Hamiltonian approach to transport in conventional and topological superconducting nanojunctions

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EXCELENCIA SEVERO OCHOA

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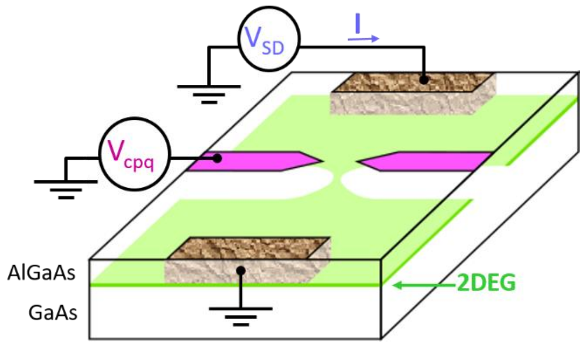
Outline:

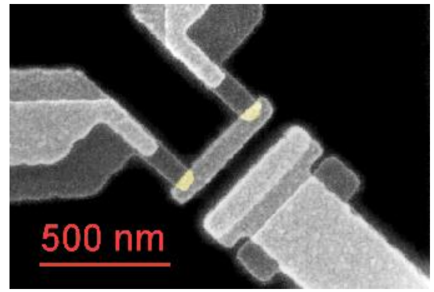
First Lecture: general introduction, application to conventional SC junctions (non-int, steady-state)

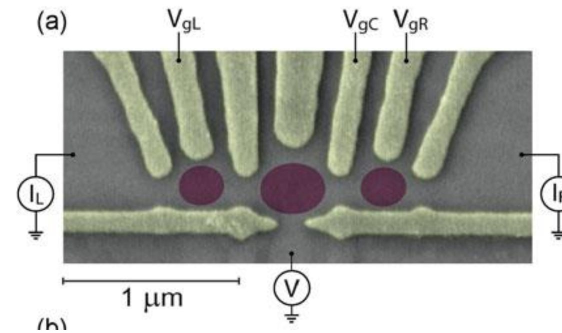
Second Lecture: effect of interactions (DCB), quench dynamics, time-dependent FCS

Third Lecture: topological superconductors featuring MBS, two terminal, multiterminal, Interaction effects

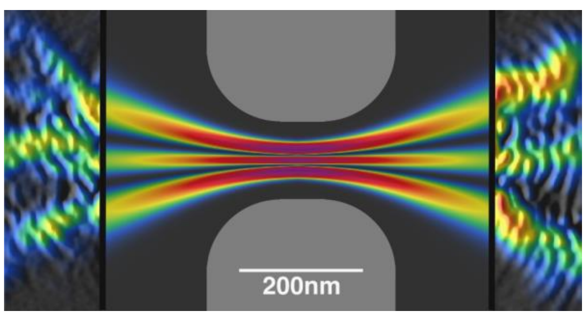
reduced dimensionality (nanoscale devices)





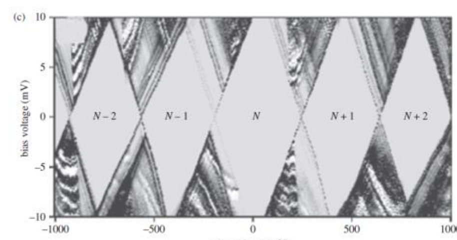


quantum transport



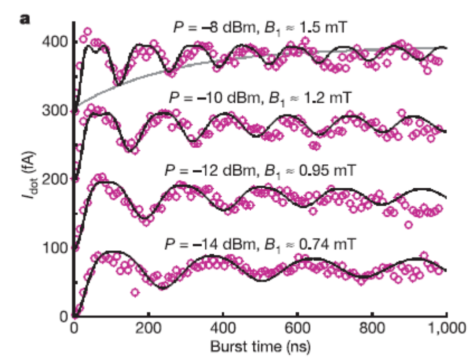
(Conductance quantization, AB effect, Quantum noise, etc)

interactions

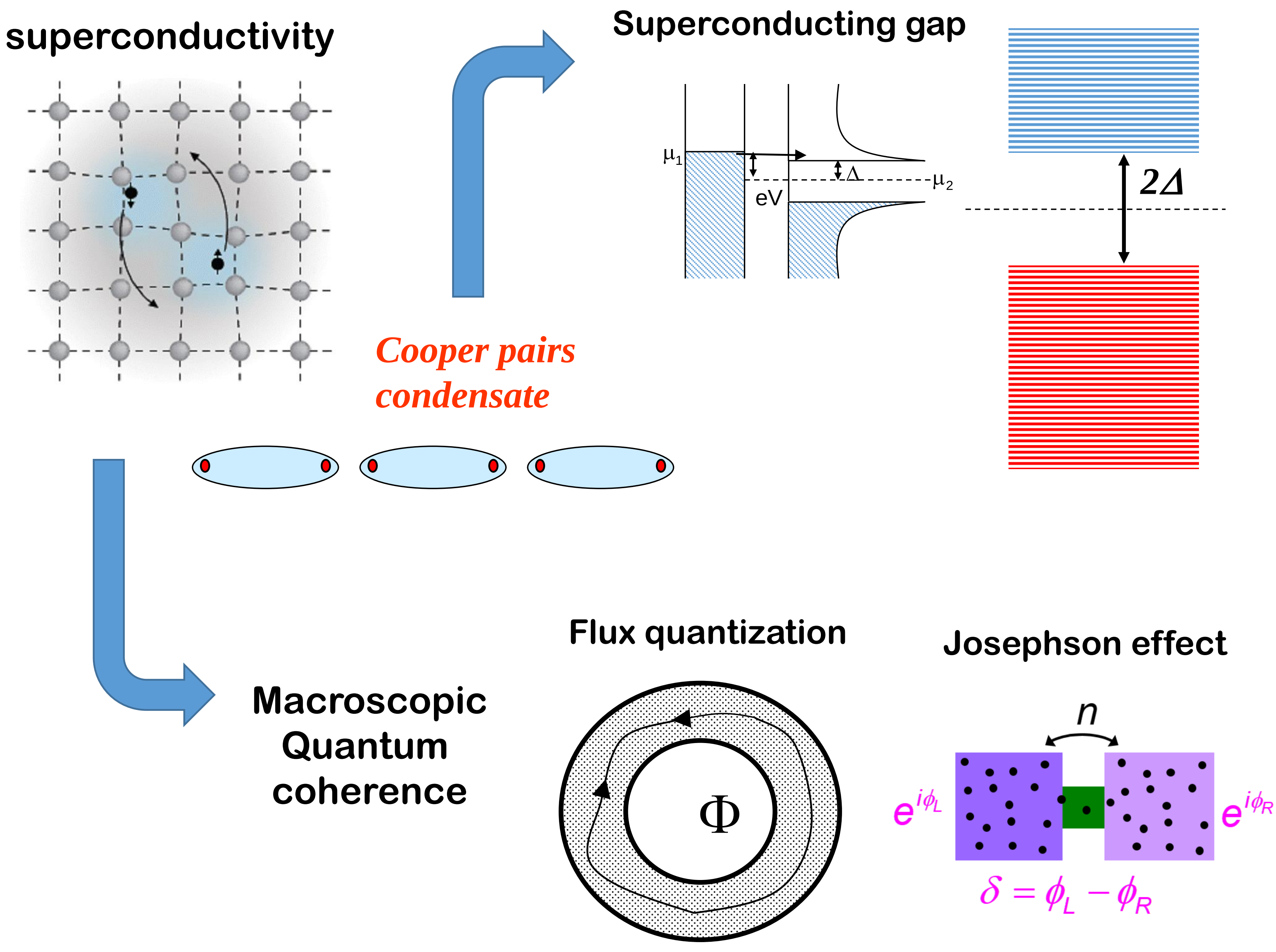


(CB, DCB, Kondo, etc)

coherent dynamics

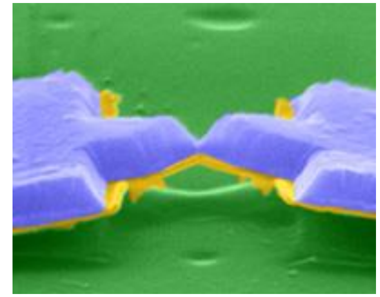


(charge and spin qubits, etc)

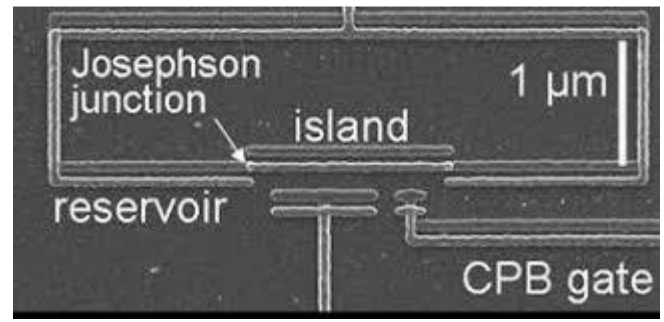

superconductivity
Superconducting gap
μ_1
Δ
μ_2
eV
2Δ
Cooper pairs condensate
Macroscopic Quantum coherence
Flux quantization
Φ
Josephson effect
n
$e^{i\phi_L}$
$e^{i\phi_R}$
$\delta = \phi_L - \phi_R$

superconductivity at nanoscale

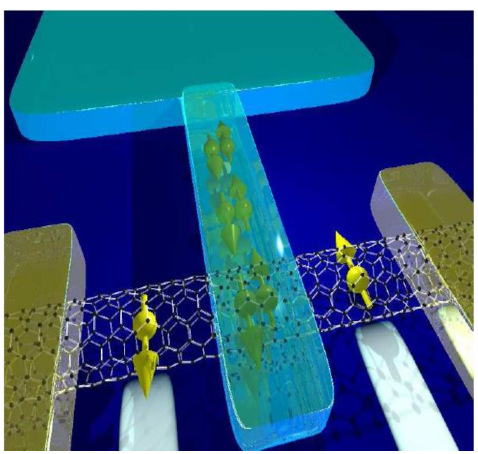
Superconducting atomic contacts



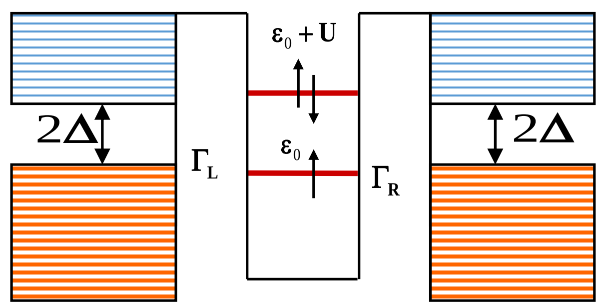
Cooper pair box

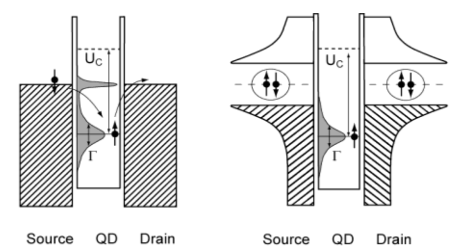


Cooper pair splitter

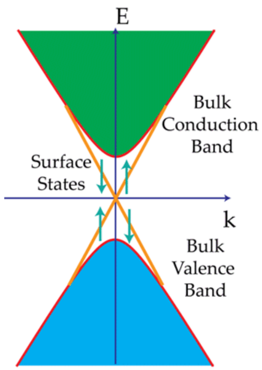


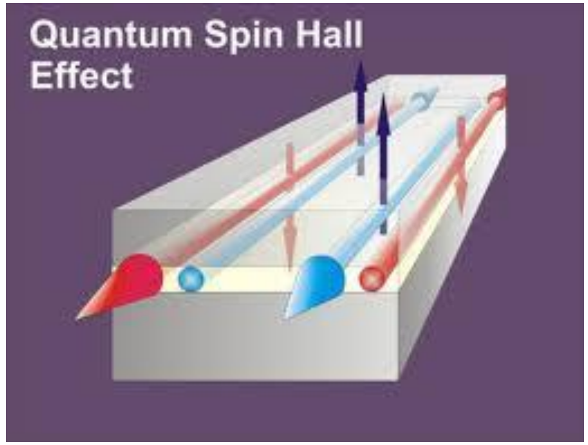
Superconducting QDs

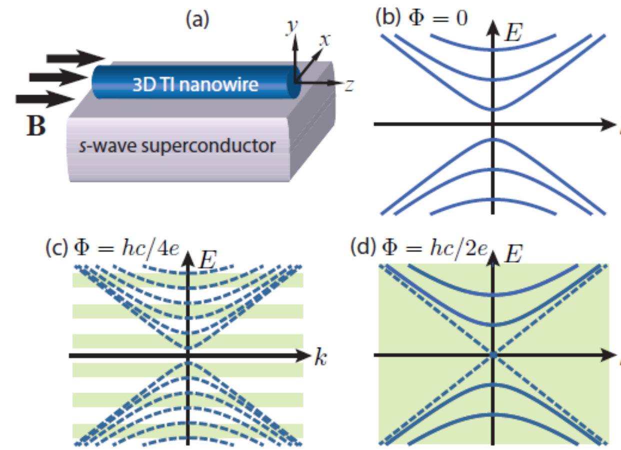




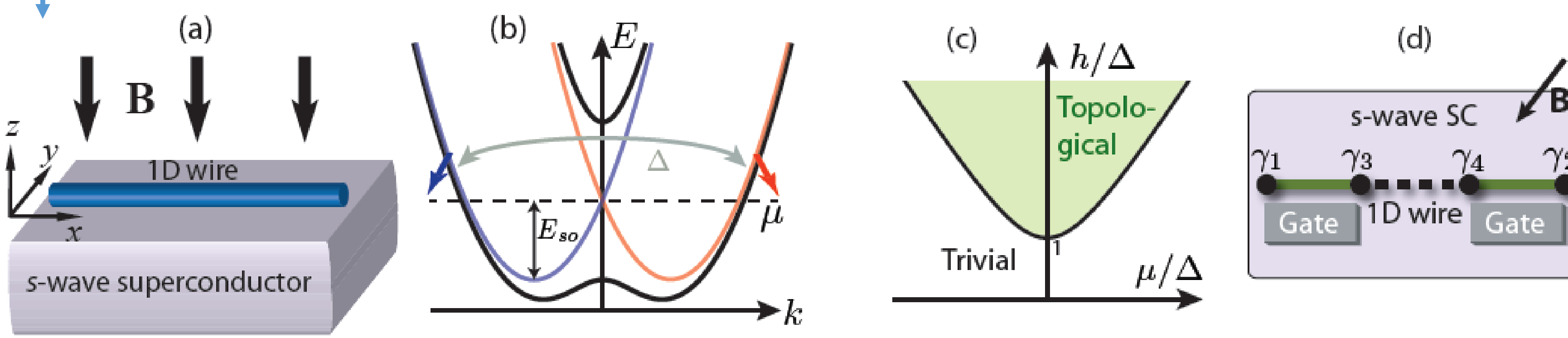
Topology



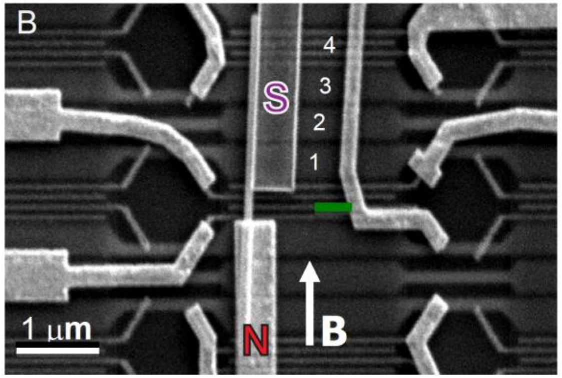


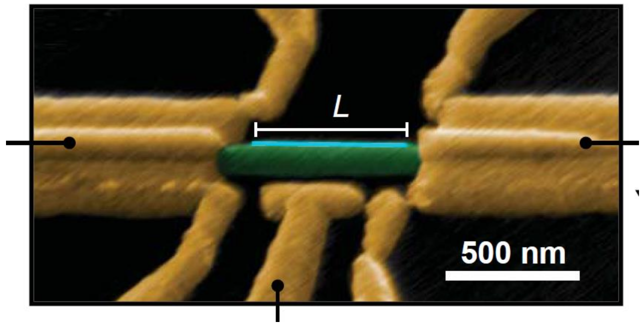


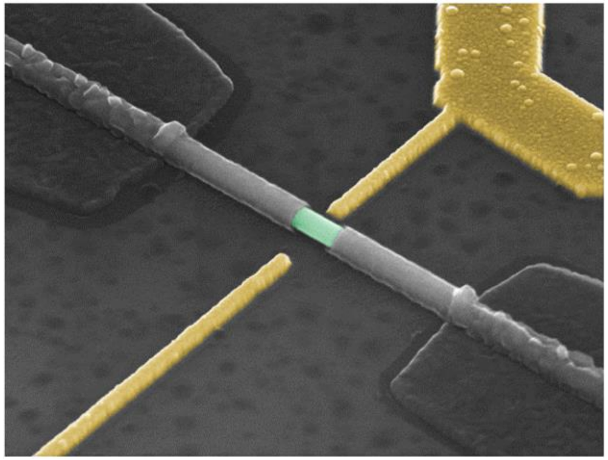
Inducing Topological Superconductivity in nanowires

