Outline:

First Lecture: general introduction, application to conventional SC junctions (non-int, steady-state)

Second Lecture: effect of interactions (DCB), quench dynamics, time-dependent FCS

Third Lecture: topological superconductors featuring MBS, two terminal, multiterminal, Interaction effects

Majorana “fermions” in Condensed Matter

Alicea, Rep. Prog. Phys. 2012
Beenakker, Annu. Rev. Con. Mat. Phys. 2013
Aguado, Nuovo Cimento 2017

General properties

$$\gamma_j = \gamma_j^\dagger$$

particle=antiparticle

$$d = (\gamma_1 + i\gamma_2)/\sqrt{2}$$

Two Majoranas form a conventional fermion

$$\{\gamma_i, \gamma_j\} = \delta_{ij}$$

Non-abelian statistics

Physical realizations: zero-energy excitations in Topological Superconductors

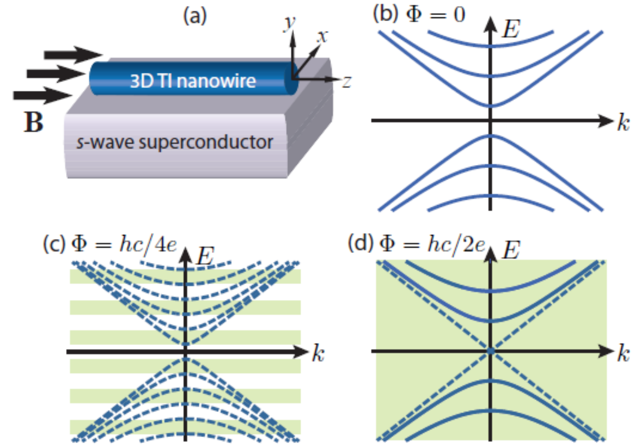
$$\gamma_i = \sum_\sigma \int dx \, \left(f_{\sigma,i}(x)\Psi_\sigma(x) + f^*_{\sigma,i}(x)\Psi^\dagger_\sigma(x)\right)$$

‘Intrinsic’ $p + ip$ superconductivity: Sr_2RuO_4

Artificial topological superconductivity in nanowires

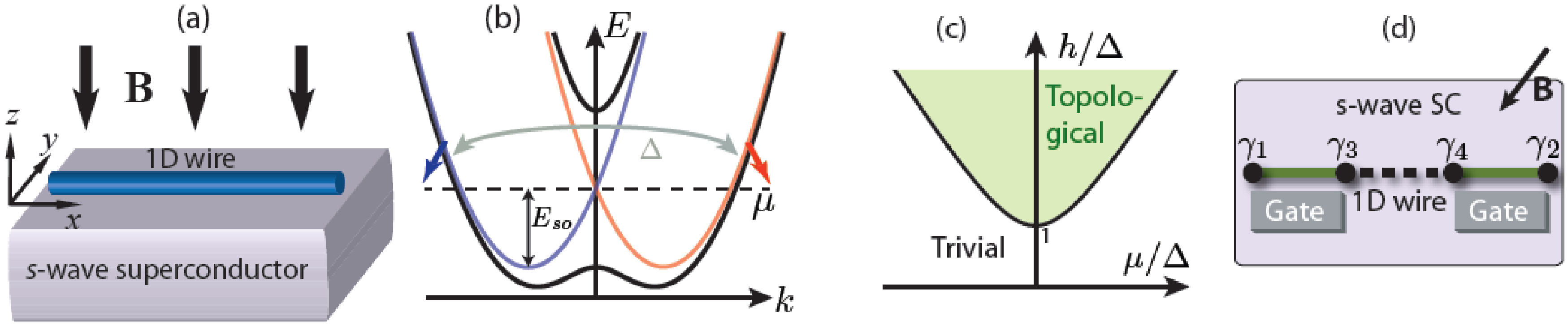
Inducing TS in 3DTI nanowires

Cook and Franz, PRB 2011



D class TS in Rashba nanowires

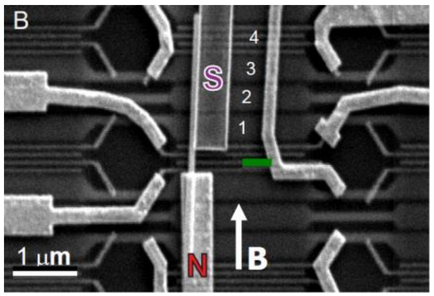
Oreg, Refael & von Oppen, PRL 2010
Lutchyn, Sau & Das Sarma, PRL 2010

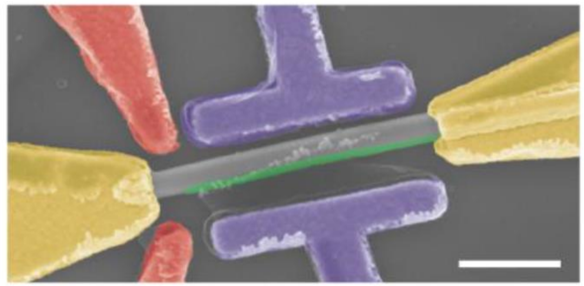


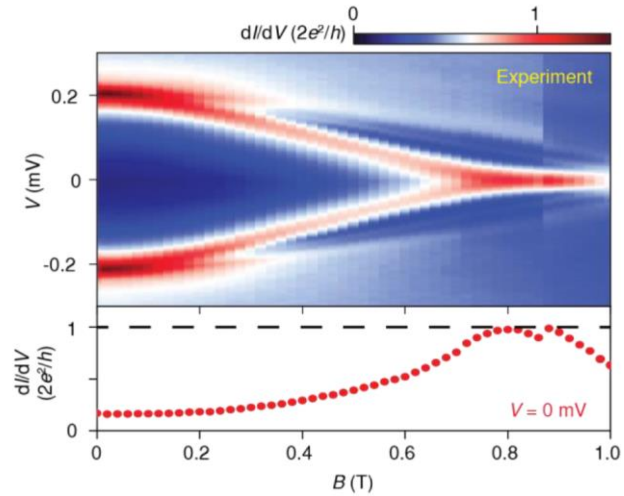
Search for Majorana bound states in semiconducting nanowires

InSb nanowires

Experimental evidence:

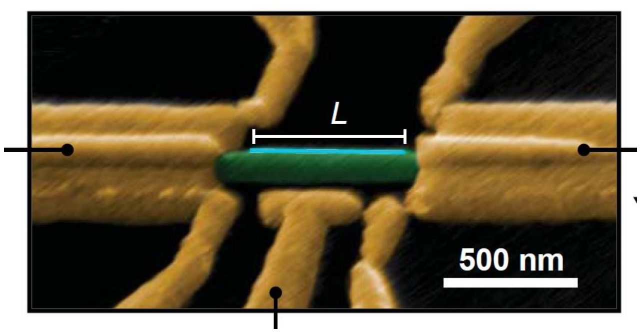


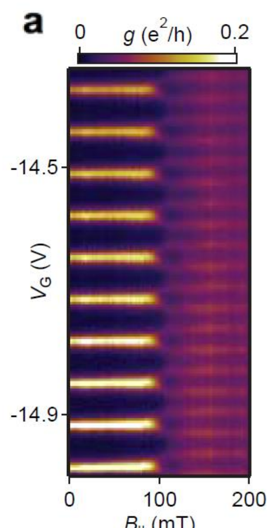


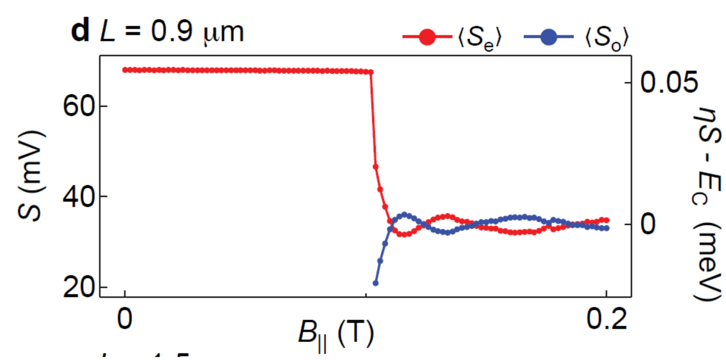


Mourik et al, Science 2012................... Zhang et al, Nature 2018

InAs nanowires, epitaxial Al



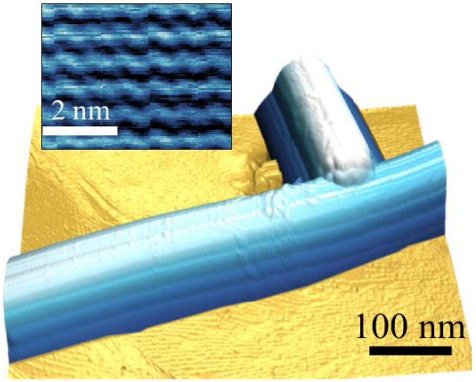




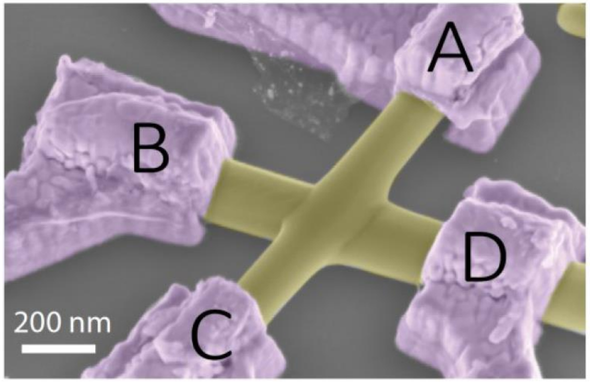
Albretch et al, Nature 2016

Multiterminal nanowire devices

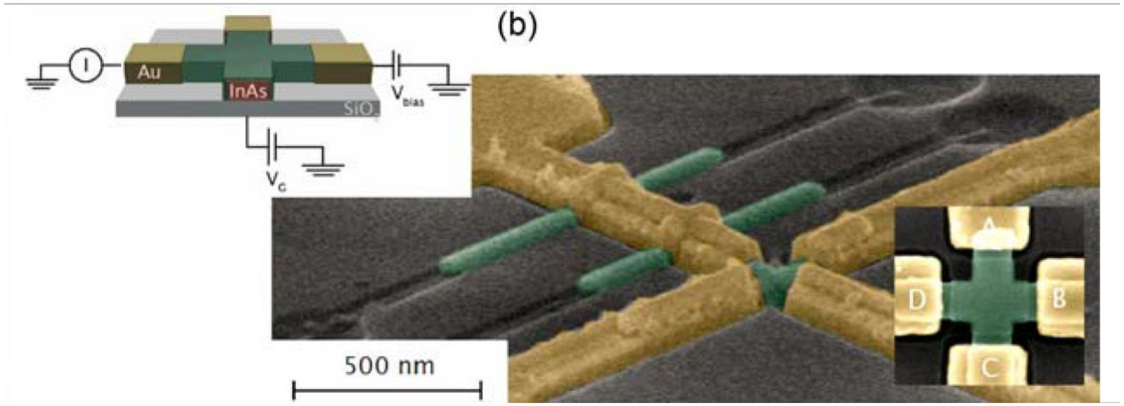
Weizmann



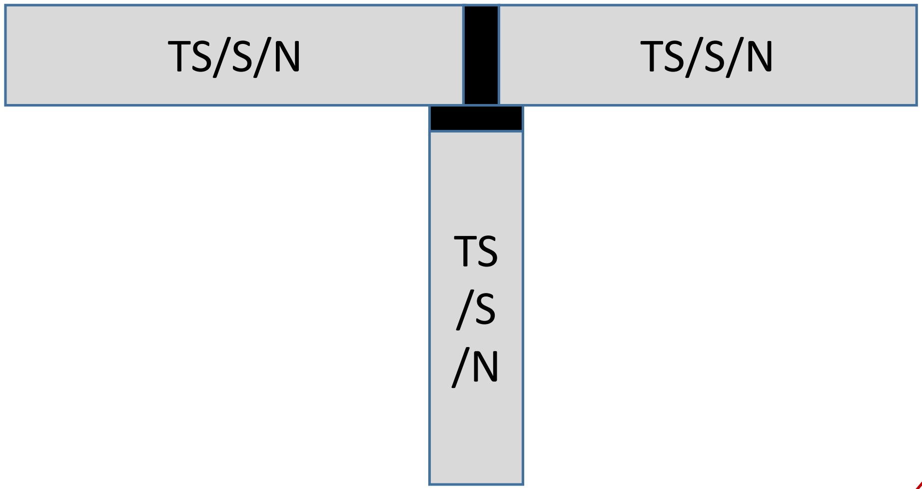
Delft



IBM Zurich



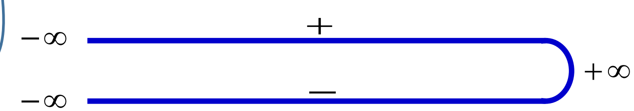
Hamiltonian approach



Appropriate bGF for TS?

$$S_{eff}\left[\hat{\bar{\Psi}}, \hat{\Psi}\right] = \sum_{ij} \int_{\mathcal{C}} dt \, (\bar{\Psi}_i, \ \bar{\Psi}_j) \begin{pmatrix} \hat{g}_i^{-1} & -\hat{T}_{ij} \\ -\hat{T}_{ji} & \hat{g}_j^{-1} \end{pmatrix} \begin{pmatrix} \Psi_i \\ \Psi_j \end{pmatrix}$$

Keldysh contour



TS case: Boundary GF for the Kitaev model

L/R chains in real space

$$H_{L/R} = \sum_{j \in L/R} t c_j^\dagger c_{j+1} + \Delta c_j c_{j+1} + \text{h.c.}$$

infinte chain (k space, Nambu)

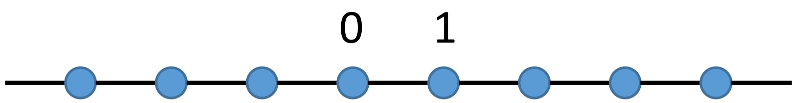
$$H_0 = \sum_k \Psi_k^\dagger \underbrace{\begin{pmatrix} t\cos k & -i\Delta \sin k \\ i\Delta \sin k & -t\cos k \end{pmatrix}}_{\mathcal{H}_k} \Psi_k \qquad \Psi_k^T = \begin{pmatrix} c_k & c_{-k}^\dagger \end{pmatrix}$$

infinite chain GF in real space

$$\hat{G}^0_{ij} = \sum_k \left[\omega - \mathcal{H}_k\right]^{-1} e^{ik|i-j|}$$

$$\hat{G}^0_{00} = \frac{-\omega}{\sqrt{(\omega^2-\Delta^2)(\omega^2-t^2)}}\sigma_0$$

$$\hat{G}^0_{01} = \frac{t(z_1^2+1)+\Delta(z_1^2-1)\sigma_x}{\sqrt{(\omega^2-\Delta^2)(\omega^2-t^2)}}\sigma_z$$

