherent states break valley U(1): translation symmetry at graphene scale.

Intervalley-Coherent (IVC)



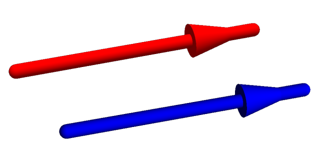
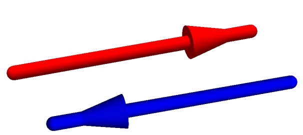
***T*-IVC**

Ground State of Chiral Model-Generalized Ferromagnet

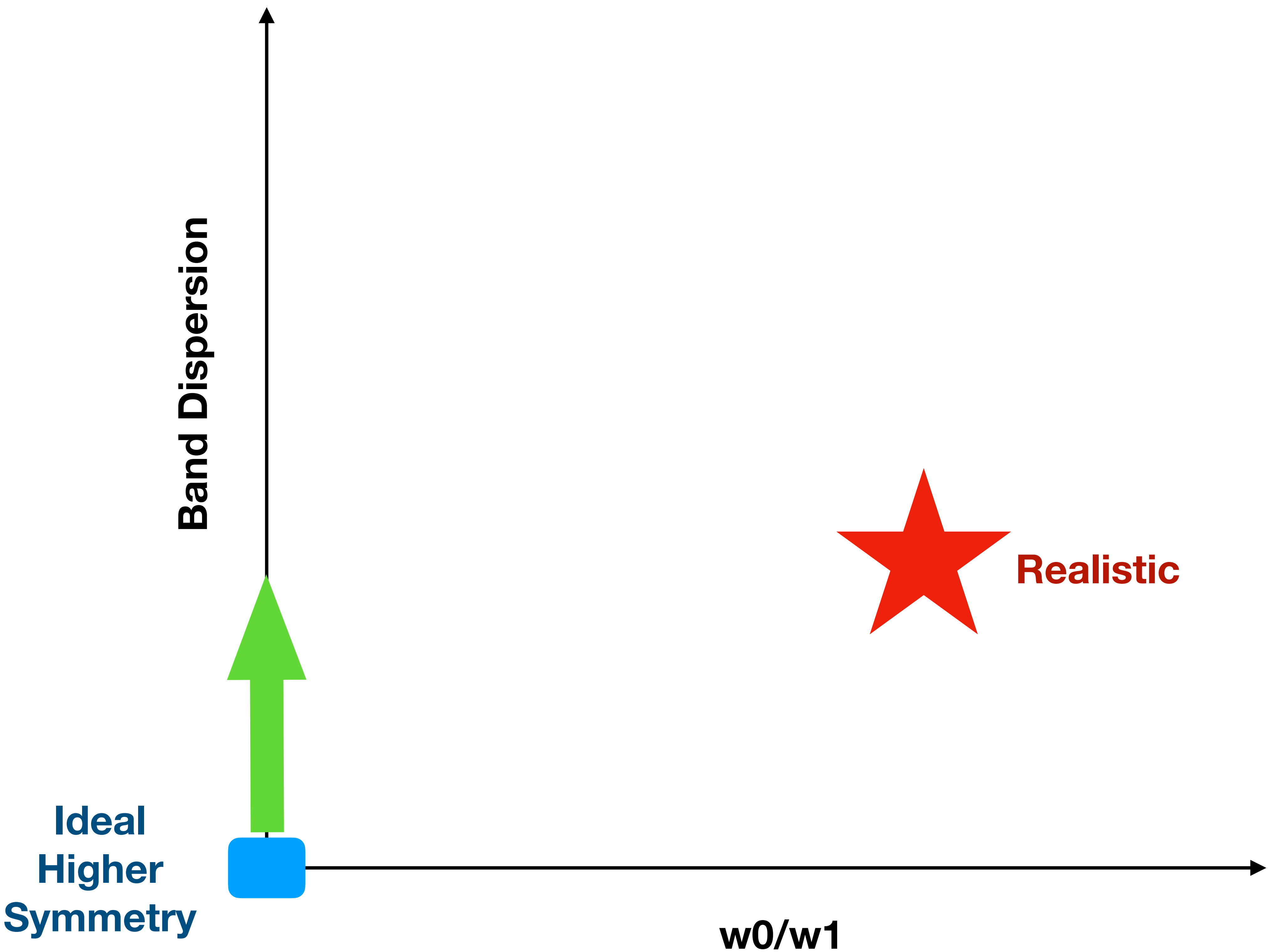
Sublattice (A/B) σ^z

Valley (K/K') τ^z

Some IVCs allowed and others ruled out

	CDW	$Q = \Delta_R \tau_x + \Delta_I \tau_y$	✗
	**T-IVC**	$Q = \sigma_x \left(\Delta_R \tau_x + \Delta_I \tau_y \right)$	✓
	**K-IVC**	$Q = \sigma_y \left(\Delta_R \tau_x + \Delta_I \tau_y \right)$	✓

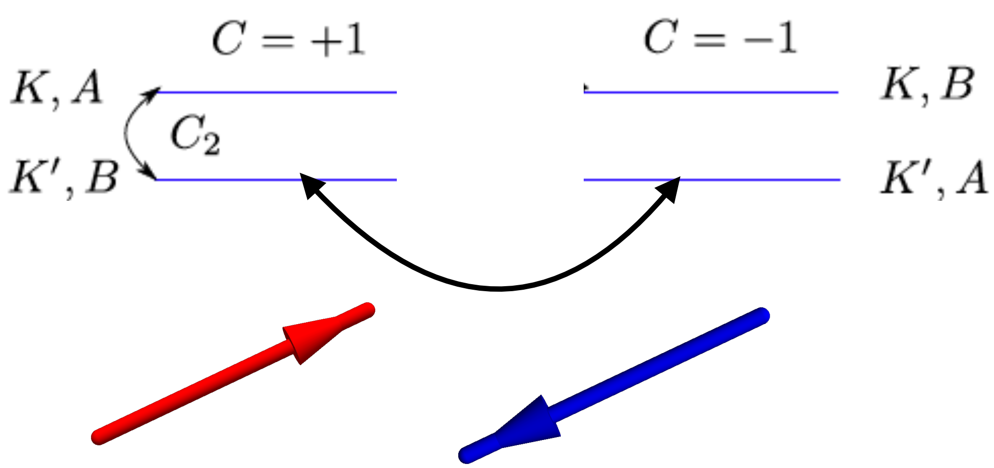
Including Interactions



Breaking the Degeneracy

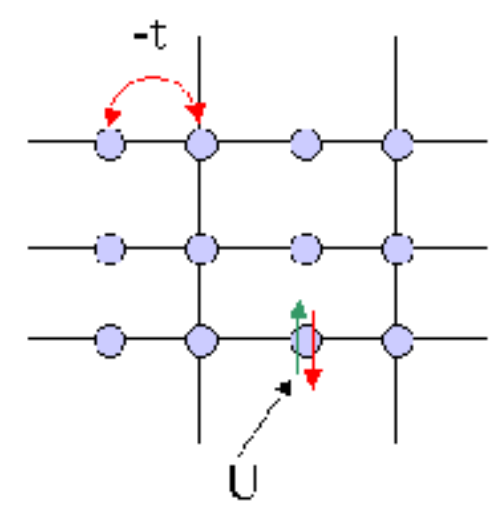
Dispersion:

Favors states that can fluctuate.



***Antiferro*-psuedospin coupling**

$\color{red}{J}\mathbf{n}_+ \cdot \mathbf{n}_-$ **J~t²/V~1-2 meV**



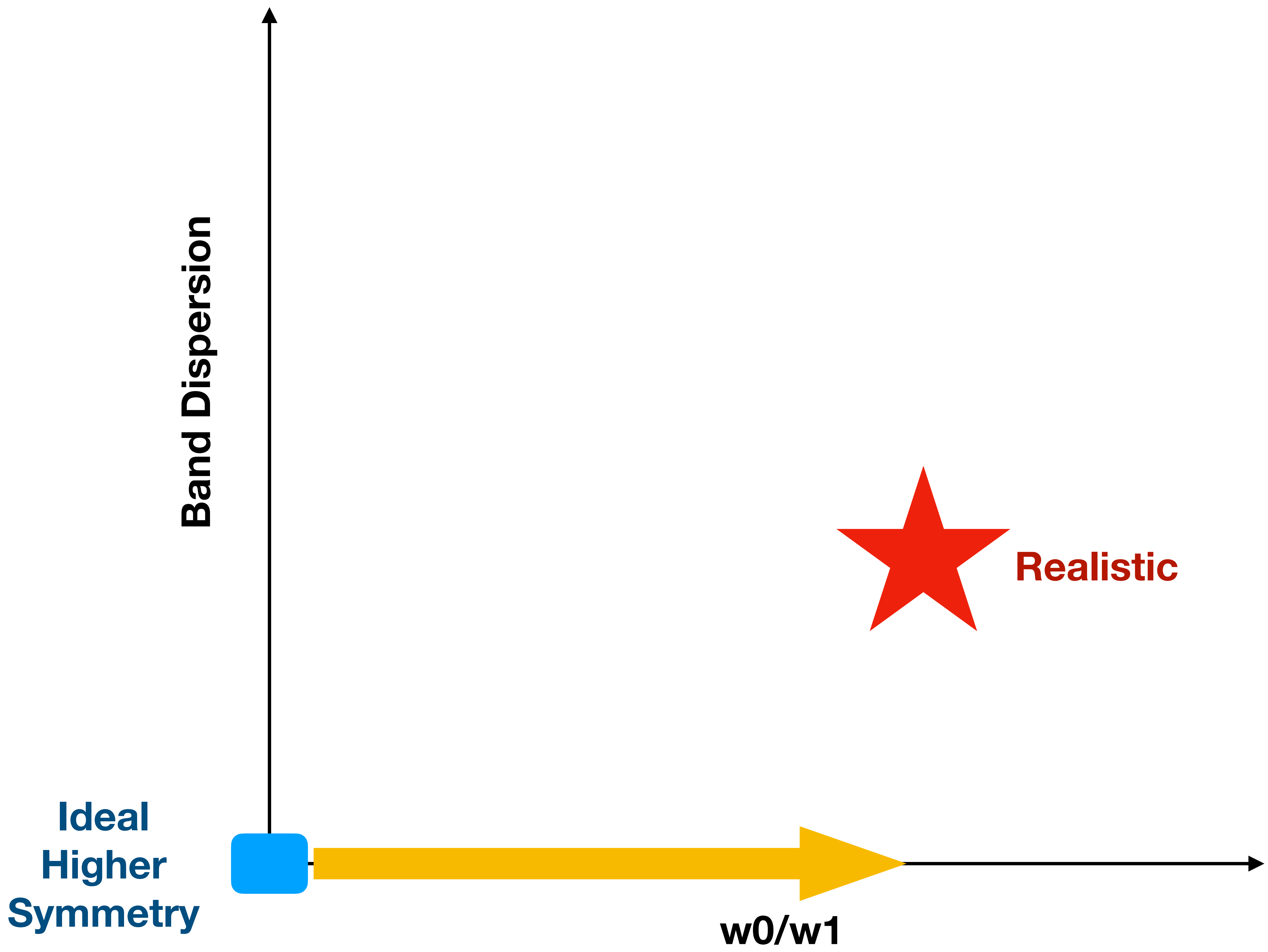
J~ 4t²/U

Superexchange

Valley Hall $Q = \sigma_z$

K-IVC $Q = \sigma_y\left(\Delta_R\tau_x + \Delta_I\tau_y\right)$

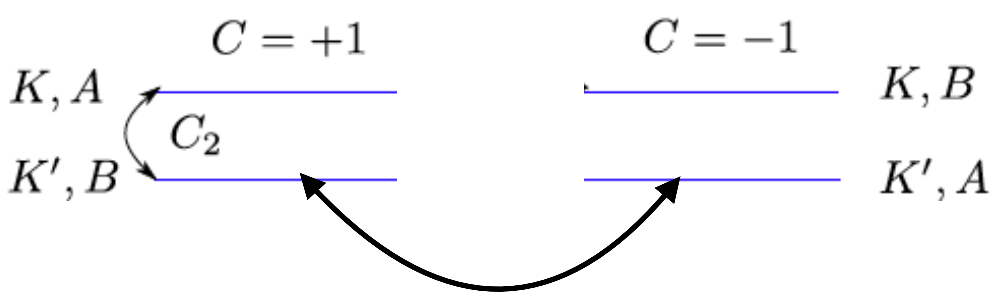
Breaking the Degeneracy



Breaking the Degeneracy

Dispersion:

Favors states that can fluctuate.



Away from Chiral Limit:
(Retains a different U(2) symmetry)

Valley Hall $Q = \sigma_z$

K-IVC $Q = \sigma_y \left(\Delta_R \tau_x + \Delta_I \tau_y \right)$

Valley polarized $Q = \tau_z$

K-IVC $Q = \sigma_y \left(\Delta_R \tau_x + \Delta_I \tau_y \right)$

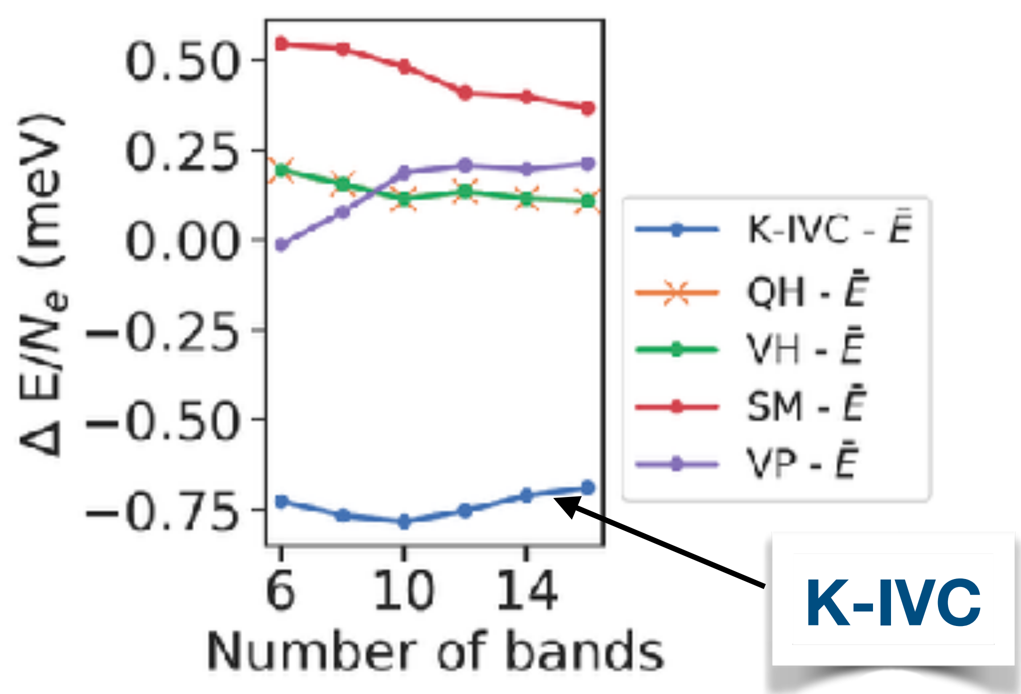
Ground State - Kramers IVC

Both perturbations pick the same state
Unfrustrated -

K-IVC

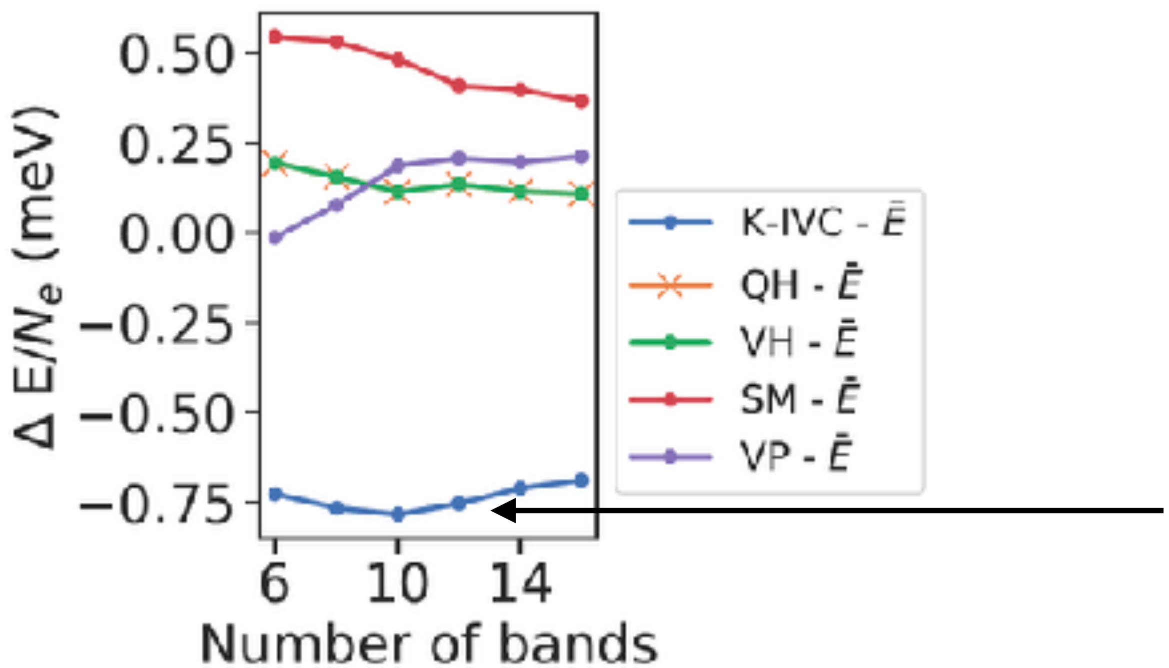
$$Q = \sigma_y \left(\Delta_R \tau_x + \Delta_I \tau_y \right)$$

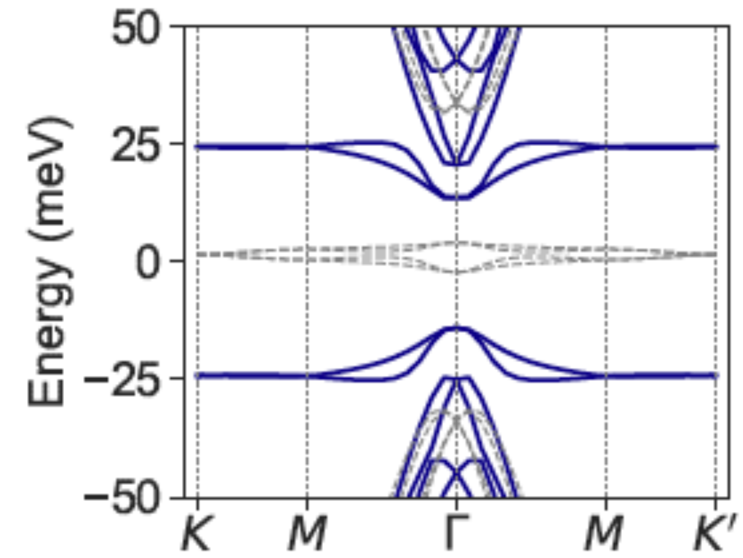
Hartree Fock Numerics-Confirms This Picture



$\theta = 1.05^{o}\, w_0 = 80\text{meV};\ w_1 = 110\text{meV};\ \epsilon = 7$

Hartree Fock & DMRG Numerics -Confirms This Picture

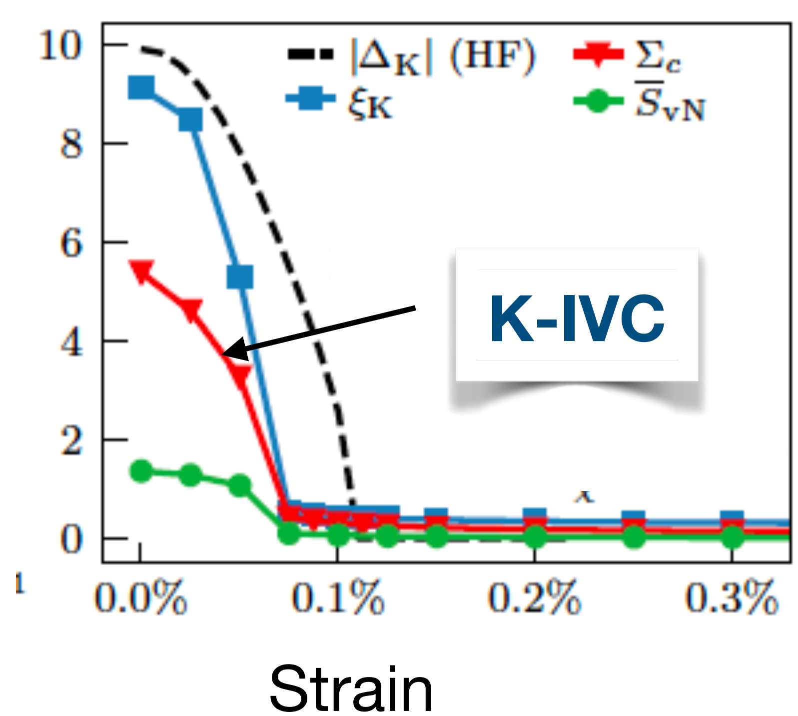


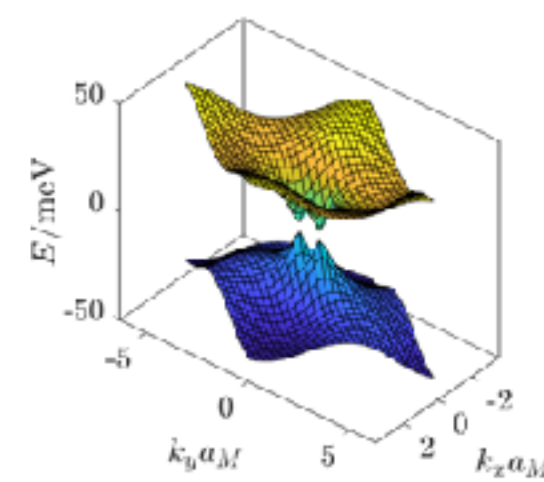


$\theta = 1.05^{o}\; w_0 = 80\text{meV};\; w_1 = 110\text{meV};\; \epsilon = 7$

Bultnick et al. arXiv:1911.02045
Kang and Vafek `19

$$Q = \sigma_y \left(\Delta_R \tau_x + \Delta_I \tau_y \right)$$





Competing state - *nematic* semimetal.