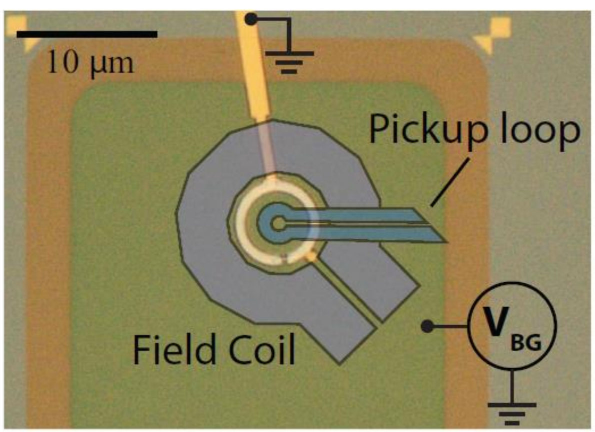


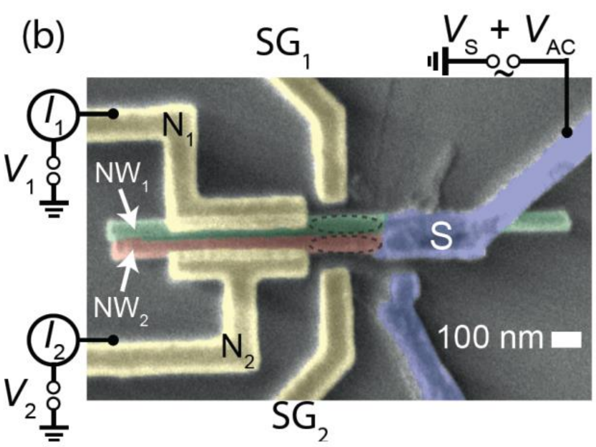
Hybrid nanowire devices

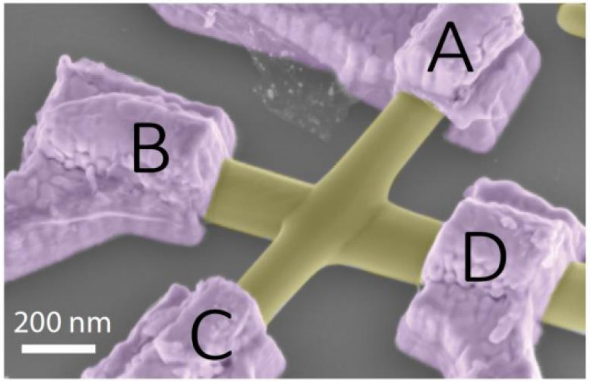






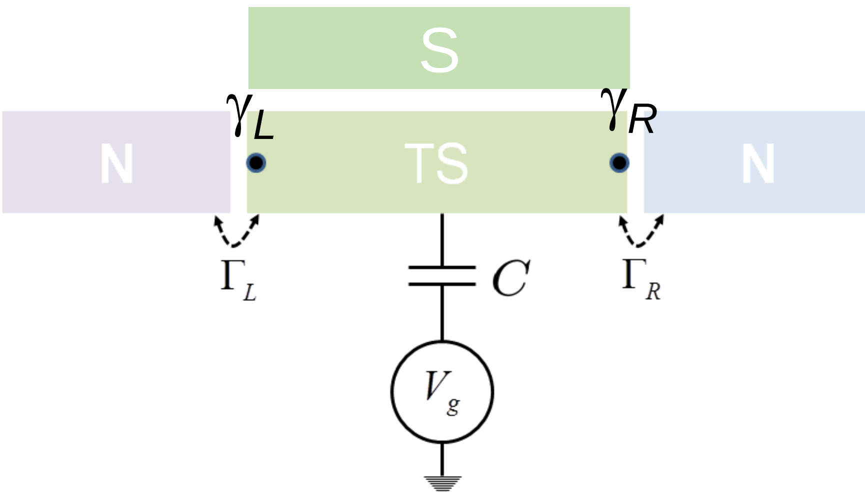




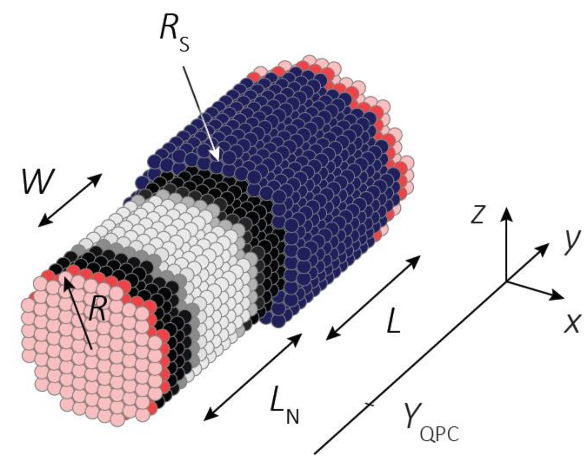


Theoretical modeling

Minimal: lowest energy states
Role of interactions
Analytical results



Extended: large discrete basis
Role of disorder
Numerical

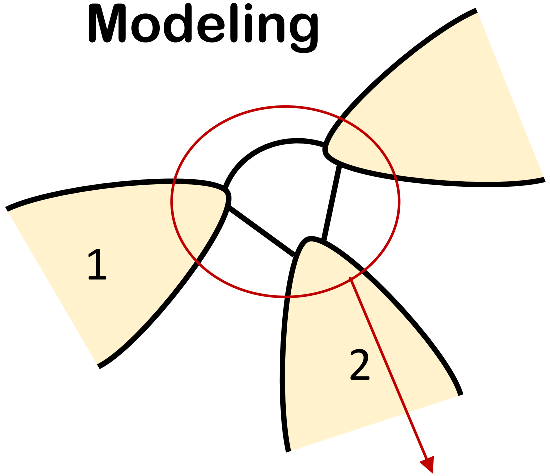


Intermediate: effective low energy theory

Transport, Subgap+continuum
possible analytical results

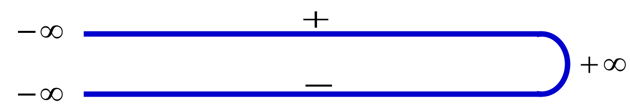
Hamiltonian approach

Modeling



$$H = H_1 + H_2 + \ldots + H_T$$

Methods



Keldysh-Nambu formalism
Non-equililbrium GFs

BCS-junctions, Cuevas et al. PRB 96
Difussive FS, Bergeret et al. PRB 05
Graphene/S, Burset et al. PRB 08
Topological, Zazunov et al. PRB 16
ABS dynamics, Seoane et al. PRL 16
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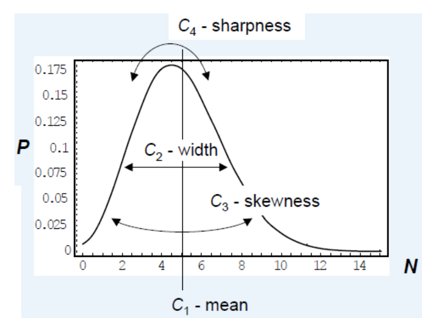
Aim: calculate

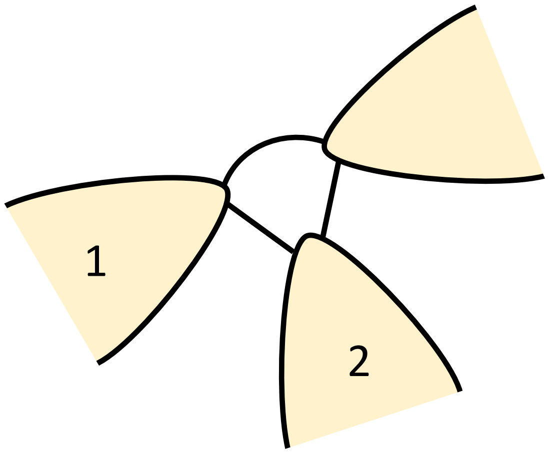
$$< I_j >$$

$$< I_i(t) I_j(t') >$$

FCS...

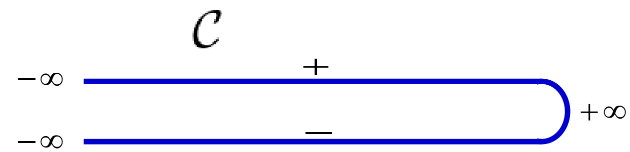


Keldysh Nambu formalism



$$\hat{\Psi}_i = \begin{pmatrix} c_{i\uparrow} \\ c^\dagger_{i\downarrow} \end{pmatrix}$$ Nambu spinors

Keldysh contour



Keldysh-Nambu GFs $\hat{G}_{i,j}(t,t') = -i\langle T_{\mathcal{C}} \hat{\Psi}_i(t) \hat{\Psi}^\dagger_j(t')\rangle$

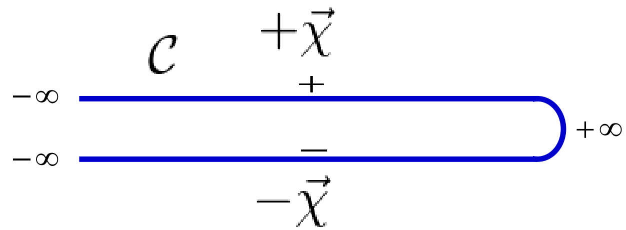
$$H_T = \sum_{ij} \hat{\Psi}^\dagger_i \hat{T}_{ij} \hat{\Psi}_j + \text{h.c.} \qquad \hat{T}_{ij} = \begin{pmatrix} T_{ij} & 0 \\ 0 & -T^*_{ij} \end{pmatrix}$$

Keldysh-Nambu-Leads Dyson Eqs $\check{\hat{G}} = \check{\hat{g}} + \check{\hat{g}} \otimes \check{\hat{\Sigma}} \otimes \check{\hat{G}}$

Mean currents $\langle I_{ij}\rangle(t) = \frac{e}{\hbar} \mathrm{Tr}\left[\sigma_z \left(\hat{T}_{ij}\hat{G}^{+-}_{ij}(t,t) - \hat{T}^\dagger_{ij}\hat{G}^{+-}_{ji}(t,t)\right)\right]$

Functional integral representation

$$H_T(\vec{\chi}) = \sum_{ij} \hat{\Psi}_i^\dagger \hat{T}_{ij}(\vec{\chi}) \hat{\Psi}_j + \mathrm{h.c.} \qquad \hat{T}_{ij}(\vec{\chi}) = T_{ij} \begin{pmatrix} e^{i\chi_{ij}} & 0 \\ 0 & -e^{-i\chi_{ij}} \end{pmatrix}$$



$$Z(\vec{\chi}) = \langle e^{-i\int_{\mathcal{C}} H_T(\vec{\chi})dt} \rangle$$

Partition or Generating function

$$Z(\vec{\chi}) = \int \mathcal{D}\hat{\bar{\Psi}} \mathcal{D}\hat{\Psi} e^{iS_{eff}(\hat{\bar{\Psi}},\hat{\Psi},\vec{\chi})}$$

Boundary Green functions

$$S_{eff}(\hat{\bar{\Psi}}, \hat{\Psi}, \vec{\chi}) = \sum_{ij} \int_{\mathcal{C}} dt \left(\hat{\bar{\Psi}}_i, \hat{\bar{\Psi}}_j \right) \begin{pmatrix} \hat{g}_i^{-1} & -\hat{T}_{ij} \\ -\hat{T}_{ij}^\dagger & \hat{g}_j^{-1} \end{pmatrix} \begin{pmatrix} \hat{\Psi}_i \\ \hat{\Psi}_j \end{pmatrix}$$

$$S(\vec{\chi}) = \log Z(\vec{\chi}) \qquad \langle I_{ij} \rangle \propto \frac{i}{2} \frac{\delta S}{\delta \chi_{ij}} \qquad \langle I_{ij} I_{kl} \rangle \propto \left(\frac{i}{2} \right)^2 \frac{\delta^2 S}{\delta \chi_{ij} \delta \chi_{kl}}$$

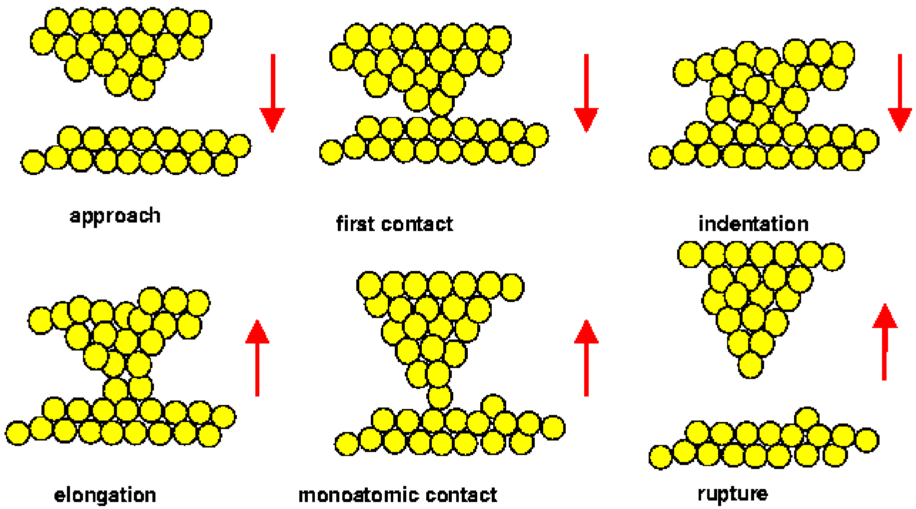
Cumulant Generating function

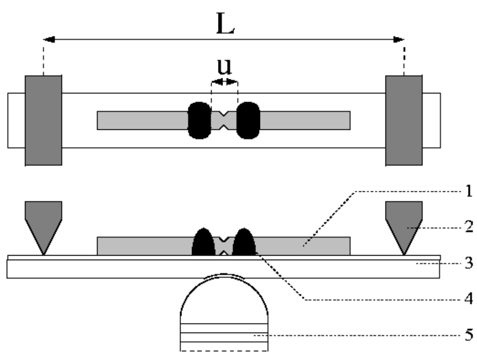
Conventional superconductors:

Application to Superconducting Atomic Contacts

Fabrication techniques

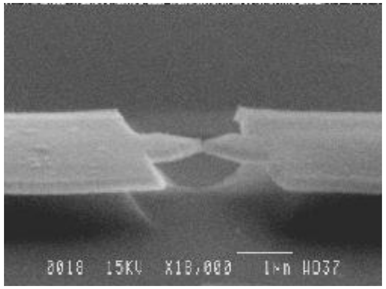
Contact formation with STM

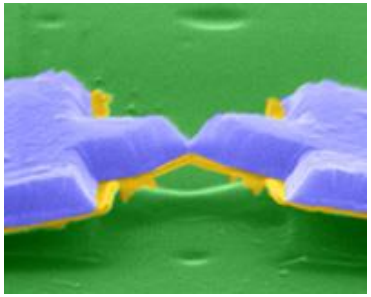




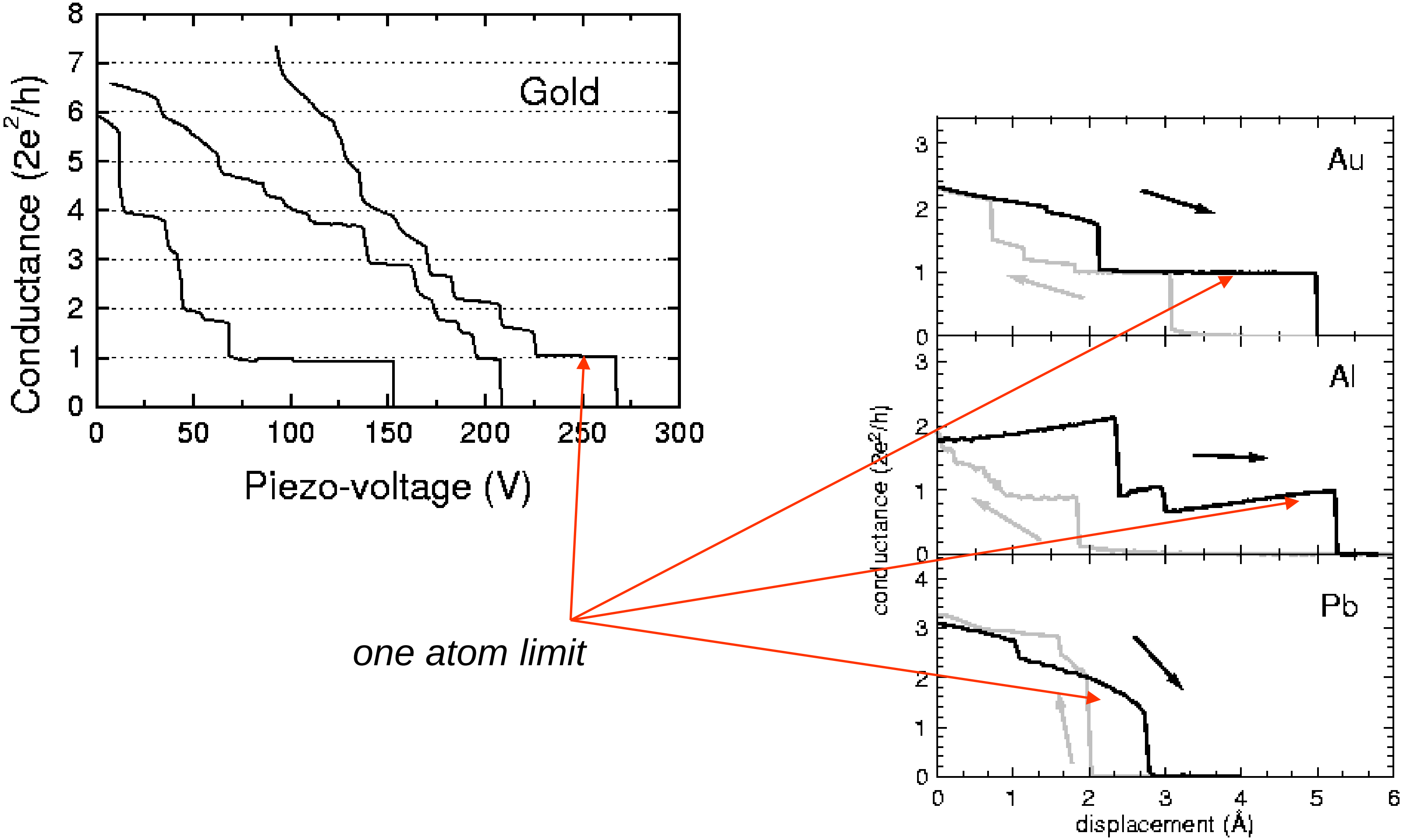
Mechanically controlable break-junctions (MCBJ)

Nanofabricated break-junctions



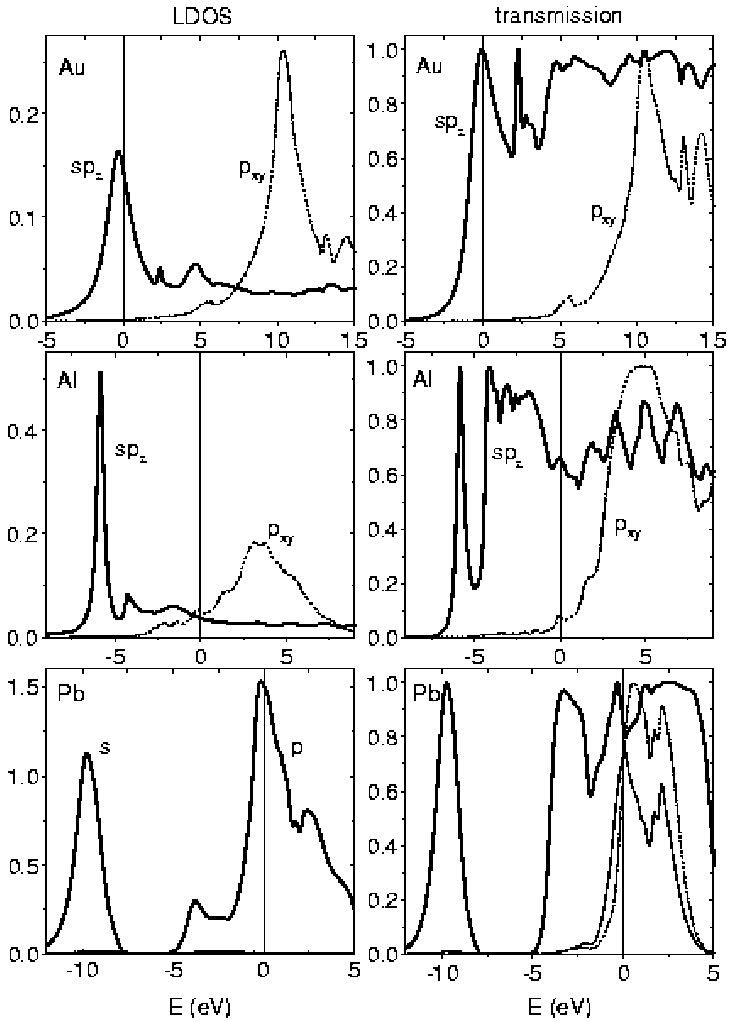


Conductance steps



Results for one-atom

theoretical results



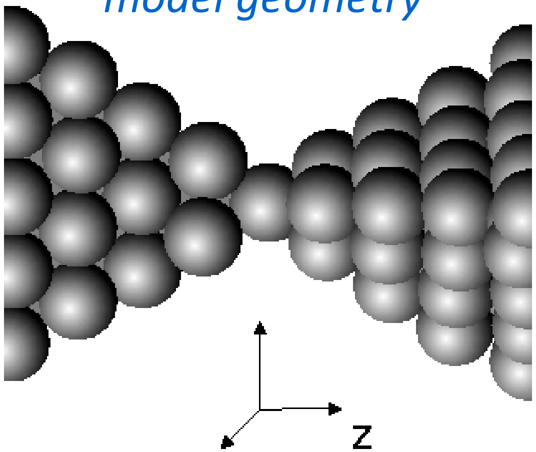
atomic configurations

Au → $5d^{10}\,6s^1$

Al → $3s^2\,3p^1$

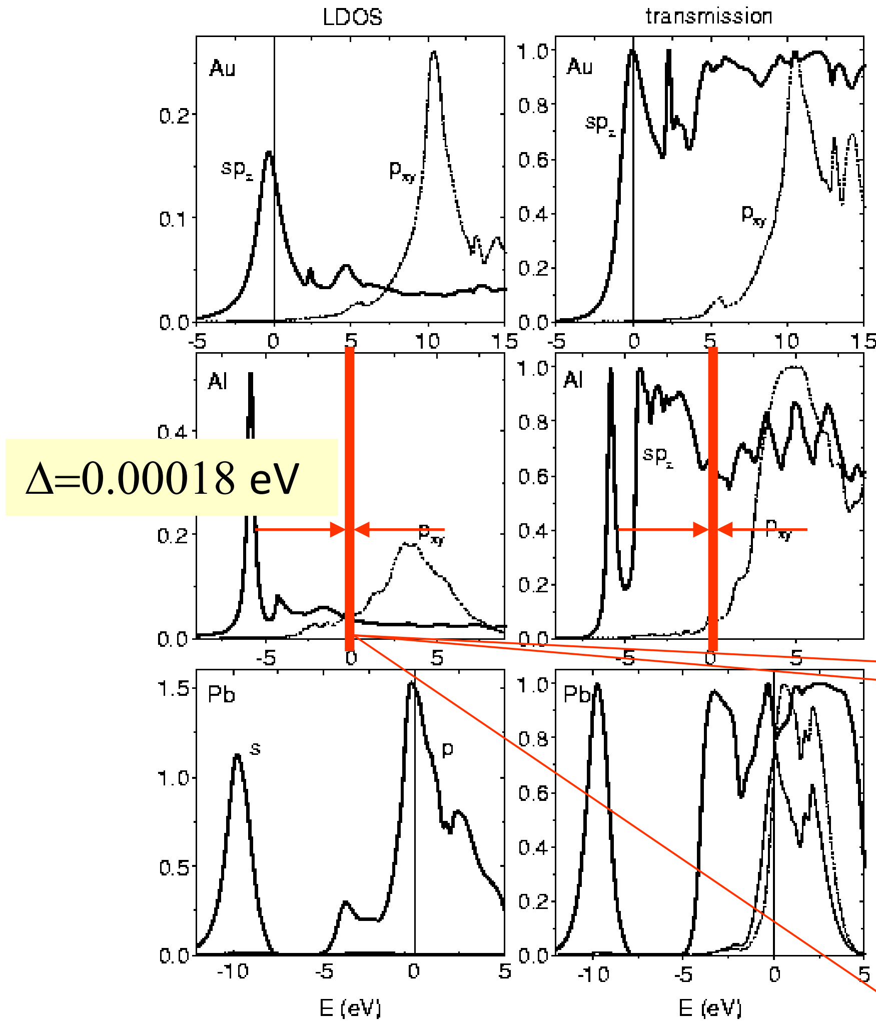
Pb → $4s^2\,4p^2$

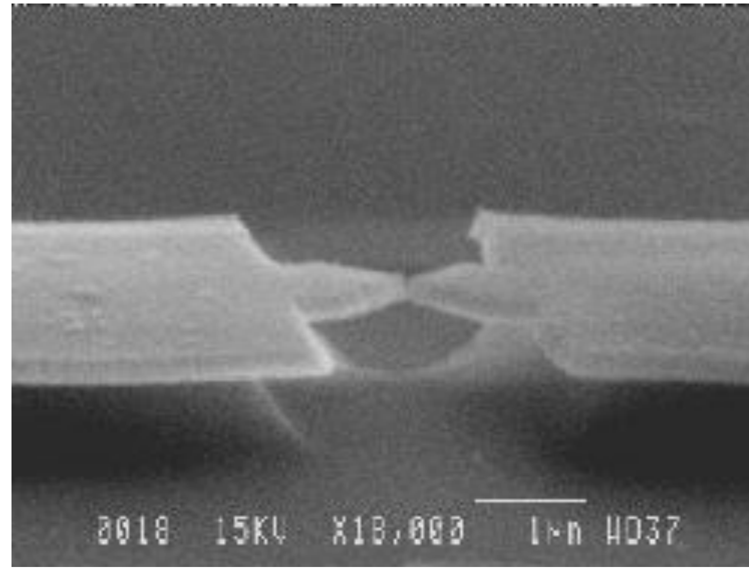
model geometry



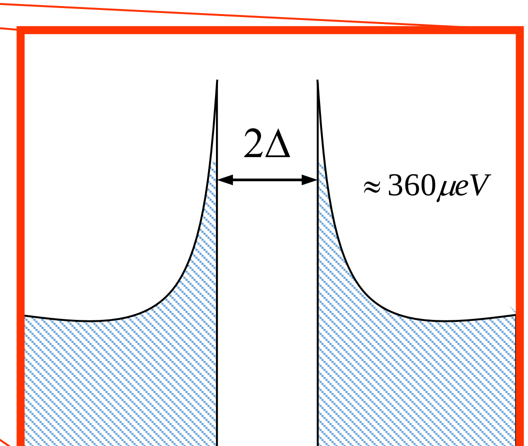
J.C. Cuevas, A. Levy Yeyati and A. Martín-Rodero, PRL 80, 1066 (1998)

