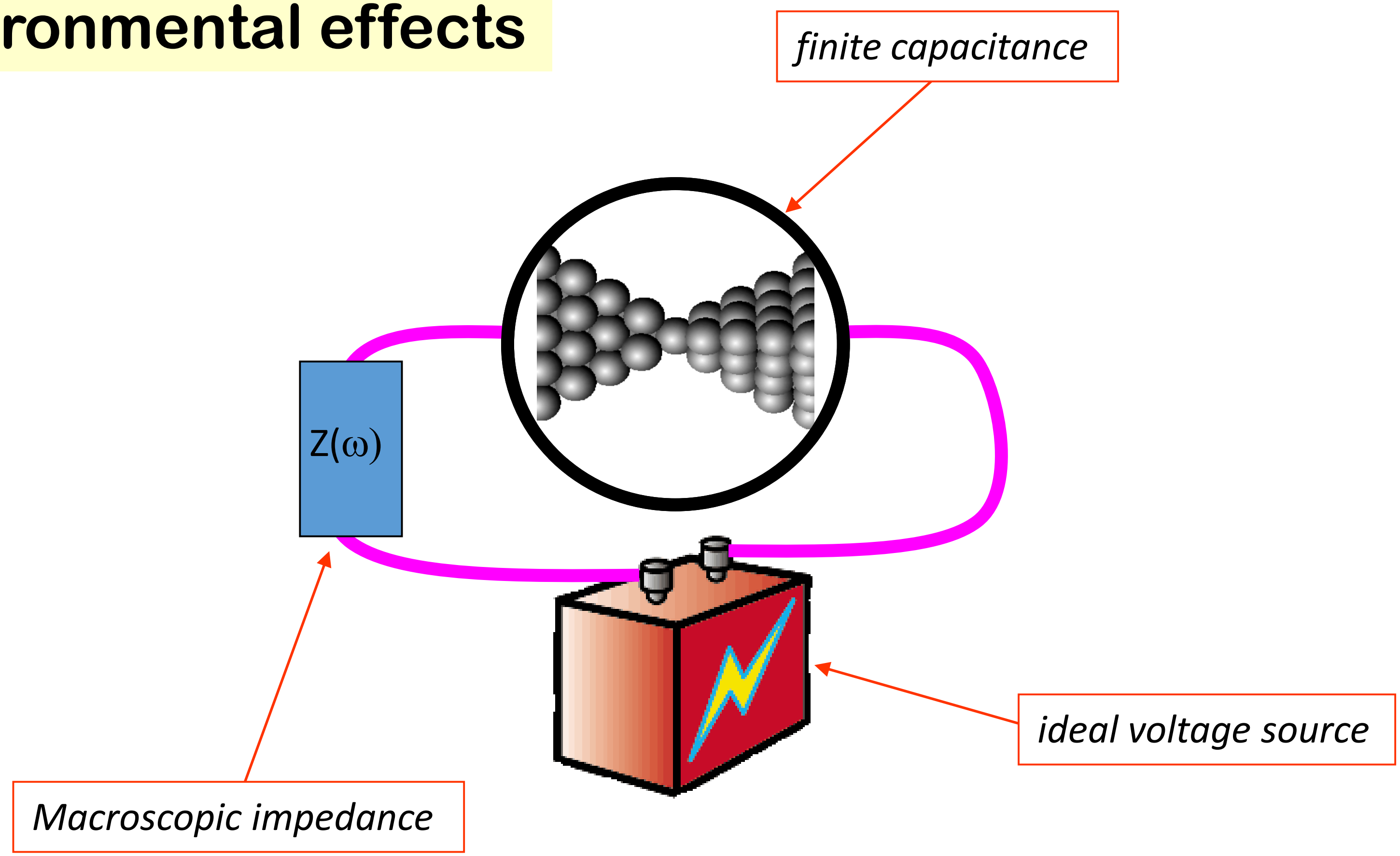
Outline:

First Lecture: general introduction, application to conventional SC junctions (non-int, steady-state)

Second Lecture: effect of interactions (DCB), quench dynamics, time-dependent FCS

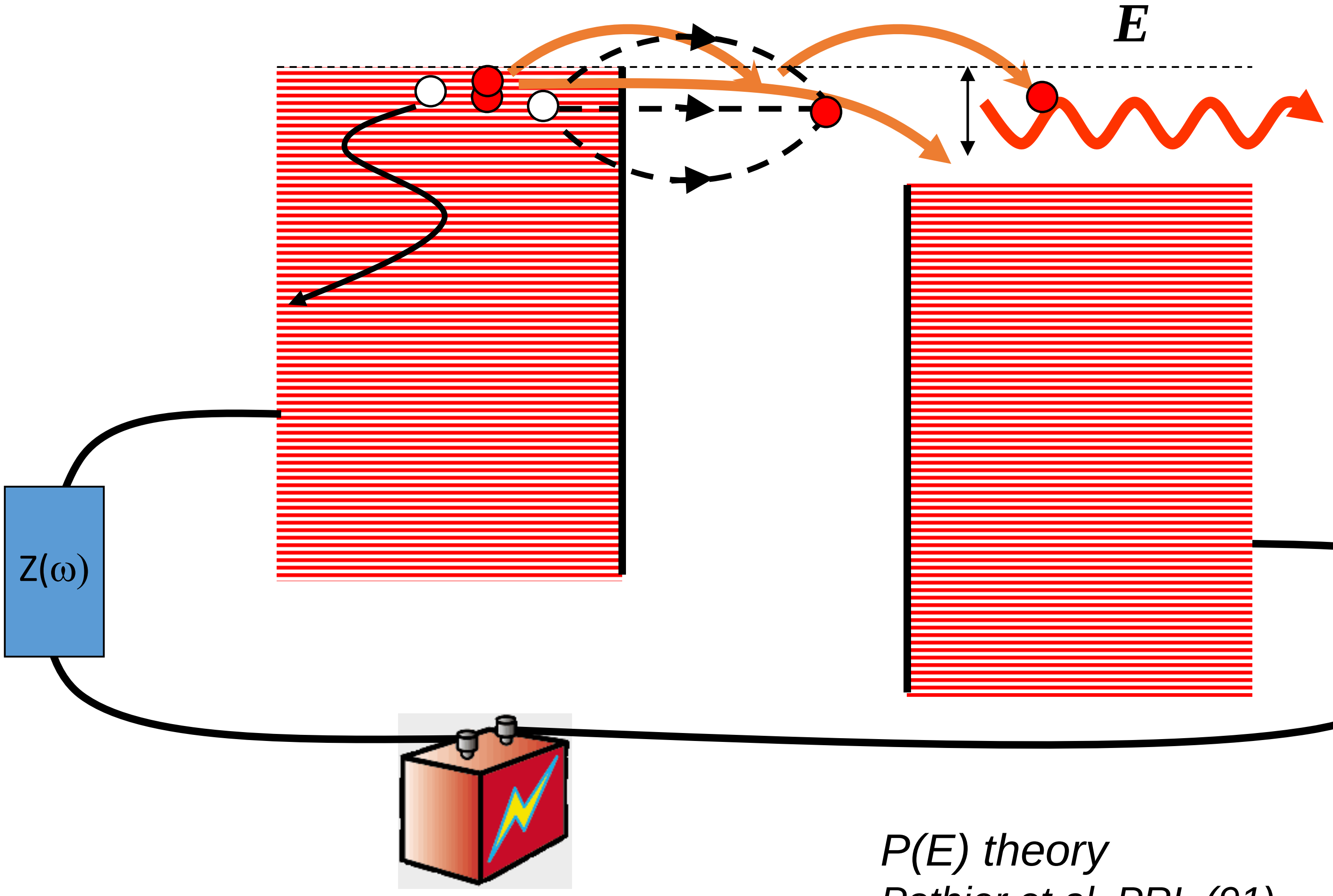
Third Lecture: topological superconductors featuring MBS, two terminal, multiterminal, Interaction effects

Environmental effects



- *Classical effects* → *Phase diffusion*
- *Quantum effects* → *Dynamical CB*

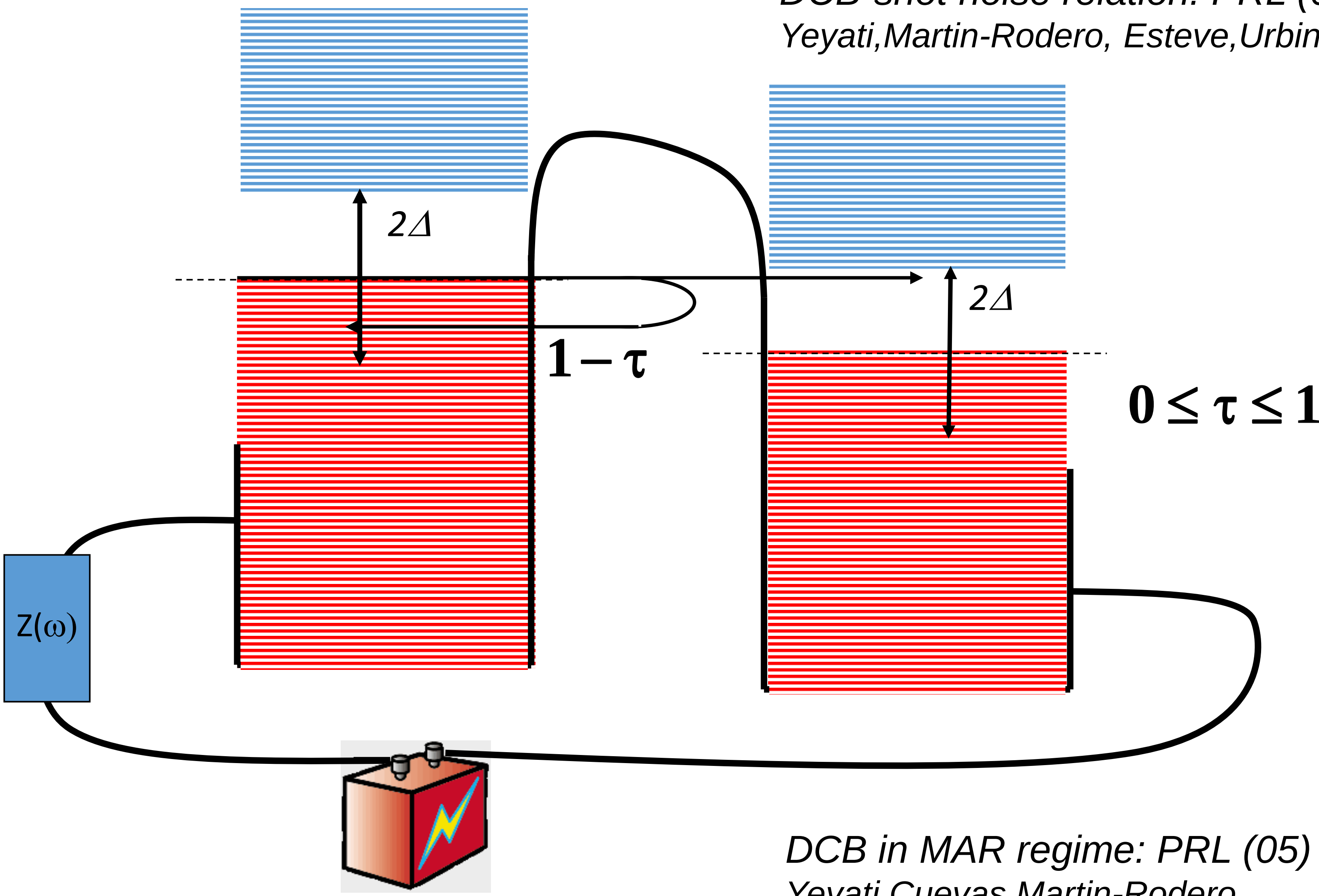
Dynamical Coulomb blockade in tunnel junctions



P(E) theory
Pothier et al. PRL (91)

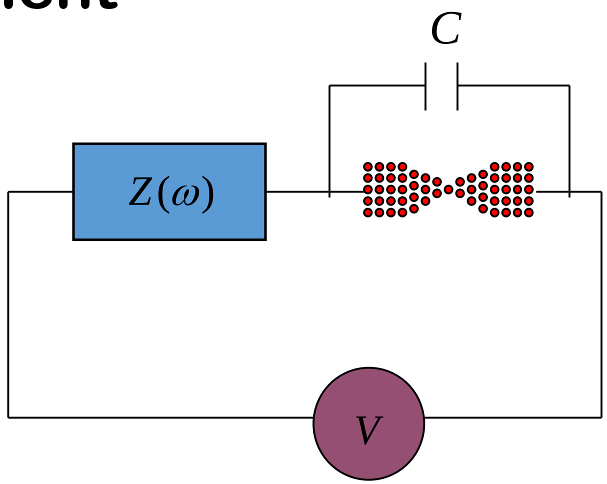
Aim: understanding DCB in more general situations