Shang Liu et al. arXiv:1905.07409
Kang& Vafek `19
Parker et al. arXiv:2012.09885

Kramers IVC - Properties

U(1)$_{\text{valley}}$ spontaneously broken - Goldstone modes (lattice translation)

K-IVC

$$Q = \sigma_y \left(\Delta_R \tau_x + \Delta_I \tau_y \right)$$

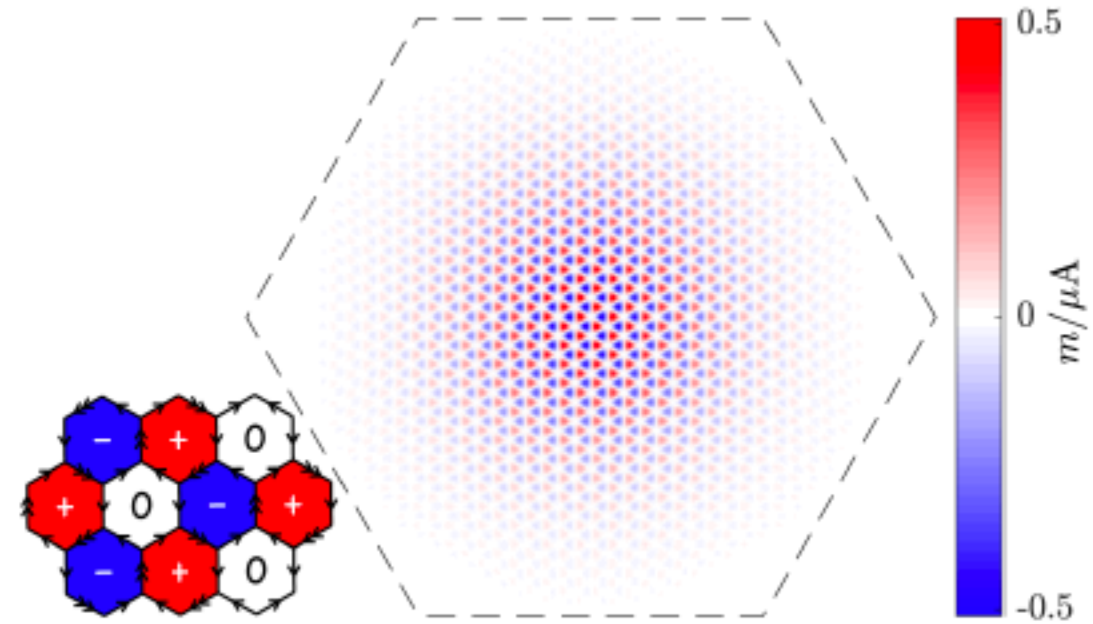
- Spontaneously breaks $\mathcal{T}$, but preserves a combination of $\mathcal{T}$ and π valley rotation $\mathcal{T}' = \tau_z \mathcal{T} = \tau_y \mathcal{K}$.

$$\mathcal{T}'^2 = -1$$

Same symmetries as topological insulator

Involves opposite Chern number band

Nontrivial Z_2 topology!

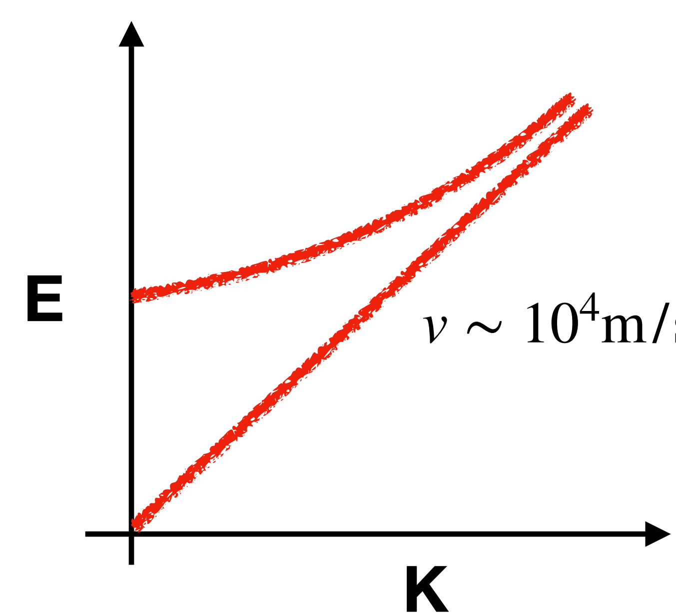


Current pattern*

“Spontaneous” topological insulator

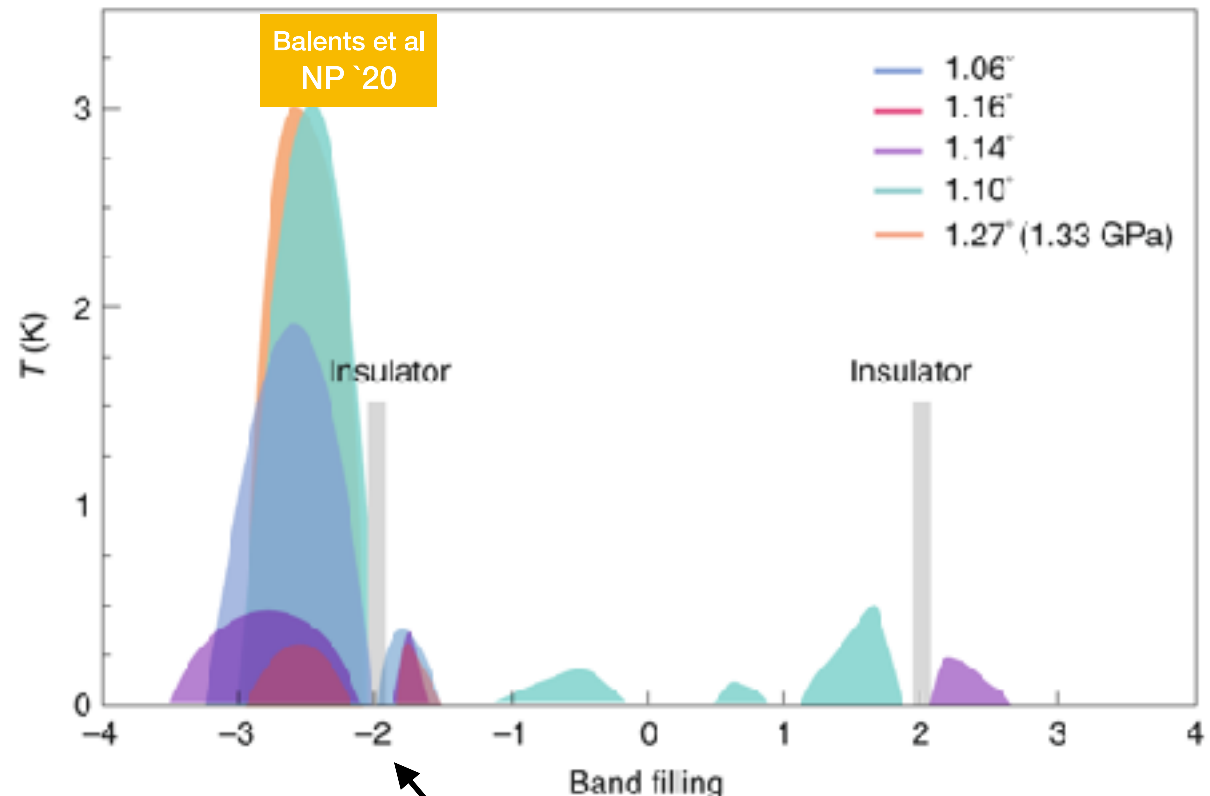
(Topological “Mott” Insulator - Raghu, Qi, Honerkamp, Zhang)

Edge states? Requires `smooth’ edge *or* spin-valley locking

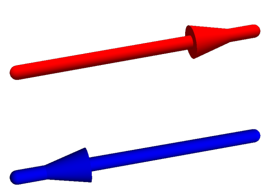


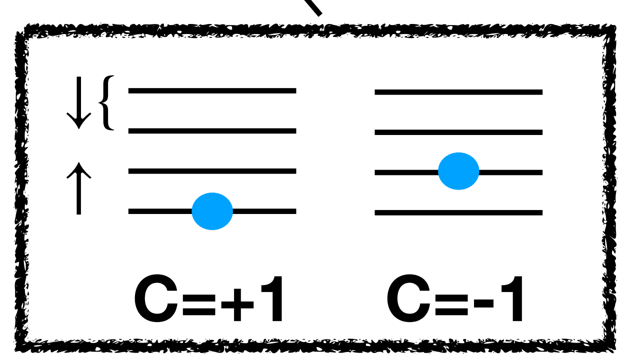
*Chakravarty, Laughlin, Morr, Nayak. Zhu, Aji, Varma.

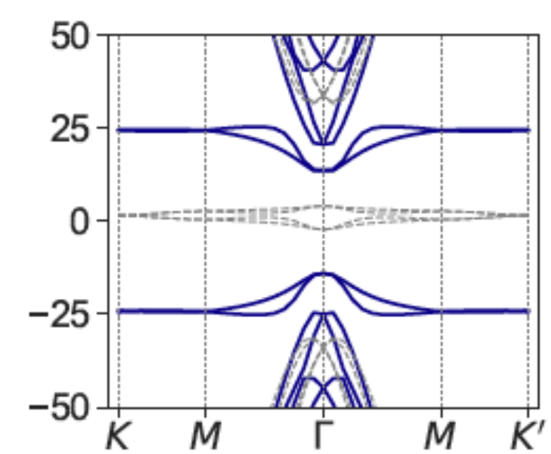
Flat Band Topology and Flavor Magnetism



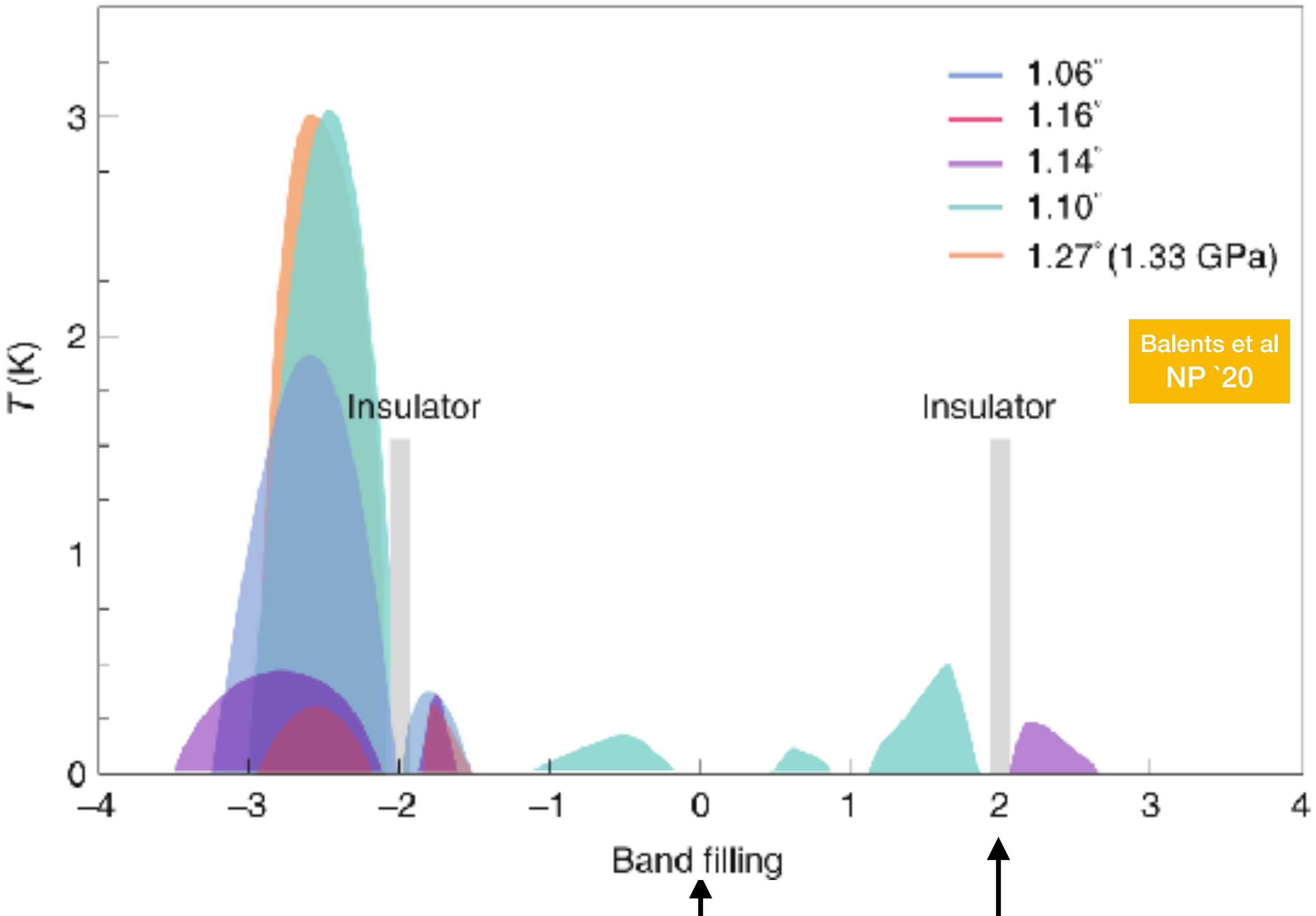
$$\hat{n}_+ = -\hat{n}_-$$

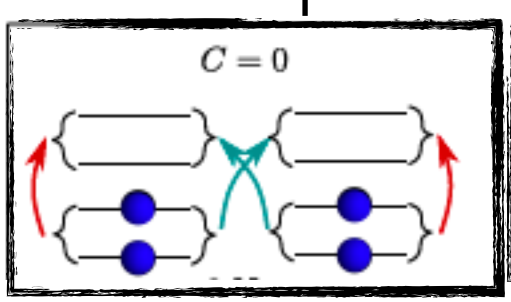


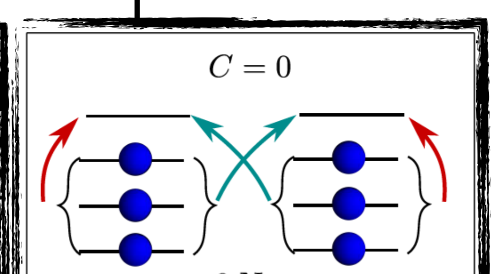


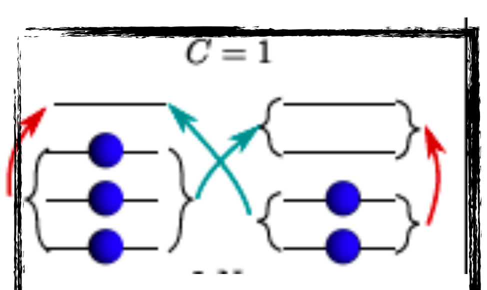


Generalized Flavor Ferromagnets









Generalized sigma model

$$Q_+ = 4 - \sum |z^+_{filled}\rangle\langle z^+_{filled}|$$

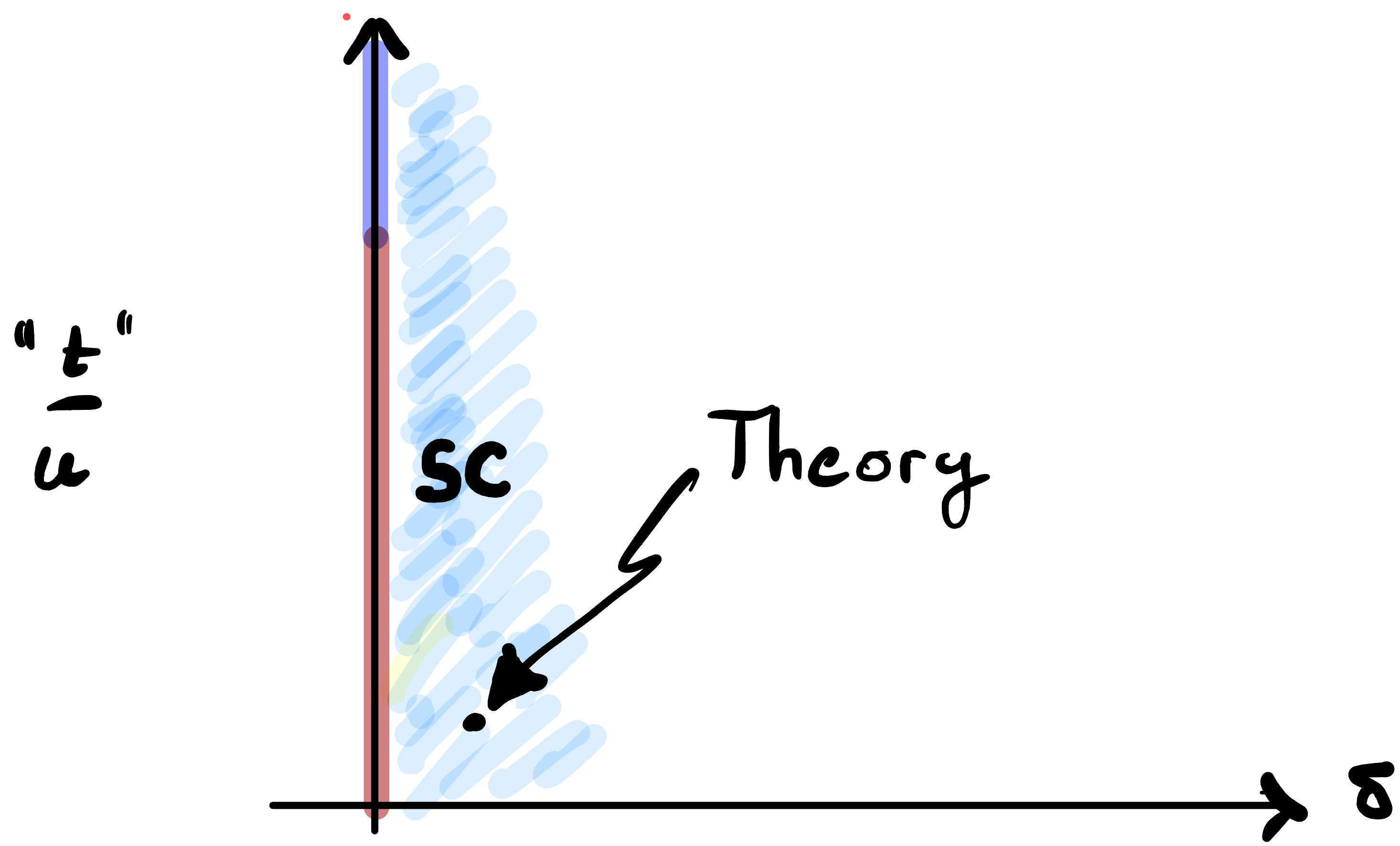
$$Q_- = 4 - \sum |z^-_{filled}\rangle\langle z^-_{filled}|$$

Eslam Khalaf, Bultinck, *AV*, Zaletel arxiv:2009.14827
Kang and Vafek arXiv:2009.09413
Kumar, Xie, MacDonald arXiv:2010.05946
Bernevig, Lian, Cowsik; Xie, Regnault, Song arXiv:2009.14200

OUTLINE

- Lecture 1 - Preliminaries, the chiral model, *wave functions,* from bilayer to n=3,4,5..
- Lecture 2 - Correlated Insulators - exact solutions, Hartree Fock, topology and σ model.
- Lecture 3 - Superconductivity - disordered σ model.
- Lecture 4 - Fractional Chern insulators in magic angle graphene

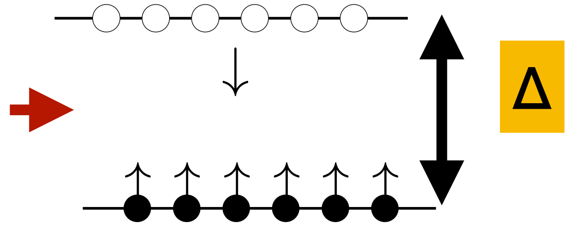
Strong Coupling Approach to Superconductivity in TBG



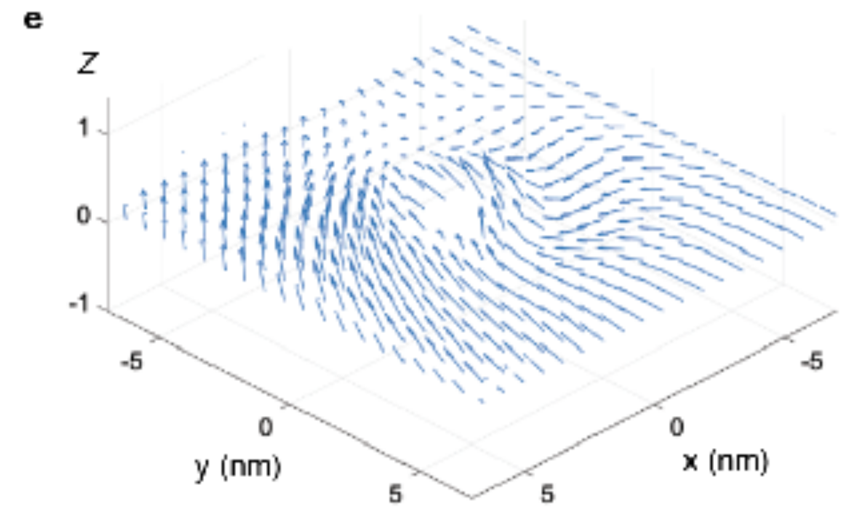
Quantum Hall Ferromagnet and Skyrmions

E

E$_{\textbf{Skx}}$
$= 8\pi\rho$



A spin-degenerate Landau level



- Quantum Hall Ferromagnet

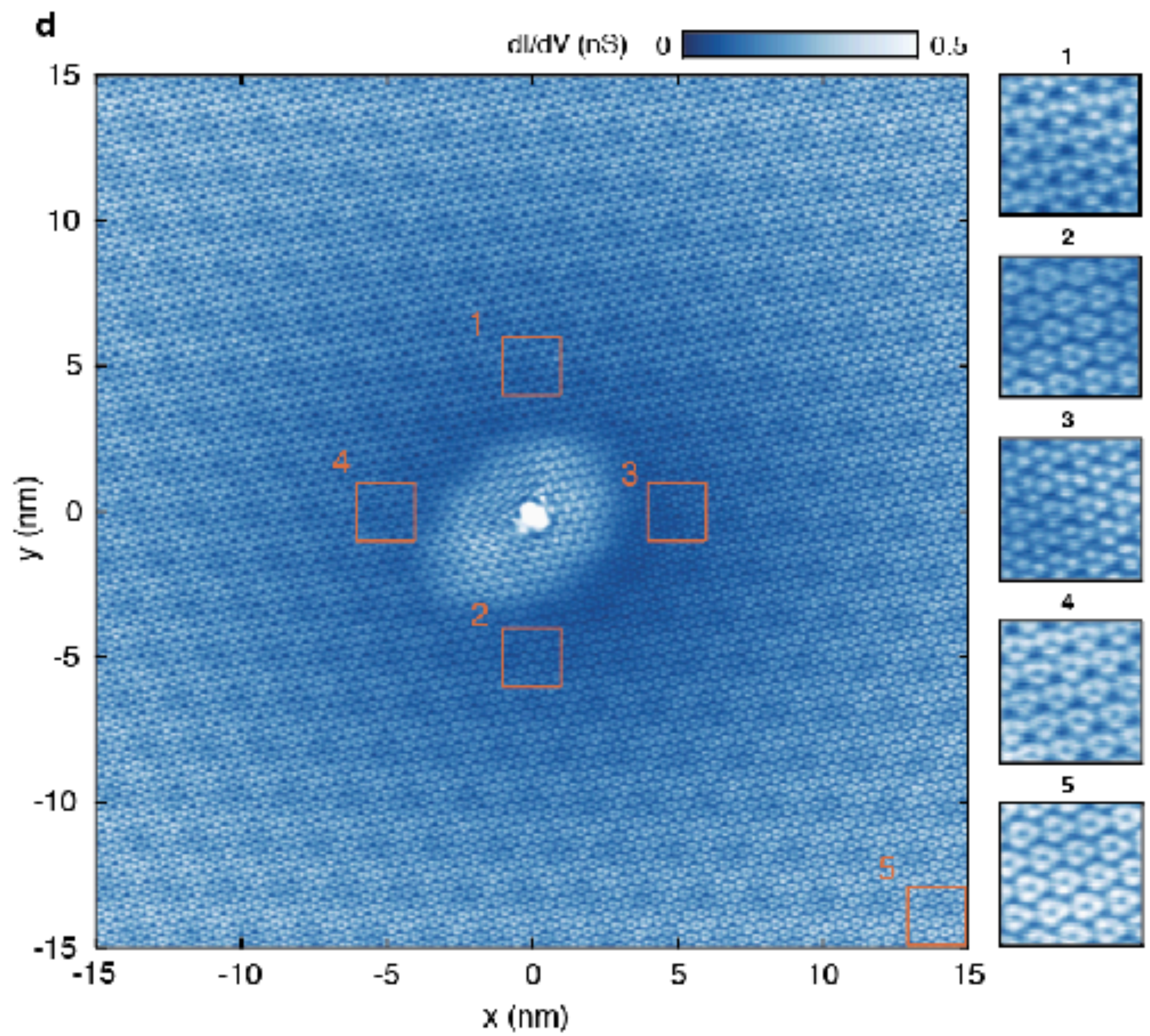
Skyrmions are **Charged**

Cheapest charge excitation is the *Skyrmion*

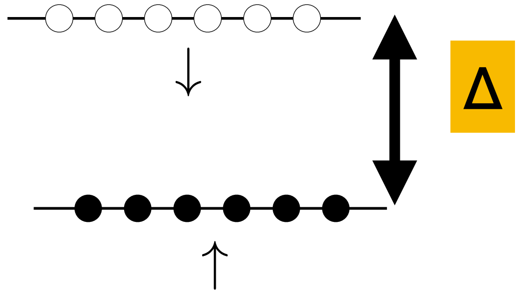
$$\mathbf{E_{Skx}} = 8\pi\rho_s = \frac{\Delta}{2}$$

Theory: Sondhi, Karlhede, Kivelson, Rezayi; Lee & Kane
Experiments: NMR - Barrett et al.
STM - Liu…Zaletel, Yazdani.

arXiv:2109.11555



What is Fundamental?



