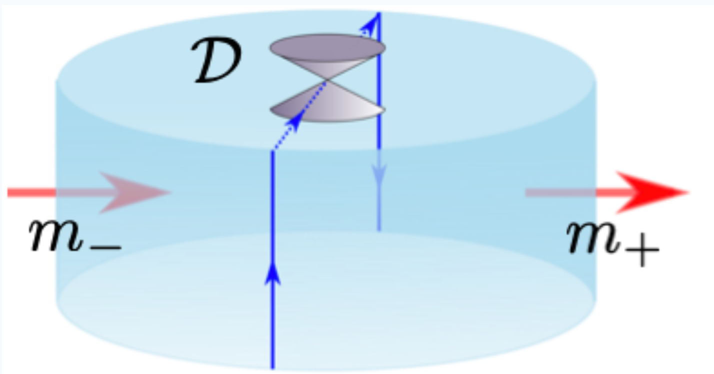
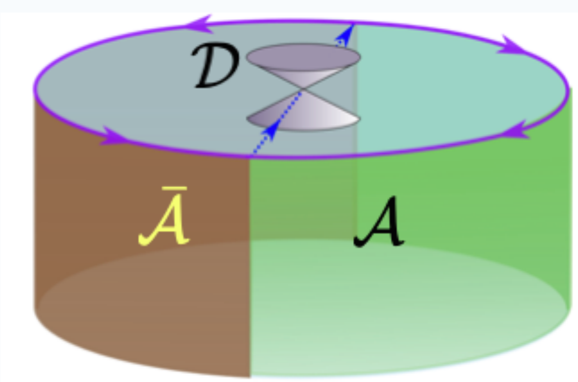
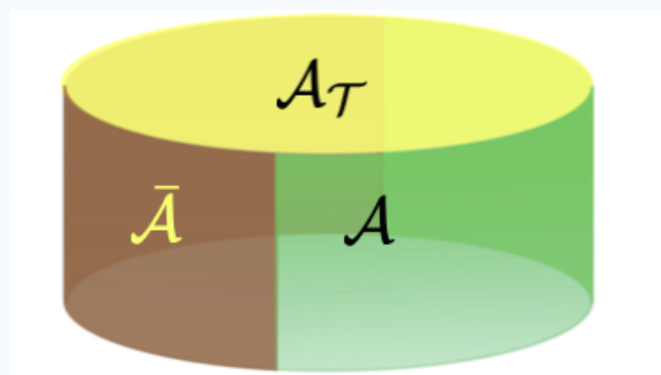
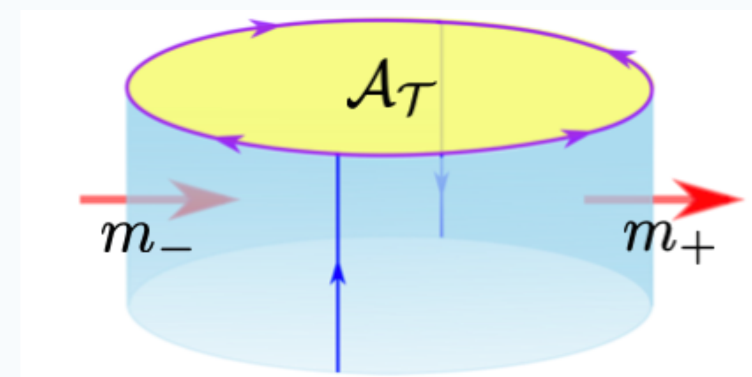
‘Unhinging’ the surface of Higher-Order Topological Phases of Matter

Appearing soon on arXiv 1905.*****









Perfect beam-splitter

Apoorv Tiwari @ University of Zurich UZH

The 15th Capri Spring School on Transport in Nanostructures 2019

Collaborators

Ming-Hao Li
ETH Zurich

B.A Bernevig
Princeton

Titus Neupert
Univ. of Zurich

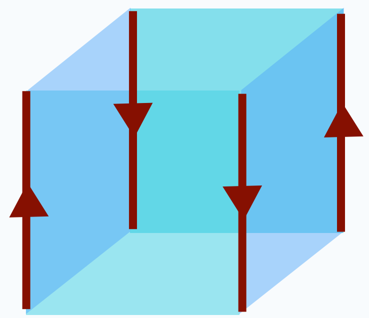
S.A Parmeswaran
Oxford

Motivation and underlying question:

To establish a generalized bulk boundary correspondence for higher-order topological phases of matter (HOTPs).

- What are the possible surface terminations for HOTPs?

- Is there a generalized notion of anomalies for HOTPs?



In this work, we focus on hinge HOTPs.

Higher-Order Topological Phases:

Defining feature : **Bulk Boundary correspondence**

pth order phases $\Longrightarrow$ symmetry protected **gapless** modes on codimension-p corners.