DCB-shot noise relation: PRL (01)
Yeyati,Martin-Rodero, Esteve,Urbina



DCB in MAR regime: PRL (05)
Yeyati,Cuevas,Martin-Rodero

Coupling to the environment



$$H_{contact} = H_L + H_R + \sum_\sigma T_0 \left(e^{i\hat{\phi}} c^\dagger_{L\sigma} c_{R\sigma} + \text{h.c.} \right)$$

$$\left[\hat{\phi}, \hat{Q} \right] = ie \Longrightarrow e^{i\hat{\phi}} \hat{Q} e^{-i\hat{\phi}} = \hat{Q} - e$$

phase-phase correlation function

$$J(t) = \langle \hat{\phi}(t)\hat{\phi}(0) \rangle - \langle \hat{\phi}^2 \rangle = 2 \int \frac{d\omega}{\omega} \frac{Z^{eff}(\omega)}{R_K} \frac{e^{-i\omega t} - 1}{1 - e^{-\beta\hbar\omega}}$$

$$Z^{eff}(\omega) = \frac{1}{Z^{-1}(\omega) + i\omega C}$$

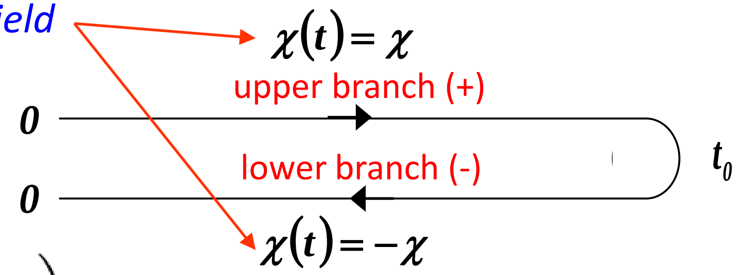
contact capacitance

Ingold & Nazarov '92
in NATO ASI "Single Charge Tunneling"

Blockade and Full Counting Statistics

counting field

$$\mathcal{Z}(\chi) = e^{-S(\chi)} = \langle e^{-\frac{i}{\hbar}\int_{\mathcal{C}} \hat{H}_\chi(t)dt} \rangle$$



$$\hat{H}_{T,\chi} = \sum_\sigma T_0 \left(e^{i(\phi+\chi)/2} c^\dagger_{L\sigma} c_{R\sigma} + \text{h.c.} \right)$$

$S(\chi)$ → *cumulant generating function* → $C_n = -(-i)^n \left[\frac{\partial^n S}{\partial \chi^n} \right]_{\chi=0}$ →

$$C_1 = -\frac{t_0}{e}\langle \hat{I} \rangle$$

$$C_2 = \frac{t_0}{2e^2} S_I(0)$$

Weak blockade $z = Z_{eff}/R_K \ll 1$

$$\delta S(\chi) = \frac{1}{e^2}\int_0^{t_0} dt_1 dt_2 \sum_{\alpha,\beta=\pm} J^{\alpha\beta}(t_1,t_2) K^{\alpha\beta}(t_1,t_2,\chi)$$

$$J^{\alpha\beta}(t,t') = \langle T_{\mathcal{C}} \left[\phi(t_\alpha)\phi(t'_\beta) \right] \rangle - \langle \phi^2 \rangle \qquad K^{\alpha\beta}(t,t',\chi) = \langle T_{\mathcal{C}} \hat{I}(t_\alpha) \hat{I}(t'_\beta) e^{-\frac{i}{\hbar}\int_{\mathcal{C}} \hat{H}_\chi(t)dt} \rangle$$

$$\delta I = \frac{ie}{t_0}\frac{\partial \delta S}{\partial \chi}$$ → *current blockade*

Current blockade in a normal single channel contact

(energy independent transmission)

ALY et al. PRL (01)

$$\delta I(V) = \frac{e}{h}\tau(1-\tau)\int d\omega \left[n_R(n_L^- - n_L^+) - n_L(n_R^- - n_R^+)\right]$$
$$+\frac{e}{h}\tau^2\int d\omega \left[n_L(n_L^- - n_L^+) - n_R(n_R^- - n_R^+)\right]$$

$$n_{L,R}^{\pm}(\omega) = \int d\omega' \, J(\omega') \, n_{L,R}(\omega \pm \omega')$$

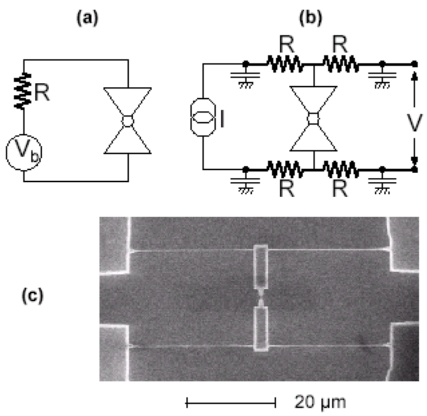
$$S_I(V,\Omega) = \frac{2e}{h}\tau(1-\tau)\sum_{\pm}\int d\omega\left[n_R(\omega)(1-n_L(\omega\pm\Omega)) + n_L(\omega)(1-n_R(\omega\pm\Omega))\right]$$
$$+\frac{2e}{h}\tau^2\sum_{\pm}\int d\omega\left[n_L(\omega)(1-n_L(\omega\pm\Omega)) + n_R(\omega)(1-n_R(\omega\pm\Omega))\right]$$

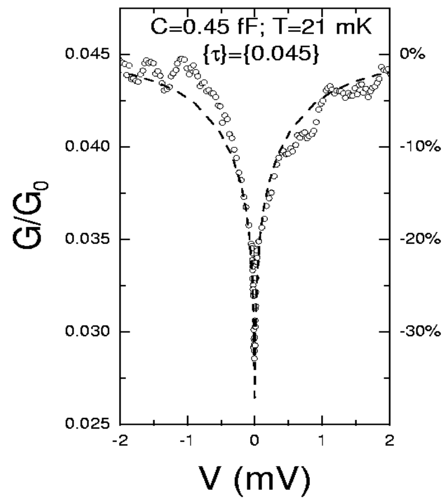
Noise spectrum

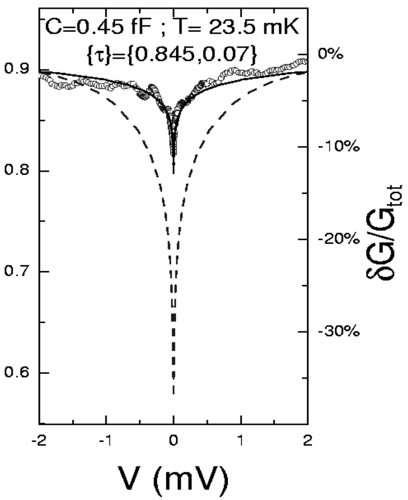
$$\delta I(V) = -\frac{1}{2e^2}\int_{-\infty}^{\infty} d\omega \, J(\omega)\int_0^{\omega} d\omega' \frac{\partial S_I(V,\omega')}{\partial V}$$

Suppression as shot noise for $\tau \to 1$

Experimental test: ASC in resistive envinvironment







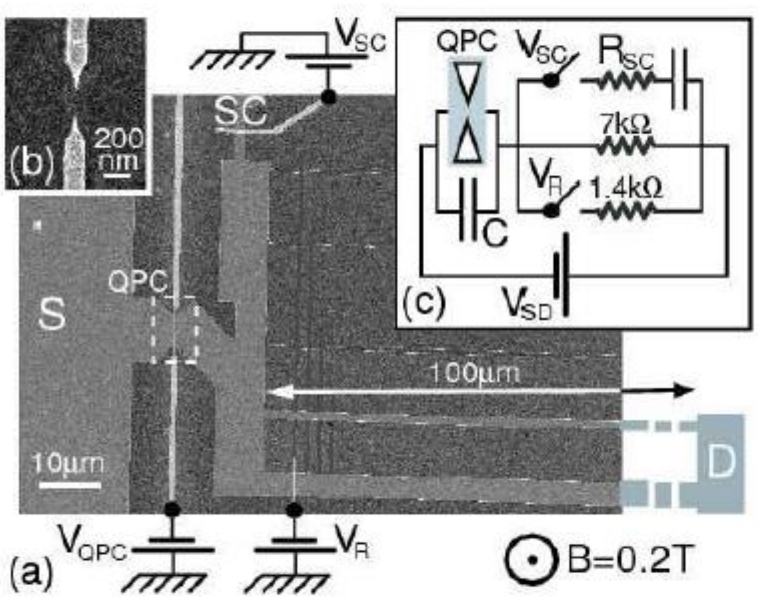
tunnel regime

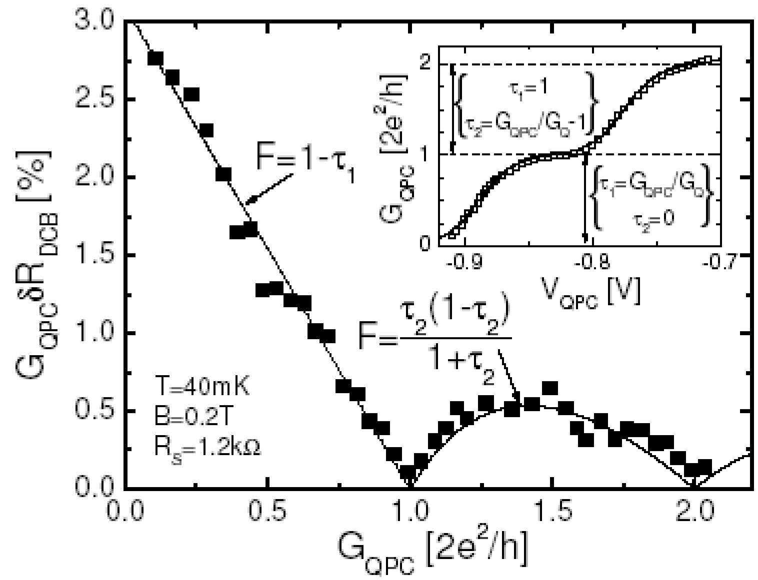
well transmitted channel

R. Cron, E. Vecino, M.H. Devoret, D. Esteve, P. Joyez, A. Levy Yeyati, A. Martín-Rodero & C. Urbina, "Electronic Correlations: from meso to nano Physics, EDPSciences (2001)

Test in QPCs

Experimental Test of the Dynamical Coulomb Blockade Theory for Short Coherent Conductors



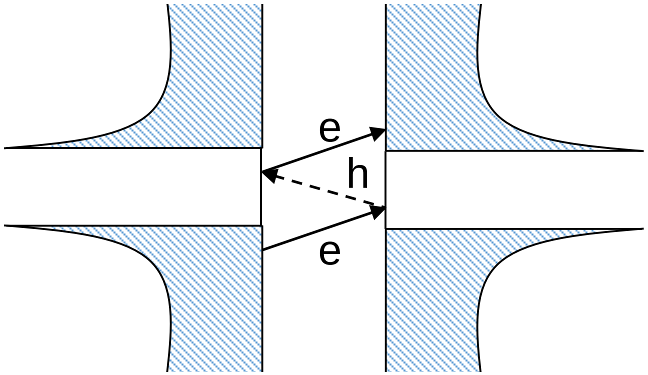


C. Altimiras, U. Gennser, A. Cavanna, D. Mailly, and F. Pierre*

PRL 99, 256805 (2007)

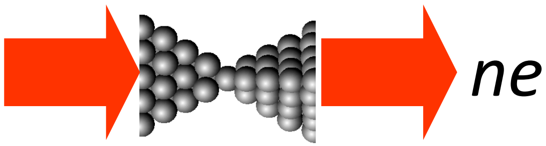
Extension to superconducting contacts: DCB in the MAR regime?

coherent MAR



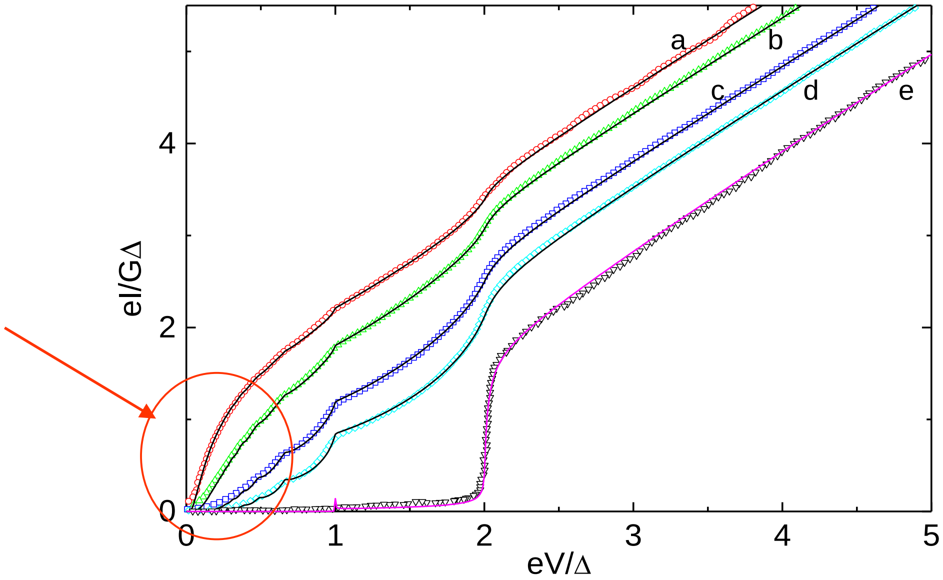
Divergent effective charge

$$\mathbf{ne} \approx 2\Delta / \mathbf{V} \rightarrow \infty \quad \textit{for} \quad \mathbf{V} \rightarrow 0$$



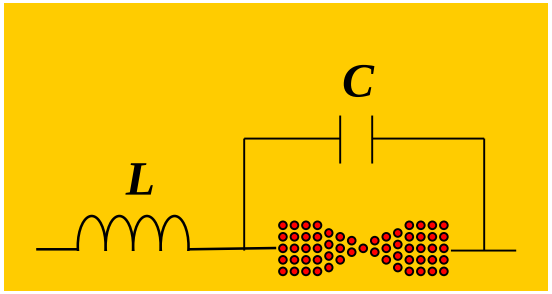
Why are charging effects negligible?