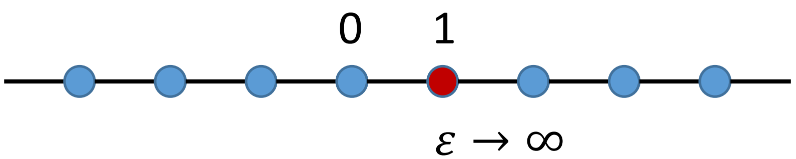


$$z_1^2 = \frac{2\omega^2-(t^2+\Delta^2)}{t^2-\Delta^2} - \text{sign}\left(2\omega^2-(t^2+\Delta^2)\right)\sqrt{\left(\frac{2\omega^2-(t^2+\Delta^2)}{t^2-\Delta^2}\right)^2-1}$$

Dyson equation for chain breaking

$$\hat{g}_L = \hat{G}^0_{00} - \hat{G}^0_{01}\left(\hat{G}^0_{00}\right)^{-1}\hat{G}^0_{10}$$

$$\hat{g}_R = \hat{G}^0_{00} - \hat{G}^0_{10}\left(\hat{G}^0_{00}\right)^{-1}\hat{G}^0_{01}$$



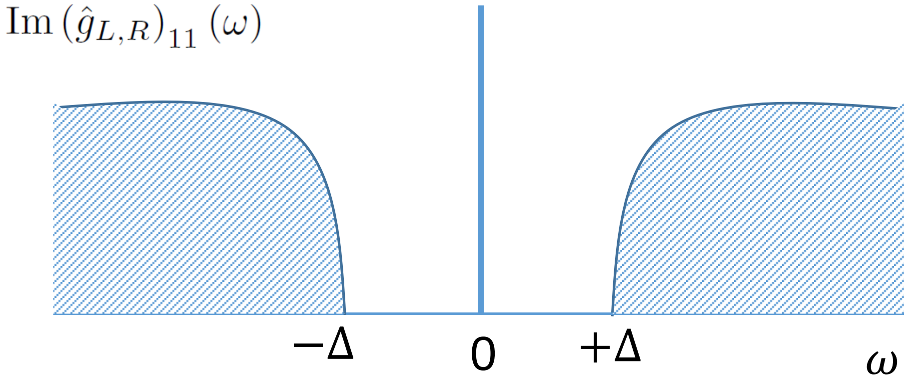
Boundary GF for the Kitaev model

Zazunov, Egger & ALY, PRB (2016)

Boundary GFs in $t >> \Delta$ *limit*

$$\hat{g}_L = \frac{2}{|t|\omega}\begin{pmatrix} \sqrt{\Delta^2-\omega^2} & \Delta \\ \Delta & \sqrt{\Delta^2-\omega^2} \end{pmatrix}$$

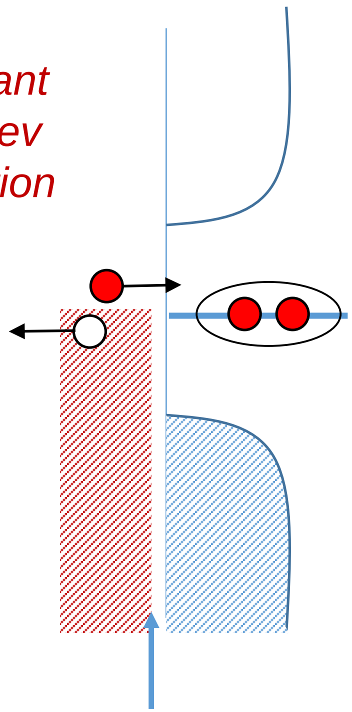
$$\hat{g}_R = \frac{2}{|t|\omega}\begin{pmatrix} \sqrt{\Delta^2-\omega^2} & -\Delta \\ -\Delta & \sqrt{\Delta^2-\omega^2} \end{pmatrix}$$



N-TS case: conductance and noise

Zazunov, Egger & ALY, PRB (2016)

resonant Andreev reflection



τ

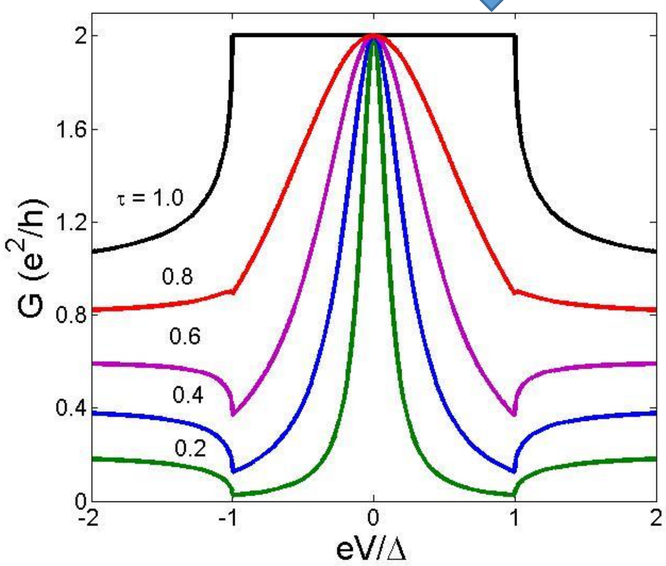
transmission coefficient

$$G(V, T=0) = \frac{2e^2}{h} J(eV)$$

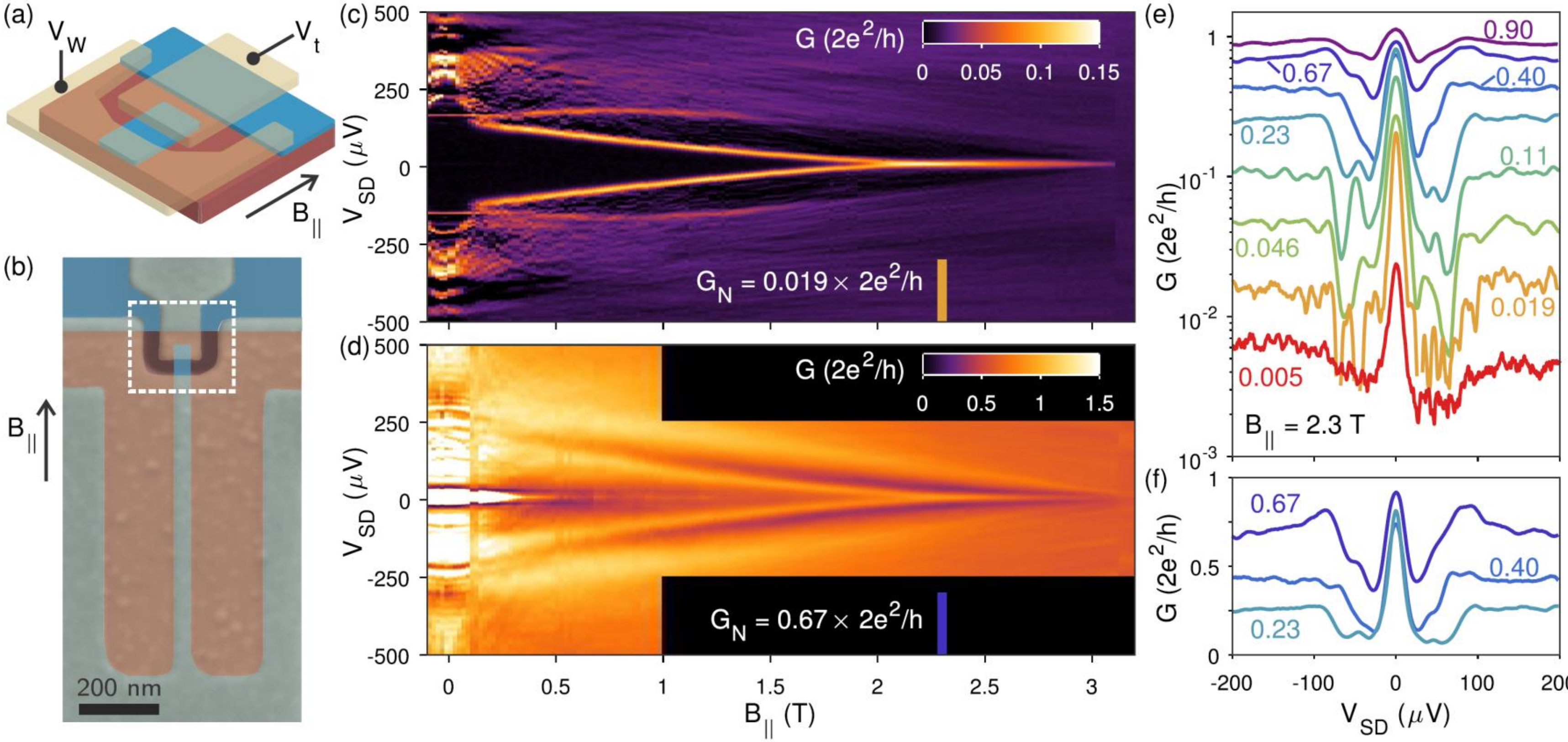
zero-temperature conductance

$$J(\omega) = \begin{cases} 1/(1+\omega^2/\Gamma^2), & |\omega| < \Delta, \\ \tau \frac{\tau + (2-\tau)\sqrt{1-(\Delta/\omega)^2}}{\left[2-\tau+\tau\sqrt{1-(\Delta/\omega)^2}\right]^2}, & |\omega| \geq \Delta, \end{cases}$$

$$\Gamma = \frac{\tau \Delta}{2\sqrt{1-\tau}}$$



Nagging issue: $2e^2/h$ or not?



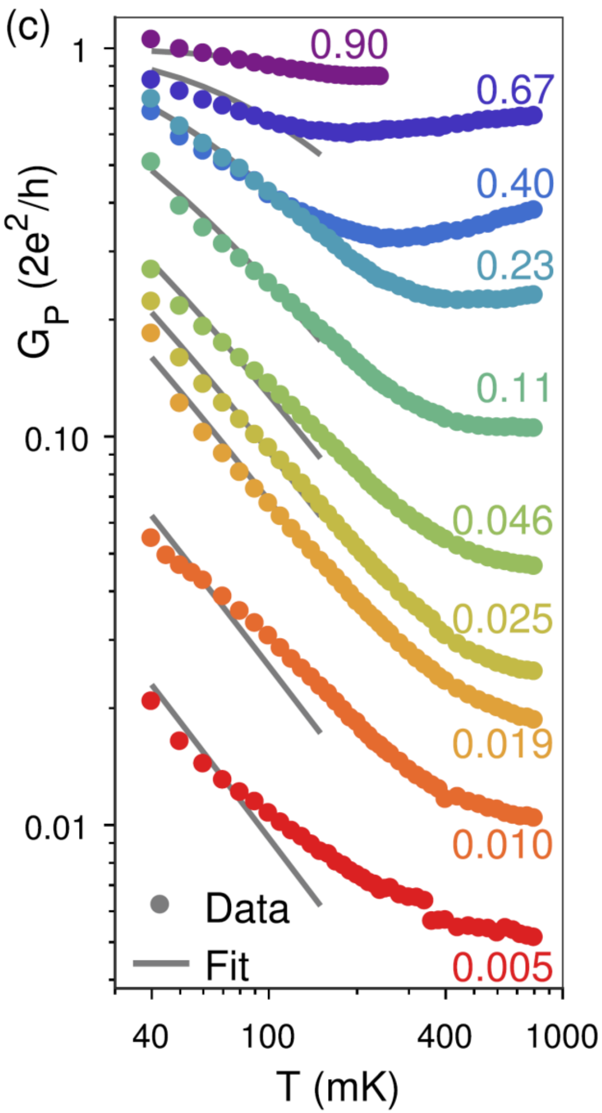
Al/InGaAs/InAs

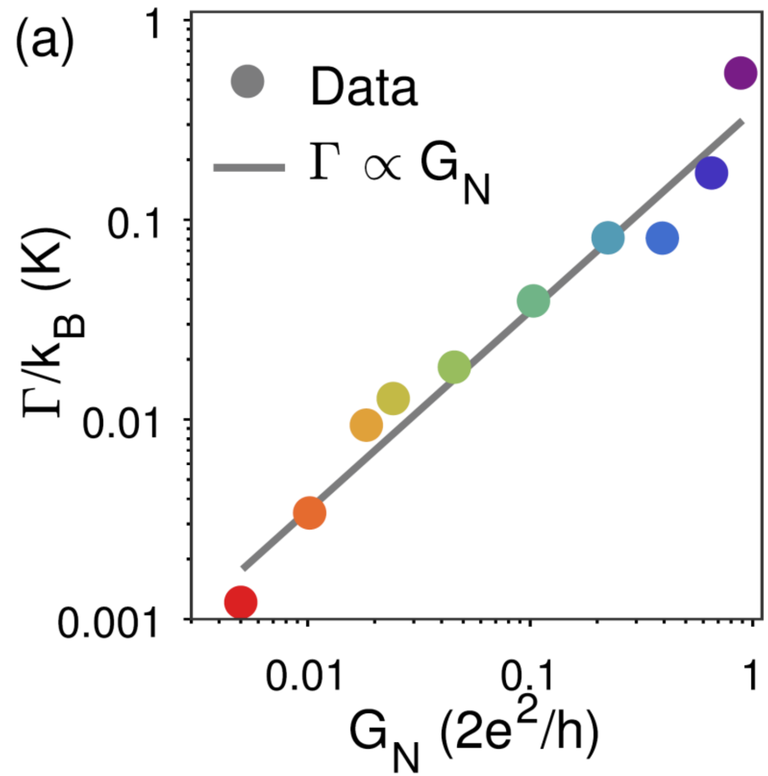
Similar results in Zhang et al, Nature (2018) for InSb nanowires