Energy Scales

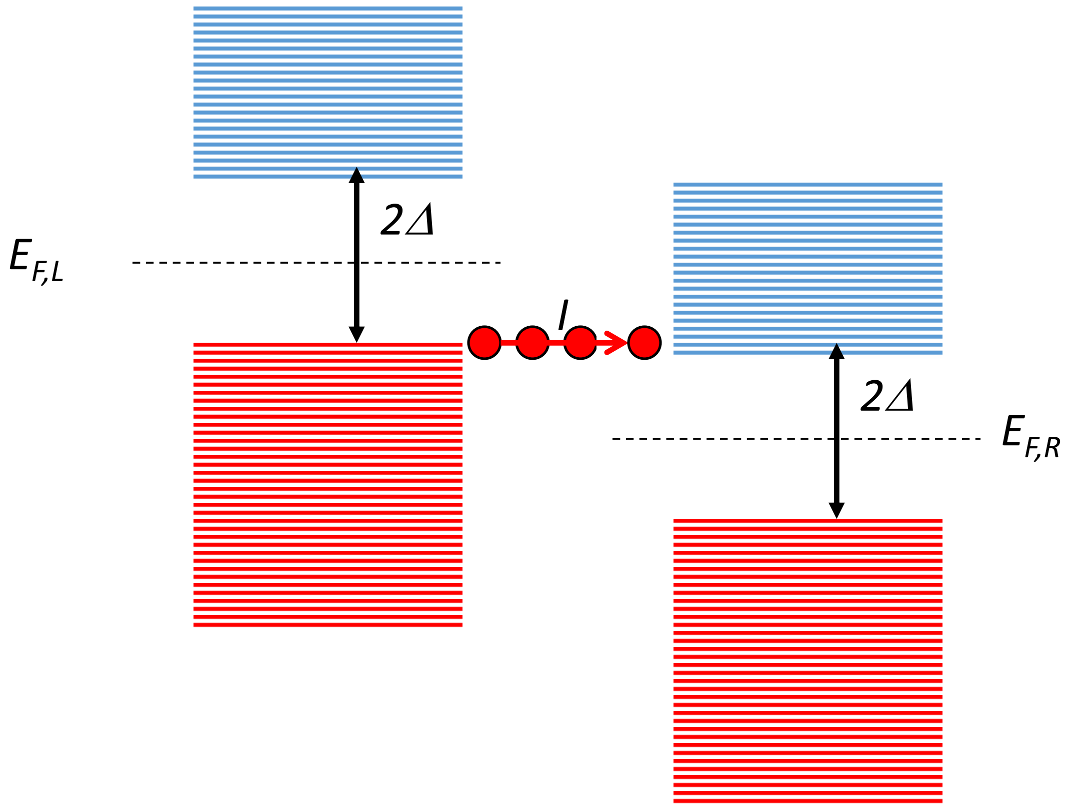


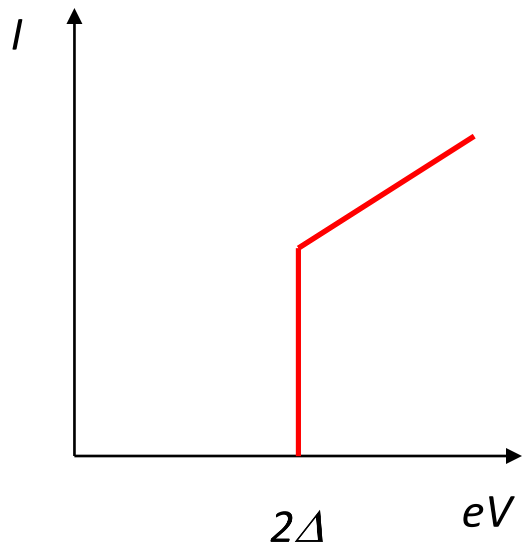


conduction channels not affected!



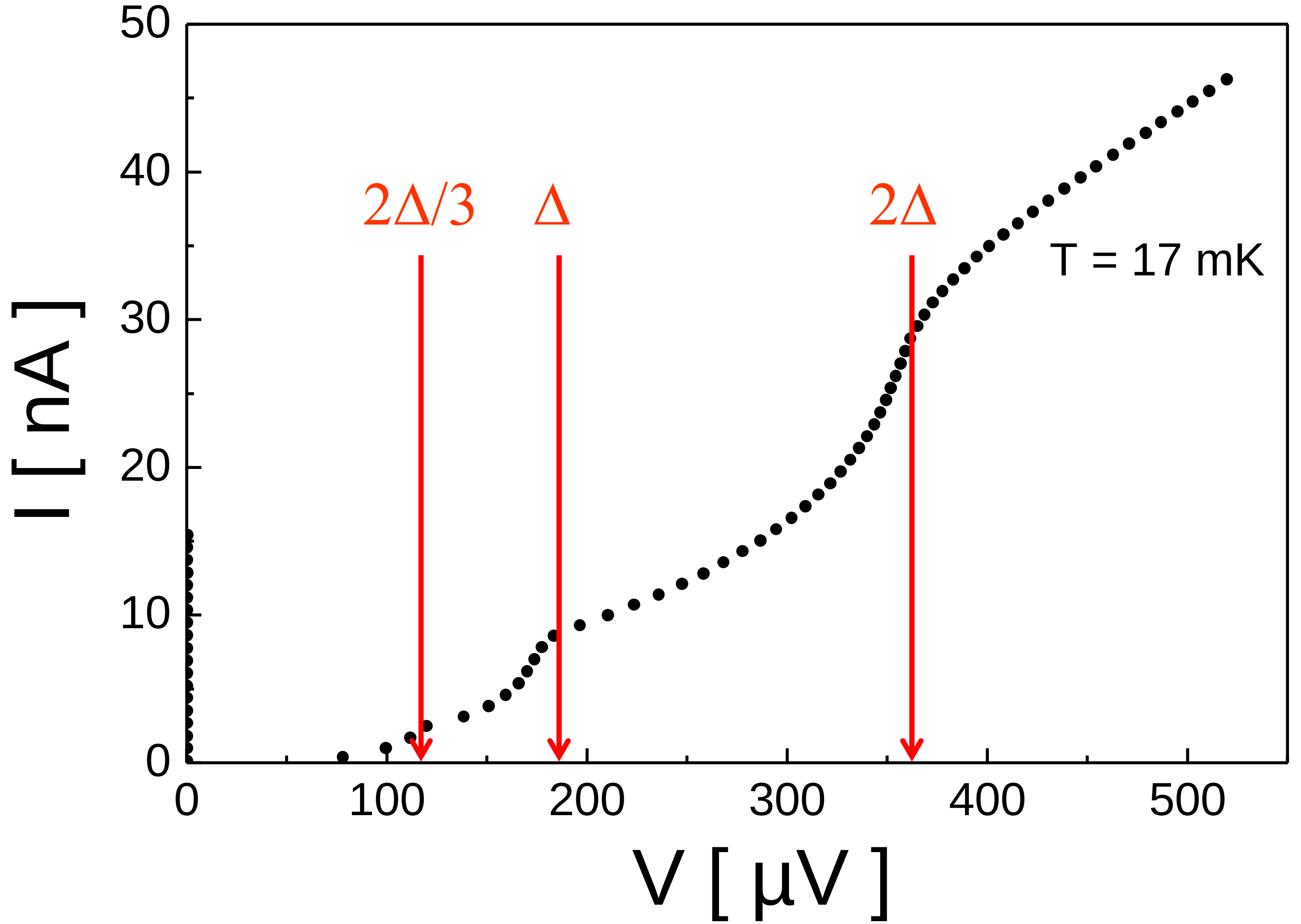
Transport between superconducting electrodes





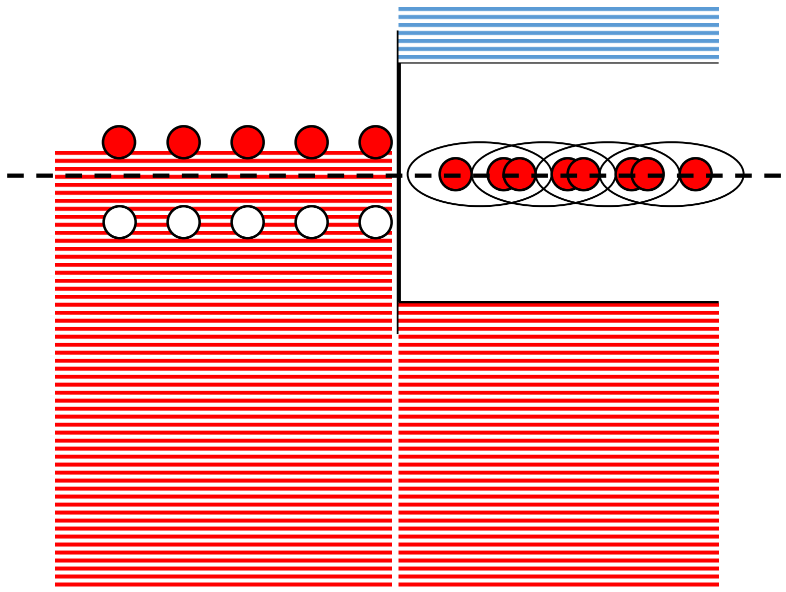
$E_{F,L} - E_{F,R} = eV > 2\Delta$

Experimental IV curves in superconducting contacts



one-atom
Al contact

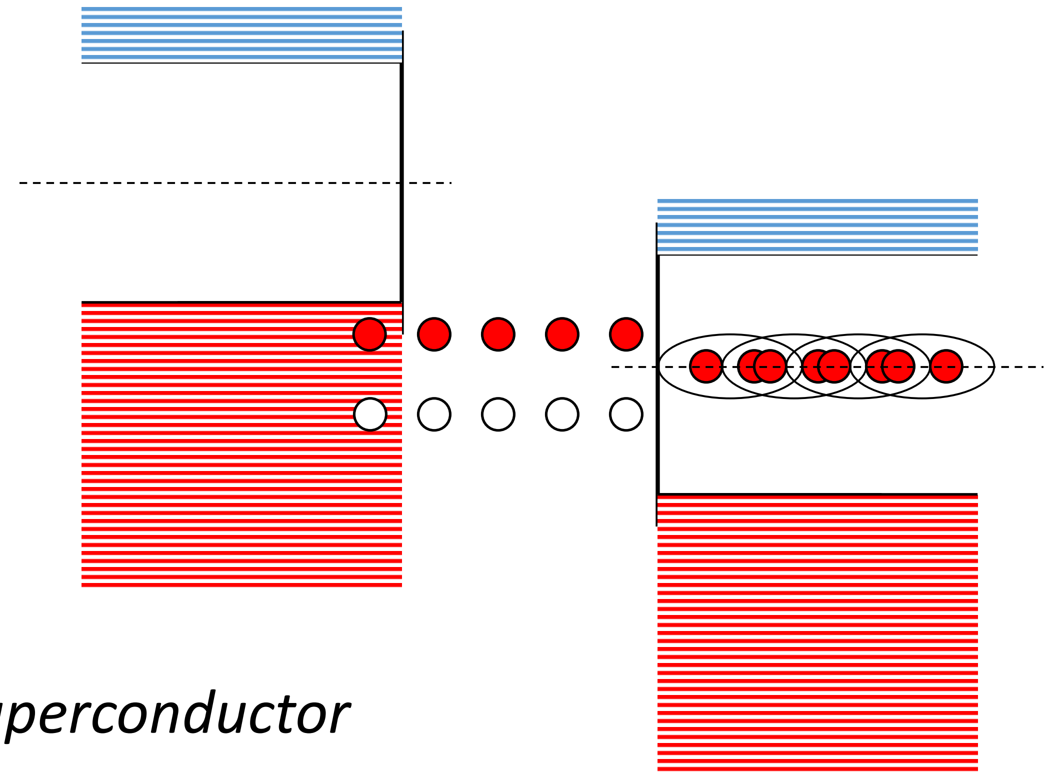
Andreev Reflection



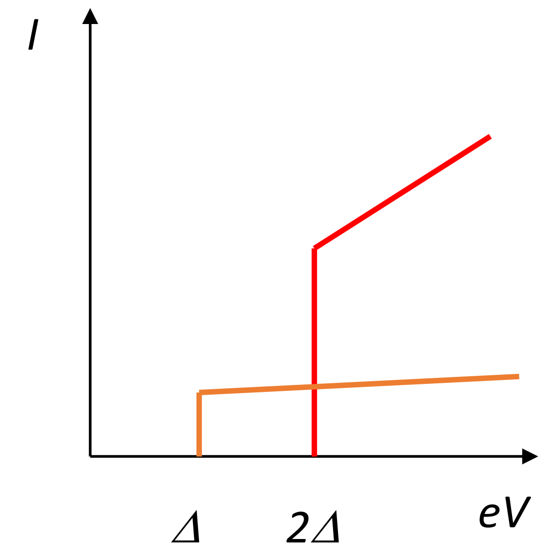
Normal metal *Superconductor*

Transmitted charge $2e$ *Probability* $\approx \tau^2$

Andreev reflection between superconducting electrodes



Superconductor

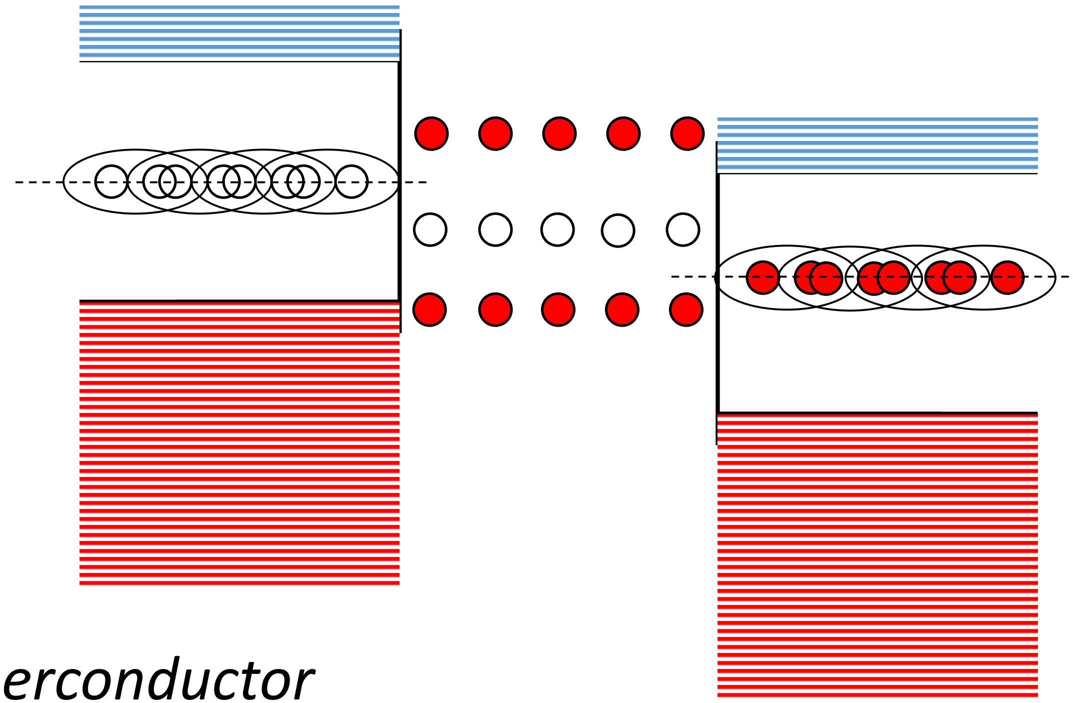


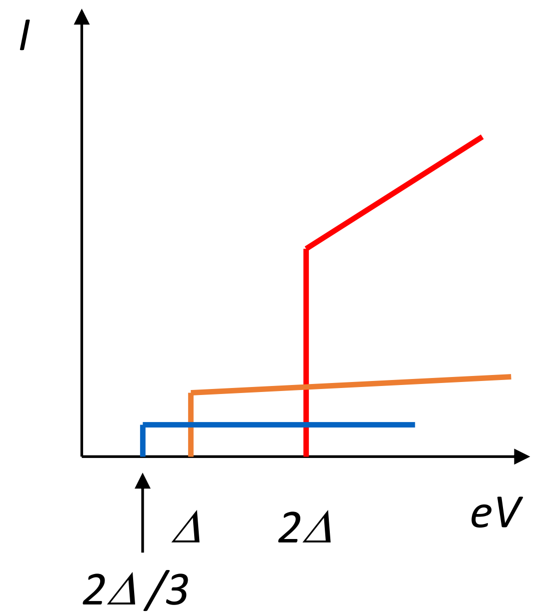
$eV>\Delta$

Superconductor

probability τ^2 *transmitted charge* $2e$

Multiple Andreev Reflection



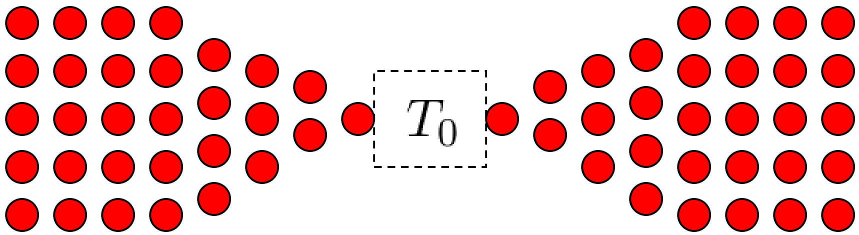


$eV > 2\Delta/3$

probability τ^3 transmitted charge $3e$

Model for single channel contact

Left lead



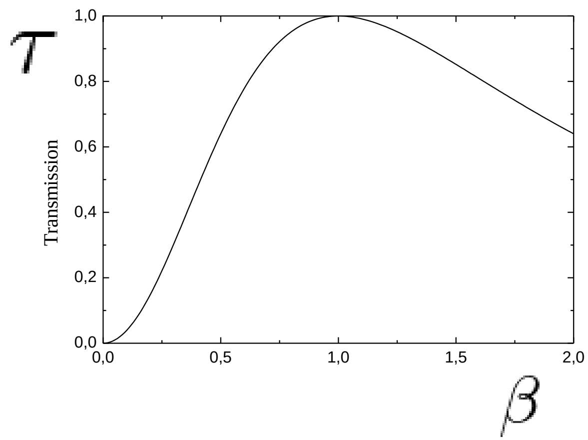
Right lead

$$H_{contact} = H_L + H_R + \underbrace{\sum_\sigma T_0 \left(c^\dagger_{L\sigma} c_{R\sigma} + \text{h.c.} \right)}_{H_T}$$

normal case $\quad H_{L,R} \rightarrow \rho(\omega) \simeq \dfrac{1}{\pi W}$

transmission coefficient →

$$\tau = \frac{4\beta}{(1+\beta)^2} \qquad \beta = \left(\frac{T_0}{W} \right)^2$$