Signatures of DCB in MAR regime?

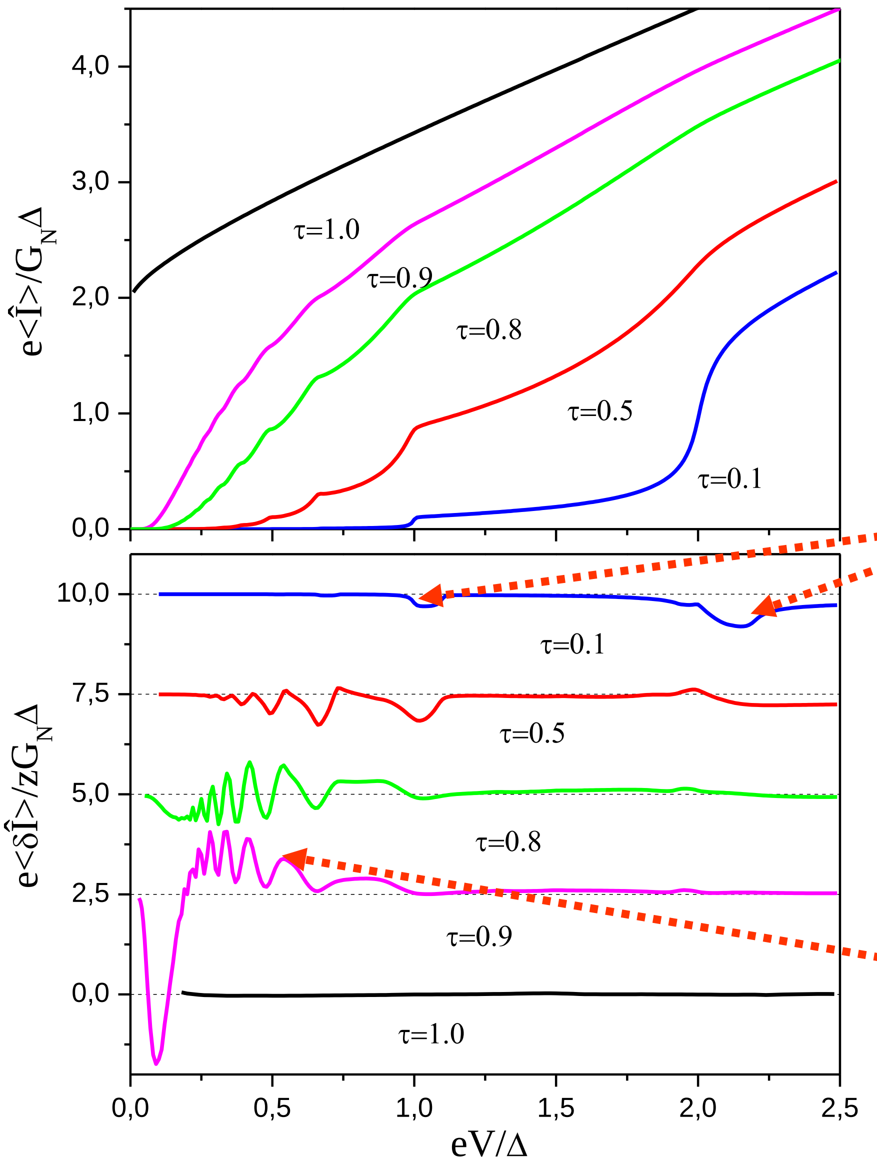


Blockade for single mode environment

ALY et al. PRL (05)



$$Z^{eff}(\omega) = \frac{i\omega L}{1+\omega^2 LC} \quad \omega_0 = \frac{1}{\sqrt{LC}} \quad Z_0 = \sqrt{\frac{L}{C}}$$



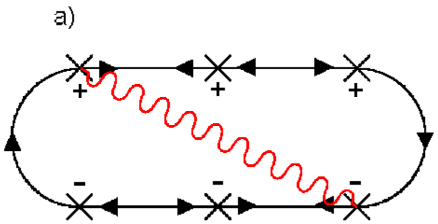
$\omega_0 = 0.2\Delta$

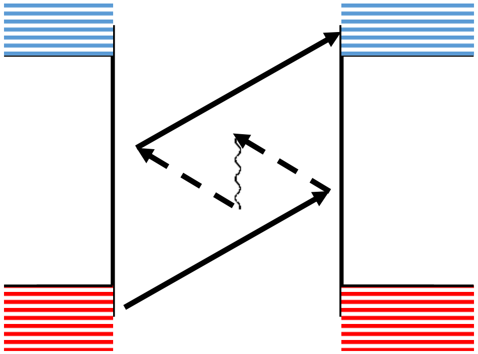
- *blockade around* $eV \approx 2\Delta / n$ *for* $\tau \to 0$
- *blockade shifts towards* $eV \to 0$ *for increasing* τ
- *"anti-blockade"*

Tunnel limit

$$n = 3$$

$$eV \approx 2\Delta/n$$





$$\delta S^{(a)} \approx -\frac{ze\pi}{h}\left(\frac{T_0}{4}\right)^{2n} e^{-in\chi}\left\{\int_{\Delta-neV}^{-\Delta} d\omega\, \mathrm{Im}g(\omega)\mathrm{Im}g(\omega+neV)K_n(\omega,\omega) - \int_{\Delta-neV}^{-\Delta-\omega_0} d\omega\, \mathrm{Im}g(\omega+\omega_0)\mathrm{Im}g(\omega+neV)K_n(\omega+\omega_0,\omega)\right\}$$

$$K_n(\omega_1,\omega_2) = \left|\sum_{j=1}^{n}\prod_{k=1}^{n-j} f(\omega_1+keV)\prod_{l=n-j-1}^{n-1} f(\omega_2+leV)\right|^2$$

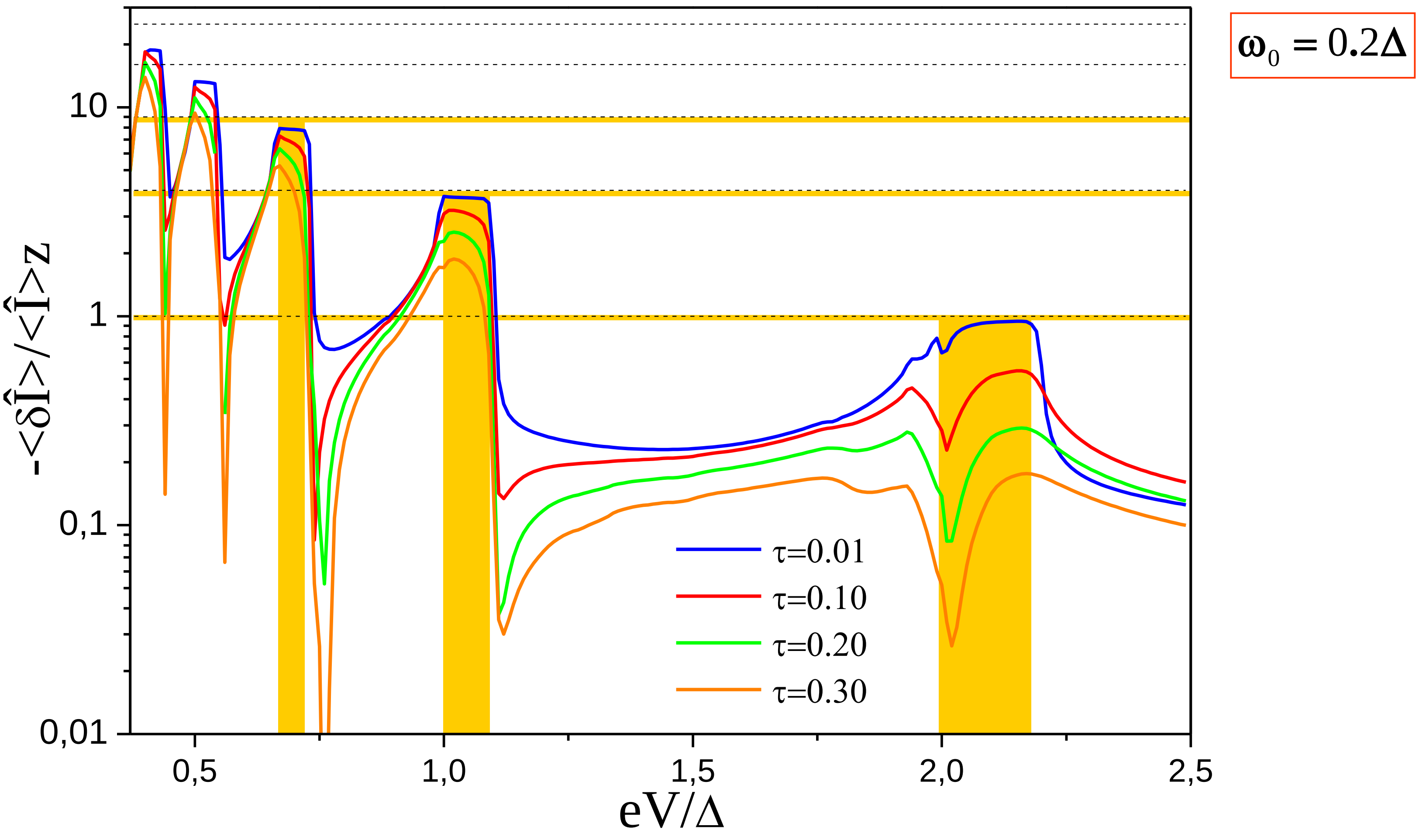
$$<\hat{I}> \approx \frac{ne}{h}\left(\frac{T_0}{4}\right)^{2n}\int_{\Delta-neV}^{-\Delta} d\omega\, \mathrm{Im}g(\omega)\mathrm{Im}g(\omega+neV)\left|\prod_{k=1}^{n-1} f(\omega_1+keV)\right|^2$$

$$\frac{2\Delta}{n} < eV \le \frac{2\Delta+\omega_0}{n}$$

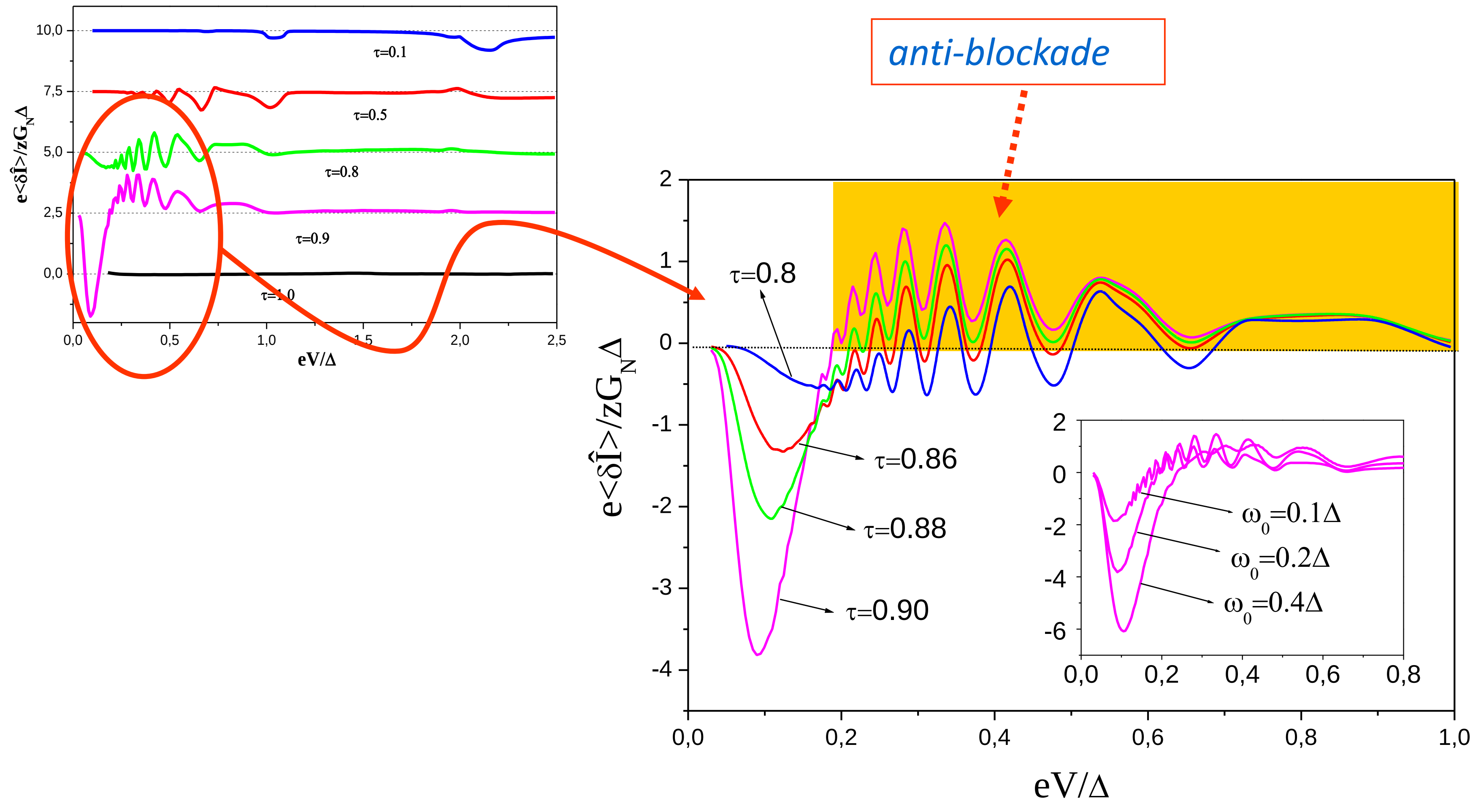
$$\max\left\{-\delta I/z <\hat{I}>\right\} \approx n^2$$