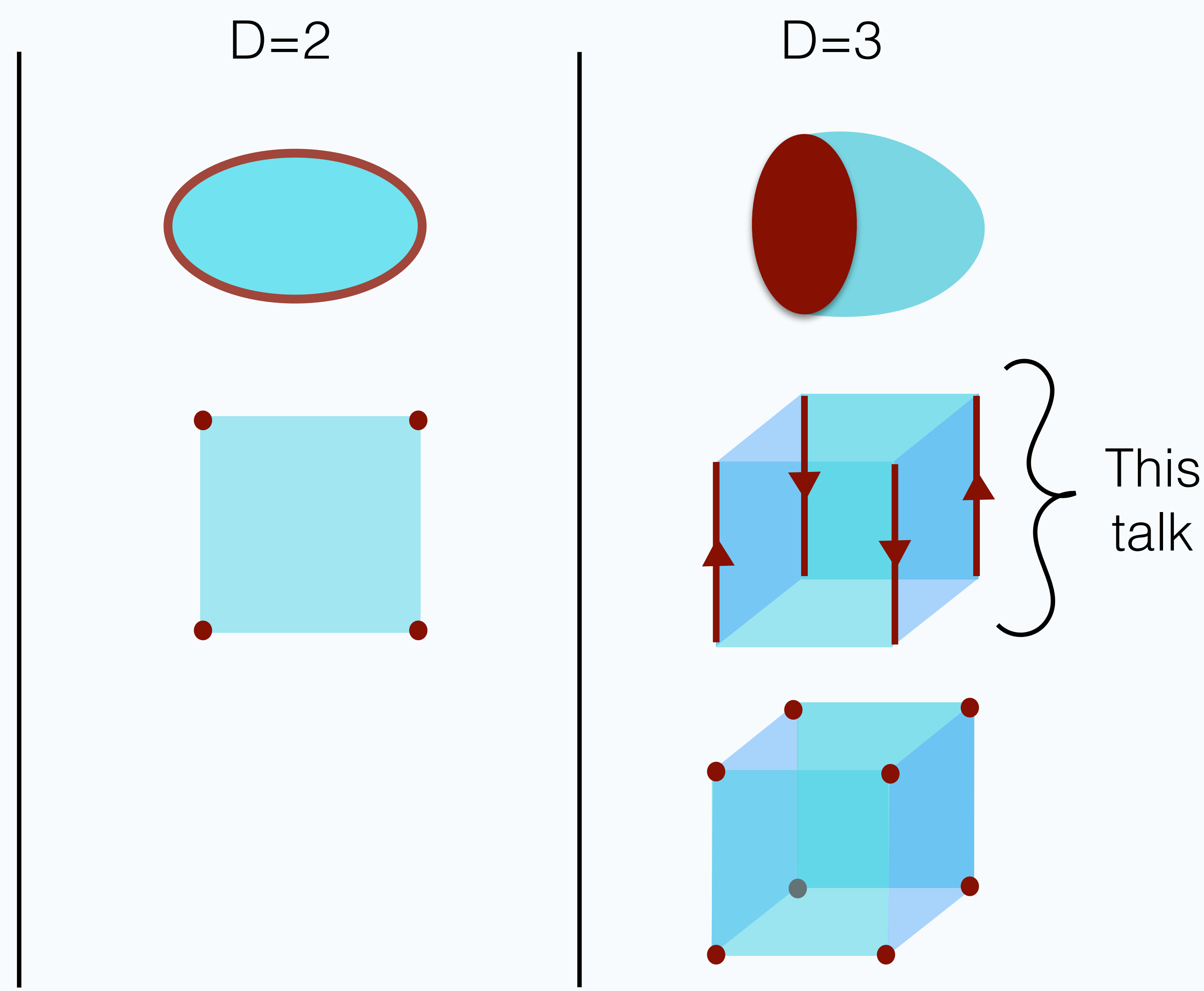
p>1 require spatial symmetries that map one surface to another.

- 1st Order Topological Phases:

  eg. TIs, TSCs, SPTs

- 2nd Order Topological Phases:

- 3rd Order Topological Phases* :



*** Benalcazar et al; Schindler et al; Brouwer et al; Khalaf; …**

1st-Order Topological Phases: Bulk-Boundary correspondence

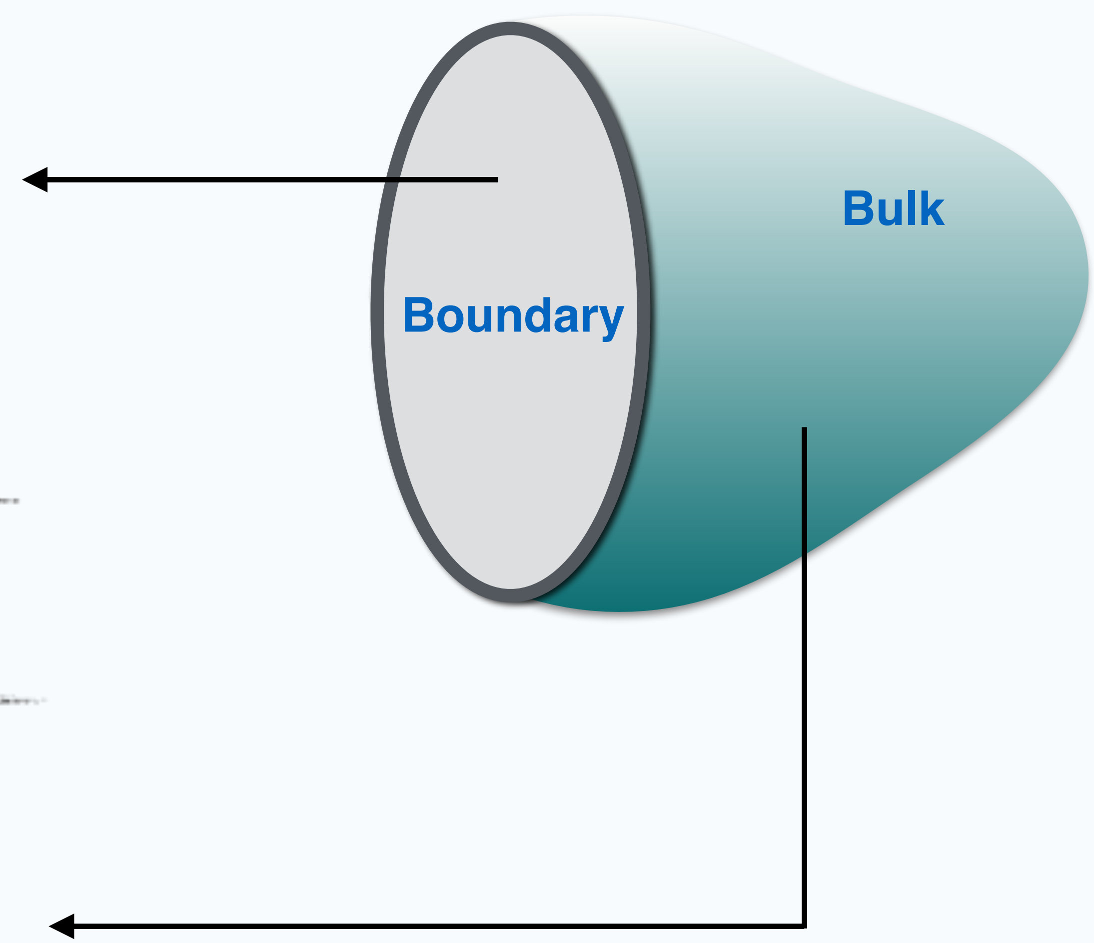
Equivalence classes of gapped (and symmetric) short-range entangled systems.

