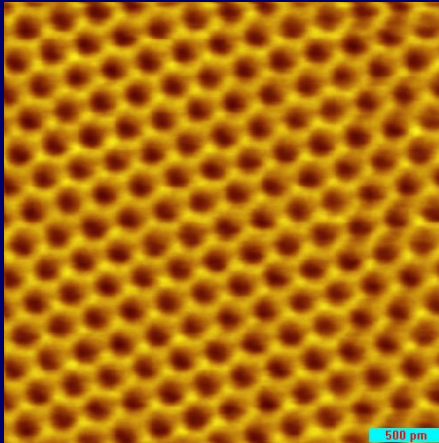
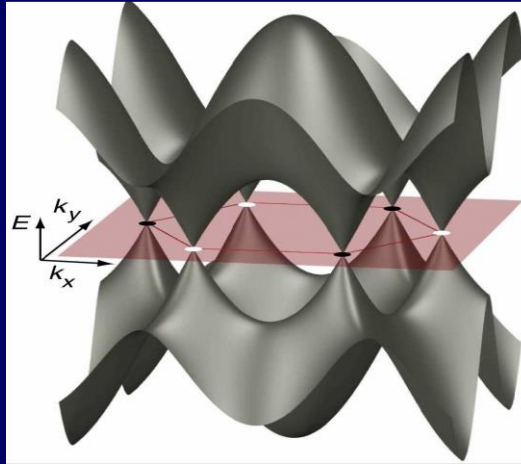


Electronic properties of Graphene and 2-D materials

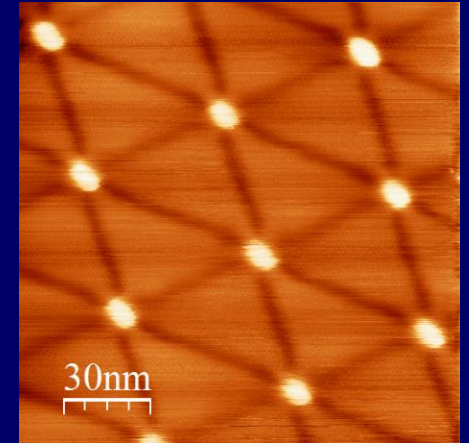
Graphene



Graphene band structure



Twisted Bilayer Graphene



- ❖ 2D materials
- ❖ Graphene
- ❖ STM and transport measurements
- ❖ Flat bands and correlations
- ❖ Twisted bilayer graphene

Eva Y. Andrei
Rutgers University



2D crystals do not exist in nature

David Mermin



Herbert Wagner

...continuous symmetries cannot be spontaneously broken at finite temperature in systems with sufficiently short-range interactions in dimensions $D \leq 2$.

ABSENCE OF FERROMAGNETISM OR ANTIFERROMAGNETISM IN ONE- OR TWO-DIMENSIONAL ISOTROPIC HEISENBERG MODELS*

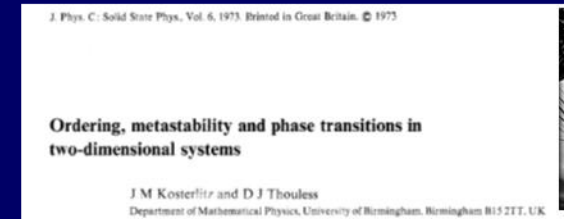
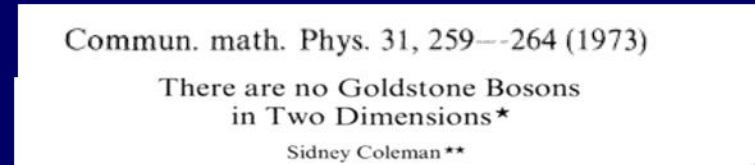
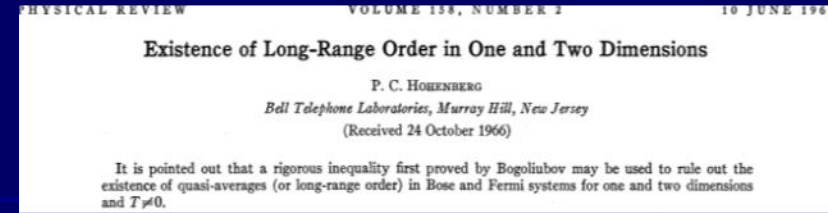
N. D. Mermin[†] and H. Wagner[‡]

Laboratory of Atomic and Solid State Physics, Cornell University, Ithaca, New York
(Received 17 October 1966)

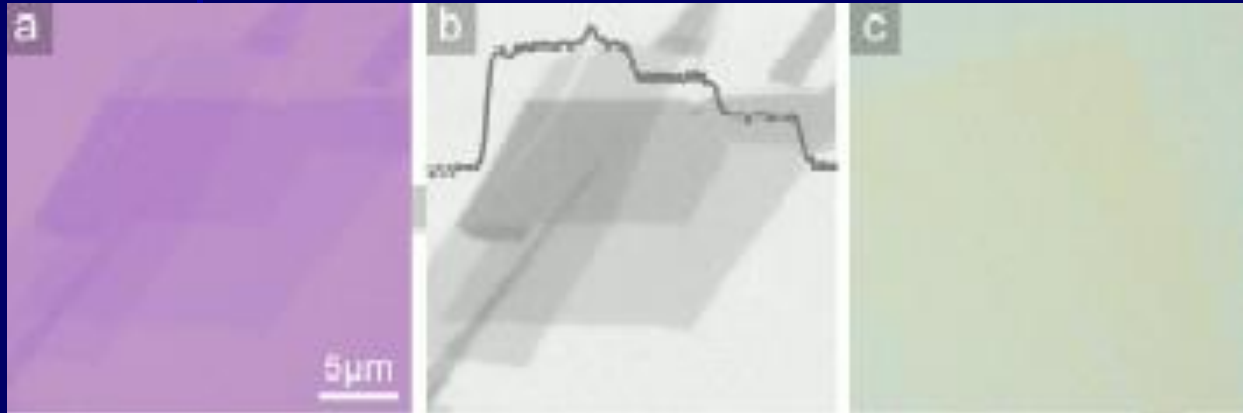
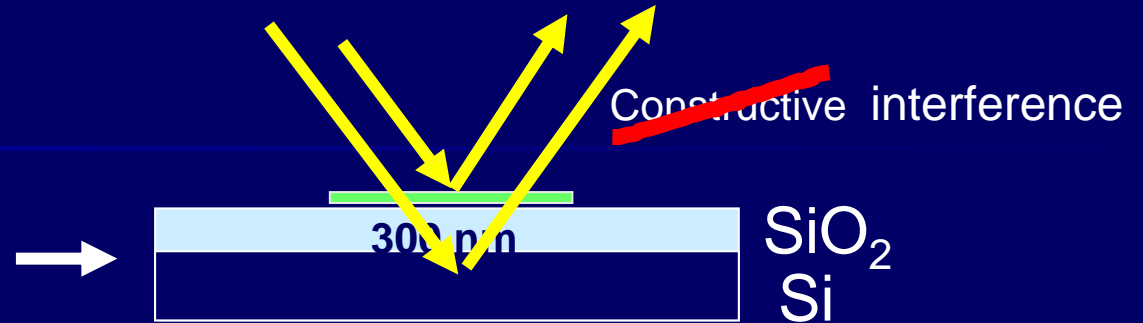
long-range fluctuations can be created with little energy cost and since they increase the entropy they are favored.

No long range order in 2D

... No Magnets
....No superfluids
...No superconductors
... **No 2D crystals**



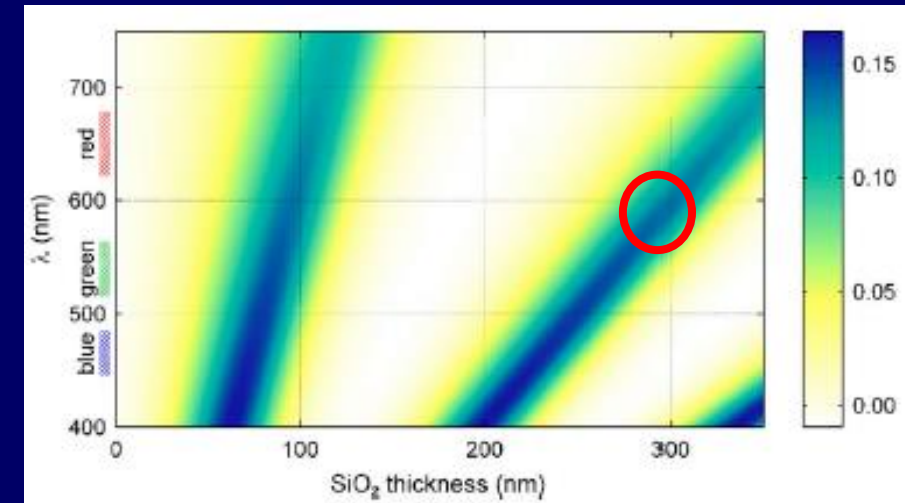
How to see an atomically thin layer?