A spin-degenerate Landau level

Electron -> ferromagnet

Ferromagnet -> electron

Tony Skyrme

A UNIFIED FIELD THEORY OF MESONS AND BARYONS

T. H. R. SKYRME †

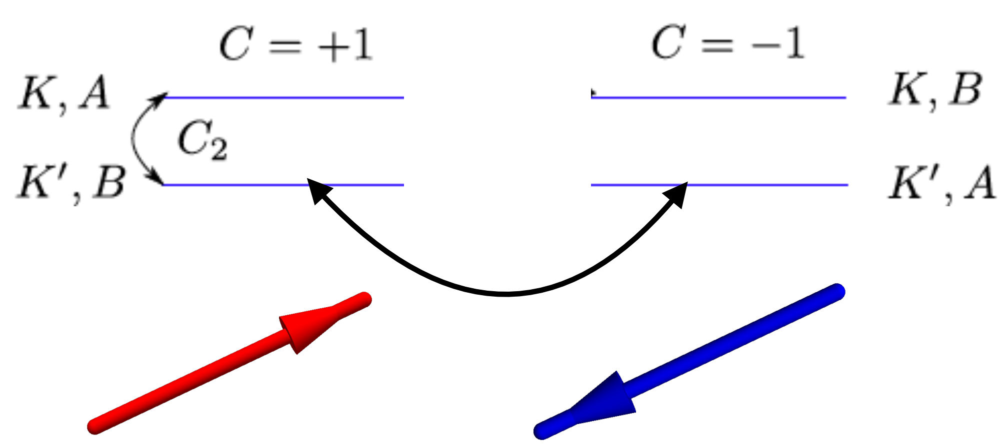
A.E.R.E., Harwell, England

Nuclear Physics **31** (1962) 556—569;

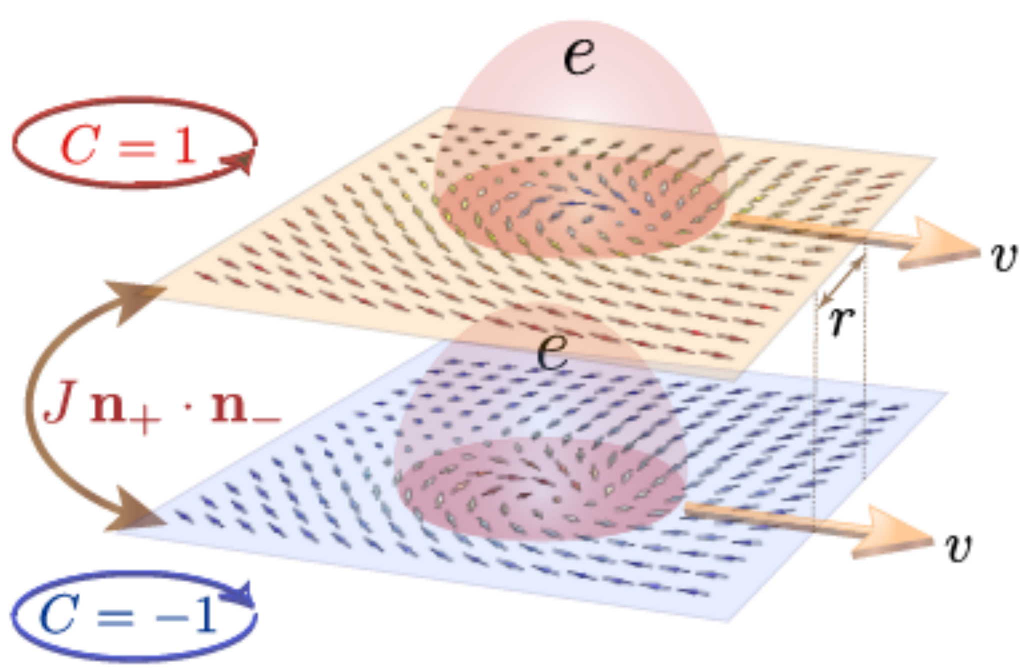
† Now at Department of Mathematics, University of Malaya, Pantai Valley, Kuala Lumpur, Malaya.

We are indebted to Mr. A. J. Leggatt for carrying out the calculations reported in sect. **3**, while a vacation student at A.E.R.E.

Skyrmions in Magic Angle Graphene?



Antiferro-psuedospin coupling $J\mathbf{n}_+ \cdot \mathbf{n}_-$ **J~h²/V~1-2 meV**



Charge 2e Skyrmion
Boson
Pairing due to `J'

Repelled by Coulomb
But attracted by J

An all electron mechanism?

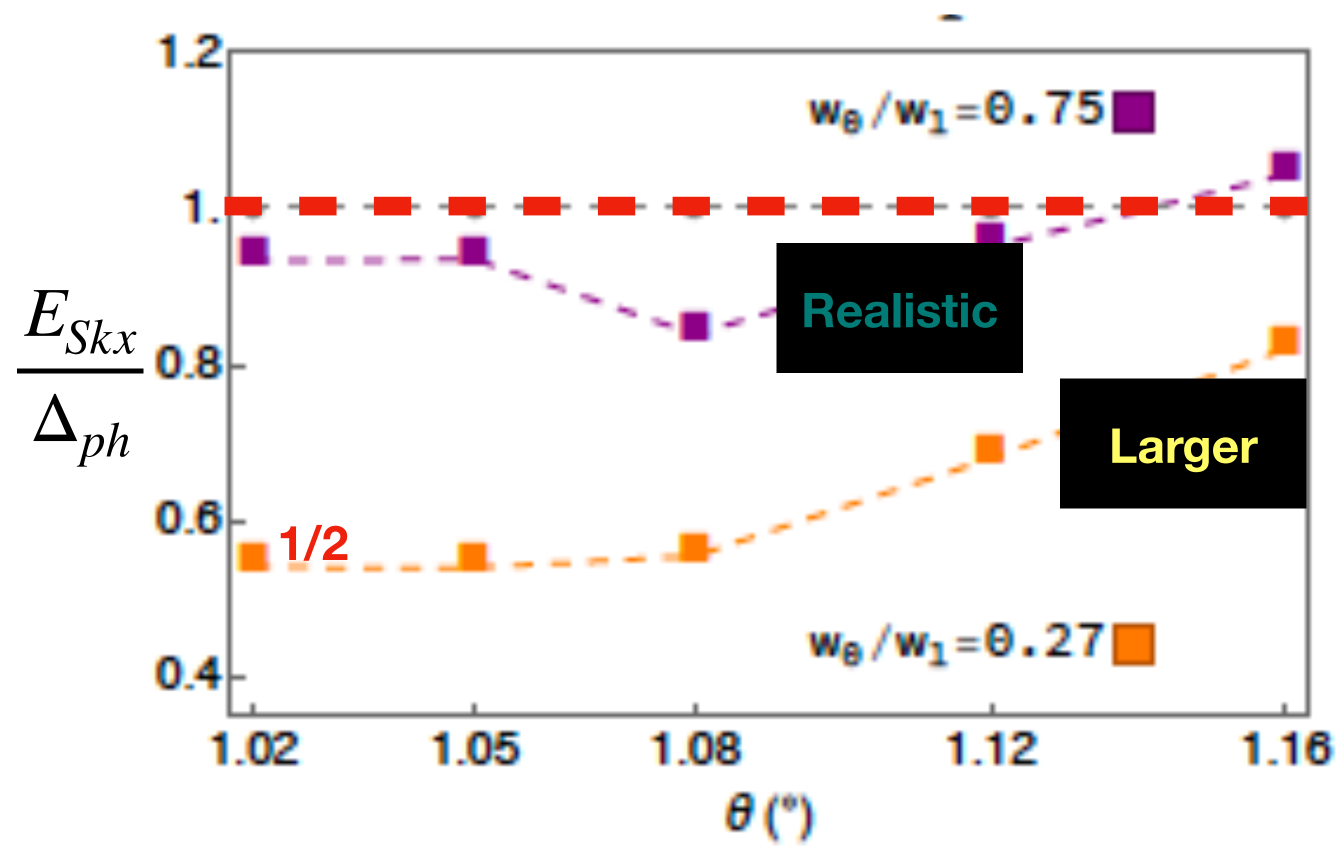
Doping the Insulator

- Use microscopically obtained values of parameters to:
    - Compare energy of skyrmions to 1 particle band gap
    - `Inertia' of skyrmions- sets condensation temperature.

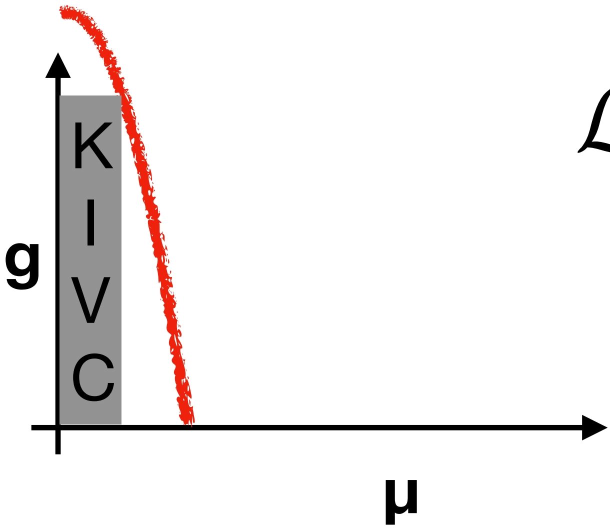
- Elastic energy $E_{\rm sk,pair} = 8\pi\rho_{\rm ps}$ vs $\Delta_{\rm PH}$

$$E = \frac{\rho}{2}\left[(\nabla n_+)^2 + (\nabla n_-)^2\right] + J n_+ \cdot n_-$$

*Neglects anisotropies, fermion dispersion

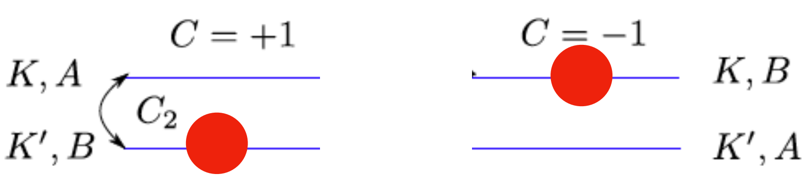


Phase Diagram & Stiffness



$$\mathcal{L}[\mathbf{n}] = \rho_n(\nabla \mathbf{n})^2 + \chi_n(\partial_t \mathbf{n})^2 - \mu \frac{2e}{4\pi} \mathbf{n} \cdot \partial_x \mathbf{n} \times \partial_y \mathbf{n}$$
$$+V_{Coulomb}[\mathbf{n}]$$

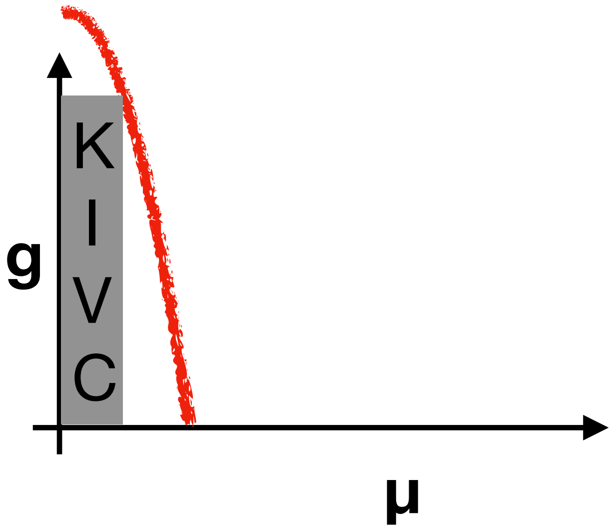
A Dual Representation:



$$|\psi\rangle = z_1 |K, A\rangle + z_2 |K', B\rangle$$

$$z = (z_1, z_2), \quad \boldsymbol{n} = z^\dagger \boldsymbol{\sigma} z,$$

Phase Diagram & Stiffness

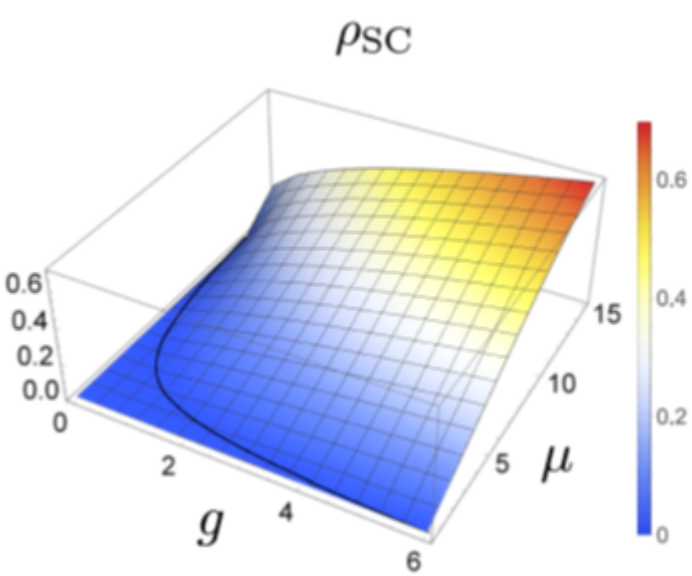


Solve using large-N CP$_N$

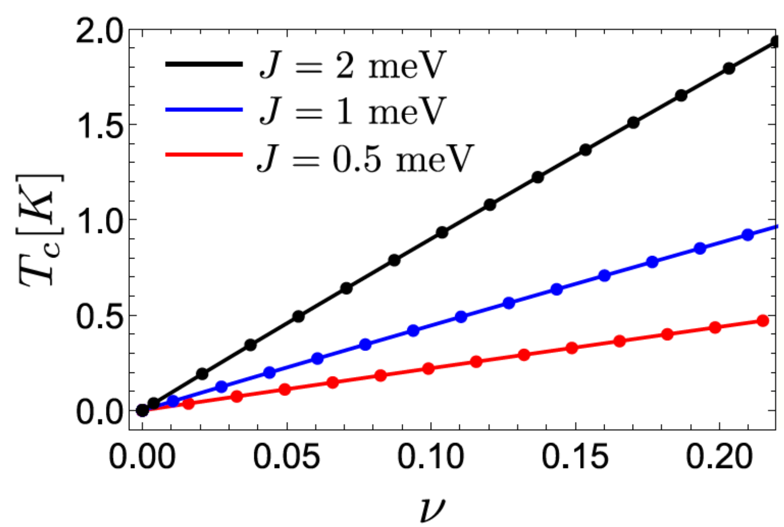
$$\mathcal{L} = \frac{\Lambda}{g}|(\partial_\mu - ia_\mu)z|^2 + \quad \mu\frac{b}{\pi}$$

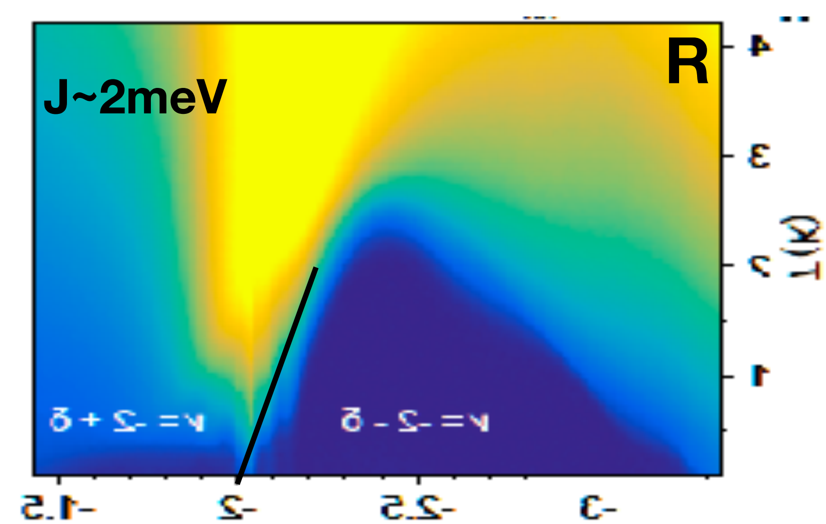
$$\mathscr{L} = |(\partial_\mu - i\alpha_\mu)w|^2 + iA_v \wedge d\alpha$$

$$\Delta = w_1^* w_2$$



$$T_c = \frac{3\nu A_M J}{4Nk_B} + O(\nu^2)$$





J. Park, Y. Cao … Pablo `20

Numerical DMRG evidence - Shubhayu's talk

Chatterjee *et al.* arxiv:2010.01144

EK, Chatterjee, Bultinck, Zaletel, Vishwanath arXiv:2004.00638

Skyrmion Properties

- Effective Mass

Skyrmion -antiskyrmion pair.

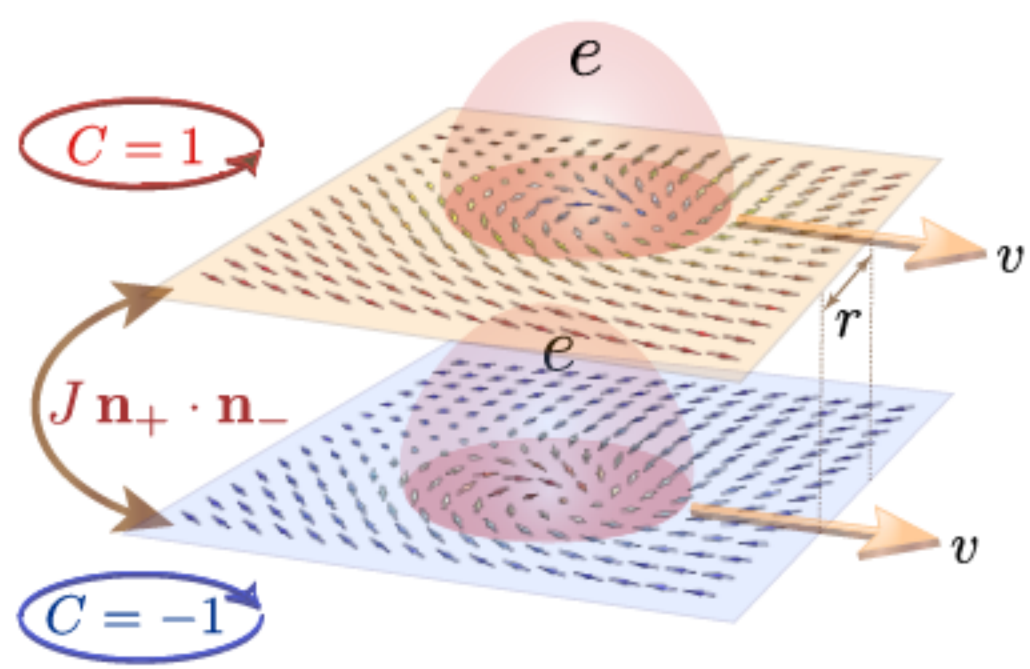
Individually - experience opposite Magnus dynamics.
Together - inertial dynamics.
What is effective mass?

$$\mathscr{L} = \frac{1}{2}(r_1 \wedge \dot{r}_1 - r_2 \wedge \dot{r}_2) - V(r_1 - r_2)$$

$$\mathscr{L}[R = \frac{r_1 + r_2}{2}, r = r_1 - r_2] = r \wedge \dot{R} - V(r)$$

$$V(r) = \frac{J}{2} r^2$$

$$\frac{1}{m} = \frac{\partial^2 E}{\partial P^2} \propto \frac{\partial^2 V}{\partial r^2} \sim J$$



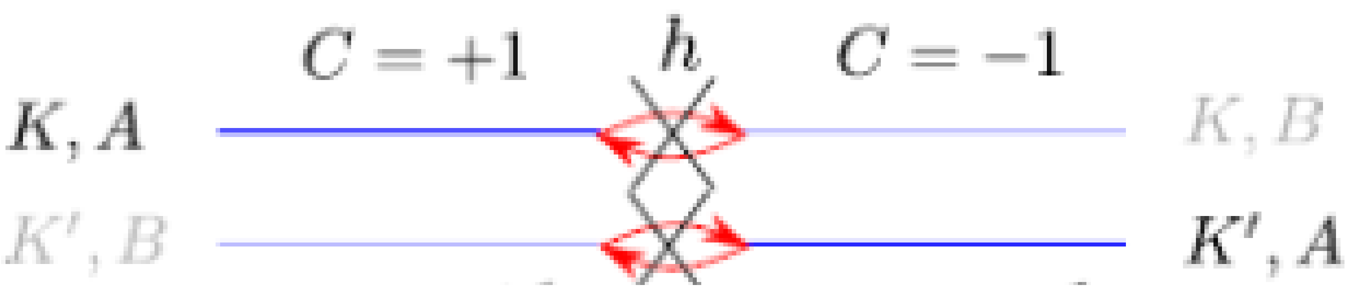
Sets superconductor Tc on doping

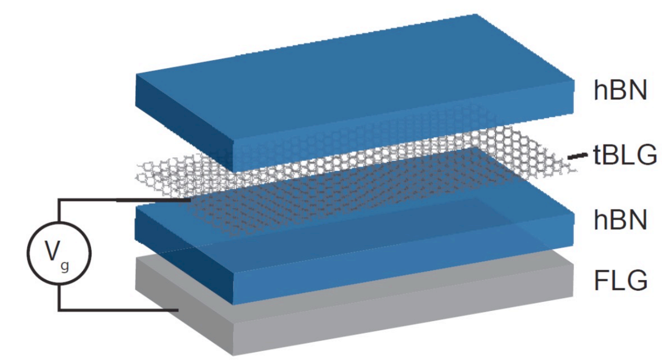
$$T_c \sim \nu J$$

- Khalaf, Chatterjee, Bultinc, Zaletel, AV arxiv: 2004.00638

Essential Ingredients for Superconductivity?