Nichele et al. PRL 2017

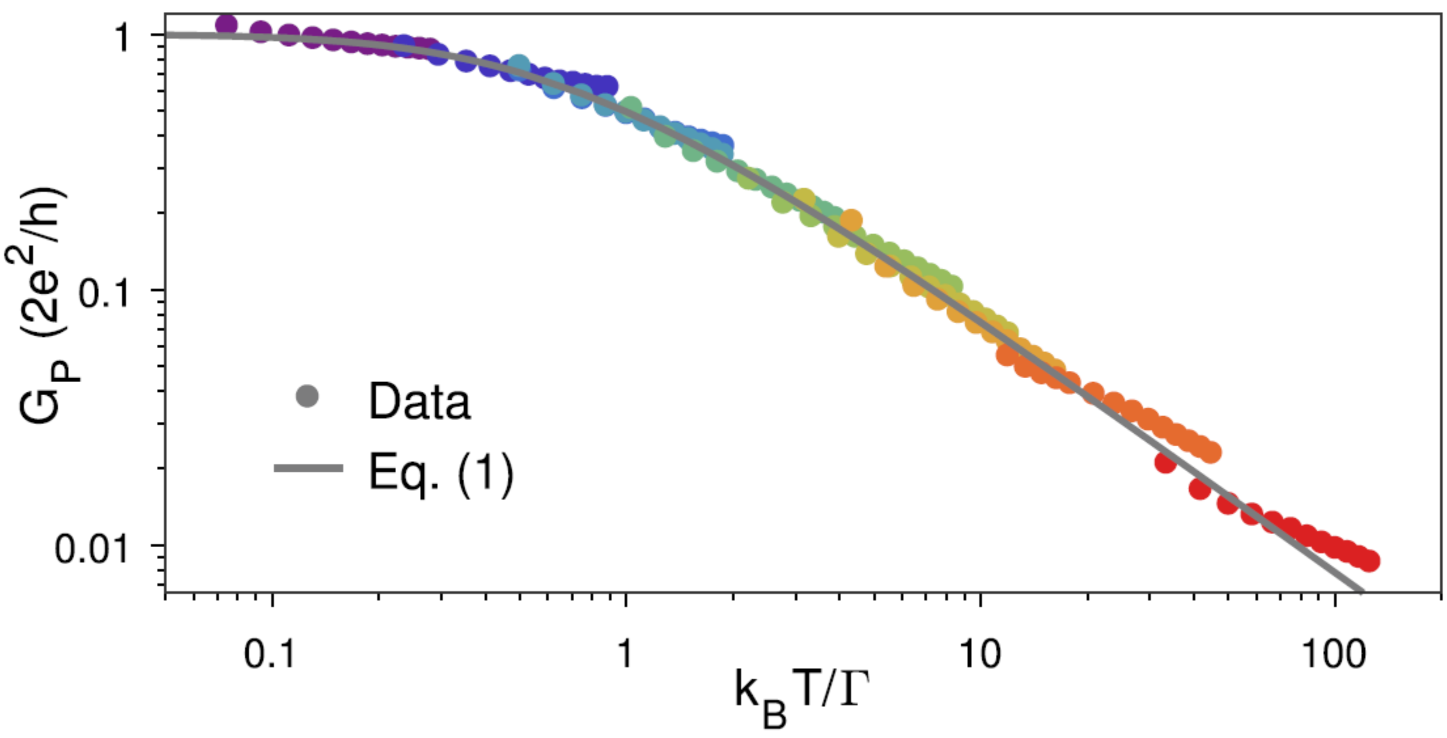
Nichele et al. PRL 2017

$$G_{\mathrm{P}} \approx \frac{e^2}{h} \int_{-\infty}^{\infty} d\omega \frac{2\Gamma^2}{\omega^2 + \Gamma^2} \frac{1}{4k_{\mathrm{B}}T \cosh^2(\omega/(2k_{\mathrm{B}}T))}$$
$$= \frac{2e^2}{h} f(k_{\mathrm{B}}T/\Gamma),$$





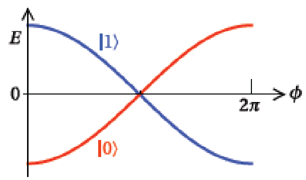
$$\Gamma = \frac{\tau \Delta_{\mathrm{topo}}}{2\sqrt{1-\tau}}$$



Nichele et al. PRL 2017

Equilibrium TS-TS case: frequency dependent noise

Zazunov, Egger & ALY, PRB (2016)



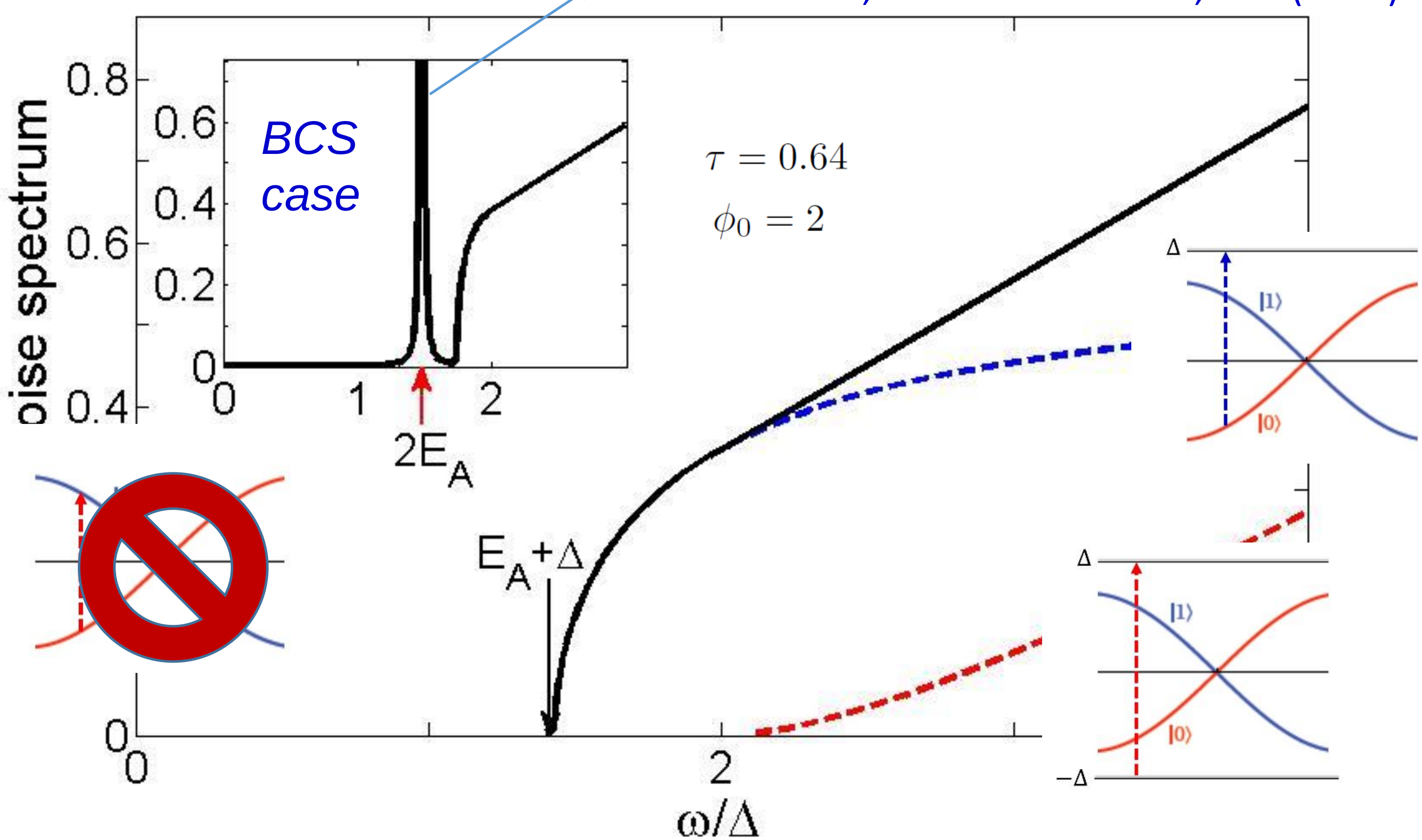
$$E_A(\phi_0) = \sqrt{\tau}\Delta\cos(\phi_0/2)$$

Andreev bound states (ABS): 4π periodicity

$$I(\phi_0) = \pm\frac{e\sqrt{\tau}\Delta}{2\hbar}\sin(\phi_0/2)$$

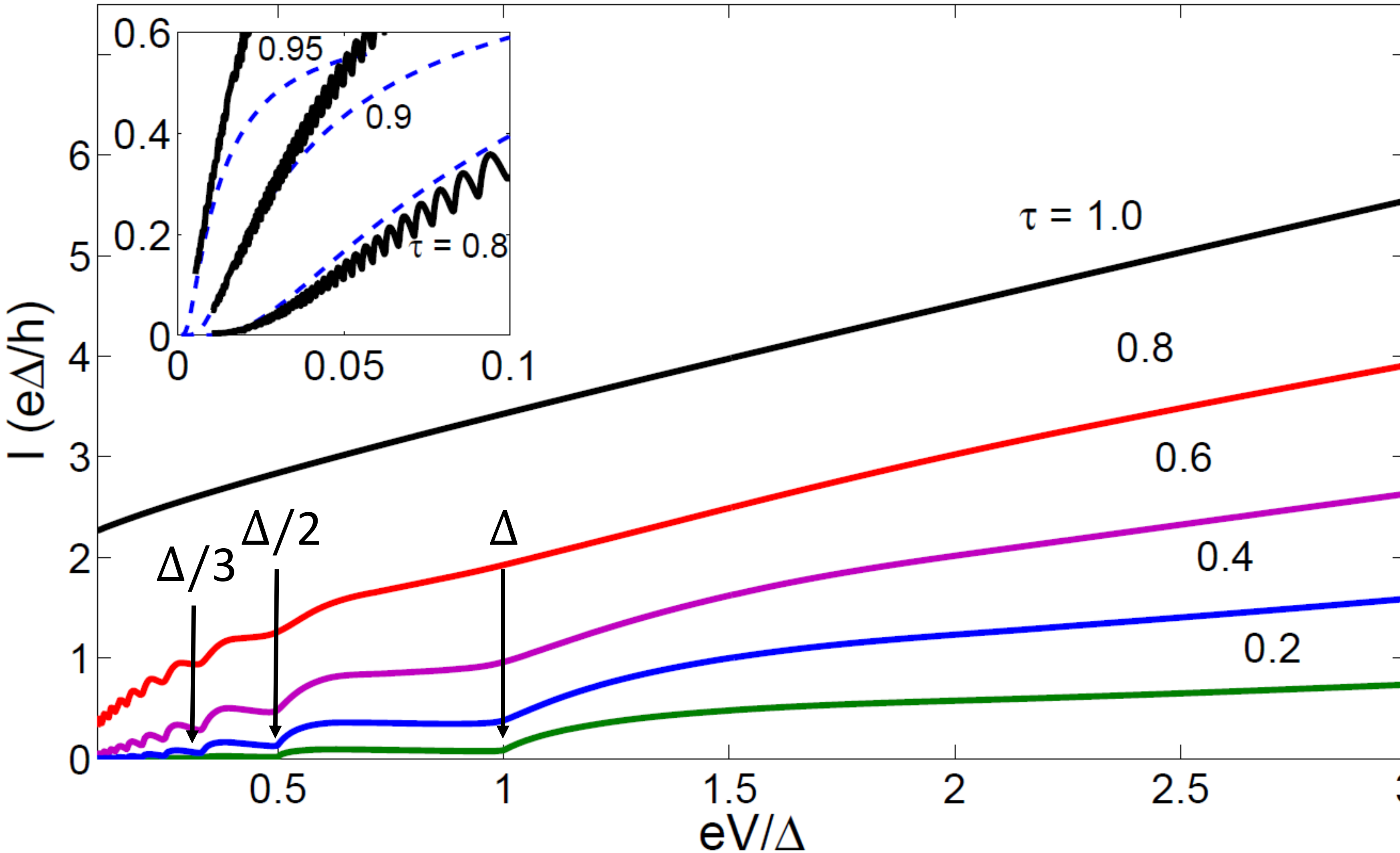
zero-temperature Josephson current

Martín-Rodero, ALY & García-Vidal, PRB (1996)



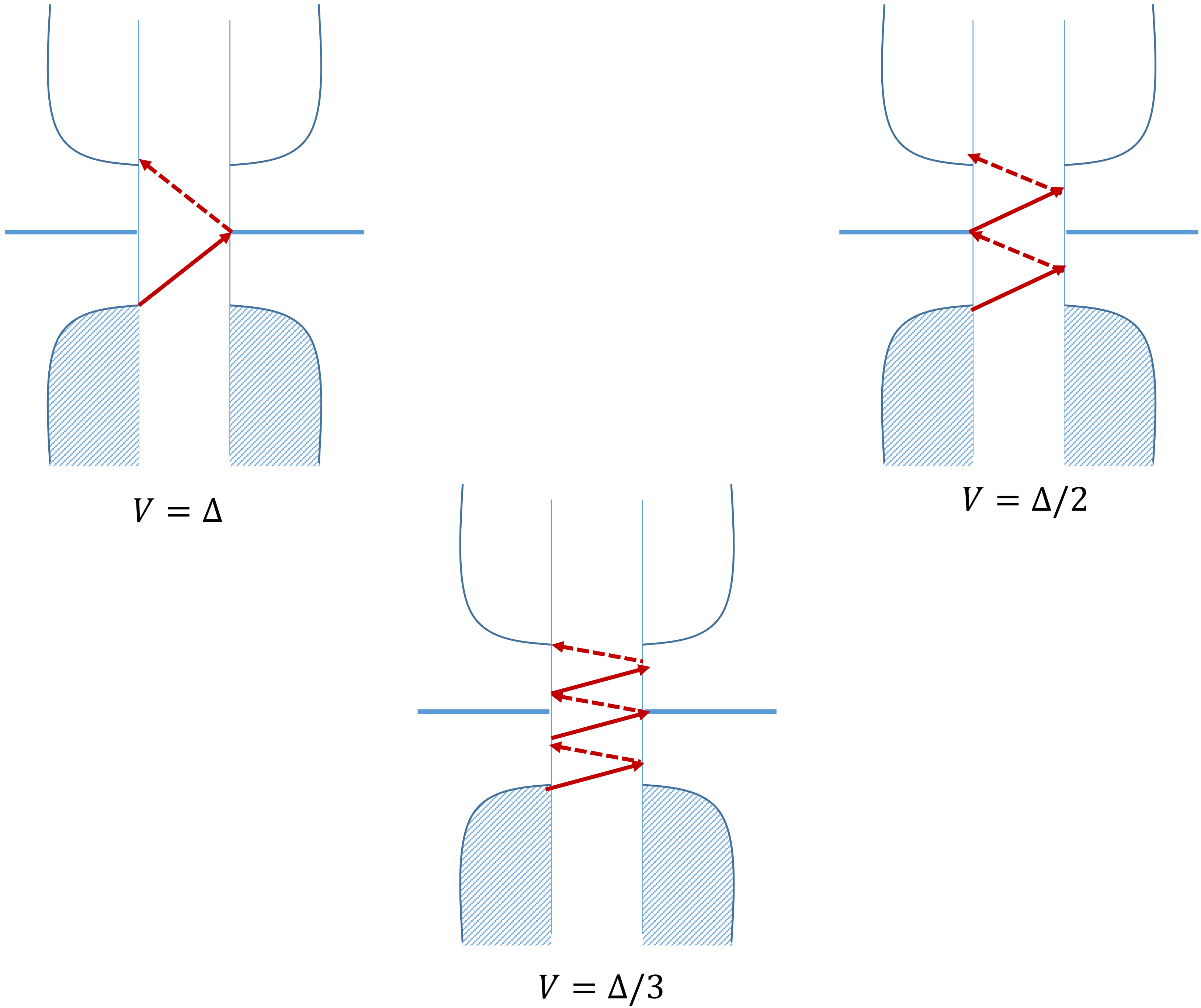
Non-equilibrium TS-TS case: MAR regime

Zazunov, Egger & ALY, PRB (2016)