J.C. Cuevas, A. Martín-Rodero and A. Levy Yeyati, PRB 54, 7366 (1996)

BCS leads

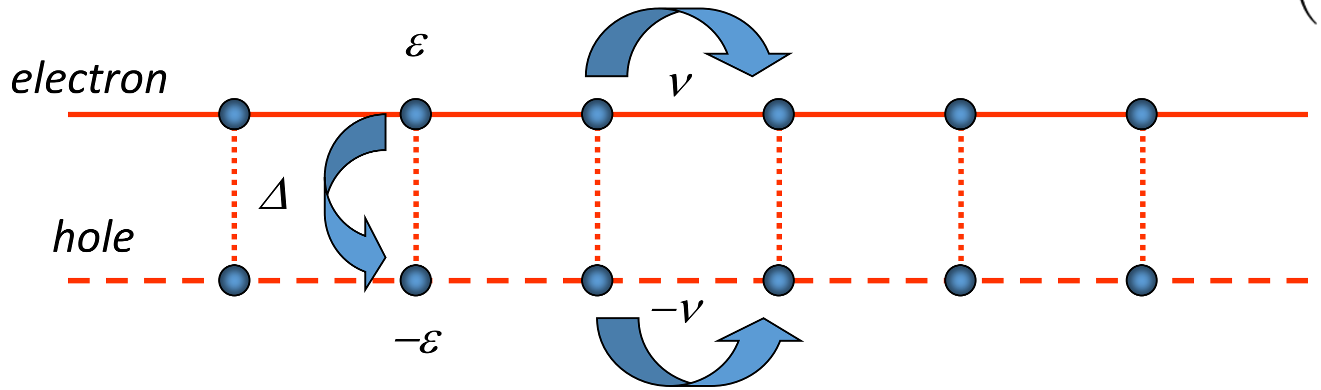
$$H_{BCS} = \sum_{k\sigma} \epsilon_k c^\dagger_{k\sigma} c_{k\sigma} + \sum_k \Delta c^\dagger_{k\uparrow} c_{-k\downarrow} + \mathrm{H.}c.$$ → *bulk leads*

Local basis

$$H_{L,R} = \sum_{i\sigma} \epsilon_i c^\dagger_{i\sigma} c_{i\sigma} + v c^\dagger_{i\sigma} c_{i\pm 1\sigma} + \sum_i \Delta_i c^\dagger_{i\uparrow} c_{i\downarrow} + \mathrm{h.c.}$$

Nambu rep $$H_{L,R} = \sum_i \Psi_i^\dagger \hat{\epsilon}_i \Psi_i + \Psi_i^\dagger \hat{v} \Psi_{i\pm 1}$$

$$\Psi_i = \begin{pmatrix} c_{i\uparrow} \\ c^\dagger_{i\downarrow} \end{pmatrix} \qquad \hat{\epsilon}_i = \begin{pmatrix} \epsilon_i & \Delta_i \\ \Delta_i^* & -\epsilon_i \end{pmatrix} \qquad \hat{v} = \begin{pmatrix} v & 0 \\ 0 & -v \end{pmatrix}$$



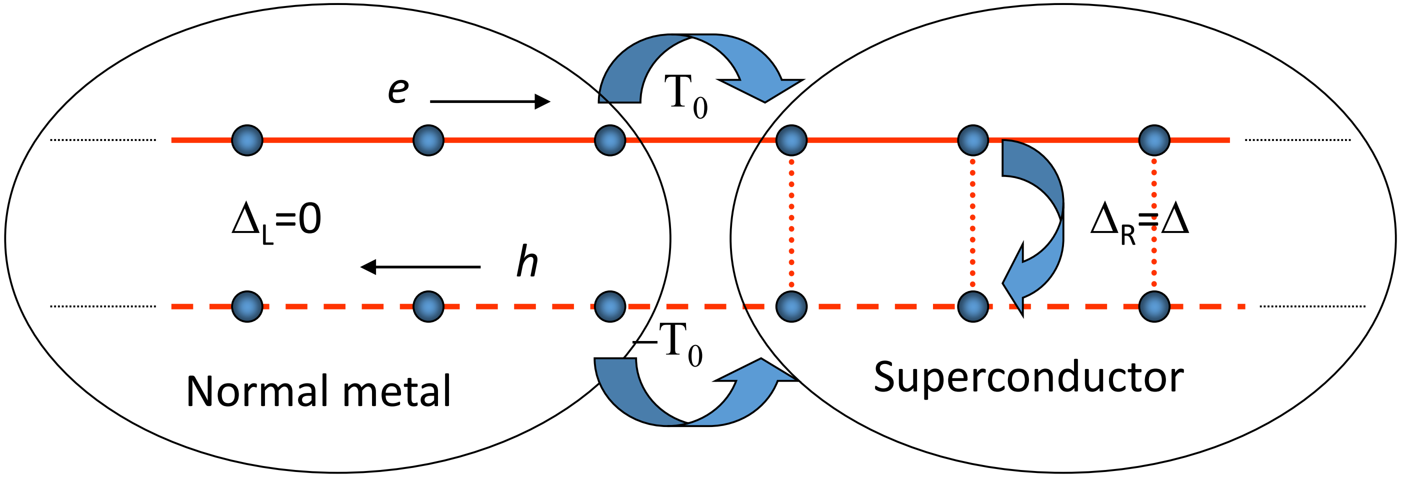
Boundary GFs

$$\omega, \Delta \ll v$$

$$\hat{g}^{r,a}(\omega) = [\omega \pm i0 - \hat{\epsilon} - \hat{v}\hat{g}^{r,a}(\omega)\hat{v}]^{-1}$$

$$\hat{g}^{r,a}(\omega) \simeq \frac{1}{v} \left(\frac{-(\omega \pm i0)\sigma_0 + \Delta\sigma_x}{\sqrt{\Delta^2 - (\omega \pm i0)^2}} \right)$$

Coupling two leads: NS interface

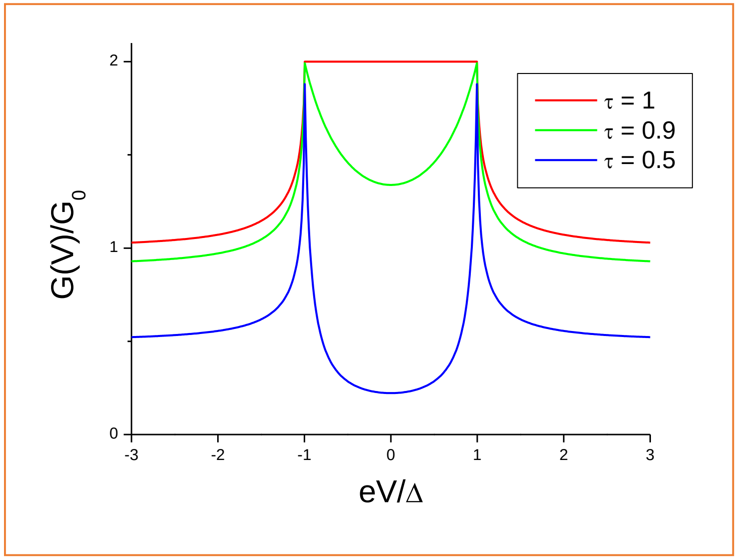


Andreev reflection

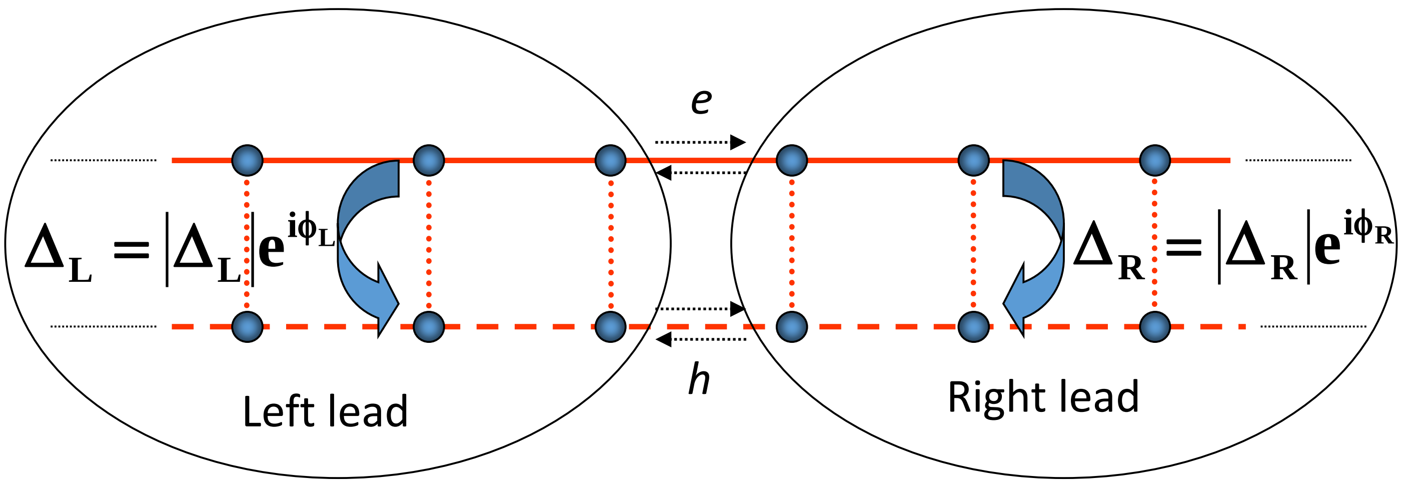
$$R_A(E) = \frac{\tau^2\Delta^2}{\tau^2E^2 + (\Delta^2 - E^2)(2-\tau)^2} \quad |E| \leq \Delta$$

$$G_{NS}(V) = \frac{4e^2}{h} R_A(eV) \quad e|V| \leq \Delta$$

Blonder, Tinkham & Klapwijk, PRB 25, 4515 (1982)

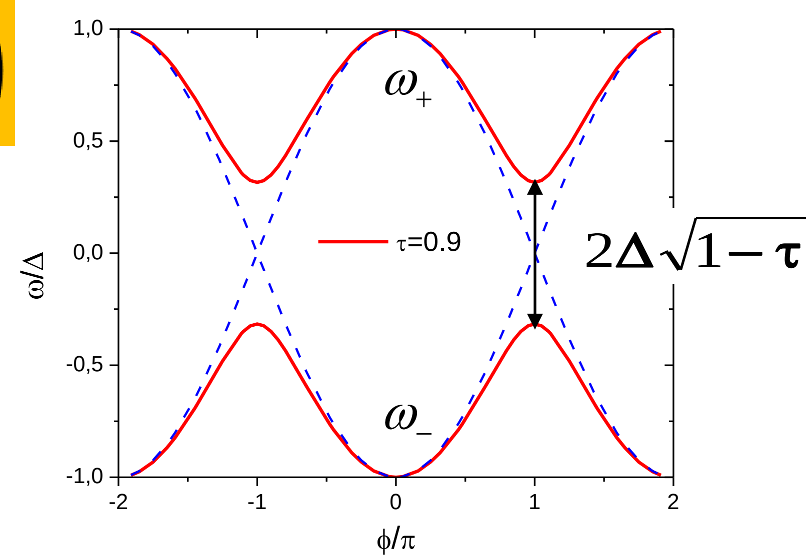


Coupling two superconductors: relevance of phase difference



$$\phi = \phi_L - \phi_R$$

$$\omega_{\pm} = \pm\Delta\sqrt{1 - \tau\sin^2\left(\frac{\phi}{2}\right)}$$



supercurrent (T=0)

$$I = \frac{e}{\hbar}\frac{\partial\omega_-}{\partial\phi} = \frac{e\Delta\tau}{2\hbar}\frac{\sin(\phi)}{\sqrt{1 - \tau\sin^2\left(\frac{\phi}{2}\right)}}$$

$I \propto \sin(\phi)$

$\tau \to 0$

$I \propto \mathrm{sgn}(\pi - \phi)\sin\left(\frac{\phi}{2}\right) \quad (\tau \to 1)$

Voltage biased SC contact: intrinsic time dependence

$$H_{L,R} \to H_{L,R} - \mu_{L,R} N_{L,R} \qquad eV = \mu_L - \mu_R$$

$$\phi_{L,R} \to \phi_{L,R} + \int dt \frac{2e\mu_{L,R}}{\hbar} \qquad \Rightarrow \frac{\partial \phi}{\partial t} = \frac{2eV}{\hbar} = \omega_0$$

Josephson frequency

Gauge transformation: eliminate time dependence from leads

$$H_T \to H_T = \sum_\sigma T_0 \left(c^\dagger_{L\sigma} c_{R\sigma} e^{i\phi(t)/2} + c^\dagger_{R\sigma} c_{L\sigma} e^{-i\phi(t)/2} \right)$$

Nambu form $\quad H_T = \Psi^\dagger_L \hat{T}_{LR}(t) \Psi_R + \Psi^\dagger_R \hat{T}_{RL} \Psi_L$

$$\hat{T}_{LR} = T_0 \begin{pmatrix} e^{i\phi(t)/2} & 0 \\ 0 & -e^{-i\phi(t)/2} \end{pmatrix} = T^*_{RL}$$