300nm oxide
White light

300nm oxide
560nm green light

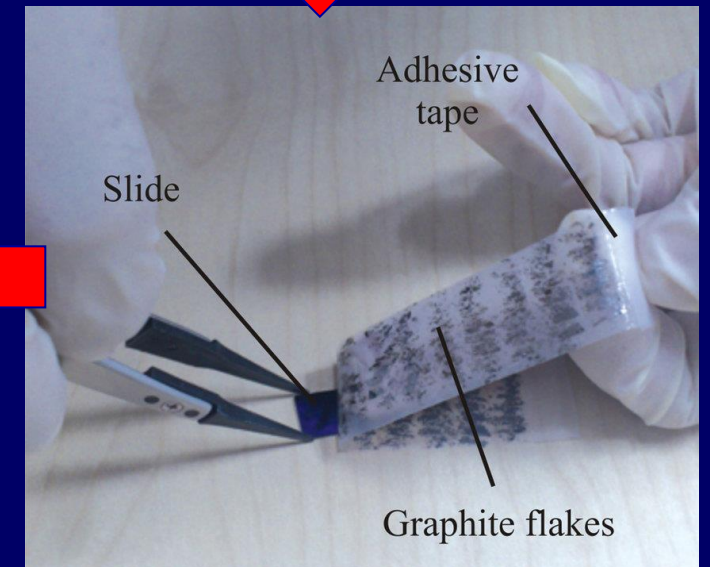
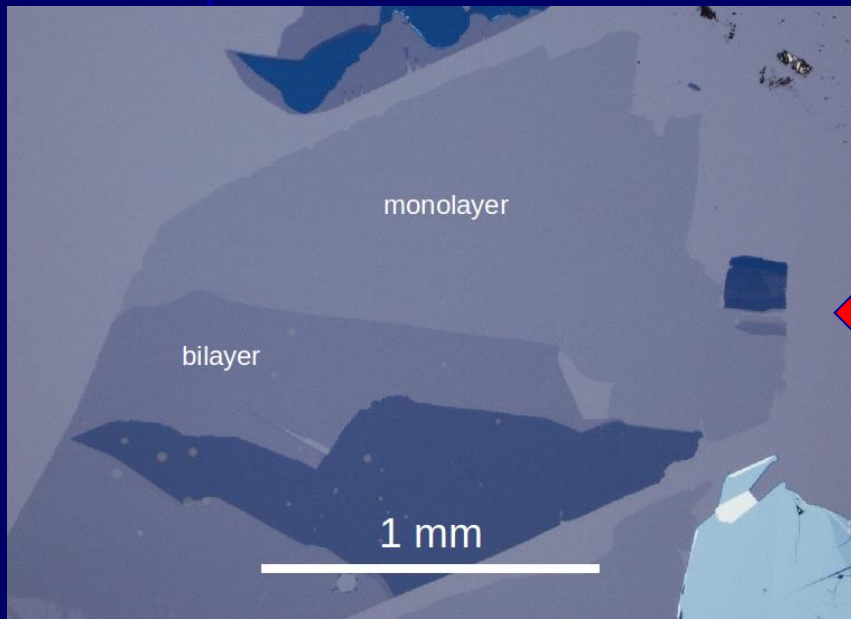
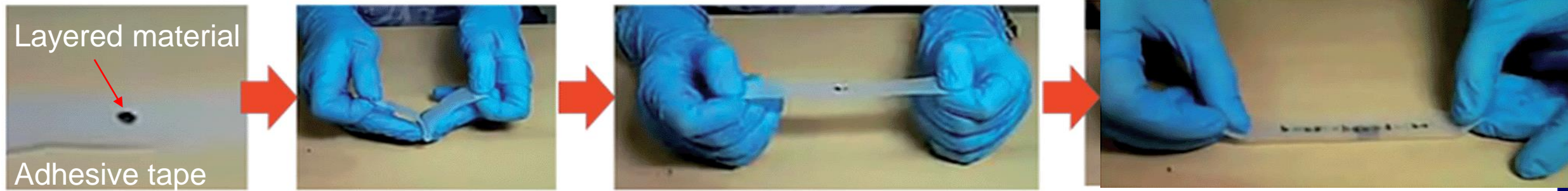
200nm oxide
White light



P. Blake et.al, APL, 91, 063124 (2007)



Scotch tape exfoliation



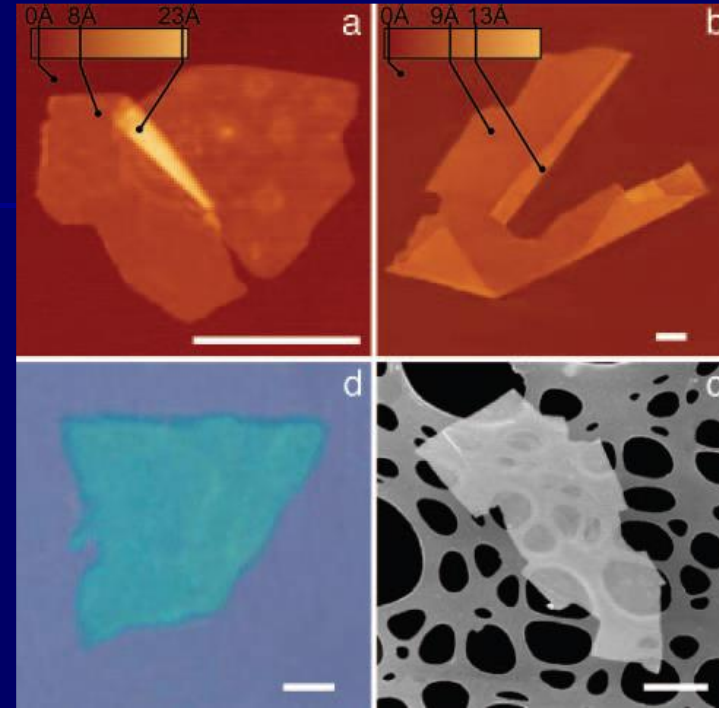
First 2D materials

Andre Geim

Konstantin Novoselov



Graphene, hBN, MoS₂,
NbSe₂, Bi₂Sr₂CaCu₂O_x,



Two-dimensional atomic crystals

K. S. Novoselov*, D. Jiang*, F. Schedin*, T. J. Booth*, V. V. Khotkevich*, S. V. Morozov†, and A. K. Geim**

*Centre for Mesoscience and Nanotechnology and School of Physics and Astronomy, University of Manchester, Manchester M13 9PL, United Kingdom; and †Institute for Microelectronics Technology, Chernogolovka 142432, Russia

Edited by T. Maurice Rice, Swiss Federal Institute of Technology, Zurich, Switzerland, and approved June 7, 2005 (received for review April 6, 2005)

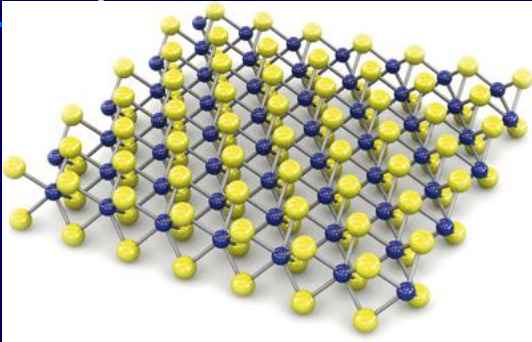
We report free-standing atomic crystals that are strictly 2D and can be viewed as individual atomic planes pulled out of bulk crystals or as unrolled single-wall nanotubes. By using micromechanical cleavage, we have prepared and studied a variety of 2D crystals including single layers of boron nitride, graphite, several dichalcogenides, and complex oxides. These atomically thin sheets (essentially gigantic 2D molecules unprotected from the immediate environment) are stable under ambient conditions, exhibit high crystal quality, and are continuous on a macroscopic scale.

wafer (Fig. 1d), because even a monolayer adds up sufficiently to the optical path of reflected light so that the interference color changes with respect to the one of an empty substrate (phase contrast). The whole procedure takes literally half an hour to implement and identify probable 2D crystallites. Their further analysis was done by atomic force microscopy (AFM), for which single-layer crystals were selected as those exhibiting an apparent (12) thickness of approximately the interlayer distance in the corresponding 3D crystals.



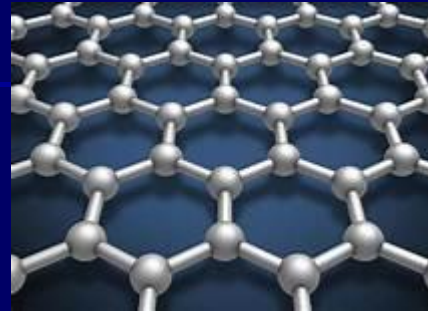
Atomically thin crystals

semiconductor



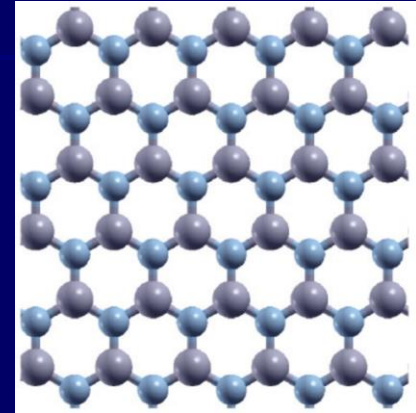
(WSe₂ MoS₂, MoSe₂)

Semi-metal



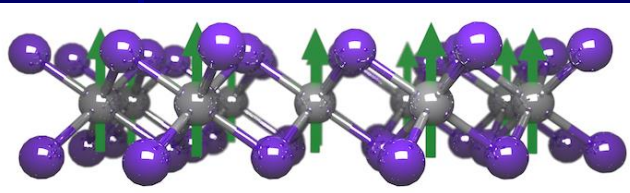
(graphene)

insulator



(boron nitride)

magnet



(FeTaS₂, CrI₃)

ARTICLES

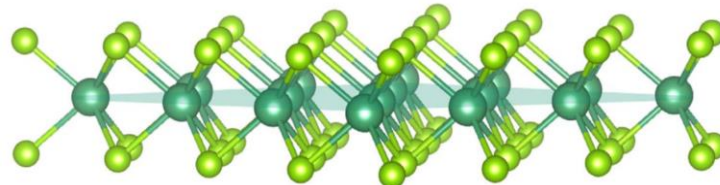
<https://doi.org/10.1038/441003a>

nature
nanotechnology

Two-dimensional materials from high-throughput computational exfoliation of experimentally known compounds

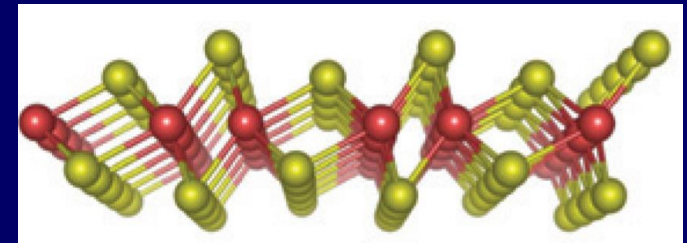
Nicolas Mounet^{1*}, Marco Gibertini², Philippe Schwaller³, Davide Campi², Andrius Merkys^{1,2}, Antimo Marrazzo², Thibault Sohier², Ivano Eligio Castelli², Andrea Cepellotti², Giovanni Pizzi² and Nicola Marzari^{1*}

superconductor



(NbSe₂ WTe₂, BiSrCaCuO)

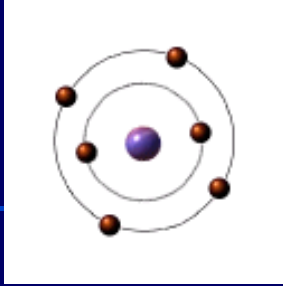
topological insulator



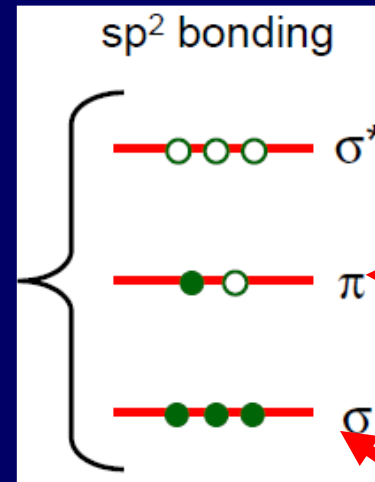
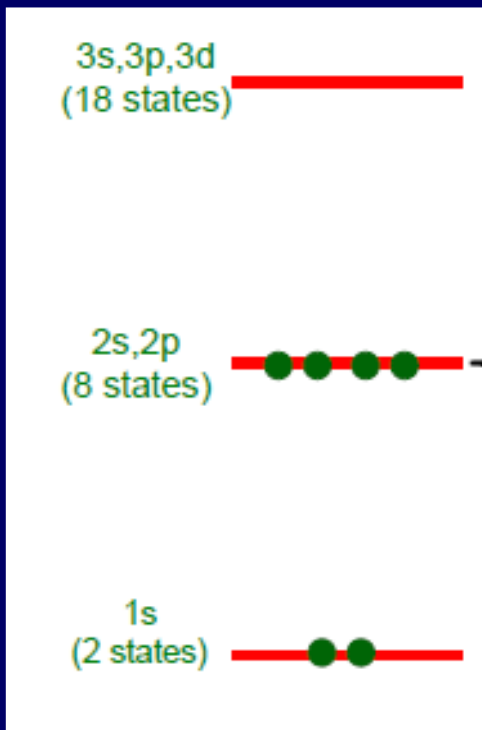
(WTe₂)