Badiane et al., PRL (2011)
San José et al., NJP (2013)

Subgap features at Δ/n instead of 2Δ/n

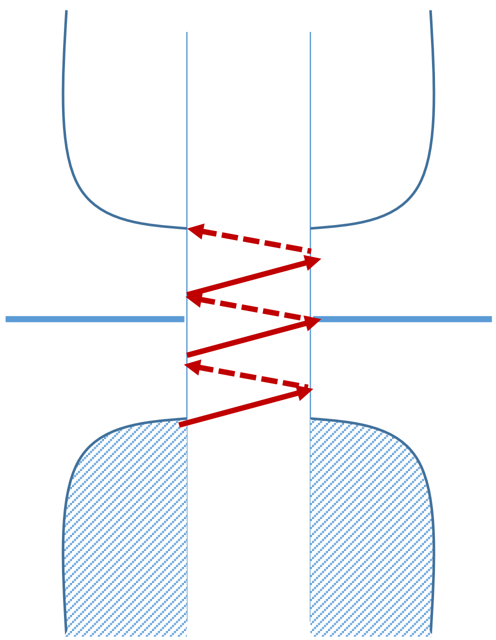

$V = \Delta$
$V = \Delta/2$
$V = \Delta/3$

$$\Gamma_n = \int dE\ \rho_{ini}(E)\ \rho_{fin}(E)\ \ \mathrm{Prob_n}(E)$$

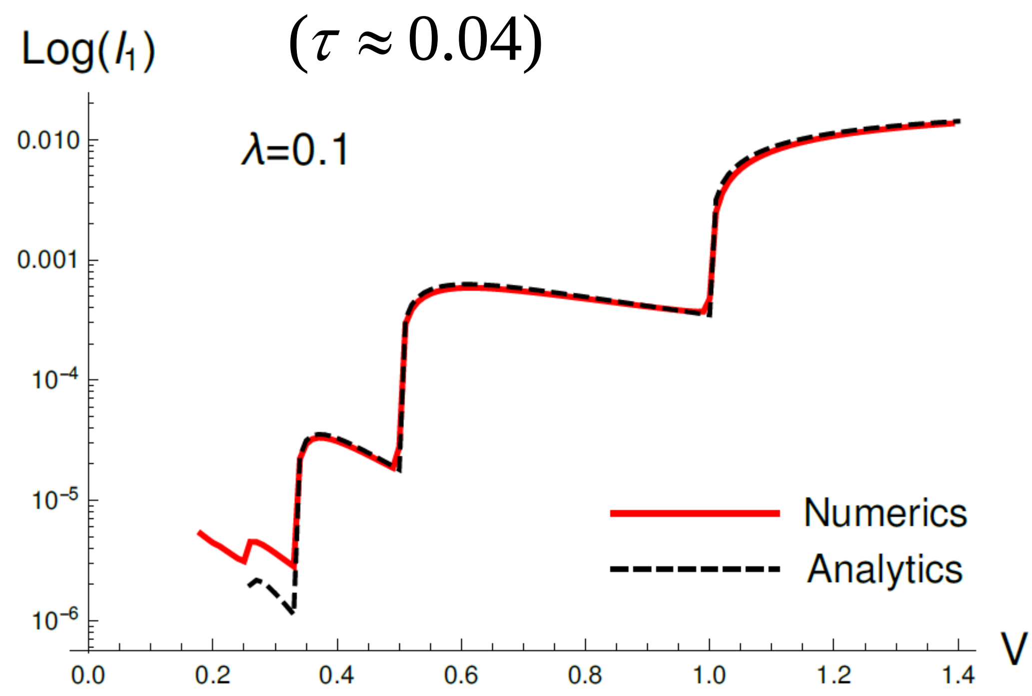
$$I = \sum_{n=1}^{\infty}(n\ \Gamma_n)$$

$$\Gamma_n = 2\lambda^2\sqrt{1-\frac{\Delta^2}{(nV)^2}}\ (\lambda^2)^{n-1}(\Delta^2)^{n-1}(1/(n-1)V)^2(1/(n-2)V)^2\cdots(1/V)^2$$

$$= 2\lambda^2\sqrt{1-\frac{\Delta^2}{(nV)^2}}\ \left(\frac{\lambda^2\Delta^2}{V^2}\right)^{n-1}\ \left(\frac{1}{(n-1)!}\right)^2$$

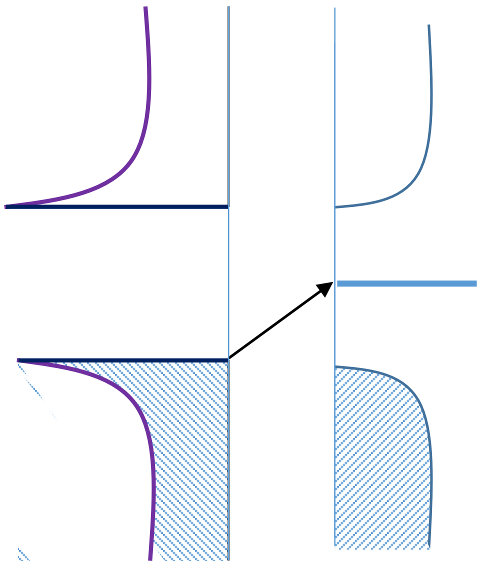


$V = \Delta/3$

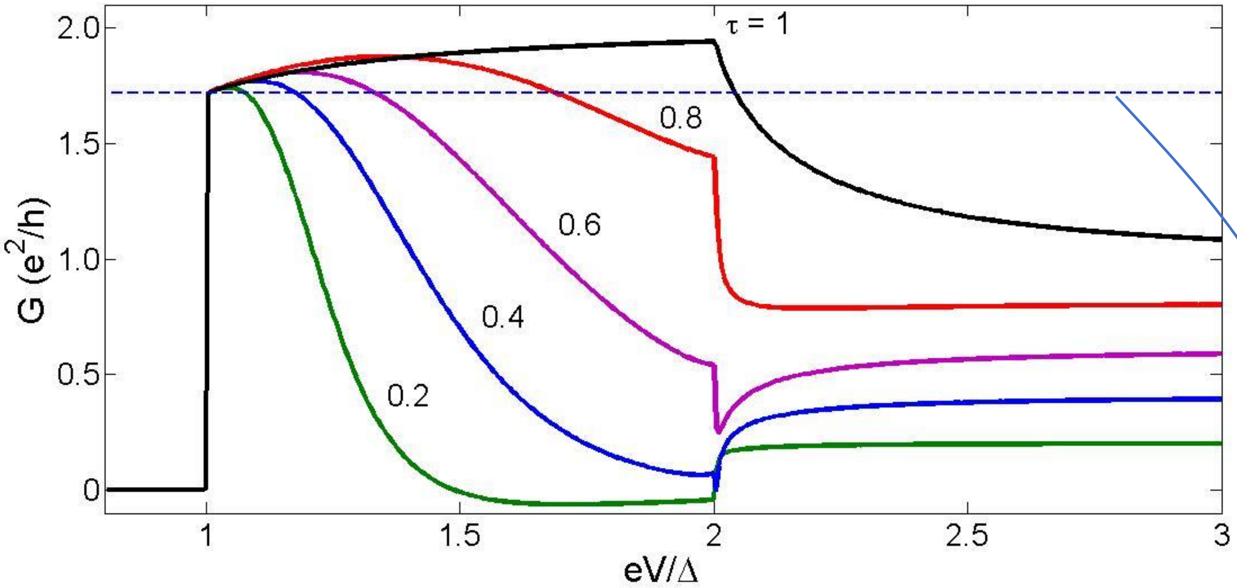


S-TS case: differential conductance

Zazunov, Egger & ALY, PRB (2016)



$V = \Delta$



$$G = (4-\pi)\,\frac{2e^2}{h}$$

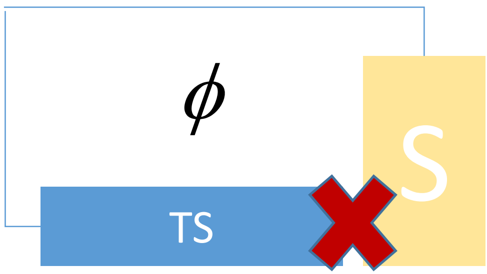
Peng et al., PRL (2015)

Behavior for spinful model → Setiawan et al., PRB (2017)

Multiterminal S-TS junctions

Previous work: topological states from multiterminal

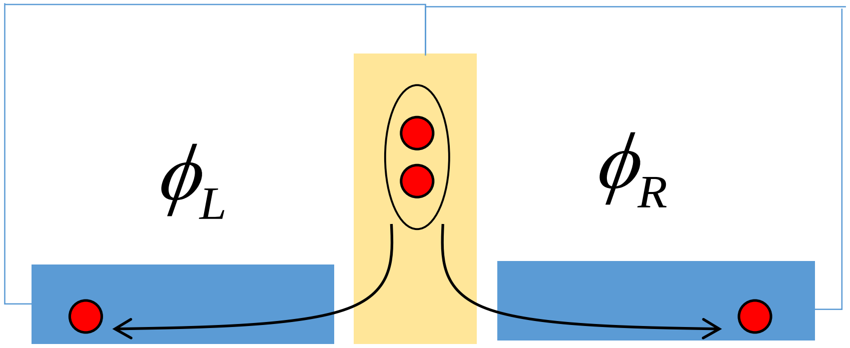
Heck et al., PRB (2014)
Riwar et al., Nature Comm. (2016)



Two-terminal S-TS: Josephson blockade

Zazunov & Egger, PRB (2012)
Zazunov, Egger & ALY, PRB (2016)

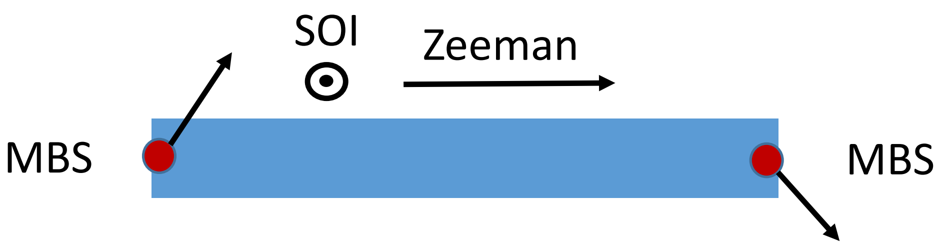
Three-terminal S-TS: lifting of Josephson blockade?



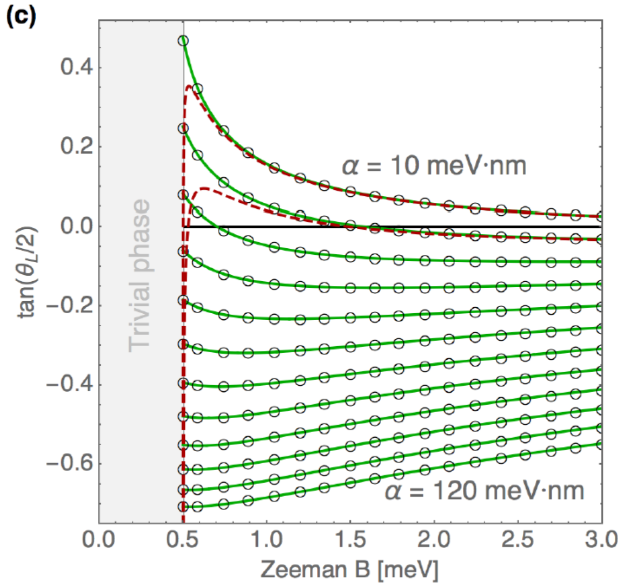
Multiterminal S-TS junctions: role of MBS spin structure

MBS spin-structure in single wire:

Sticlet, Bena & Simon, PRL (2012)
Prada, Aguado & San-José, PRB (2017)

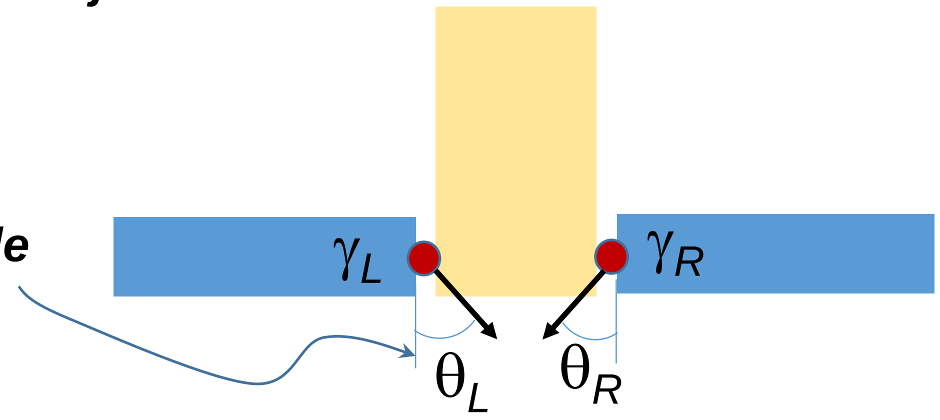


MBS spin canting angle



MBS spin-structure in multiterminal junction:

Spin canting angle



Multiterminal S-TS junctions: modeling

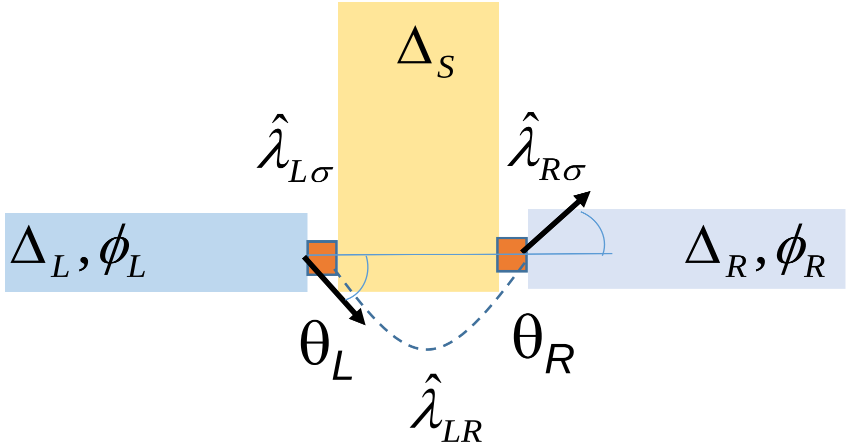
Zazunov, Egger, Alvarado & ALY, PRB (2017)

$$H_T = \sum_{\mu \equiv L,R;\sigma} \hat{\psi}^\dagger_{s\sigma} \hat{\lambda}_{\mu\sigma} \hat{\psi}_\mu + \hat{\psi}^\dagger_L \hat{\lambda}_{LR} \hat{\psi}_R \; + \; \text{h.c.}$$

$$\hat{\lambda}_{LR} = \lambda_{LR} \tau_z e^{i\tau_z(\phi_L - \phi_R)/2}$$

$$\hat{\lambda}_{\mu\uparrow} = \lambda_\mu \begin{pmatrix} e^{i\phi_\mu/2}\cos\frac{\theta_\mu}{2} & 0 \\ 0 & -e^{-i\phi_\mu/2}\sin\frac{\theta_\mu}{2} \end{pmatrix}$$

$$\hat{\lambda}_{\mu\downarrow} = \lambda_\mu \begin{pmatrix} e^{i\phi_\mu/2}\sin\frac{\theta_\mu}{2} & 0 \\ 0 & -e^{-i\phi_\mu/2}\cos\frac{\theta_\mu}{2} \end{pmatrix}$$