D-1 dimensional surface:

Anomalous i.e cannot be realized as a symmetric phase of matter in D-1-dimensions

D dimensional bulk:

Characterized by topological response/ index.

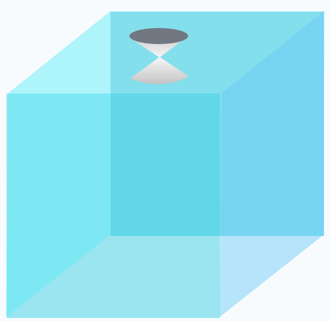


Bulk-boundary correspondence for TIs

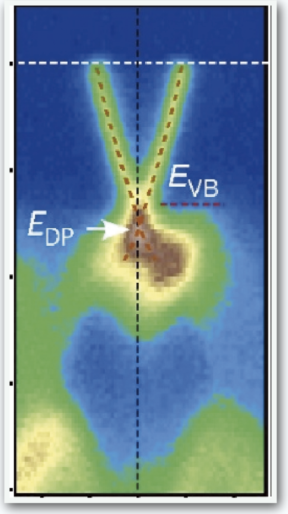
Bulk topological response: $$S_{\text{em-resp}}[A] = \frac{\theta}{8\pi^2}\int_{\mathcal{M}} F \wedge F \qquad ; \theta = 0 \text{ or } \pi.$$

Anomalous surface terminations:

- **Gapless:**



Single Dirac Cone



$$Z_\psi[A] = |Z_\psi[A]| \exp\left\{-\frac{i\pi}{2}\sum_k \text{sgn}(\lambda_k[A])\right\}$$

$$=: |Z_\psi[A]| \exp\left\{-\frac{i\pi\eta[A]}{2}\right\}$$

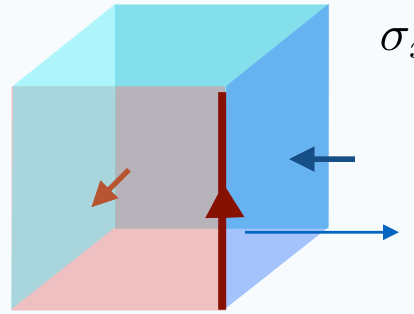
→ Anomalous

$$Z_{\text{TI}}[\mathcal{M}, A] = |Z_\psi[A]| \exp\left\{-\frac{i\pi\eta[\partial\mathcal{M}, A]}{2} + iS^{\theta=\pi}_{\text{em-resp}}[\mathcal{M}, A]\right\}$$

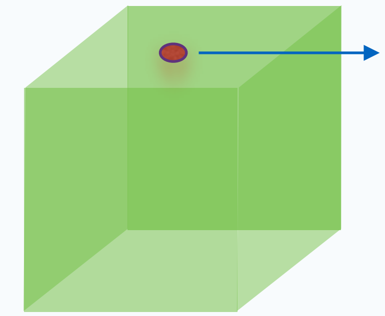
→ Non-Anomalous

* Witten RMP (2016); Alvarez-Gaume Della Pietra, Moore (1985)

- **Symmetry broken:**



Time reversal breaking (ferromagnetic coating)



U(1) breaking (s-wave pairing)

- **Anomalous Surface Topological Order (STO):**

➡ By physical requirements the topological order needs to have:

  - A local fermion, i.e cannot be described by a modular tensor category.
  - Chiral central charge $c_- = 1/2.$
  - Hall conductance $\sigma_{xy} = 1/2.$

➡ Minimal realization known as T-Pfaffian: $\text{T-Pfaffian} = \left[\mathsf{U}(1)_8 \times \overline{\text{Ising}}\right]/\mathbb{Z}_2$

$j \rightarrow$	0	1	2	3	4	5	6	7
1	1		i		1		i	
ψ	-1		$-i$		-1		$-i$	
σ		1		-1		-1		1
Charge	0	$e/4$	$e/2$	$3e/4$	e	$5e/4$	$3e/2$	$2e$

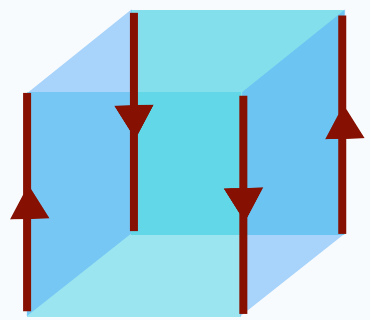
local fermion

$$\mathcal{T}: \begin{array}{l} 1_2 \longleftrightarrow \psi_2 \\ 1_6 \longleftrightarrow \psi_6 \\ \text{else} \circlearrowleft \end{array}$$

Similarly Anomalous STOs have been proposed for Topological superconductors and SPTs in general.

* **Vishwanath-Senthil; Bonderson-Nayak-Qi; Chen-Fidkoski-Vishwanath; Metlitski et al; …**

Back to the central question:



What are the other possible symmetric surface terminations of hinge higher order topological phases?

$\mathsf{C}_{2n}\mathcal{T}$ symmetric second-order topological phases:

Higher-order phase	Symmetry	Chiral Hinge mode	Surface pasting
Fermionic HOTI	$\mathsf{C}_{2n}\mathcal{T} \ltimes \mathsf{U}(1)$	Dirac $q=1;\ c_-=1$	IQHE
Fermionic HOTSC	$\mathsf{C}_{2n}\mathcal{T} \times \mathbb{Z}_2^f$	Majorana $c_-=1/2$	$\mathsf{p}\pm i\mathsf{p}$
Bosonic HOSPT	$\mathsf{C}_{2n}\mathcal{T}$	Bosonic $c_-=8$	$\mathbb{E}_8$ phase

} $\mathbb{Z}_2$ classified.