Graphitic bond



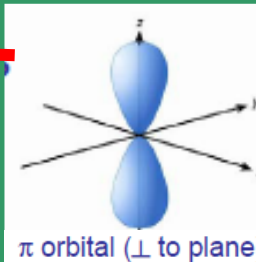
Carbon: $Z=6$
4 valence electrons $2s^2 2p^2$



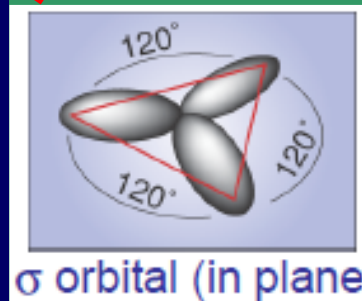
sp²

$2S + 2p_x + 2p_y$ hybridize

3 planar σ orbitals: tetragon
1 Out of plane $2p_z$ [" π " orbital]



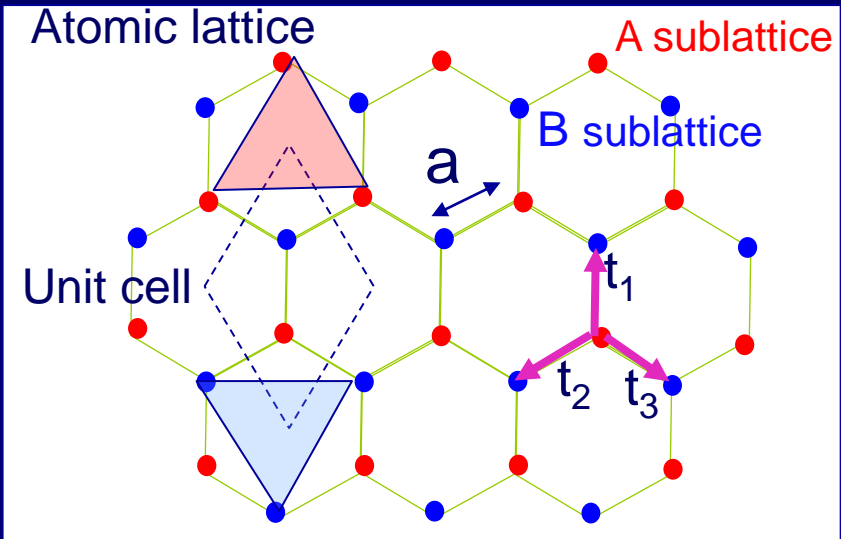
π electrons allow
conduction



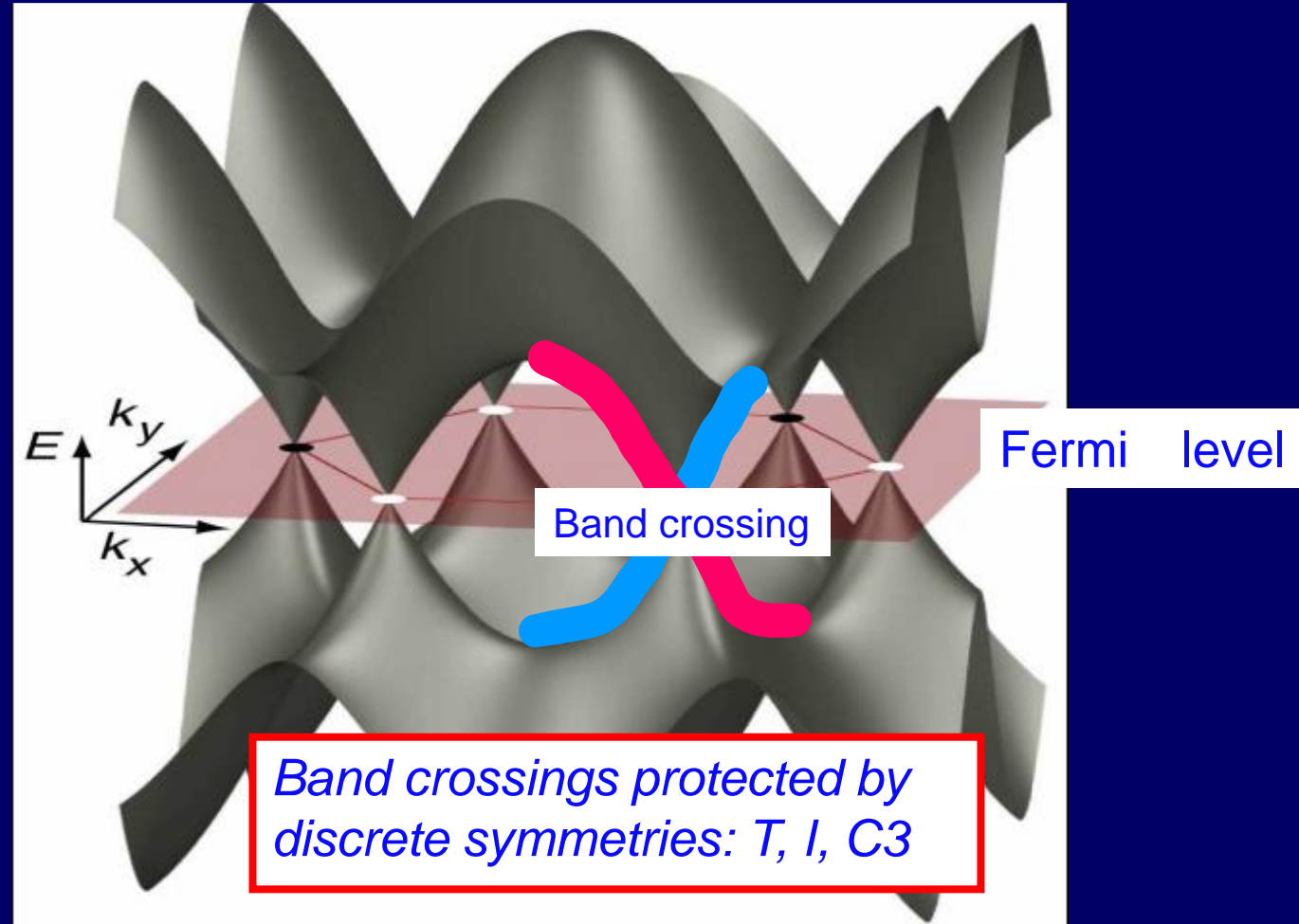
σ bonds: 2D and
exceptional rigidity



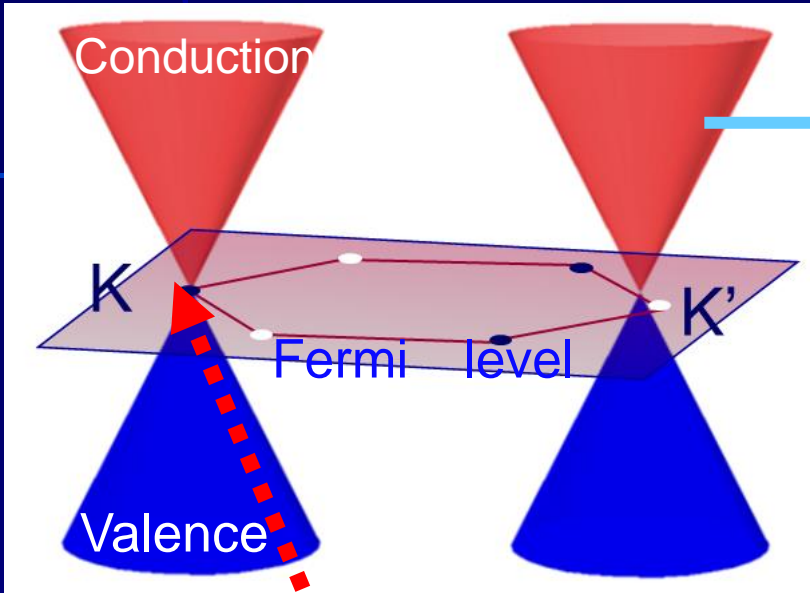
Graphene: Tight binding model



1. 2D
2. Honeycomb structure (non-Bravais)
3. 2 identical atoms/cell
4. Identical NN hopping



Graphene



Dirac point

$$v_F = \frac{3}{2\hbar} a t \sim 10^6 \text{ m/s}$$

Dirac Weyl Hamiltonian

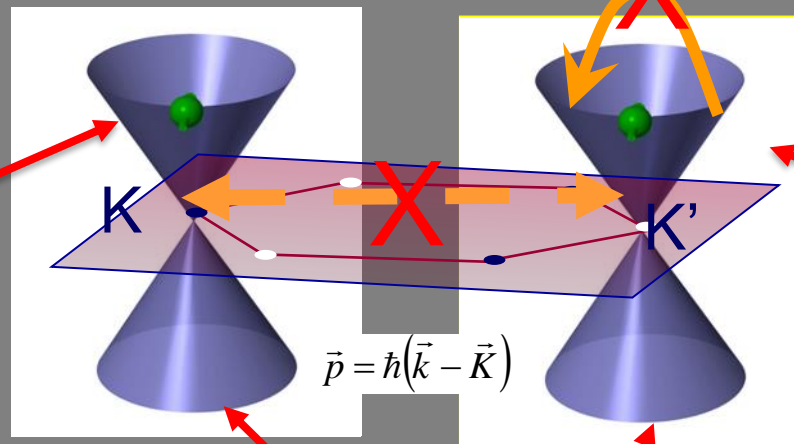
$$H = v_F \begin{pmatrix} \vec{\sigma} \cdot \vec{p}_K & 0 \\ 0 & -\vec{\sigma}^* \cdot \vec{p}_{K'} \end{pmatrix}$$

\vec{p} momentum relative to K point
 σ Pauli matrices operate on
 sublattice degree of freedom