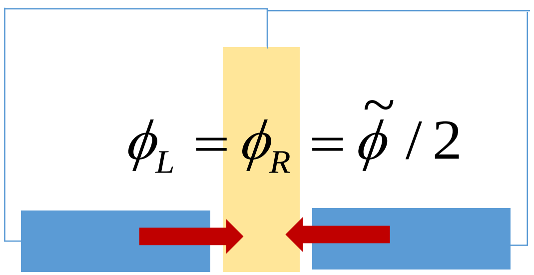
$$\hat{G}^{-1} = \hat{g}^{-1} - \hat{\Sigma} \qquad \hat{g} = \begin{pmatrix} g_L & 0 \\ 0 & g_R \end{pmatrix} \qquad \hat{\Sigma} = \begin{pmatrix} \Sigma_{LL} & \Sigma_{LR} \\ \Sigma_{RL} & \Sigma_{RR} \end{pmatrix}$$

$$I_j = \frac{e}{h} \int d\omega n_F(\omega) \text{Re Tr} \left[\sigma_z \left\{ \hat{\Sigma}^A, \hat{G}^A \right\}_{jj} \right]$$

Multiterminal S-TS junctions: CPR results

$$\lambda_L = \lambda_R = \lambda \qquad \Delta_L = \Delta_R = \Delta \qquad \lambda_{LR} = 0 \qquad \theta = \theta_L - \theta_R$$

"parallel" case



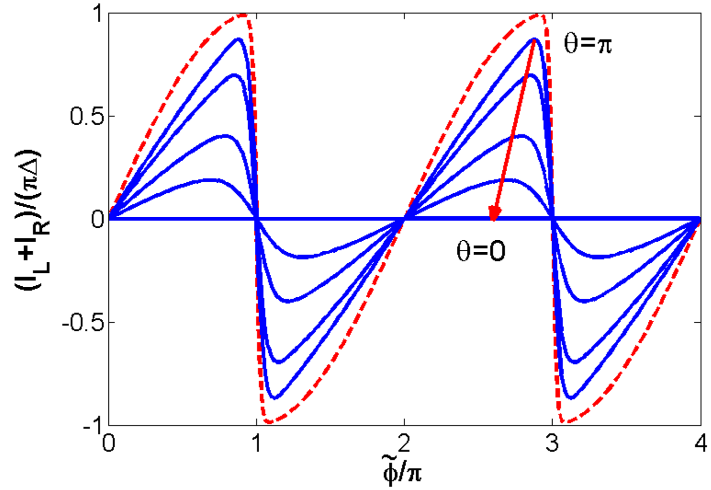
$$\phi_L = \phi_R = \tilde{\phi}/2$$

$\Delta_s \to \infty$ *limit*

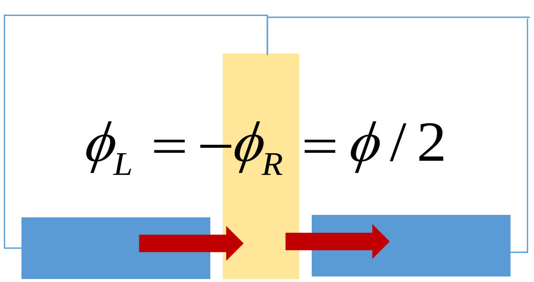
$$\epsilon_A = \sqrt{\tau}\Delta\cos(\tilde{\phi}/2)$$

$$\tau = 4\Lambda_\theta^2/(1+\Lambda_\theta^2)^2$$

$$\Lambda_\theta = \lambda^2 \sin(\theta/2)$$



"serial" case



$$\phi_L = -\phi_R = \phi/2$$

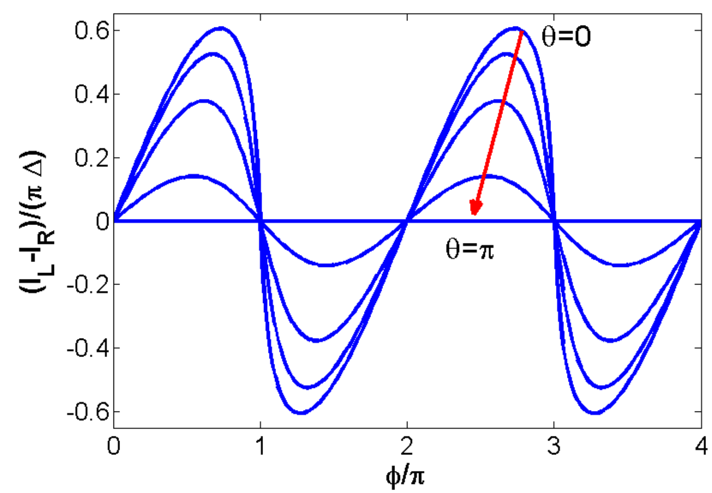
$\Delta_s \to 0$ *limit*

$$\epsilon_A(\phi) = \tilde{\Delta}\sqrt{1-\tau\sin^2(\phi/2)}$$

$$\tau = \cos^2(\theta/2)$$

$$\tilde{\Delta} = \Delta/\sqrt{1+x^2}$$

$$x = \frac{1+\lambda^4\sin^2(\theta/2)}{2\lambda^2}$$



Zazunov, Egger, Alvarado & ALY, PRB (2017)

Boundary GF for the spinful wire model

Zazunov, Egger, Alvarado & ALY, PRB (2017)

infinte wire (k space, Nambu)

$$H_0 = \sum_k \Psi_k^\dagger \underbrace{\left((\epsilon(k)-\mu)\sigma_0\tau_z + \alpha\sin(k)\sigma_z\tau_z + V_x\sigma_x\tau_0 + \Delta\sigma_0\tau_x\right)}_{\mathcal{H}_k} \Psi_k$$

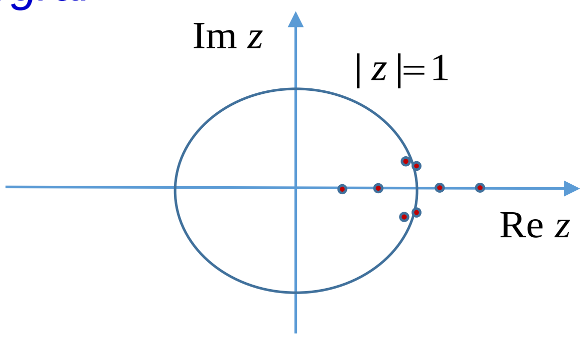
$$\epsilon(k) = -2t\left(\cos(k)-1\right)$$

$$\Psi_k^T = \left(c_{k\uparrow}\ ,\ c_{k\downarrow}\ ,\ c^\dagger_{-k\downarrow}\ ,\ -c^\dagger_{-k\uparrow}\right)$$

Infinite wire: Real space GF as contour integral

$$\hat{G}^0(k,\omega) = [\omega - \mathcal{H}_k]^{-1} \qquad z = e^{ik}$$

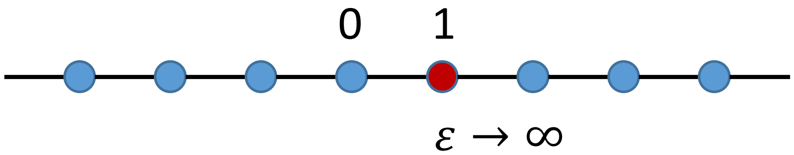
$$\hat{G}^0_{lm}(\omega) = \oint_{|z|=1} \frac{dz}{iz}\hat{G}^0(z,\omega) z^{(l-m)}$$



Dyson equation for chain breaking

$$\hat{g}_L = \hat{G}^0_{00} - \hat{G}^0_{01}\left(\hat{G}^0_{00}\right)^{-1}\hat{G}^0_{10}$$

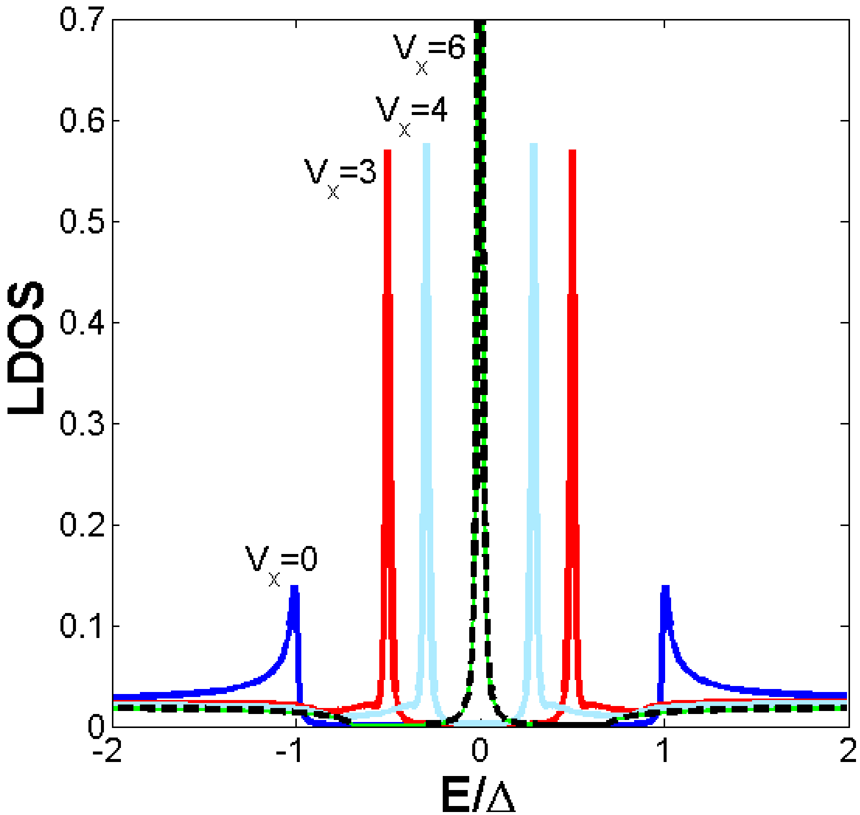
$$\hat{g}_R = \hat{G}^0_{00} - \hat{G}^0_{10}\left(\hat{G}^0_{00}\right)^{-1}\hat{G}^0_{01}$$



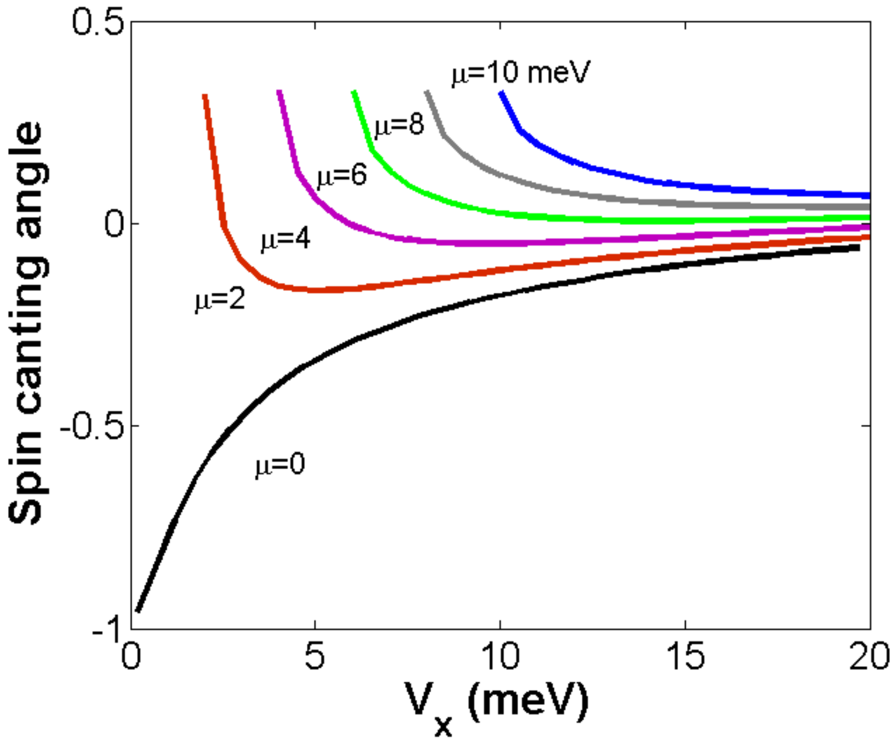
Boundary GF: LDOS and spin canting angle

$$\mu = 5meV$$

$$V_c = \sqrt{\mu^2 + \Delta^2} \approx 5\,\text{meV}$$



$$S_{\alpha,j}(\omega) = \frac{1}{2\pi i}\mathrm{Tr}\left[(1+\tau_z)\,\sigma_\alpha\left(G_j^A(\omega) - G_j^R(\omega)\right)\right]$$