→ MAR processes as shots of charge *ne*

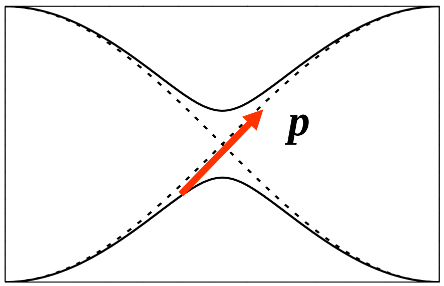
stronger blockade for $\frac{2\Delta}{n} < eV \leq \frac{2\Delta + \omega_0}{n}$



Evolution towards ballistic limit



Antiblockade and Landau-Zener transitions



$$p = e^{-\frac{\pi \Delta r}{eV}}$$

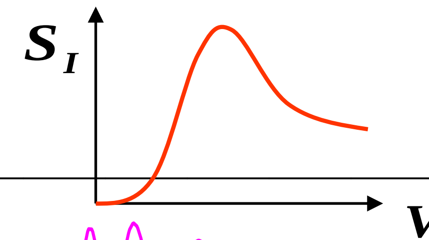
$$<\hat{I}> = \frac{4e\Delta}{h}\, p$$

effective charge

partition

$$S_I \propto \frac{2\Delta}{V}\, p(1-p)$$

$$\tau \to 1$$

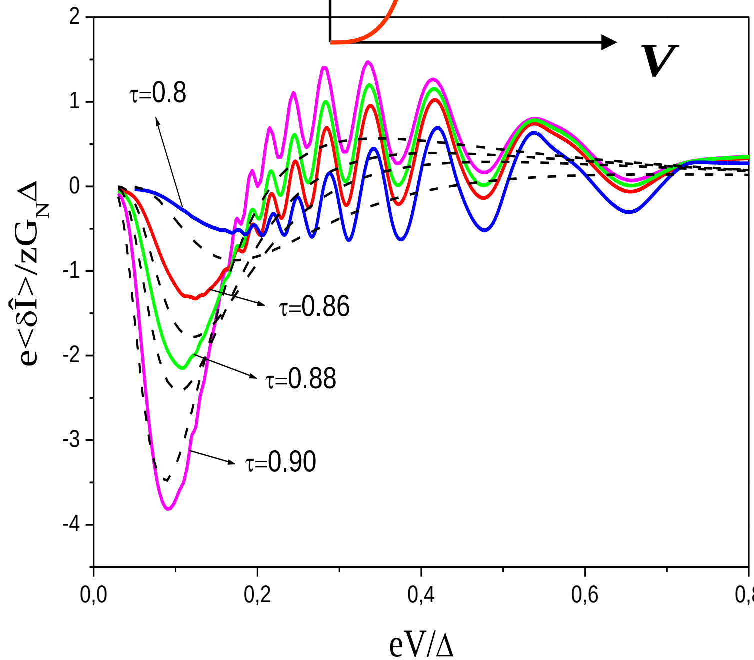


$$\delta I(V) \approx -\frac{1}{2e^2}\int_{-\infty}^{\infty} d\omega\, J(\omega) \int_0^{\omega} d\omega' \frac{\partial S_I(V,\omega')}{\partial V}$$

$$\omega_0 \to 0 \quad \Rightarrow \quad \delta I \propto -\omega_0 \frac{\partial S_I}{\partial V}$$

$$\delta I \propto \omega_0 \frac{2\Delta}{V^2}\, p\left[(1-p) - \frac{\pi \Delta r}{eV}(1-2p)\right]$$

positive *negative*

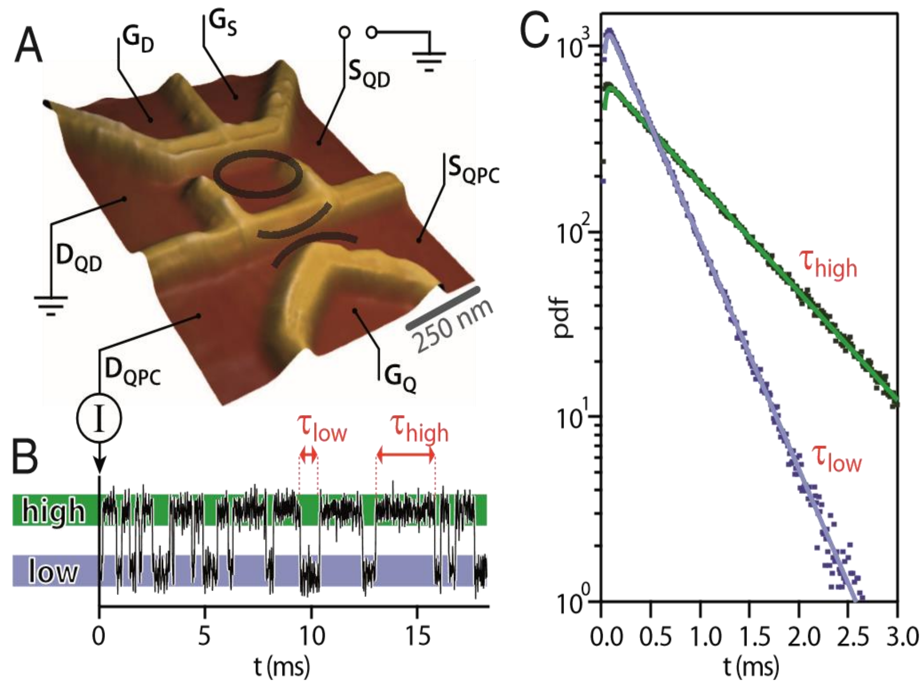


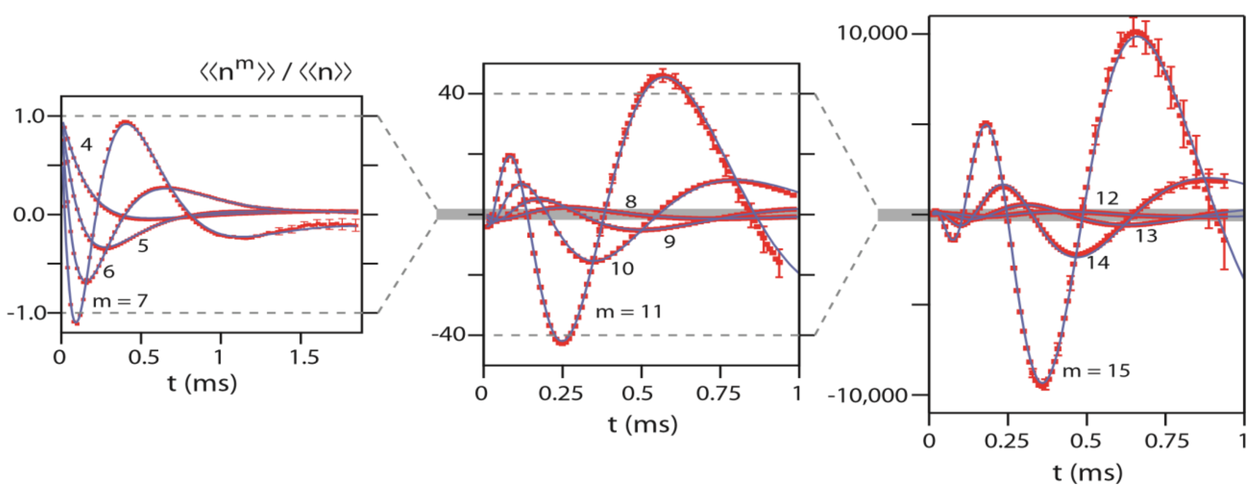
Transient Dynamics and Time-dependent FCS

Time resolved measurements: QD sequential regime

Gustavsson et al., PRL (2006)

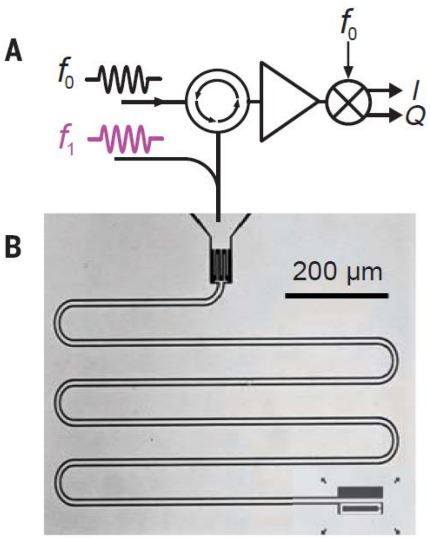
Flindt et al., PNAS (2009)

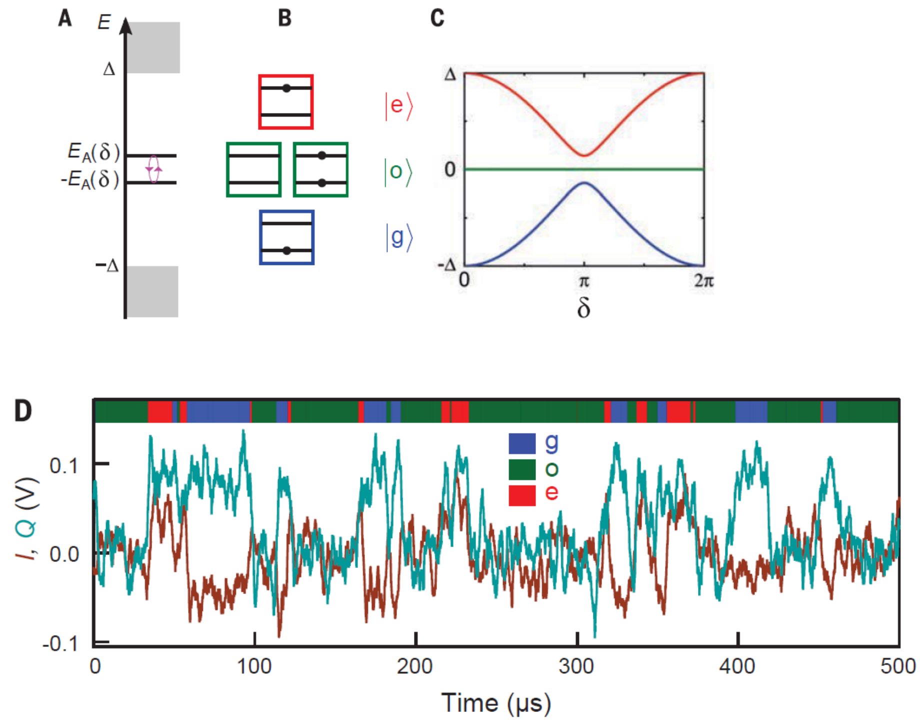


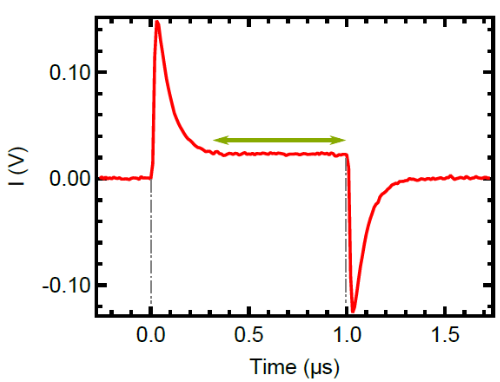


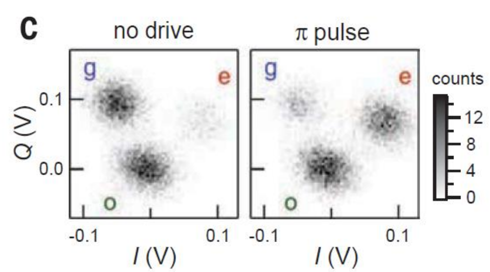
Time resolved measurements: superconducting atomic contacts

Janvier et al., Science (2015)