- $C_2\mathcal{T}$ symmetry crucial for superconductivity
- Sublattice potential *suppresses* the coupling J





- Twisted bilayer graphene aligned on hBN → no superconductivity (Sharpe et al. Science 19, Serlin et al. Science 20)

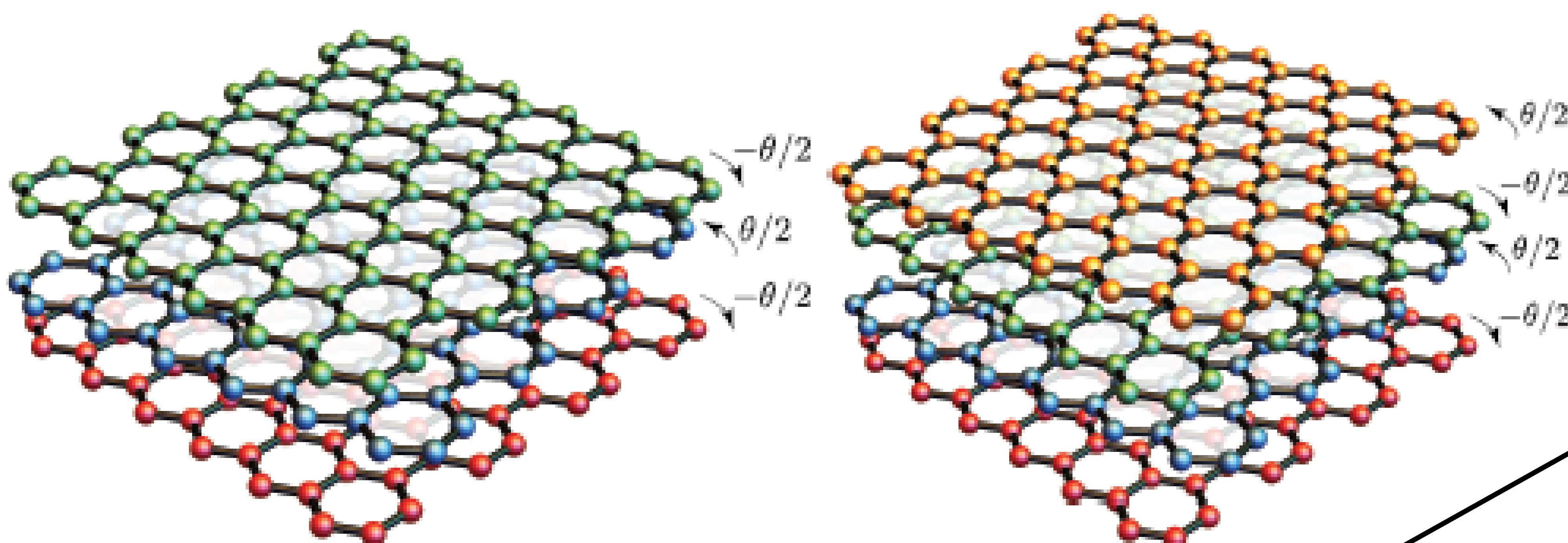
- Relatively *few* Moire materials apart from magic angle graphene with this symmetry.

Alternating Twist Multilayers

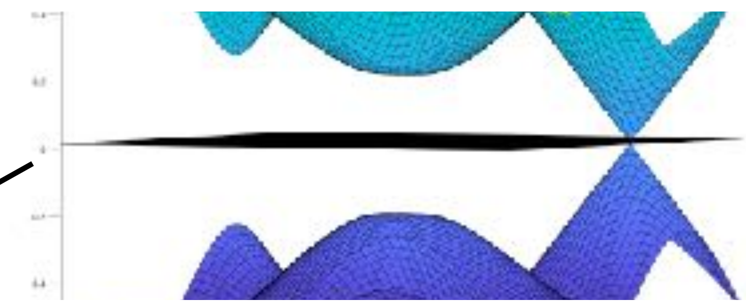
- Other systems with C2 symmetry

Eslam Khalaf

Alternating twist sandwich

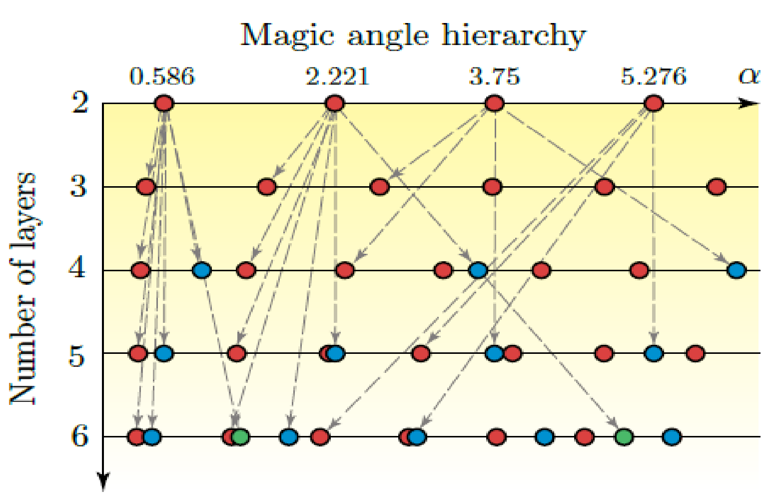


Magic angle = $\sqrt{2}\,1.1^o \sim 1.55^o$



Flat band+Dirac

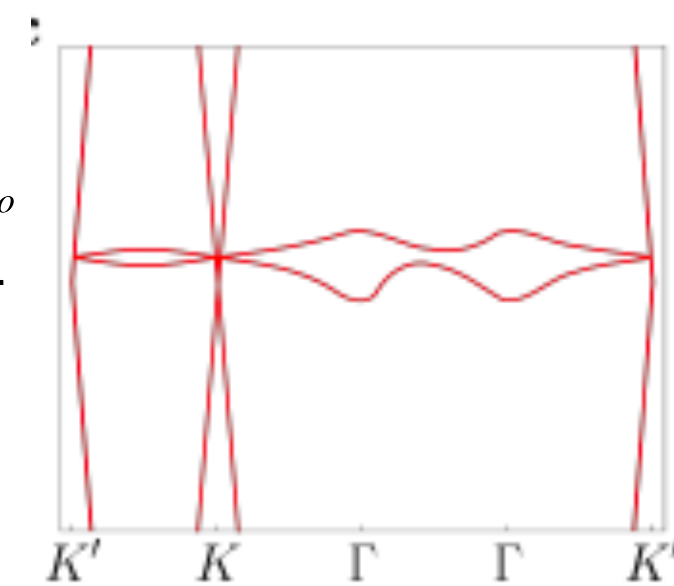
Carr et al. stability `19



$\sqrt{2}$

$\varphi = \frac{1+\sqrt{5}}{2}$

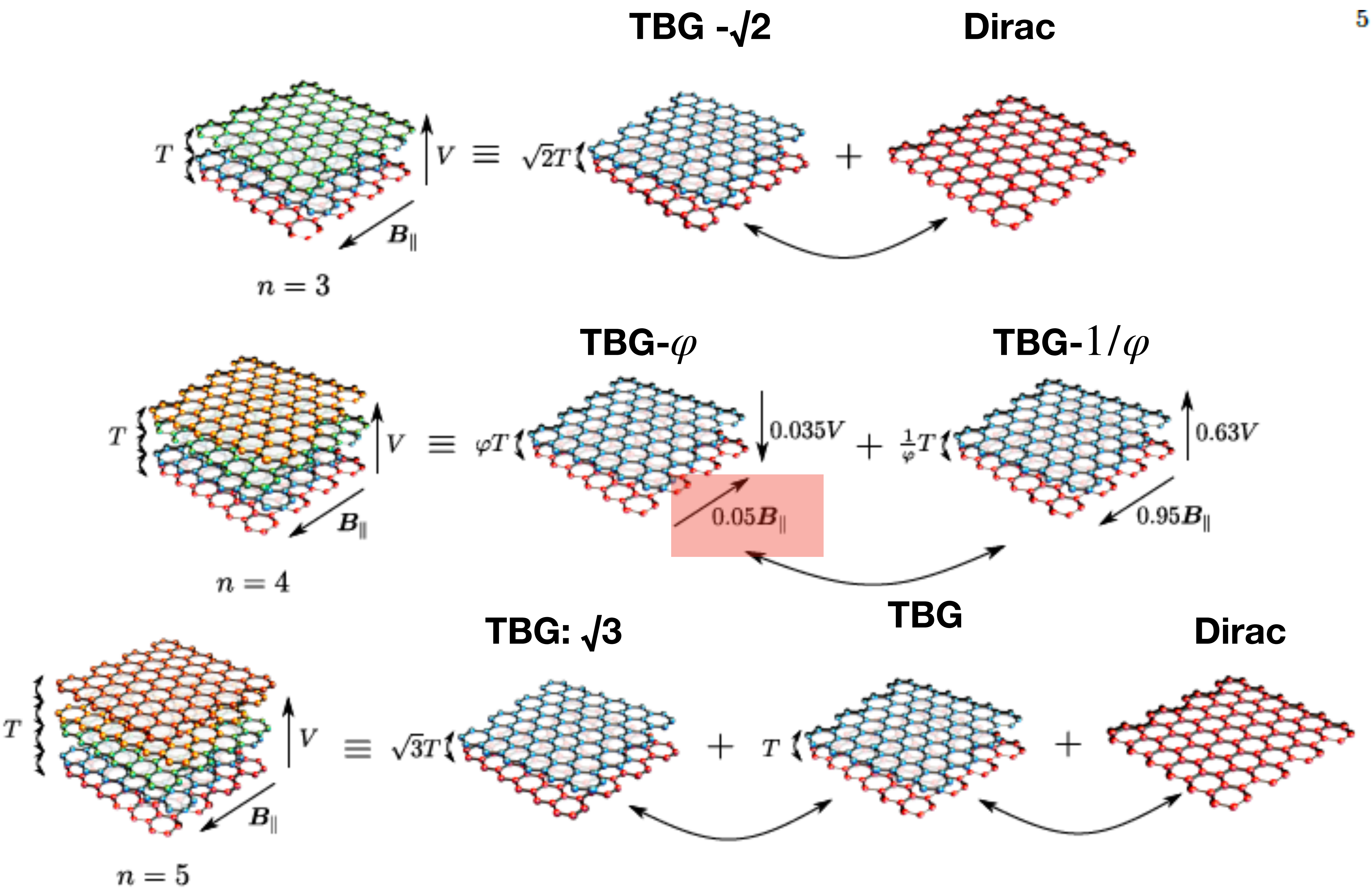
Magic angle = $\frac{1+\sqrt{5}}{2}\,1.1^o \sim 1.78^o$



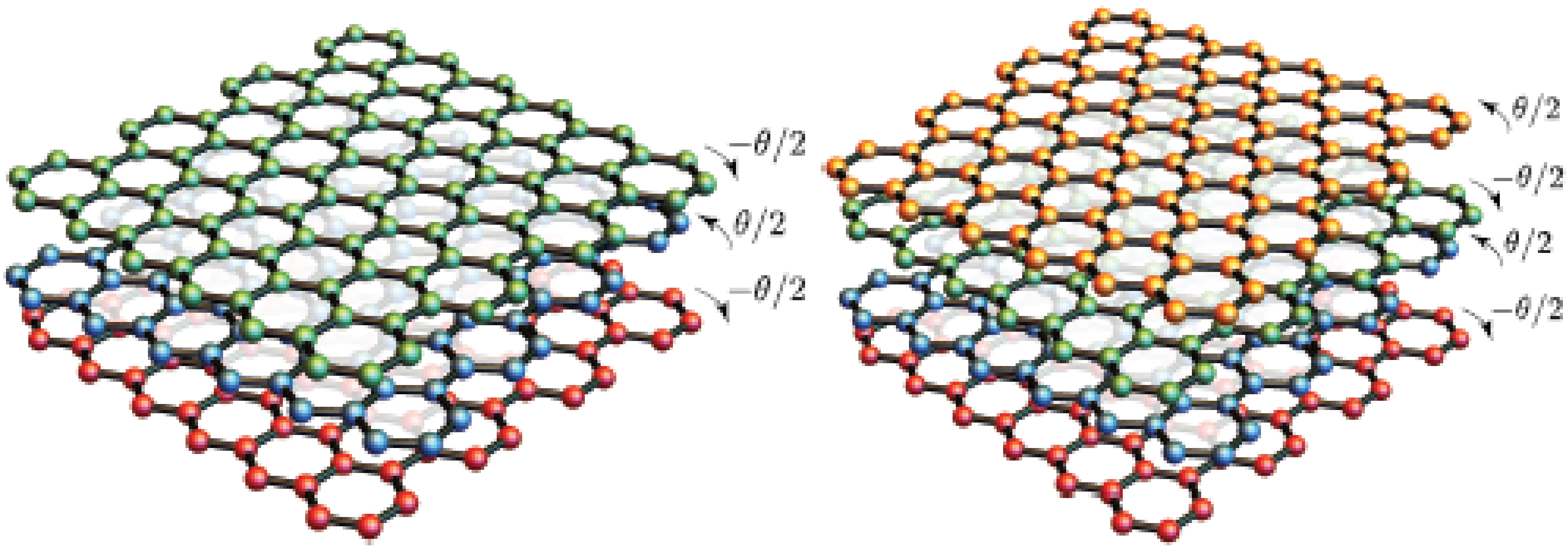
Khalaf, Kruchkov, Tarnopolsky, AV, 1901.10485

Larger magic angles!

Deconstructing n=3, 4, 5

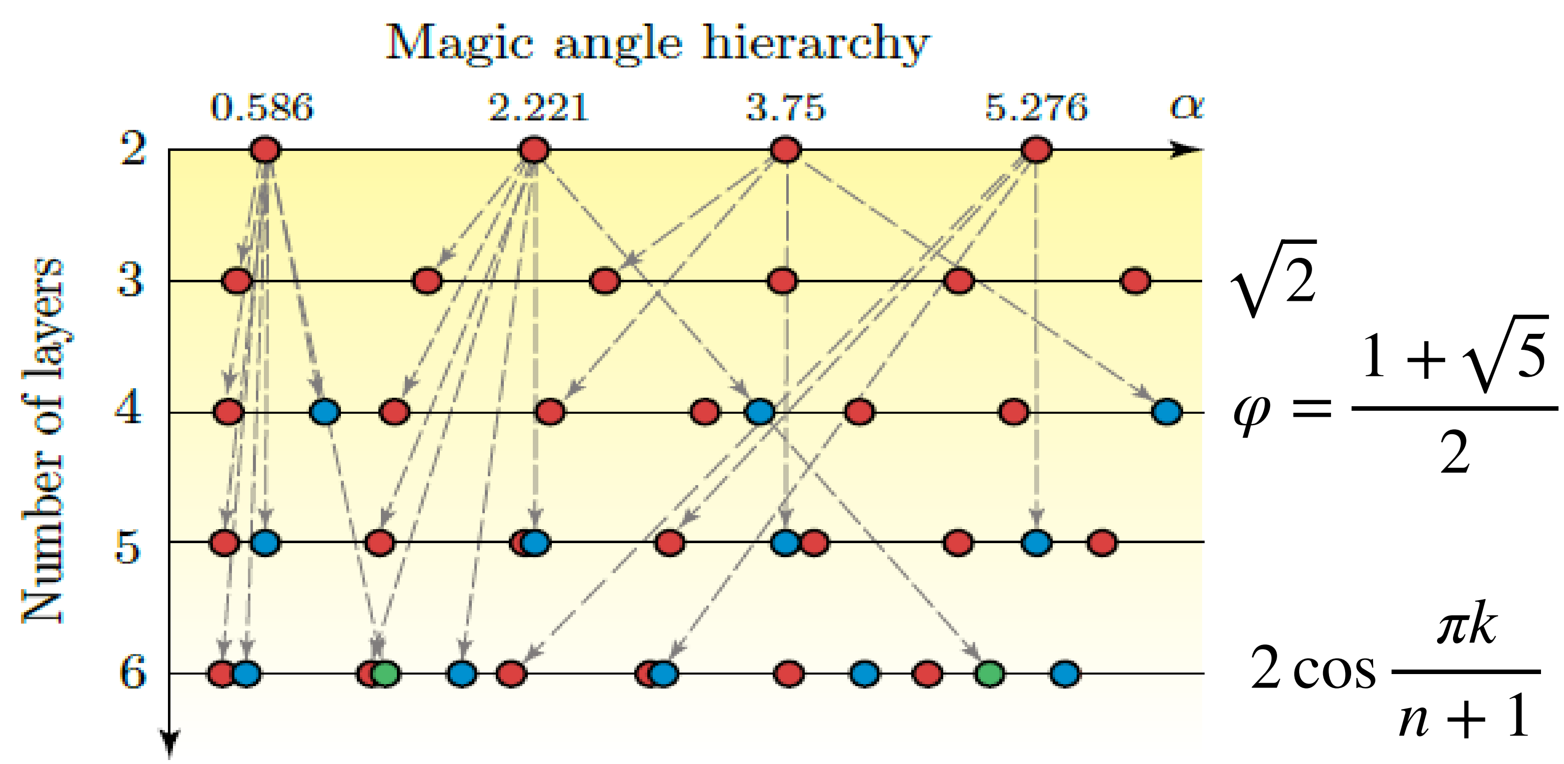


ASIDE: Alternating-twist multilayer graphene



Eslam Khalaf

- Alternating twist:
  - Magic angles *simply* related to the bilayer case.



Khalaf, Kruchkov, Tarnopolsky, AV, 1901.10485

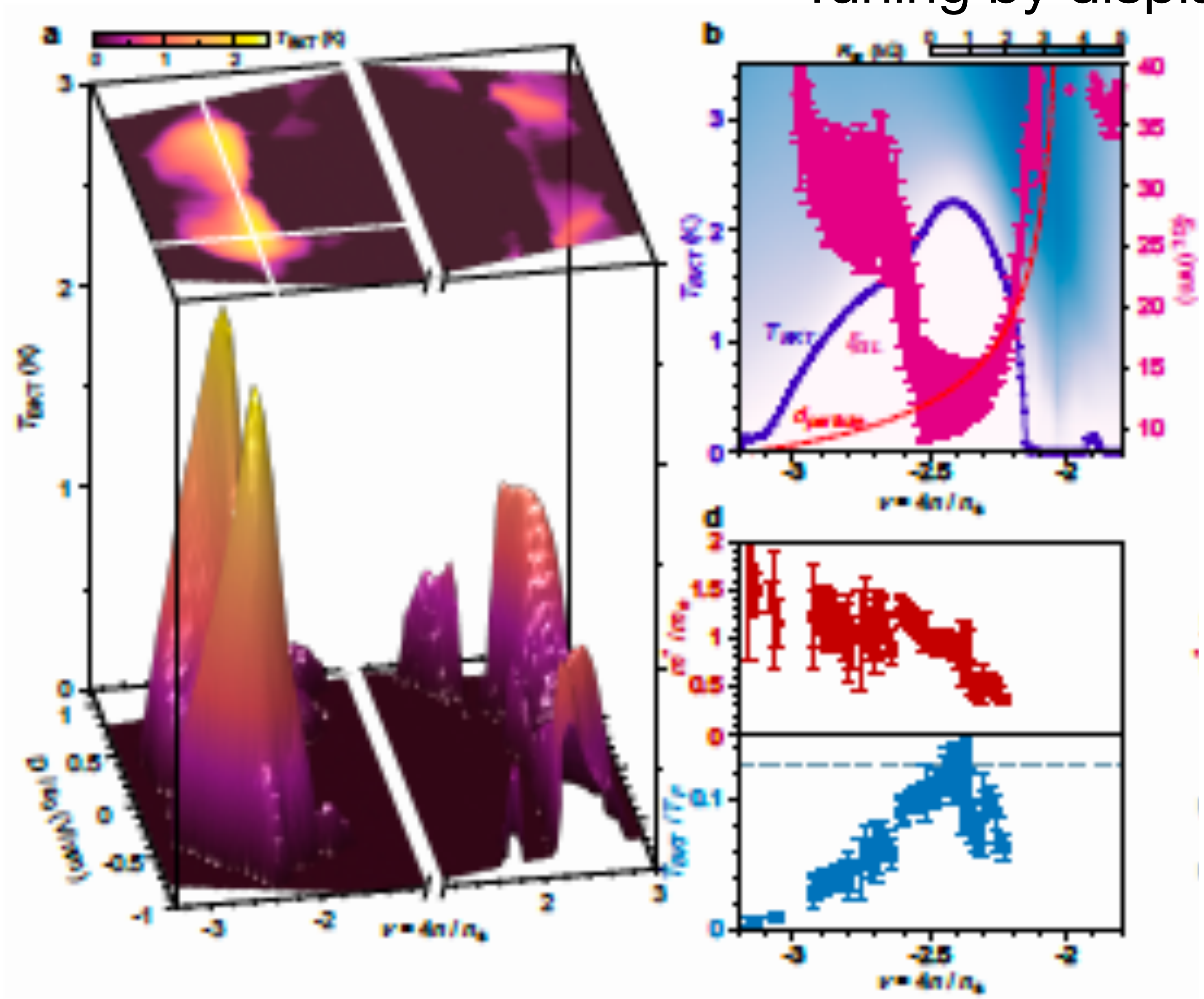
Matthias' Rules for Superconductivity

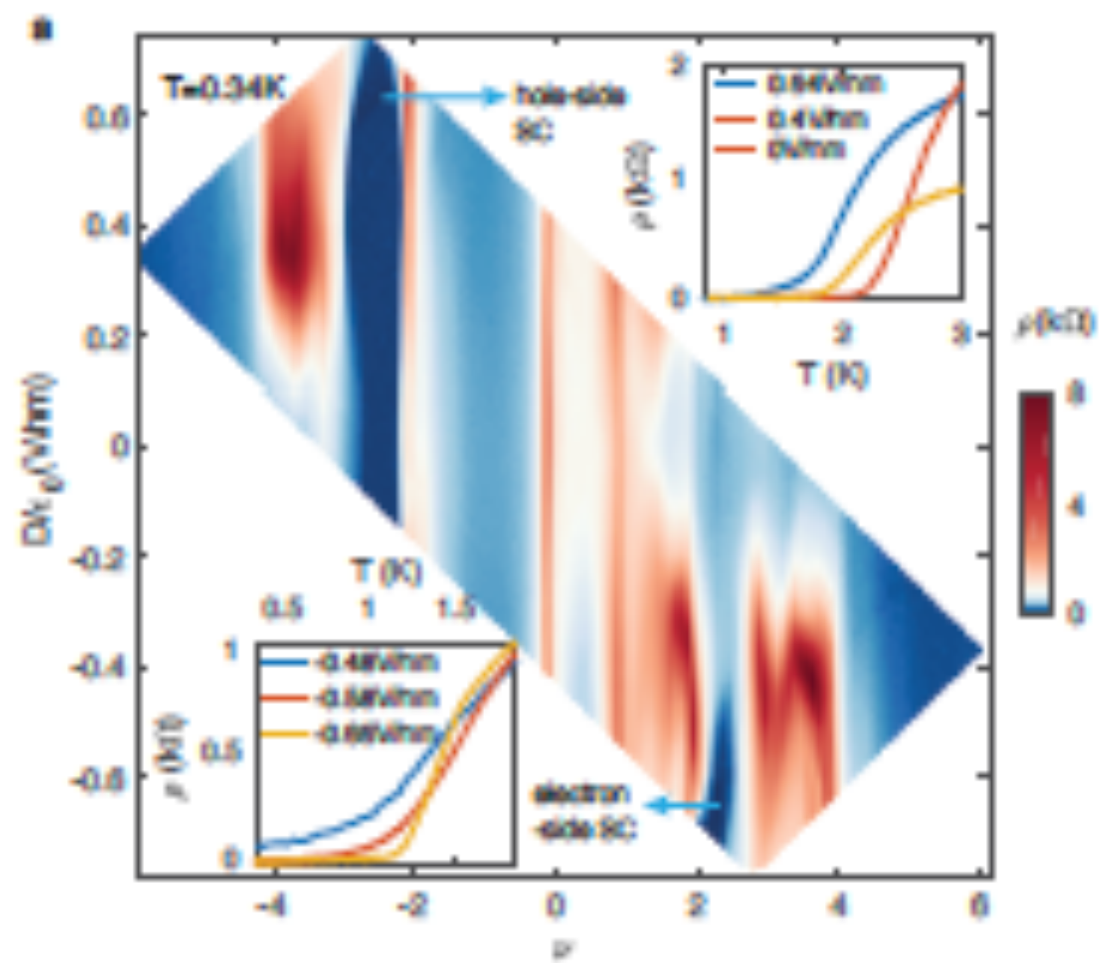
Bernd Matthias (1918-1980)

1. high symmetry is good, cubic symmetry is the best
2. high density of electronic states is good
3. stay away from oxygen
4. stay away from magnetism
5. stay away from insulators
6. stay away from theorists.

