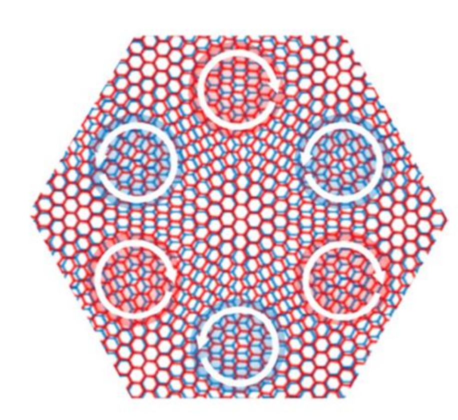
# Chern insulators



- ❖ Magic angle Twisted bilayer graphene
    - Chern insulators
- ❖ Strain induced Flat bands
    - Pillars
    - Buckled graphene
- ❖ Vacancy states:
    - Atomic collapse
    - Kondo screening

E.Y. Andrei

# Flat band superconductivity and topology

London equation $\vec{j}_s = -D_s \vec{A}$

Superfluid current

Superfluid weight

**Superfluidity in topologically nontrivial flat bands**

Sebastiano Peotta & Päivi Törmä

*Nature Communications* **6**, Article number: 8944 (2015)

$$\psi_{\vec{k}}(\vec{r}) = e^{i\vec{k}\cdot\vec{r}} u_{\vec{k}}(\vec{r})$$

$$D_s = D_s^{conv} + D_s^{Geom}$$

Multi-band

Ginsburg Landau - isolated band

$$D_s^{conv} = e^2 n_s / m^*$$

Flat band: $m^* \to \infty$

$$D_s^{conv} \to 0$$

no supercurrent

Geometric contribution

$$D_s^{Geom} \propto C$$

Topological invariant

Chern number

Flat band+nontrivial band topology

$$D_s \geq D_s^{Geom} \propto |C|$$

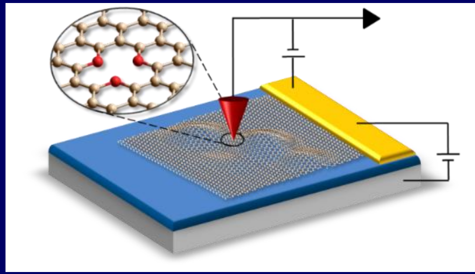
Finite supercurrent

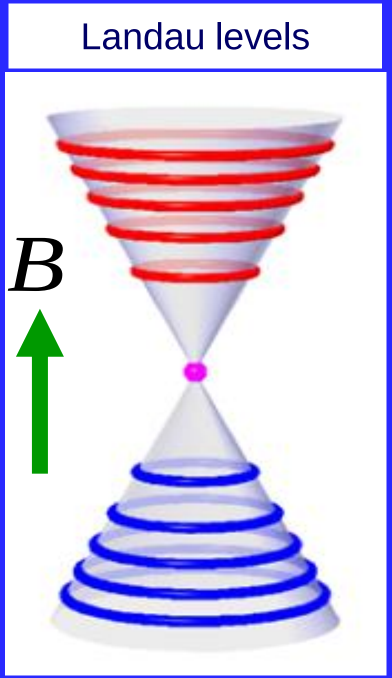
**Needed: flat bands with non-trivial topology**

E.Y. Andrei

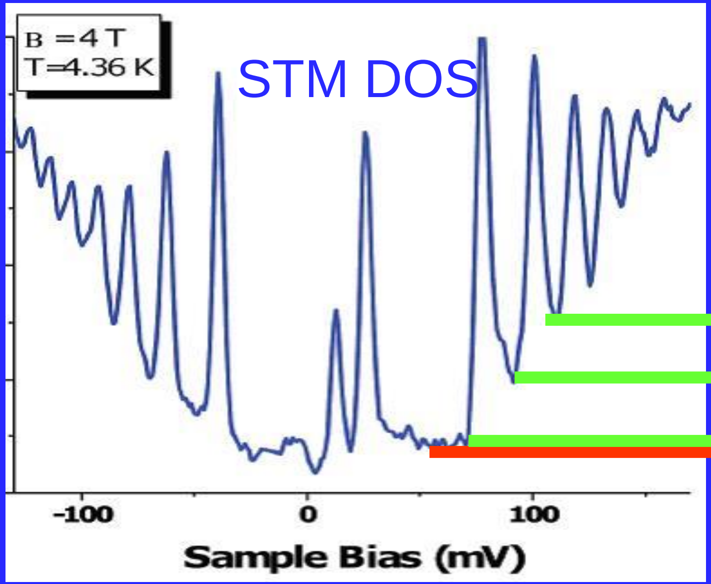
# Landau levels – topological flat bands

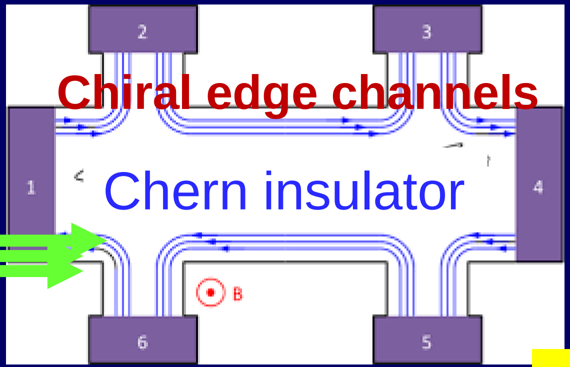
G. Li, A. Luican, E.Y.A, PRL, 102, (2009)
G. Li, E.Y.A., Nat. Phys , 3 (2007), 623

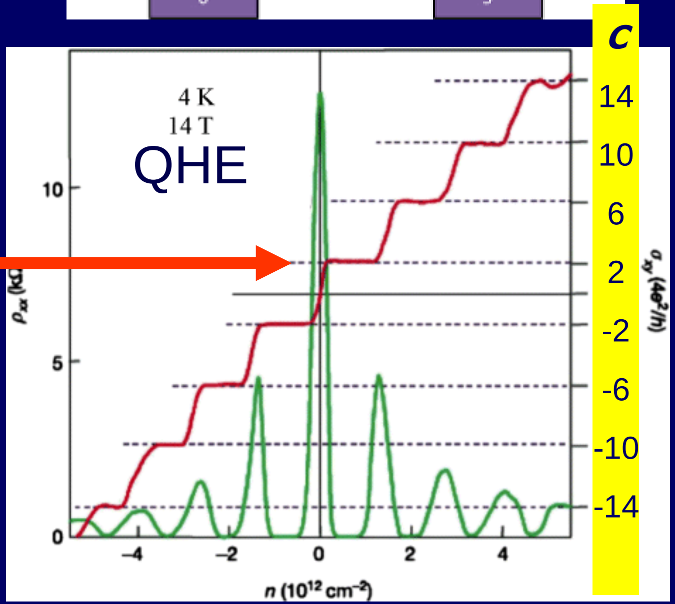





STM - Landau levels




flat bands + non-trivial topology

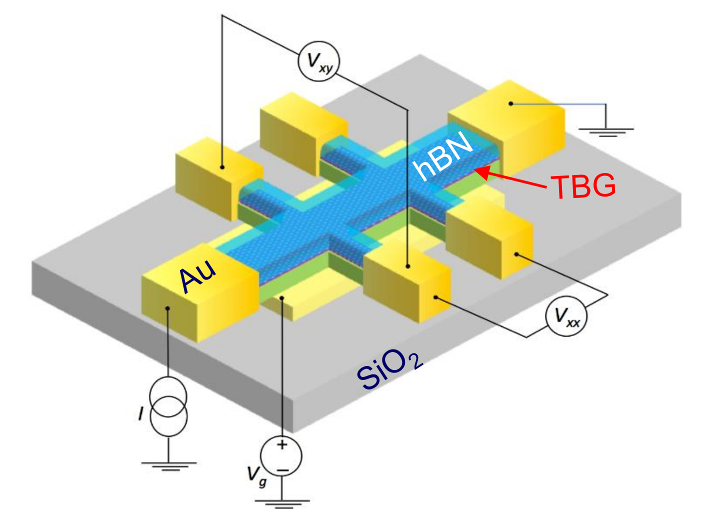
$$\sigma_{xy} = \frac{e^2}{h} C$$

Flat band in MATBG – Non-trivial topology?

E.Y. Andrei

# Filling the flat band – and correlation induced gaps

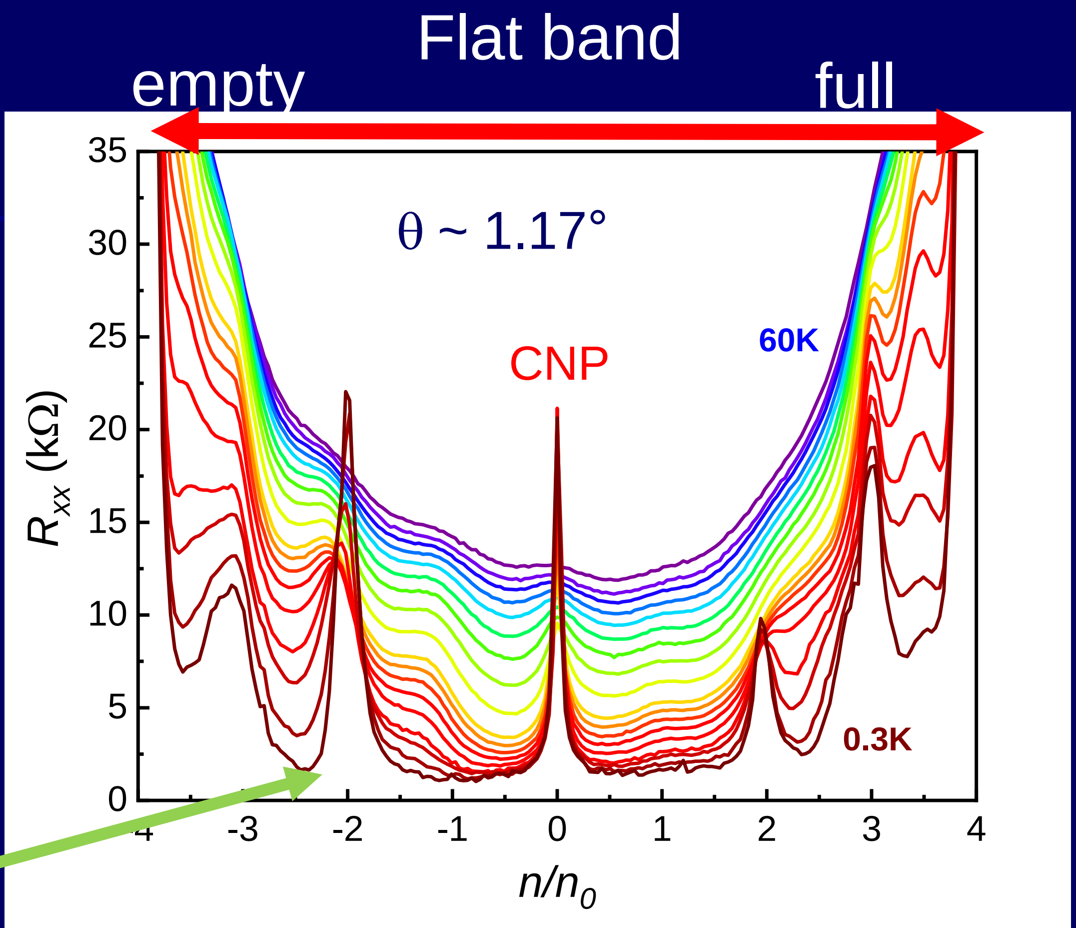
S. Wu,.. EYA, *Nature Mat* 20,488 (2021)

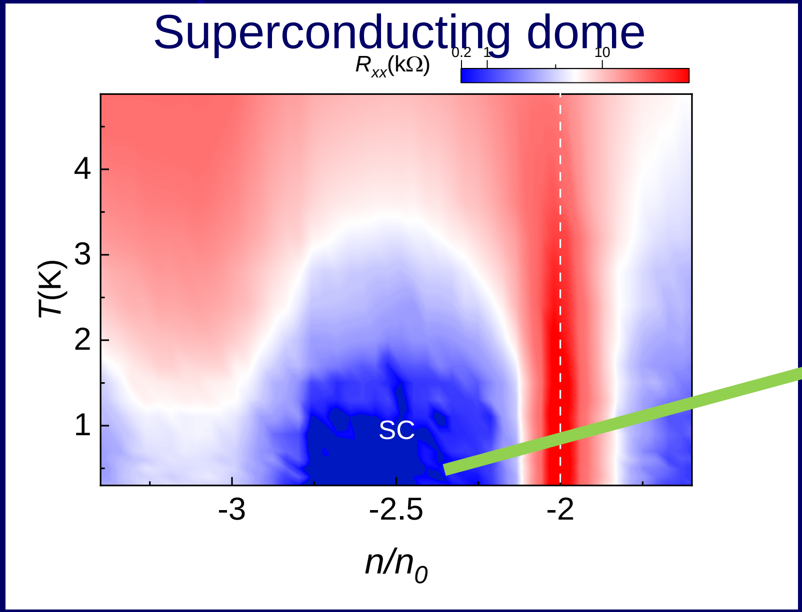



$n/n_0$ = # of carriers/moire cell

$n_0$ = one carrier/ moire cell

| $n/n_0$ | state |
|---|---|
| -3 | x |
| -2 | Insulating |
| -1 | x |
| 0 | Insulating |
| 1 | x |
| 2 | Insulating |
| 3 | x |