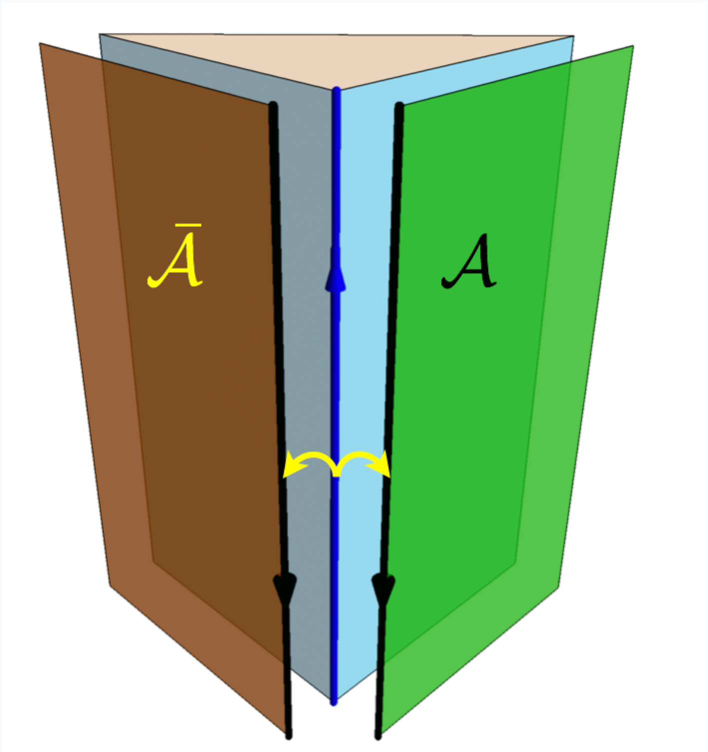
General strategy to “unhinge” HOTPs

- Start from hinge HOTP.

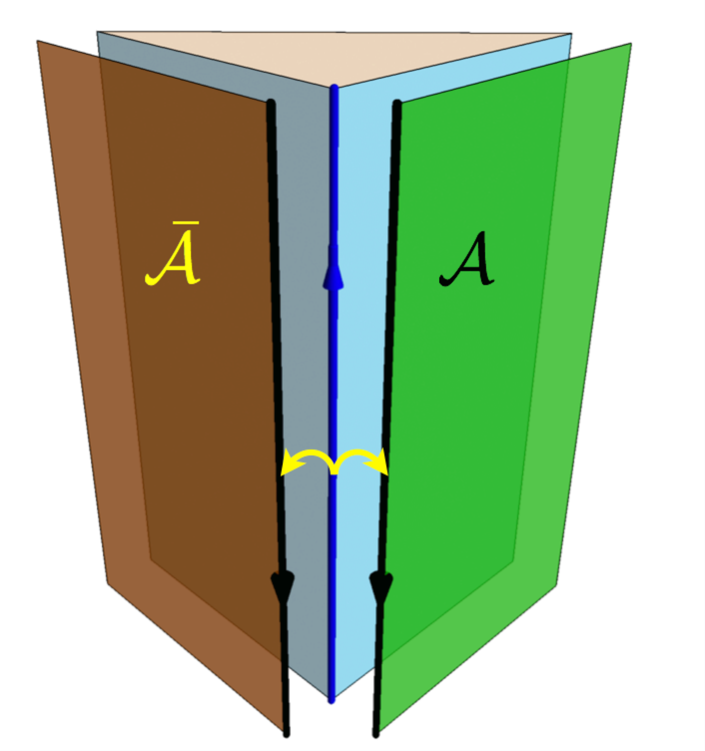
- Introduce $C_{2n}\mathcal{T}$ symmetric topological order on the surface.

- Properties of surface topological order (STO) can be read-off from properties of hinge it needs to absorb.

- Look for symmetric gapping channels. (Haldane gapping criteria and anyon condensation)