Alternating twist trilayer -EXPT

Strong coupling superconductivity
associated with |nu|=2
Tuning by displacement field.





Hao et al.

Park et al.

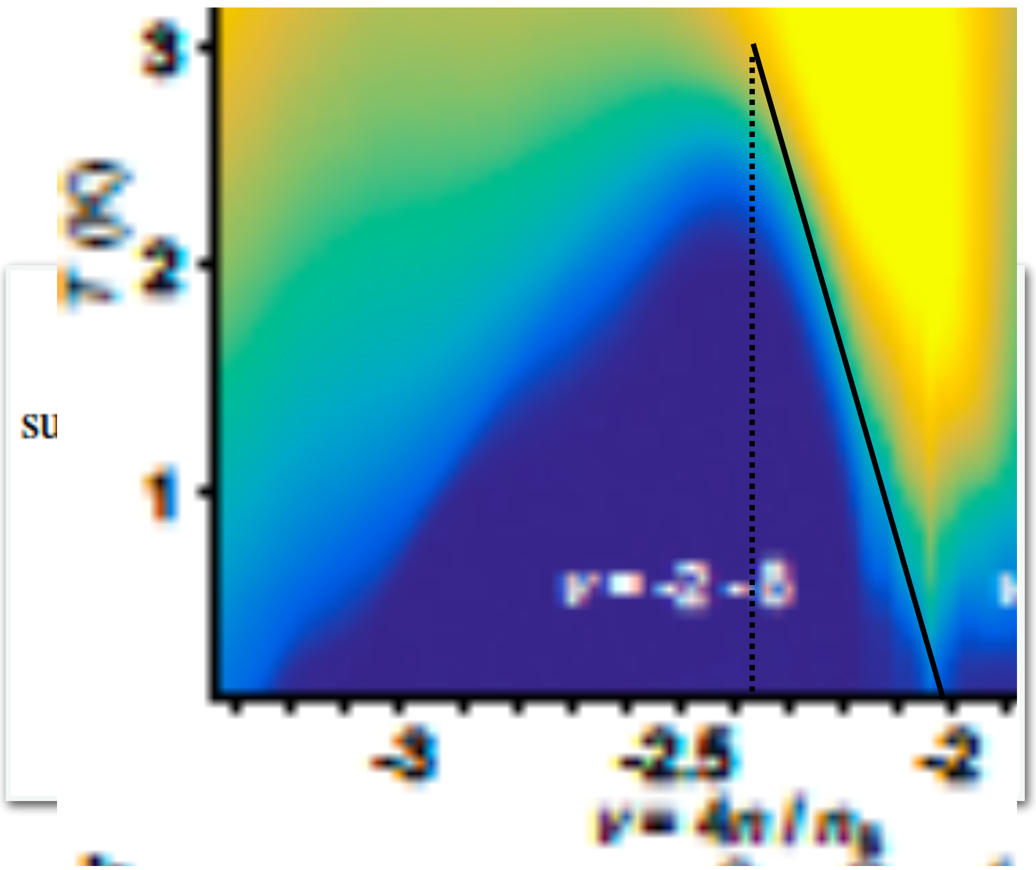
Tunable Phase Boundaries and Ultra-Strong Coupling Superconductivity in Mirror Symmetric Magic-Angle Trilayer Graphene

Jeong Min Park,[1,*] Yuan Cao,[1,*,†] Kenji Watanabe,[2]
Takashi Taniguchi,[2] and Pablo Jarillo-Herrero[1,†]

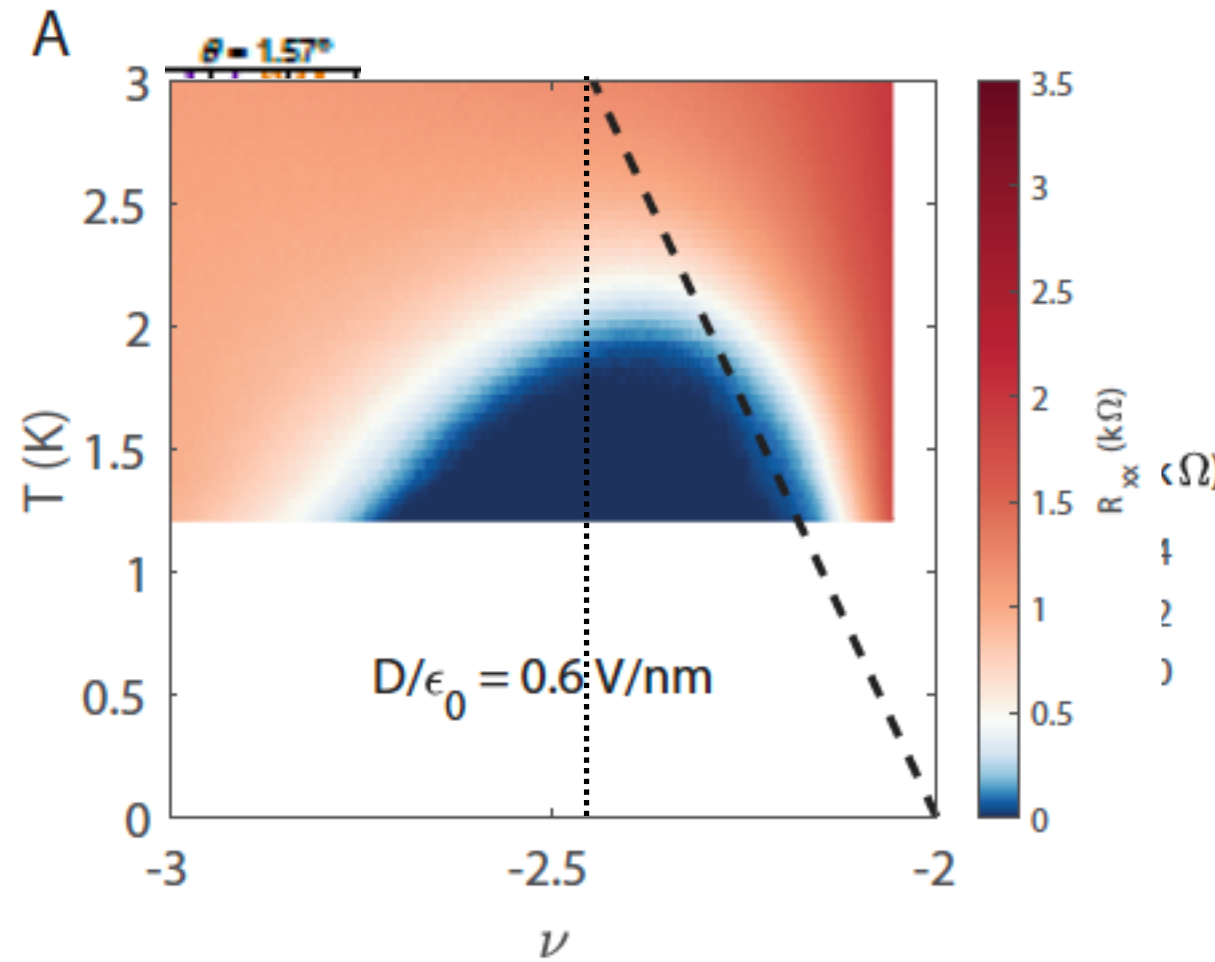
Electric field tunable unconventional superconductivity in alternating twist magic-angle trilayer graphene

Zeyu Hao[1†], A. M. Zimmerman[1†], Patrick Ledwith[1], Eslam Khalaf[1],
Danial Haie Najafabadi[1], Kenji Watanabe[2], Takashi Taniguchi[3],
Ashvin Vishwanath[1] & Philip Kim[1*]

Experiments on Alternating Twist Trilayer

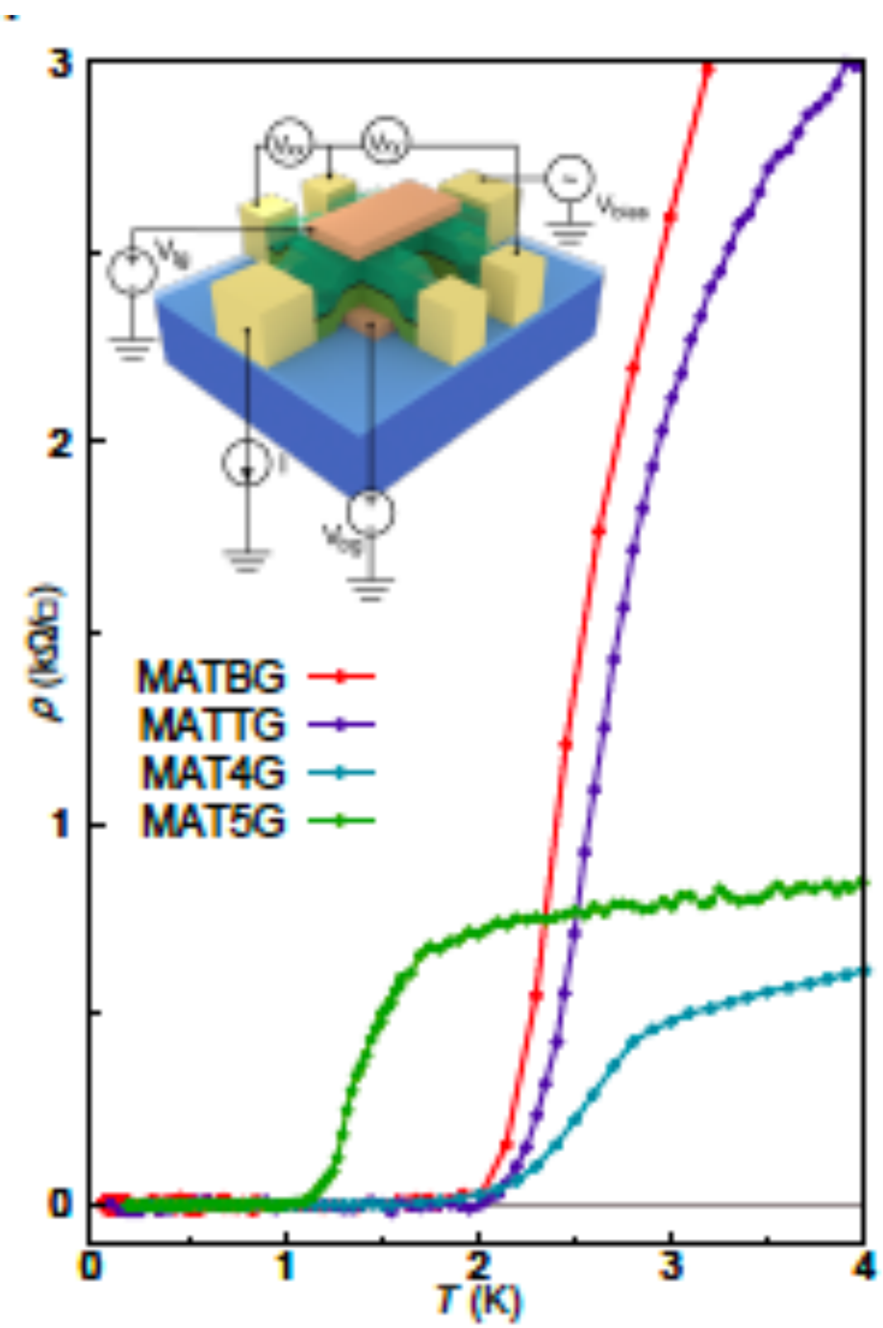


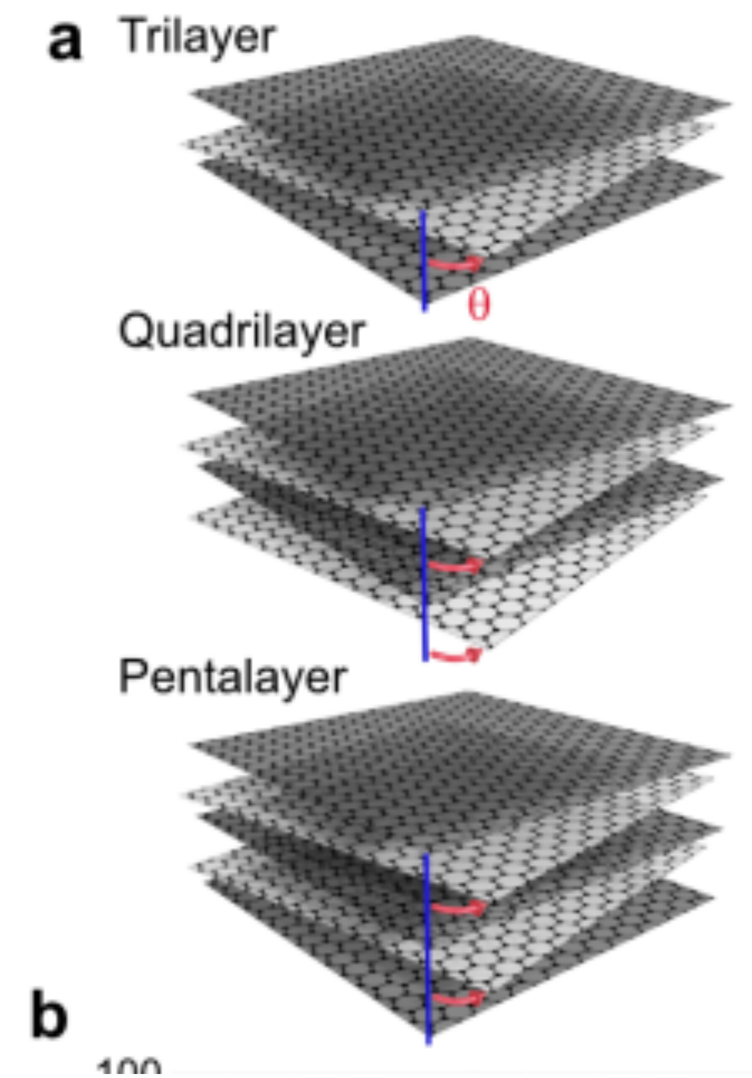
Pablo’s talk - Moire 3.0



Hao et al.

n=4, 5 Alt. Twist Multilayers



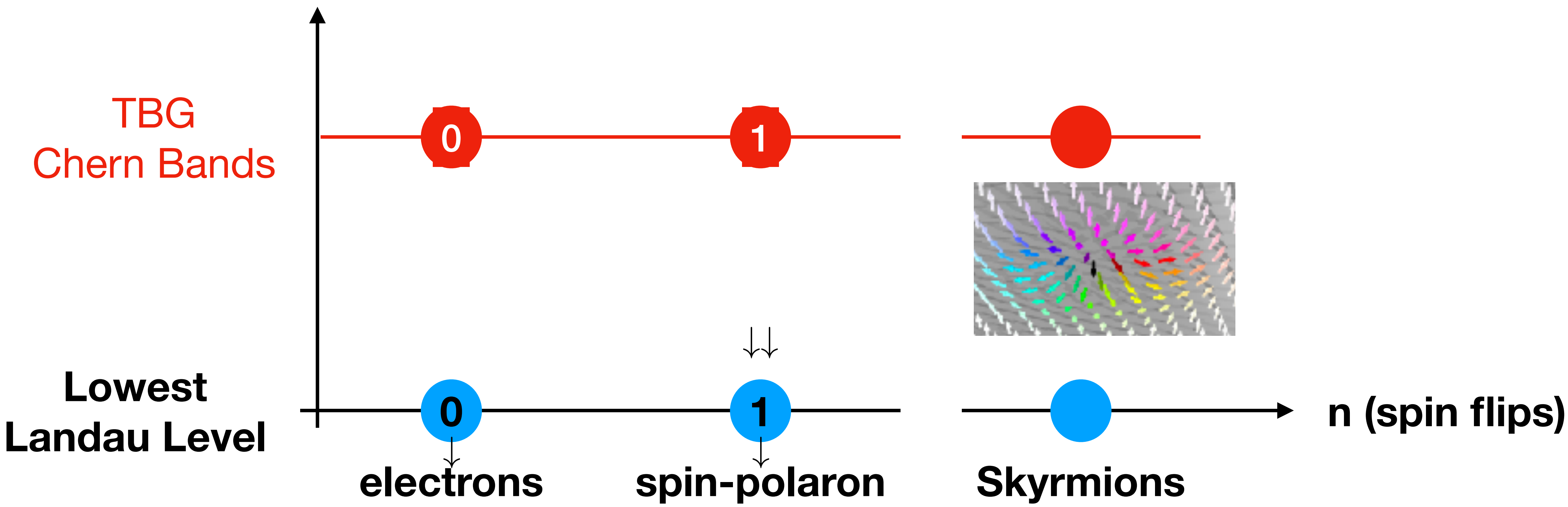


Magic-Angle Multilayer Graphene: A Robust Family of Moiré Superconductors

Jeong Min Park,[1,*,†] Yuan Cao,[1,2,*] Liqiao Xia,[1] Shuwen Sun,[1]
Kenji Watanabe,[3] Takashi Taniguchi,[3] and Pablo Jarillo-Herrero[1,†]

Also Stevan Nadj-Perge group

From Skyrmions to Spin-Polarons
&
From LLL to TBG

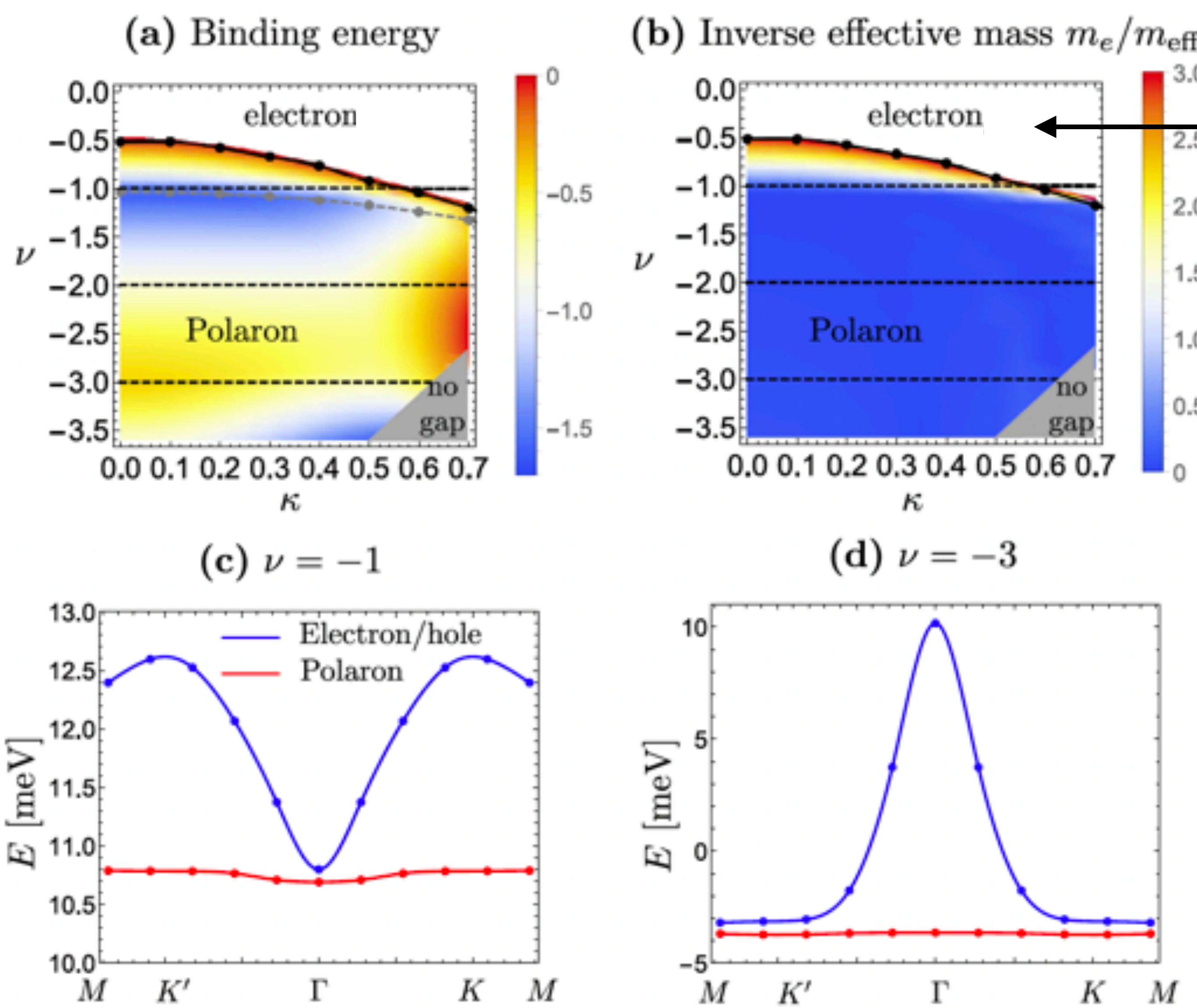


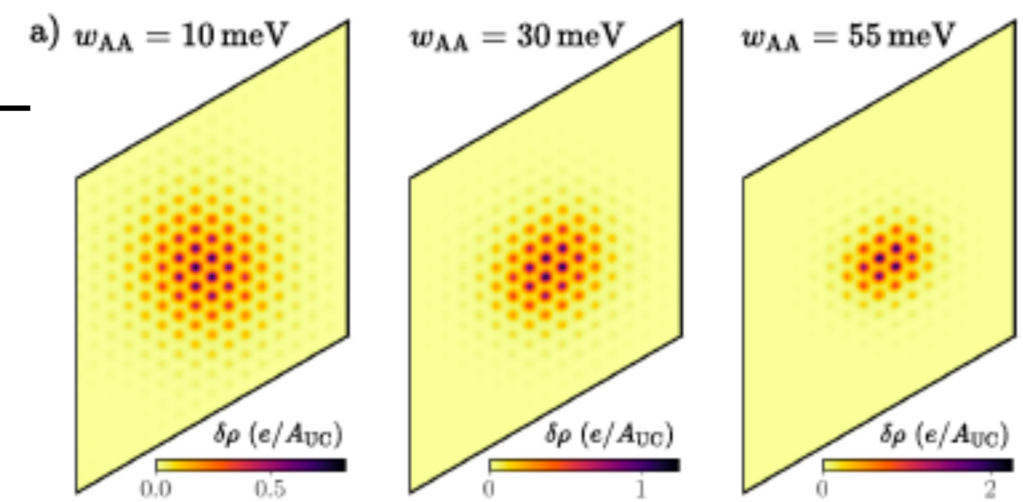
- Skyrmions = electrons+ many spin flips

- Landau Levels versus TBG bands:

Bound States with Dispersion

- Spin-Polaron bound states appear on the `low dispersion' side [away from charge neutrality].