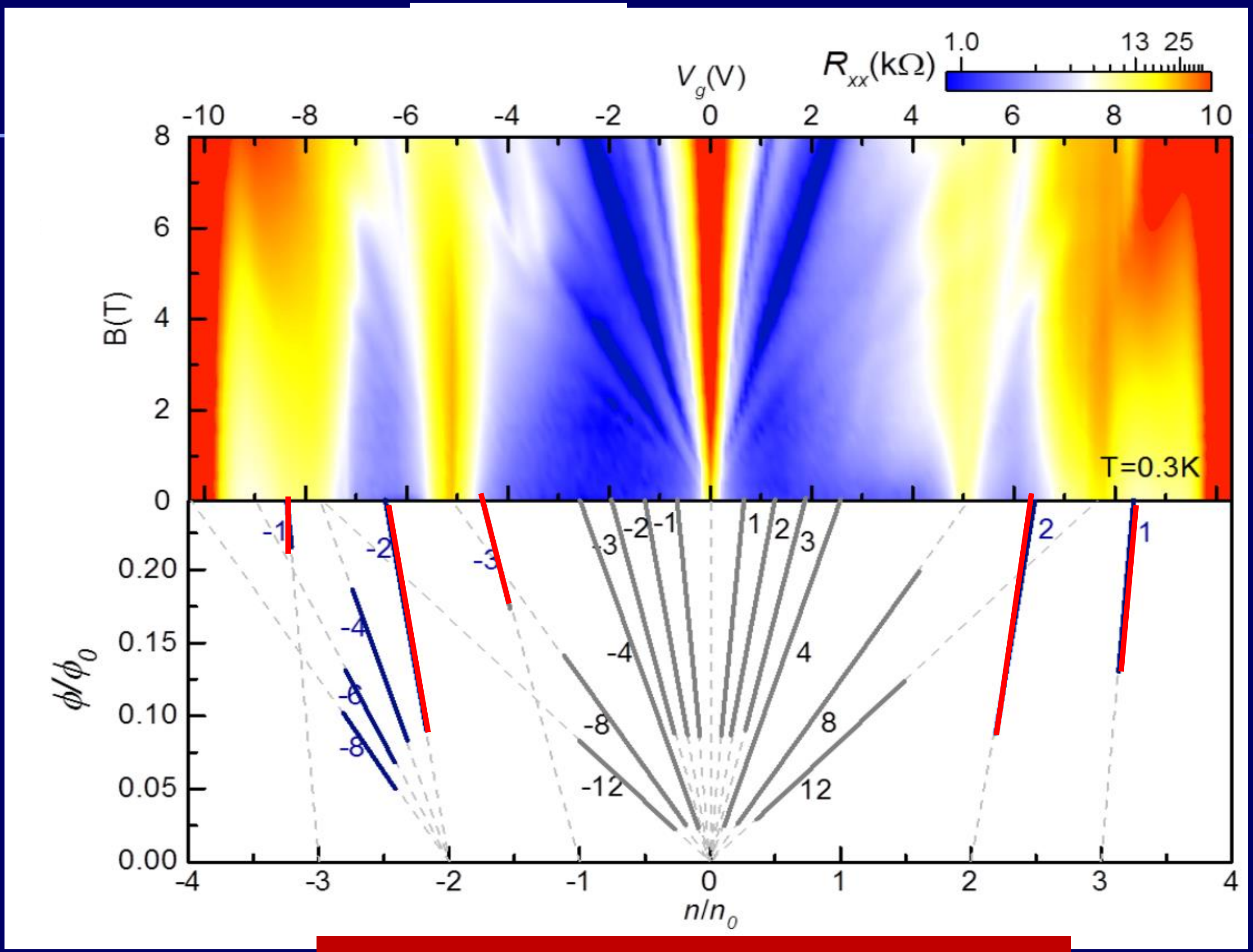




Non-trivial topology?

E.Y. Andrei

# Unilateral Landau fans

S. Wu,.. EYA, Nature Mat 20,488 (2021)




Magnetic flux lattice + periodic potential + interactions

$$n/n_0 = s + \nu(\phi/\phi_0)$$

$$s = 0 \text{ or } s \cdot \nu > 0$$

s: bloch-band index
ν: Landau level index
$\phi_0$ : quantum flux unit

Rigid bands produce bilateral Landau fans
G/hBN ↦ Hofstadter butterfly

Unilateral Landau fans
- ➢ malleable band structure

# Flat band in Chern basis

## Layer basis

$$H_K = v_F \begin{pmatrix} \vec{\sigma}_{\theta/2}\cdot\vec{p} & U(\vec{r}) \\ U^{+}(\vec{r}) & \vec{\sigma}_{-\theta/2}\cdot\vec{p} \end{pmatrix} \begin{pmatrix} 1A \\ 1B \\ 2A \\ 2B \end{pmatrix}$$

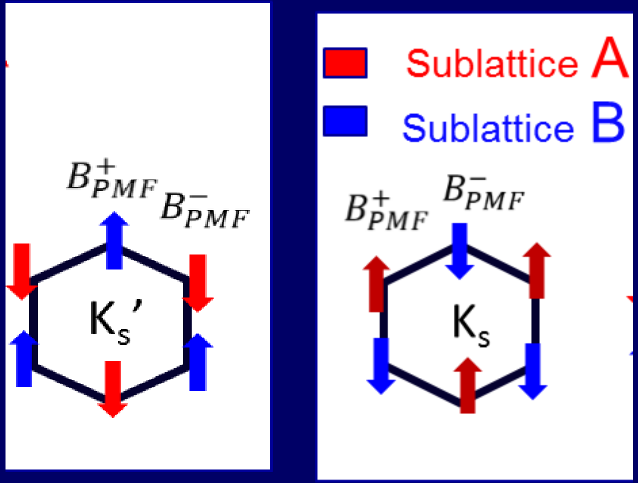
Layer polarized

**Change Basis** →

## Sublattice basis

$$H_K = v_F \begin{pmatrix} \vec{\sigma}\cdot(\vec{p}-e\vec{A}) & \sim 0 \\ \sim 0 & \vec{\sigma}\cdot(\vec{p}+e\vec{A}) \end{pmatrix} \begin{pmatrix} A+ \\ A- \\ B+ \\ B- \end{pmatrix}$$

Sublattice polarized




Pseudo magnetic field

$$\vec{B}_{pmf} = \pm\nabla\times\vec{A} \sim 120T$$

PMF-Sublattice Locking $\mapsto$ 8 states

Chern basis

| $+B_{PMF}$ C = +1 | | $-B_{PMF}$ C = -1 | |
|---|---|---|---|
| $\uparrow\downarrow, K, A$ | $\uparrow\downarrow, K', B$ | $\uparrow\downarrow, K', A$ | $\uparrow\downarrow, K, B$ |


Liu *et al.* PRB **99**, 155415 (2019)
Butnick *et al.* PRL **124**, 166601 (2020)
Kang & Vafek PRL 2019
Tarnopolski.. Viswanath , PRL (2019)

# *Flat band in Chern basis $\mapsto$ N=0 pseudo-Landau level*

Layer basis

$$H_K = v_F \begin{pmatrix} \vec{\sigma}_{\theta/2}\cdot\vec{p} & U(\vec{r}) \\ U^+(\vec{r}) & \vec{\sigma}_{-\theta/2}\cdot\vec{p} \end{pmatrix} \begin{pmatrix} 1A \\ 1B \\ 2A \\ 2B \end{pmatrix}$$

Layer polarized

Change Basis

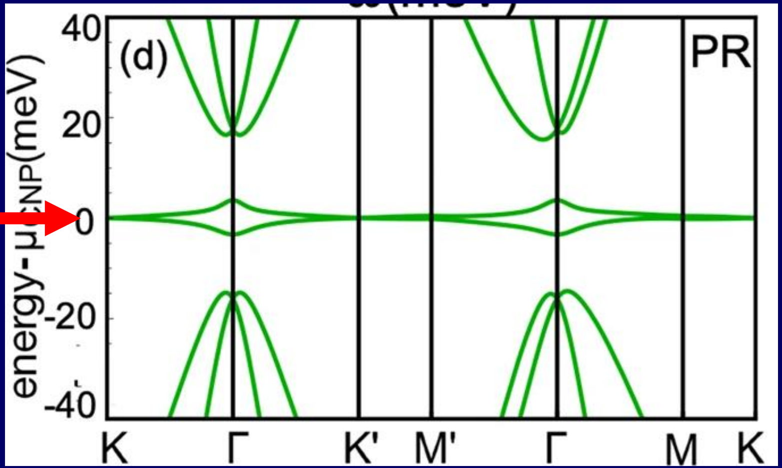
Sublattice basis

$$H_K = v_F \begin{pmatrix} \vec{\sigma}\cdot(\vec{p}-e\vec{A}) & \sim 0 \\ \sim 0 & \vec{\sigma}\cdot(\vec{p}+e\vec{A}) \end{pmatrix} \begin{pmatrix} A+ \\ A- \\ B+ \\ B- \end{pmatrix}$$

Sublattice polarized

N=0 pseudo Landau level
8-fold Degeneracy protected
by $C_{2z}T$ symmetry

180° rotation    Time reversal

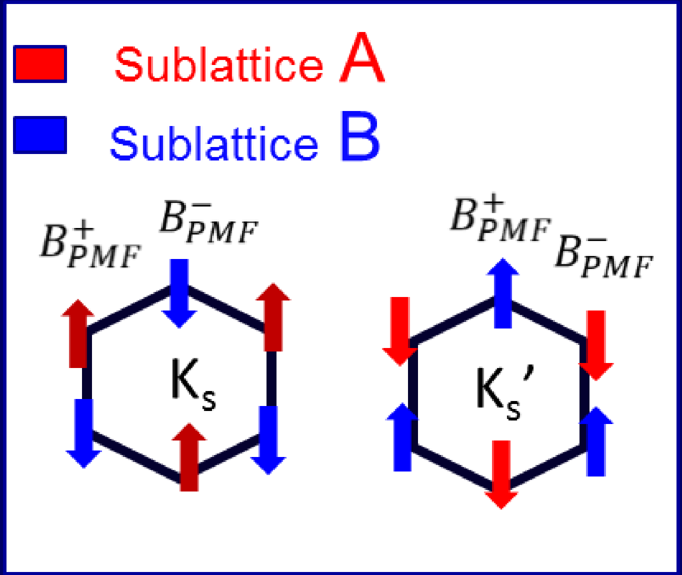



Pseudo magnetic field

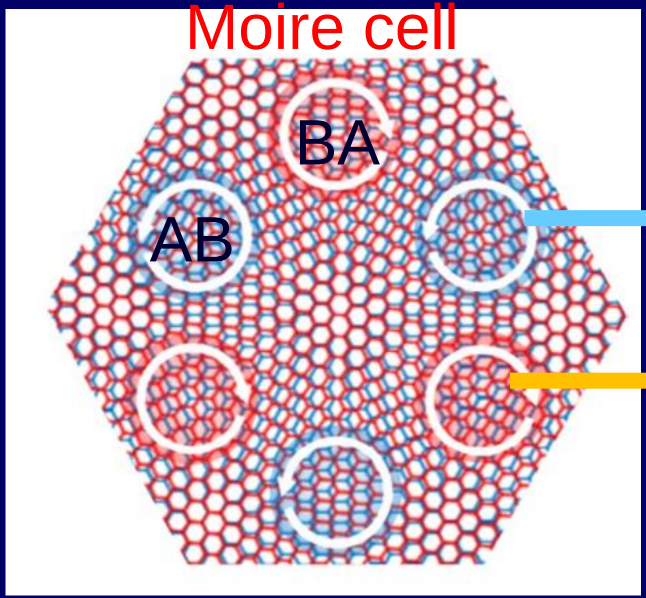
$$\vec{B}_{pmf} = \pm\nabla\times\vec{A} \sim 120T$$

Liu *et al.* PRB **99**, 155415 (2019)
Butnick *et al.* PRL **124**, 166601 (2020)
Kang & Vafek PRL 2019
Tarnopolski.. Viswanath , PRL (2019)

# Pseudo-magnetic fields induced by Moire potential




PMF-Sublattice Locking




Chiral currents

$B_{PMF}$ C=+1

$-B_{PMF}$ C= -1

Diamagnetic current loops: alternating orbital magnetic moments

Chern basis

| $+B_{PMF}$ C = +1 | $-B_{PMF}$ C = -1 |
|---|---|
| $\uparrow\downarrow, K, A$   $\uparrow\downarrow, K', B$ | $\uparrow\downarrow, K', A$   $\uparrow\downarrow, K, B$ |

Liu *et al.* PRB **99**, 155415 (2019)
Butnick *et al.* PRL **124**, 166601 (2020)
Kang & Vafek PRL 2019
Tarnopolski.. Viswanath , PRL (2019)

# Breaking $C_2$ symmetry to reveal chirality

- Break $C_{z2}$
- staggered potential –hBN substrate

Filling $n/n_0 = +3$

$+B_{PMF}$ C = +1

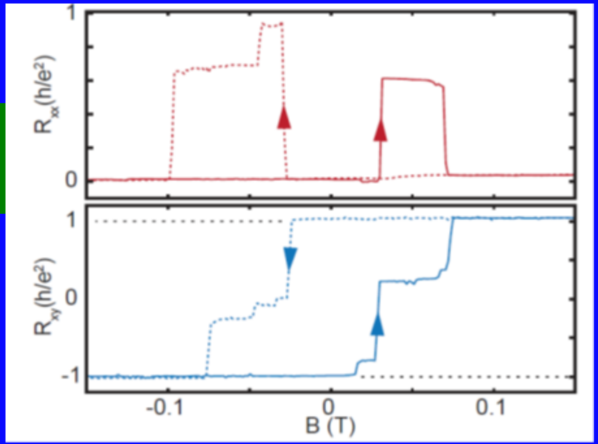
$\uparrow, K, A$ $\downarrow, K, A$ $\uparrow, K', B$ $\downarrow, K', B$