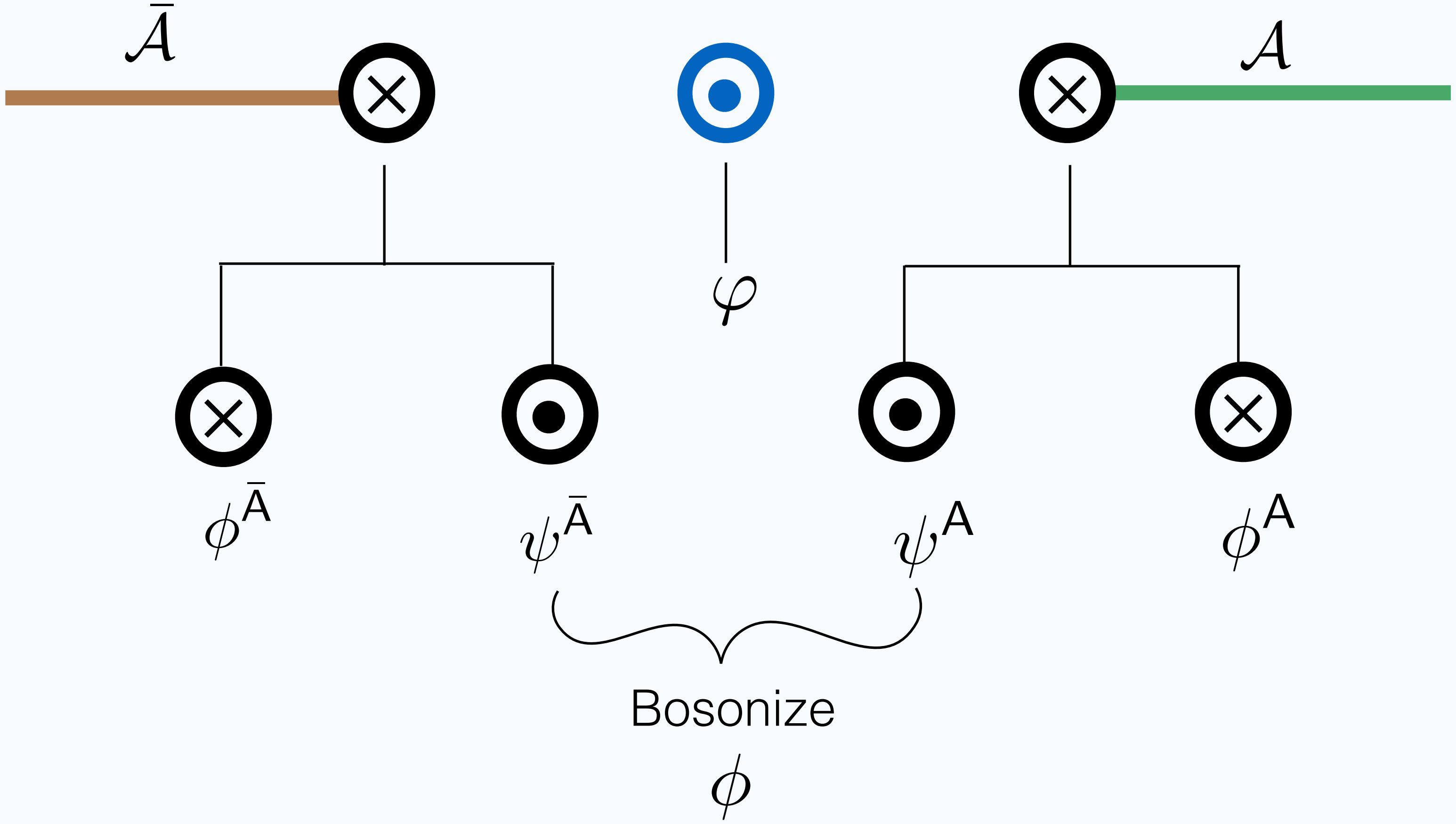


Unhinging the hinge HOTI



- Properties of $\mathcal{A}$:
  - ➡ Chiral central charge, $c_- = 1/2.$ Therefore Non-abelian!
  - ➡ Hall conductance, $\sigma_{xy} = 1/2.$
- Same constraints as STO for TI, therefore we can use T-Pfaffian.

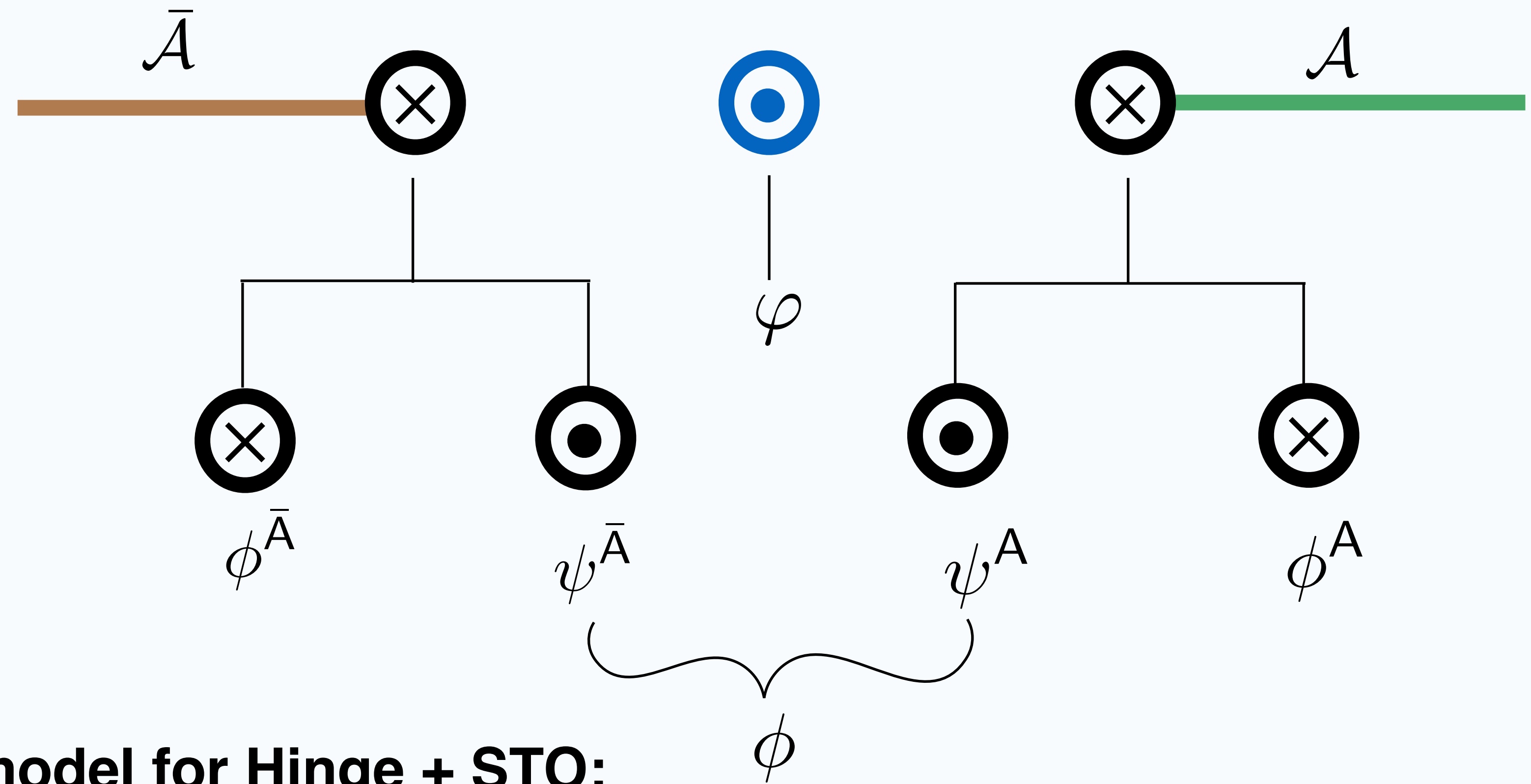
$$\text{T-Pfaffian} = \left[\mathsf{U}(1)_8 \times \overline{\text{Ising}}\right]/\mathbb{Z}_2$$



	$\lvert c \rvert$	$\lvert \sigma_{xy} \rvert$
φ	1	1
$\phi^{A/\bar{A}}$	1	1/2
$\psi^{A/\bar{A}}$	1/2	0
ϕ	1	0

- Effective model for Hinge + STO, multicomponent chiral Luttinger liquid.