*6 Dirac cones, but only 2
 independent at K and K' valleys*



No Backscattering



Dirac Weyl Hamiltonian

$$H = v_F \begin{pmatrix} \vec{\sigma} \cdot \vec{p} & 0 \\ 0 & -\vec{\sigma}^* \cdot \vec{p} \end{pmatrix}$$

Helicity

$$H_K \propto \vec{\sigma} \cdot \vec{p}$$

→ projection of pseudospin
along momentum conserved

No backscattering between cones

No backscattering within cones

➤ High carrier mobility

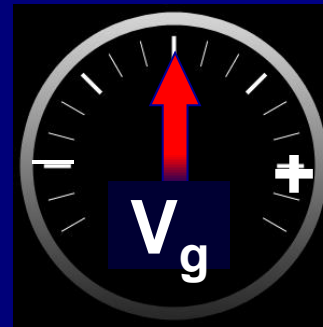
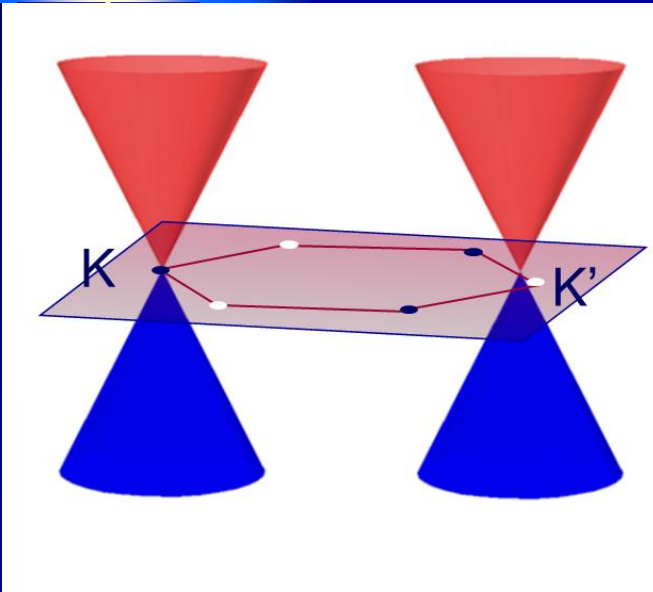
Two massless Dirac fermions
with opposite chirality (helicity)

4 flavors: 2 Spin , 2 valley



Density of states

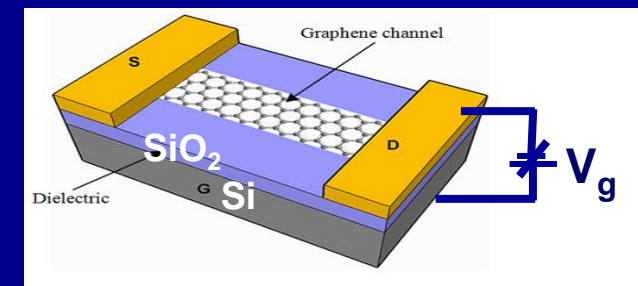
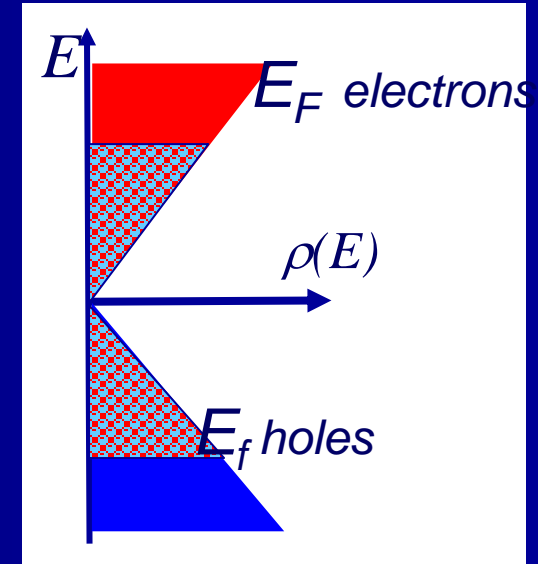
Band structure



$$1\text{V} \mapsto 7 \times 10^{10} \text{ cm}^{-2}$$
$$\Delta E \sim 80 \text{ meV}$$

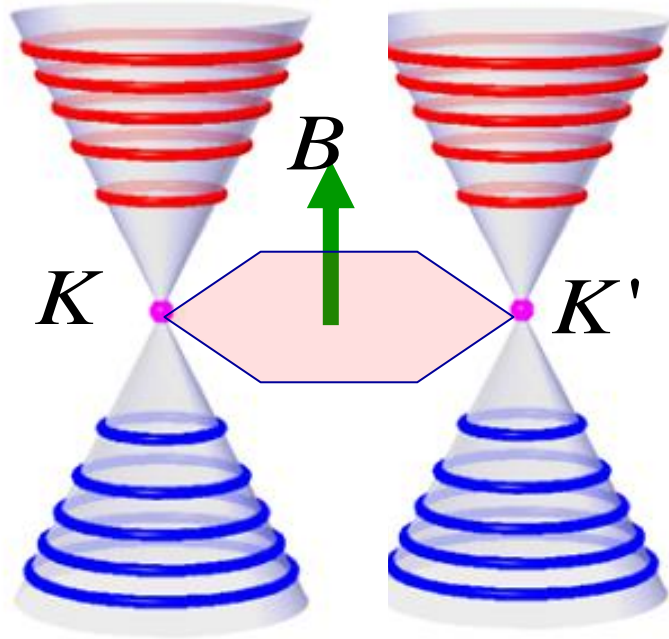
Density of states

$$\rho(E) = 3^{3/2} a^2 \frac{|E|}{\pi (\hbar v_F)^2}$$



Landau levels

Band structure



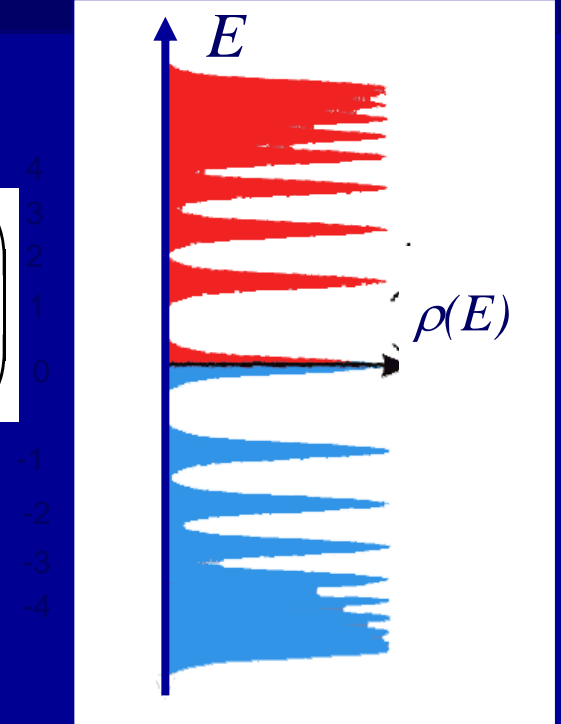
$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$H_{K,K'} = v_F \begin{pmatrix} \vec{\sigma} \cdot (\vec{p} - e\vec{A}) & 0 \\ 0 & -\vec{\sigma}^* \cdot (\vec{p} - e\vec{A}) \end{pmatrix}$$

Landau Levels

$$E_N = \pm v_F \sqrt{2e\hbar B |N|} \quad N = 0, \pm 1, \dots$$

Density of states

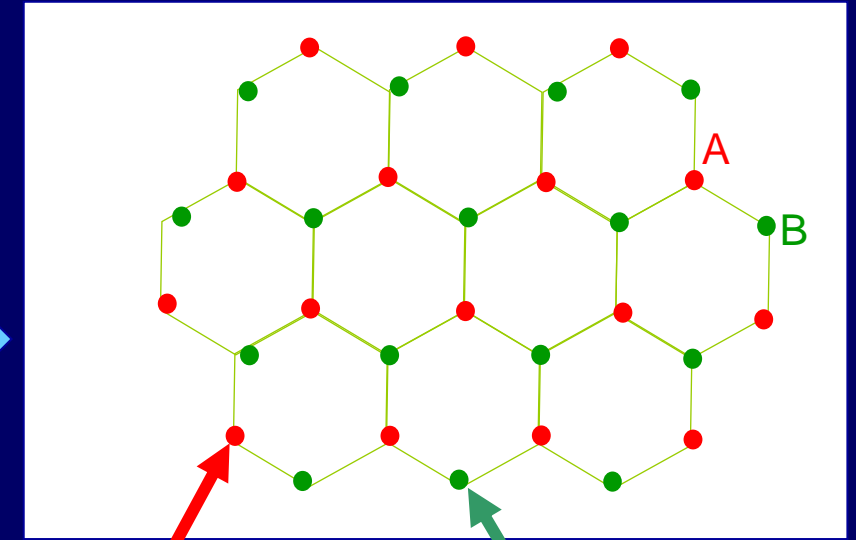
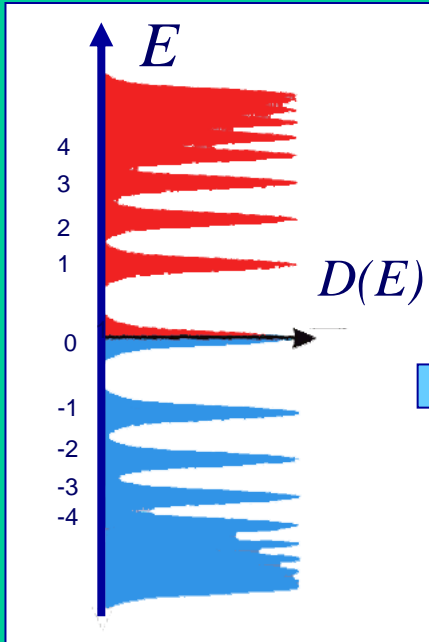
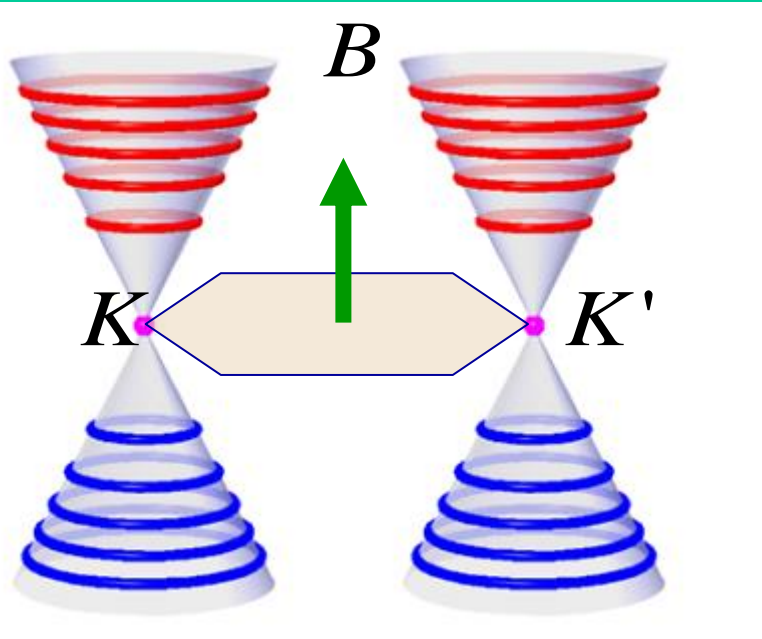