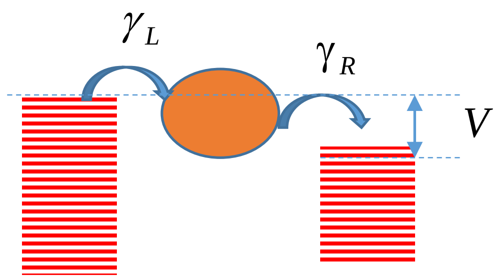


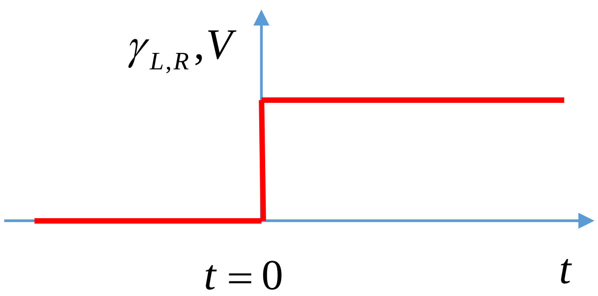


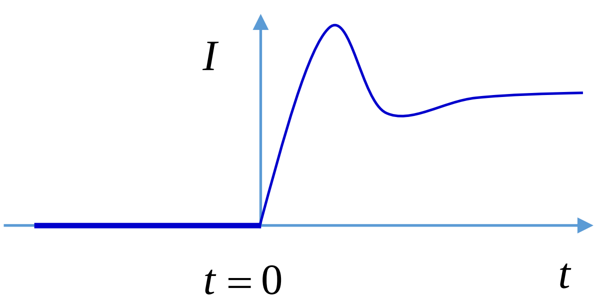




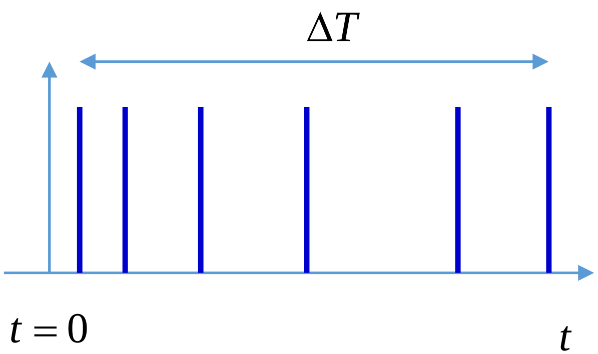
Transient dynamics



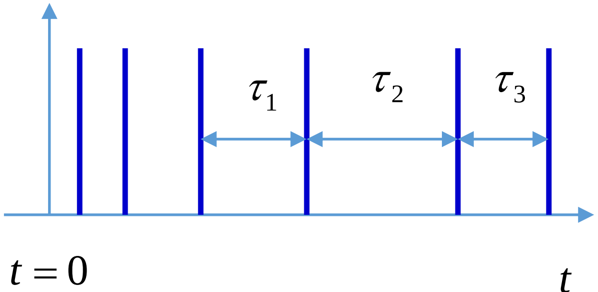




Counting Statistics (time resolved)



$$P(n, \Delta T)?$$

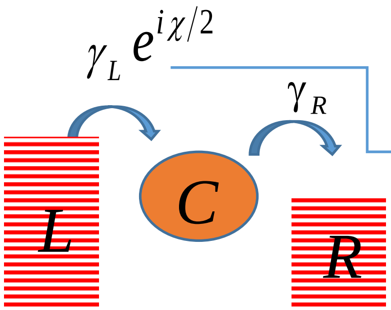


$$W(\tau)$$

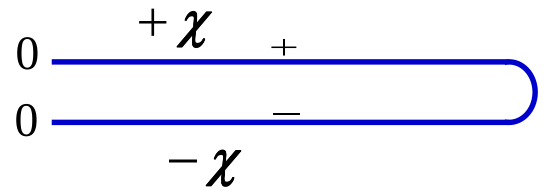
General framework: model + Keldysh formalism

Generic nanodevice model

$$H = H_C + H_L + H_R + H_T$$



counting field



Keldysh contour

$$\mathcal{Z}(\chi, t) = \langle T_C \exp\left\{ -i \int_C H_{T,\chi}(t')dt' \right\} \hat{\rho}(0) \rangle$$

Generating function

$$\hat{\rho}(0) = \hat{\rho}_L \otimes \hat{\rho}_C \otimes \hat{\rho}_R$$

$$\mathcal{Z}(\chi, t) = \sum_n P_n(t) e^{in\chi}$$

"Probability" of n charges at time t

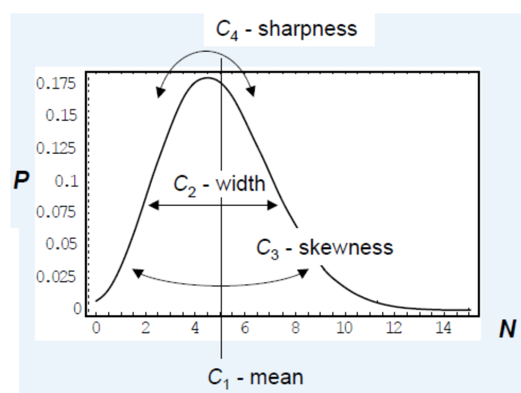
$$S(\chi, t) = \ln \mathcal{Z}(\chi, t)$$

Transient Cumulant Generating Function

$$S(\chi, t) = \sum_n C_n(t) \frac{(i\chi)^n}{n!}$$

Transient current cumulants

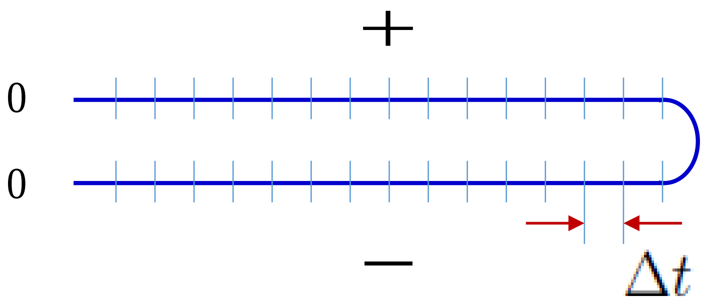
$$\langle\langle I^n(t) \rangle\rangle = \frac{\partial C_n}{\partial t}$$



Single level dot: non-interacting case

$$H_C \equiv H_d = \epsilon d^\dagger d \qquad H_{T,\chi} = \sum_{jk} \gamma_j e^{i\chi_j} c^\dagger_{jk} d + \text{h.c.}$$

$$Z(\chi, t) = \det\left[\hat{G}_{dd}(\chi=0)\hat{G}^{-1}_{dd}(\chi)\right] = \det\left[\hat{G}_{dd}(\chi=0)\left(\hat{g}_d^{-1} - \hat{\Sigma}_L(\chi) - \hat{\Sigma}_R\right)\right]$$



$$n_d = \frac{\rho}{1+\rho}$$

Initial occupation

$$N = t/\Delta t$$

$$h_\pm = 1 \mp i\epsilon\Delta t$$

$$i\hat{g}_d^{-1} = \left(\begin{array}{cccc|cccc} -1 & & & & & & & -\rho \\ h_- & -1 & & & & & & \\ & h_- & -1 & & & & & \\ & & \ddots & \ddots & & & & \\ \hline & & & 1 & -1 & & & \\ & & & & h_+ & -1 & & \\ & & & & & \ddots & \ddots & \\ & & & & & & h_+ & -1 \end{array}\right)_{2N\times 2N}$$

Contour closing

Single level dot: transient charge and current

$$n_d(t) = \frac{\Gamma}{\pi}\theta(t)e^{-\Gamma t}\sum_j \int d\omega \frac{f_j(\omega+\epsilon)}{\Gamma^2+\omega^2}\left[\cosh(\Gamma t) - cos(\omega t)\right]$$

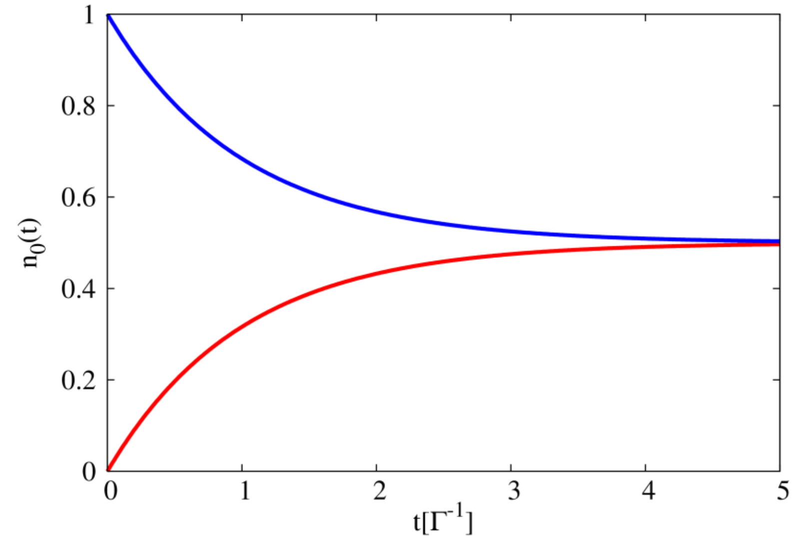
Riwar & Schmidt, PRB (2009)

ε=0 V=6Γ

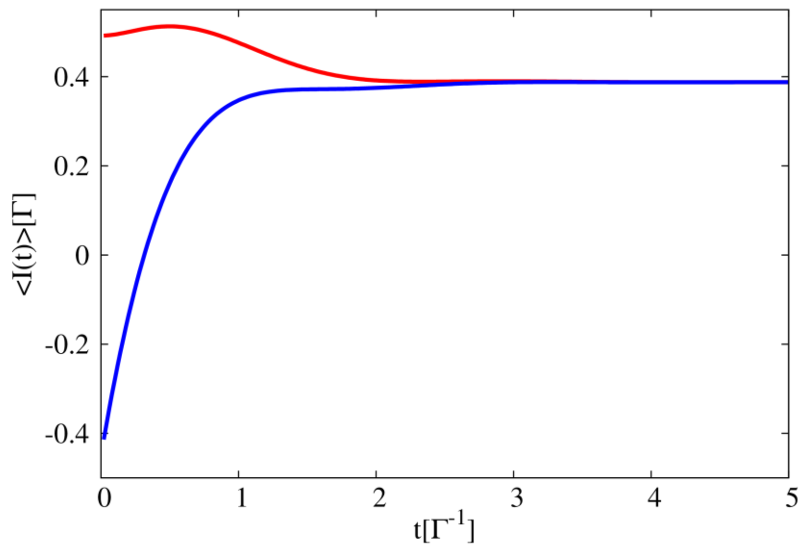
Initially full (blue)

Initially empty (red)

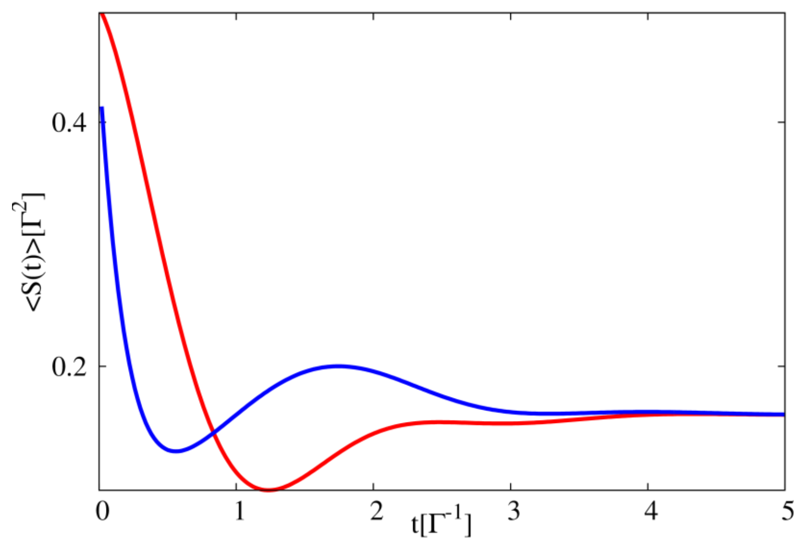
charge



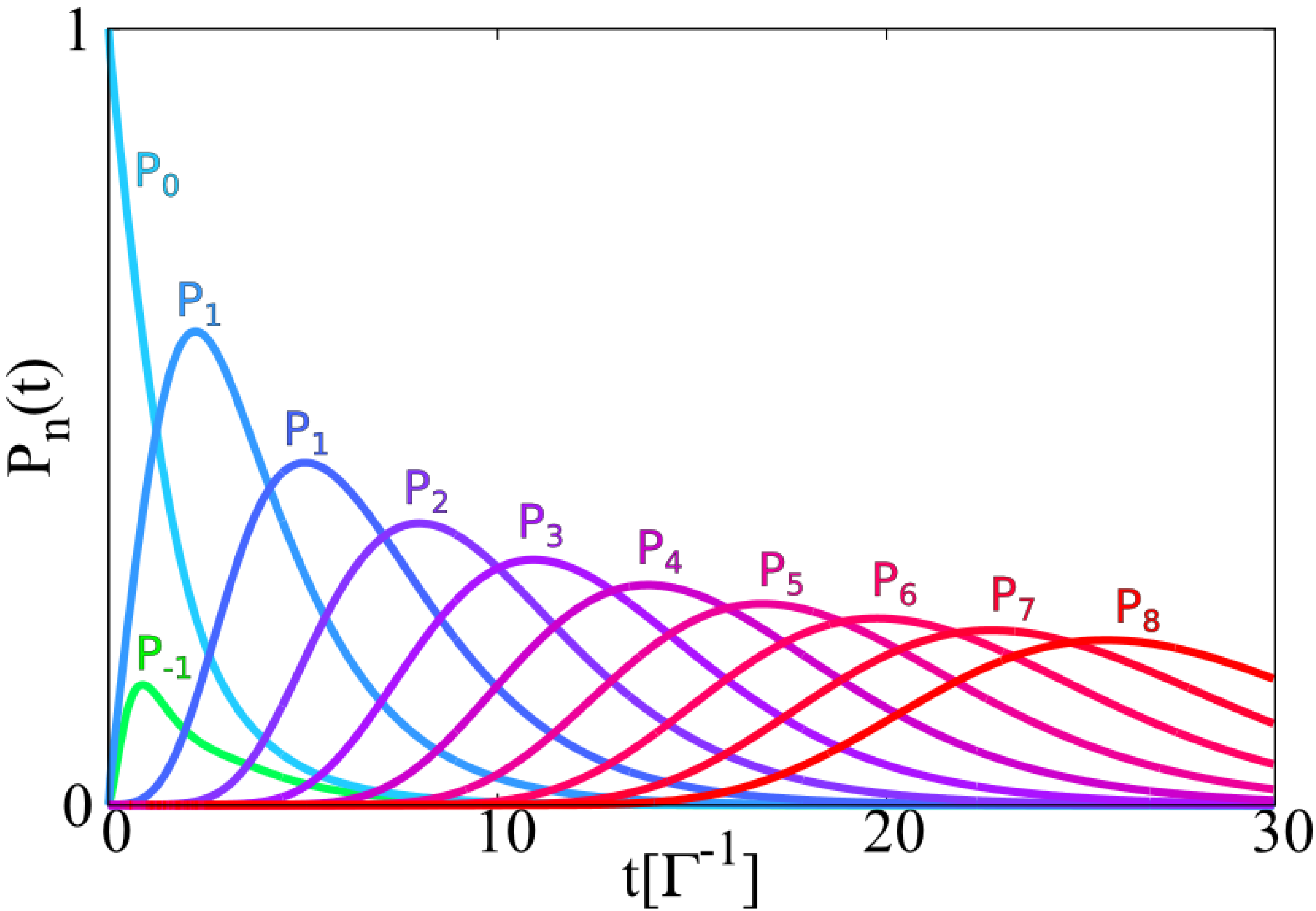
current



noise



Single level dot: charge transfer probabilities



ε=0 V=4Γ $n_d(0) = 1$

Yang-Lee zeros and higher order cumulants

$$\mathcal{Z}(z,t) = \sum_n P_n(t) z^n \qquad z = e^{i\chi} \qquad c_j(t) = \left(\frac{\partial}{\partial i\chi}\right)^j \log \mathcal{Z}(\chi, t)$$