Unhinging the hinge HOTI



- **Effective model for Hinge + STO:**

$$\mathcal{L}_{\text{Hinge}} = \frac{1}{4\pi}\partial_x \Phi^T K \partial_t \Phi - \frac{V}{4\pi}\partial_x \Phi^T \partial_x \Phi + \sum_I \lambda_I \cos[\ell_I^T \Phi + \alpha_I]$$

$$\Phi^T = [\phi, \phi^{\mathsf{A}}, \phi^{\bar{\mathsf{A}}}, \varphi]; \quad K = \text{diag}[1, -2, -2, 1]; \qquad q = [0, 1, 1, 1].$$

- **Haldane gapping criteria:**
  - ➡ Condensability: $\ell_I^T K^{-1} \ell_I = 0.$
  - ➡ Mutual locality: $\ell_I^T K^{-1} \ell_J = 0.$
  - ➡ No Spontaneous symmetry breaking: $\ell_I^T K^{-1} q = 0.$

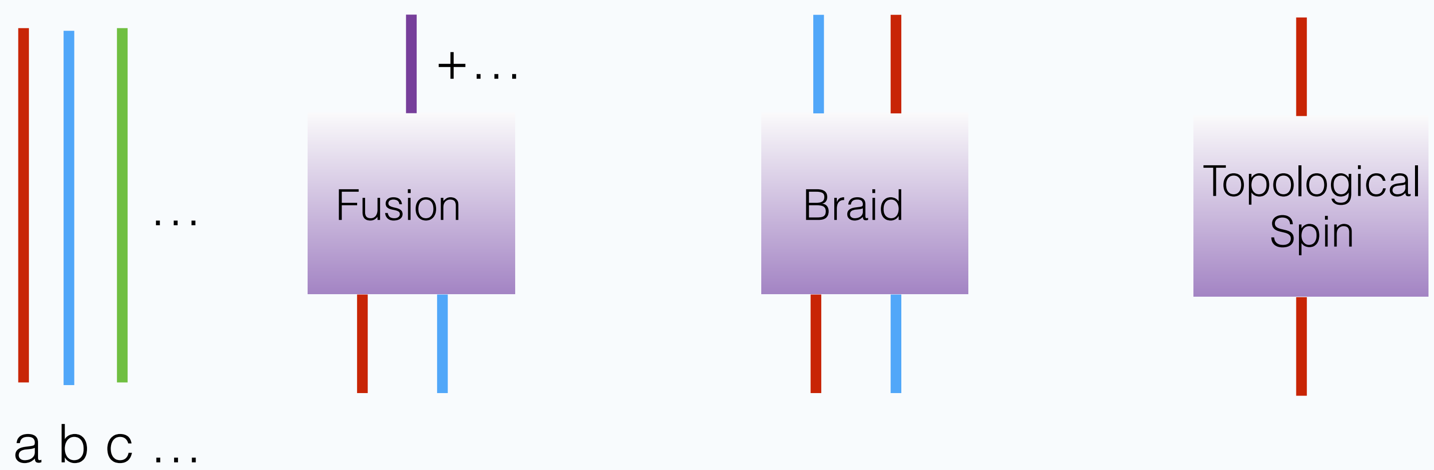
$$\ell_1 = [1, 1, -1, 0]^T$$

$$\ell_2 = [0, 1, 1, 1]^T$$

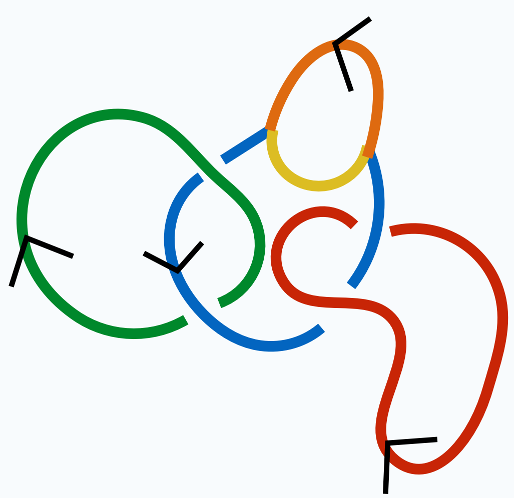
Algebraic formulation of $\mathcal{A}$ as a **Modular Tensor Category (MTC)**:

- **particle types**, a,b,c…i.e **Anyon types** in the bulk TQFT and **conformal blocks** in edge CFT.
- **Fusion rules** a x b = ….
- **Braiding phases** and **topological spins** (`S’ and `T’ matrices).

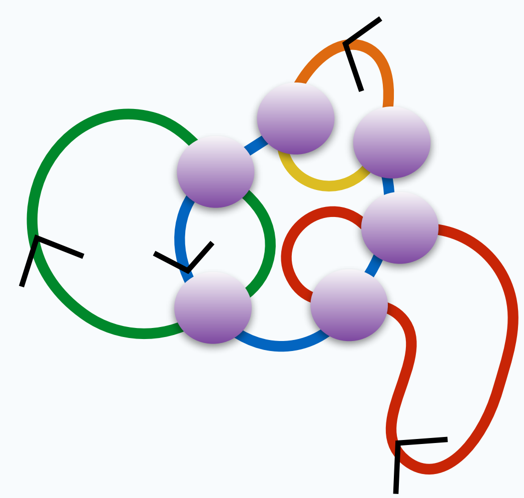
Several consistency conditions between this data.



- Can compute Ribbon diagrams using MTC:

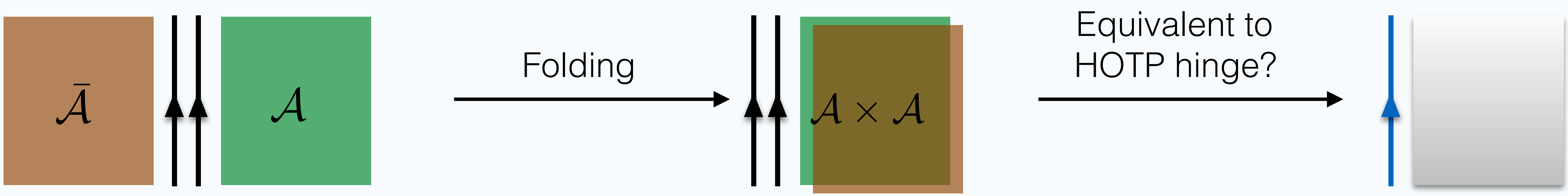


Ribbon diagram



MTC computation sketch

Unhinging via edge condensation



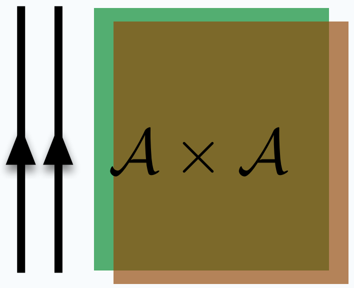
Edge condensation between $\mathcal{A}$ and $\bar{\mathcal{A}}$. $\simeq$ Anyon condenation in $\mathcal{A} \times \mathcal{A}$.