$-B_{PMF}$ C = -1

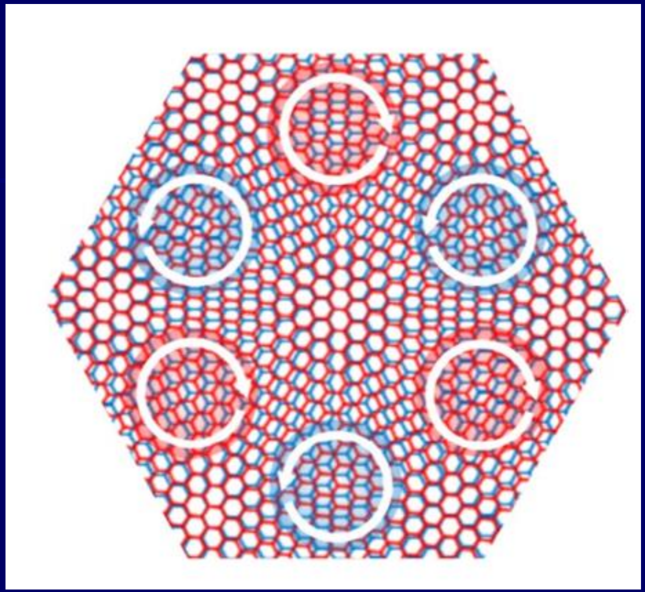
$\uparrow, K', A$ $\downarrow, K', A$ $\uparrow, K, B$ $\downarrow, K, B$

$$AQHE : n/n_0 = +3 \Rightarrow C = +1$$

Sharpe et al 2020
Serlin et al 2019

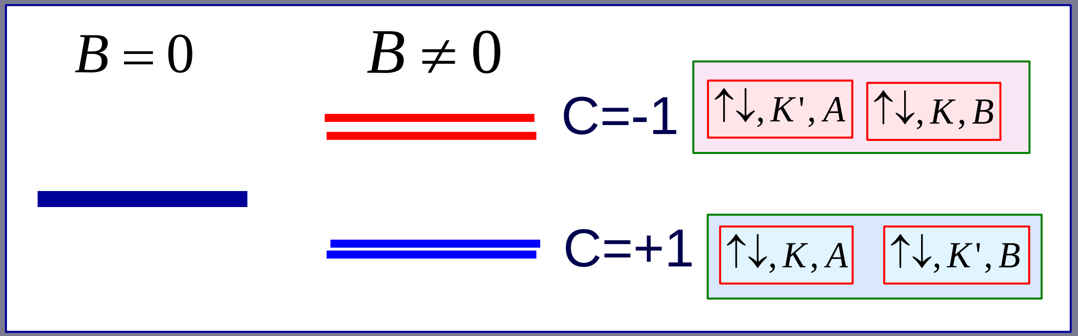



Orbital magnetism

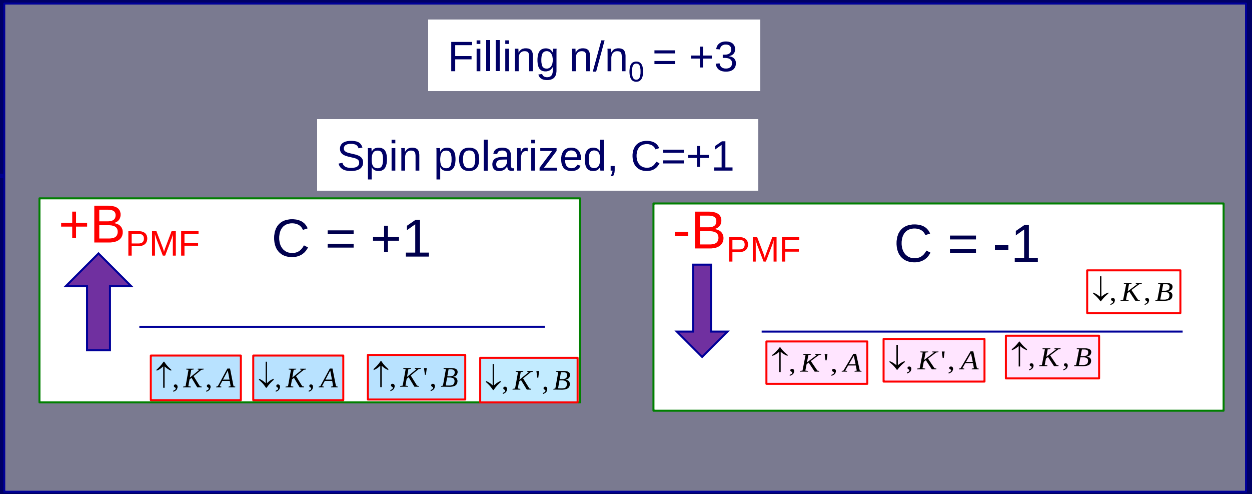


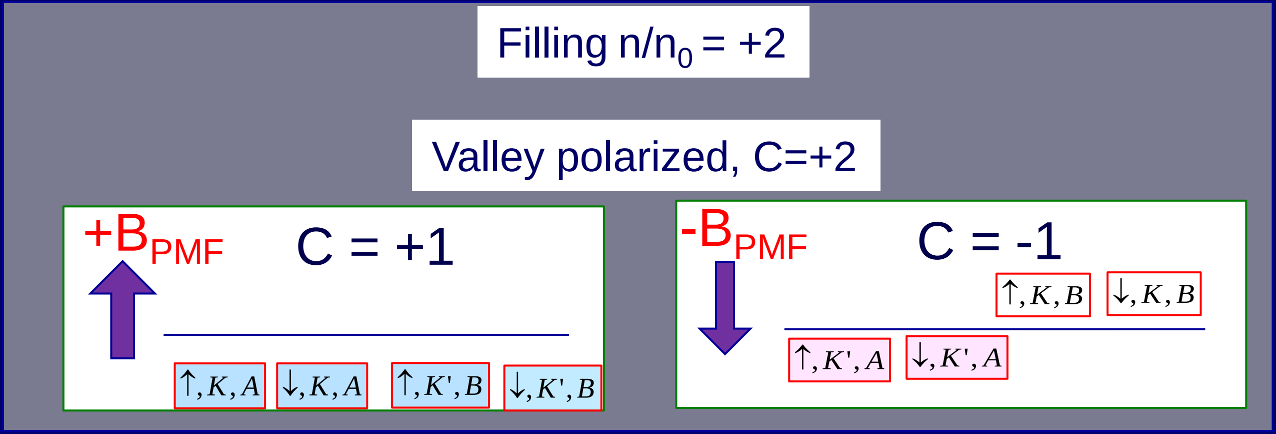
# Breaking T symmetry to reveal chirality

## Breaking T with magnetic field




| $n/n_0$ | C | |
|---|---|---|
| 3 | 1 | Spin polarized |
| 2 | 2 | Valley polarized or IVC |
| 1 | 3 | Spin polarized |
| 0 | | |
| -1 | -3 | Spin polarized |
| -2 | -2 | Valley polarized or IVC |
| -3 | -1 | Spin polarized |

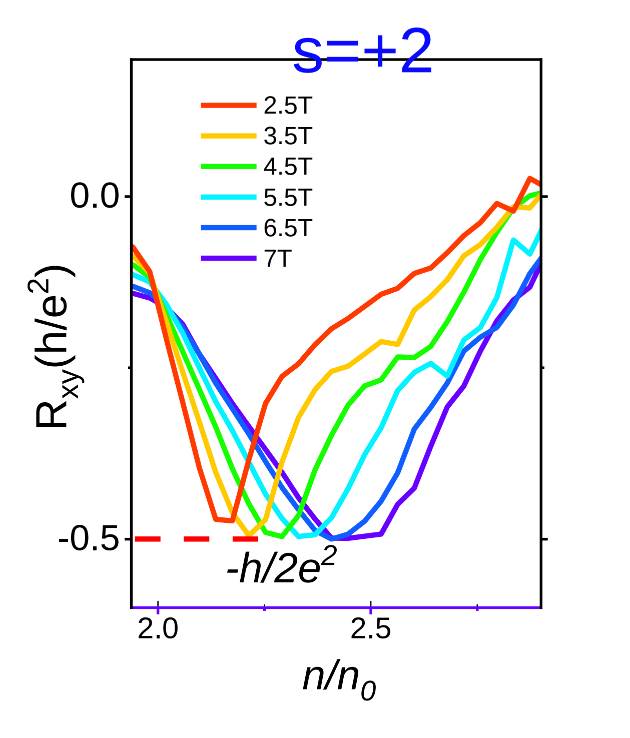


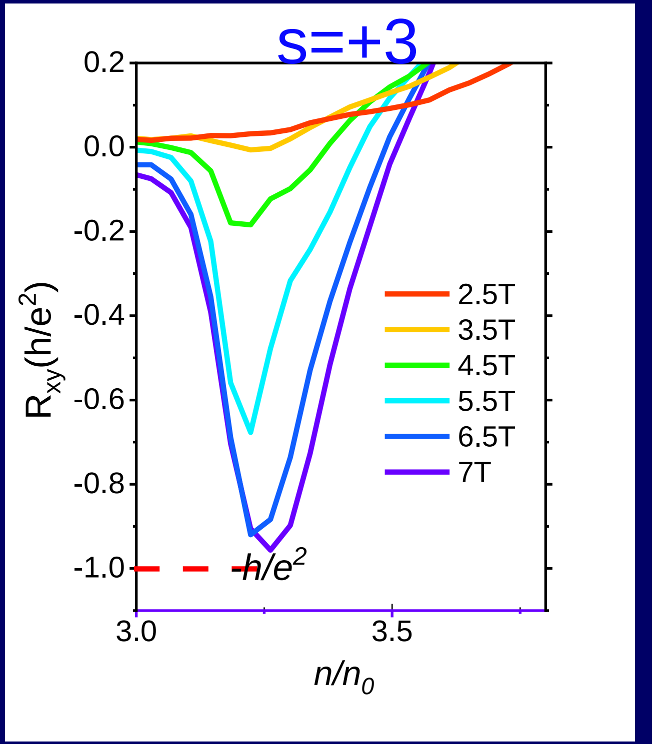



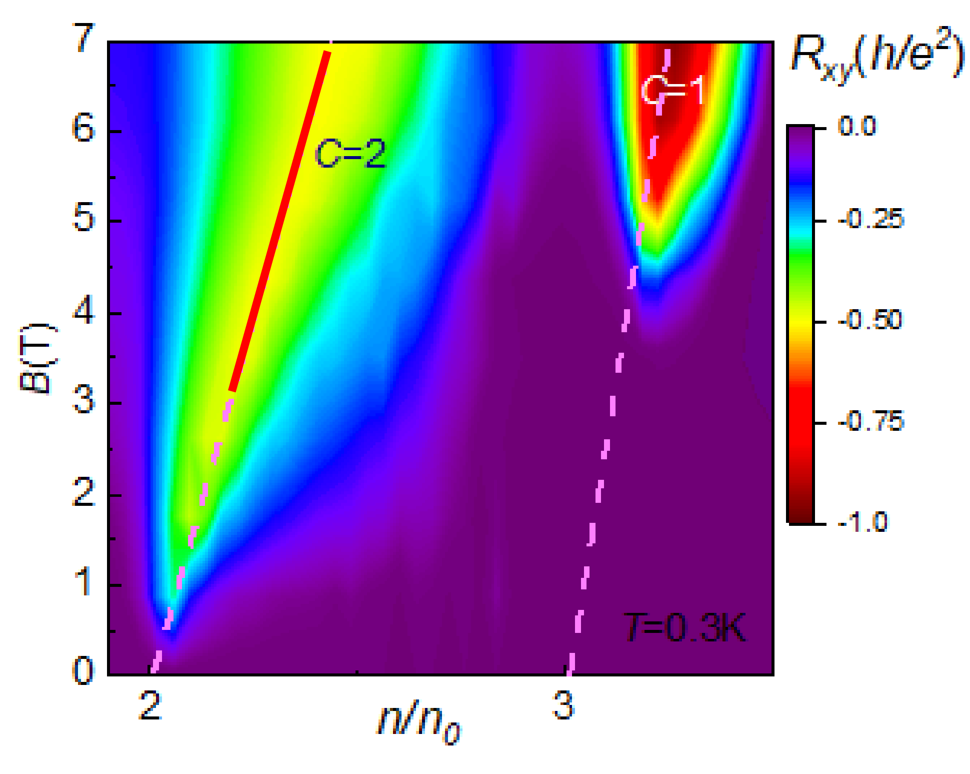
# Field Induced Chern insulators

## Electron sector

### QHE




$$R_{xy}^{QHE} = -\frac{1}{C}\frac{h}{e^2}$$

| s | C | |
|---|---|---|
| 3 | 1 | Spin polarized |
| 2 | 2 | Valley polarized or IVC |
| 1 | 3 | Spin polarized |