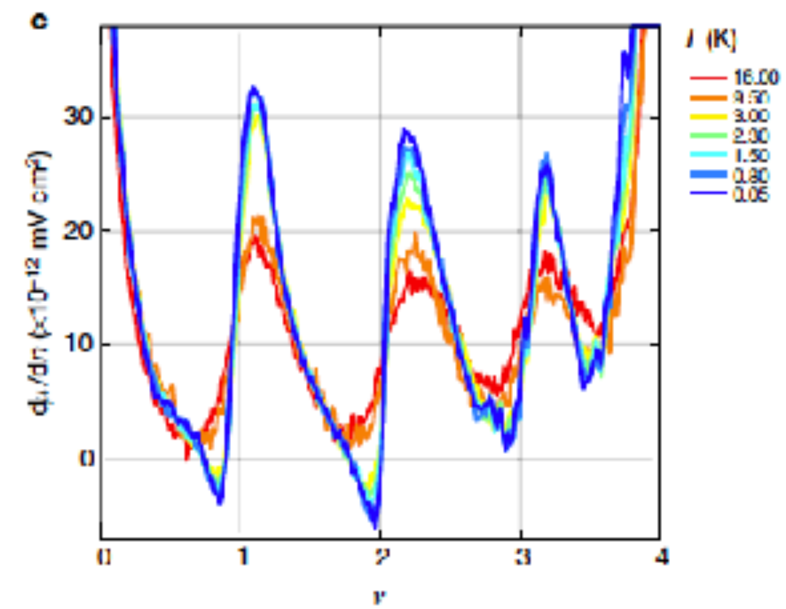


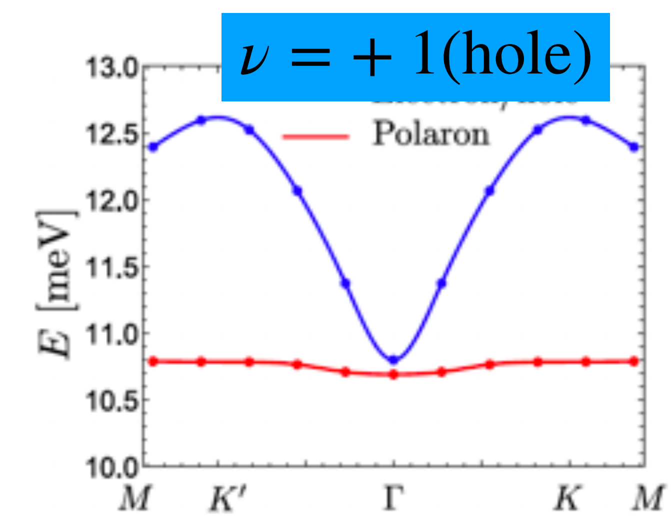
Could be unstable to further spin flip bound states.
(Kwan…Parameswaran arXiv:2112.06936)

- Very small dispersion.

Phenomenology

- Consequences for Experiment? STM, Cascade features?
- Superconductivity: “Spin-Bipolarons” paired via magnetic superexchange.
    - Role of strain?





Also Vafek-Bernevig

OUTLINE

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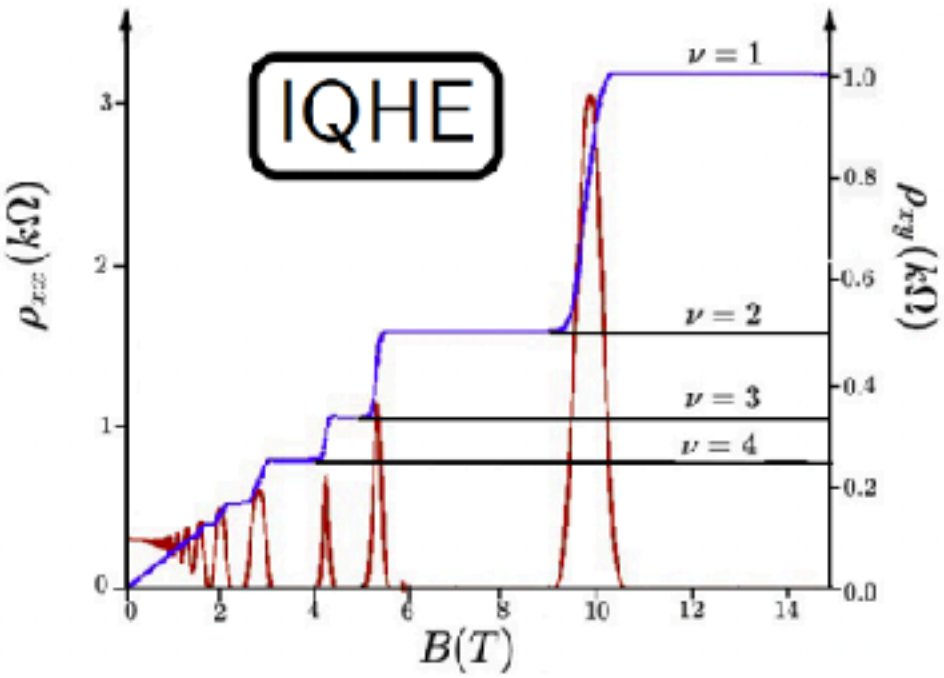
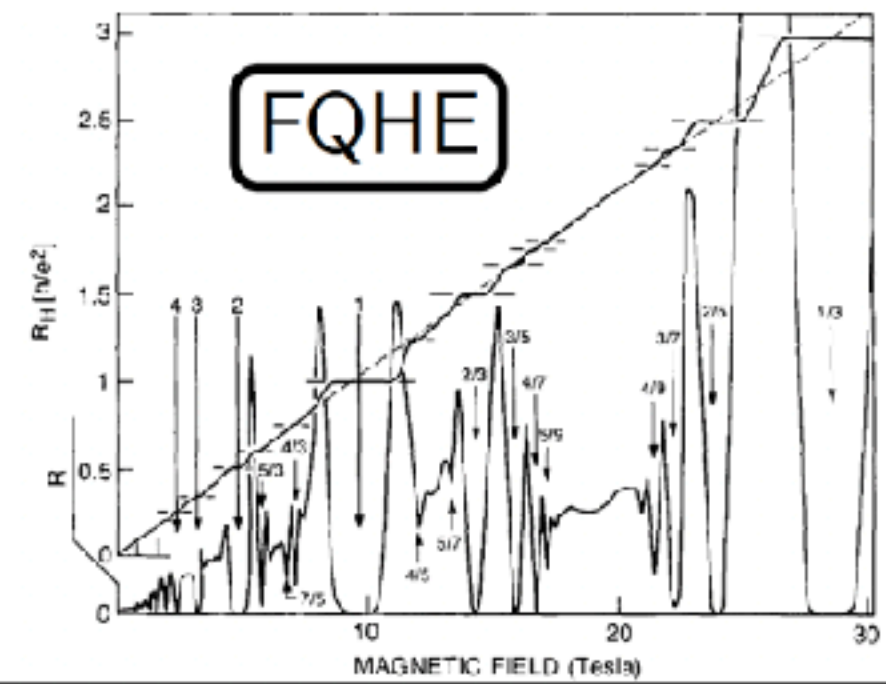
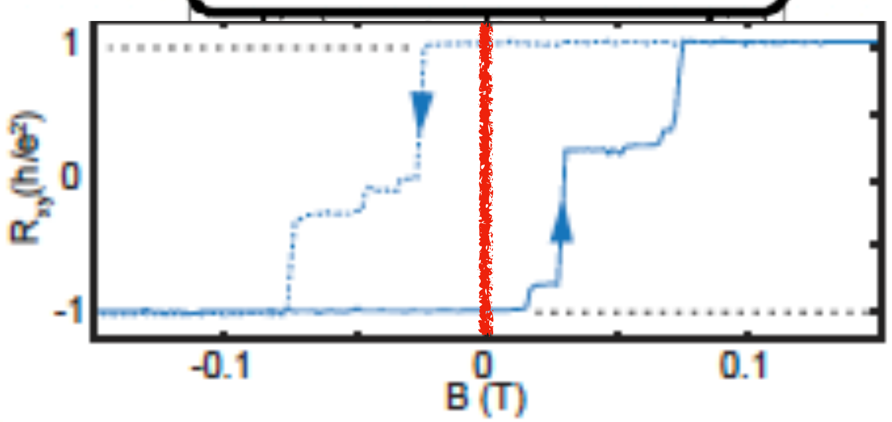
- Lecture 4 - Fractional Chern insulators in magic angle graphene

Overview

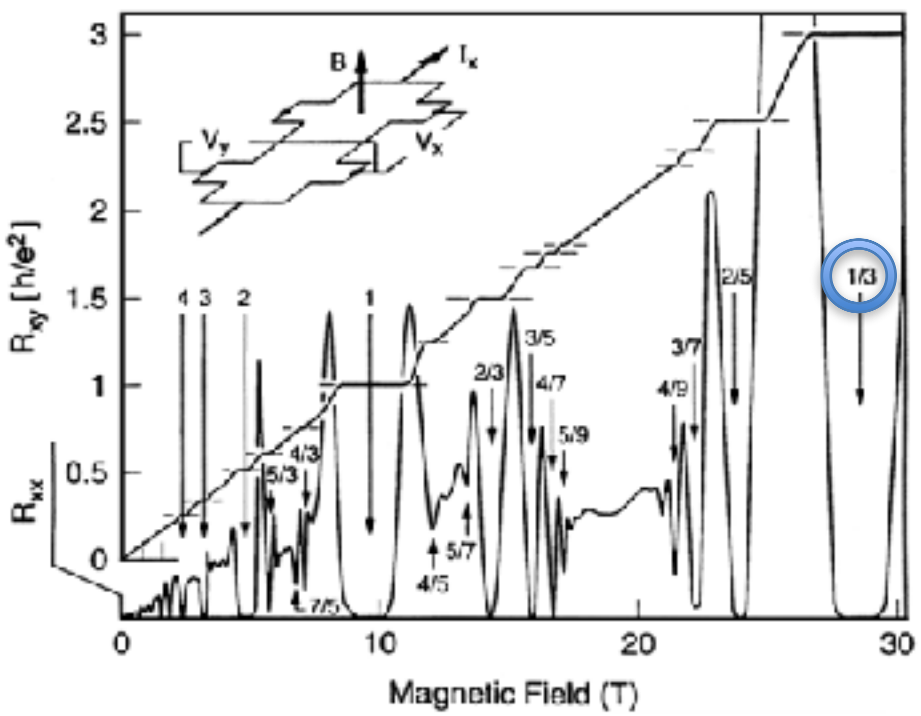
	Integer Charge	Fractional Charge
(B Field)		
(Quantum Geometry)	Chern Insulator 	Fractional Chern Insulator FCIs by fractional filling of Chern band?

Figures: Wikipedia; Stormer (1992);

Magic angle graphene (+ aligned hBN)
Serlin et al. Sharpe et al. Science `19. Stepanov et al. PRL `21

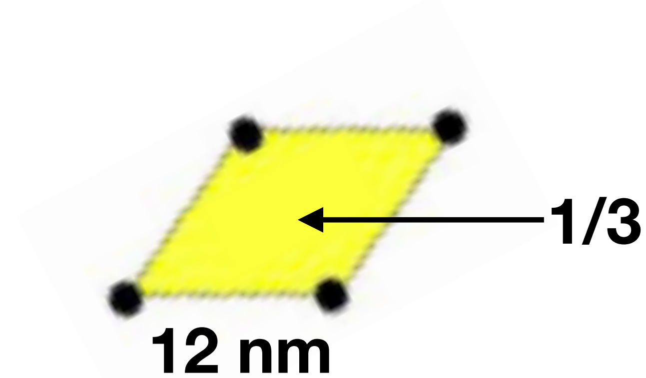
Intrinsic Topological Order

Intrinsic Topological Order - *BEYOND* Fractional Quantum Hall



B = 28 Tesla

n = 0.25 x 10^{12} cm^{-2}



1. Density of electrons *not* set by the field.
2. FQH at small or zero field.
3. Potentially enhanced energy scales ($e^2\sqrt{n}$)
4. Translation symmetry *enriched* Topological Order

PART1: Fractional Chern Insulators:

Fractional Quantum Hall effect in topological band structure

(rather than Landau levels)?

Sheng et al, `11. Neupert et al, `11. Tang et al, '11. Regnault & Bernevig `11. Qi `11.

- *Ideal* quantum geometry of chiral TBG
- Fractional Chern insulators in realistic TBG
- Experiments on TBG FCIs
- Zero field FCIs and Conclusion.

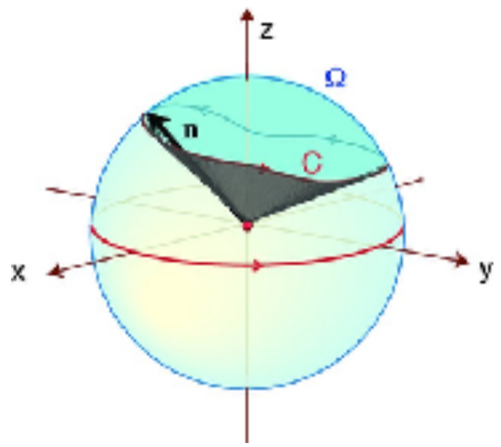
Quantum Geometry of Electronic Bands

Berry Curvature - (well known)

$A(\mathbf{k}) = \langle u_{\mathbf{k}} | i \nabla_{\mathbf{k}} | u_{\mathbf{k}} \rangle$: **Berry** connection

$\mathbf{\Omega}(\mathbf{k}) = \nabla_{\mathbf{k}} \times \mathbf{A}(\mathbf{k})$: **Berry** curvature

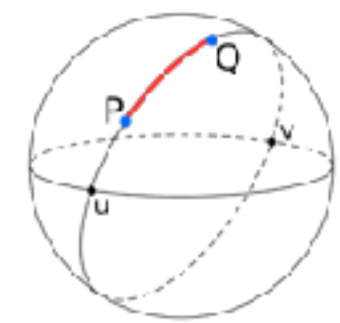
Berry phase Ω



Fubini Study metric

$$ds^2 = g_{ba}(k)\, dk_b\, dk_a$$

$g_{ab}(k)$



Metric on BZ

2-bands

Quantum Metric: $\eta_{ab} = \langle \partial_a u_k | 1 - | u_k \rangle \langle u_k | | \partial_b u_k \rangle$

Projector - eliminates trivial changes

(Hermitian) $\eta_{ab}(k) = g_{ab}(k) - \frac{i}{2} \Omega \epsilon_{ab}$

Approaching Landau Levels

- Achieve FCIs by mimicking Lowest Landau Level
  - Flat bands
  - Uniform Berry curvature [Parameswaran, Roy, Sondhi;…]
  - Trace condition - Ideal quantum geometry. [Roy, Classen et al.]

$$|\mathrm{Tr}[g]| \geq 2\sqrt{|\mathrm{Det}[g]|} \geq |\Omega|$$

Equality=Trace Condition (Lowest L. L.)

Automatically satisfied by chiral TBG and lowest LL!

Quantum Geometry of Moire Flat Bands

- What is the Quantum Geometry of the Moire flat bands?

(Ledwith et al.)

$$|\mathrm{Tr}[g]| \geq 2\sqrt{|\mathrm{Det}[g]|} \geq |\Omega|$$

Equality=Trace Condition (Lowest L. L.)

- In the chiral limit, Moire flat bands satisfy the Trace condition.

quantum metric:

$$|\mathrm{Tr}[g]| = |\Omega|$$

$$\eta(k) = \begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix} \Omega(k)$$

Previous examples lowest Landau Level

(See also Jie Wang et al.)

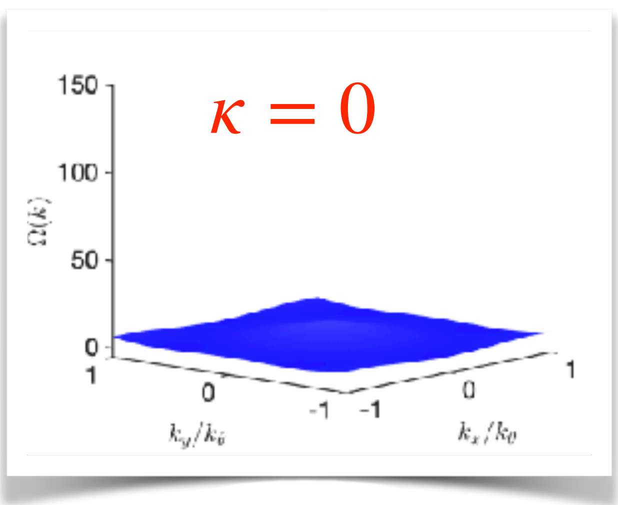
Quantum Geometry and Fractional Chern insulators

Special properties of Chiral Flat Bands:

(i) Trace Condition Satisfied

$$|\mathrm{Tr}[g]| = |\Omega|$$

(ii) Nearly *uniform* Berry curvature



(iii) Flat bands

Parameswaran et al. `13. Roy `14. Classen et al. 2014. Mera and Ozawa `21. Varjas et a

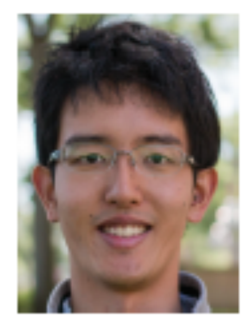
Ledwith, Tarnopolsky, Khalaf, AV `20. Jie Wang, Cano, Millis, Liu, Yang `21

Moire' Numerics: Repellin, Senthil Phys. Rev. Research. `20 Abouelkomsan, Liu, Bergholtz PRL `20 Wilhelm, Lang, Läuchli arXiv:2012.09829

Numerical DMRG Study of FCIs in TBG

arXiv: 2112.13837

Parker *et al*, 2021



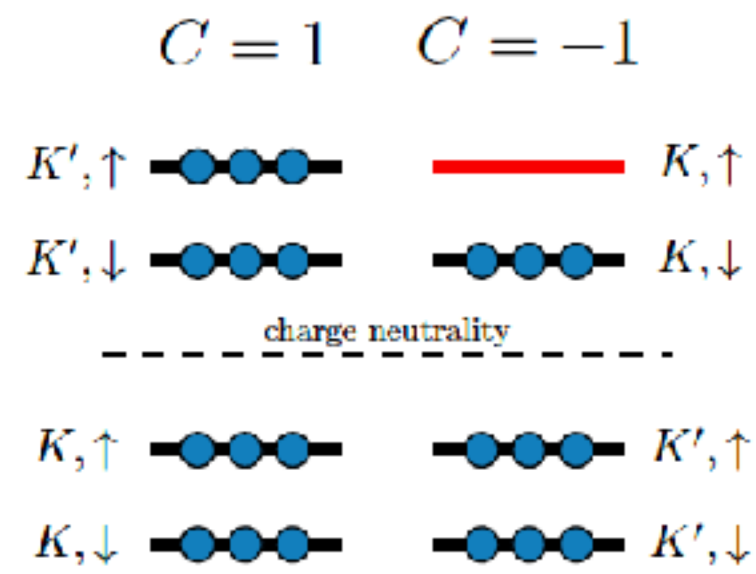
Dan Parker | Johannes Hauschild | Tomo Soejima | Mike Zalelel

Harvard | Berkeley | Berkeley | Berkeley

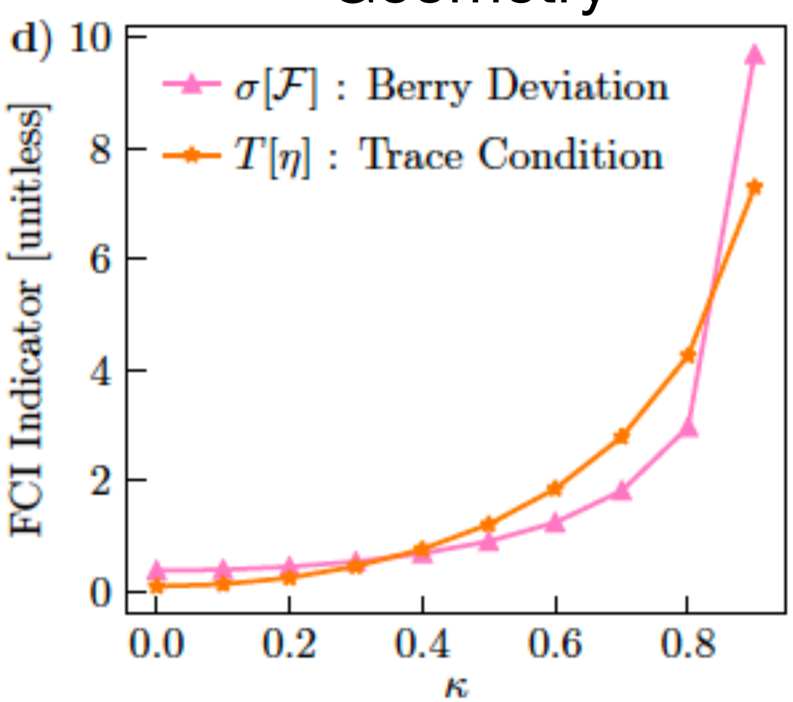
FCI Stability in TBG

Single-species model for top band:

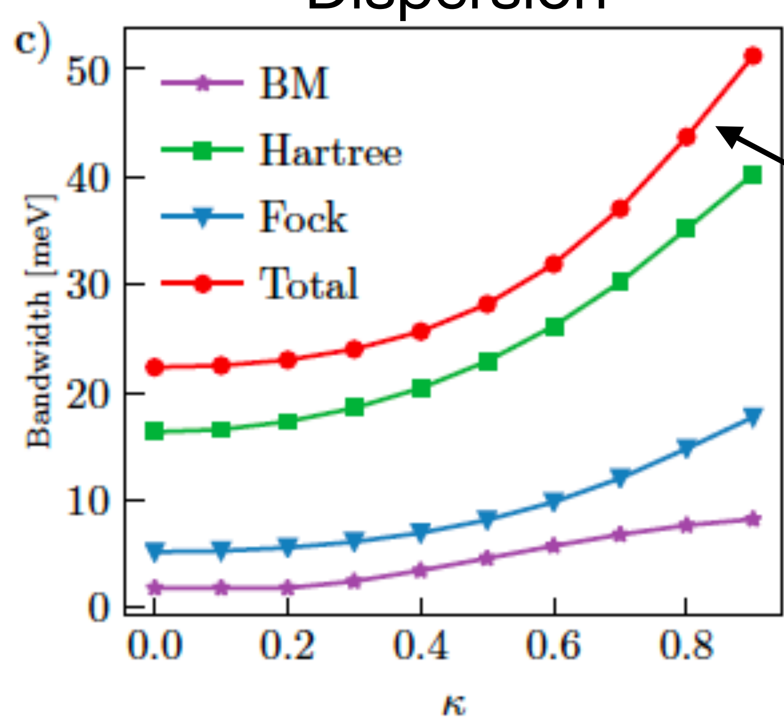
Spin and valley polarized;
C2 broken by hBN