Sublattice Valley locking

Band structure

DOS

N=0 Wave function



$$E_N = \pm v_F \sqrt{2e\hbar B |N|} \quad N = 0, \pm 1, \dots$$

$$\psi_{N=0}^K(A)$$

$$\psi_{N=0}^{K'}(B)$$



Energy and length scales

$$E_N = \pm v_F \sqrt{2e\hbar B|N|} = \pm \varepsilon_0 \sqrt{2|N|}; \quad N = 0, \pm 1, \dots$$

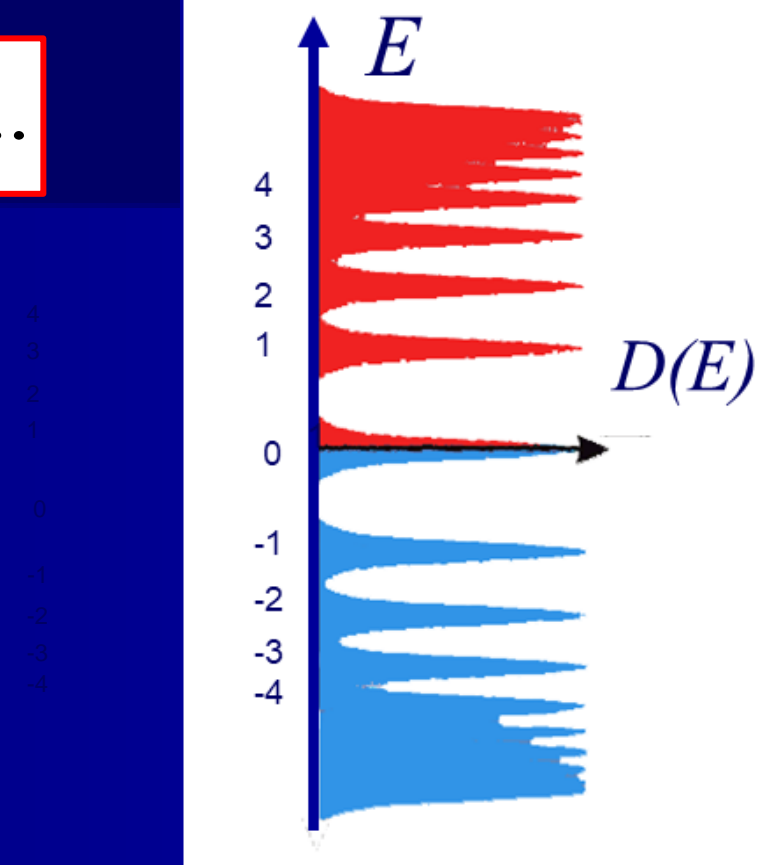
$$\varepsilon_0 = \hbar v_F / l_B \approx 35\sqrt{B} \text{ meV}$$

$$\text{Orbital Degeneracy } g_0 = B / \phi_0 = 2.5 \times 10^{14} \text{ m}^{-2} B[T]$$

$$\text{Total Degeneracy: } g = 4g_0; \quad \phi_0 = h/e$$

Compare to non-relativistic electrons

$$E_N = (N + 1/2)\hbar\omega_c$$
$$\hbar\omega_c \approx (0.1B)\text{meV}$$



Exquisite sensitivity of 2D materials

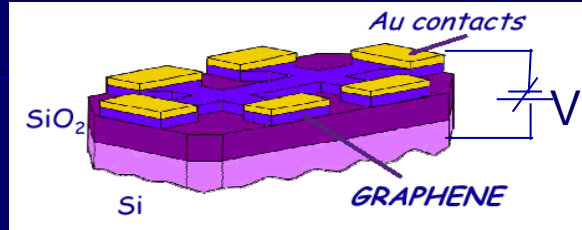
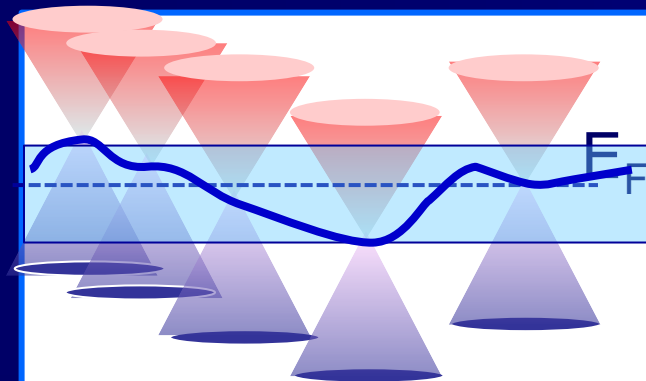
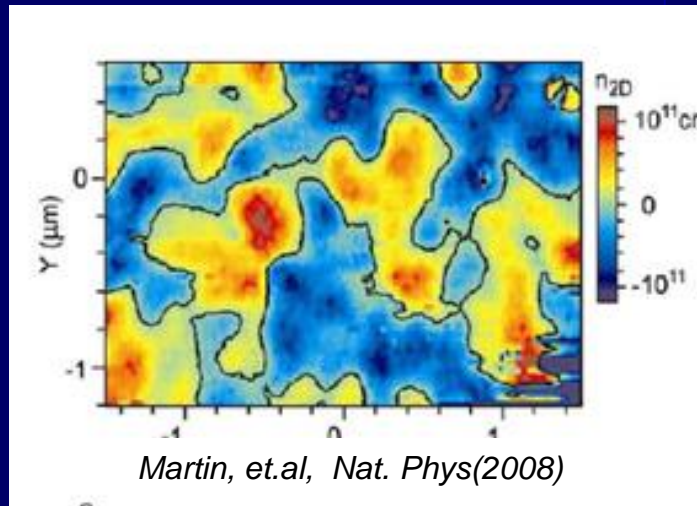
- ❖ 2D materials
- ❖ Graphene
- ❖ Transport and STM
- ❖ Flat bands and correlations
- ❖ Twisted bilayer graphene



Transport measurements: Substrate trouble

e-h puddles
smeared Dirac point

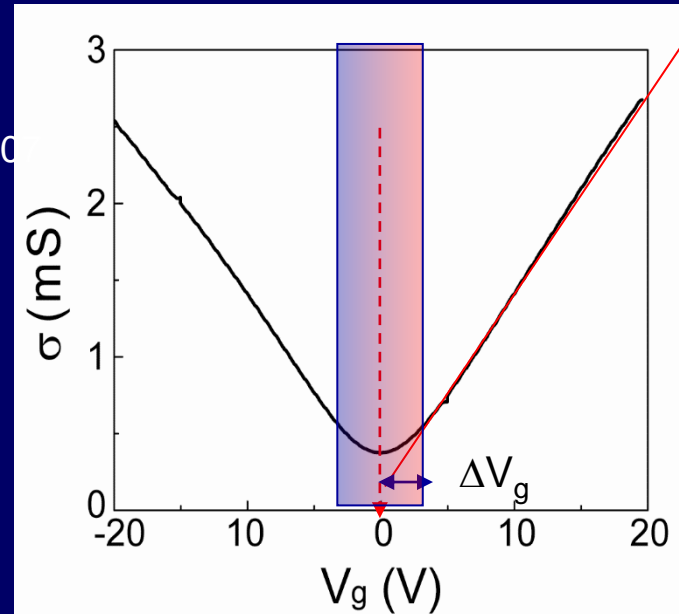
SET: Local charge density



STM

- Ishigami et al Nano letters 200
- Stolyarova et al PNAS 2007
- Geringer et al PRL 2009
- Zhang et al Nat Phys 2009
- Luican et al PRB R 2011
- S. Jung et al Nat Phys 2011

Conductivity

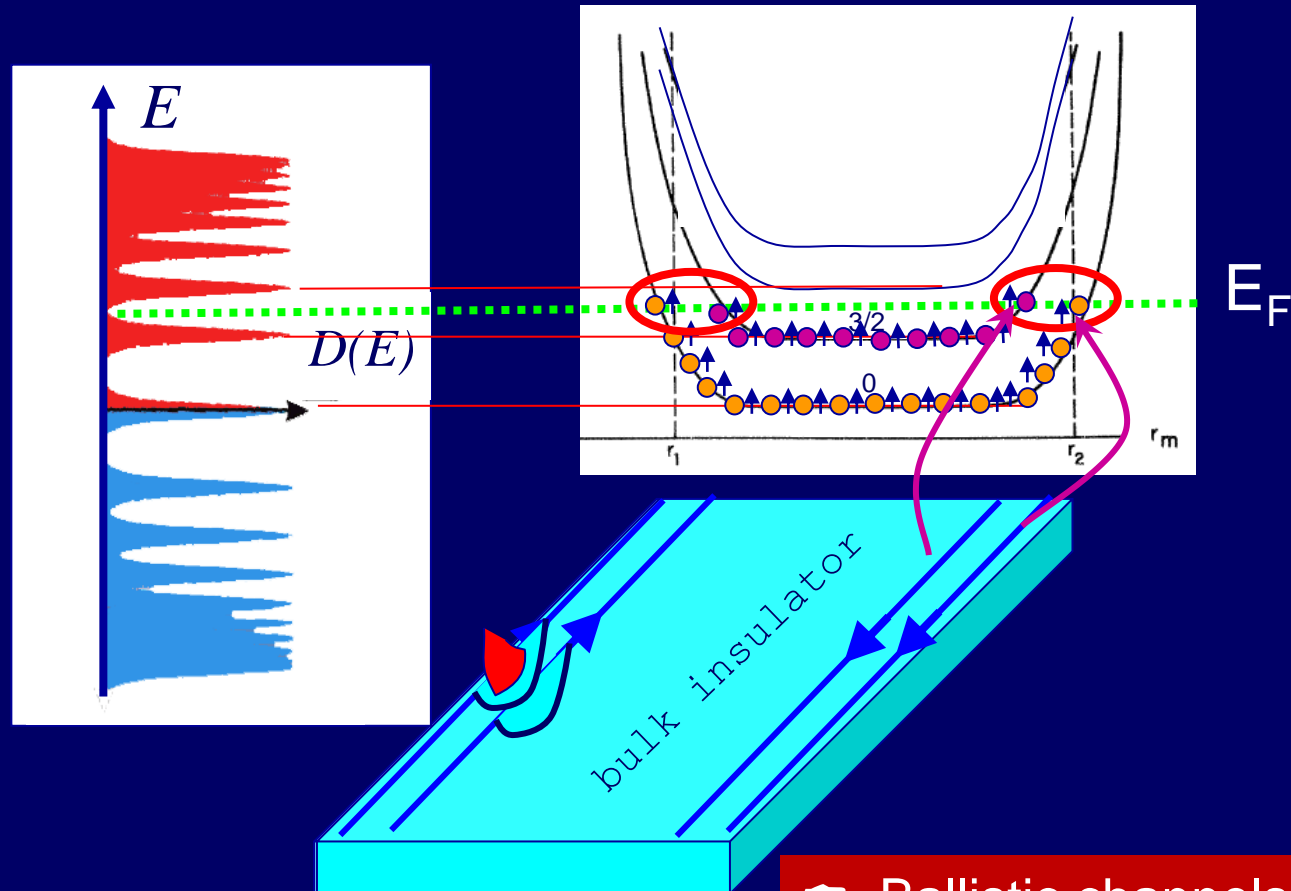


$V_{gmin} \sim 1-10V$
 $n_{min} \sim 10^{11} - 10^{12} \text{ cm}^{-2}$
($\Delta E_{RP} \sim 30-100 \text{ meV}$)