Seoane et al. Fortsch. Phys. (2017)

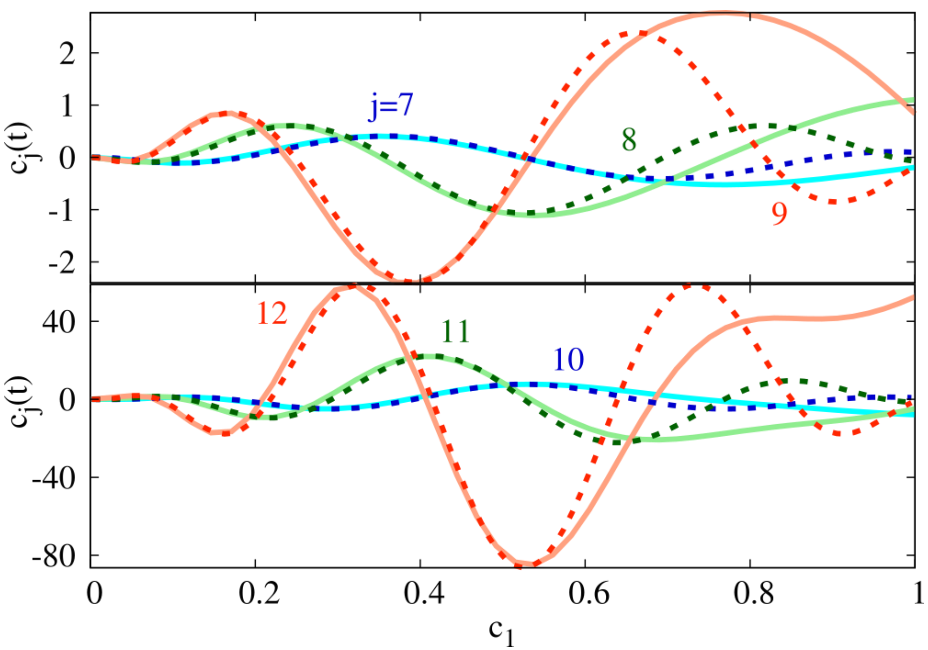
Yang-Lee zeros

polylogarithm

$$\mathcal{Z}(z,t) = \prod_k (z - z_k(t)) \qquad c_j(t) = -\sum_k \mathrm{L}i_{1-j}\left(\frac{1}{z_k(t)}\right)$$

short time-behavior (unidirectional transport)

$$c_j(t) \simeq -\mathrm{L}i_{1-j}\left(\frac{1}{z_d(t)}\right) = \left(\frac{\partial}{\partial i\chi}\right)^j \log\left[1 + \frac{e^{i\chi}}{z_d(t)}\right]_{\chi=0}$$

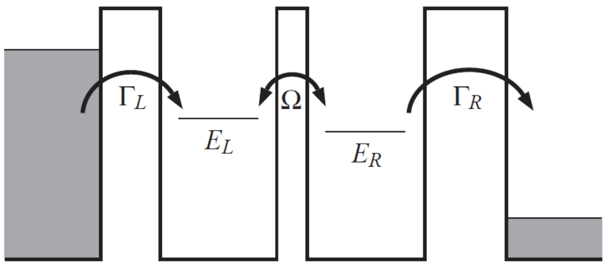


dominant zero (closer to z=1)

“Universal” scaling

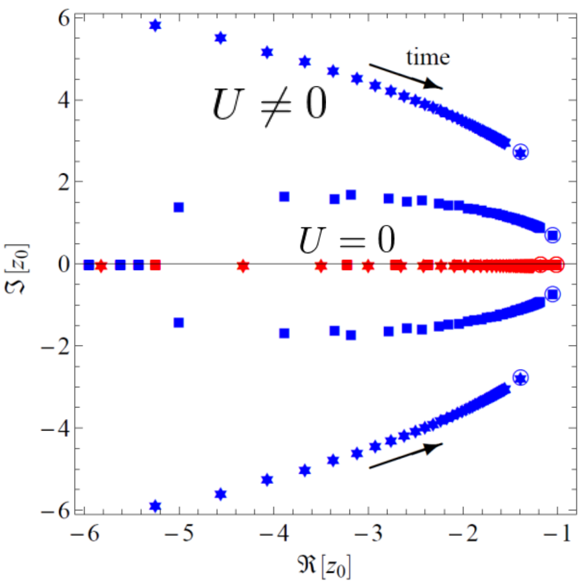
$$\max \frac{c_j}{c_1} \sim \frac{(j-1)!}{\pi^j}$$

Yang-Lee zeros: effect of interactions



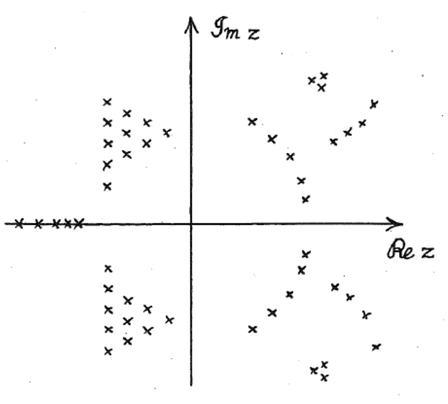
$$\hat{H}_{DQD} = E_L \hat{a}_L^\dagger \hat{a}_L + E_R \hat{a}_R^\dagger \hat{a}_R + \Omega(\hat{a}_L^\dagger \hat{a}_R + \hat{a}_R^\dagger \hat{a}_L) + U \hat{n}_L \hat{n}_R,$$

Kambly & Flindt, J. Comp. Elect. (2013)

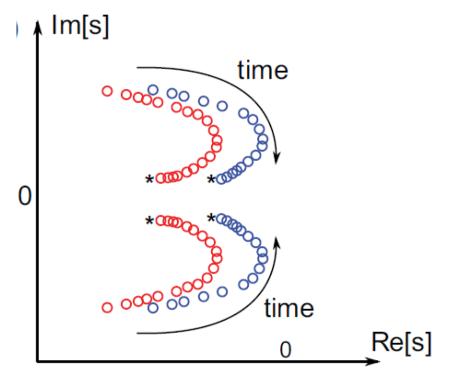


Analogy with phase transitions

Zeros of partition function converging to a branch cut on real axis for V→∞



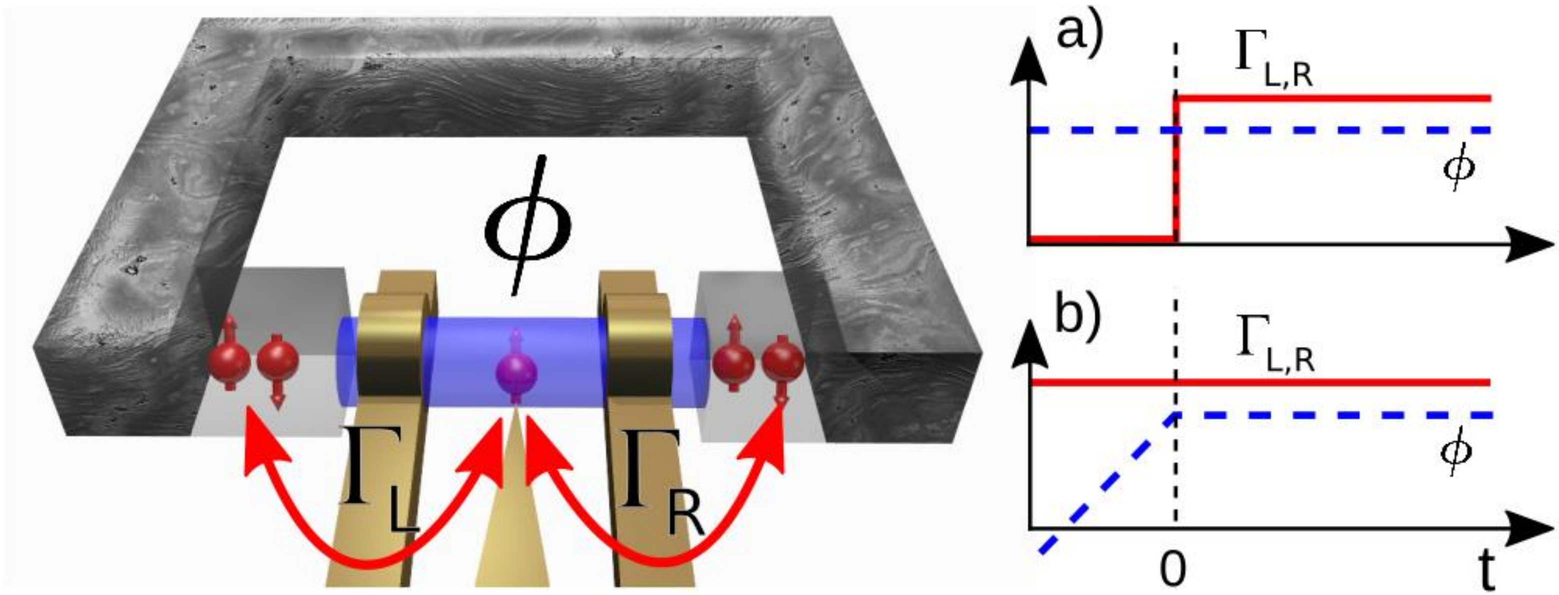
Dynamical phase transitions signaled by motion of YLZ for t→∞



Hickey et al. PRE (2017)

Quench dynamics in superconducting nanojunctions

R. Seoane, A. Martín-Rodero & ALY, PRL 2016



Basic formalism $H = H_0 + H_T + H_{leads}$

$$H_0 = \Psi_0^\dagger \epsilon \sigma_z \Psi_0 \qquad \Psi_\nu^\dagger = \left(c_{\nu\uparrow}^\dagger \, , \; c_{\nu\downarrow} \right)$$

$$Z(\chi, t) = \det \left[\mathbf{G}(\chi = 0) \mathbf{G}(\chi)^{-1} \right]$$

$$H_T = \sum_{j,k} \gamma_j \Psi_{jk}^\dagger \sigma_z e^{i\sigma_z \phi_j} \Psi_0 + \mathrm{hc}$$

