PROBING AND MANIPULATING ANDREEV STATES

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Quantronics Group
CEA-Saclay, France

PhD Thesis

C. Janvier	2016
L. Bretheau	2013
Q. Le Masne	2009
B. Huard	2006
M. Chauvin	2005
R. Cron	2001

PROBING AND MANIPULATING ANDREEV STATES

1. JOSEPHSON EFFECT
2. dc EXPERIMENTS on ATOMIC CONTACTS
3. cQED EXPERIMENTS on ATOMIC CONTACTS

PROBING AND MANIPULATING ANDREEV STATES

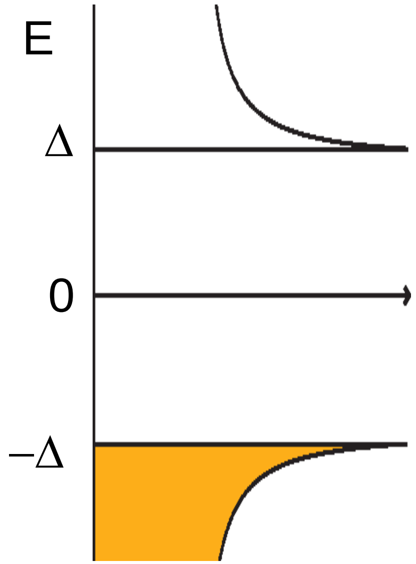
1. JOSEPHSON EFFECT
   a) Josephson Junction
   b) Mesoscopic description of Josephson effect
   c) Andreev bound states (ABS)
   d) Andreev Quantum Dot (AQD)

2. dc EXPERIMENTS on ATOMIC CONTACTS
   a. Supercurrent
   b. Current-phase relation
   c. Spectroscopy of ABS

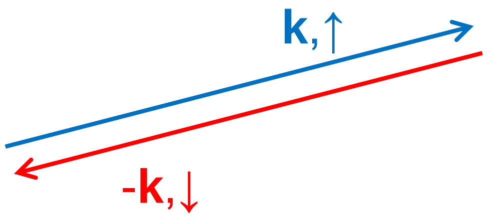
3. cQED EXPERIMENTS on ATOMIC CONTACTS
   a. Spectroscopy
   b. Coherent Manipulation of ABS
   c. Continuous monitoring of quantum jumps

BCS SUPERCONDUCTOR

$$\Delta = |\Delta| e^{i\varphi}$$



Coupling of time-reversed states



GAP IN
SINGLE PARTICLE
DENSITY OF STATES

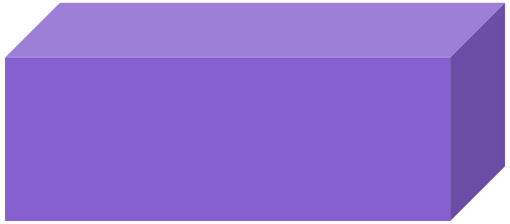
GROUND STATE LOWER BAND FULL (OF BOGOLIUBONS)

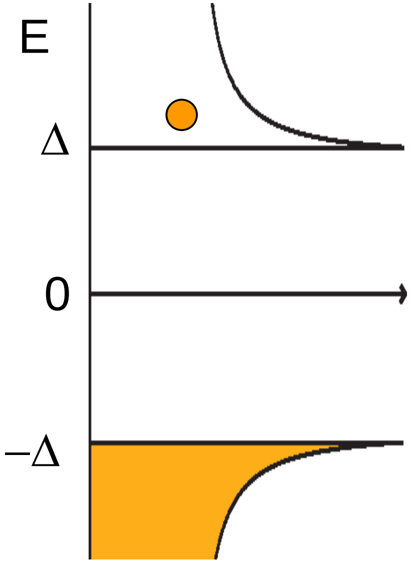
≡ BCS CONDENSATE OF "COOPER PAIRS"

$$\boxed{|\Phi_{BCS}\rangle = \prod_k \left(u_k + v_k e^{i\varphi} c_{k\uparrow}^{+} c_{-k\downarrow}^{+}\right)|0\rangle}$$

ELEMENTARY EXCITATIONS

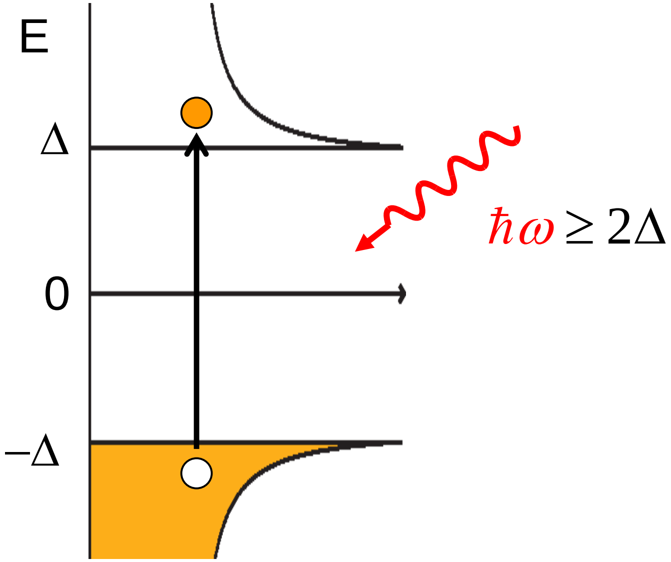
$$\Delta = |\Delta| e^{i\varphi}$$





ADD QP

energy $\geq \Delta$



CREATE PAIRS of QPs

energy $\geq 2\Delta$

COOPER PAIR BREAKING

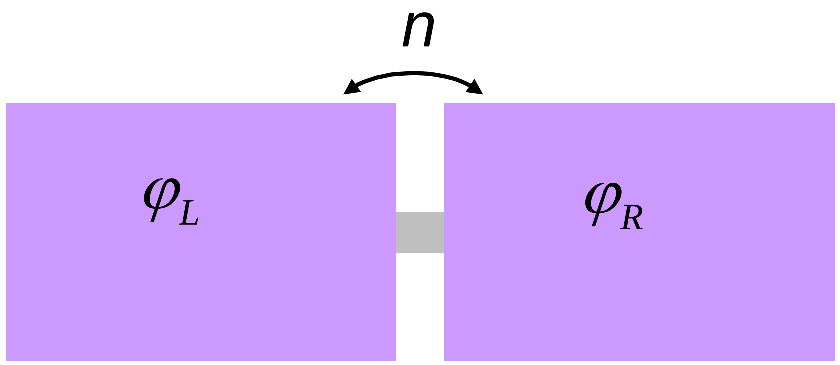
INCOHERENT EXCITATIONS OF FINITE ENERGY

JOSEPHSON EFFECT

1962

Nobel prize 1973

"WEAK-LINK"



$$\delta = \varphi_L - \varphi_R \neq 0$$

DISSIPATIONLESS CURRENT DETERMINED BY δ

SUPERCONDUCTORS

Anderson & Rowell 1963

SUPERFLUIDS

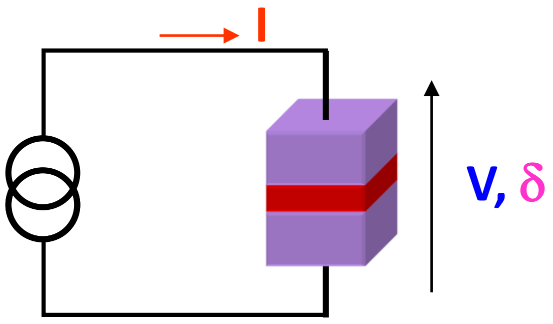
- ^{3}He: Avenel & Varoquaux 1987
- ^{4}He: Sukhatme *et al*, 2001

B.E.C.

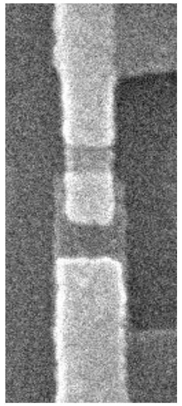
Albiez *et al*, 2005

Levy *et al*, 2007

THE JOSEPHSON JUNCTION

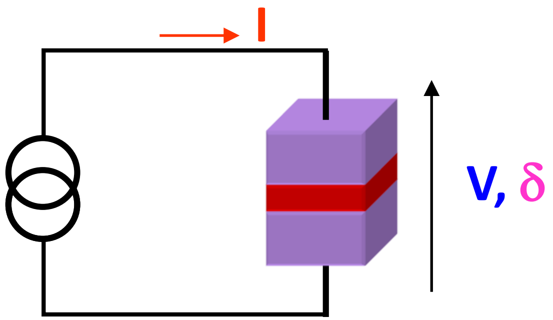


SIS Tunnel Junction



COOPER PAIR TUNNELING

JOSEPHSON EQUATIONS



SIS Tunnel Junction

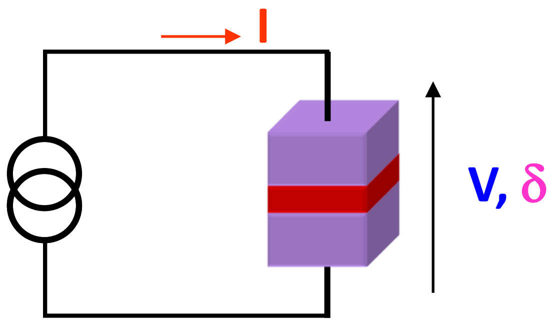
COOPER PAIR TUNNELING

$$\boxed{\begin{aligned} V &= \varphi_0 \frac{d\delta}{dt} \\ I &= I_0 \sin\delta \end{aligned}}$$

$$\varphi_0 \equiv \frac{\hbar}{2e}$$ FLUX QUANTUM

$$I_0 \propto G_N$$ CRITICAL CURRENT

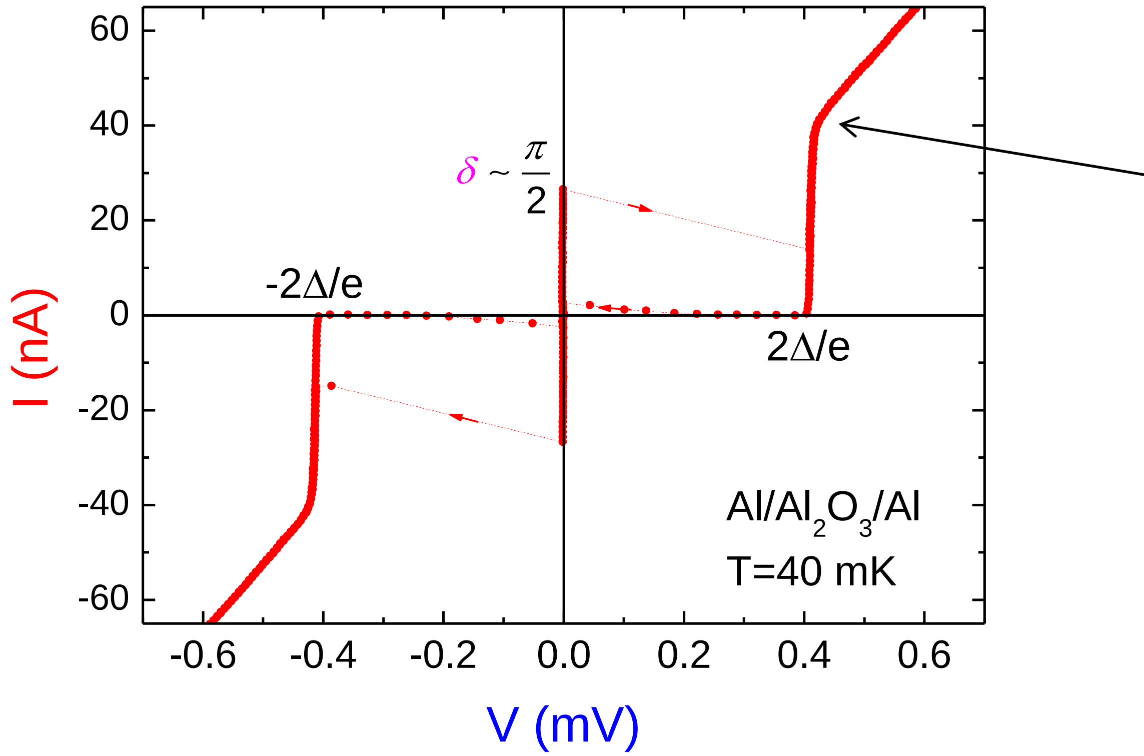
I-V CHARACTERISTICS OF JOSEPHSON JUNCTION

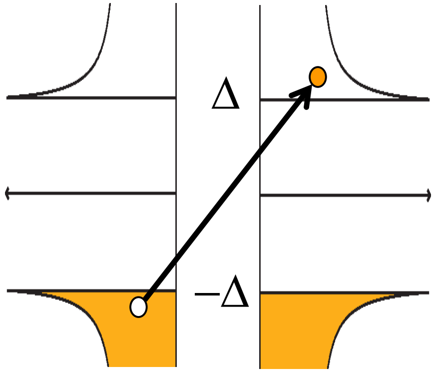


SIS Tunnel Junction

$$V = \varphi_0 \frac{d\delta}{dt}$$

$$I = I_0 \sin \delta$$





A NON-LINEAR INDUCTOR

$$I = I_0 \sin\delta$$

$$V = \varphi_0 \frac{d\delta}{dt}$$

$$V = \frac{\varphi_0}{I_0 \cos\delta}\frac{dI}{dt} = \frac{L_J}{\cos\delta}\frac{dI}{dt}$$

KINETIC INDUCTANCE

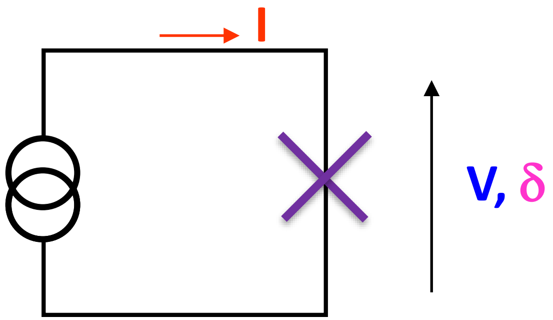
inertia of Cooper pairs
tunneling across insulator