Quantum Hall Effect

For $E_F \neq E_N$ and sharp edge each filled Landau level contributes *one (g-fold degenerate) edge state*



- ☛ Ballistic channels
- One dimensional edge states
- No backscattering: as long as channels far apart
- Robust against disorder

Prerequisites for QHE in graphene

IQHE

1. Decoupled edges

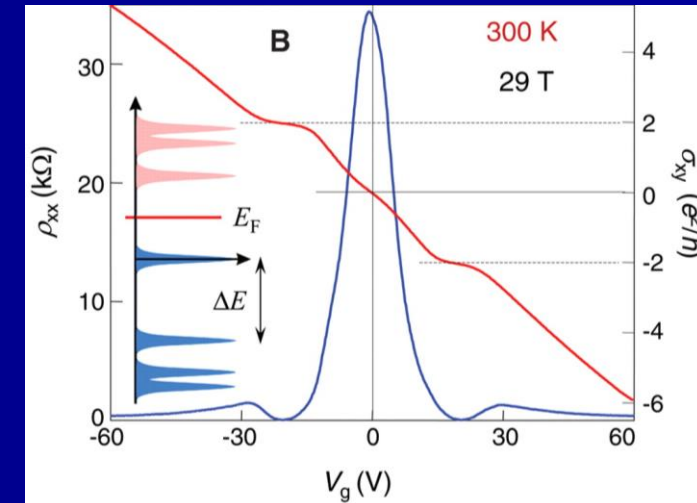
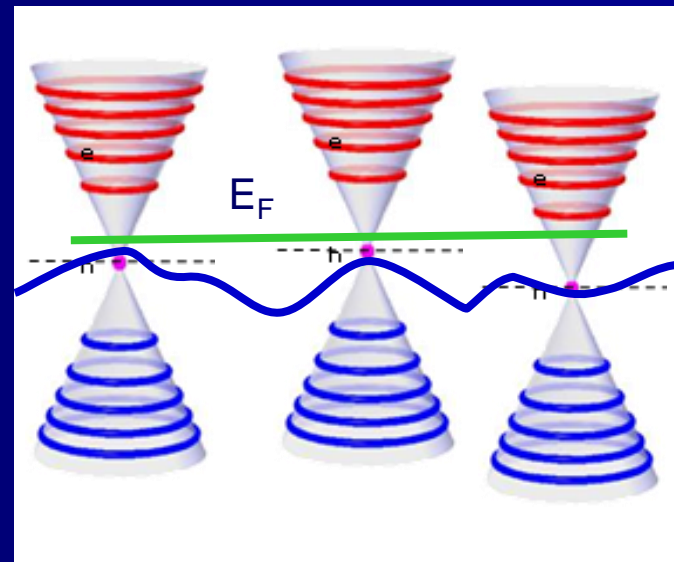
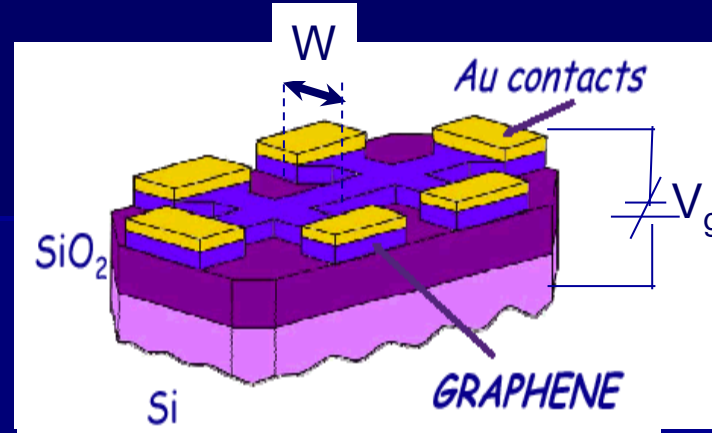
$$l_c = 25nm / (B[T])^{1/2} \ll W$$

2. Well defined Landau levels

$$E_1 - E_0 = 35meV \cdot (B[T])^{1/2} > \Delta E_{RP}, k_B T$$

$$\Delta E_{RP} \sim 100meV \rightarrow \text{IQHE for } B > 10T$$

random
potential



Room-Temperature Quantum Hall Effect in Graphene

K. S. NOVOSELOV, Z. JIANG, Y. ZHANG, S. V. MOROZOV, H. L. STORMER, U. ZEITLER, J. C. MAAN, G. S. BOEBINGER, P. KIM, [...] A. K. GEIM

+1 authors

Authors Info

Affiliations

SCIENCE • 9 Mar 2007 • Vol 315, Issue 5817 • p. 1379 • DOI: 10.1126/science.1137201



Integer and fractional QHE in graphene

IQHE

1. Decoupled edges

$$l_c = 25nm / (B[T])^{1/2} \ll W$$

2. Well defined Landau levels

$$E_1 - E_0 = 35meV \cdot (B[T])^{1/2} > \Delta E_{RP}, k_B T$$

$$\Delta E_{RP} \sim 100meV \rightarrow \text{IQHE for } B > 10T$$

random
potential

IQHE easily observed in graphene even at room temperature!

FQHE

1. Decoupled edges

$$l_c = 25nm / (B[T])^{1/2} \ll W$$

2. Well defined “ Λ ” levels

$$E_{\Lambda=1/3} = v^2 \frac{e^2}{\kappa l_c} \approx 5meV \frac{(B[T])^{1/2}}{\kappa} > \Delta E_{RP}$$

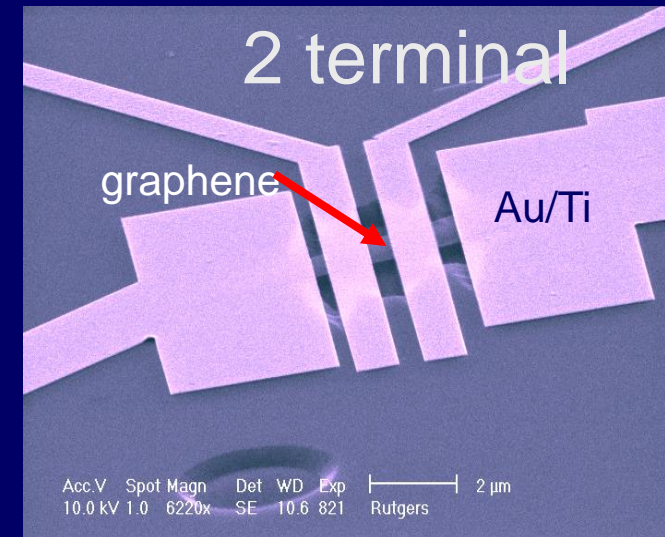
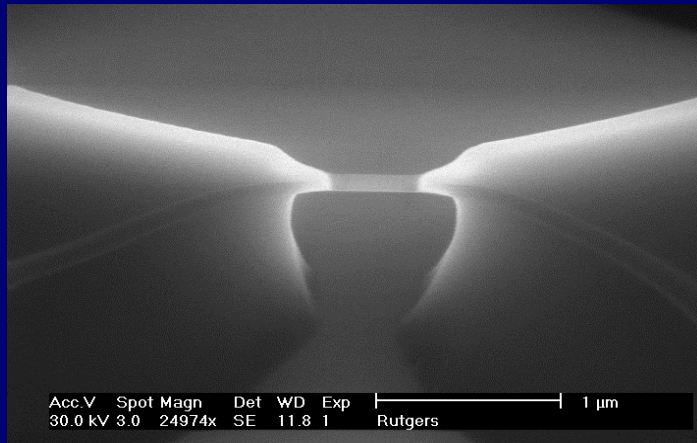
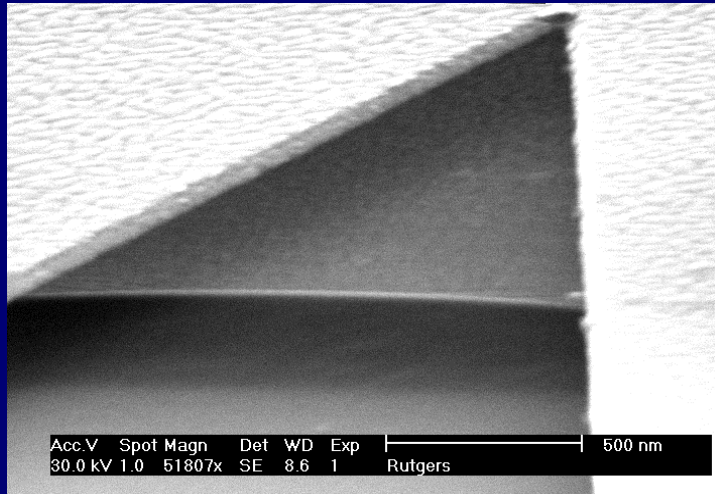
$$\Delta E_{RP} \sim 100meV \Rightarrow B > 6400 T$$