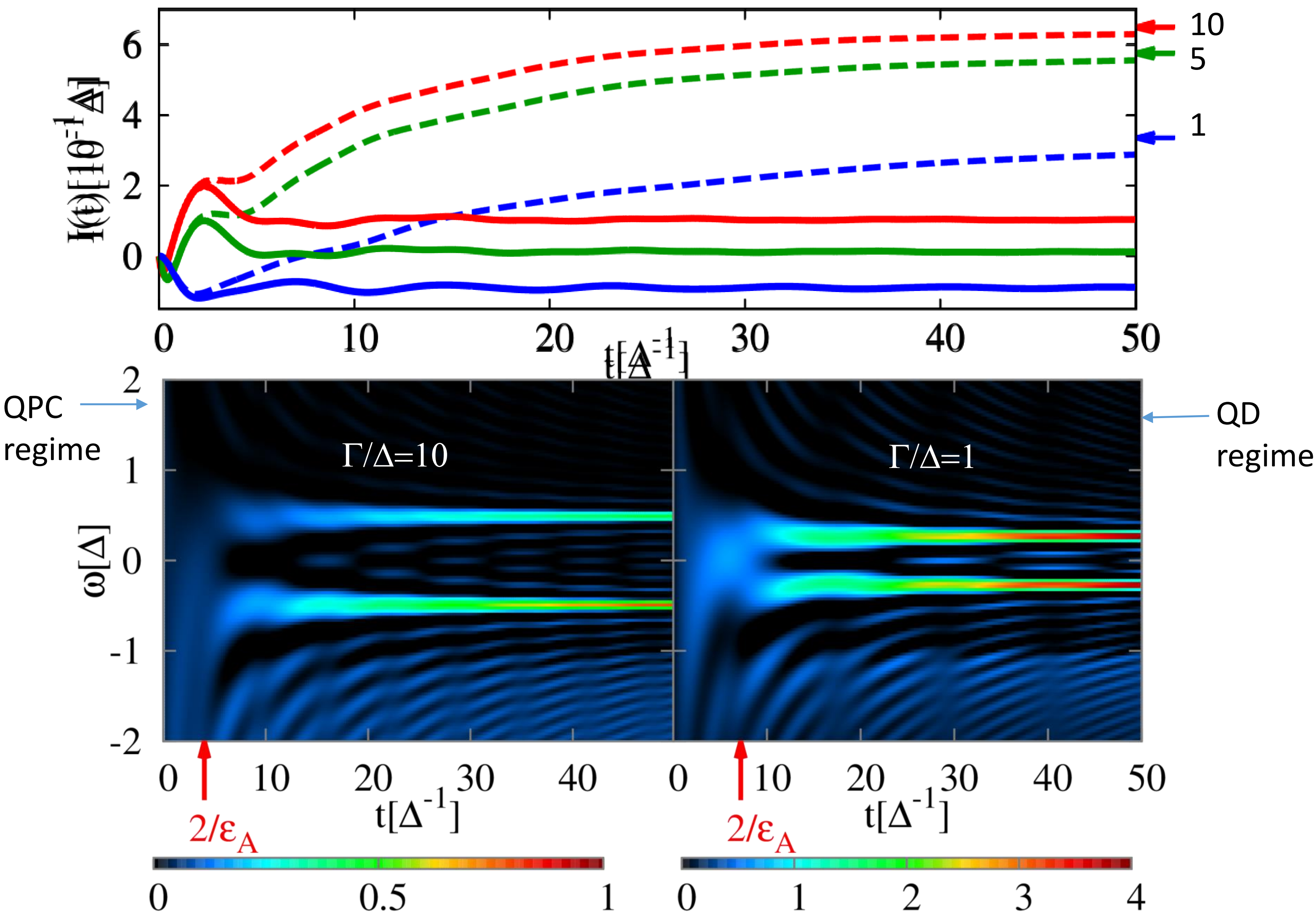
$$H_{leads} = \sum_{jk} \Psi_{jk}^\dagger \left(\epsilon_k \sigma_z + \Delta_j \sigma_x \right) \Psi_{jk}$$

$$\mathbf{G} = -i \langle T_K \Psi_0(t) \Psi_0^\dagger(t') \rangle$$

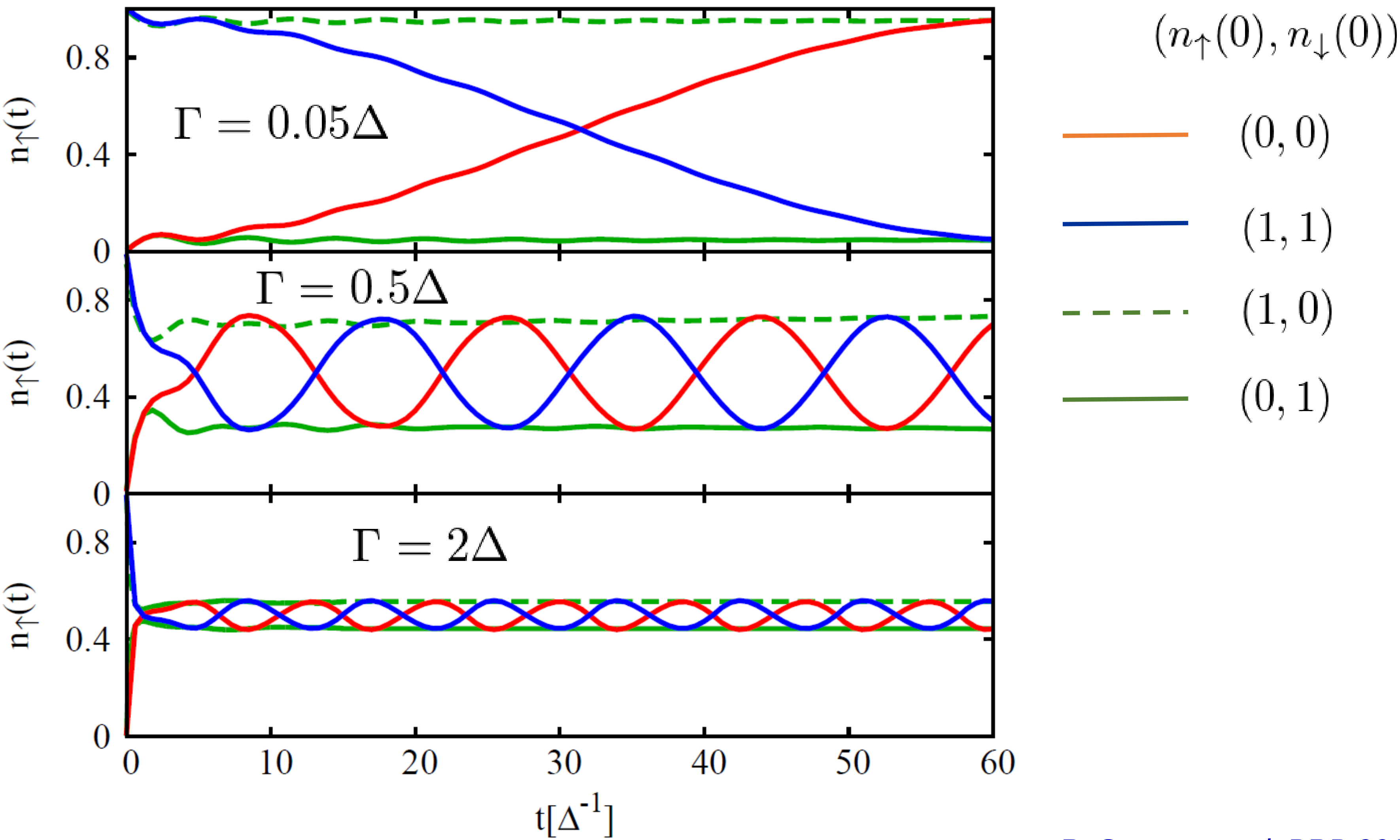
Transient current and spectral densities

$\epsilon = 0 \quad \Gamma_L = \Gamma_R = \Gamma/2$

$\phi=2 \qquad \Gamma/\Delta$

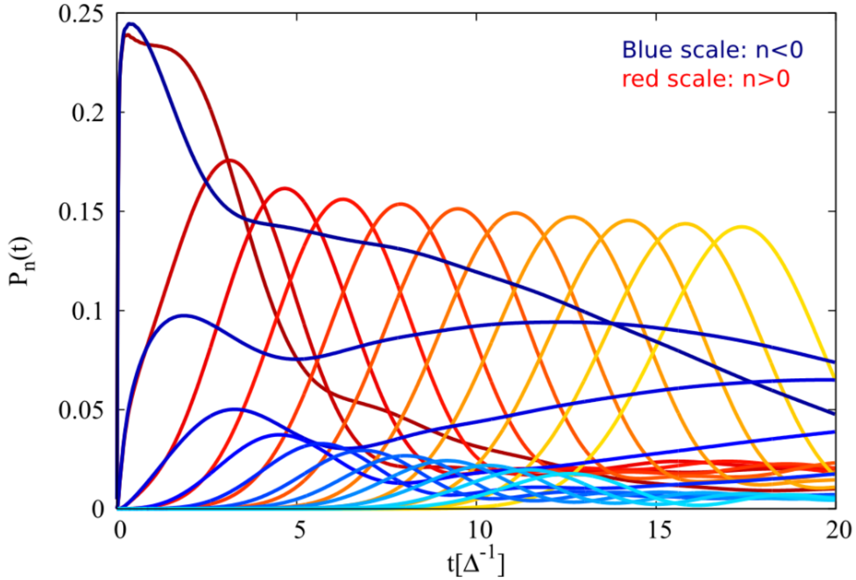


Dependence on initial conditions

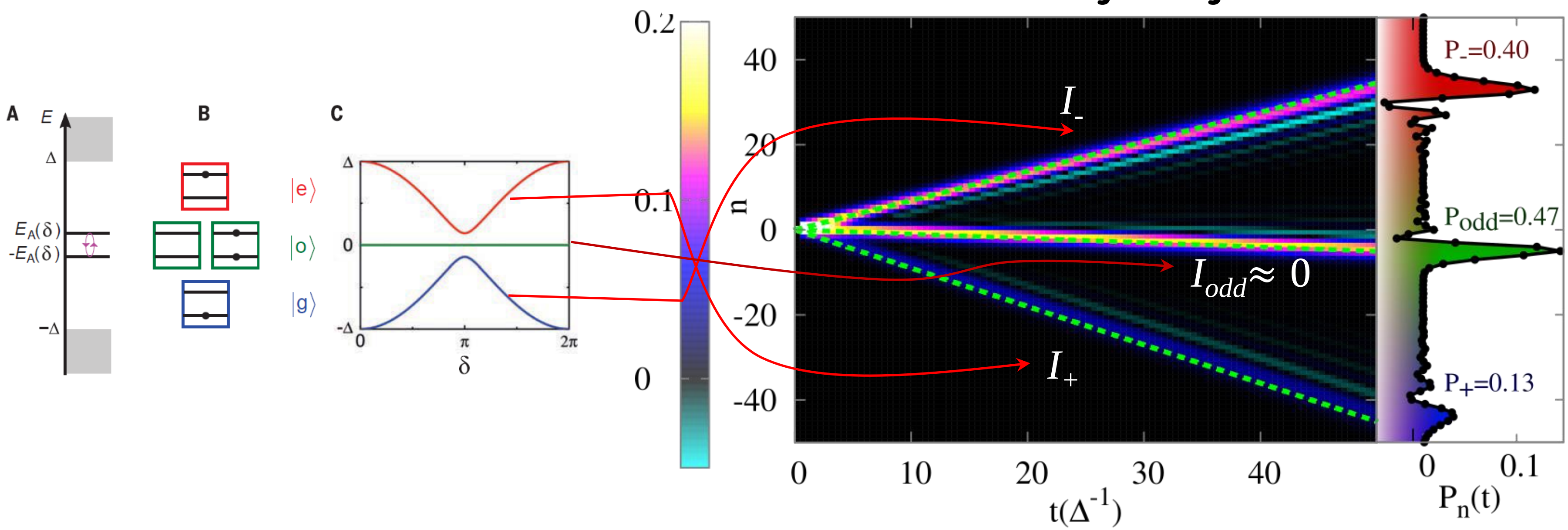


R. Seoane et. al. PRB 2017

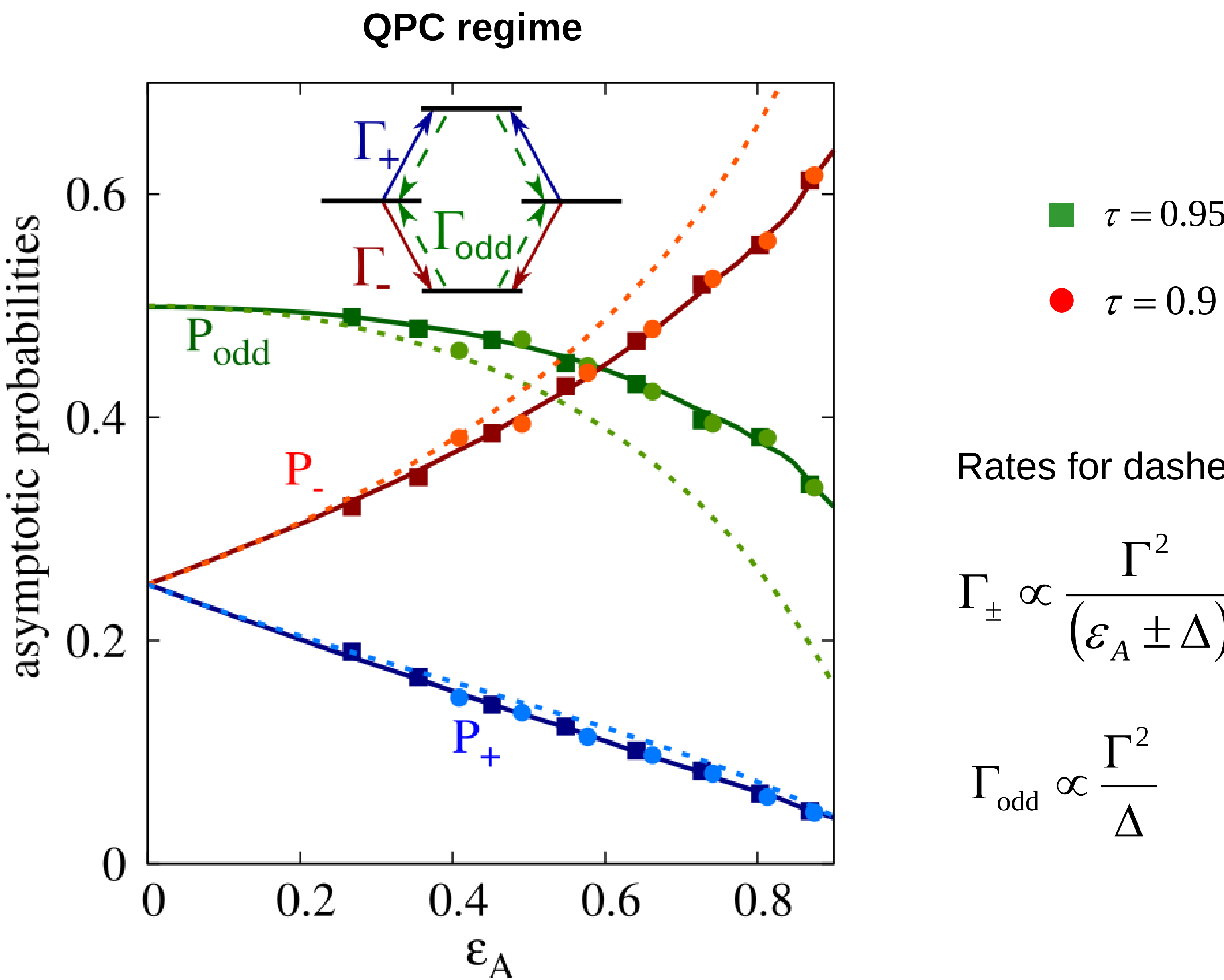
Full counting statistics analysis (QPC regime)