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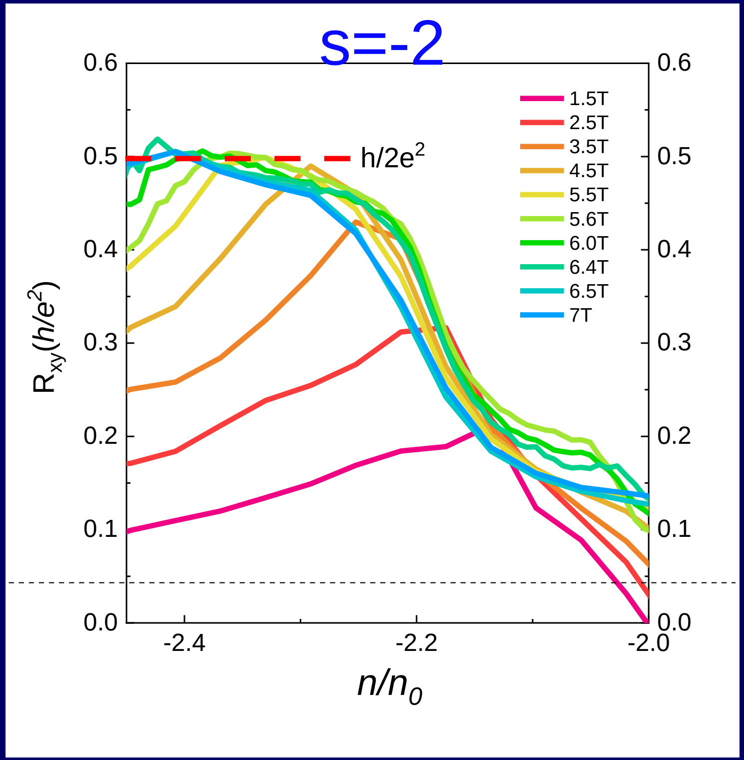
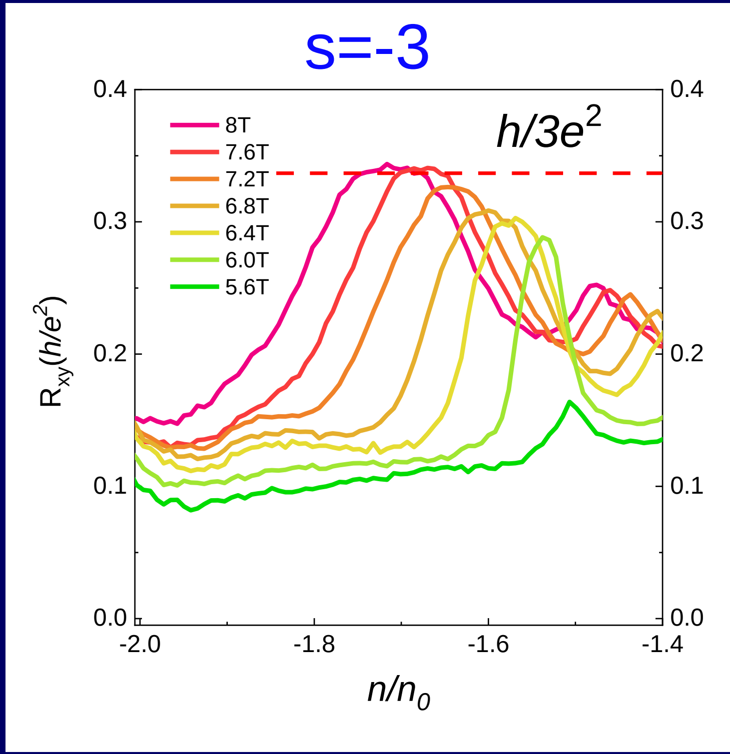
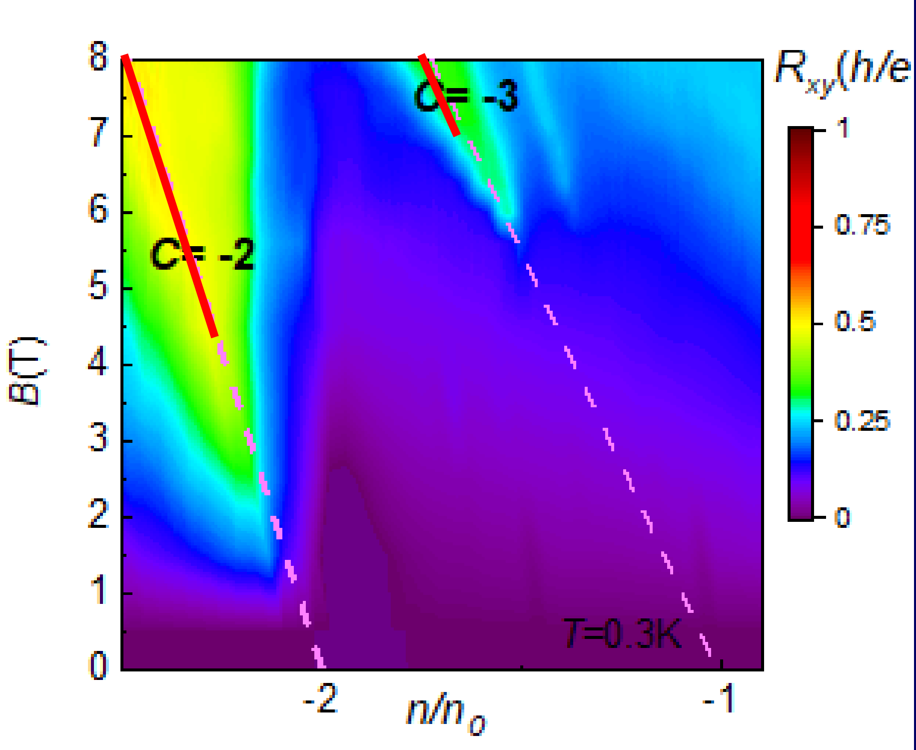
# Field Induced Chern insulators

QHE

Hole sector




$$R_{xy}^{QHE} = -\frac{1}{C}\frac{h}{e^2}$$

| s | C | |
|---|---|---|
| -1 | -3 | Spin polarized |
| -2 | -2 | Valley polarized or IVC |
| -3 | -1 | Spin polarized |

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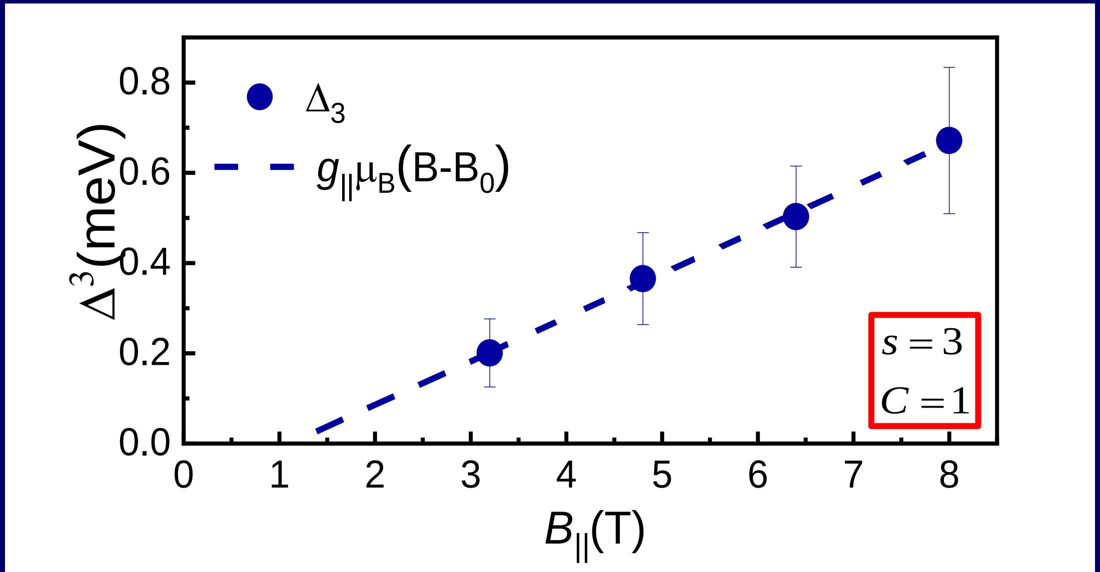
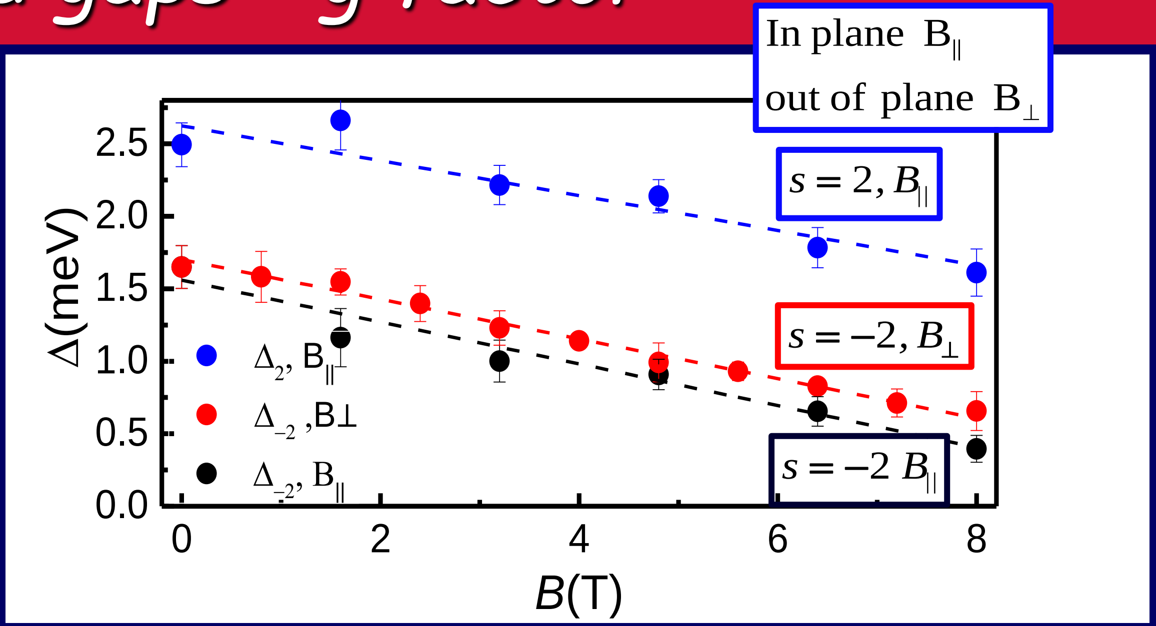
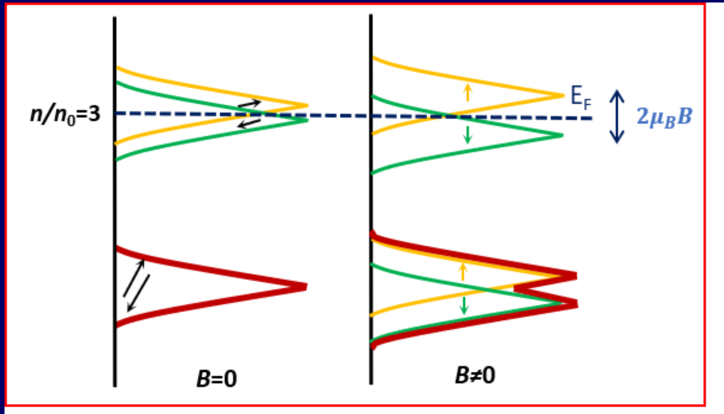
# Thermally activated gaps – g factor



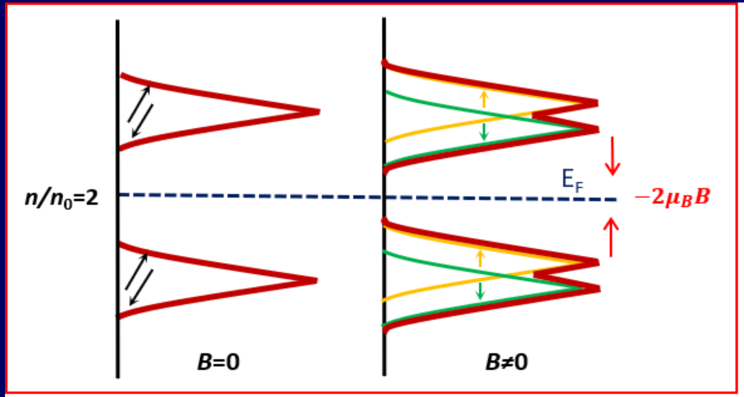

C=1 spin polarized

$C = 2$ spin unpolarized




| $n/n_0$ | $g_{||}$ |
|---|---|
| 3 | $1.5 \pm 0.2$ |

Orbital contribution?




| $n/n_0$ | $g_{||}$ | $g_{\perp}$ |
|---|---|---|
| 2 | $2.4 \pm 0.1$ | |
| -2 | $2.5 \pm 0.3$ | $2.1 \pm 0.3$ |

Zeeman splitting

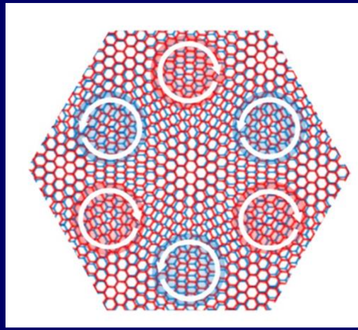
skip

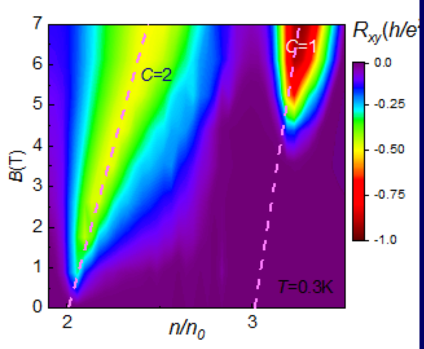
*S. Wu,,.. EYA, Nature Mat (2021)*

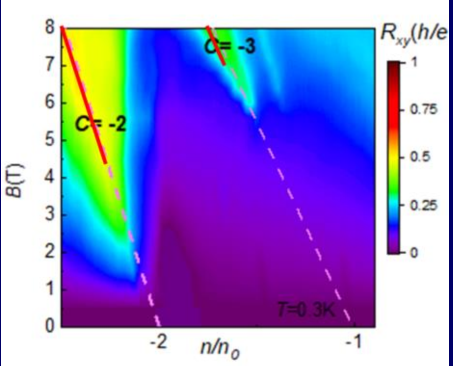
E.Y. Andrei

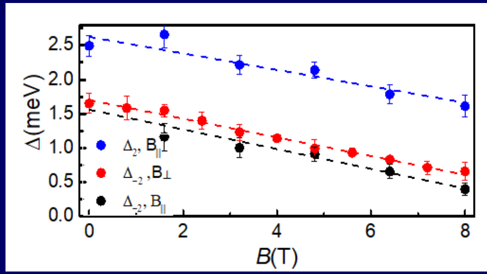
# Summary of CI

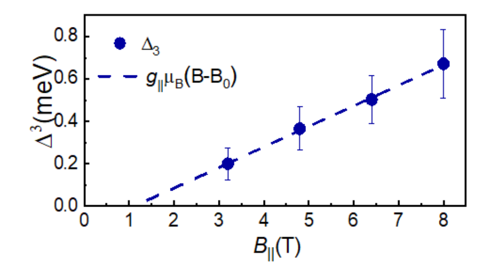
- Flat band $\mapsto$ N=0 pseudo-LL
- PMF with opposite signs on A/B and K/K' $\mapsto$ diamagnetic screening currents
- Breaking $C_2T$ symmetry $\mapsto$ Chern insulators at integer fillings $\mapsto$ QHE, AQHE
- Thermally activated gaps $\mapsto$ Filling 2 spin unpolarized; Filling 3 – spin polarized