Anyon/edge condensation: Theoretical tool to study possible phase transitions. More powerful than K-matrix Luttinger liquid approach.

Procedure:

- Identify a set $\mathcal{B}$ of bosonic mutually local anyons that may condense.
- Two anyons a_1 and a_2 identified if $a_1 \in \mathcal{B} \times a_2$.
- An anyon a splits if $a \in \mathcal{B} \times a$.
- Anyons that braid non-trivially with $\mathcal{B}$ get confined.

Unhinging the HOTI via edge condensation

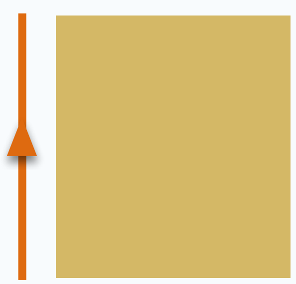


$\mathcal{A} \equiv \mathsf{T}-\text{Pfaffian}$

$$\{1_j^{\mathsf{A}}, \psi_j^{\mathsf{A}}, \sigma_j^{\mathsf{A}}\} \times \left\{1_j^{\bar{\mathsf{A}}}, \psi_j^{\bar{\mathsf{A}}}, \sigma_j^{\bar{\mathsf{A}}}\right\} \longrightarrow \text{anyons in } \mathcal{A} \times \mathcal{A}$$

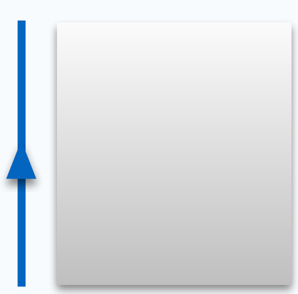
condense

$1_0^{\mathsf{A}} 1_0^{\bar{\mathsf{A}}}, \psi_0^{\mathsf{A}} 1_0^{\bar{\mathsf{A}}}, \psi_4^{\mathsf{A}} 1_0^{\bar{\mathsf{A}}}, 1_4^{\mathsf{A}} 1_0^{\bar{\mathsf{A}}}, \sigma_1^{\mathsf{A}} \sigma_3^{\bar{\mathsf{A}}}, \sigma_1^{\mathsf{A}} \sigma_7^{\bar{\mathsf{A}}}$



$\text{Toric Code} \times \{1, f\}$

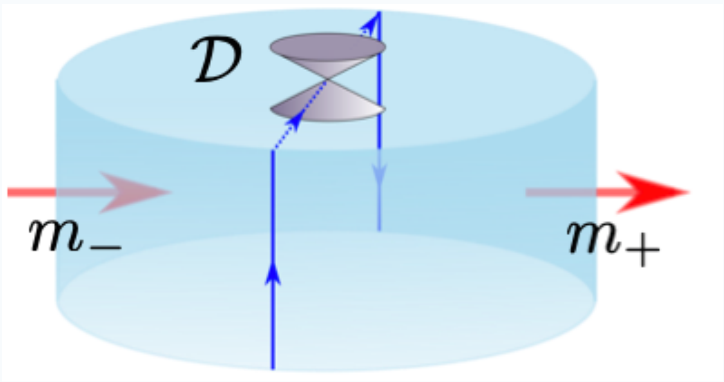
condense `e’ in Toric Code



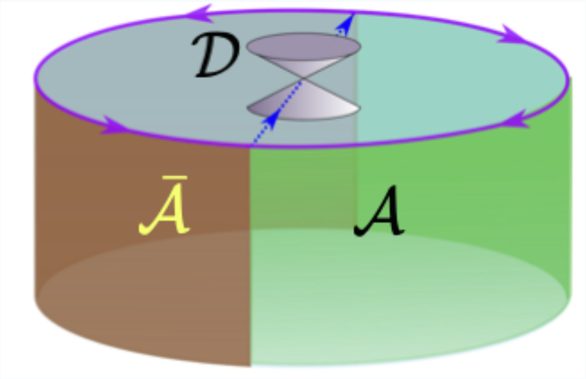
HOTI hinge

$\{1, f\}$

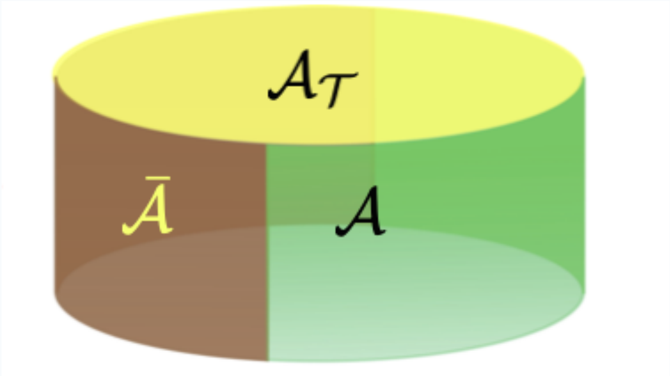
Other surface terminations



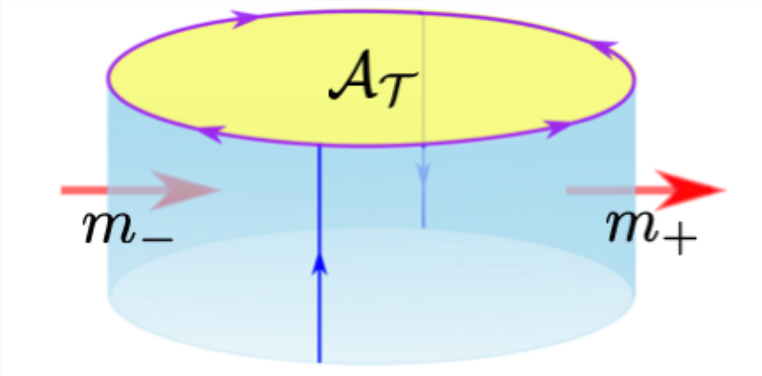
HOTI surface with no topological order.



Only side-surfaces gapped.



Completely gapped surface.