Current operator

$$I(t) = \frac{ie}{\hbar} \left[\Psi^\dagger_L \hat{T}_{LR}(t) \Psi_R - \text{h.c.} \right]$$

$$\langle I \rangle(t) = \frac{e}{\hbar} \mathrm{Tr} \left[\sigma_z \left(\hat{T}_{LR} \hat{G}^{+-}_{LR}(t,t) - \hat{T}^*_{LR} \hat{G}^{+-}_{RL}(t,t) \right) \right]$$

Coupled integral equations for full GFs

$$\hat{G}^{r,a} = \left(\hat{1} + \hat{G}^{r,a} \otimes \hat{\Sigma}^{r,a}\right) \otimes \hat{g}^{r,a}$$

$$\hat{\Sigma}^{r,a}_{LR} = \left(\hat{\Sigma}^{r,a}_{RL}\right)^* = \hat{T}_{LR}(t)$$

$$\hat{G}^{+-} = \left(\hat{1} + \hat{G}^{r} \otimes \hat{\Sigma}^{r}\right) \otimes \hat{g}^{+-} \otimes \left(\hat{1} + \hat{\Sigma}^{a} \otimes \hat{G}^{a}\right)$$

Double Fourier transformation

$$\hat{G}(t,t') = \frac{1}{2\pi} \int\int d\omega d\omega' e^{-i\omega t + i\omega' t'} \hat{G}(\omega,\omega')$$

$$\hat{G}(\omega,\omega') \rightarrow \sum_n \hat{G}_{n0}(\omega)\delta\left(\omega - \omega' + n\frac{\omega_0}{2}\right)$$

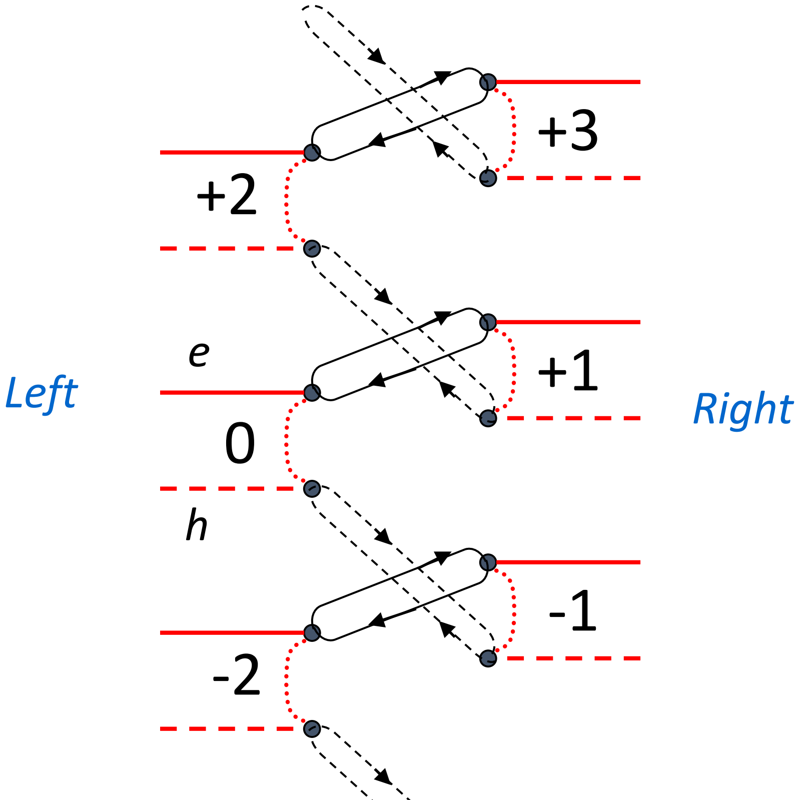
$$\langle I\rangle(t) = \frac{e}{\hbar}\mathrm{Tr}\left[\sigma_z\left(\hat{T}_{LR}G^{+-}_{LR}(t,t) - \hat{T}^*_{LR}G^{+-}_{RL}(t,t)\right)\right]$$

$$\Longrightarrow I(t,V) = \sum_n I_n(V)e^{in\omega_0 t}$$

dc + ac components

Pictorial representation

$$\hat{\mathbf{T}}^{+}_{\mathbf{LR}} = \begin{pmatrix} \mathbf{T}_0 & 0 \\ 0 & 0 \end{pmatrix} = \hat{\mathbf{T}}^{-}_{\mathbf{RL}} \qquad \hat{\mathbf{T}}^{-}_{\mathbf{LR}} = \begin{pmatrix} 0 & 0 \\ 0 & -\mathbf{T}_0 \end{pmatrix} = \hat{\mathbf{T}}^{+}_{\mathbf{RL}}$$



$$\hat{\mathbf{G}}_{00}(\omega) = \left[\hat{\mathbf{g}}_0^{-1} - \hat{\mathbf{T}}^{+}_{\mathbf{LR}} \hat{\mathcal{G}}_1 \hat{\mathbf{T}}^{-}_{\mathbf{RL}} - \hat{\mathbf{T}}^{-}_{\mathbf{LR}} \hat{\mathcal{G}}_{-1} \hat{\mathbf{T}}^{+}_{\mathbf{RL}} \right]^{-1}$$

$$\hat{\mathcal{G}}_1(\omega) = \left[\hat{\mathbf{g}}_1^{-1} - \hat{\mathbf{T}}^{+}_{\mathbf{RL}} \hat{\mathcal{G}}_2 \hat{\mathbf{T}}^{-}_{\mathbf{LR}} \right]^{-1}$$

$$\hat{\mathcal{G}}_{-1}(\omega) = \left[\hat{\mathbf{g}}_{-1}^{-1} - \hat{\mathbf{T}}^{-}_{\mathbf{RL}} \hat{\mathcal{G}}_{-2} \hat{\mathbf{T}}^{+}_{\mathbf{LR}} \right]^{-1}$$

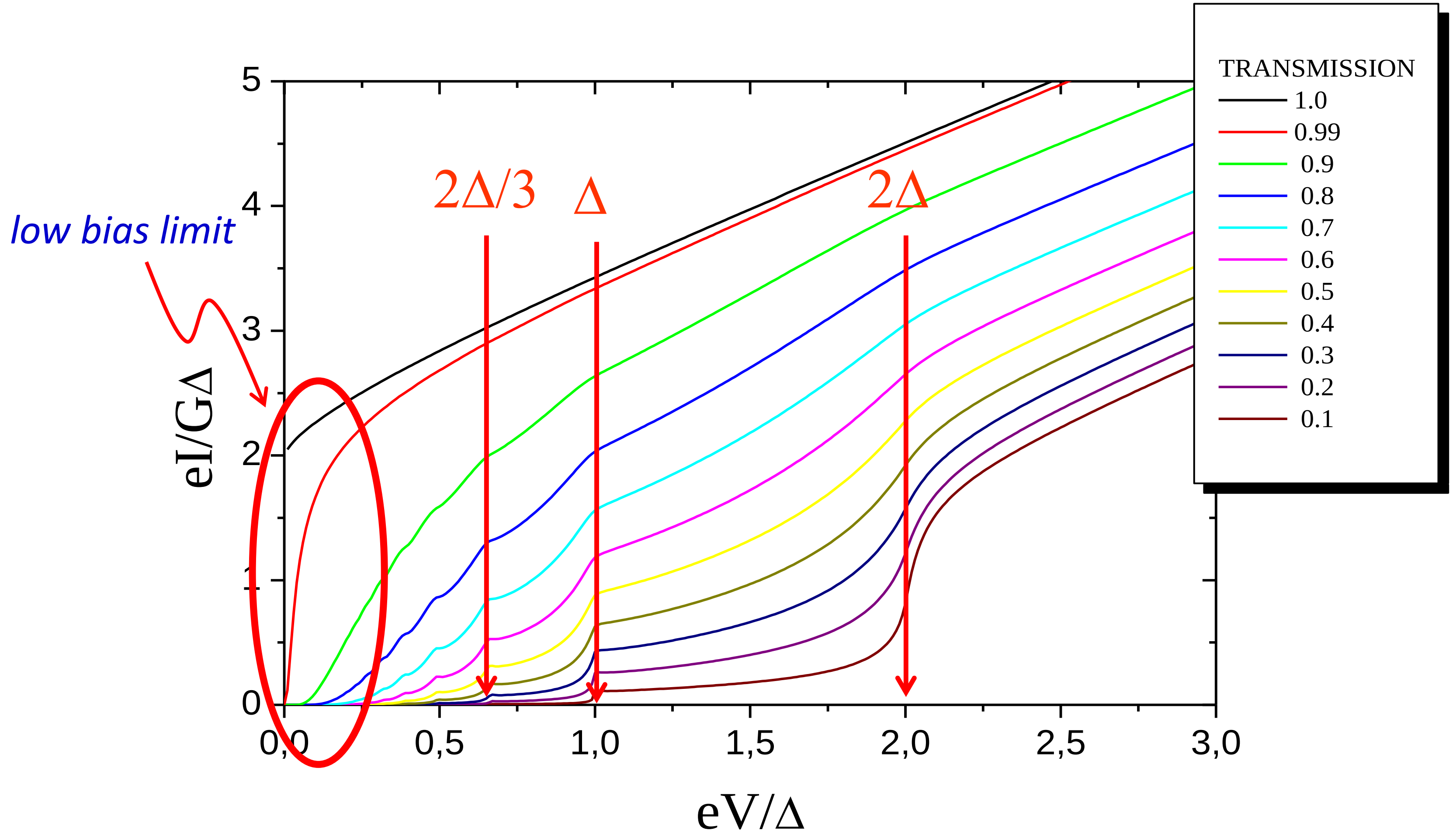
$$\vdots$$

$$\hat{\mathcal{G}}_{\mathbf{n}}(\omega) = \left[\hat{\mathbf{g}}_1^{-1} - \hat{\mathbf{T}}^{+}_{\mathbf{LR}} \hat{\mathcal{G}}_{\mathbf{n}+1} \hat{\mathbf{T}}^{-}_{\mathbf{RL}} \right]^{-1}$$

$$\hat{\mathcal{G}}_{-\mathbf{n}}(\omega) = \left[\hat{\mathbf{g}}_1^{-1} - \hat{\mathbf{T}}^{-}_{\mathbf{LR}} \hat{\mathcal{G}}_{-\mathbf{n}-1} \hat{\mathbf{T}}^{+}_{\mathbf{RL}} \right]^{-1}$$

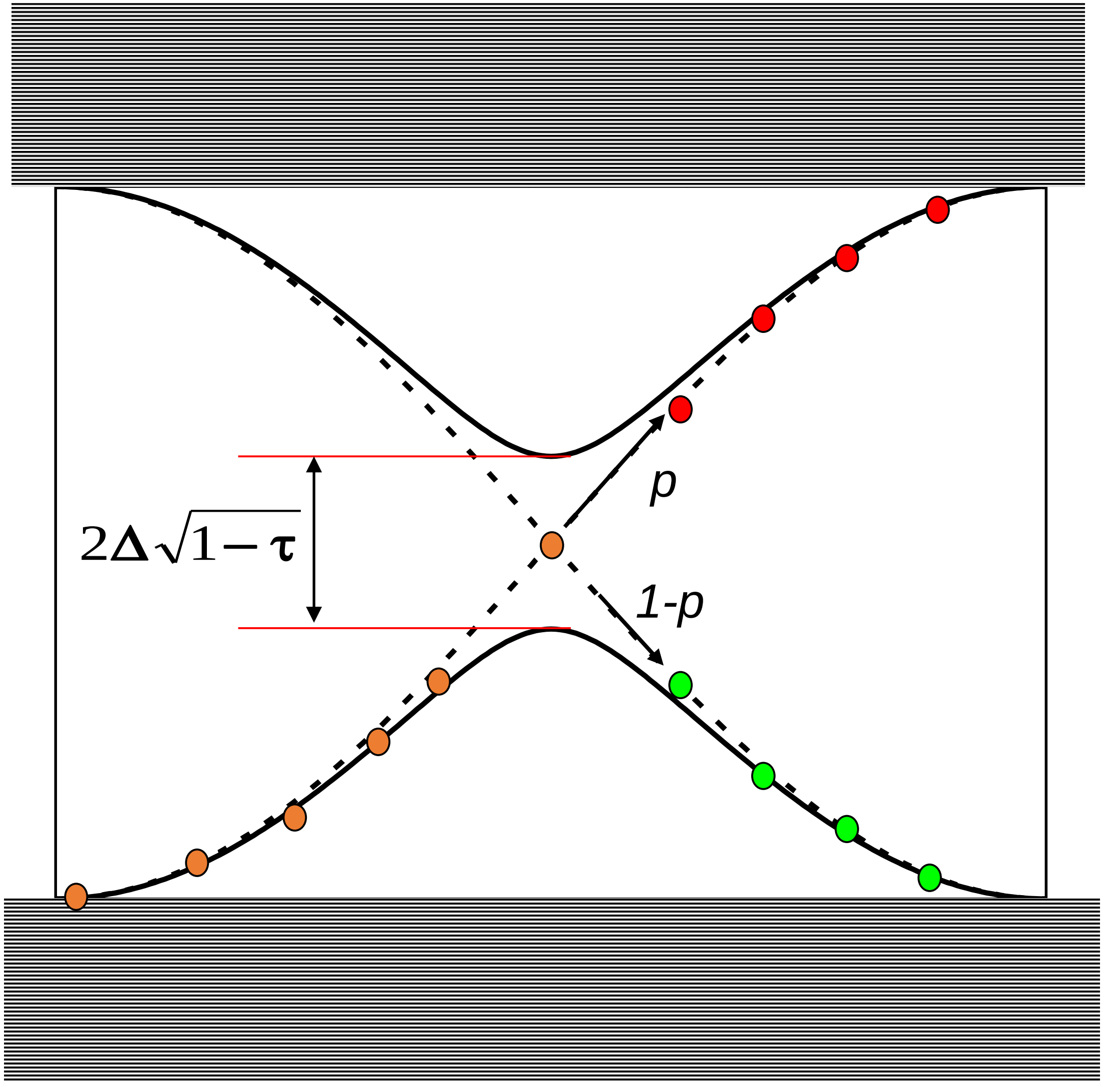
truncation $\boldsymbol{n} >> 2\Delta / \boldsymbol{V}$

Theoretical IV curves for superconducting contacts



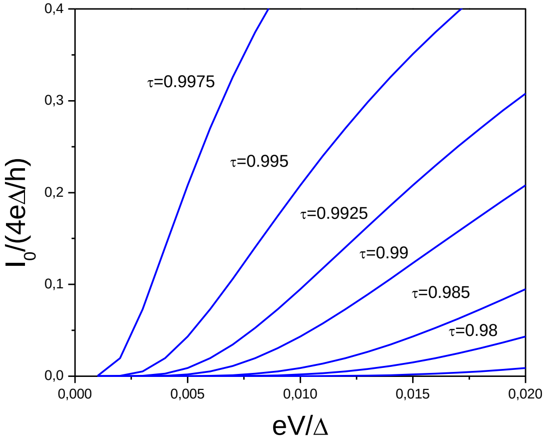
J.C. Cuevas, A. Martín-Rodero and A. Levy Yeyati, PRB 54, 7366 (1996)
same results with different approach: Averin & Bardas (95), Shumeiko et al. (97)