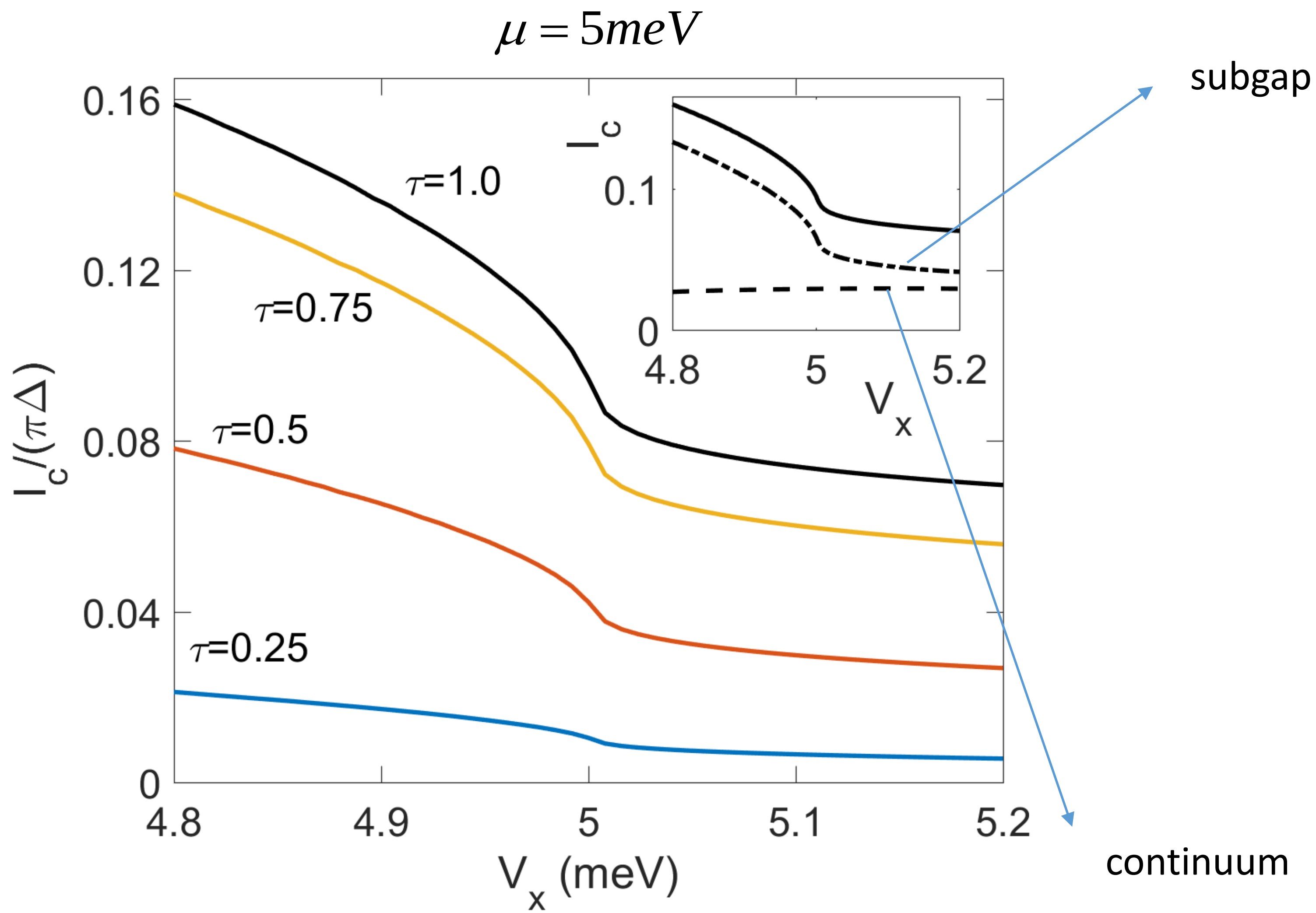


Parameters suitable for InAs/Al

$$t = 20\,\text{meV};\alpha = 4\,\text{meV};\Delta = 0.2\,\text{meV}$$

Zazunov, Egger, Alvarado & ALY, PRB (2017)

Josephson in S/TS (spinful model)

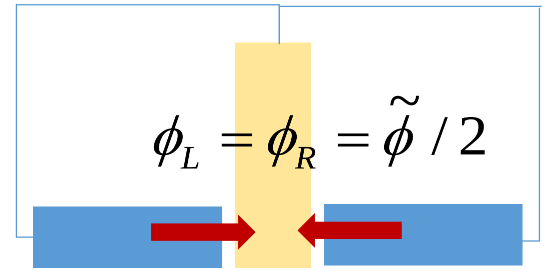


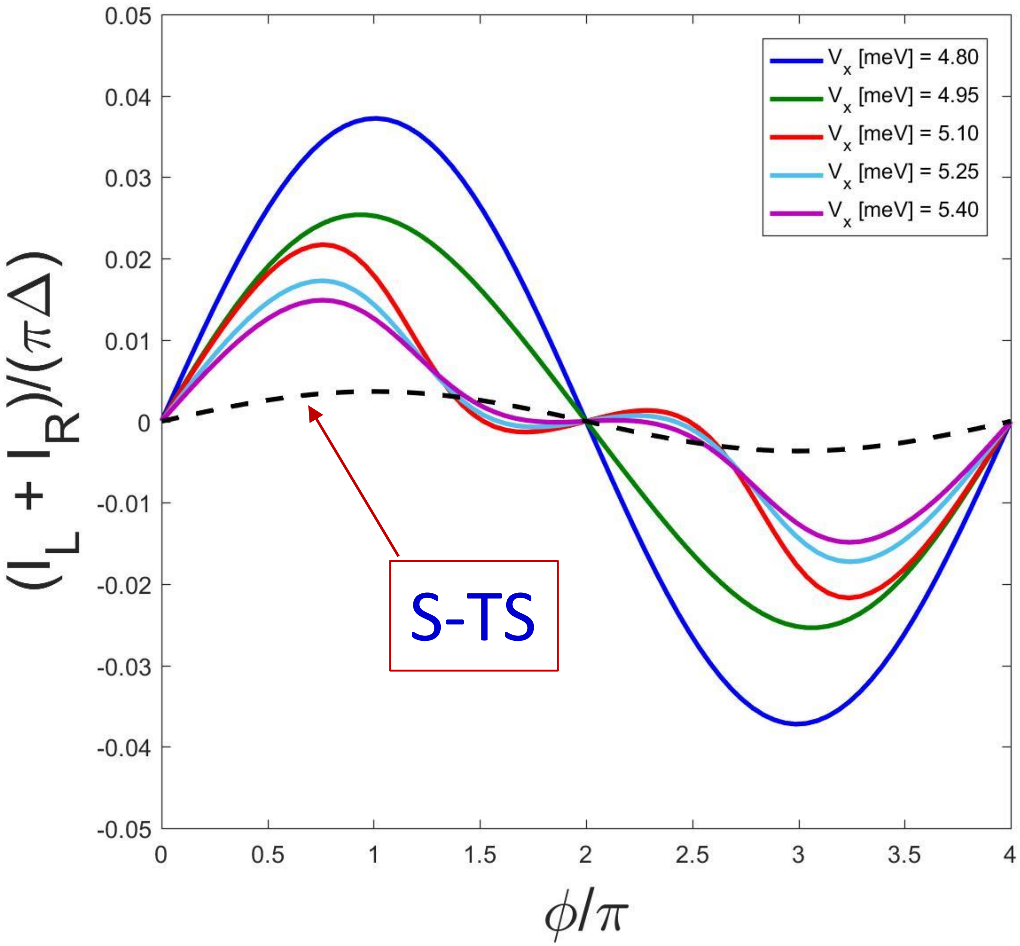
Zazunov et al. arXiv1801.10343

Multiterminal S-TS junctions: CPR across topo transition

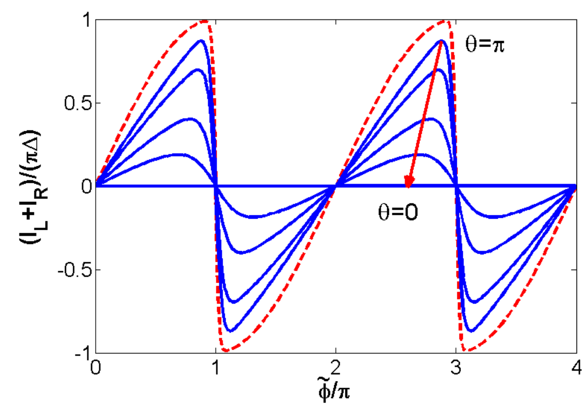
$\mu = 5meV$

"parallel" case





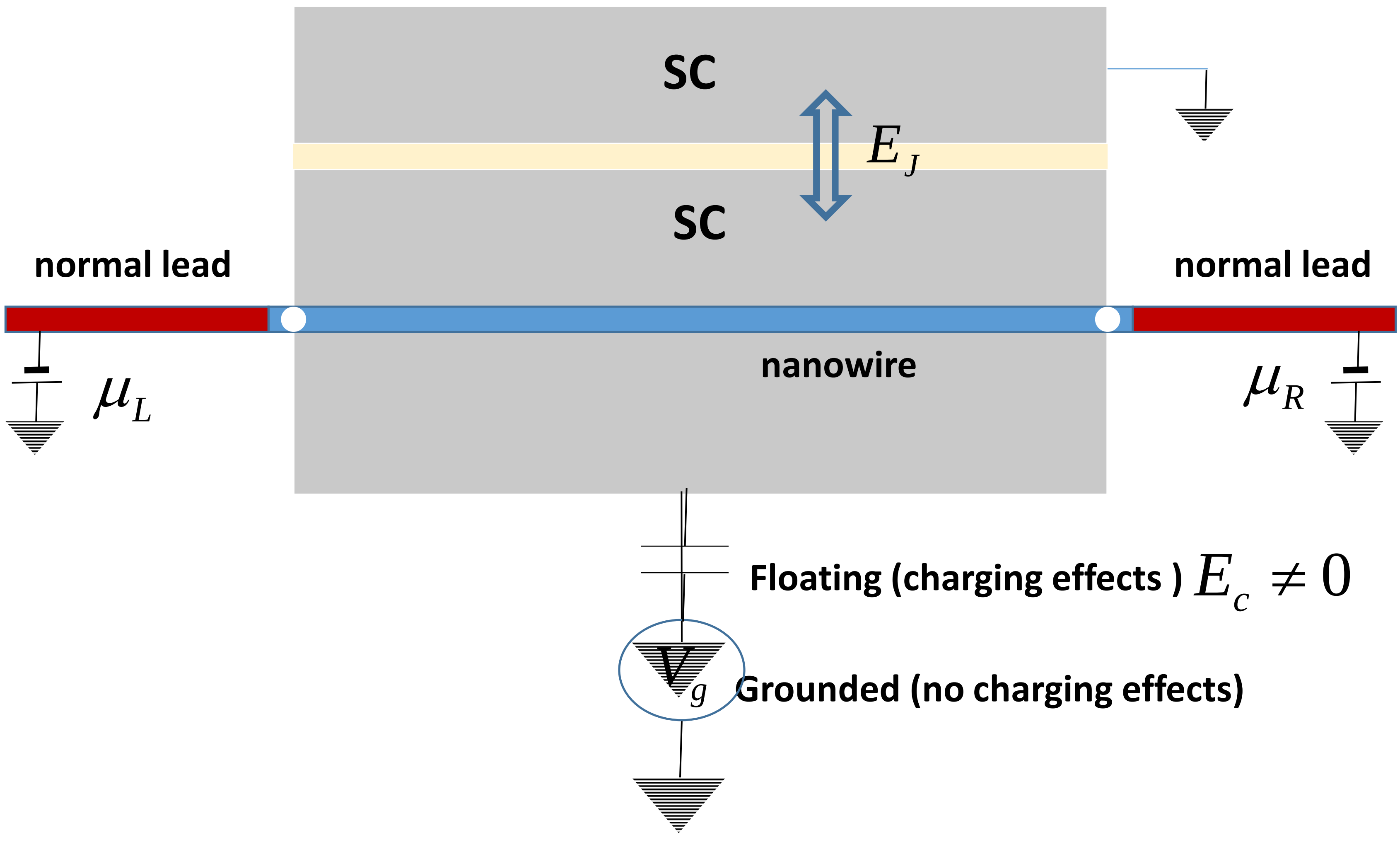
Kitaev limit



Zazunov, Egger, Alvarado & ALY, PRB (2017)

Charging and Interaction Effects

Transport through Majorana wires: effects of interactions?



The Majorana Single Charge Transistor model

Hützen et al. PRL (2012)

$$\hat{Q} = 2\hat{N} + \hat{n}_d$$

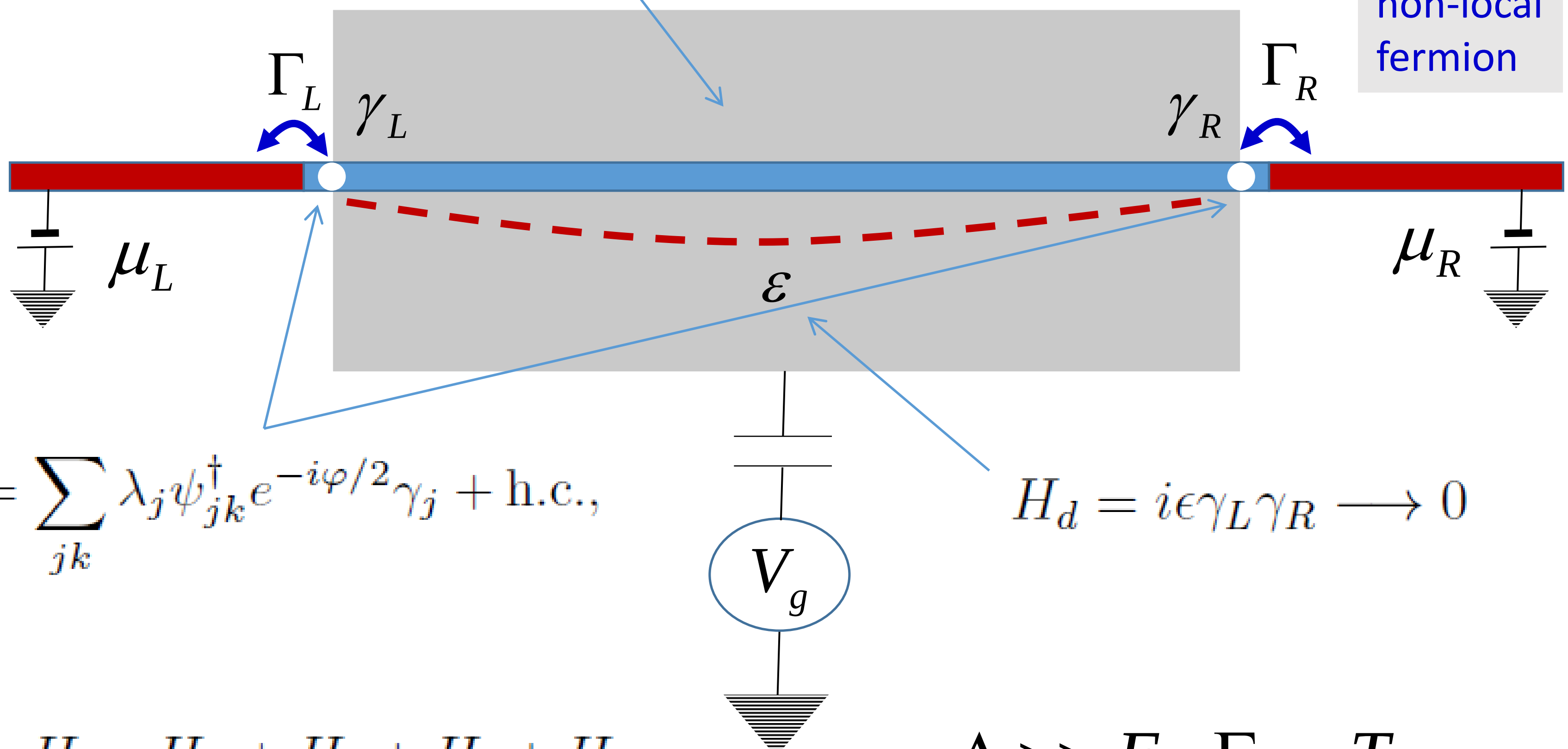
$\hat{N}$ Cooper pairs

$$H_c = E_c\left(\hat{Q} - n_g\right)^2$$

$$\hat{n}_d = d^\dagger d$$

$$d = \frac{1}{\sqrt{2}}(\gamma_L + i\gamma_R)$$

non-local fermion



$$H_t = \sum_{jk} \lambda_j \psi^\dagger_{jk} e^{-i\varphi/2} \gamma_j + \text{h.c.},$$