FQHE not observable in graphene on SiO₂ substrates

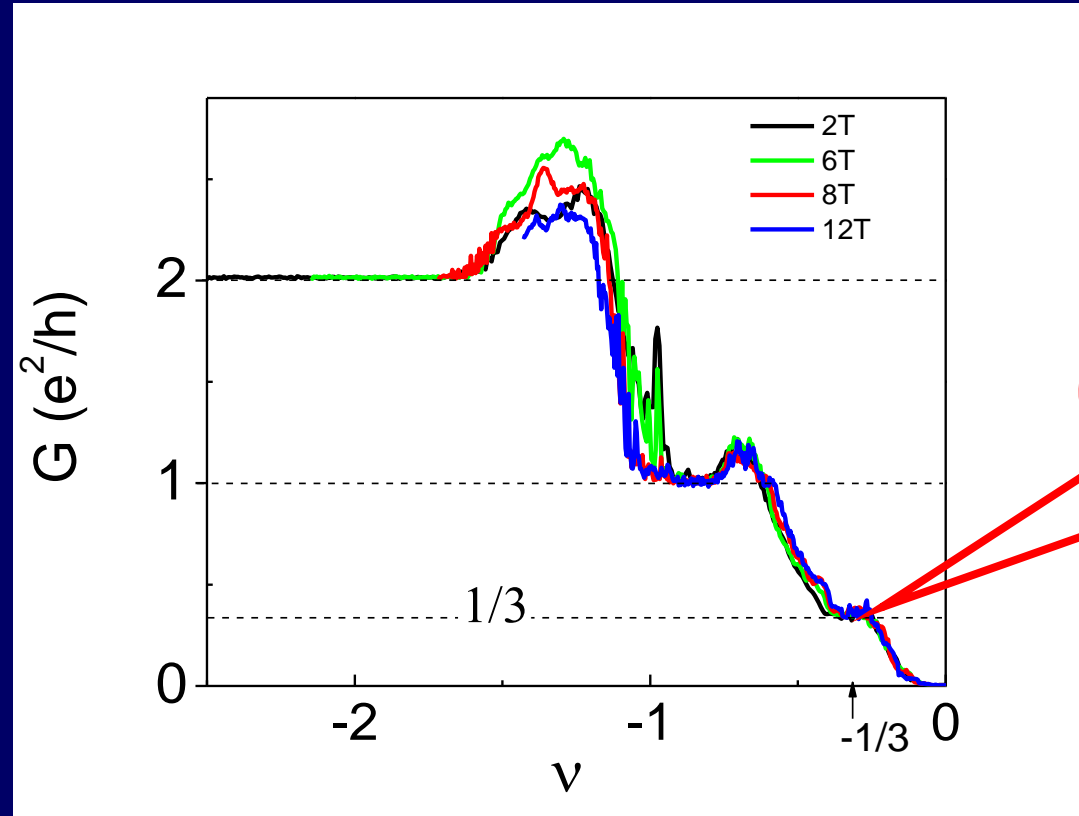


Suspended Graphene

- X. Du, I. Skachako, A. Barker, E. Y. A. Nature Nanotech. 3, 491 (2008)
- Bolotin et al , Solid State Communications (2008)



X. Du, I. Skachko, F. Duerr, A. Luican, EYA, Nature **462**, 192 (2009)



FQHE in graphene
Seen already at 2T at 0.3K
Persists up to 20K (in 12T)

- Bolotin et al (Columbia) Nature 462 (2009)
- Geim & Novoselov (Manchester)

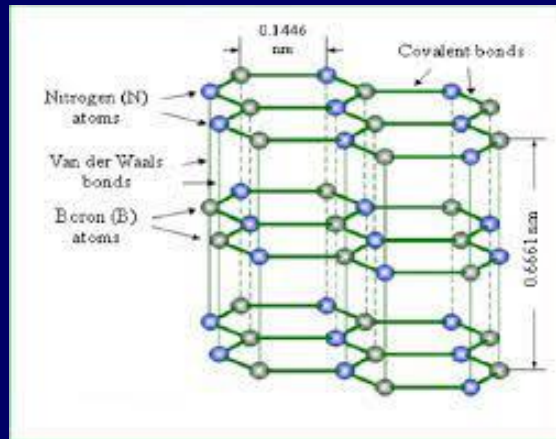
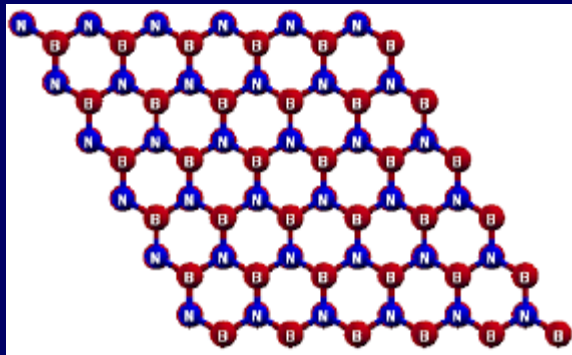
G/BN C. R. Dean et. al arxiv1010.1179




The magic of hexagonal Boron Nitride (hBN)

Hexagonal Boron Nitride - hBN

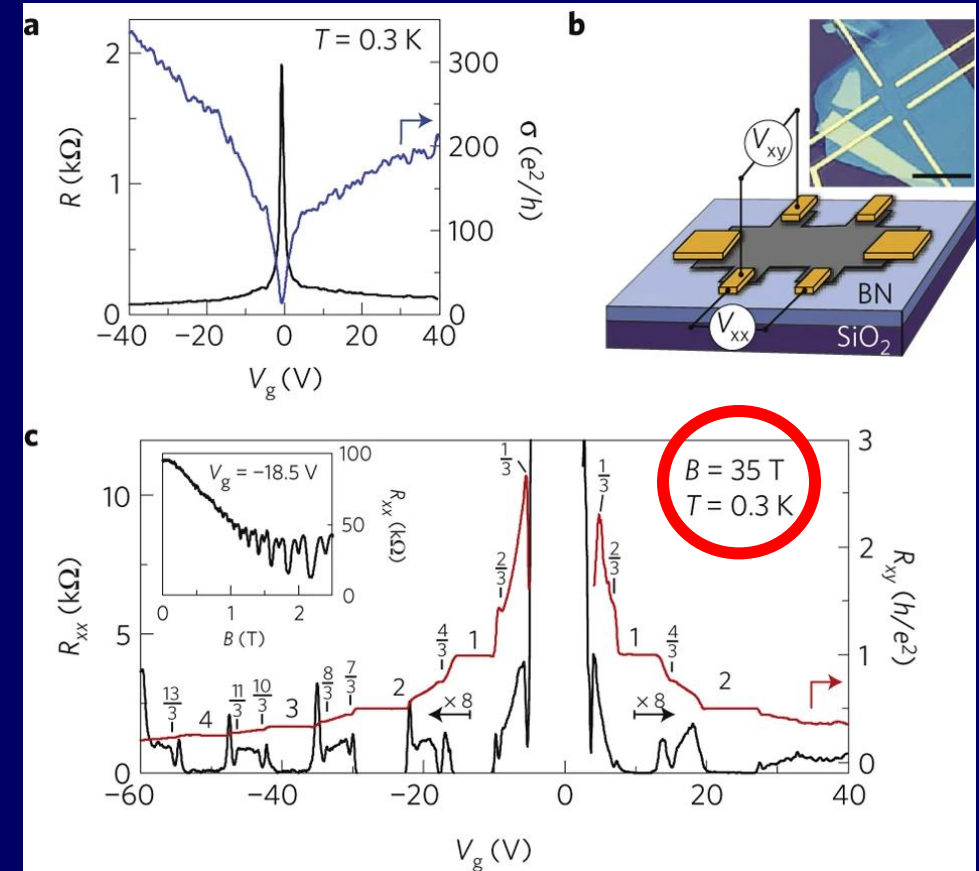
- Honeycomb structure – no dangling bonds, chemically inert.
- Protects graphene from electrical random potential, and chemical contamination



Multicomponent fractional quantum Hall effect in graphene

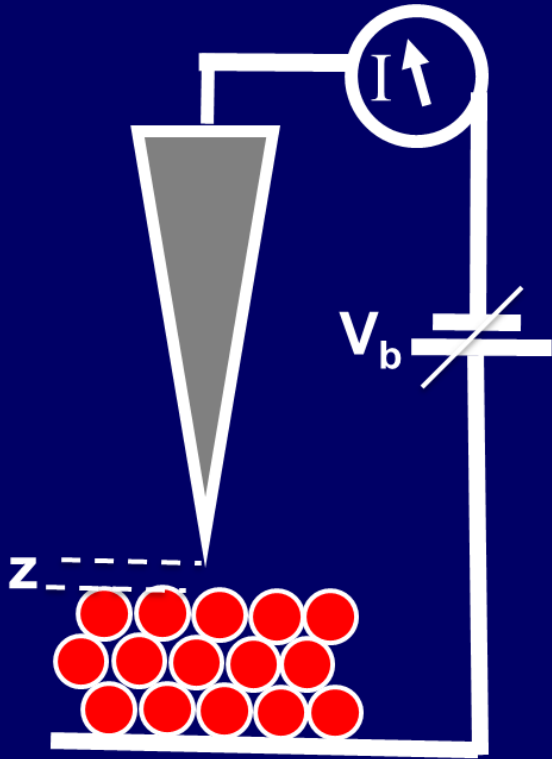
C. R. Dean, A. F. Young, P. Cadden-Zimansky, L. Wang, H. Ren, K. Watanabe, T. Taniguchi, P. Kim , J. Hone & K. L. Shepard

Nature Physics **7**, 693–696 (2011) | [Cite this article](#)