Identification of many body states



Asymptotic probabilities of many body states



Rates for dashed lines:

$$\Gamma_\pm \propto \frac{\Gamma^2}{(\varepsilon_A \pm \Delta)}$$

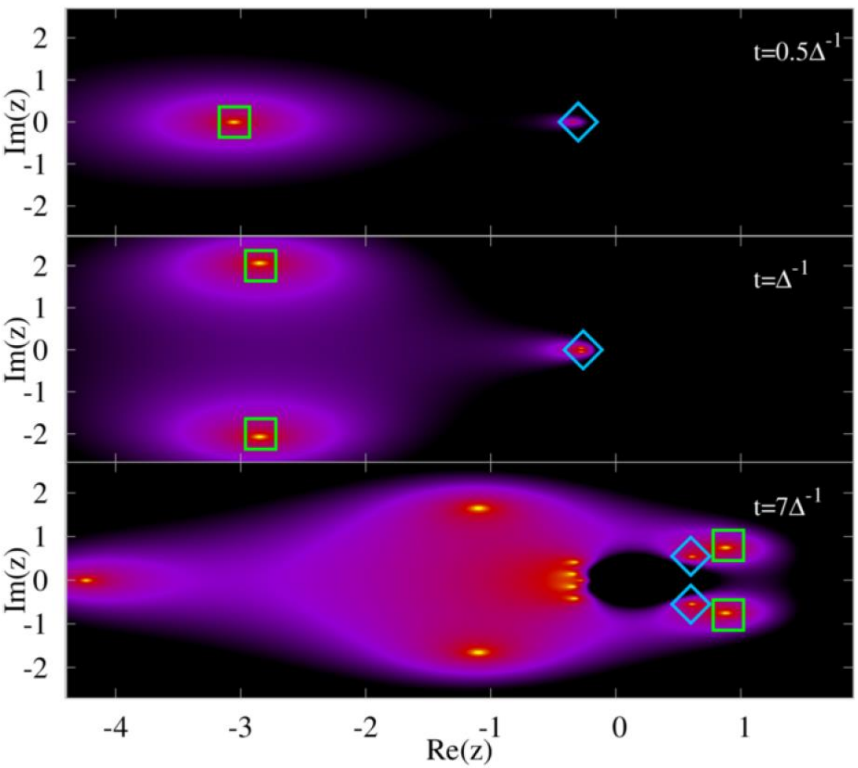
$$\Gamma_{odd} \propto \frac{\Gamma^2}{\Delta}$$

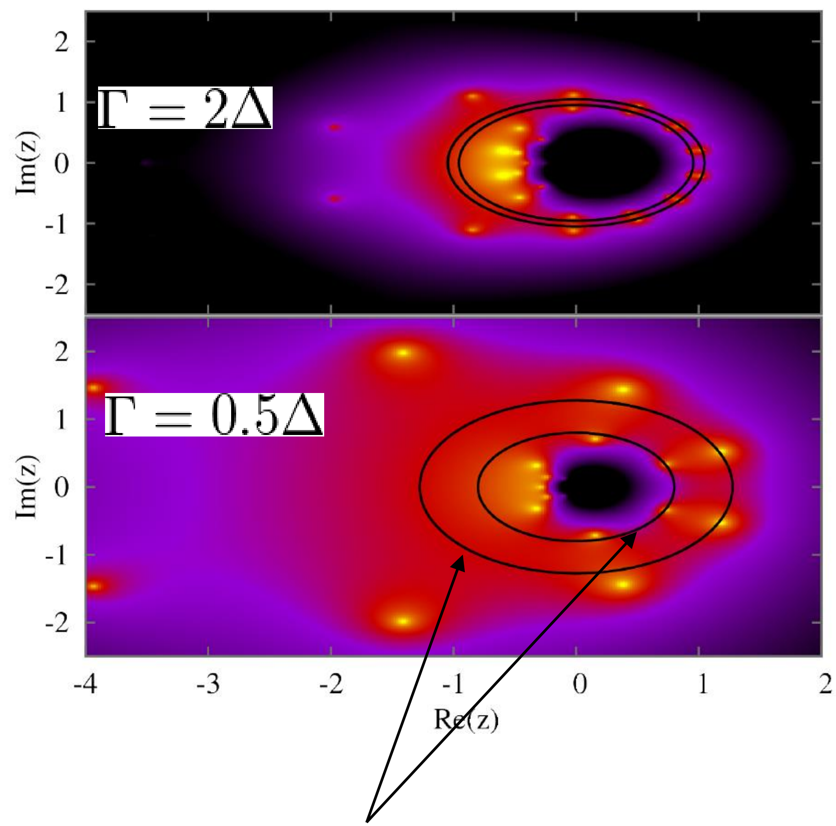
R. Seoane et. al. PRB 2017

Evolution of Yang-Lee zeros

$$\Gamma = 2\Delta$$

$$t \sim 25/\Delta$$





Coarse grained statistics

$$\mathcal{Z}(\chi, t) \simeq P_{-} e^{i\chi I_{-} t} + P_{+} e^{i\chi I_{+} t} + P_{odd} e^{i\chi I_{odd} t}$$

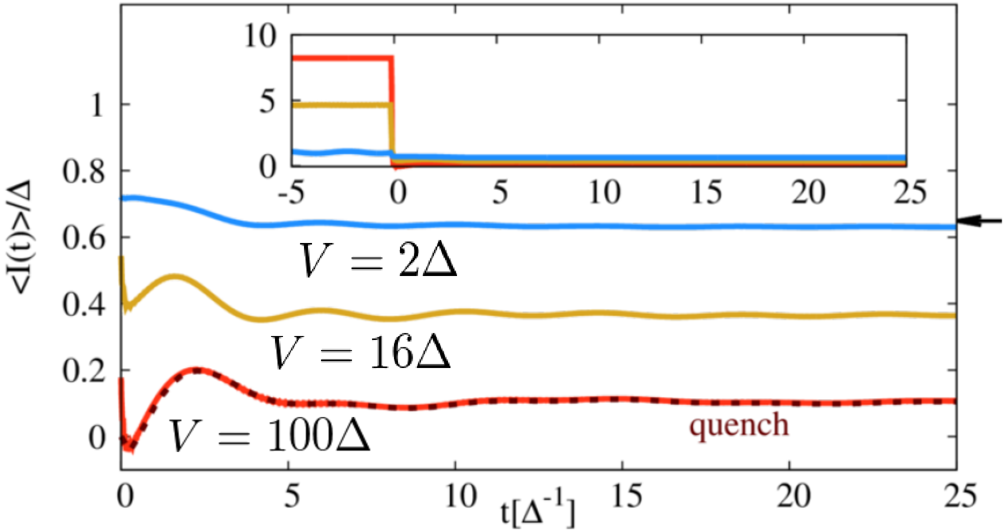
$$I_{-} = -I_{+} = I_A + I_{odd}$$

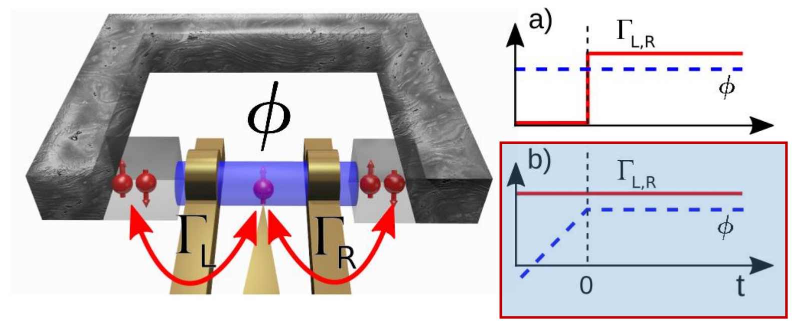
Asymptotic position of YLZ

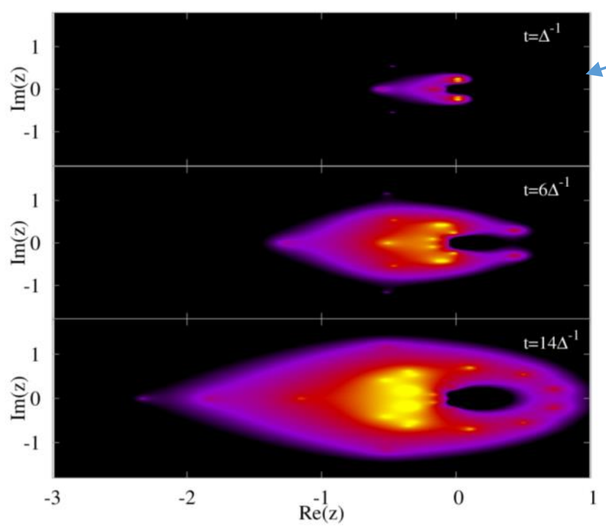
$$\alpha_{\pm} = z_{\pm}^{I_A t} \approx \frac{-P_{odd} \pm \sqrt{P_{odd} - 4P_{-}P_{+}}}{2P_{-}}$$

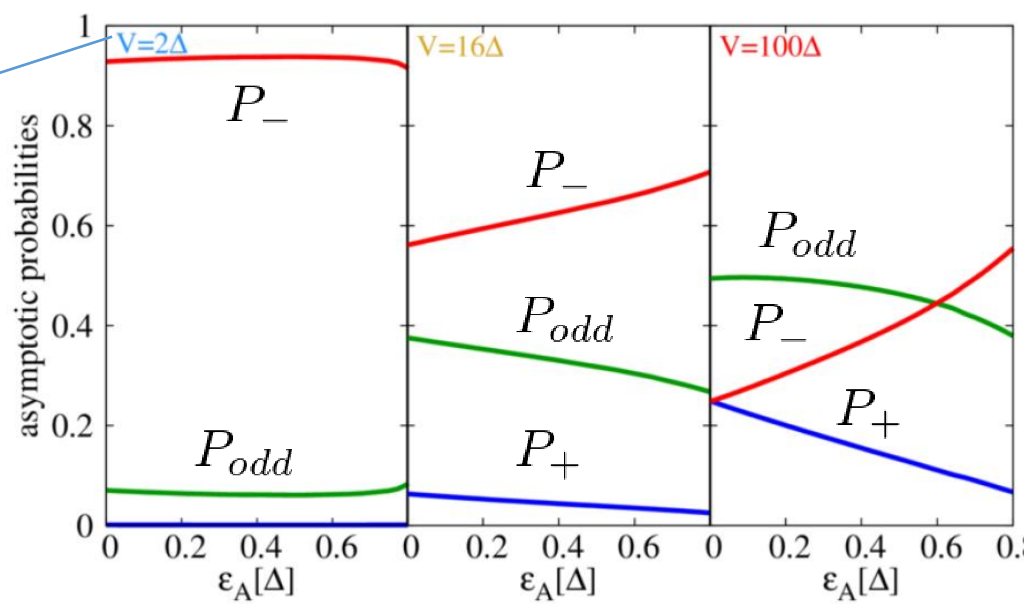
R. Seoane et. al. PRB 2017

Initialization by a sudden voltage drop (QPC regime)





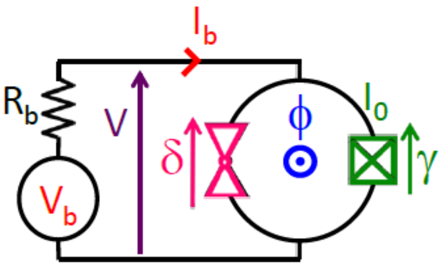




Consistency with experimental observations

Long lived trapped quasiparticle states

*M. Zgirski et al. PRL **106**, 257003 (2011)*

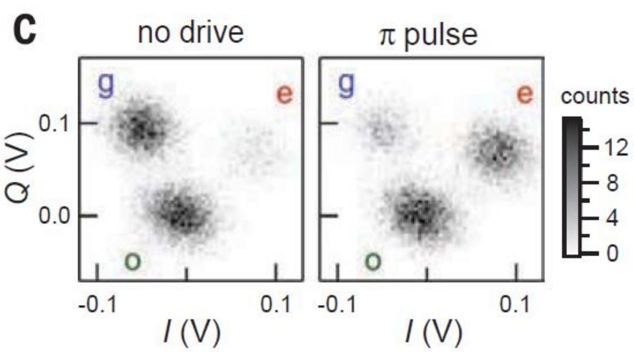


Specific theory

Olivares et al. PRB (2014)
Zazunov et al. PRB (2014)

Coherent manipulation of ABSs

Janvier et al., Science (2015)



Conclusions (Second Lecture):

Inclusion of interactions within Hamiltonian approach (illustration in terms of dynamical CB)

Time-dependent transport: transient dynamics and time-dependent counting statistics

Analysis of time-dependent FCS in terms of DYLZs

Metastabilty in superconducting nanojunctions: robustness of odd states in QPC regime