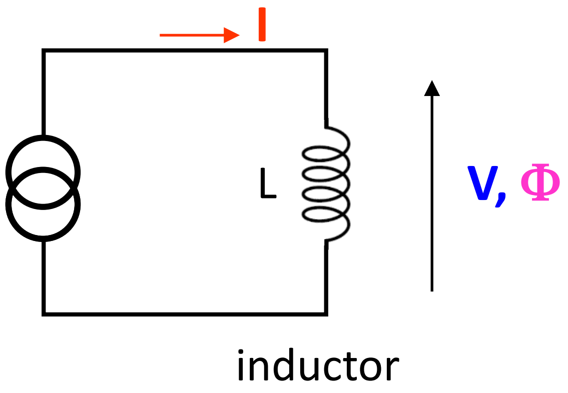
STORED ENERGY

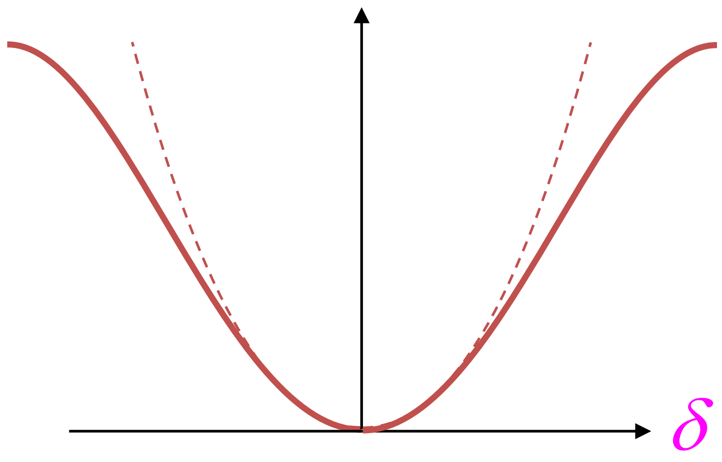
$$E(\delta) = \int_{-\infty}^{t} V(t')I(t')dt'$$
$$= -\varphi_0 I_0 \cos\delta$$
$$= -E_J \cos\delta$$

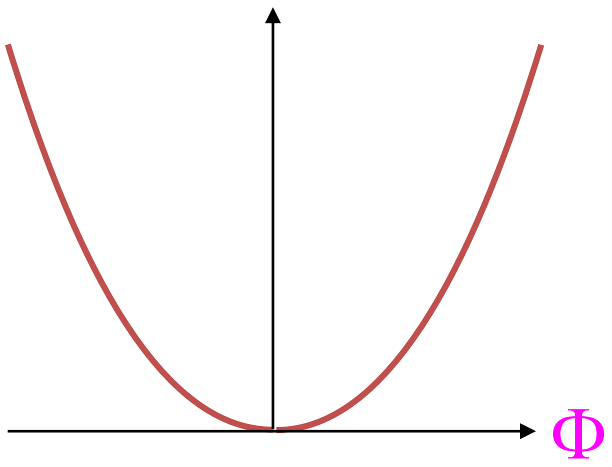
$$L_J = \frac{\varphi_0}{I_0} = \frac{\varphi_0^2}{E_J}$$

THE JOSEPHSON JUNCTION, A NON-LINEAR INDUCTOR





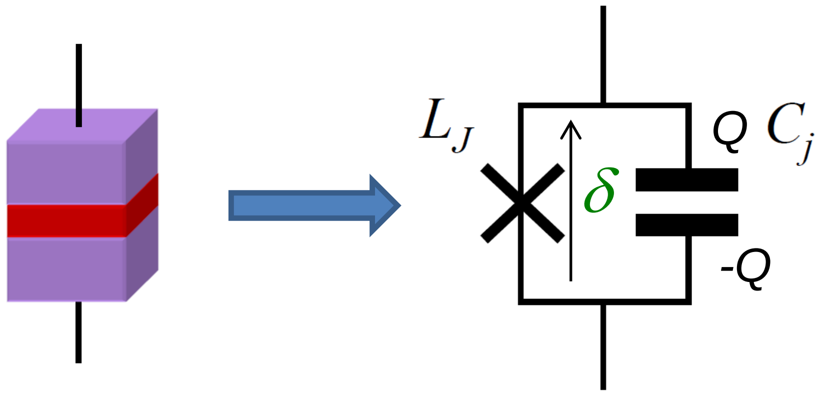


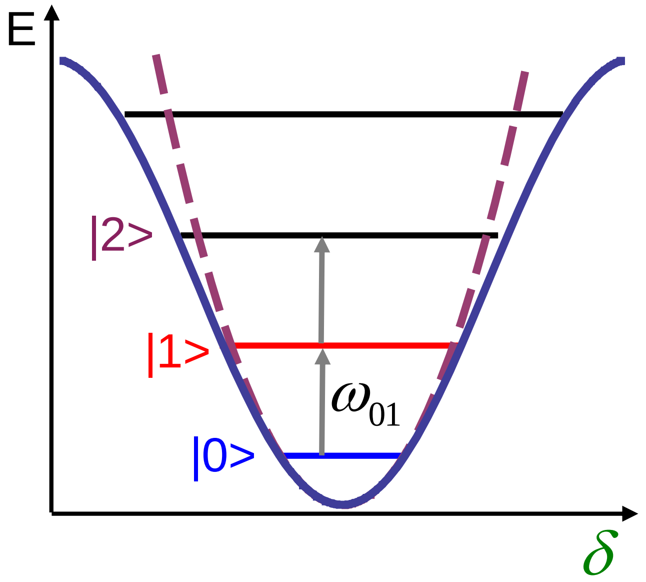


$$E(\delta) = -E_J \cos\delta \approx cste + \frac{(\varphi_0 \delta)^2}{2L_J} + \ldots$$

$$E(\Phi) = \frac{\Phi^2}{2L}$$

QUANTUM ANHARMONIC OSCILLATOR





Martinis, Devoret, Clarke (1985)

$$H = -E_J \cos\delta + \frac{Q^2}{2C}$$

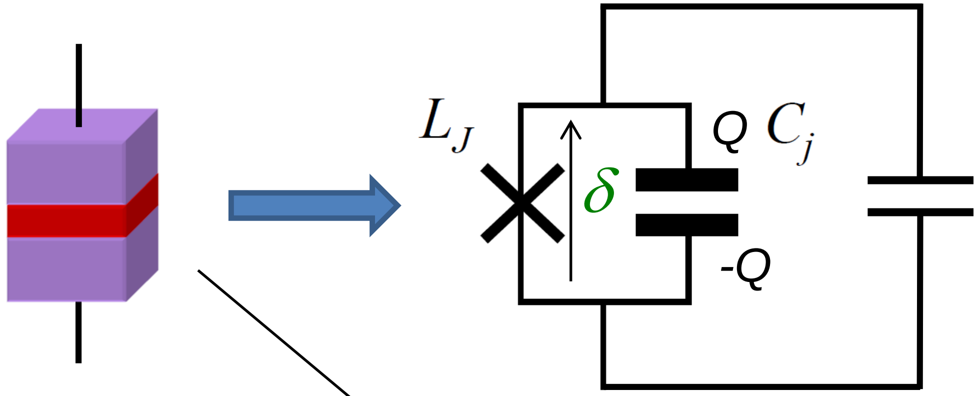
$$[\varphi_0 \delta, Q] = i\hbar$$

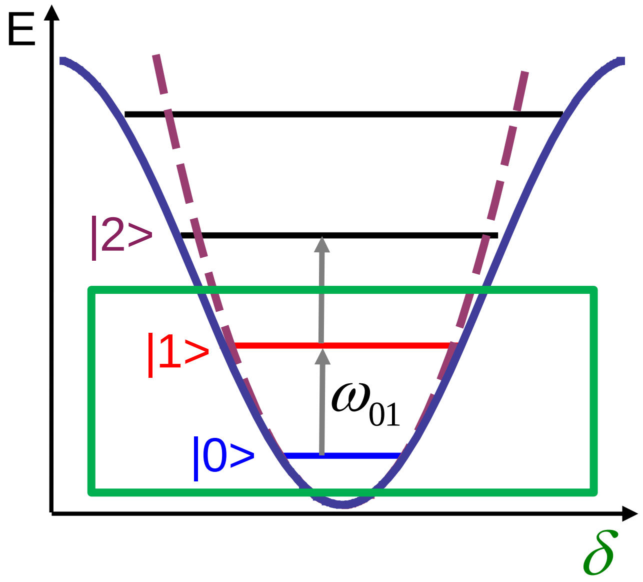
Quantized excitations of
a collective electromagnetic mode
(plasma mode)

NO microscopic excitations!

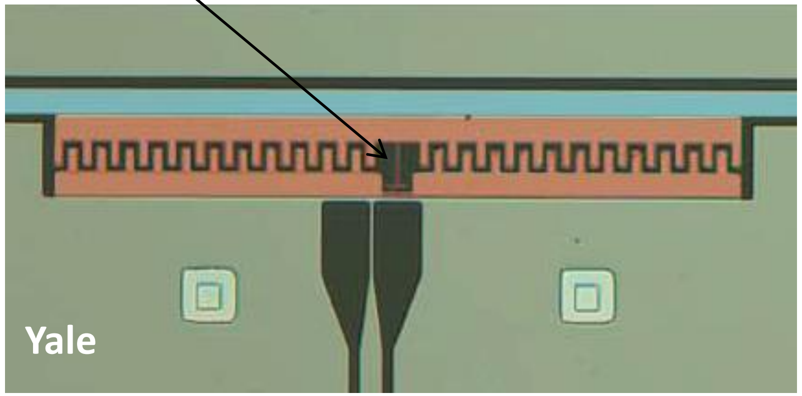
SUPERCONDUCTING QUBITS

Josephson junction





Use 2 lowest electromagnetic levels of an anharmonic LC circuit



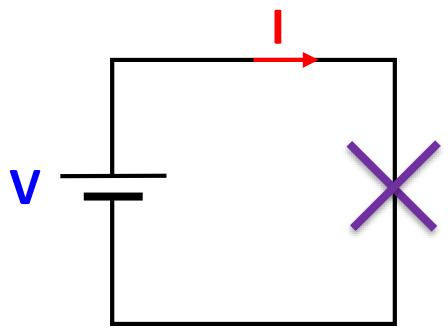
TRANSMON

Koch *et al.*, PRA 2007

$$H \approx \frac{\hbar\omega_{01}}{2}\sigma_z$$

but bosonic mode!

THE JOSEPHSON JUNCTION, A MICROWAVE SOURCE



Josephson equations

$$\begin{cases} V = \varphi_0 \dfrac{d\delta}{dt} \Rightarrow \delta = \dfrac{V}{\varphi_0} t \\ I = I_0 \sin\delta = I_0 \sin(\omega_J t) \end{cases}$$

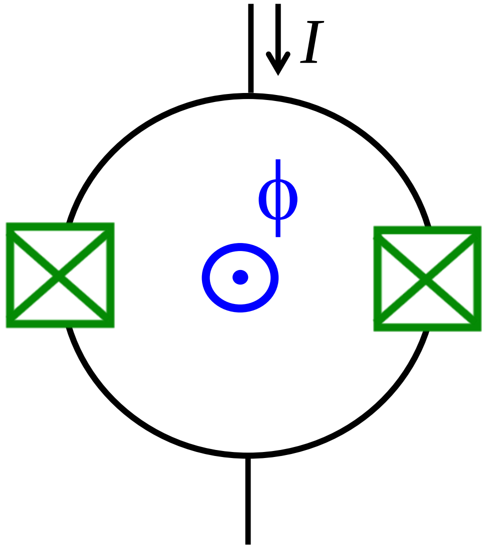
$$\frac{\omega_J}{2\pi} = \frac{2e}{h} V$$

JOSEPHSON FREQUENCY

0.5 GHz / µV

THE DC SQUID

SUPERCONDUCTING QUANTUM INTERFERENCE DEVICE



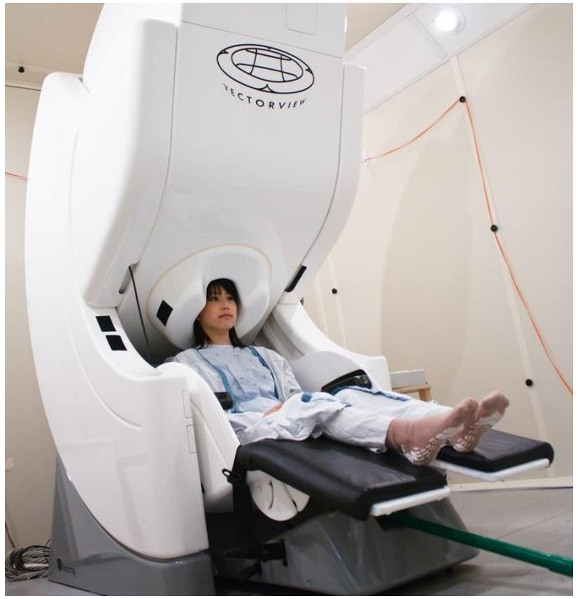
IF LOOP INDUCTANCE << JOSEPHSON INDUCTANCES

$$I_c = 2I_0 \cos\left(\pi \frac{\phi}{\phi_0}\right)$$

EXTREMELY SENSITIVE MAGNETOMETERS

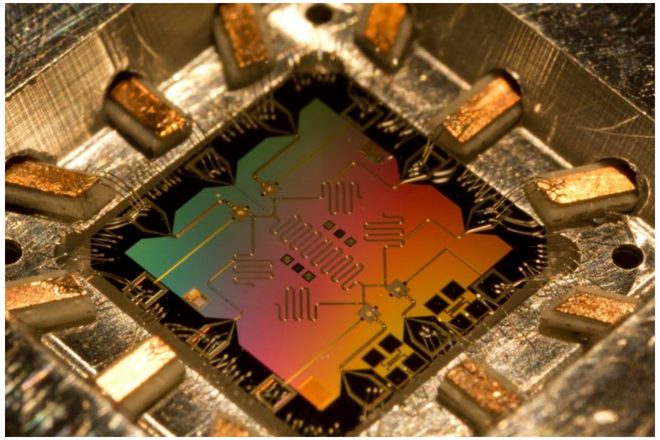
JOSEPHSON CIRCUITS

MAGNETOENCEPHALOGRAPHY

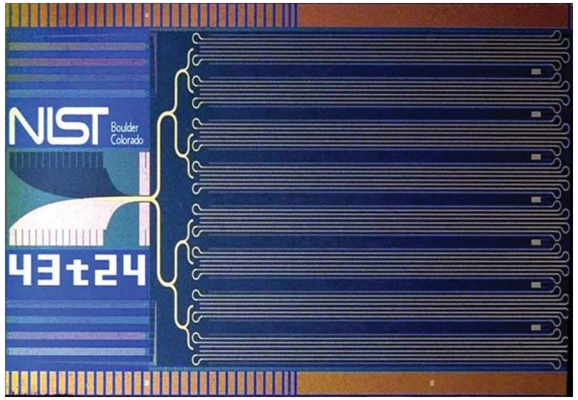


Elekta, Sweden

QUANTUM COMPUTING



UCSB, California



NIST, Colorado

METROLOGY

South Pole Obs.