Landau-Zener transitions between AS's

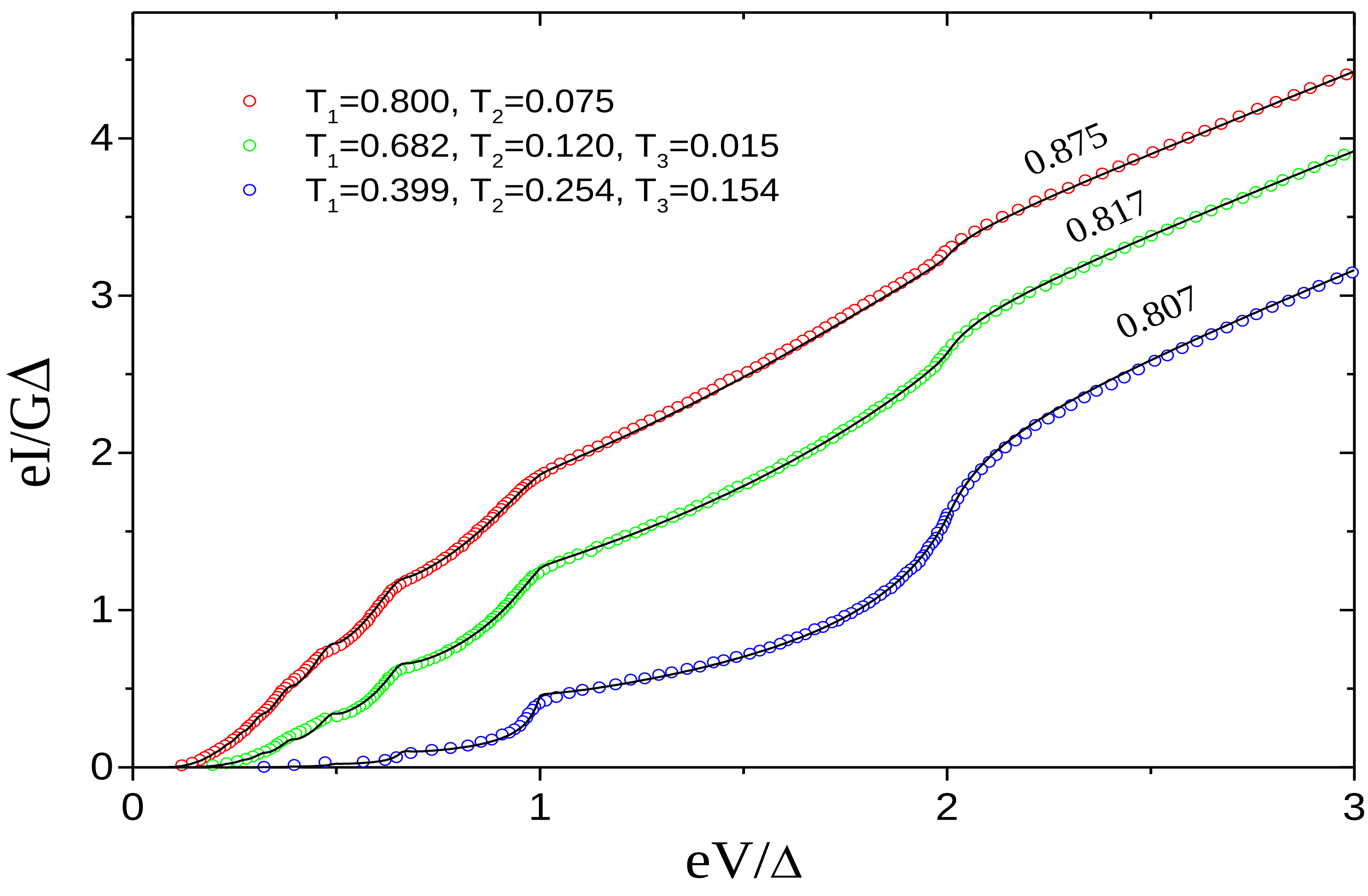


$$\mathbf{p} = \mathbf{e}^{-\frac{\pi\Delta r}{eV}} \qquad \mathbf{r} = 1 - \tau$$

$$< \mathbf{I(t)} >= \frac{4e\Delta}{h}\mathbf{p}$$

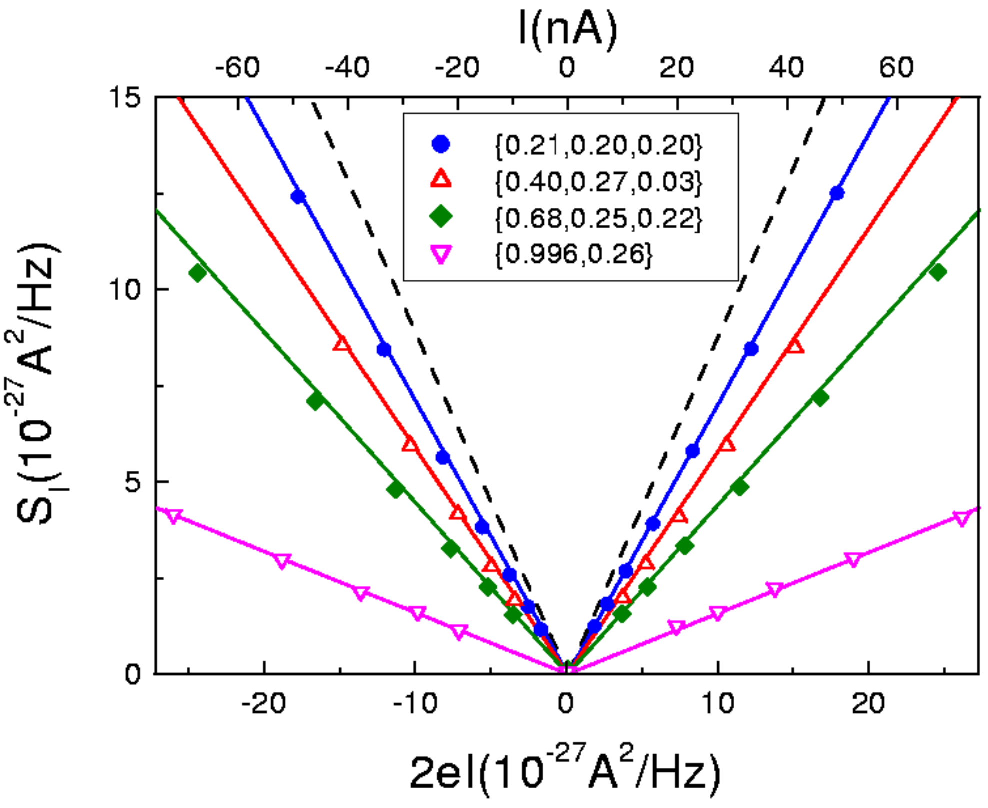


Fitting IV curves for Al contacts



E. Scheer et al, PRL 1997

Consistency with noise measurements



R. Cron et al, PRL 2001

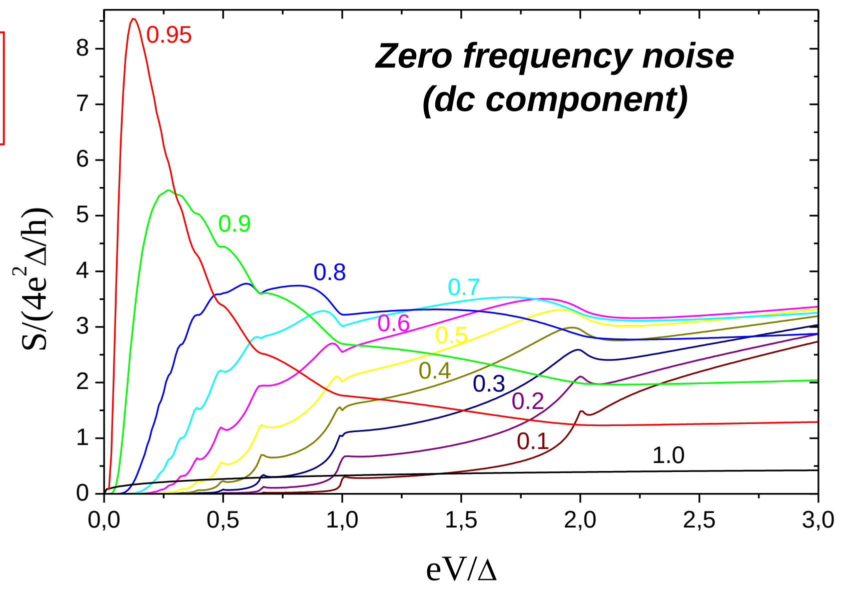
$$S(V) = \int \langle I(t)I(0)\rangle - \langle I\rangle^2 dt = 2eV\,\frac{2e^2}{h}\sum_n \tau_n(1-\tau_n)$$

Shot noise in superconducting contacts

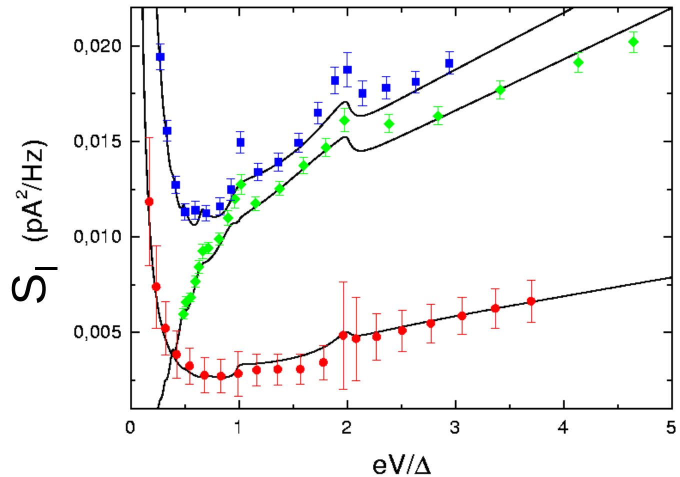
J.C.C, A.M.R and A.L.Y.
Y. Naveh and D. Averin } *PRL 82 (1999)*

$$S(0) = \int dt < \delta I(t)\delta I(0) + \delta I(0)\delta I(t) >$$

$$\delta I(t) = I(t) - < I(t) >$$



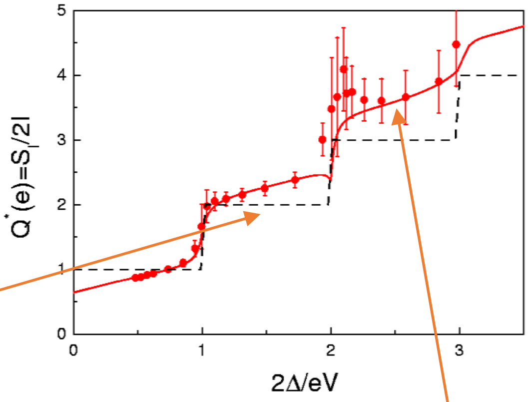
Comparison to experiments
(no adjustable parameters!)



R. Cron et al, PRL 2001

Effective charge $\mathbf{Q^* = S(0) / 2eI}$

tunnel limit $\mathbf{Q^* = Int[1 + 2\Delta / eV]}$



experimental values for $\tau_n = \{0.40, 0.27, 0.03\}$

Physical Review Focus

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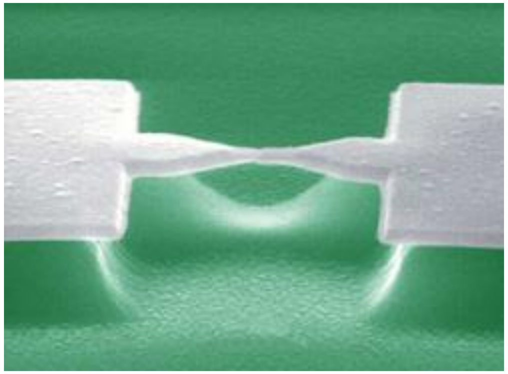
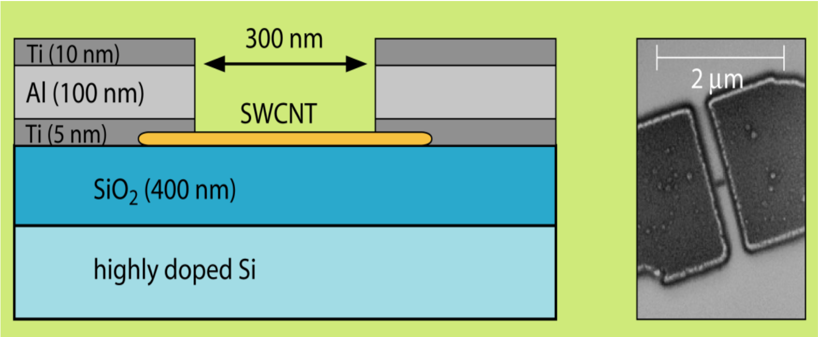
30 April 2001

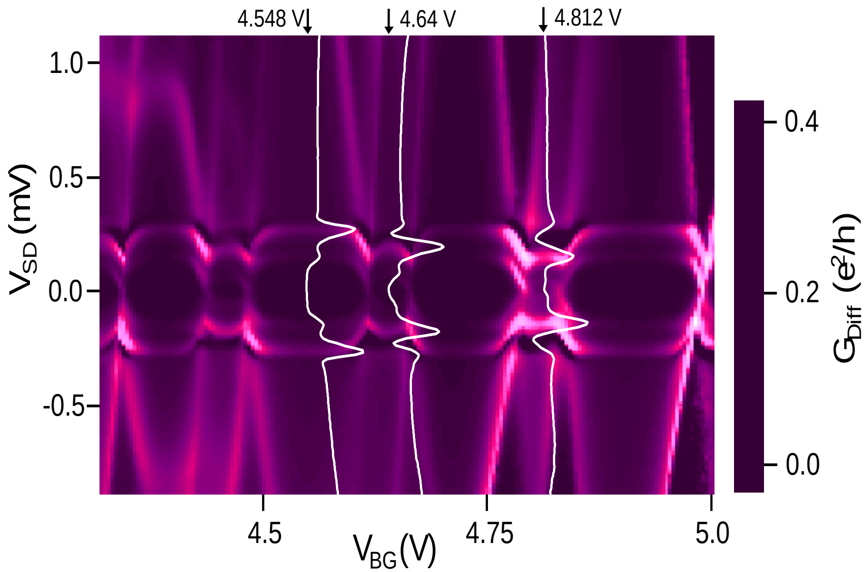
Electric Current in Big Chunks



MAR in hybrid nanostructures

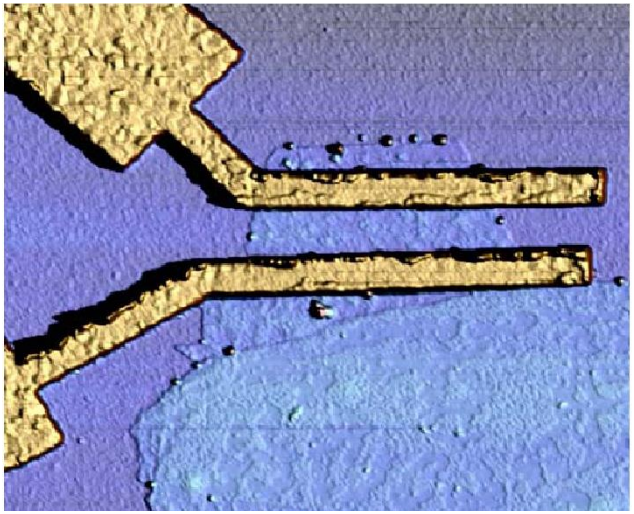
SWCNT + S leads

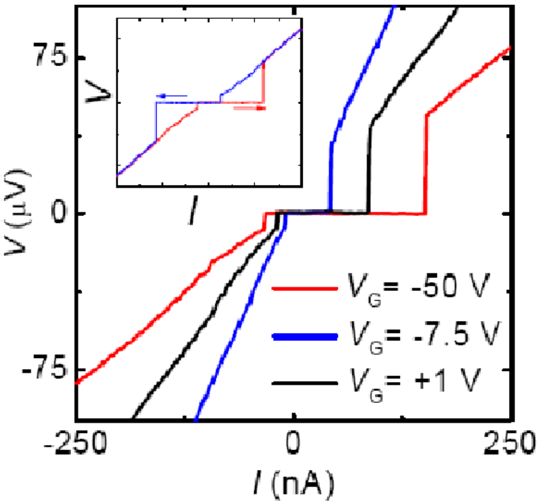




Eichler *et al.* **Phys. Rev. Lett.** (2007)