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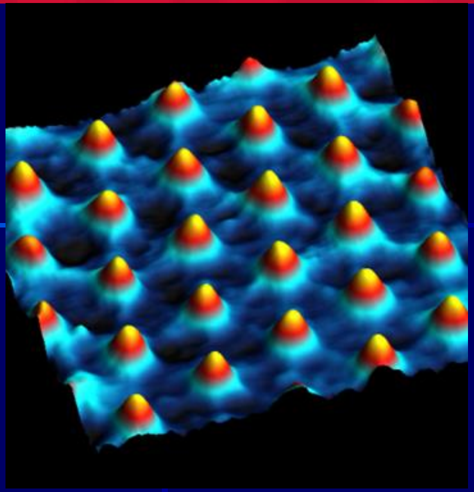
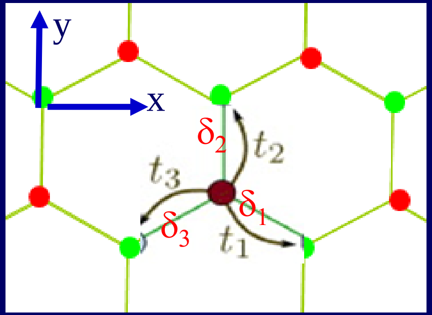
# Lecture 3 – strain and point defects



- ❖ Magic angle Twisted bilayer graphene
  - Chern insulators
- ❖ Strain induced Flat bands
  - Pillars
  - Buckled graphene
- ❖ Vacancy states:
  - Atomic collapse
  - Kondo screening

# Strain-induced Pseudo-magnetic field

## Undistorted




$\delta_{1..3} = a = 0.142nm$ NN distance

$t_{1..3} = t = 2.7eV$ NN hopping

$$H = v_F \begin{pmatrix} \vec{\sigma}.\vec{p} & 0 \\ 0 & -\vec{\sigma}^*.\vec{p} \end{pmatrix}$$

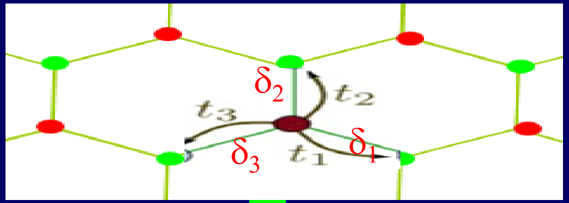
$$v_F = \frac{3}{2\hbar} at$$

$$t_n = te^{-\beta(\delta_n / a - 1)}$$

$$\beta = \partial \log(t) / \partial \log(a) \approx 3$$

β Gruneisen parameter

## Strained




$t_1 \neq t_2 \neq t_3$

Spatially modulated Hamiltonian $H = H(\vec{p}, \vec{R})$

Taylor expand $H(\vec{p}, \vec{R}) \approx H(\vec{p}, 0) + \vec{\nabla} H \cdot \vec{R}$

Reabsorb in the momentum $H(\vec{p}, \vec{R}) \approx H(\vec{p} + \vec{A}(\vec{R}), 0)$

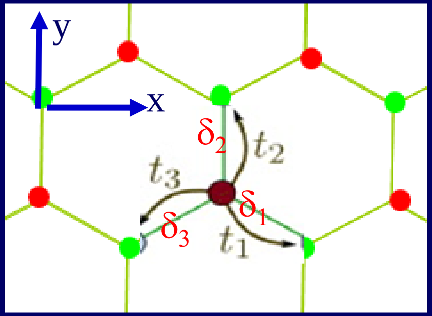
Emergent artificial gauge field $\vec{B} = \vec{\nabla} \times \vec{A}$


M. Vozmediano et al Phys. Reports 496, 109(2010)
N. Levy et al Science, **329**, **544** (2010)
F. Guinea et al, *Nature Physics* **6**, 30 (2010
Reserbat-Plantay et al, Nanoletters **14**, 5044 (2014)


E.Y. Andrei

# Strain-induced Pseudo-magnetic field

## Undistorted




$\delta_{1..3} = a = 0.142nm$ NN distance

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$$H = v_F \begin{pmatrix} \vec{\sigma}.\vec{p} & 0 \\ 0 & -\vec{\sigma}^*.\vec{p} \end{pmatrix}$$

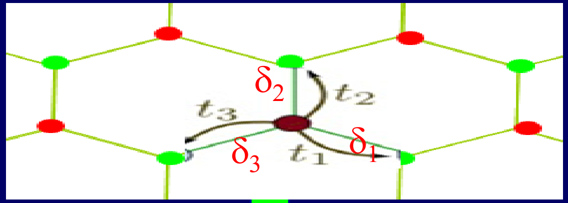
$$v_F = \frac{3}{2\hbar} at$$

$$t_n = te^{-\beta(\delta_n / a - 1)}$$

$$\beta = \partial \log(t) / \partial \log(a) \approx 3$$

β Gruneisen parameter

## Strained




$t_1 \neq t_2 \neq t_3$

$$H = v_F \begin{pmatrix} \vec{\sigma}.(\vec{p} + e\vec{A}) & 0 \\ 0 & -\vec{\sigma}^*.(\vec{p} - e\vec{A}) \end{pmatrix}$$

Pseudo-vector potential

$$\mathrm{A}_{x_{K,K'}} = \pm\phi_0 \frac{\beta}{2\pi a}(\varepsilon_{xx} - \varepsilon_{yy}); \quad \mathrm{A}_{y_{K,K'}} = \pm\phi_0 \frac{\beta}{2\pi a} 2\varepsilon_{xy}$$

Pseudo-magnetic field

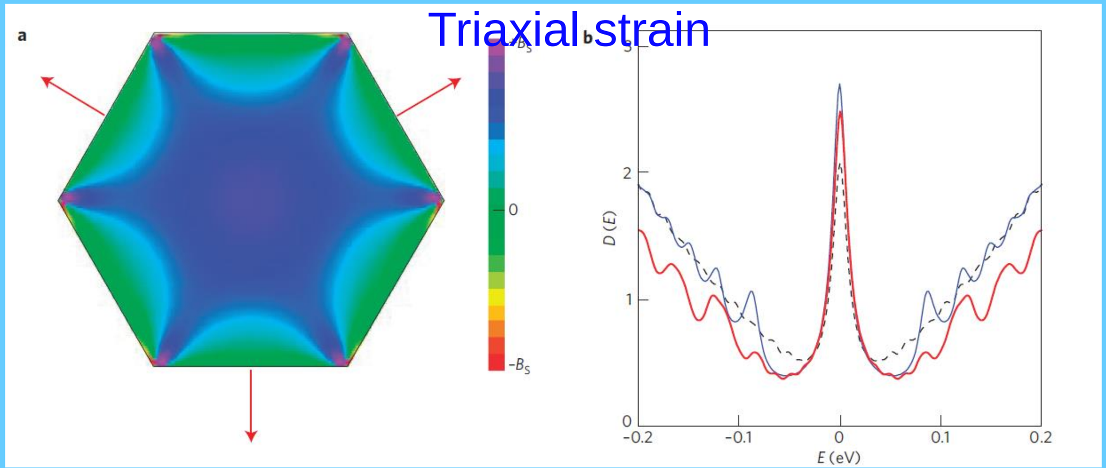
$$\vec{B}_{K,K'} = \pm \vec{\nabla} \times \vec{A}$$

opposite signs in K and K'

**Strain $\mapsto$ pseudo vector potential**

- Pseudo-magnetic field with opposite signs in K and K' valleys
- Time reversal symmetry preserved

# Strain-induced Pseudomagnetic Landau levels



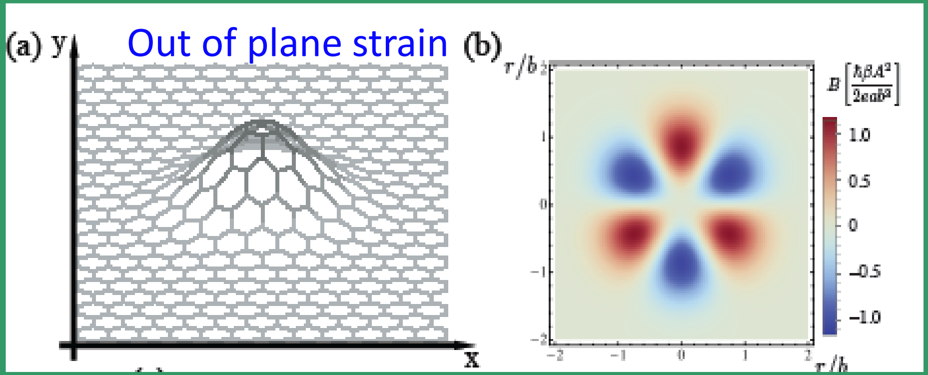

LETTERS

PUBLISHED ONLINE: 27 SEPTEMBER 2009 | DOI: 10.1038/NPHYS1420

nature physics

## Energy gaps and a zero-field quantum Hall effect in graphene by strain engineering

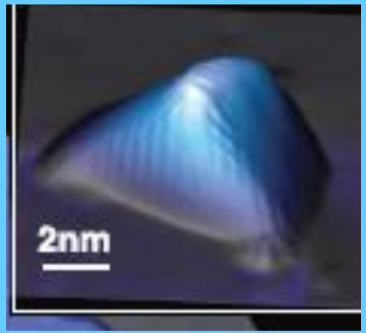
F. Guinea[1]*, M. I. Katsnelson[2] and A. K. Geim[3]*

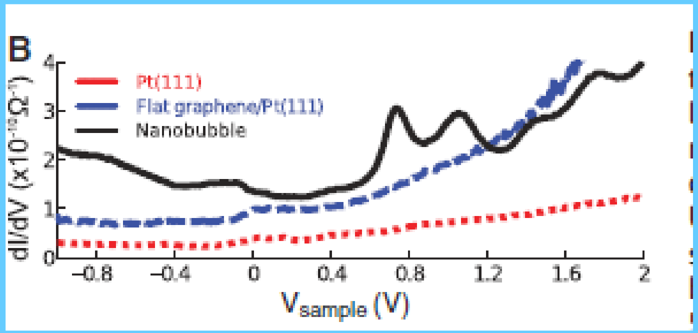



PHYSICAL REVIEW B 90, 041411(R) (2014)

## Gaussian deformations in graphene ribbons: Flowers and confinement

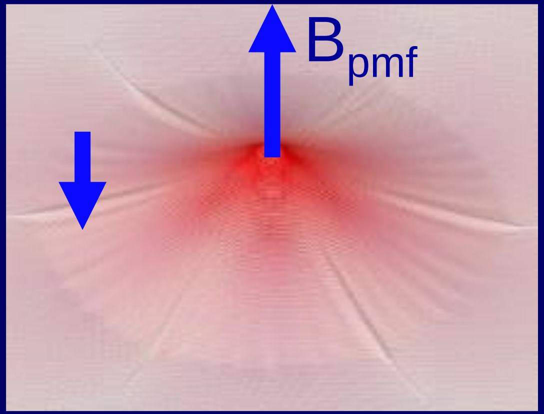
R. Carrillo-Bastos,[1,2,3,*] D. Faria,[4] A. Latgé,[4] F. Mireles,[2] and N. Sandler[3]




**Strain-Induced Pseudo Magnetic Fields Greater Than 300 Tesla in Graphene Nanobubbles**
N. Levy, *et al.*
*Science* **329**, 544 (2010);
DOI: 10.1126/science.1191700

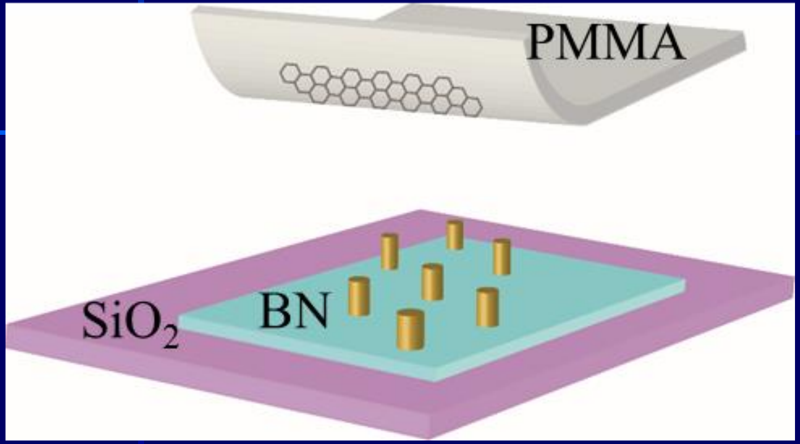
# Periodic Strain

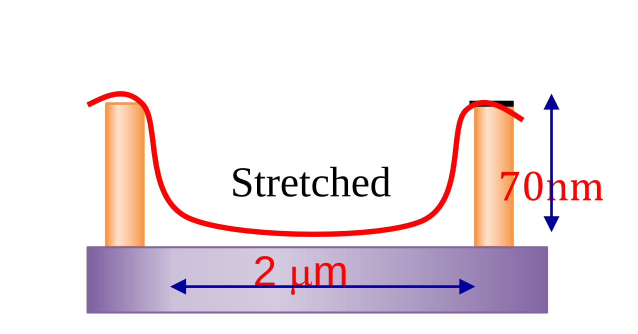



PMF is LOCALIZED

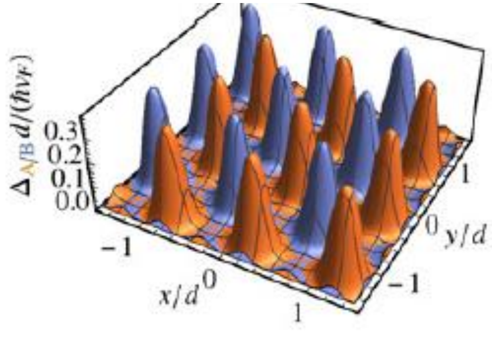
- Periodic strain structures $\mapsto$ global PMF