RADIO-ASTRONOMY

AND YET…

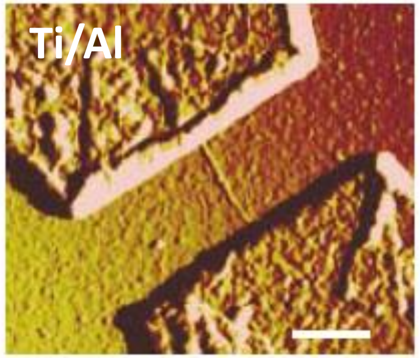
THERE IS MORE TO THE JOSEPHSON EFFECT THAN
THE JOSEPHSON RELATIONS

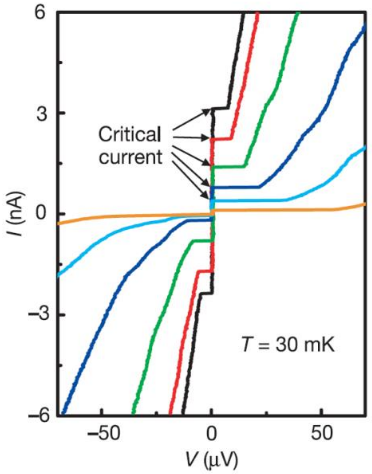
JOSEPHSON WEAK LINKS HAVE
AN INTERNAL, FERMIONIC DEGREE OF FREEDOM

A NEW RESOURCE FOR
QUANTUM PHYSICS EXPERIMENTS IN ELECTRICAL CIRCUITS

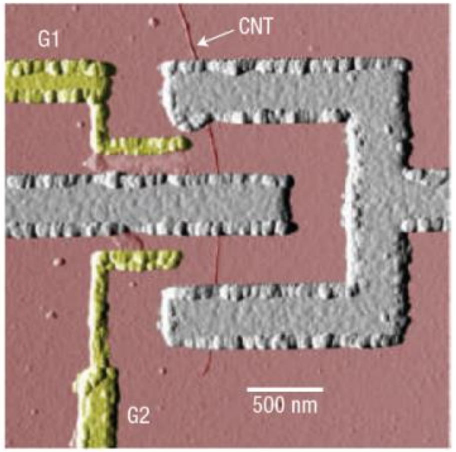
JOSEPHSON EFFECT IN ALL TYPES OF WEAK LINKS

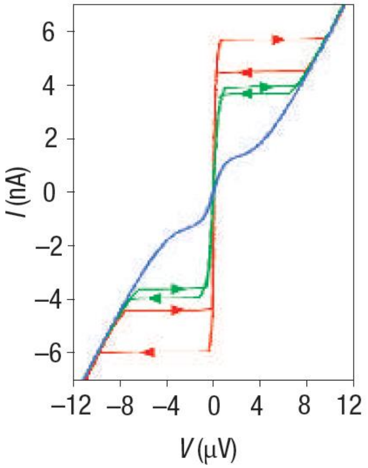
Carbon nanotube





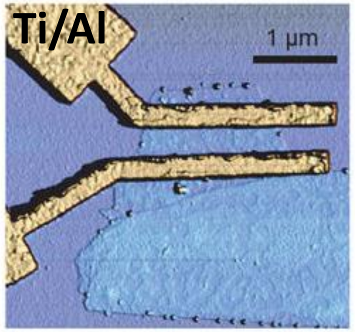
P. Jarillo-Herrero *et al.*,
Nature **439**, 953 (2006)

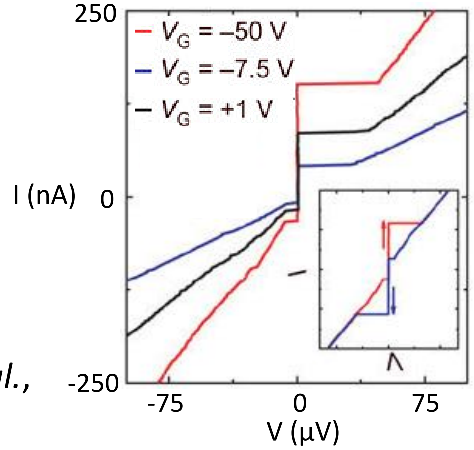




J.-P. Cleuziou *et al.*,
Nature Nano. **1**, 53 (2006)

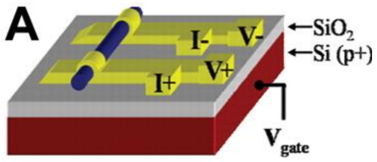
Graphene

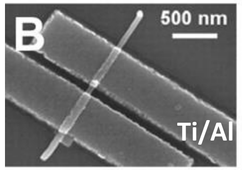


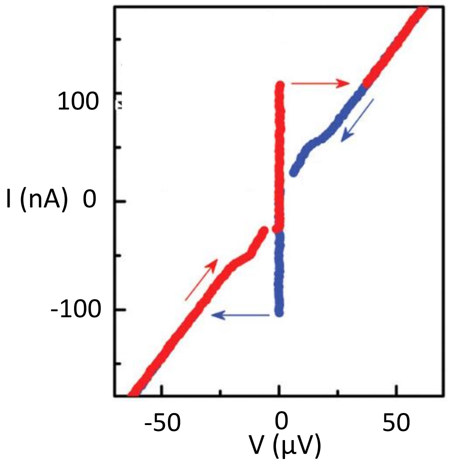


Hubert B. Heersche *et al.*,
Nature **446**, 56 (2007)

InAs nanowire



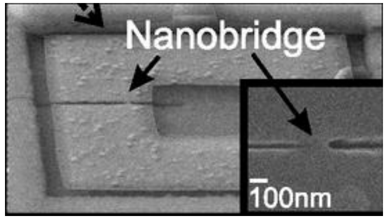


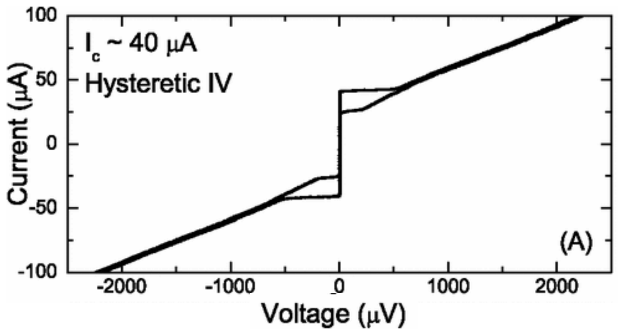


Yong-Joo Doh *et al.*,
Science **309**, 272 (2005)

JOSEPHSON EFFECT IN ALL TYPES OF WEAK LINKS

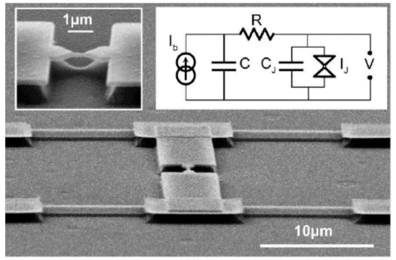
(Nb) nanobridge

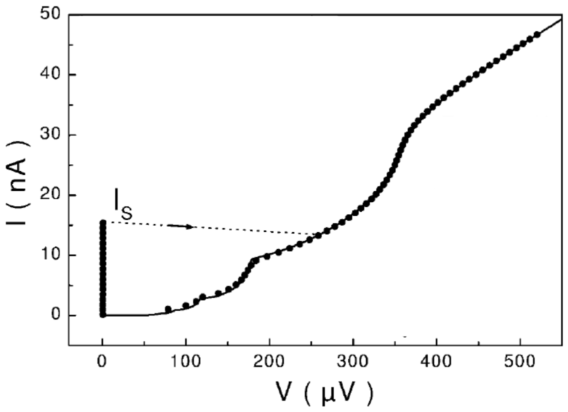




A. G. P. Troeman *et al.*,
Phys. Rev. B **77**, 024509 (2008)

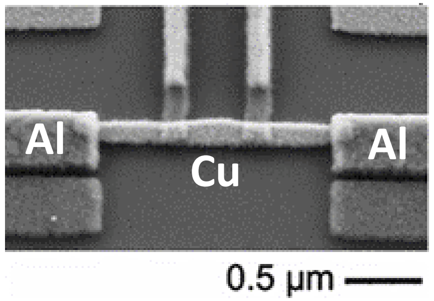
Atomic contact

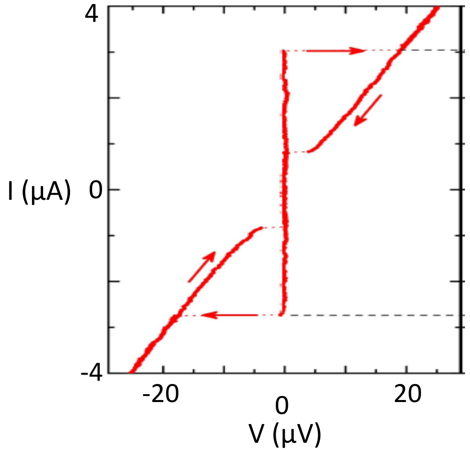




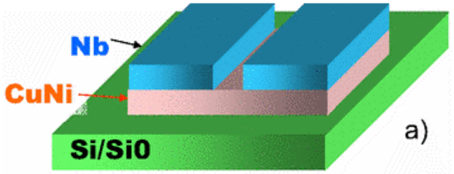
M.F. Goffman *et al.*,
Phys. Rev. Lett. **85**, 170 (2000)

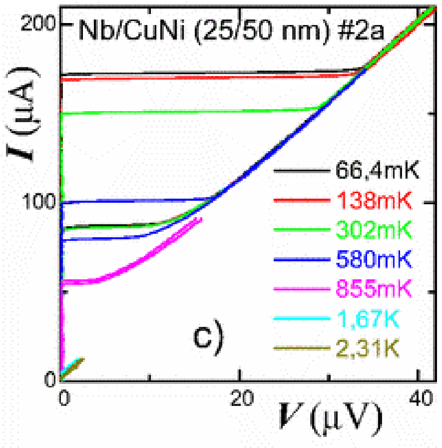
SNS junction





H. Courtois, M. Meschke,
J. T. Peltonen, and J. P. Pekola
Phys. Rev. Lett. **101**, 067002 (2008)





V. M. Krasnov *et al.*,
Phys. Rev. B **76**, 224517 (2007)

MESOSCOPIC DESCRIPTION OF JOSEPHSON EFFECT

JOSEPHSON JUNCTION

S-I-S

JOSEPHSON WEAK LINK

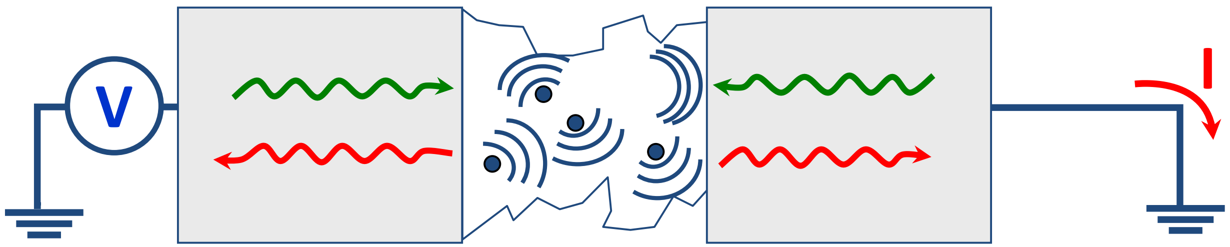
S-?-S

PHASE DRIVEN SUPERCURRENT

$$\boxed{\begin{gathered} I = f(\delta) \\[2em] V = \frac{\phi_0}{2\pi}\frac{d\delta}{dt} \end{gathered}}$$

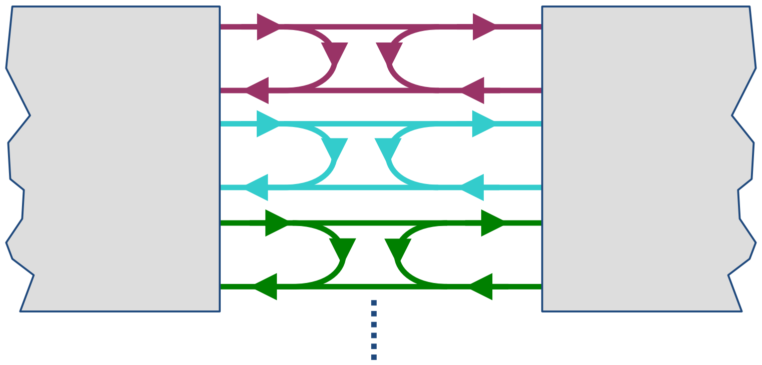
2 ingredients:	- conduction *channels*
	- Andreev reflection