Geometry



Dispersion



Hartree-Fock generated bandwidth
for electrons: $\nu = 3+$

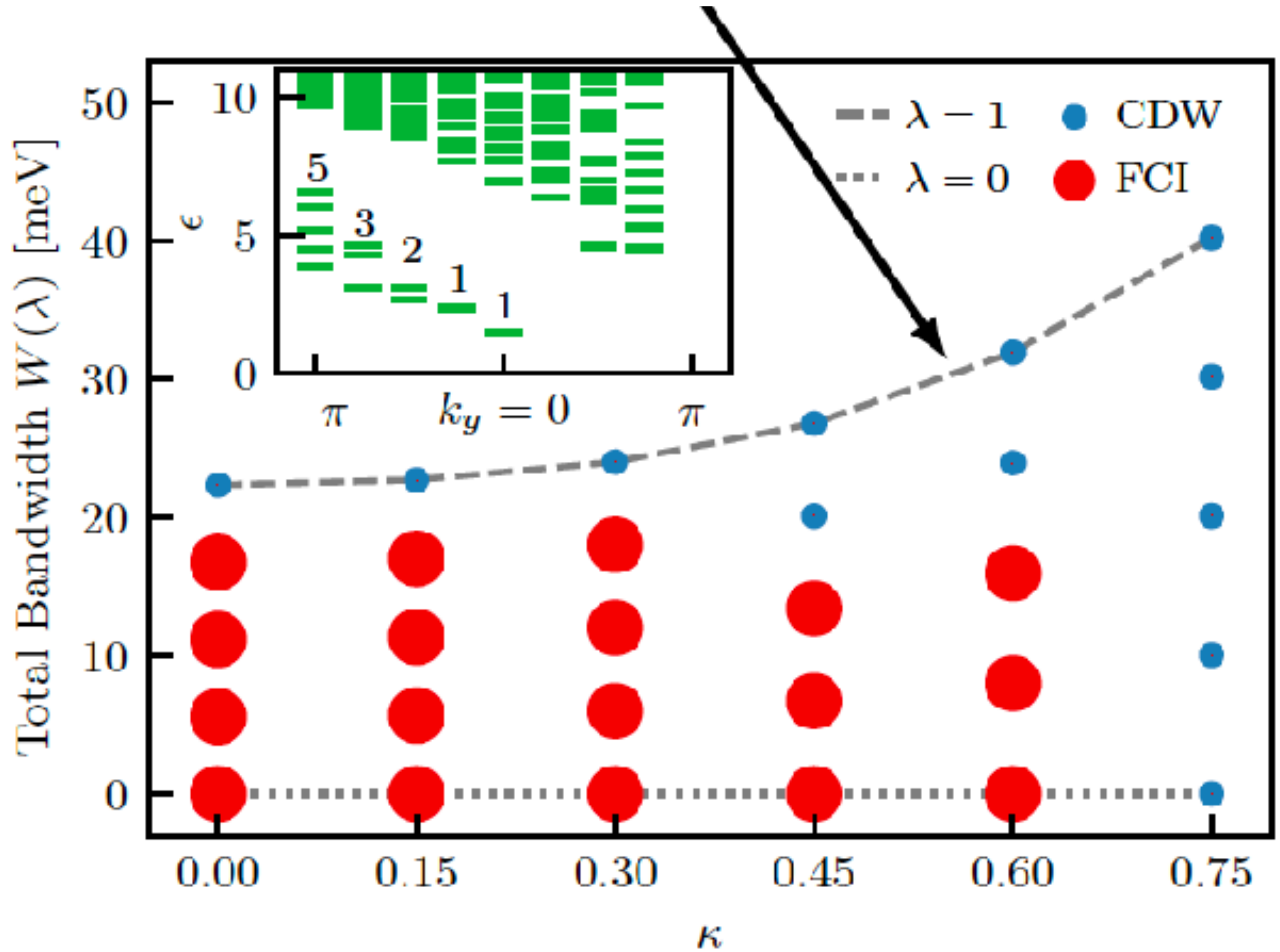
BM Model

FCI Stability in TBG

Infinite cylinder DMRG with MPO compression on

$$\hat{H}^{(\nu=3)} = \lambda \hat{h}_T + :\hat{V}_{Coulomb}\hat{\rho}\hat{\rho}:$$
$$\hat{h}_T = \hat{h}_{BM} + \hat{h}_{HF}[P]$$

Artificially reduce the bandwidth by $0 \leq \lambda \leq 1$.



Parameters:

angle $\theta \approx 1.05°$

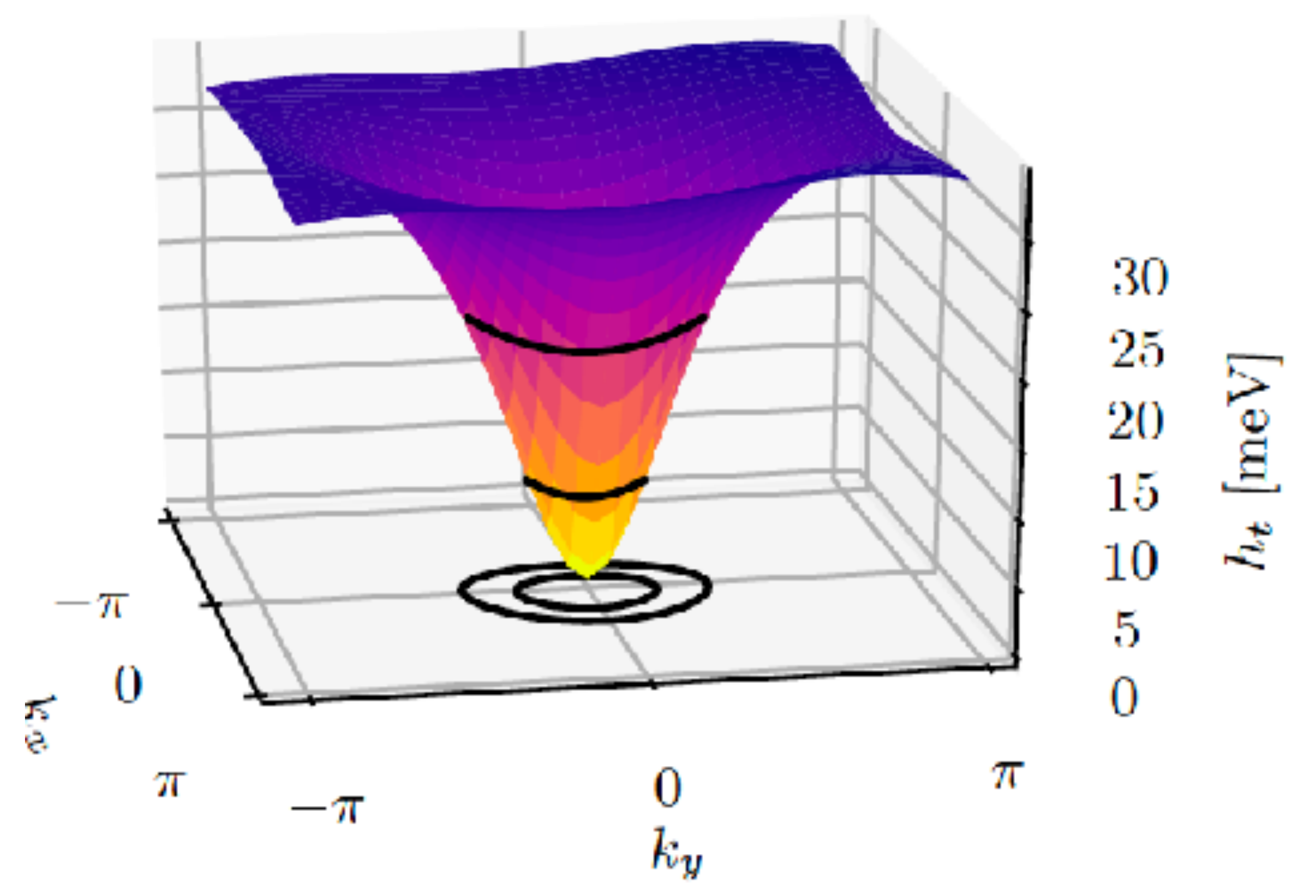
chiral ratio $\kappa \approx 0.5 - 0.8$

hBN mass a few meV

B-Field Stabilized FCI in TBG

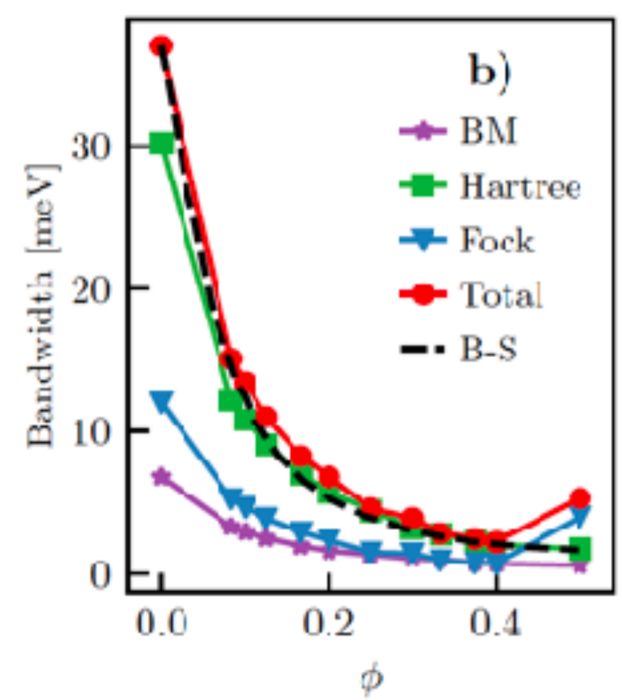
- Note, dispersion restricted to small area of BZ near Gamma.

- Effective mass at band bottom: $m^* \approx m_{\text{band}}/6$



- In a weak magnetic field - expect significant reduction of bandwidth.

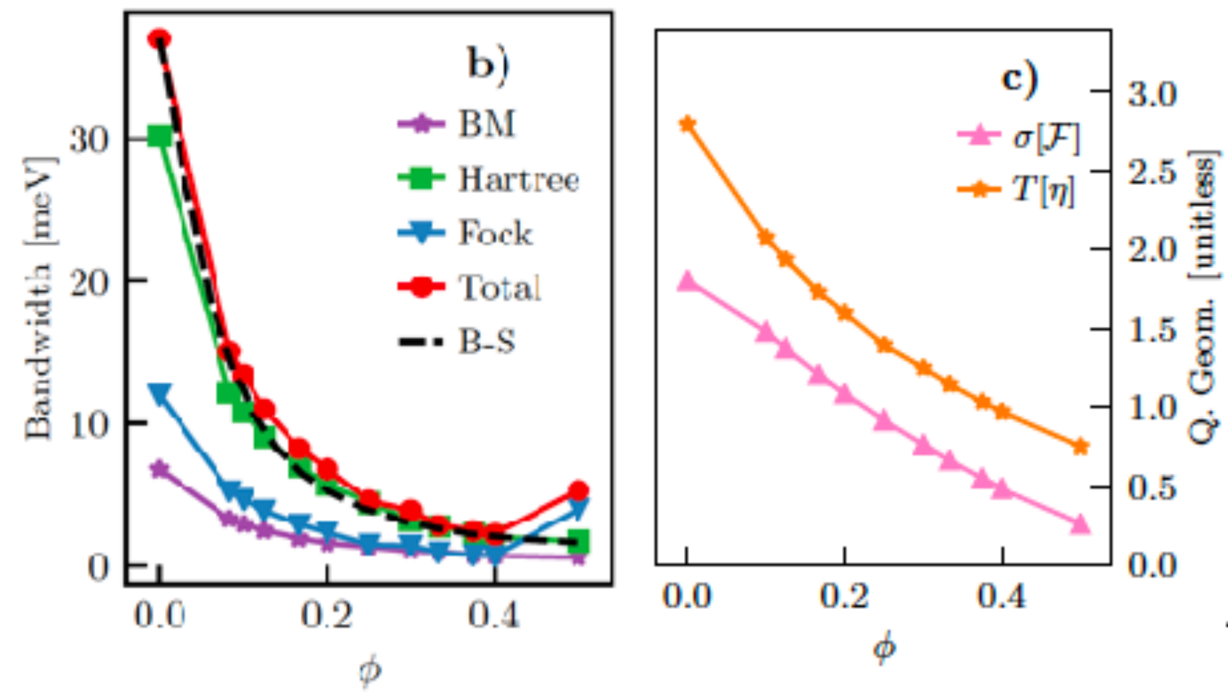
$$W = W_0 - \frac{\hbar}{2}\frac{eB}{m^*}$$



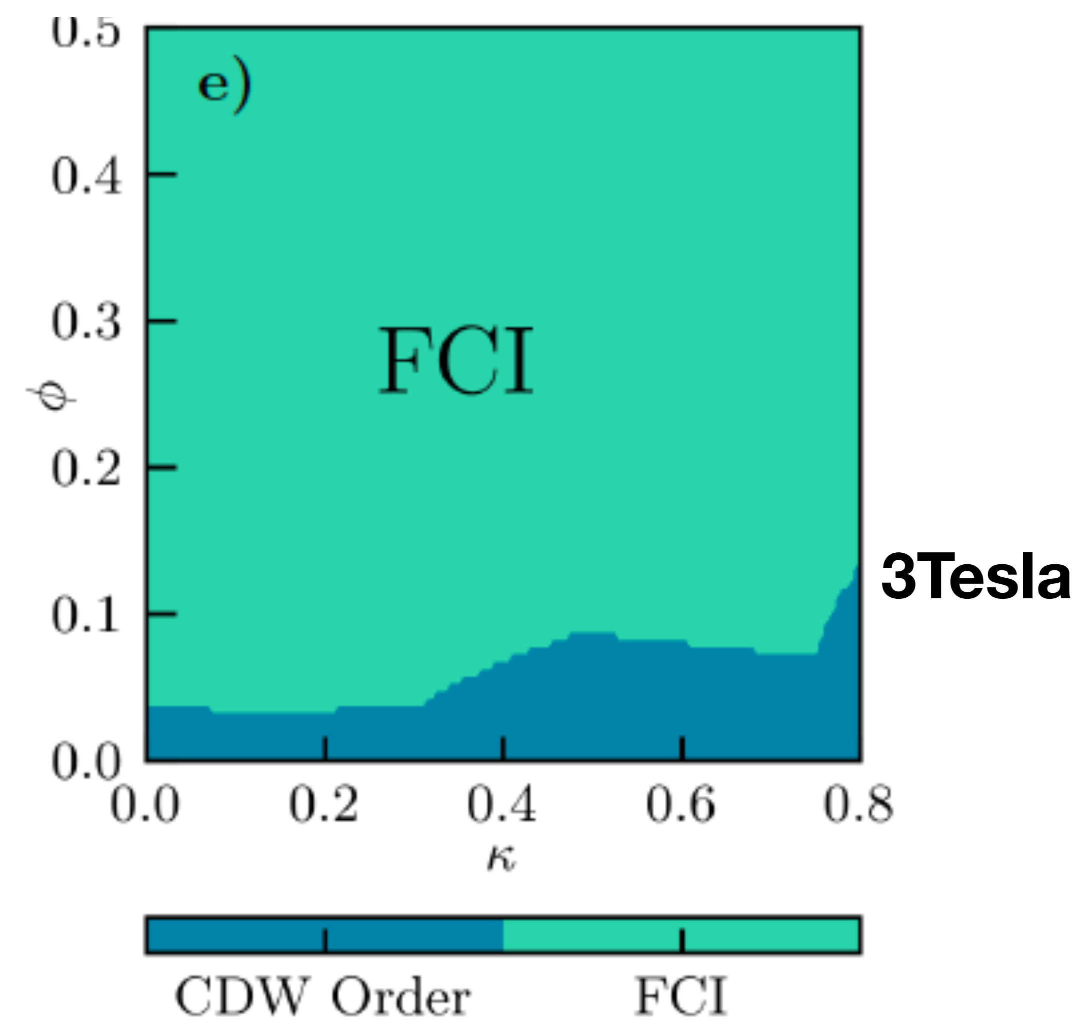
Parker et al. `21

B-Field Stabilized FCI in TBG

- Small magnetic fields *improves* band geometry and dramatically *reduces* bandwidth.



- Weak field should stabilize FCI at filling 3+2/3



Experiments - FCIs in TBG

arXiv: 2107.10854

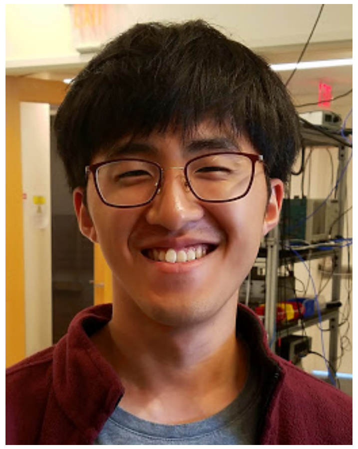
Xie *et al*, *Nature* **600**, 439–443 (2021)

Amir

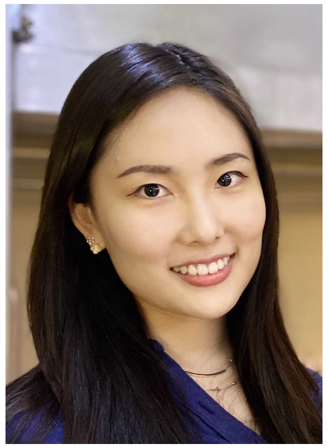
Harvard

Yonglong Xie

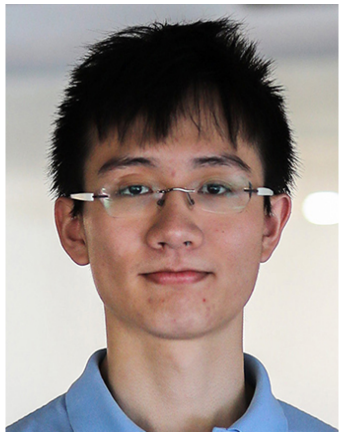
Andrew Pierce



Seung Hwan Lee



Jeong Min Park



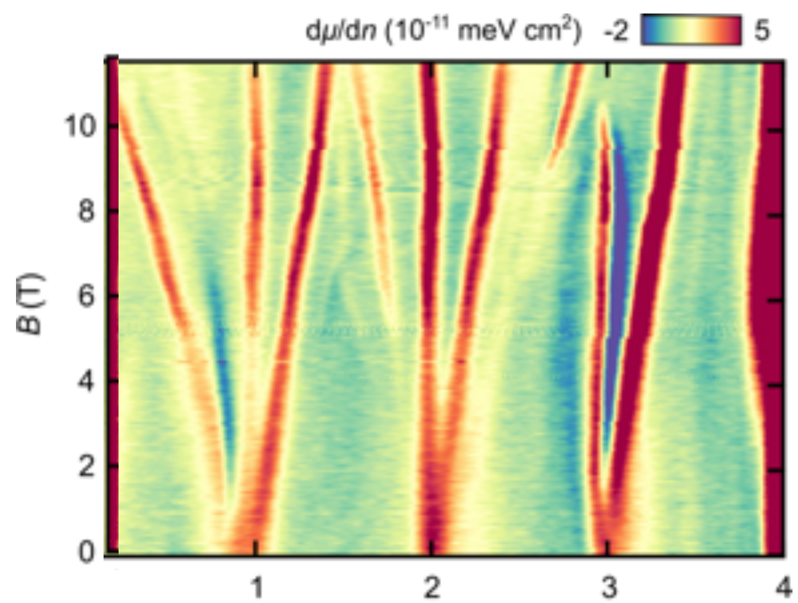
Yuan Cao

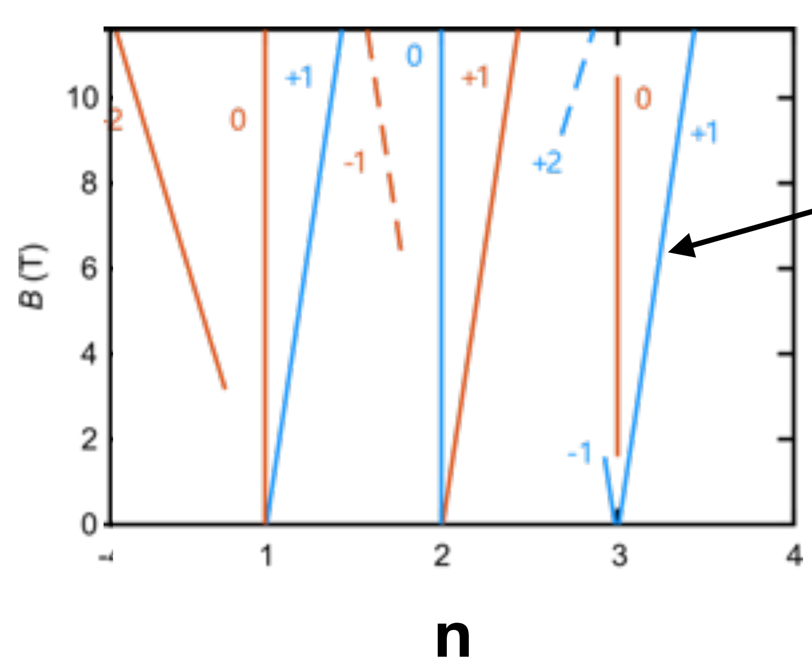
Pablo Jarillo-Herrero

Identifying FCIs

Yacoby Group - Measure compressibility
Identify incompressible states in (n, B) plane

$$n = \color{red}{C}\frac{\phi}{\phi_0} + \color{blue}{s}$$





$$(\color{red}{C}, \color{blue}{s}) = (+1, +3)$$

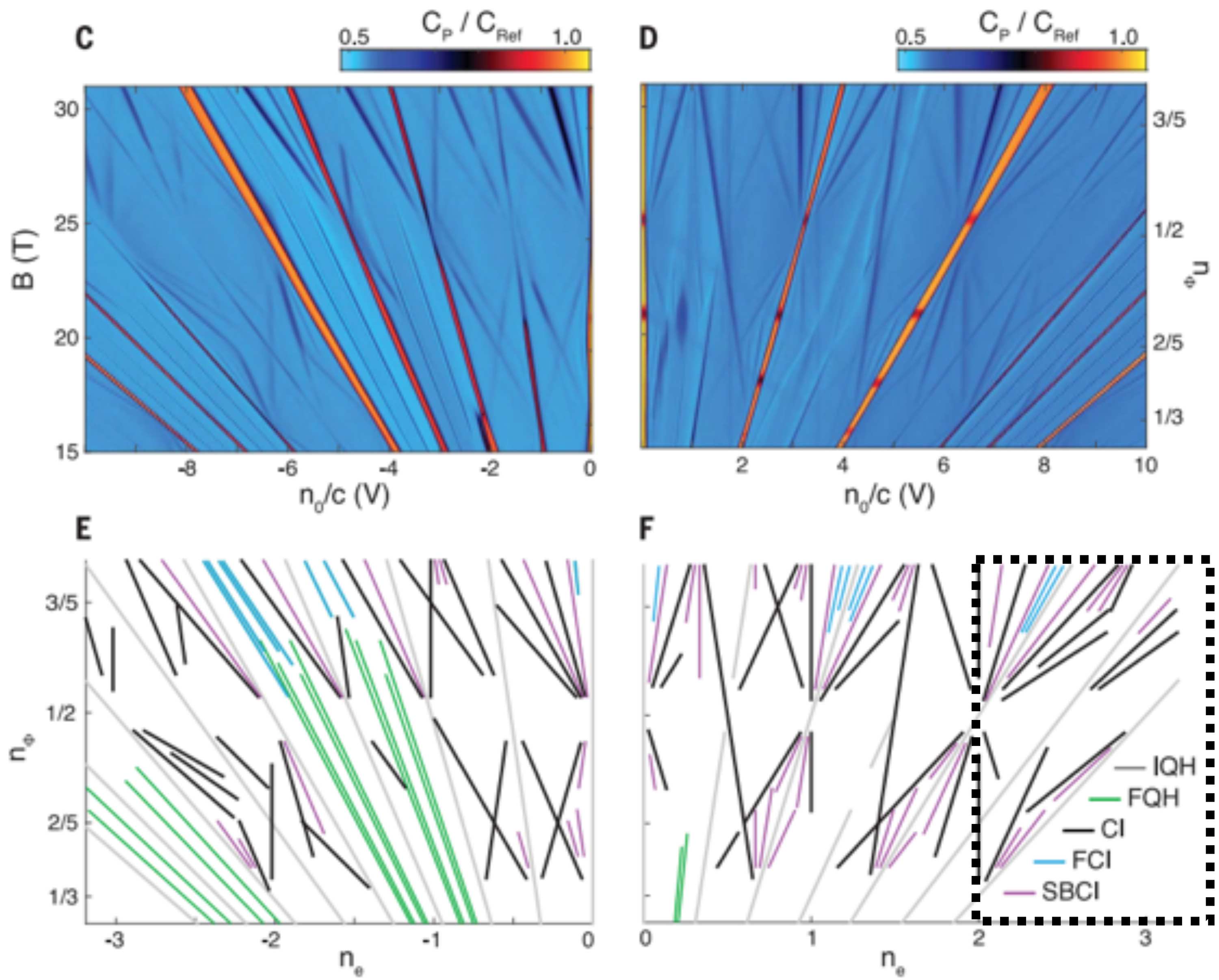
	C	s
Chern Insulator CI	Integer	Integer
CDW	0	Fractional
Symm Breaking CI	Integer	Fractional
FCI	Fractional	Fractional