# Pillar structure for periodic PMF




Superconducting gap




$B_{PMF} = 400$

$d = 8nm$

$\Rightarrow T_c^{BKT} \approx 11K$

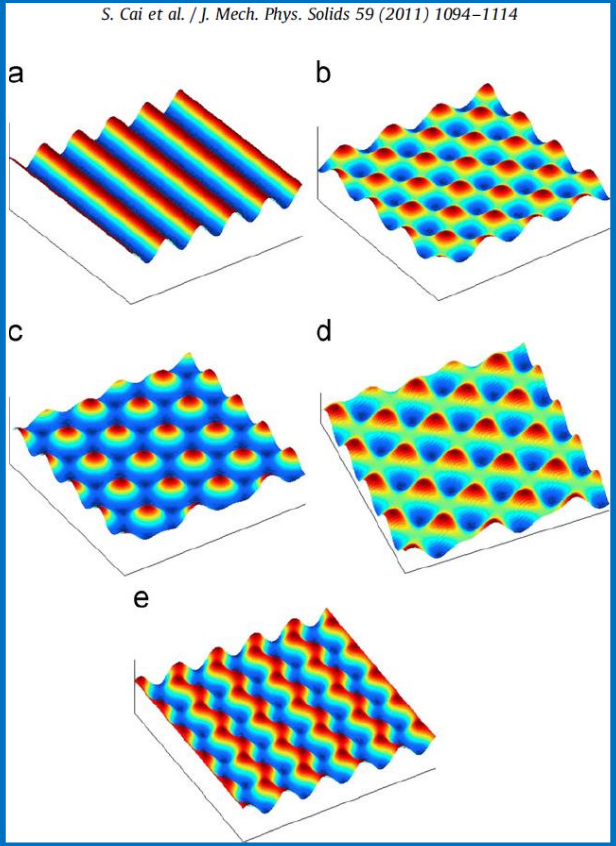
J. Peltonen T. Hekkila (2020)

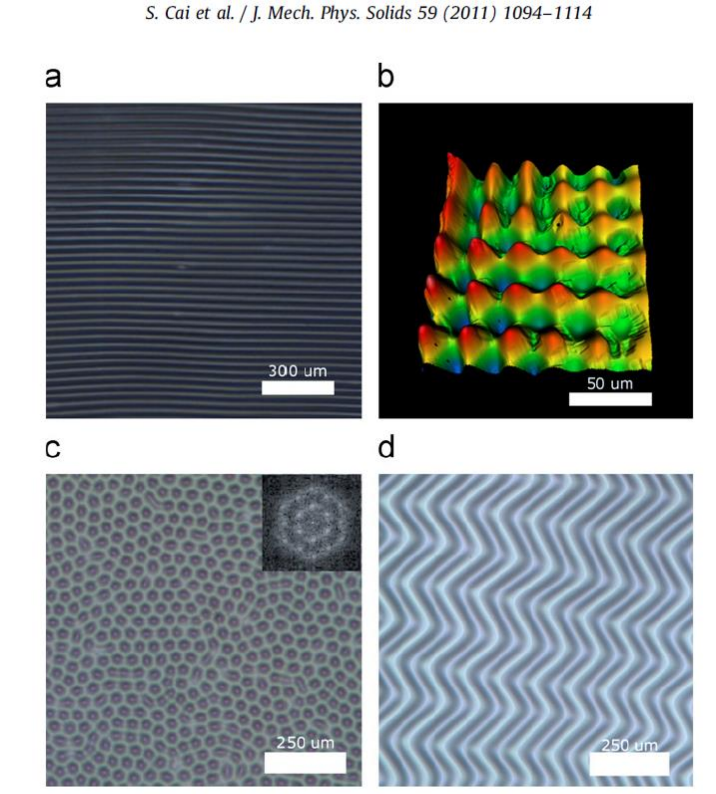
Y. Jiang,..EYA *et al*, Nano Lett. 17, 2839 (2017 )

# Buckling modes of thin membranes

Thin stiff membranes subject to biaxial compressive stress buckle into periodic mode patterns : 1D mode,  checkerboard, hexagonal and herringbone.

Finite element analysis of membrane on substrate



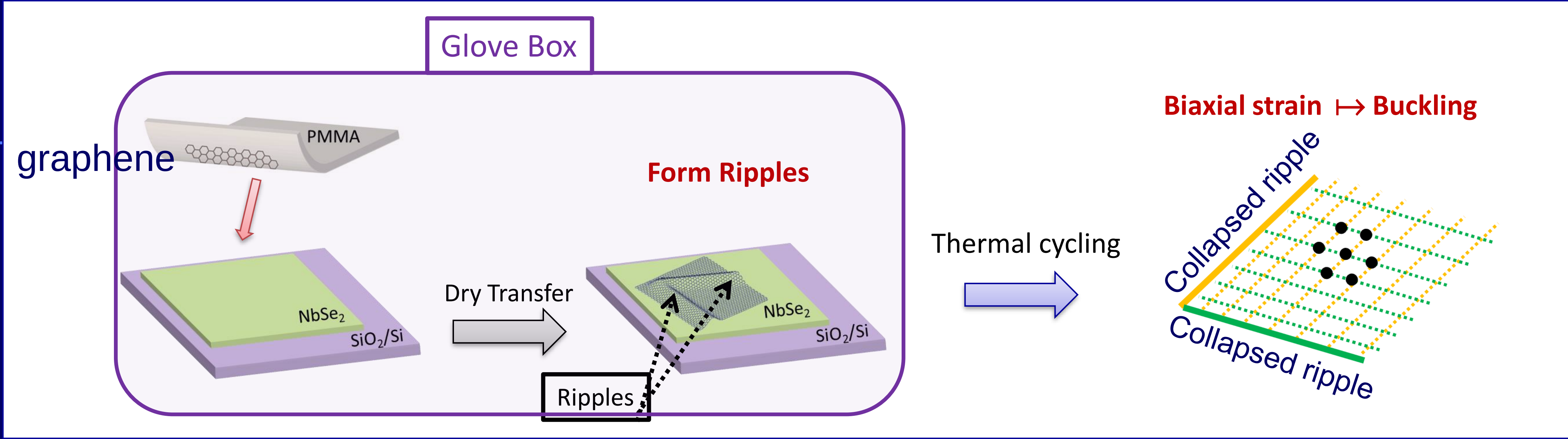




Buckling modes of biaxially compressed films on PDMS substrates

S. Cai et al. J Mech Phys Solids 59, 1094 (2011).
E. Cerda, Physical Review Letters 90, 074302 (2003

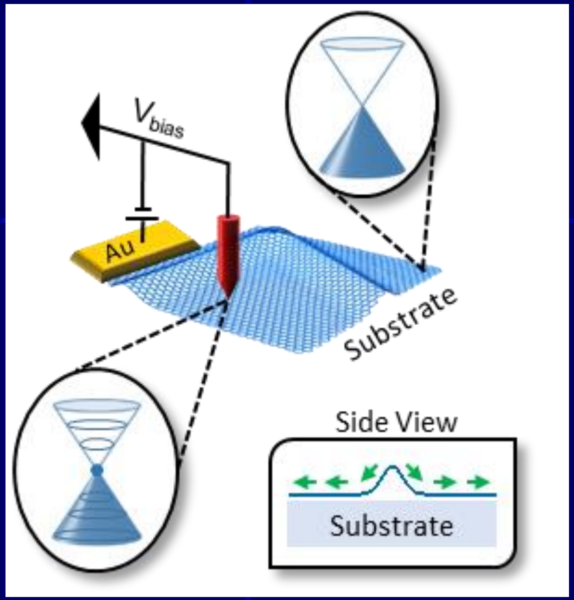
E.Y. Andrei

# Buckling transition by collapse of thermally cycled ripples

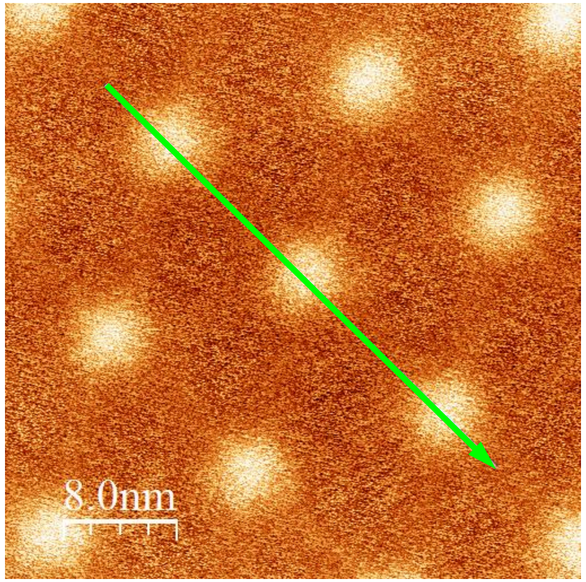



J. Mao.. EYA et al Nature, **584**, 215 (2020)

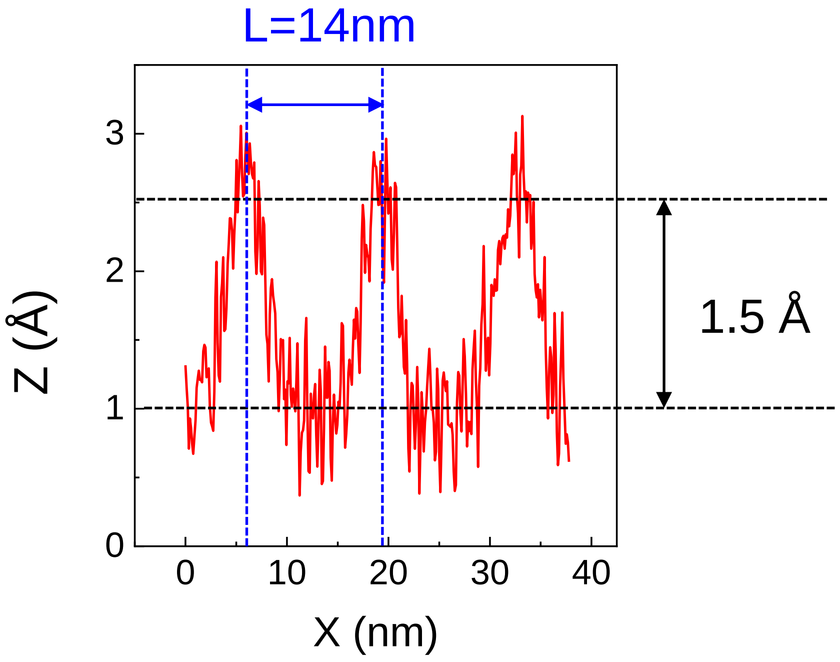
# STM on buckled superlattice




## Buckled Graphene



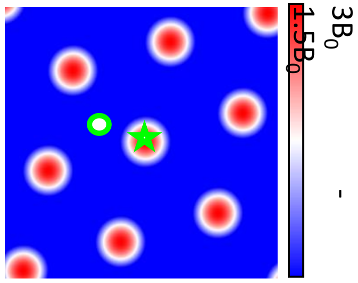

## Height Profile



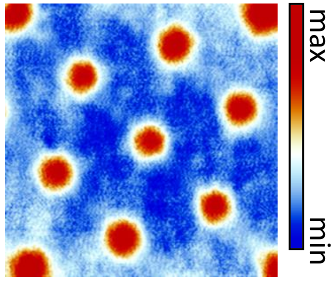

*J. Mao.. EYA et al Nature*, ***584***, *215* (2020)