SCATTERING APPROACH TO TRANSPORT



COHERENT CONDUCTOR = “WAVEGUIDE FOR ELECTRONS”

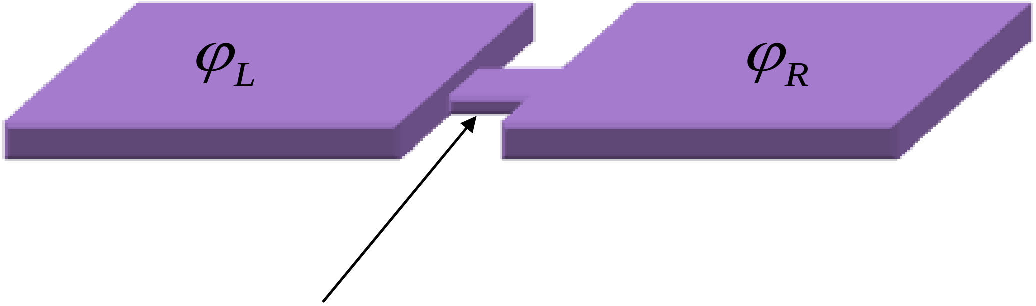
collection of independent “conduction channels” $$N \sim \frac{S}{\lambda_F{}^2}$$



$$G = \frac{2e^2}{h}\sum_i^N \tau_i$$

Transmission probabilities τ_n

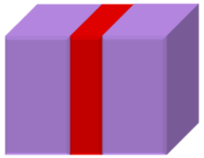
MESOSCOPIC DESCRIPTION OF JOSEPHSON WEAK LINKS



COHERENT SCATTERING REGION

➔ DECOMPOSE INTO INDEPENDENT ***CHANNELS***

WITH TRANSMISSIONS $\tau_1, \tau_2, \tau_3, \tau_4 \dots$



TUNNEL JUNCTION $N \gg 1$ and $\tau_1, \dots, \tau_N \ll 1$

PHASE-BIASED ELEMENTARY WEAK-LINK

$$\delta \equiv \varphi_L - \varphi_R \neq 0$$

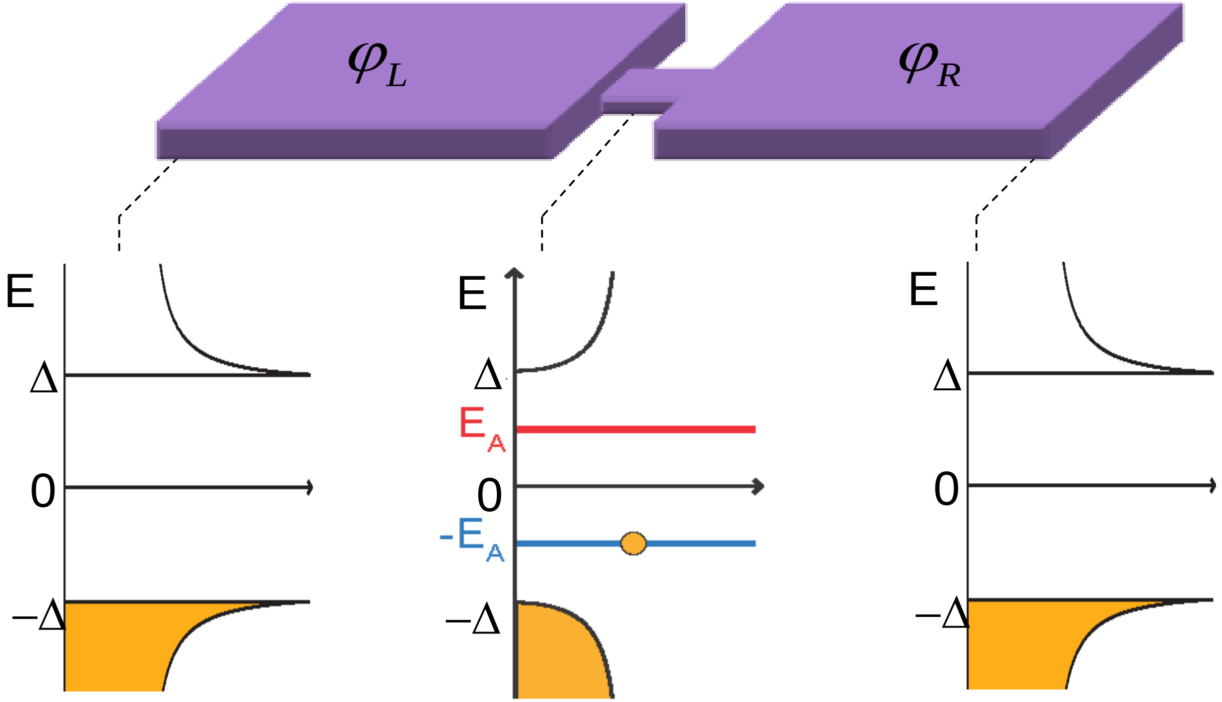
SINGLE CONDUCTION CHANNEL
SHORT (L < ξ)

TRANSMISSION PROBABILITY τ

SUPERCONDUCTIVITY LOCALLY FRUSTRATED

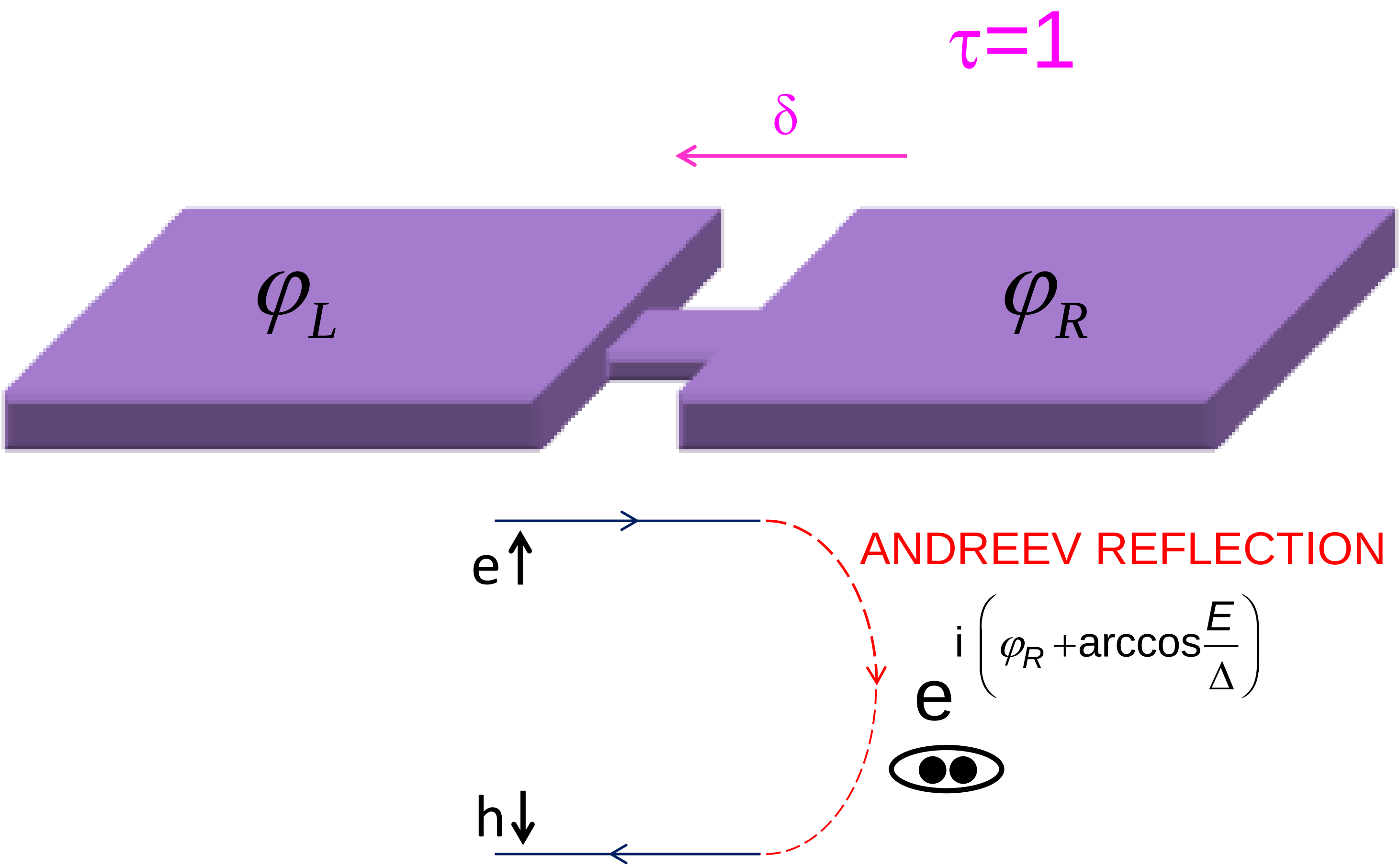
ANDREEV BOUND STATES

$$\delta \equiv \varphi_L - \varphi_R \neq 0$$



$$\boxed{E_A = \Delta\sqrt{1-\tau\sin^2\left(\delta/2\right)}}$$

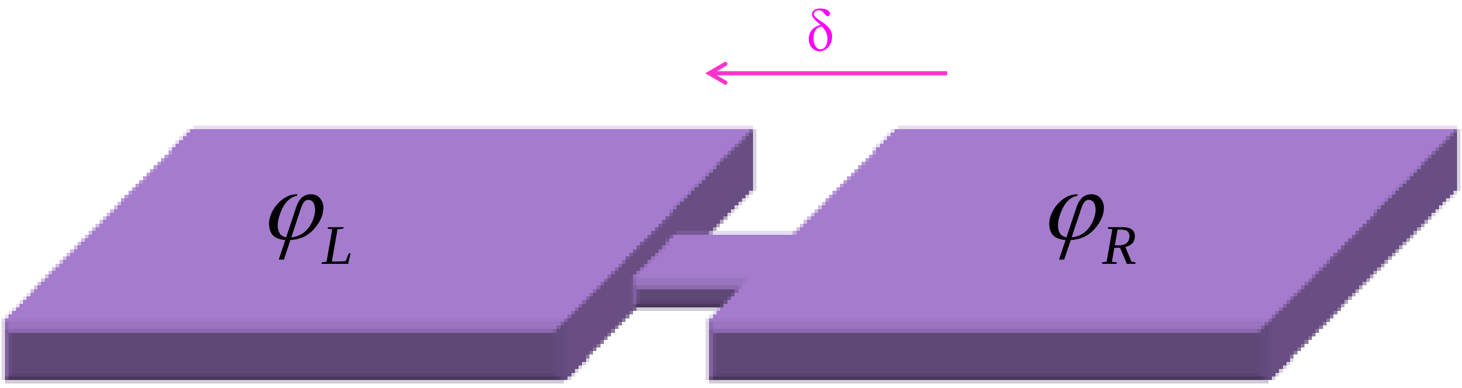
BALLISTIC CHANNEL



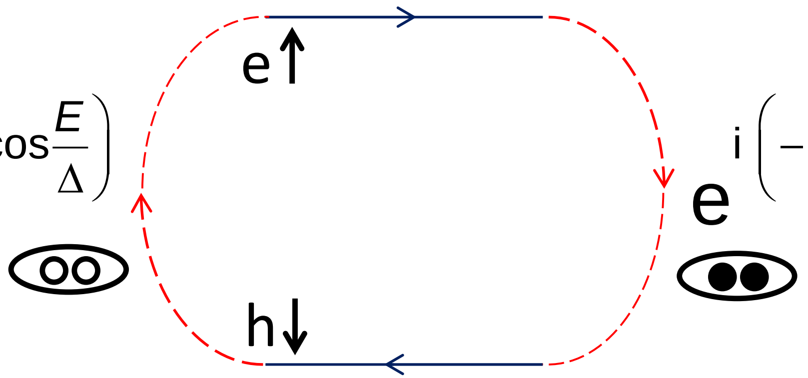
Saint-James, J. Phys. France 1964
Andreev, Sov. Phys. JETP 1964

LOCALIZED RESONANCE

τ=1



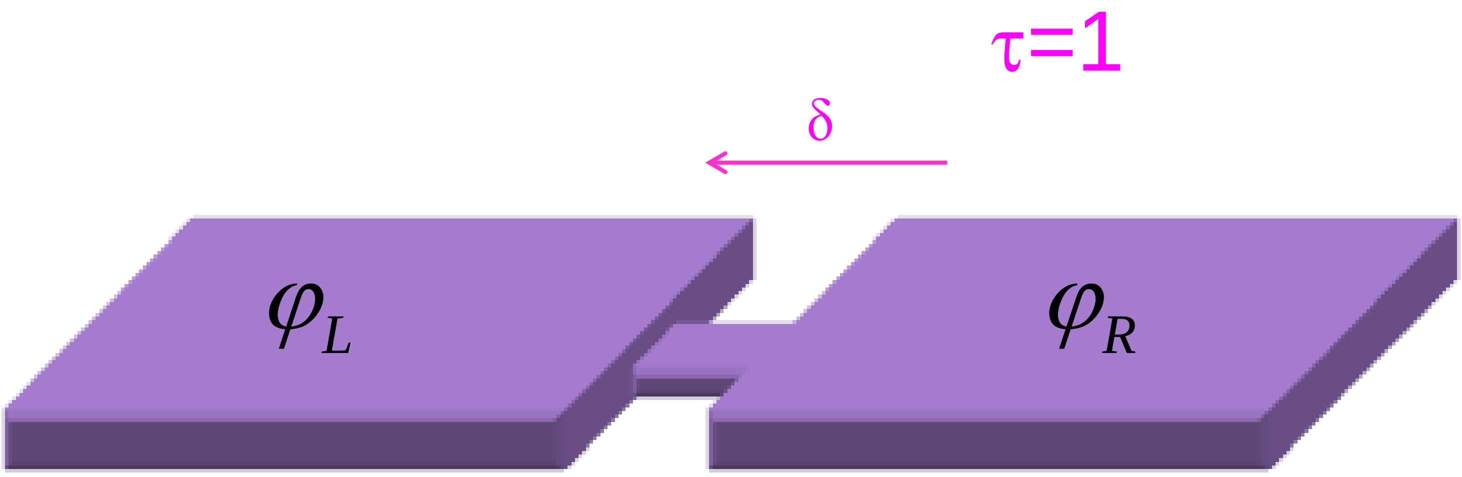
$$e^{i\left(\varphi_L - \arccos\frac{E}{\Delta}\right)}$$