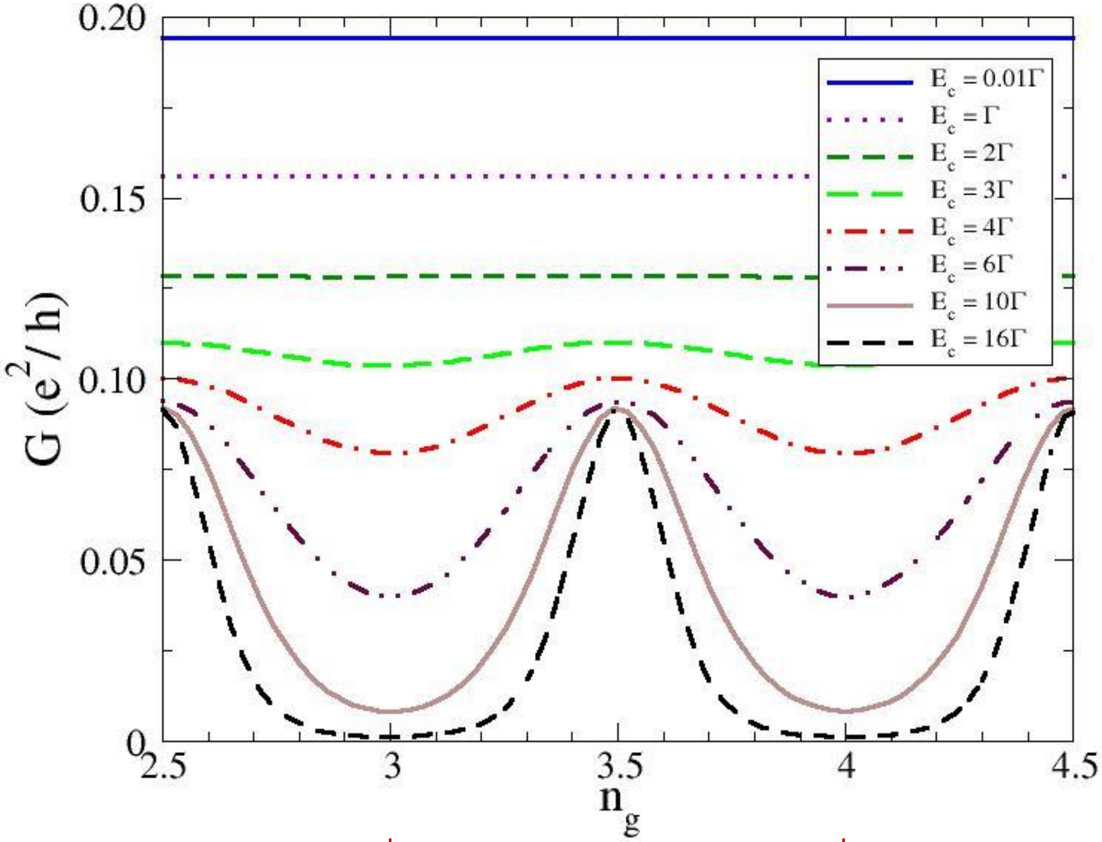
$$H_d = i\epsilon\gamma_L\gamma_R \longrightarrow 0$$

$$H = H_d + H_c + H_t + H_{leads}$$

$$\Delta >> E_c, \Gamma_{L,R}, T$$

Results from Master equation approach

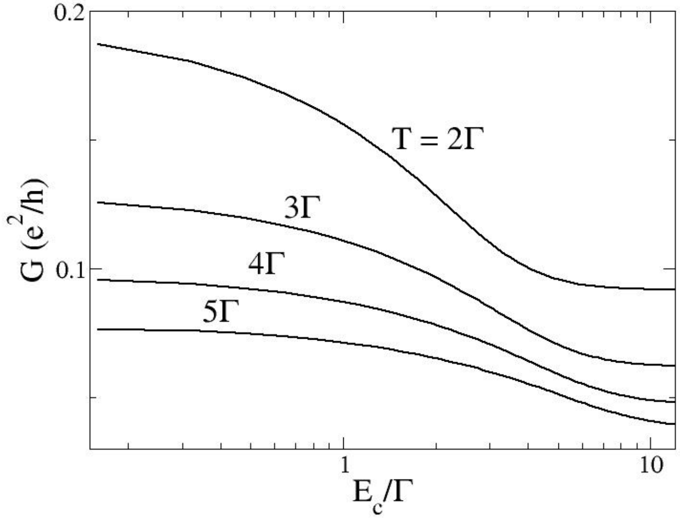
CB oscillations $T = 2\Gamma$



1e periodicity

$$G_{\text{valley}}(\delta) = \frac{e^2}{h}\frac{\Gamma_L\Gamma_R}{E_c^2}\frac{1}{(1-4\delta^2)^2}$$

Peak conductance



halving of peak conductance

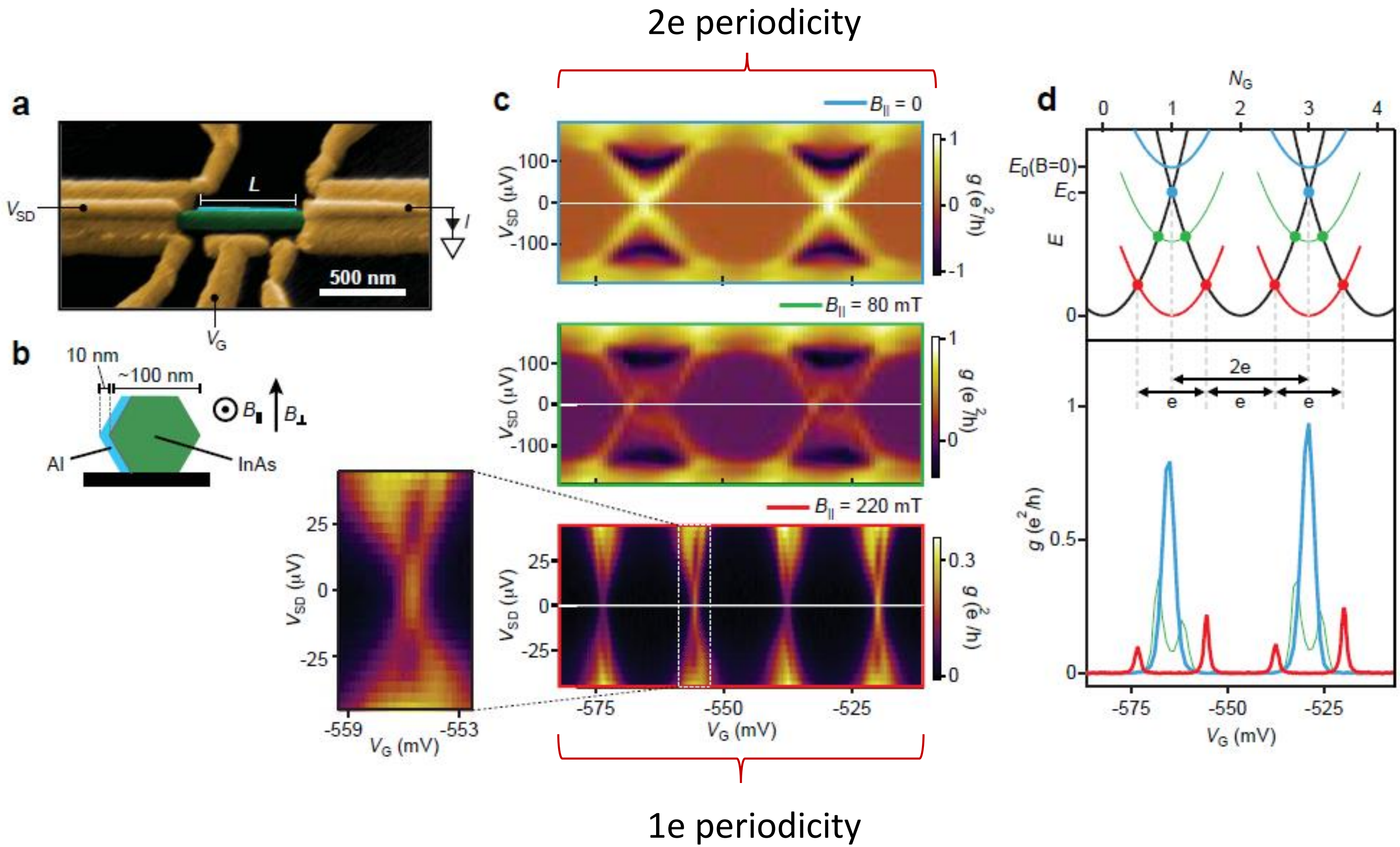
$$G_{\text{peak}}(\delta) = \frac{e^2}{h}\frac{\pi\Gamma}{16T}\frac{1}{\cosh^2(\delta E_c/T)}$$

Hützen et al. PRL (2012)

Experimental realization of a MSCT

InAs nanowires + epitaxial Al

Albretch et al. Nature (2016)

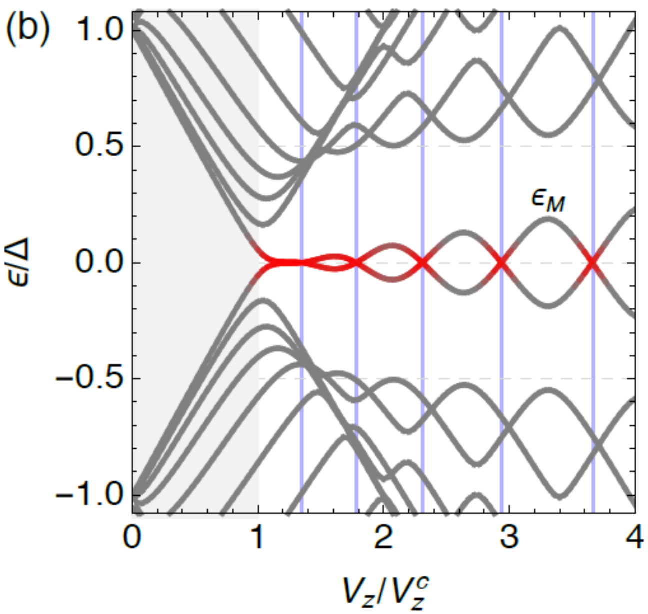


Zero-energy pinning from interactions

Dominguez, Cayao, San-José, Aguado, ALY & Prada, NPJ QM (2017)

Apparent absence of MBS hibridization in finite wires

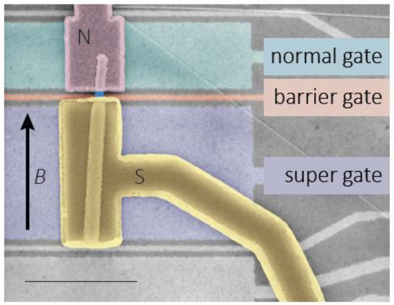
Theory (non-interacting)

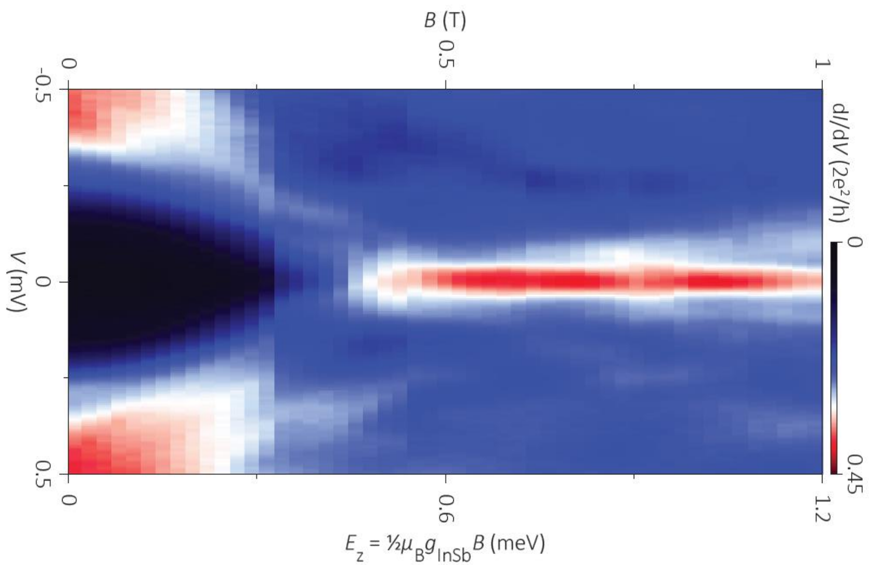


$$L = 1\mu\text{m}; \alpha = 20\,\text{meVnm};$$

$$\Delta = 0.5\,\text{meV}; m^* = 0.015 m_e$$

Exp: InSb/NbTiN





Zhang et al., Nature Nano (2018)

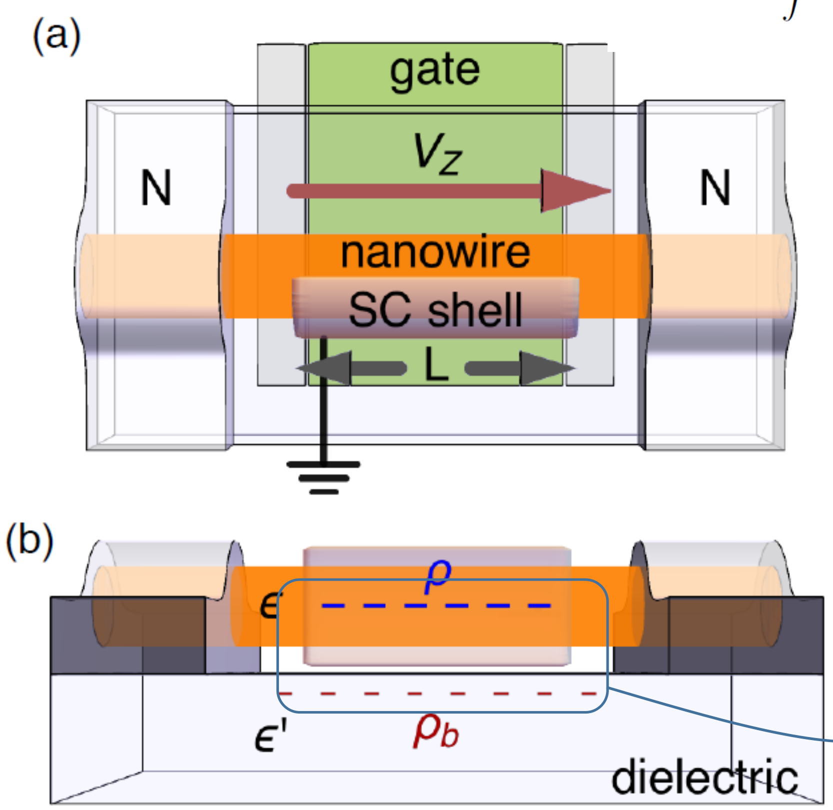
Zero-energy pinning from interactions

Dominguez, Cayao, San-José, Aguado, ALY & Prada, NPJ QM (2017)

electrostatic potential

$$H_{wire} = \int dx \Psi^\dagger(x) \left[\left(-\frac{\hbar^2}{2m^*}\frac{\partial^2}{\partial x^2} - \mu + \phi(x) \right) \tau_z + V_Z \sigma_x + i\sigma_z \tau_z \alpha \frac{\partial}{\partial x} + \Delta \tau_x \right] \Psi(x)$$

$$\Psi^T(x) = \left(\psi_\uparrow(x),\ \psi_\downarrow(x),\ \psi_\downarrow^\dagger(x),\ -\psi_\uparrow^\dagger(x) \right)$$