# Measured local DOS

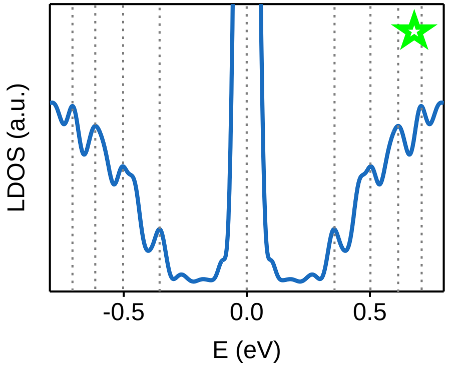
## PMF image

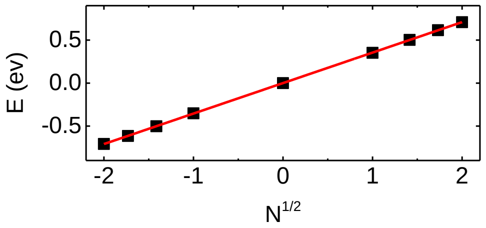



## STM Topography




## Crest Area



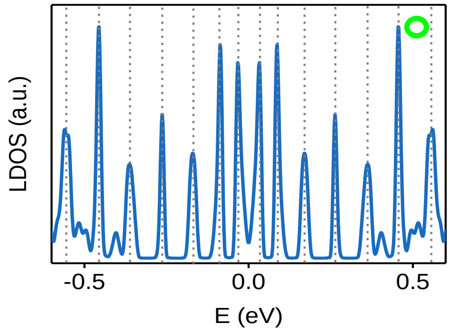



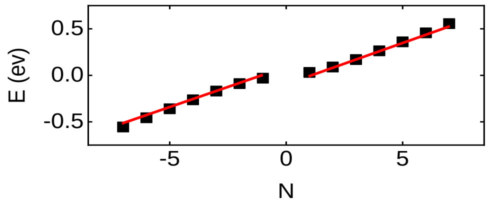

Pseudo-Landau levels:

$$E_N = sgn(N) v_F \sqrt{2e\hbar |N| B_{PMF}}$$

$B_{eff}$ ~ 112 T

## Trough Area




$B_{eff}$ ~ -56 T

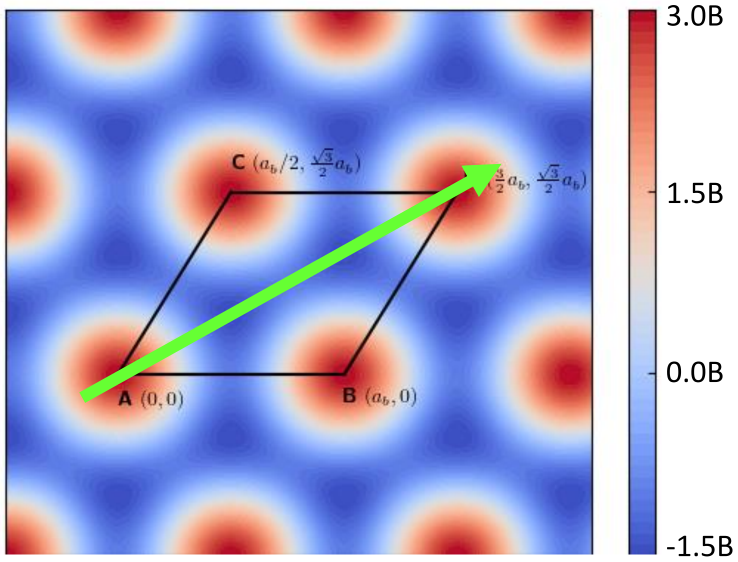
**Evidence of flat bands and correlated states in buckled graphene superlattices**

Jinhai Mao, Slaviša P. Milovanović, Miša Anđelković, Xinyuan Lai, Yang Cao, Kenji Watanabe, Takashi Taniguchi, Lucian Covaci, Francois M. Peeters, Andre K. Geim, Yuhang Jiang & Eva Y. Andrei

*Nature* **584**, 215–220 (2020) | Cite this article

# Calculated PMF Superlattice

**PMF Landscape**
**(B=120T)**



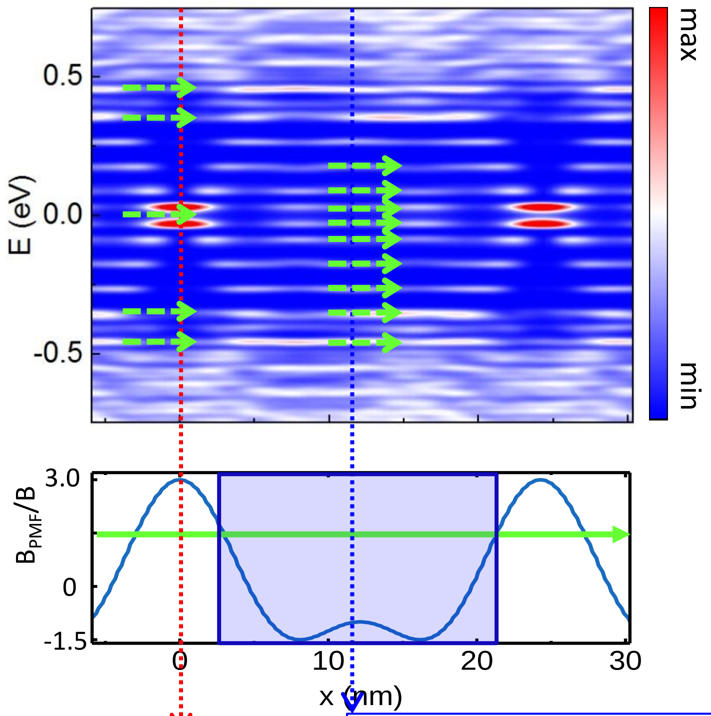

$$\boldsymbol{B_{PMF}}(x,y) = B[\cos(\boldsymbol{b_1 r}) + \cos(\boldsymbol{b_2 r}) + \cos(\boldsymbol{b_3 r})]$$

$$\boldsymbol{b_1} = \frac{2\pi}{a_b}\left(1, -\frac{1}{\sqrt{3}}\right);\ \boldsymbol{b_2} = \frac{2\pi}{a_b}\left(0, \frac{2}{\sqrt{3}}\right);$$
$$\boldsymbol{b_3} = \boldsymbol{b_1} + \boldsymbol{b_2}$$

Crests:
Pseudo-
Landau levels:
$B_{eff}$ = 112 T

Expt:
$B_{PMF}$ = 108±8 T.

**Averaged LDOS Map**
**(A+B Sublattice)**




Troughs:
Peak spacing
ΔE = 89mV

Expt:
ΔE = 94±0.5 mV

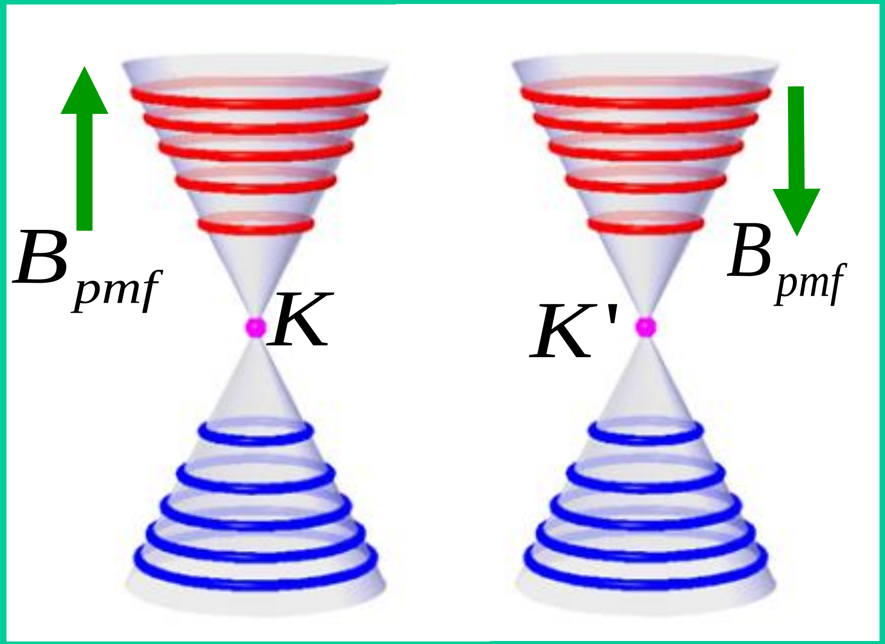
**Crest → PMF**

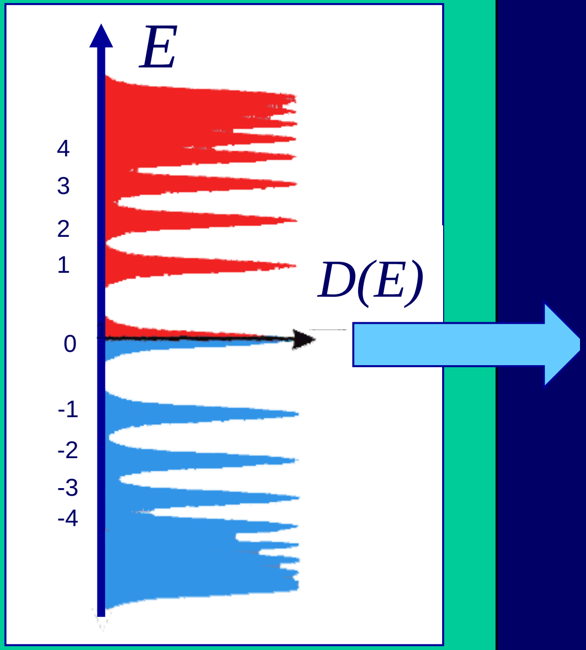
**Trough → Confinement States**

# Sublattice PMF locking

Band structure

DOS

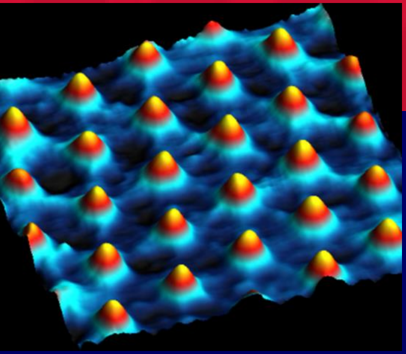


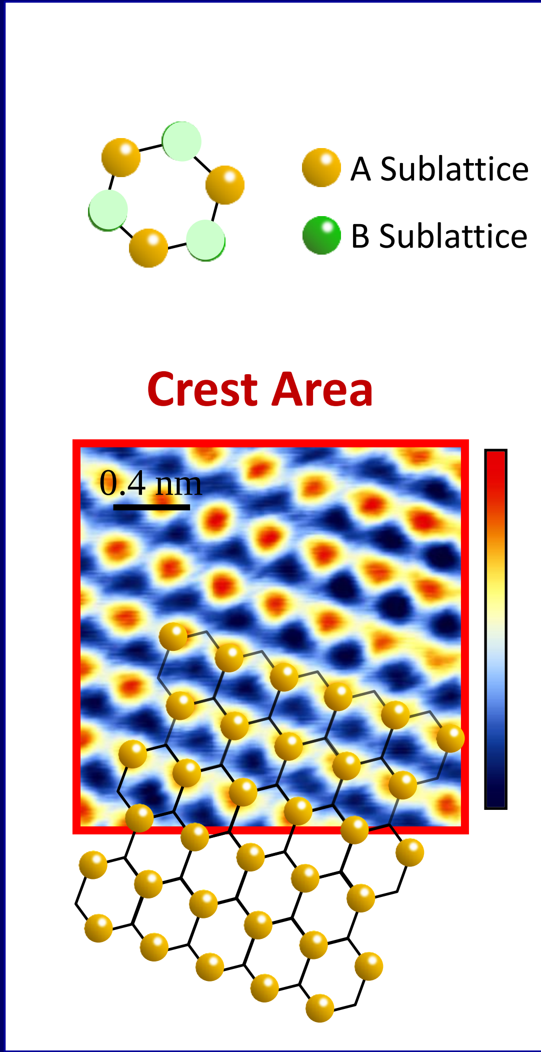

$$E_N = \pm v_F \sqrt{2e\hbar B|N|} \quad N = 0, \pm 1, ..$$

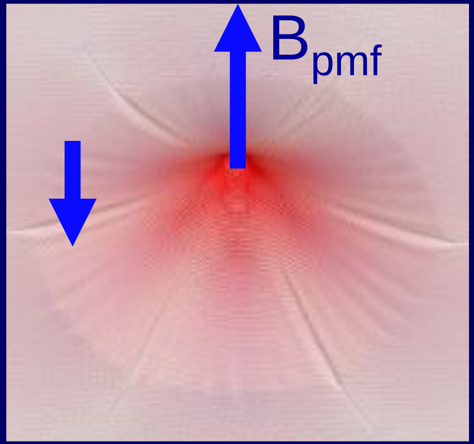
Same as in real magnetic field

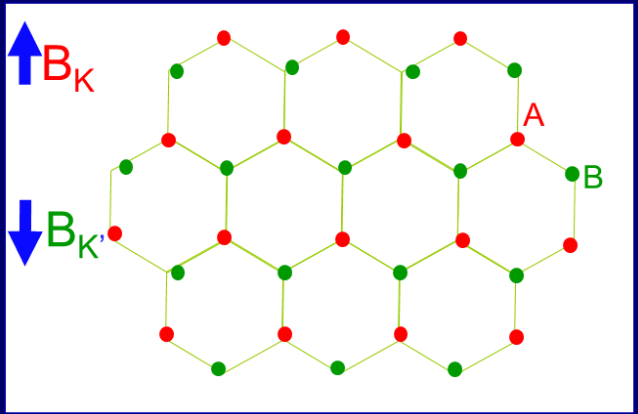
# Valley-Sublattice Locking

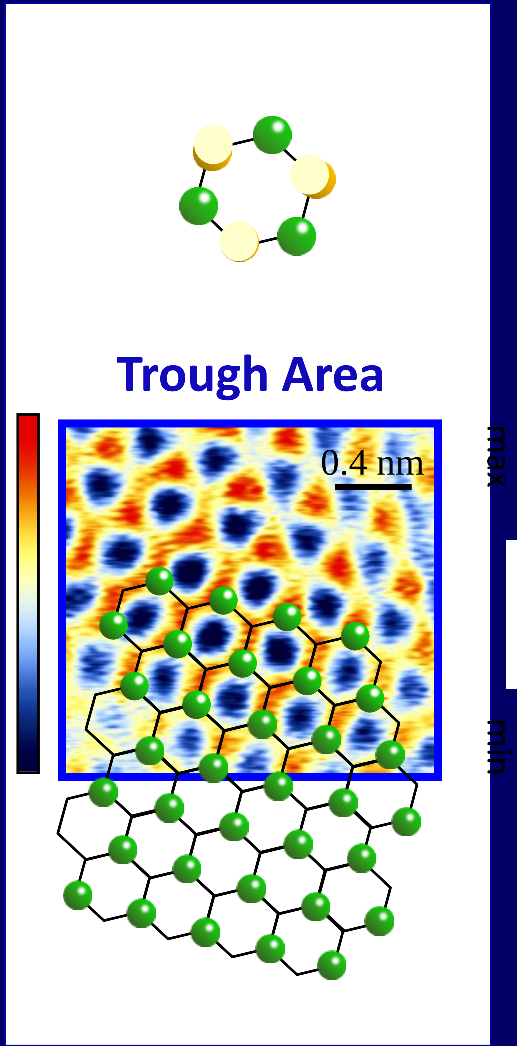


## Atomic resolution maps at N=0



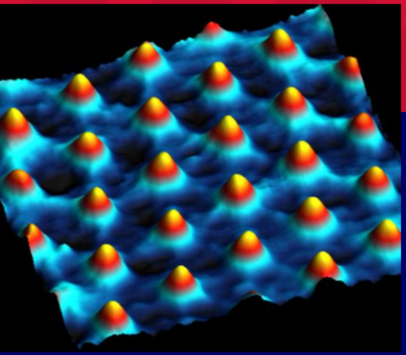

Wavefunction on sublattice A

Wavefunction on sublattice B

*J. Mao.. EYA et al Nature*, ***584***, *215* (2020)