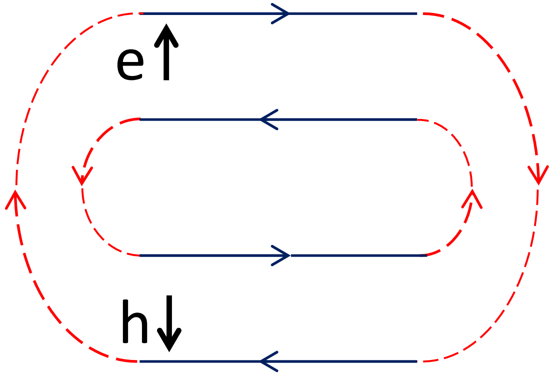


$$e^{i\left(-\varphi_R - \arccos\frac{E}{\Delta}\right)}$$

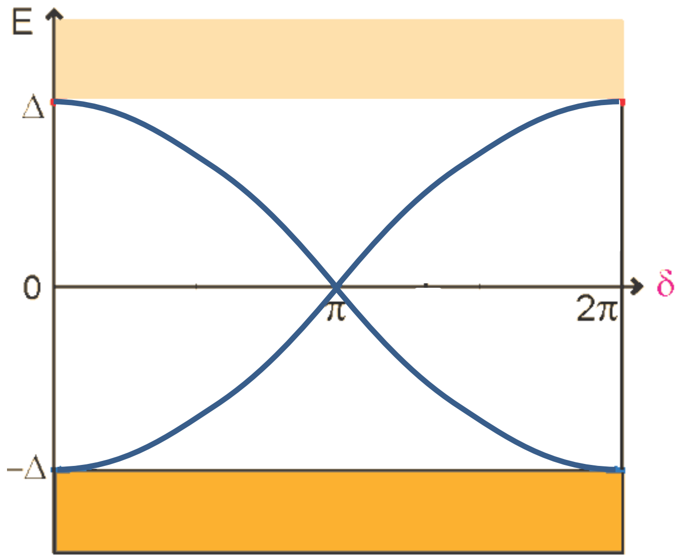
$$\varphi_R - \varphi_L + 2\arccos\frac{E}{\Delta} = 0 \quad \mathrm{mod}(2\pi)$$