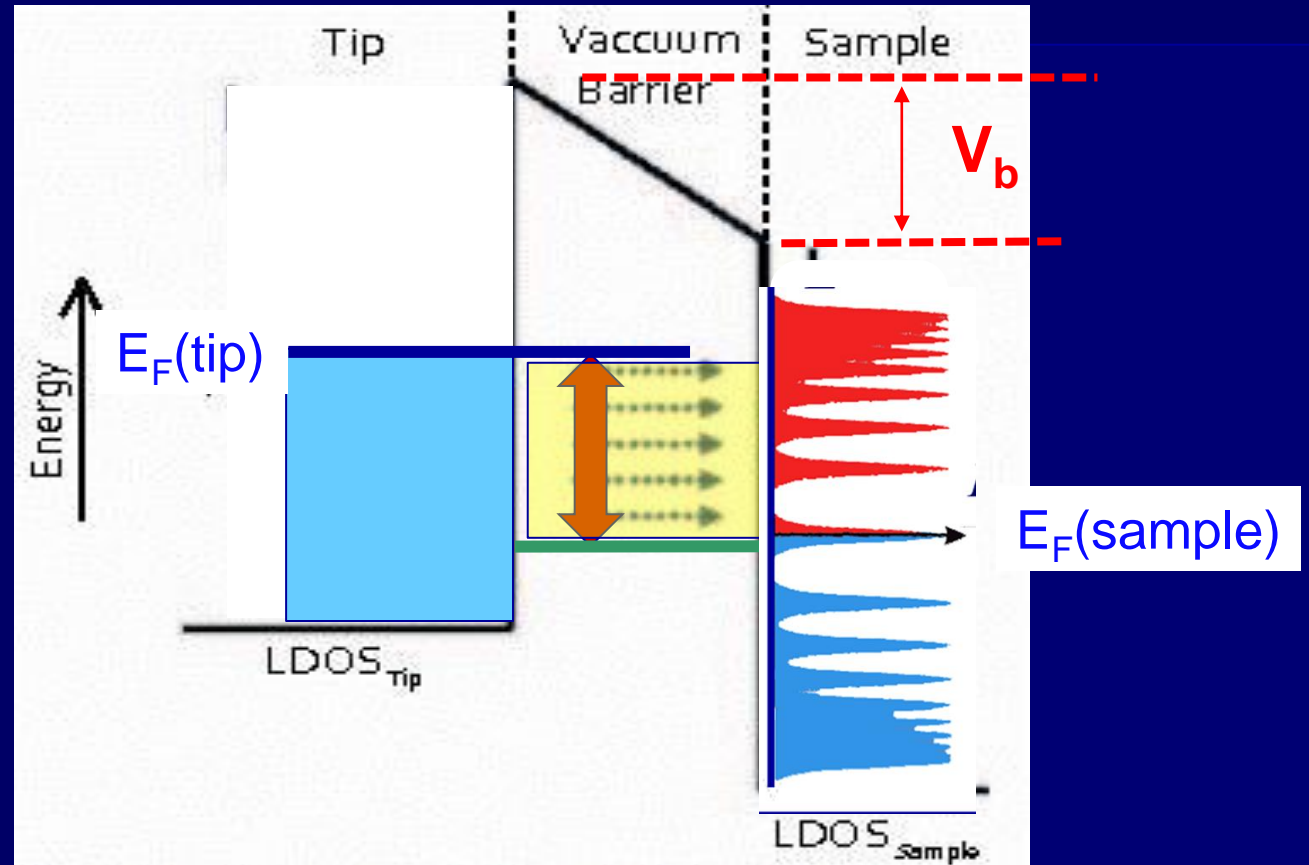
Scanning Tunneling Microscopy (STM)

STM measures tunnel current across a vacuum barrier

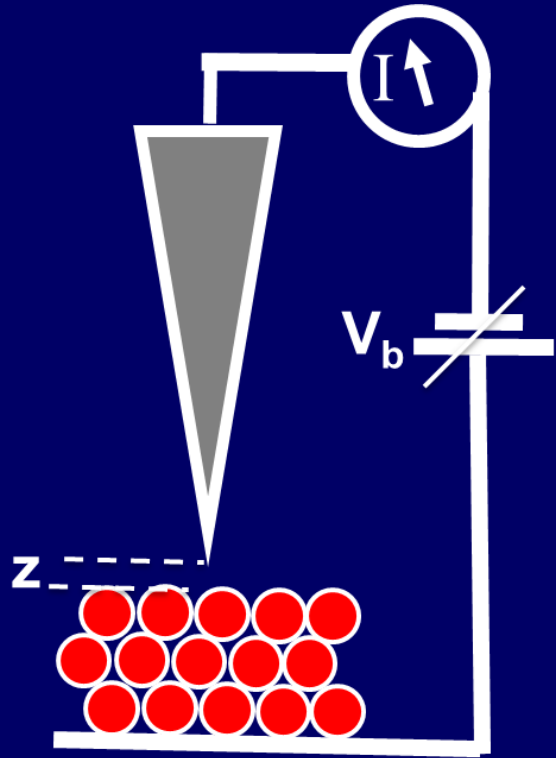


STM probes all states between $E_F(\text{tip})$ and $E_F(\text{Sample})$

$$I(r, z, V) \propto \left[\int_0^{eV_b} \rho(r, \varepsilon) d\varepsilon \right] \exp^{-z(r)\kappa}$$



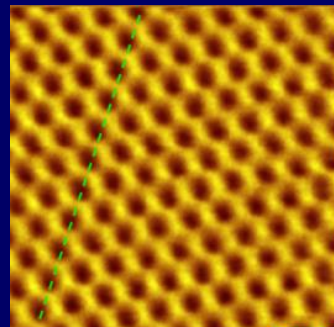
Graphene: STM and STS



$$I(r, z, V) \propto \left[\int_0^{eV_b} \rho(r, \varepsilon) d\varepsilon \right] \exp^{-z(r)K}$$

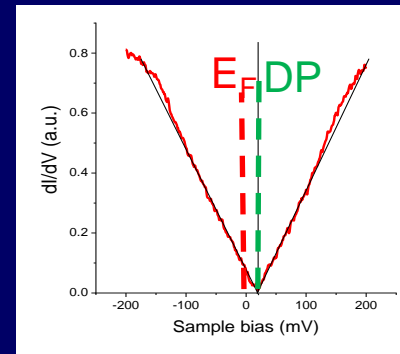
I, V_b constant
Topography:
Imaging Atoms

$$I(r, z) \propto \exp^{-z(r)K}$$



z constant
Spectroscopy:
Density of states

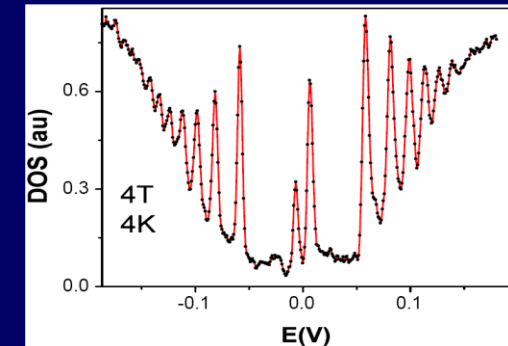
$$dI/dV_b \propto \rho(r, V_b)$$



Landau level
Spectroscopy:

- Local Fermi velocity
- Quasiparticle lifetime
- Coupling to substrate

$$E_N = \pm v_F \sqrt{2e\hbar B |N|}$$



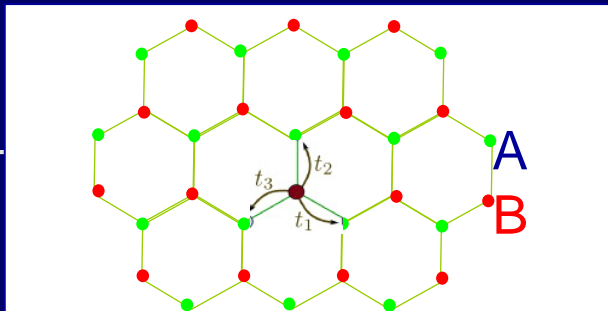
Graphene on graphite

G. Li, A. Luican, E. Y. A.
Phys. Rev. Lett 102, 176804 (2009)

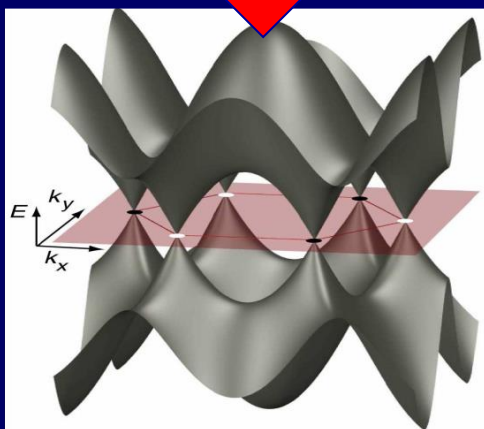
E.Y. Andrei



Controlling the band structure?



1. 2D
2. Honeycomb structure
3. 2 identical atoms/cell
4. $t_1=t_2=t_3=t$



Can one control/manipulate the band structure?

- Relax the condition: $t_1=t_2=t_3$
 - Strain
- Impose external potential
 - Substrate
- Point Defects
- Electrostatic potential
- Magnetic field



2-terminal measurement of QHE

Conformal invariance and shape-dependent conductance of graphene samples

Dmitry A. Abanin and Leonid S. Levitov
Phys. Rev. B **78**, 035416 – Published 10 July 2008

Fractional quantum Hall effect in suspended graphene: Transport coefficients and electron interaction strength

D. A. Abanin, I. Skachko, X. Du, E. Y. Andrei, and L. S. Levitov
Phys. Rev. B **81**, 115410 – Published 8 March 2010

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MATHEMATICAL, PHYSICAL AND ENGINEERING SCIENCES

Fractional quantum Hall effect in suspended graphene probed with two-terminal measurements

I. Skachko, X. Du, F. Duerr, A. Luican, D. A. Abanin, L. S. Levitov and E. Y. Andrei

Published: 13 December 2010 | <https://doi.org/10.1098/rsta.2010.0226>

Berry phase of π

Electronic properties of graphene: a perspective from scanning tunneling microscopy and magnetotransport

Eva Y Andrei¹, Guohong Li¹ and Xu Du²

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[Reports on Progress in Physics](#), Volume 75, Number 5

Onsager: Area of N'th orbit in k space

$$S(k_N) = \pi k_N^2 l_B^2 = 2\pi(N + 1/2 - \gamma / 2\pi)$$

γ = Berry Phase

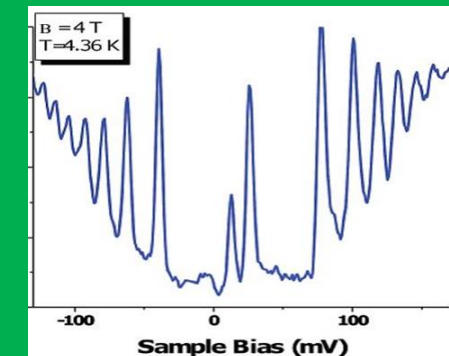
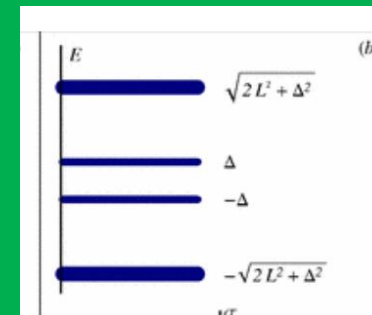
$$\oint_C d\lambda \langle \psi(\lambda) | i(\partial/\partial\lambda) | \psi(\lambda) \rangle$$

Excitonic gap (Magnetic catalysis)

Excitonic gap, phase transition, and quantum Hall effect in graphene

V. P. Gusynin, V. A. Miransky, S. G. Sharapov, and I. A. Shovkovy

Phys. Rev. B **74**, 195429 – Published 22 November 2006



More references

Graphene monolayer and bilayer tutorials

REVIEW ARTICLE

The electronic properties of bilayer graphene

Edward McCann¹ and Mikito Koshino²

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[Reports on Progress in Physics](#), [Volume 76](#), [Number 5](#)

McCann E 2012 *Graphene Nanoelectronics: Metrology, Synthesis, Properties and Applications* Raza H (ed) (Berlin: Springer-Verlag) 237-275