Poisson equation

$$-\nabla \cdot [\epsilon(r) \nabla \phi(r)] = 4\pi \rho(r)$$

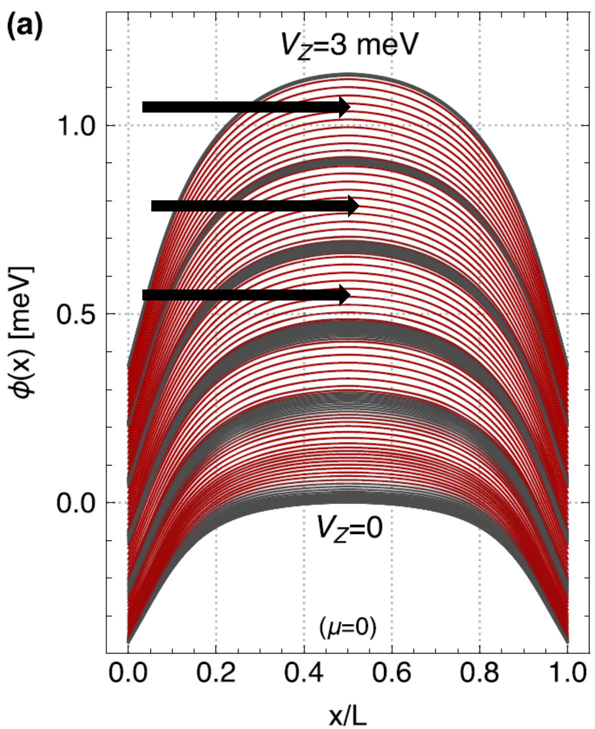
repulsive for $\varepsilon > \varepsilon'$!

$$\varepsilon(\text{nanowire}) \approx 18$$

$$\varepsilon(\text{substrate}) \approx 4$$

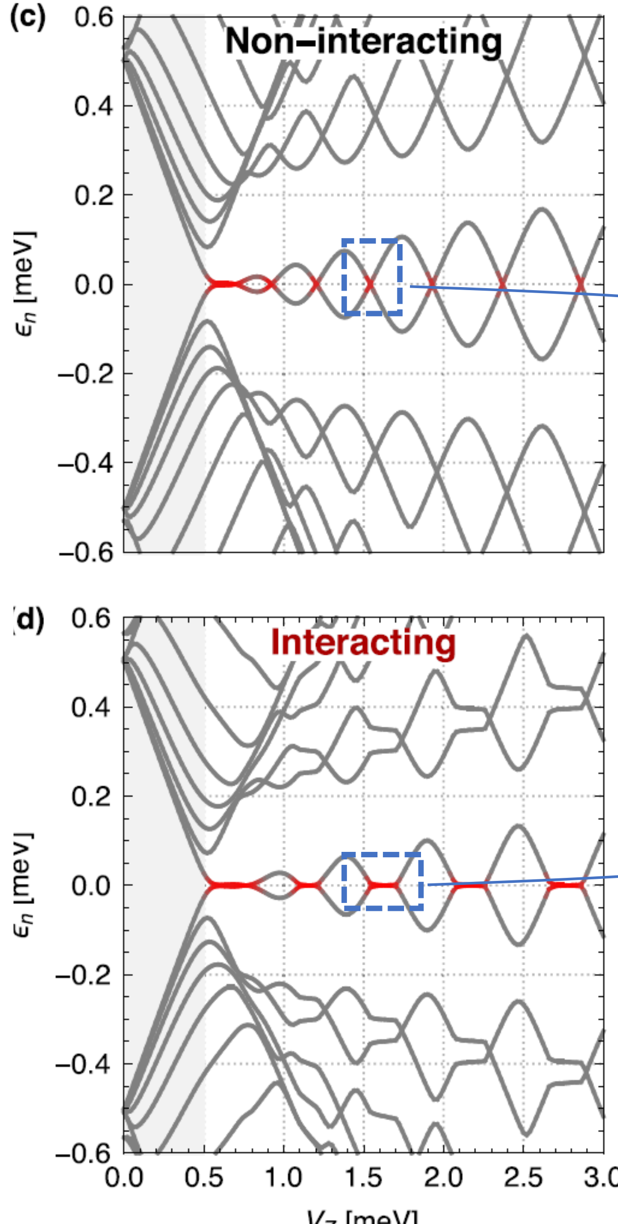
Zero-energy pinning from interactions

Self-consistent potential



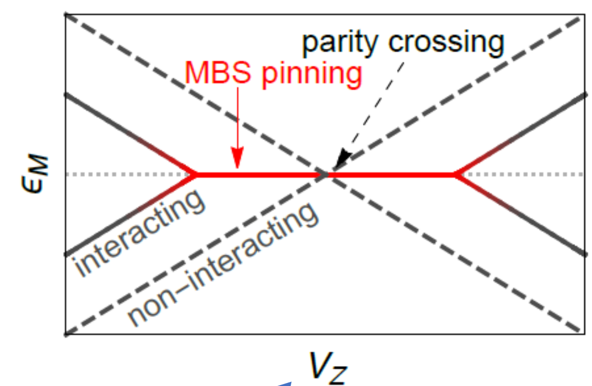
R=50 nm; L=1 µm

parameters for
a InSb/Nb wire



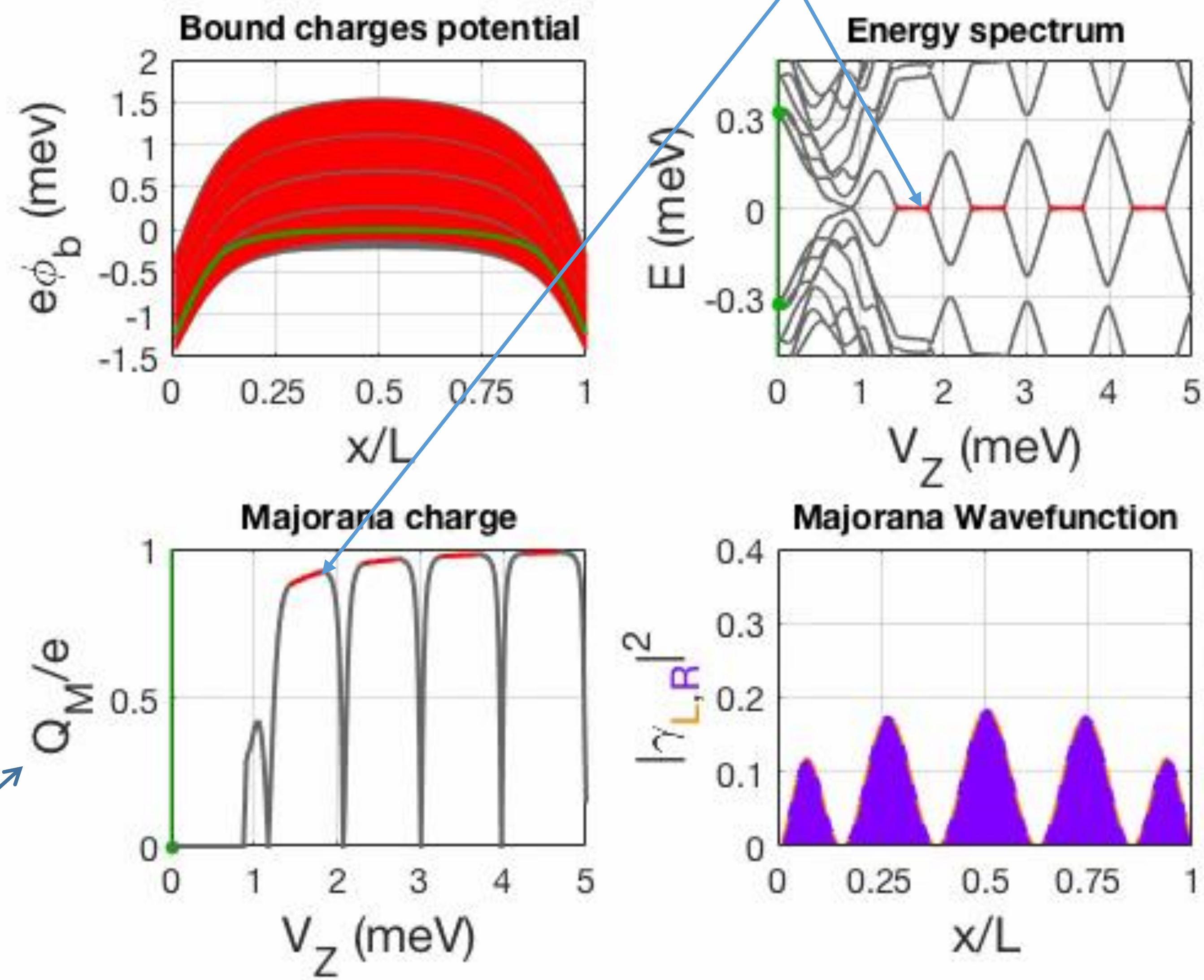
Energy spectrum

Dominguez et al., NPJ QM (2017)



$$Q_M \simeq e\frac{\partial E_{hyb}}{\partial \mu}$$

Knapp et al, PRB (2018)



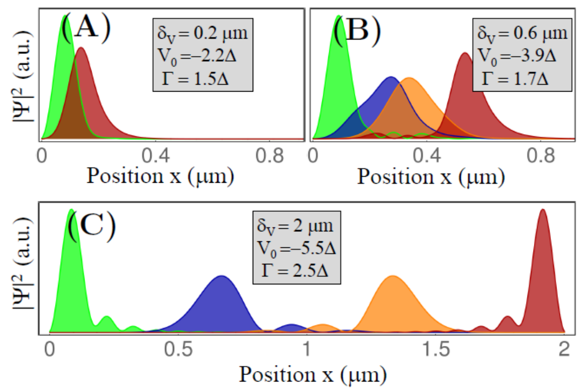
$$Q_M = |Q_{+1} - Q_{-1}| = \left| e\int dx u_L(x) u_R(x) \right|$$

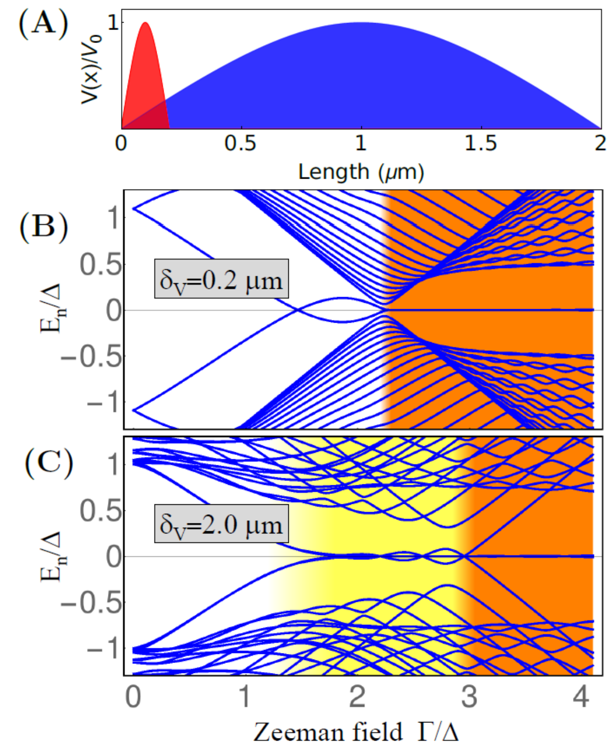
$$\gamma_L = \gamma_{+1} + \gamma_{-1}$$
$$\gamma_R = -i\left(\gamma_{+1} - \gamma_{-1}\right)$$

Related work

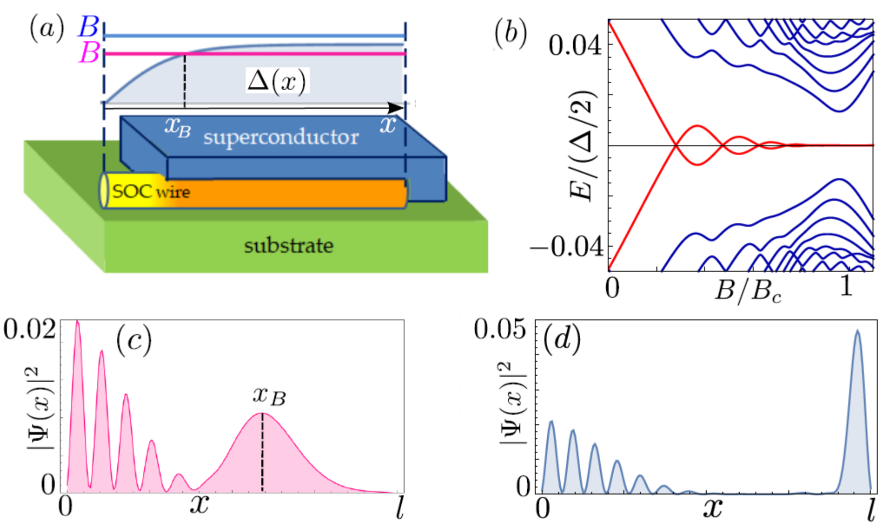
Effect of non-uniform electrostatic potential

C. Moore, T. Stanescu, S. Tewari
arXiv:1711.06256





Effect of non-uniform pairing potential



Fleckenstein, Dominguez, Traverso Ziani, Trauzettel
arXiv:1710.08866

Conclusions (third lecture):

Extension of Hamiltonian Approach to TS case: boundary GFs for Kitaev and spinful models

Analytical results for NTS, TSTS, STS, etc

Lifting of supercurrent blockade in STS (multiterminal and S-QD-TS)

Charging effects: the Majorana single charge transistor

Interaction effects: MBS zero energy pinning and quantum dot formation

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Thank you!