BALLISTIC ANDREEV BOUND STATES

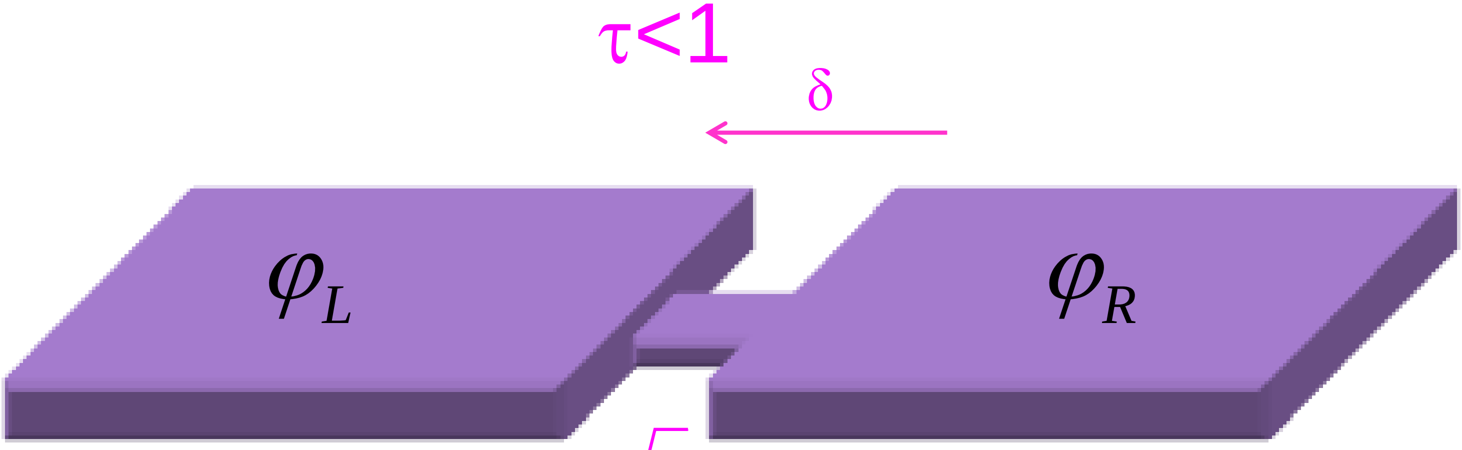


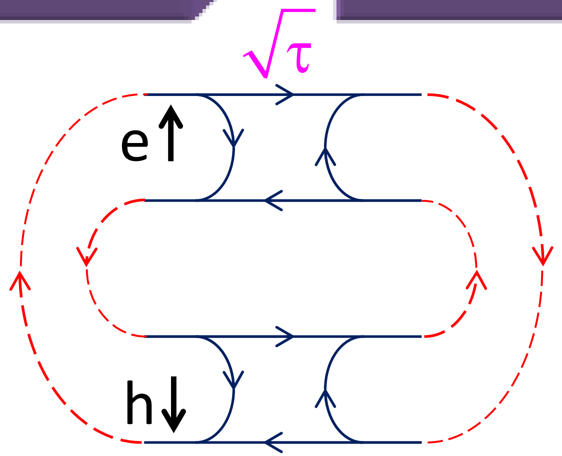


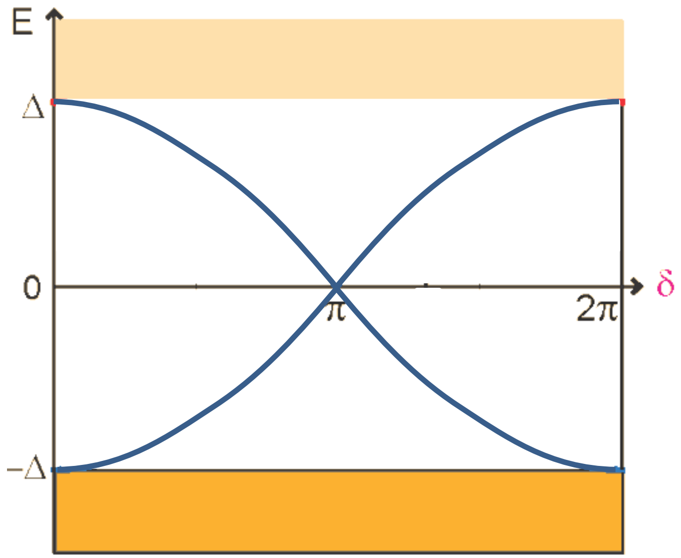
$$E = \pm\Delta\cos(\delta/2)$$



FINITE REFLECTION CHANNEL

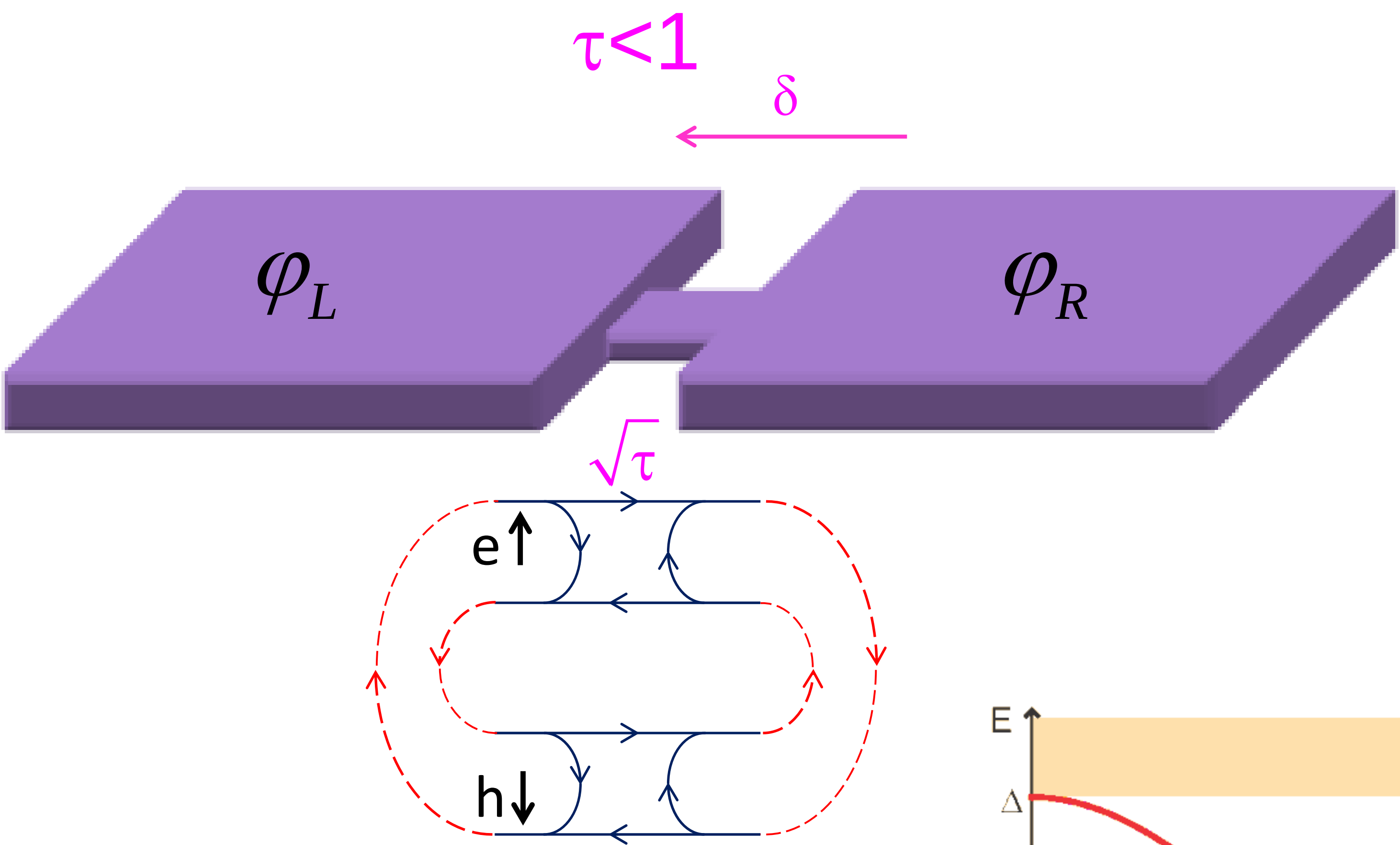






HYBRIDIZATION OF THE TWO BALLISTIC STATES

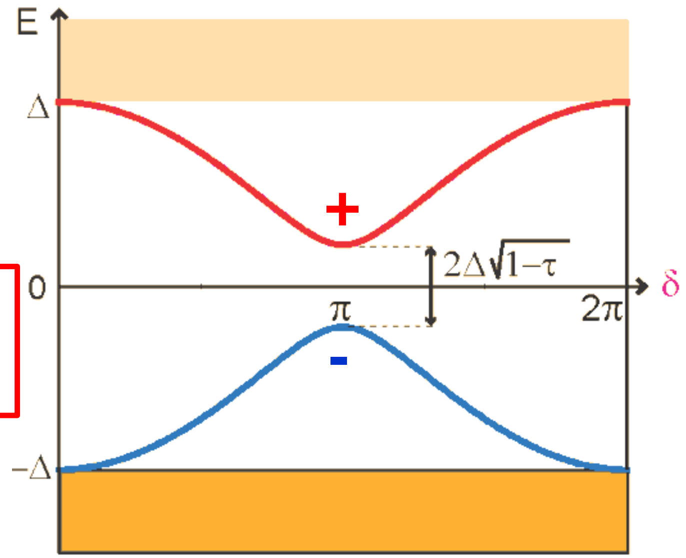
ABS FOR ARBITRARY TRANSMISSION



Bound states: $E = \pm E_A$

$$\boxed{E_A = \Delta\sqrt{1 - \tau \sin^2(\delta/2)}}$$

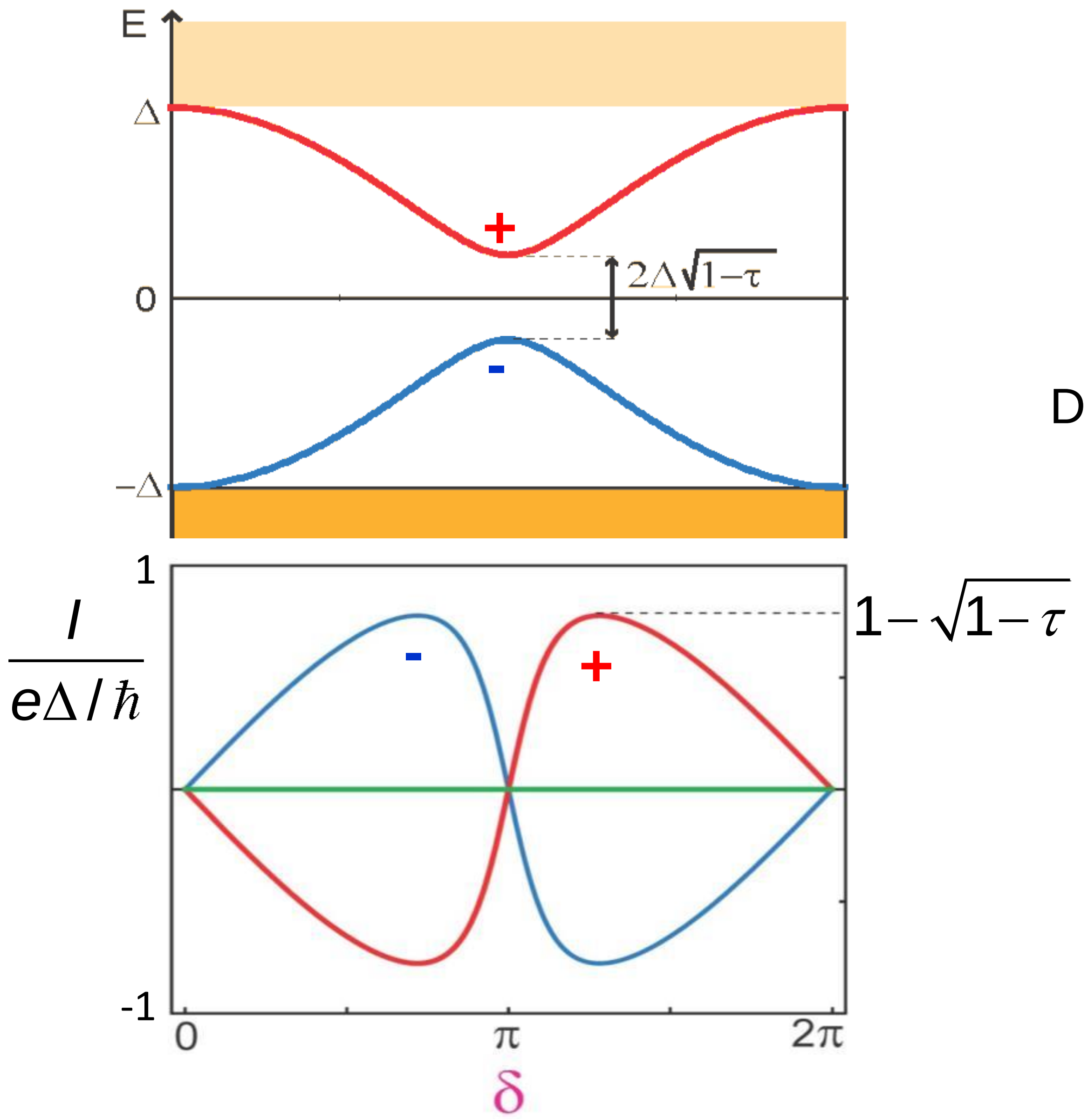
Beenakker 1991



ABS & SUPERCURRENT

ENERGY

CURRENT



$$I = 2e\dot{n} = \frac{1}{\varphi_0}\frac{\partial E}{\partial \delta}$$

AN INTERNAL FERMIONIC DEGREE OF FREEDOM

FROM ABS TO STANDARD JOSEPHSON EFFECT

Weak link = tunnel junction

$N \gg 1$ channels

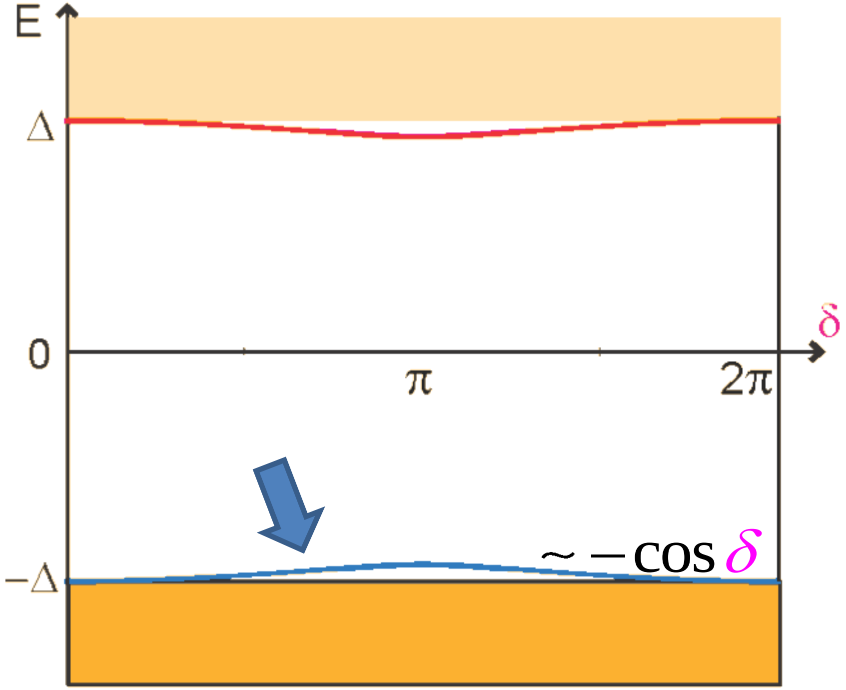
$\tau_i \ll 1$

$$E = -\Delta\sqrt{1-\tau\sin^2(\delta/2)}$$

$$\underset{(\tau \ll 1)}{\simeq} -\Delta\left(1-\frac{\tau}{4}+\frac{\tau}{4}\cos\delta\right)$$

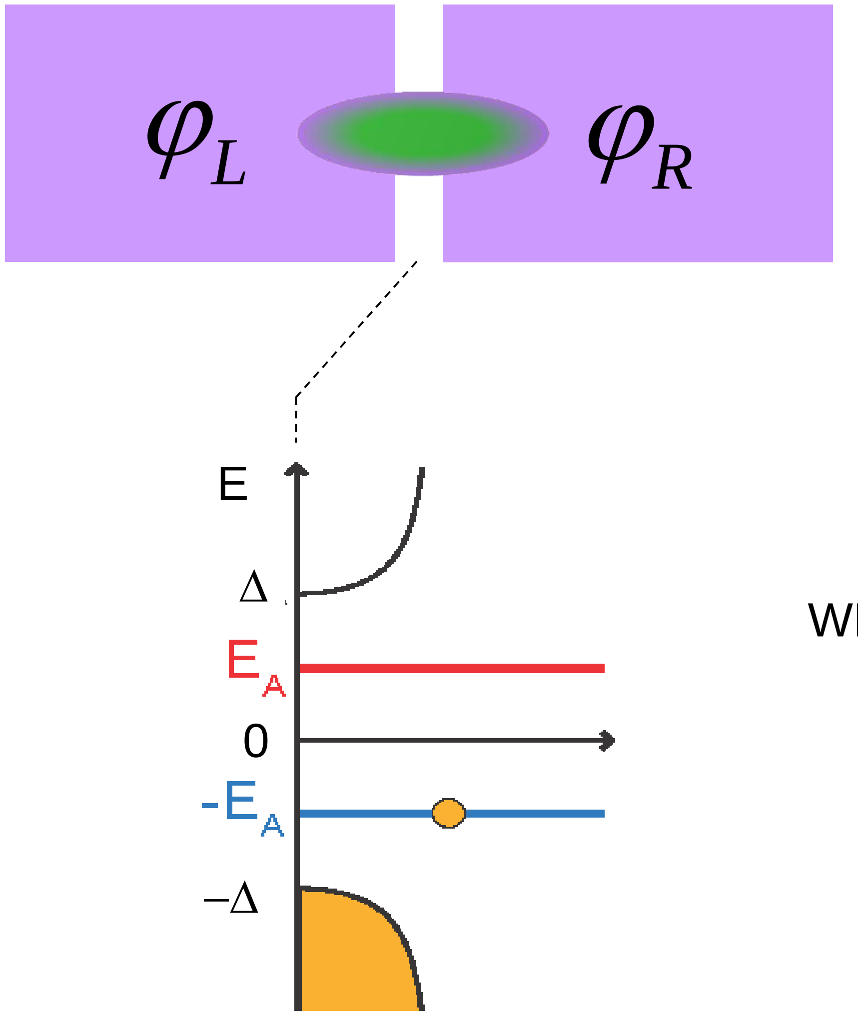
n channels: $E = cst - E_J\cos\delta$

$$\left(E_J = n\Delta\frac{\tau}{4}\right)$$



Only the ground state plays a role

LOCALIZED STATES



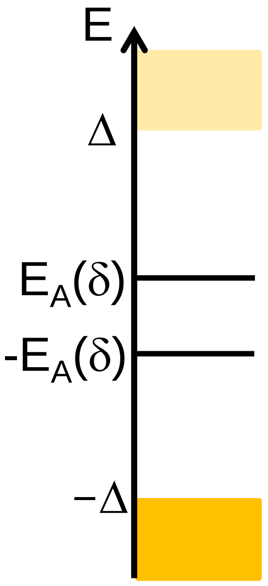
$$\ell = \frac{\xi}{\sqrt{1-\left(E_A/\Delta\right)^2}}$$

WELL LOCALIZED AS LONG AS

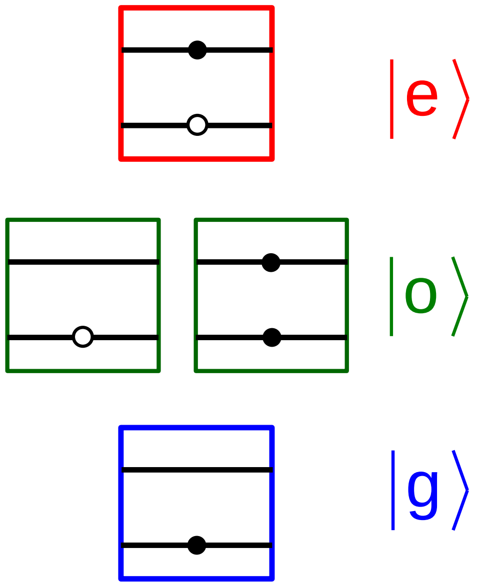
$$E_A \ll \Delta$$

$$\tau \to 1 \quad \text{and} \quad \delta \sim \pi$$

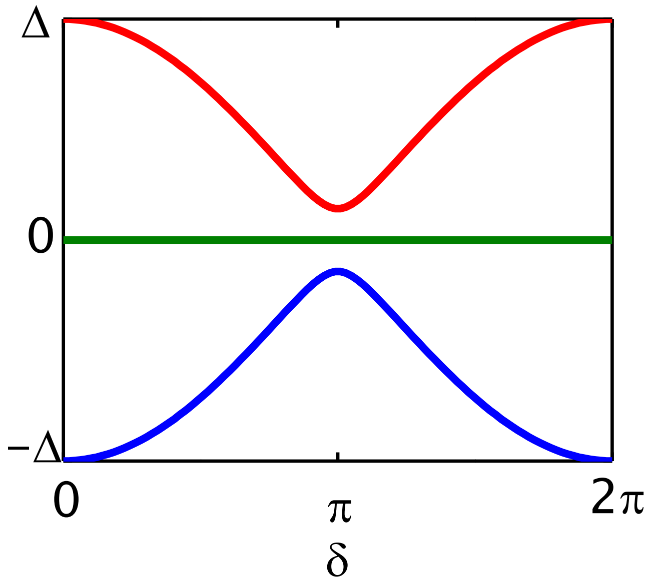
ANDREEV QUANTUM DOT



LEVELS



OCCUPATION CONFIGURATION



ENERGY OF STATES

$|e\rangle$ DOUBLE EXCITATION

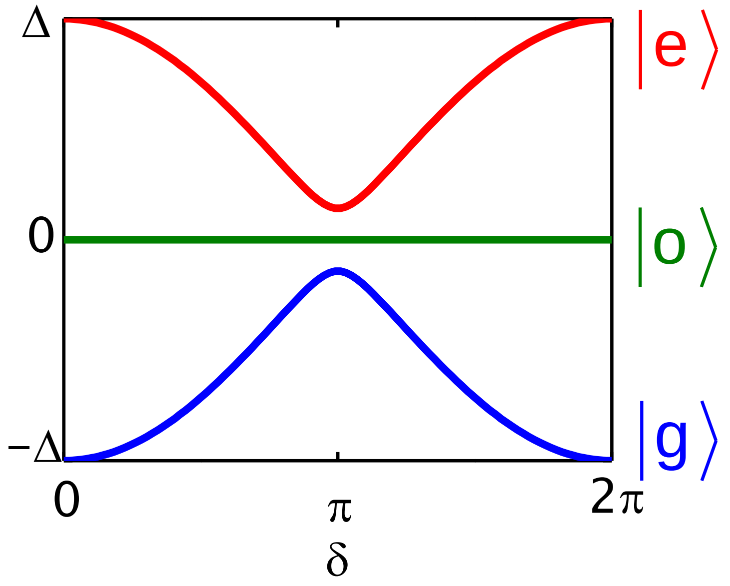
$|o\rangle$ SINGLE EXCITATION

$|g\rangle$ NO EXCITATION

Chtchelkatchev & Nazarov,
Phys. Rev. Lett. **90**, 226806 (2003)