Earlier Work - FCIs in Hofstadter Bands

Spanton et al, Science (2018)

id	t	s	B [T] (min,max)
F1	2/3	-1/3	(28,32)
F2	4/3	1/3	(28,39)
F3	5/3	1/6	(29,31*)
F4	7/3	-1/6	(35,40)
F5	8/3	-1/3	(29,31*) (35,40)
F6	10/3	1/3	(28,39)
F7	11/3	1/6	(28,31*)
F8	8/3	-2/3	(35,40)
F9	4/3	2/3	(36,42)
F10	10/3	2/3	(36,42)
F11	-13/3	1/3	(25,36)
F12	-22/5	2/5	(27,32*)
F13	-23/5	3/5	(27,32*)
F14	-14/3	2/3	(26,38)
F15	-11/3	2/3	(28,32*) (35,39)
F16	-10/3	1/3	(28,32*) (35,39)
F17	-11/3	-1/3	(30,36)
F18	-10/3	-2/3	(30,38)
F19	-2/3	1/3	(29.5,31.5)

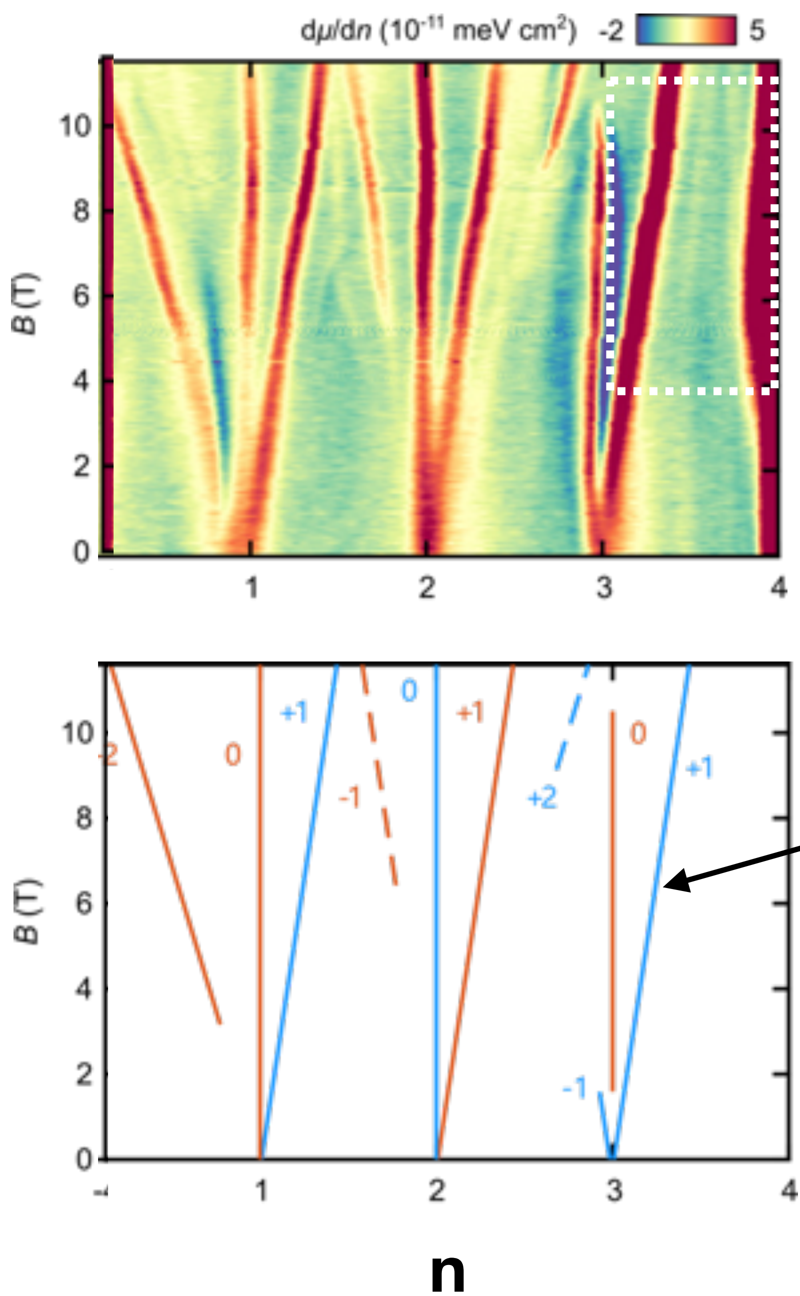
s,



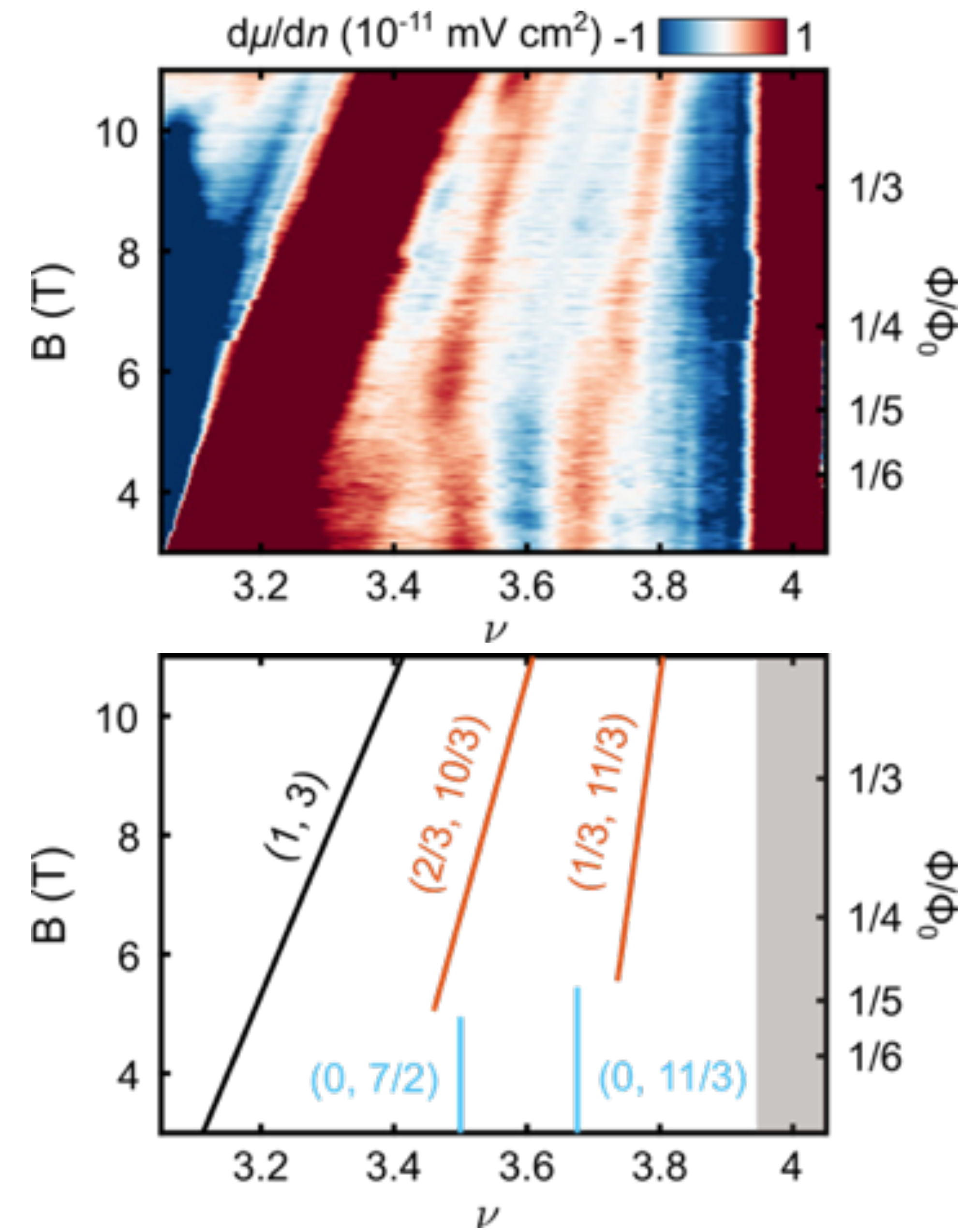
High field FCIs at ≥ 25 Tesla

Magnetic Field Creates Chern bands

FCIs in Chern Bands



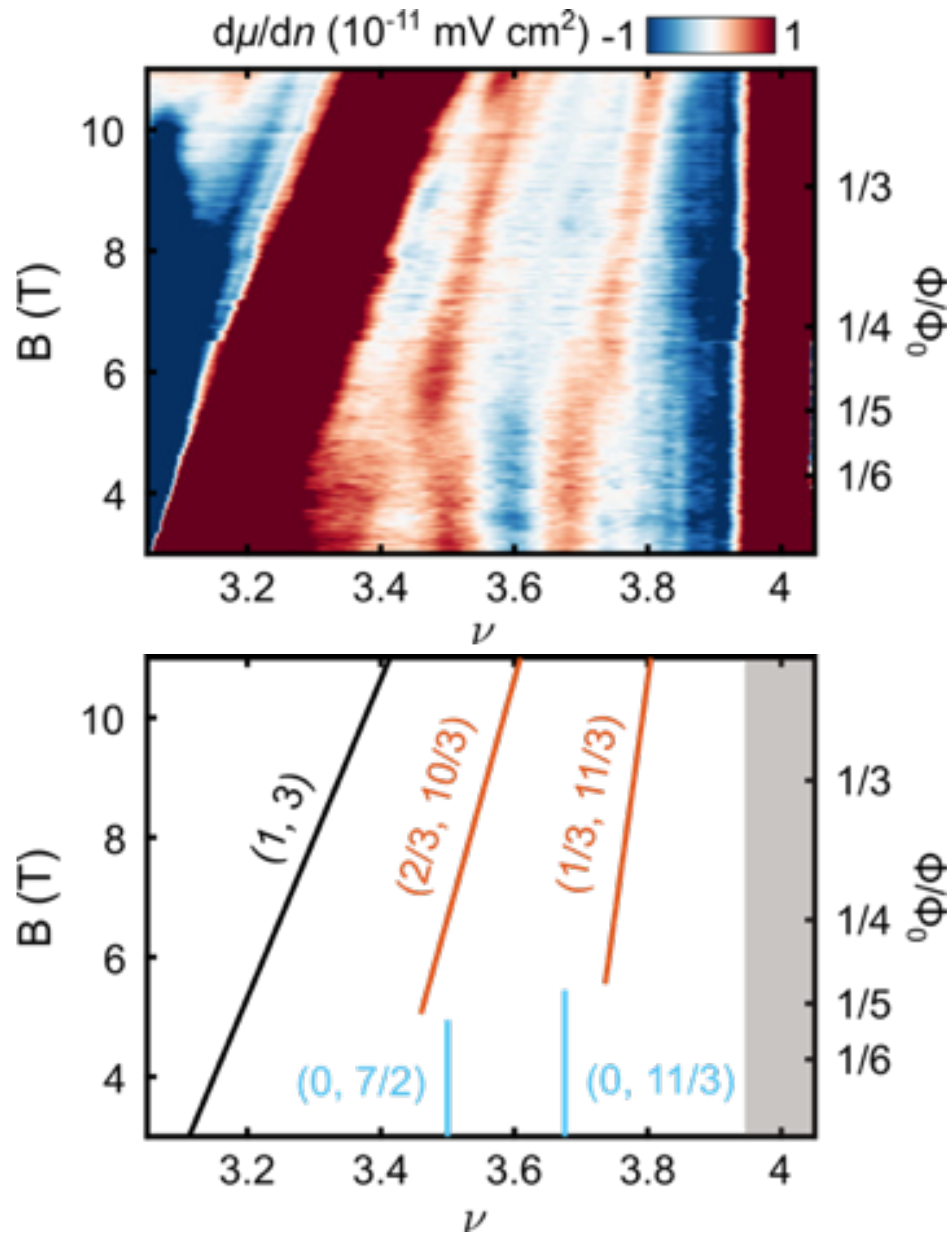
$(\textcolor{red}{C}, \textcolor{blue}{s}) = (+1, +3)$

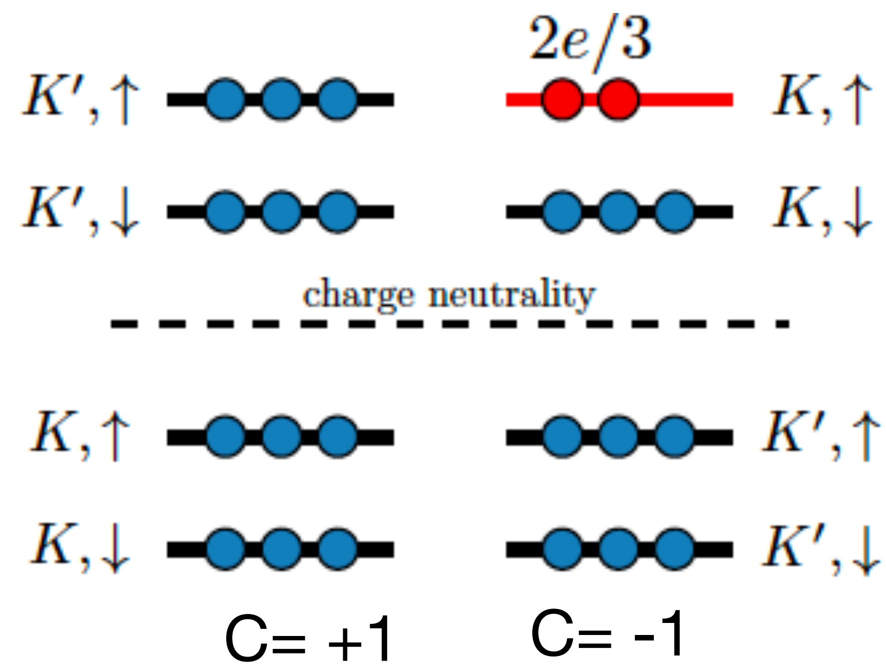


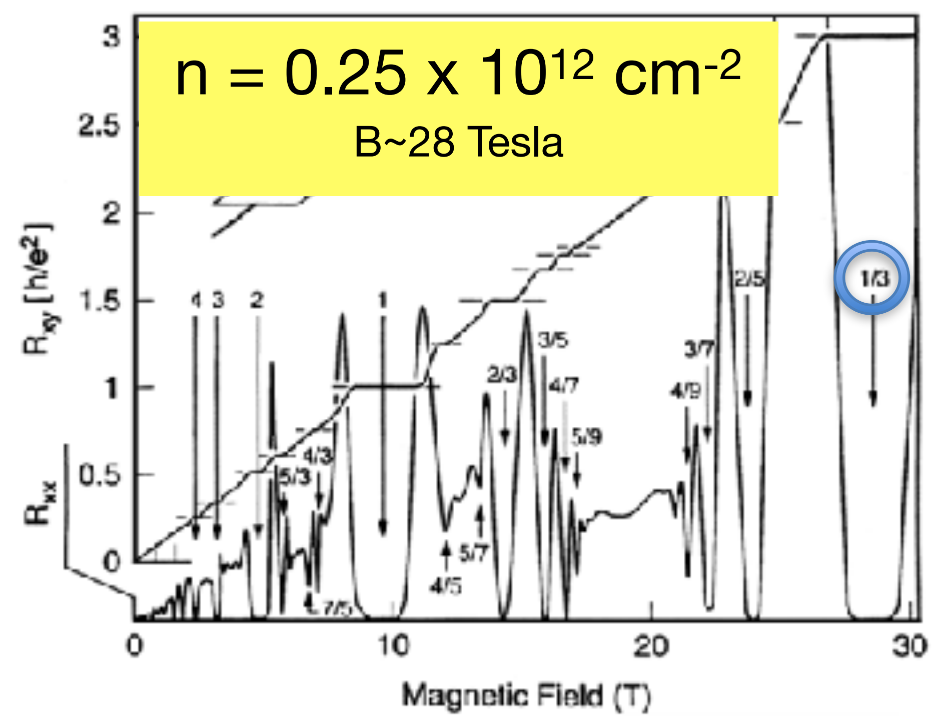
Y. Xie, A. Pierce…Y. Yacoby. arxiv:2107:10854

FCIs in Moire Graphene

FCIs in 5-6 Tesla







Numerics + theory: ~2-3 Tesla

Same density of holes
Requires ~28Tesla for FQH state

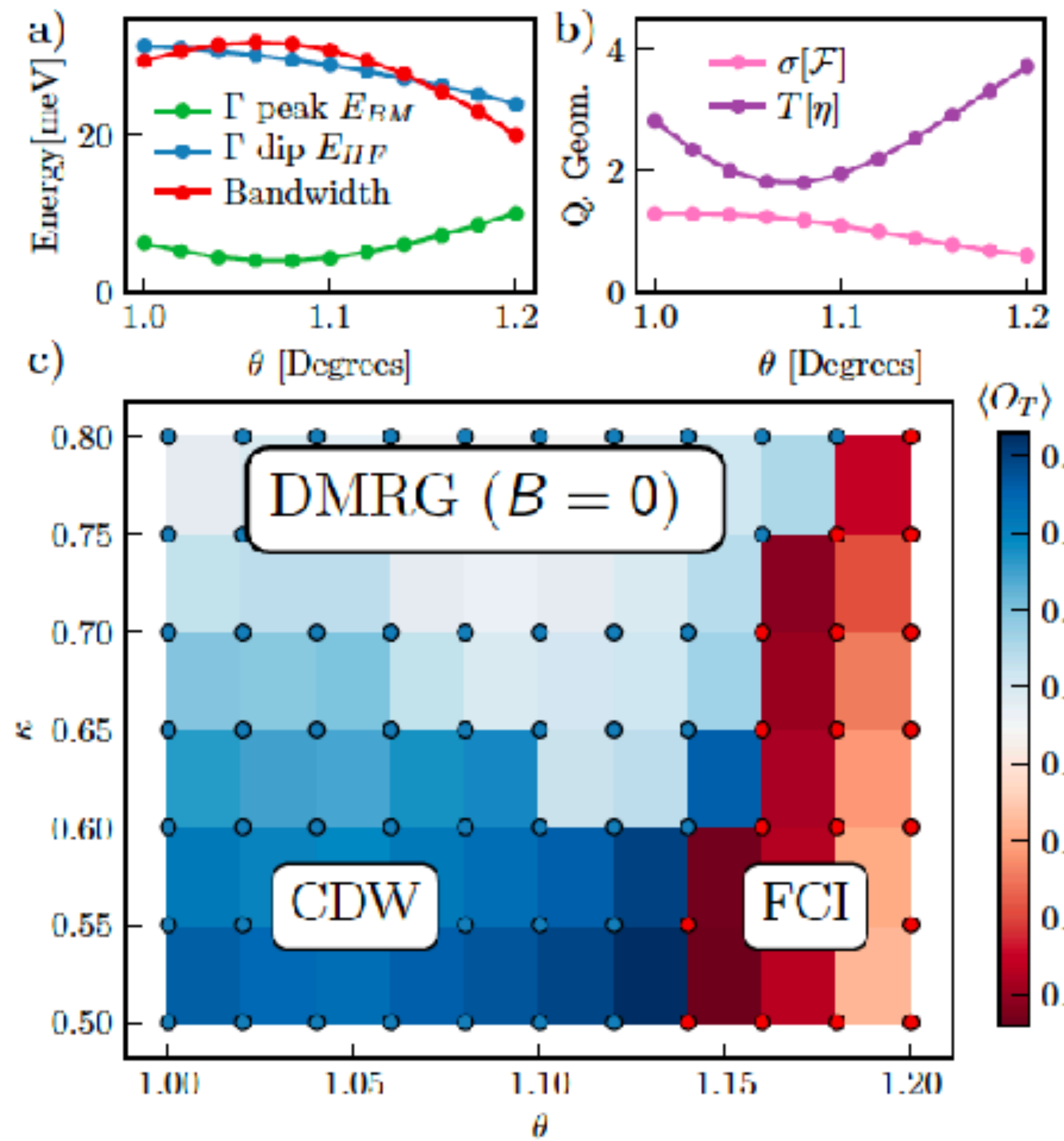
Zero Field FCI?

FCIs at $B = 0$

- Reducing bandwidth slightly gives an FCI
- Increasing angle reduces bandwidth.
- Quantum geometry still good enough
- Therefore we *predict*:

> Zero-field FCIs in hBN-aligned TBG near $\theta \approx 1.17^\circ$

- DMRG confirms
- Experimentally observable?



arXiv: 2112.13837
Parker *et al*, 2021

Conclusions and Future Directions

- Unique Quantum Band geometry of Magic angle graphene holds the seed to realizing FCIs!
  - Zero field @magic angle - FCIs seem close but need a weak field to stabilize
  - Slightly larger angles - DMRG predicts zero field FCIs.
    - Role of strain?
  - Many other fractions including even denominator experimentally observed!

Collaborators

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Experiment: Kim group, Yacoby & Pablo Jarillo Herrero group, Yazdani group.