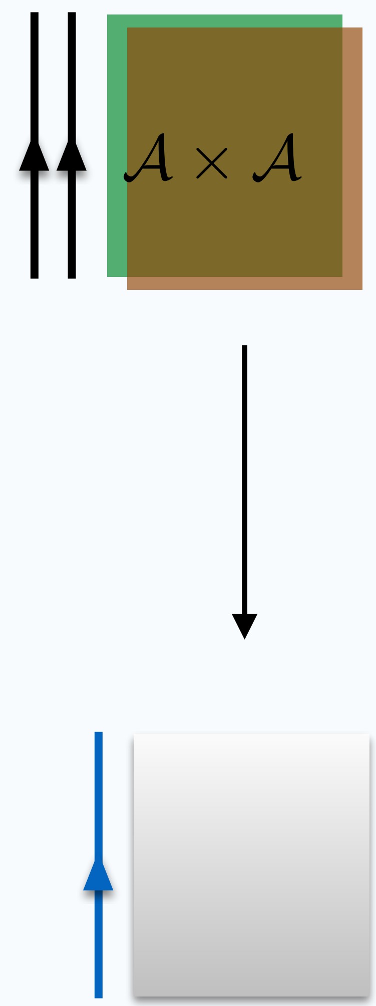
Higher-Order surface as a **beam splitter**

Unhinging the HOTSC and HOSPT

HOTSC

$\mathcal{A} \equiv \mathsf{SO}(3)_6$ anyon model

Single Majorana mode



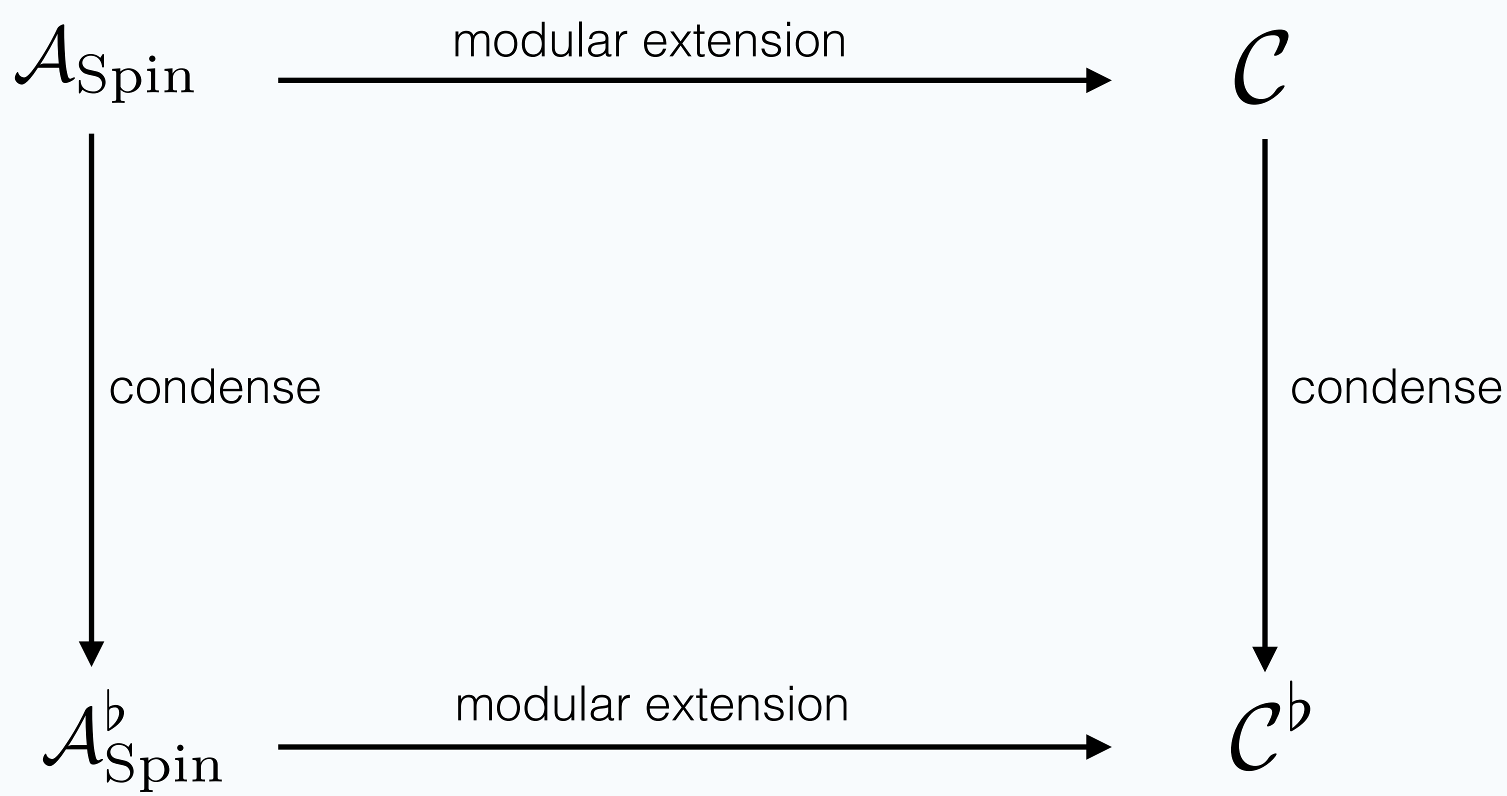
Bosonic HOSPT

$\mathcal{A} \equiv \mathsf{SO}(8)_1$ anyon model

c=8 chiral Boson

A technical aside

- Modular tensor categories don't have transparent fermions!
- Technical difficulty: extracting chiral central charge of condensed theory {1,f} as there are sixteen such theories with $c_-^{\nu} = \nu/2$, where $\nu \in \mathbb{Z}_{16}$.



- Can use commutativity of this diagram to compute chiral central charge.

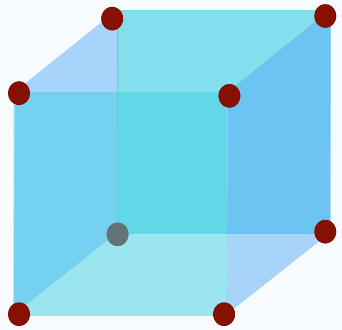
Summary

- Surfaces of Hinge Topological phases can be gapped out at the close of introducing surface topological order.

- Additionally there are various other surface terminations possible.

Future directions

- Surface topological order for 3rd-order topological phases?



- Topological order enriched by spatial symmetries. Fractionalized higher-order topological phases.

Thank you for your attention!