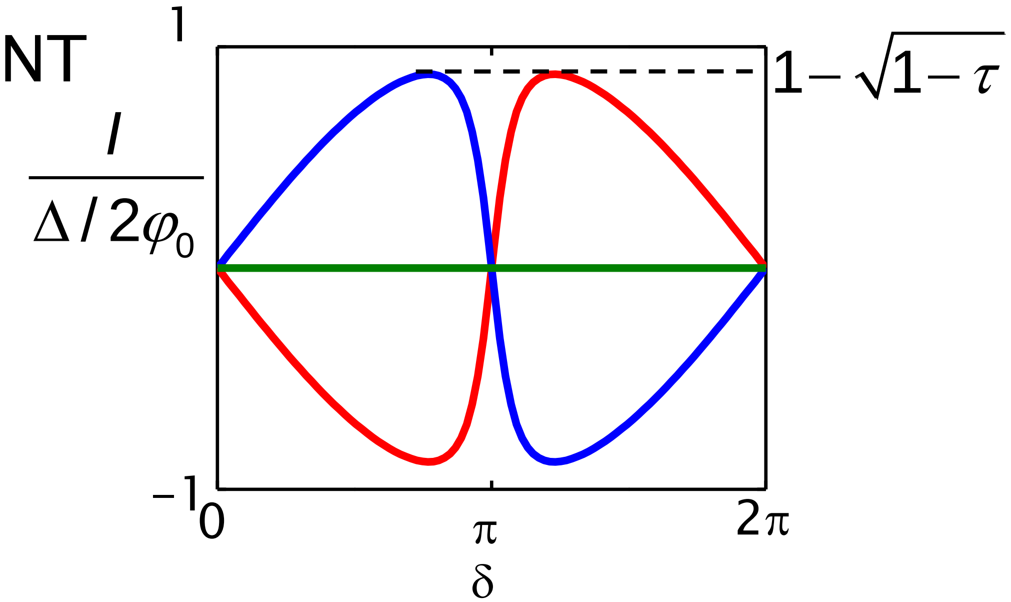
3 STATES, 3 CURRENTS

ENERGY



$$E_A = \Delta\sqrt{1-\tau \sin^2\left(\delta/2\right)}$$

CURRENT

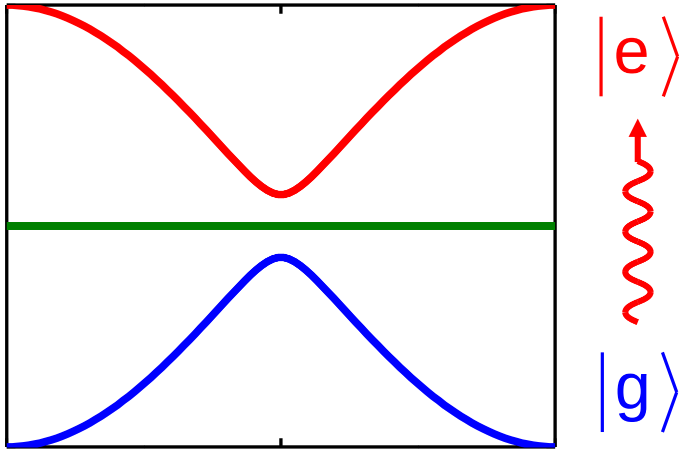


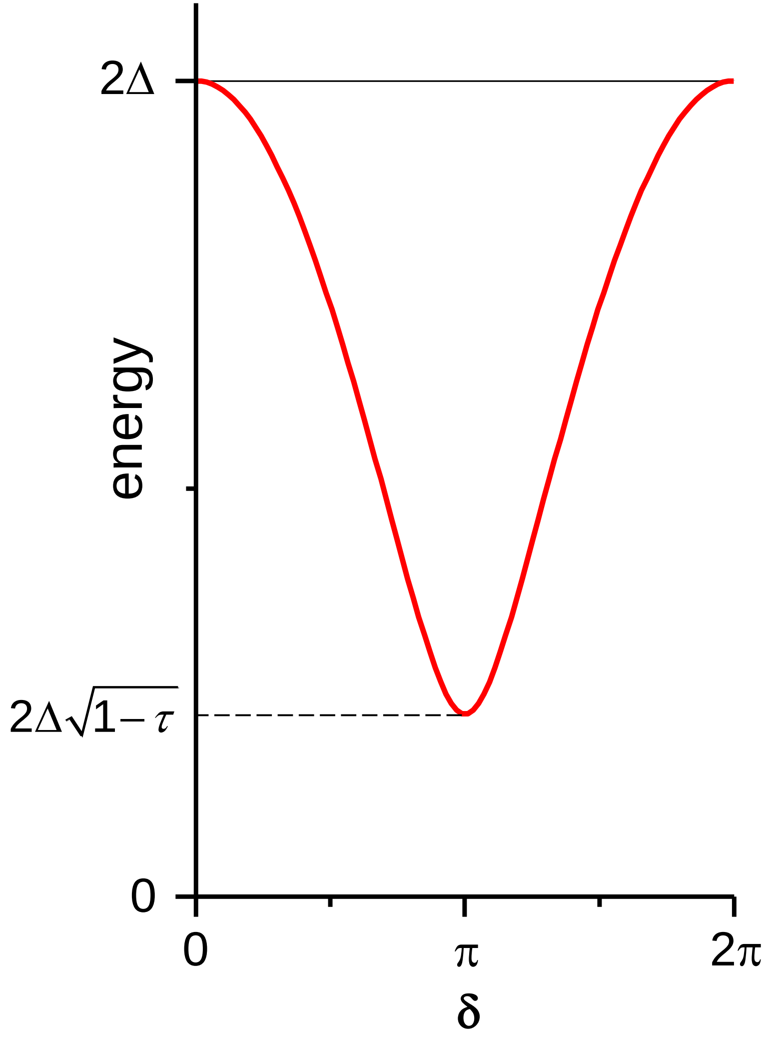
$$I_A\left(\delta,\tau\right) = \pm\frac{\Delta}{4\varphi_0}\frac{\tau\sin\delta}{\sqrt{1-\tau\sin^2\left(\delta/2\right)}}$$

$$\text{Al: } \frac{\Delta}{2\varphi_0} \approx 50\text{ nA}$$

ANDREEV QUBIT

A TWO LEVEL SYSTEM
AS LONG AS PARITY
IS CONSERVED

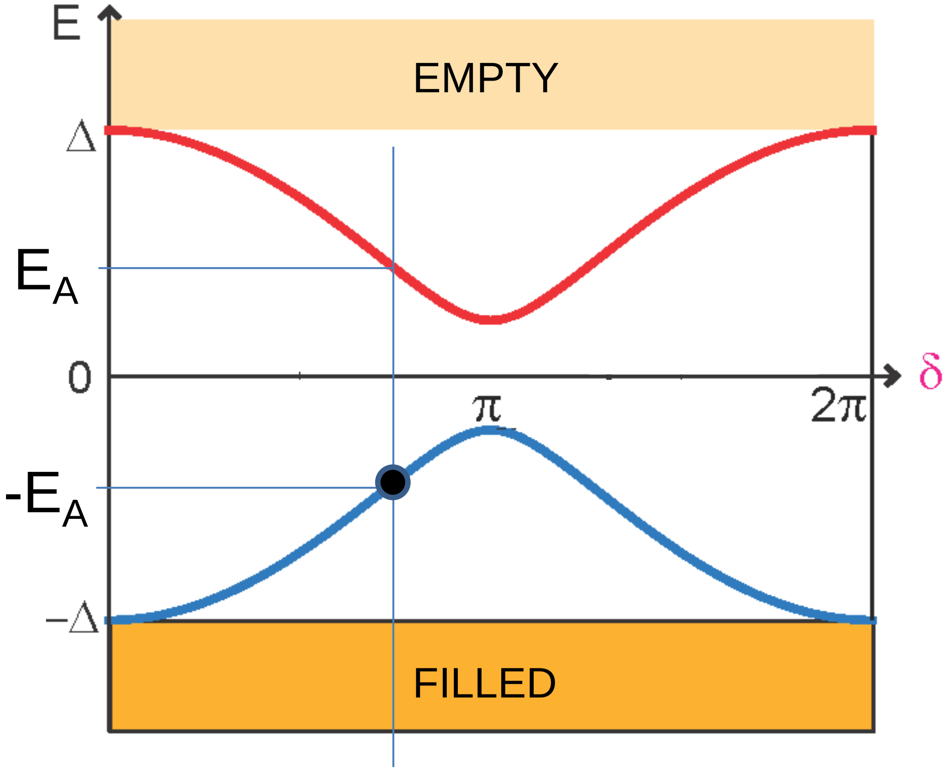




Desposito & Levy Yeyati PRB 2001
Zazunov *et al* PRL 2003
Chtchelkatchev & Nazarov PRL 2003