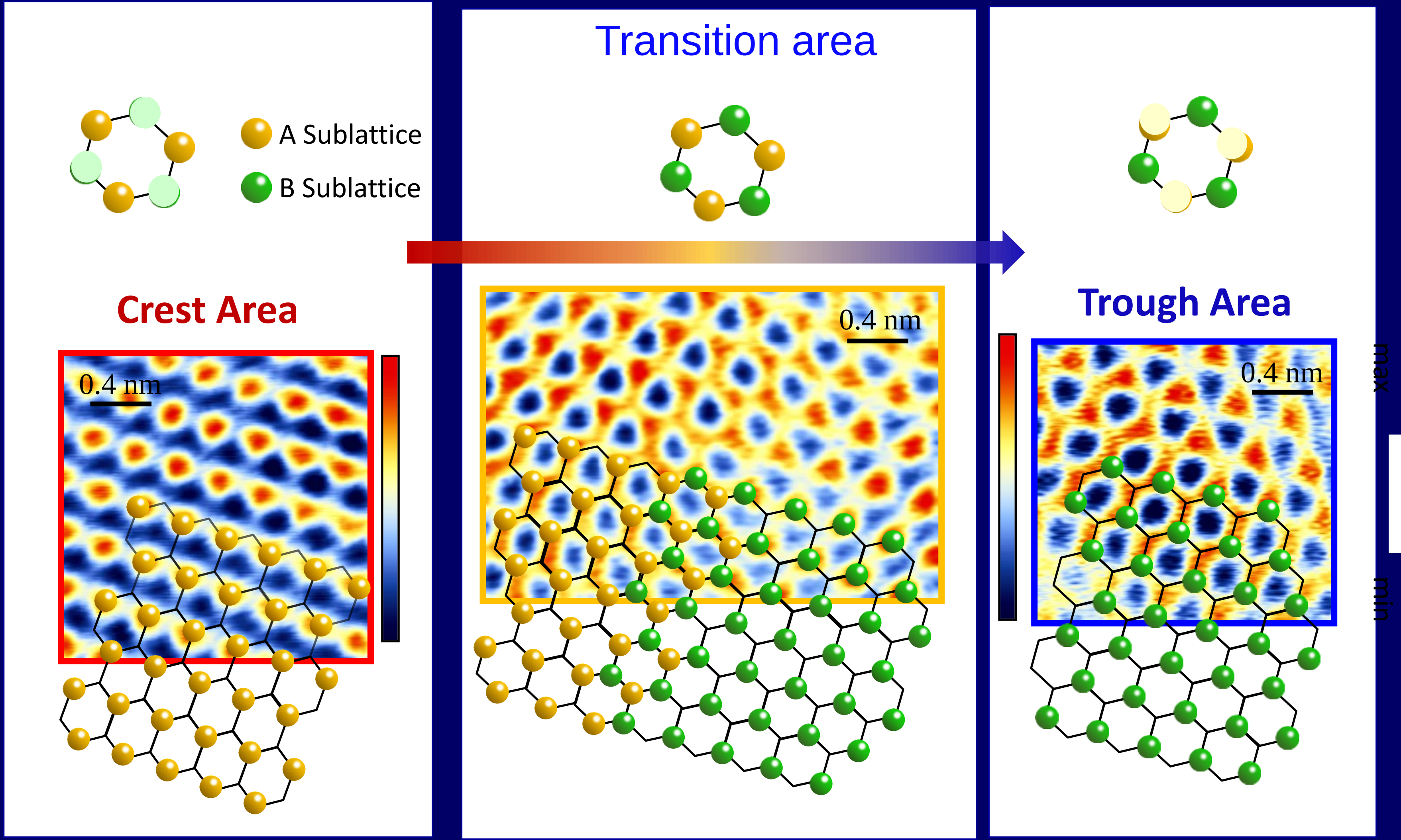
E.Y. Andrei

# Valley-Sublattice Locking



Atomic resolution maps at N=0

Wavefunction on sublattice A




Wavefunction on sublattice B

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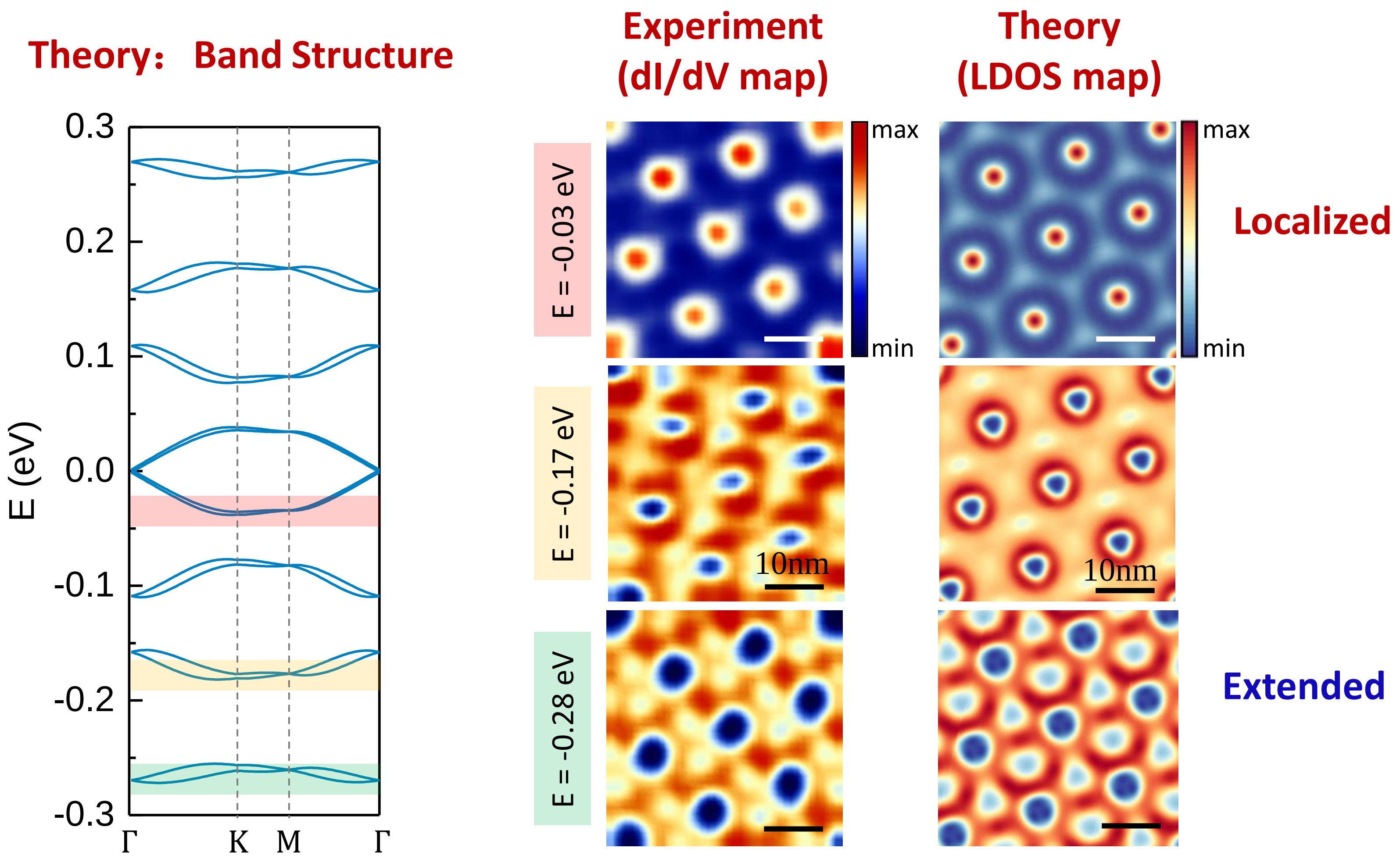
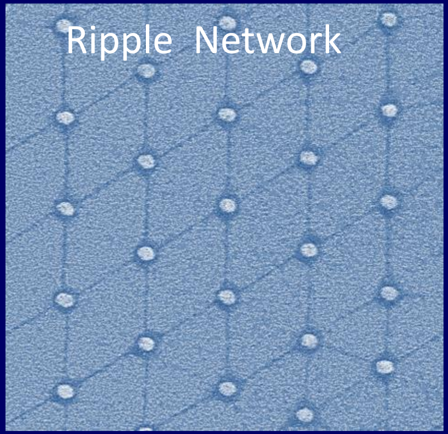
# Strain induced Flat bands

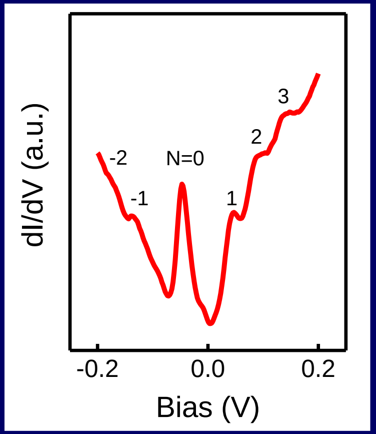


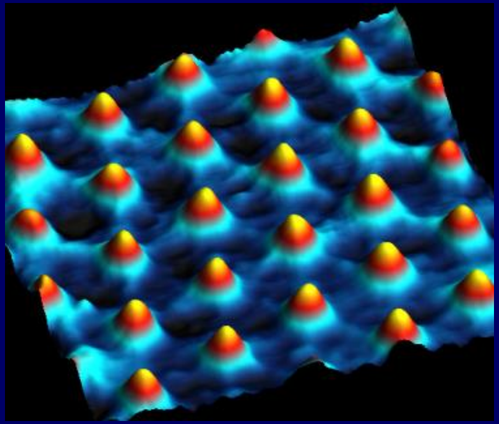

*J. Mao.. EYA et al Nature*, ***584***, *215* (2020)

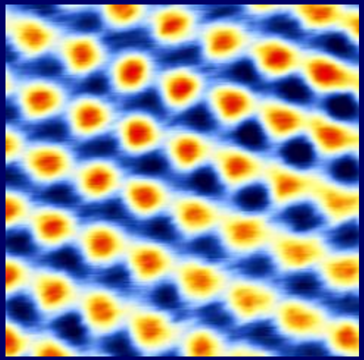
# Pseudo Magnetic fields Summary

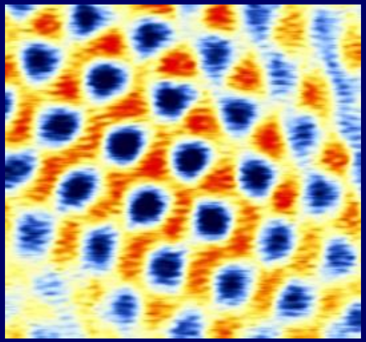
- Graphene on pillar array
  - ✓ Ordered ripple network
  - ✓ Strain-induced periodic pseudo-magnetic fields ~6T
- Buckling superlattice and flat bands in graphene
  - ✓ Buckling transition ↦ Hexagonal buckling mode
  - ✓ PMF sub-lattice polarization $B_{PMF}$ ~+ 108T ; - 60V
  - ✓ Flat bands without broken TRS and without fine tuning
  - ✓ Pseudogap in flat band

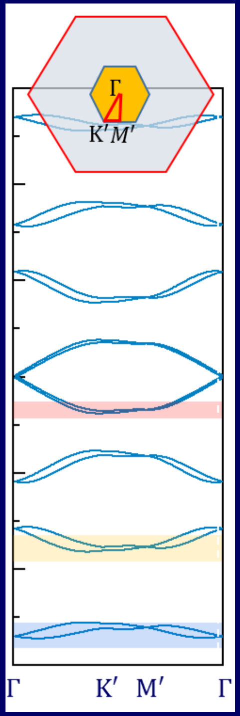


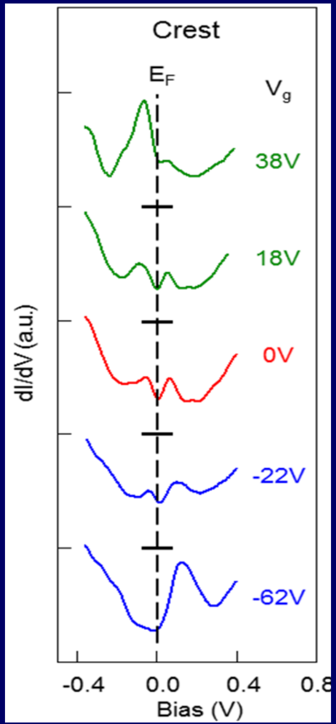










E.Y. Andrei