GROUND STATE

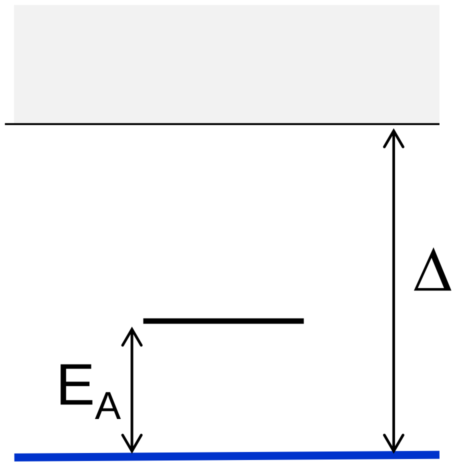
SINGLE PARTICLE REPRESENTATION



δ

EXCITATION REPRESENTATION

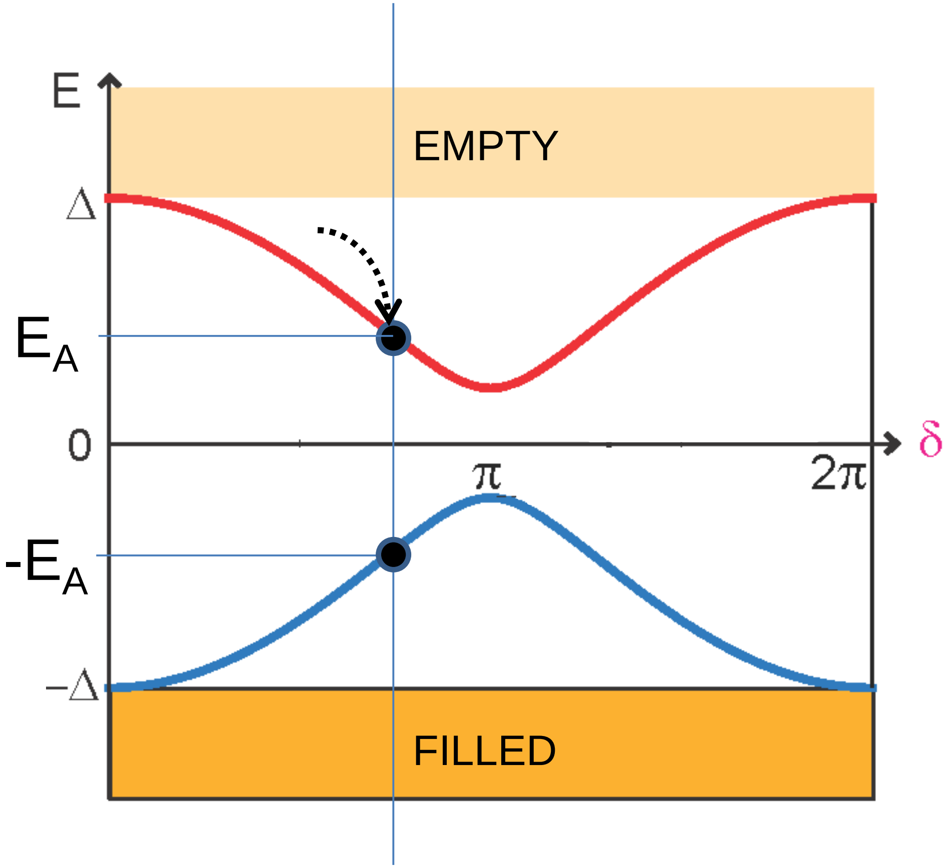
no excitation



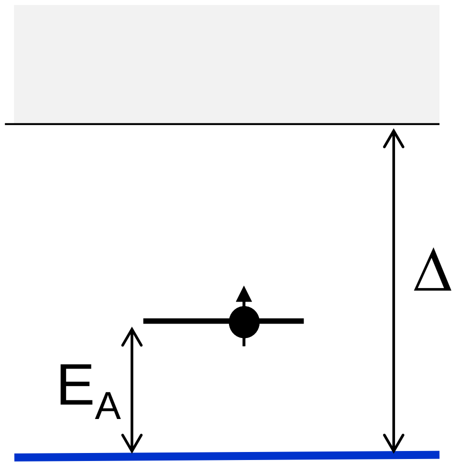
$$-E_A(\delta)$$

$$-I_A(\delta)$$

SINGLE QP EXCITATION



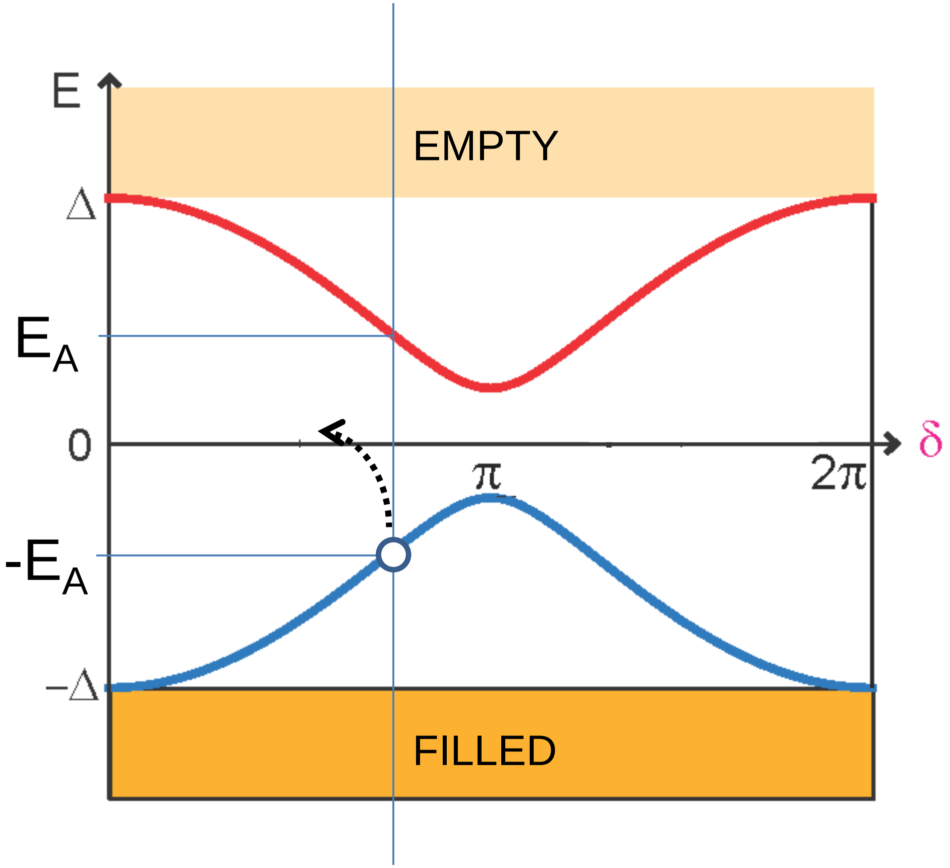
Excited *odd* state



0 energy

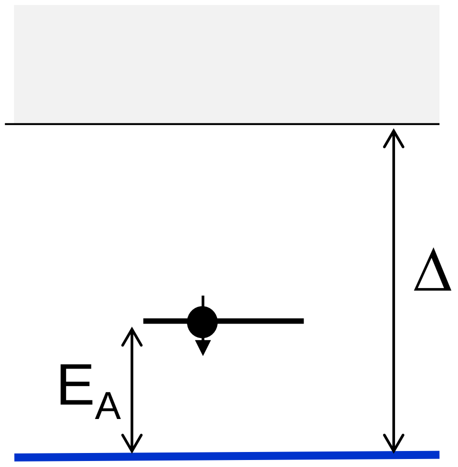
0 supercurrent

SINGLE QP EXCITATION



Excited *odd* state

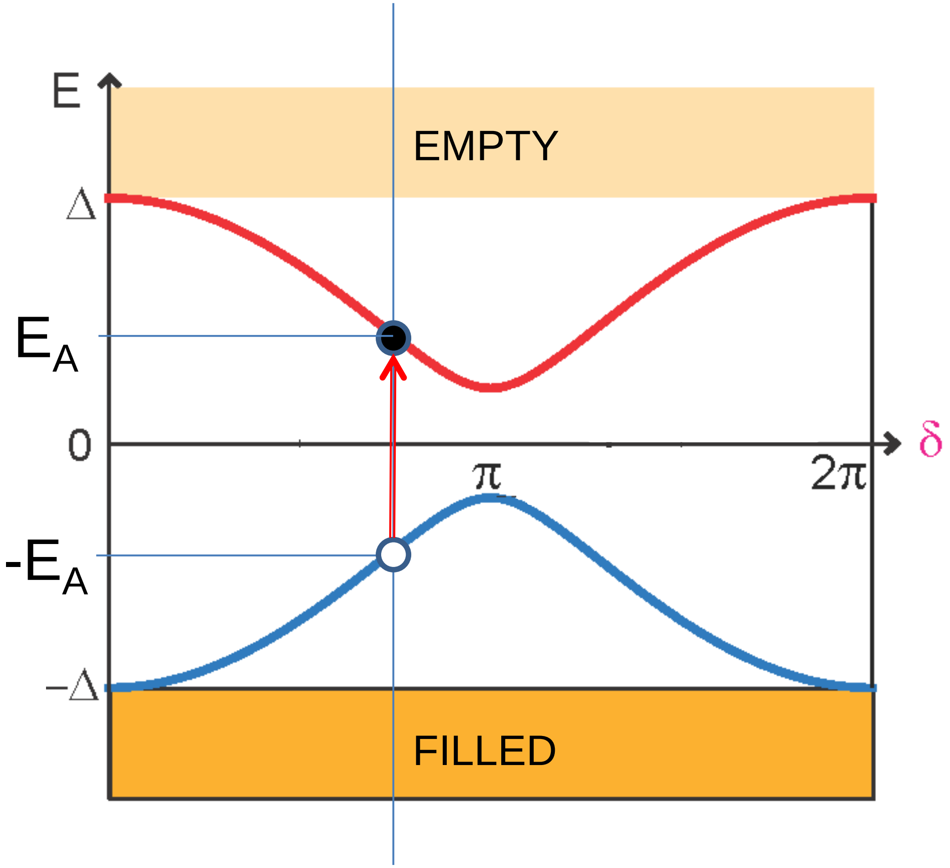
One excitation
E_A above ground state



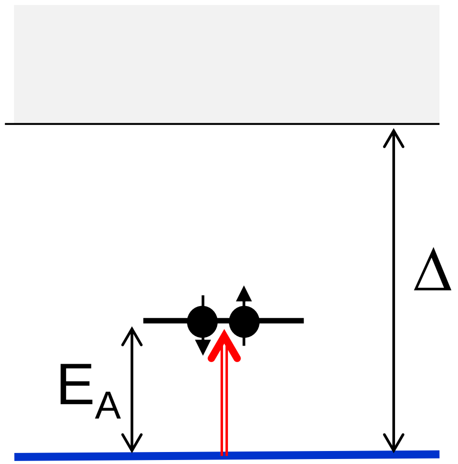
0 energy

0 supercurrent

EXCITED PAIR STATE



Two excitations
$2E_A$ above ground state

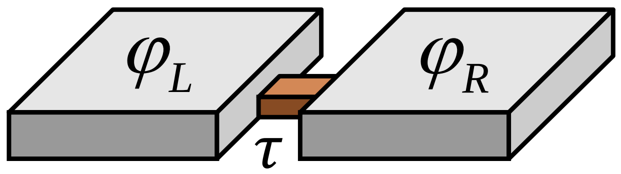


$$+E_A(\delta)$$

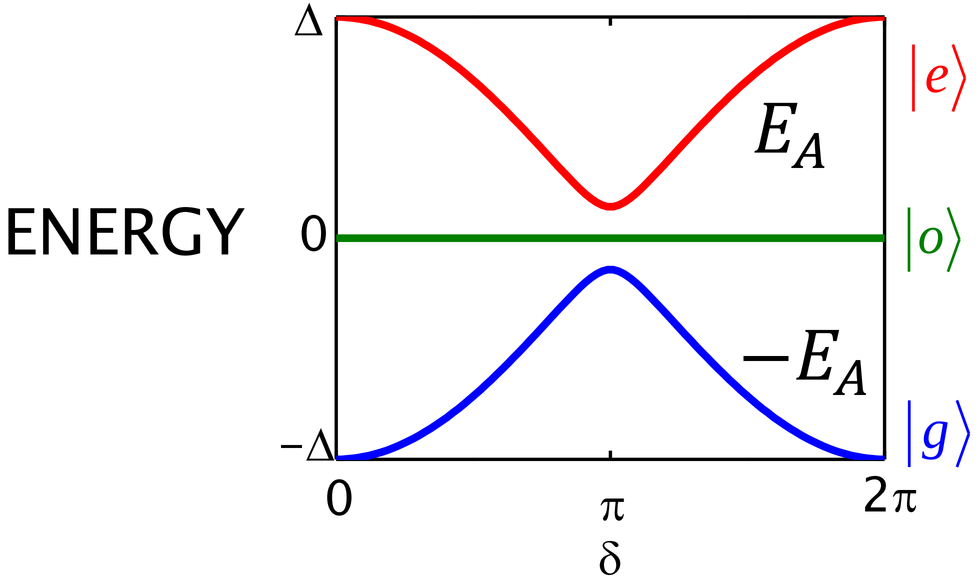
$$+I_A(\delta)$$

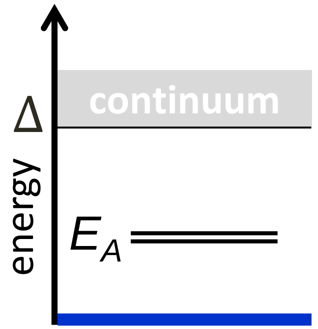
SUPERCURRENT REVERSAL

ANDREEV QUANTUM DOT



$\delta \equiv \varphi_L - \varphi_R \neq 0$

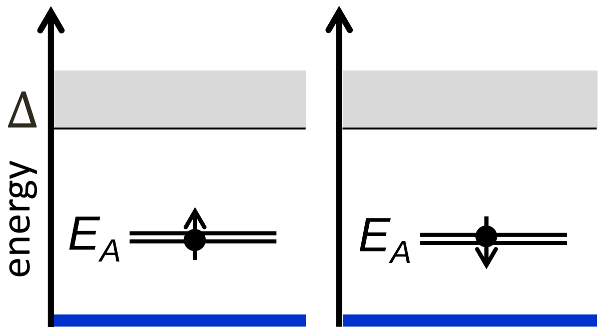




Ground state

$|g\rangle$

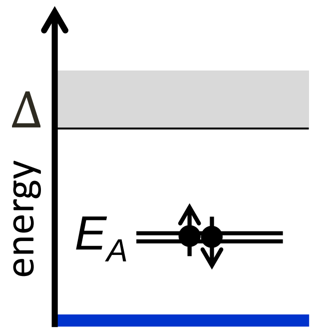
$E = -E_A$



Odd states

$|o\uparrow\rangle$ $|e\rangle$

$E = 0$

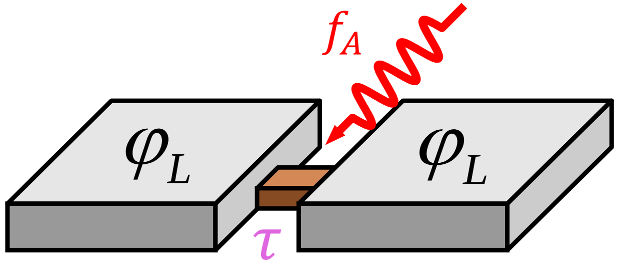


Excited even state

$|e\rangle$

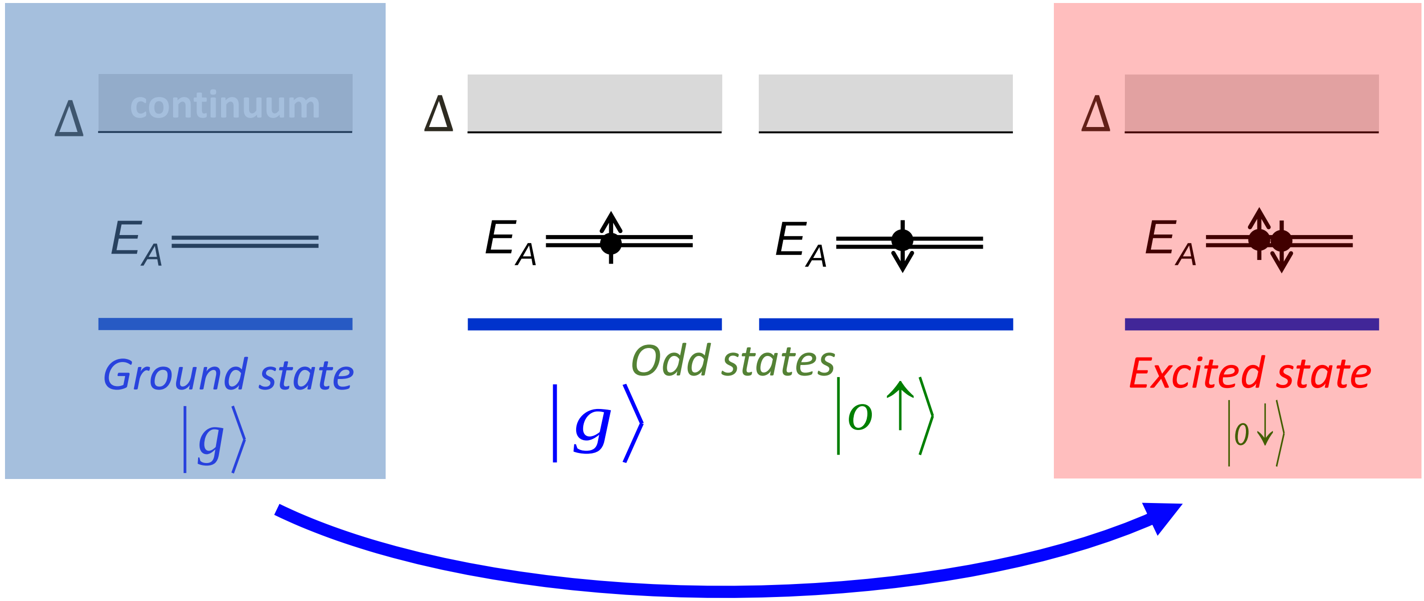
$E = E_A$

ANDREEV QUBIT

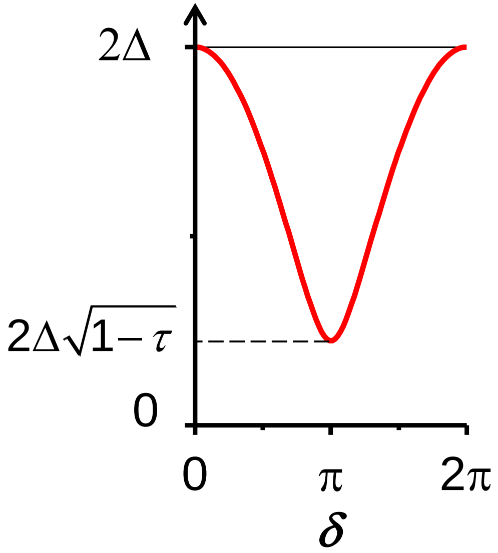


$$\delta \equiv \varphi_L - \varphi_R \neq 0$$

$$E_A = \Delta\sqrt{1 - \tau \sin^2(\delta/2)}$$



$$hf_A = 2E_A$$



ATOMIC CONTACTS:
A TEST-BED FOR ANDREEV PHYSICS

→ I_0

→ $\mathbf{I}(\delta)$

→ $\mathbf{E_A}(\delta,\tau)$

→ $|g\rangle, |e\rangle, |o\rangle$