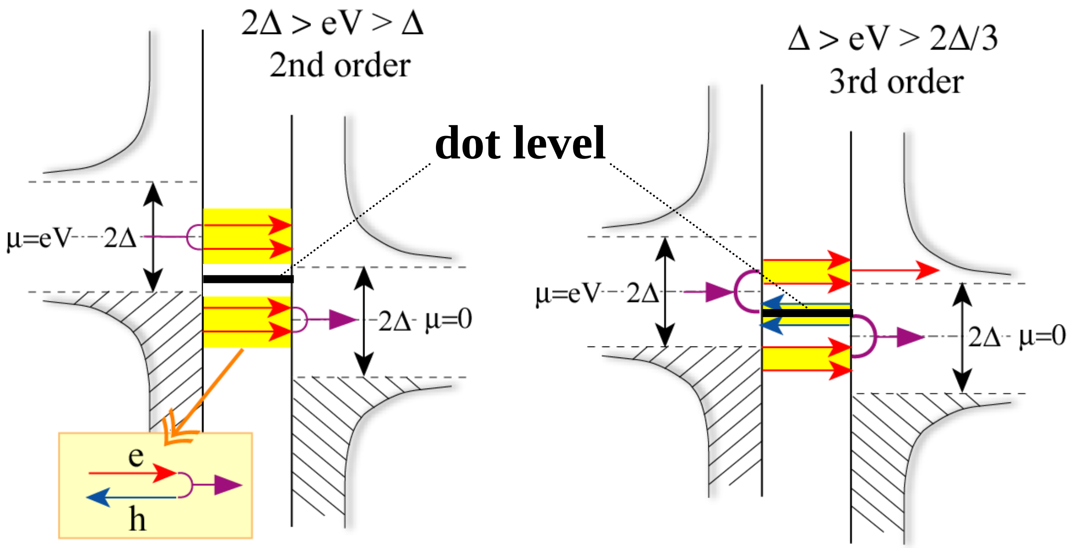
Graphene + S leads



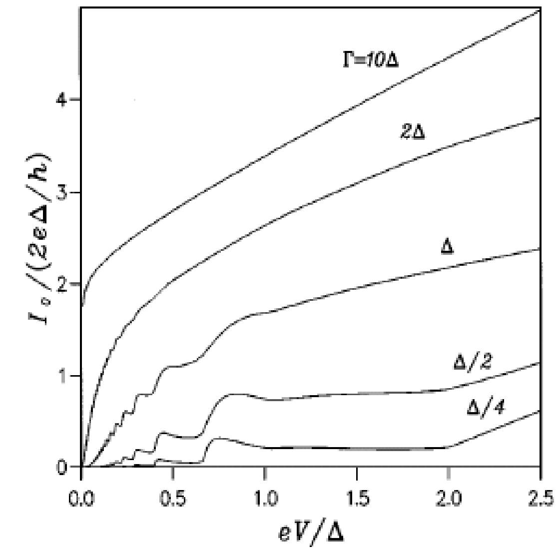


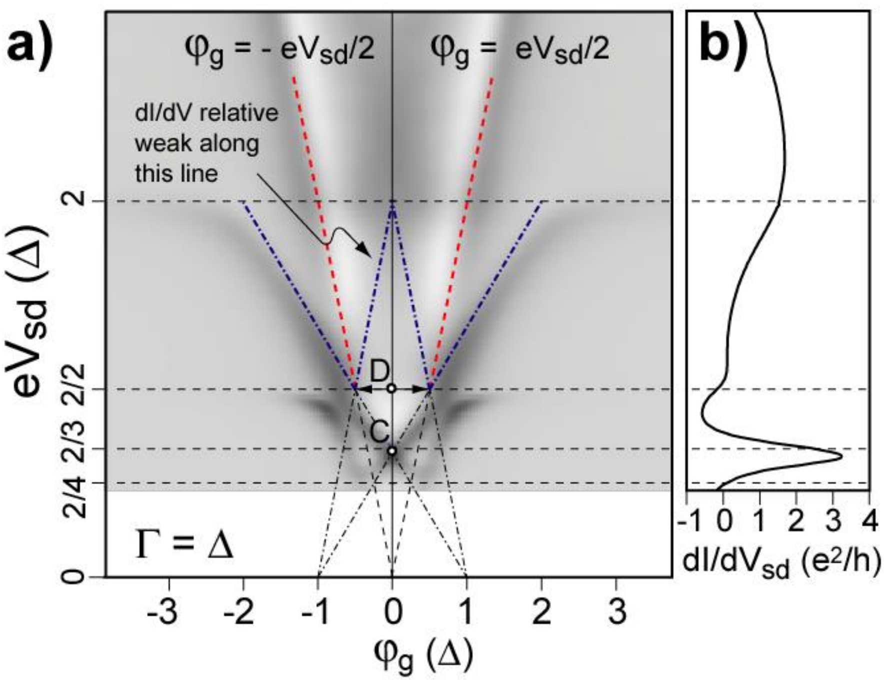
Heersche *et al.* **Nature** (2007)

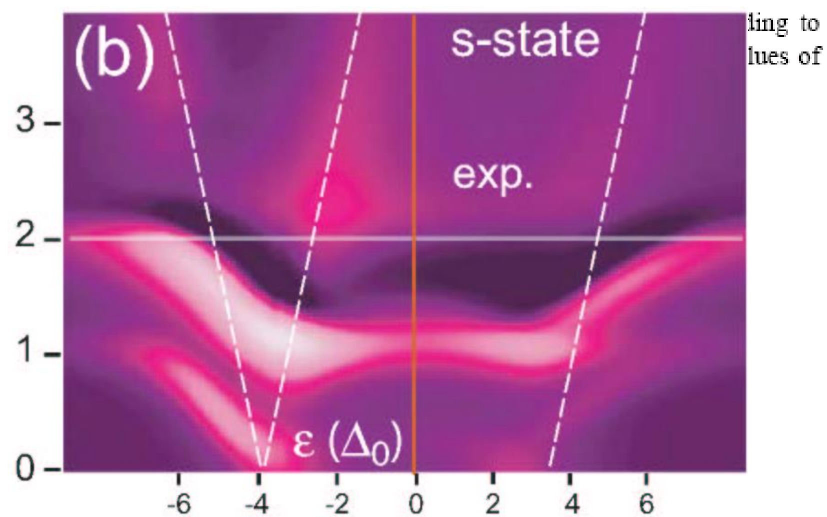
SQDS: resonant MAR



ALY et. al. PRB **55**, R3167 (97)

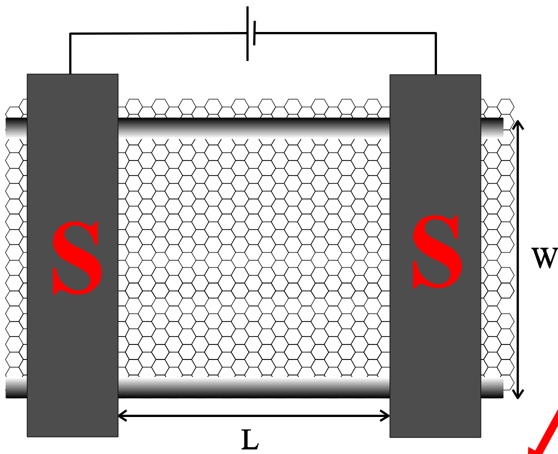


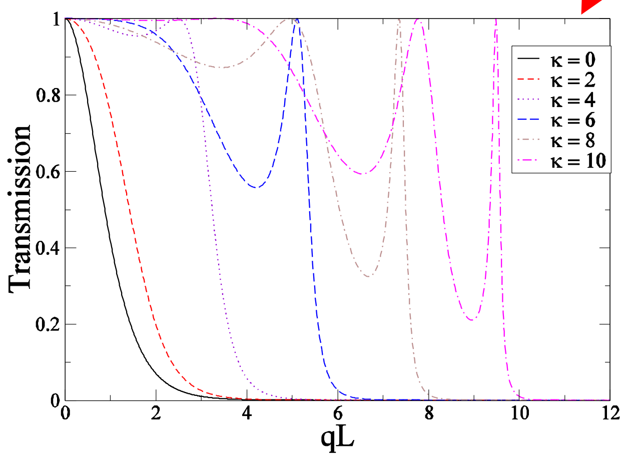




MAR in Graphene

J.C. Cuevas & A. Levy Yeyati PRB ***74***, R180501 (06)





$$\kappa = e|V_{\rm gate}|L/\hbar v_F$$

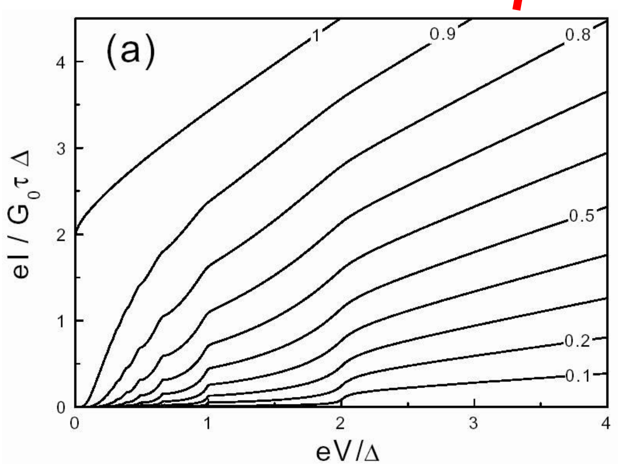
Normal Transmission

$$\tau_n = \left| \frac{2\delta^2 - 2\left(q_n - k_n\right)^2}{e^{k_n L}\left(q_n - k_n + i\delta\right)^2 + e^{-k_n L}\left(q_n - k_n - i\delta\right)^2} \right|^2$$

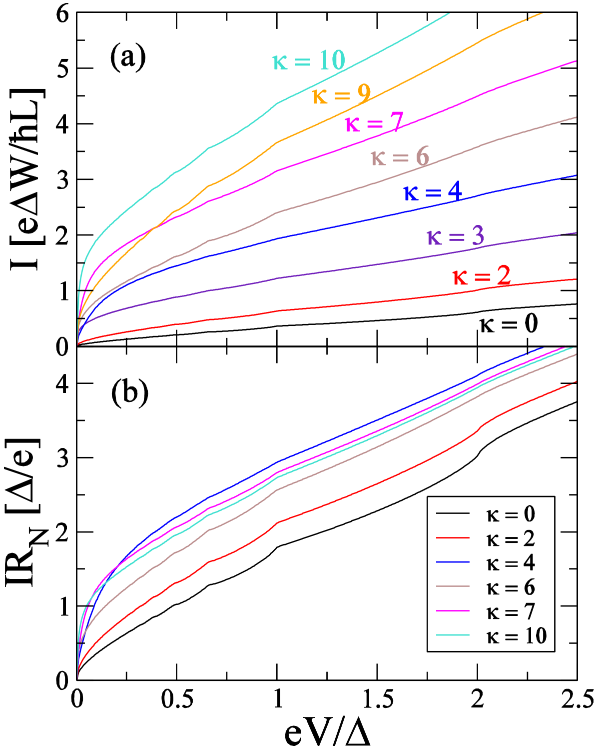
$$q_n = \frac{\pi}{W}\left(n + \frac{1}{2}\right) \qquad k_n = \sqrt{q_n^2 - \delta^2} \qquad \delta = e|V_{\rm gate}|/\hbar v_F$$

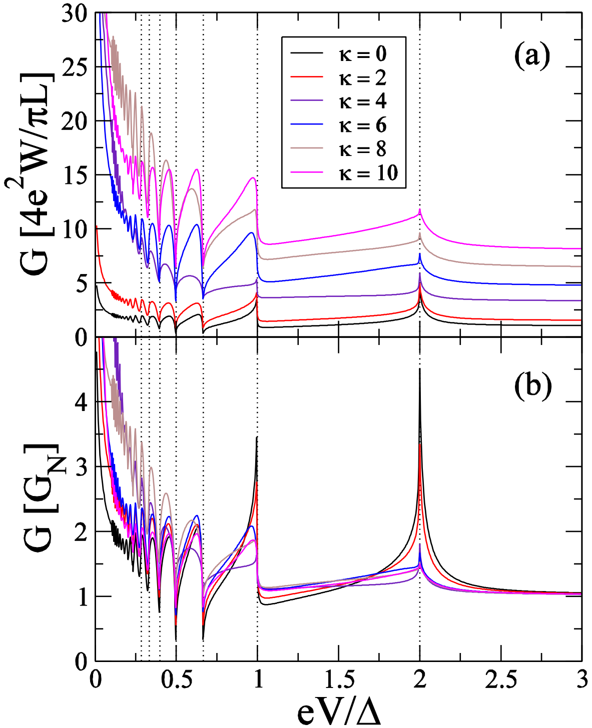
Channel superposition

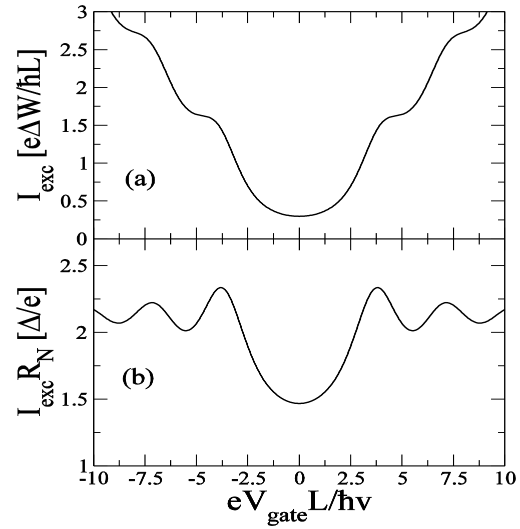
$$\mathbf{I(V,t) = 2\sum_{n=0}^{\infty} I(V,t,q_n)}$$



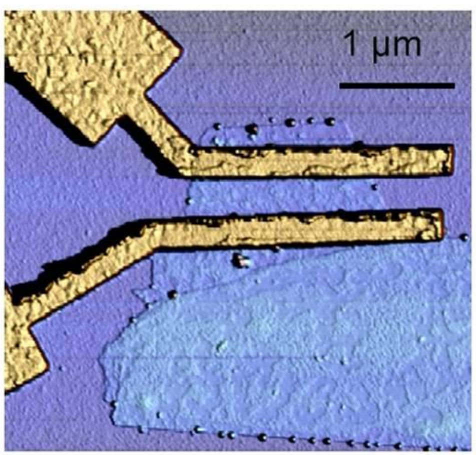
Excess current

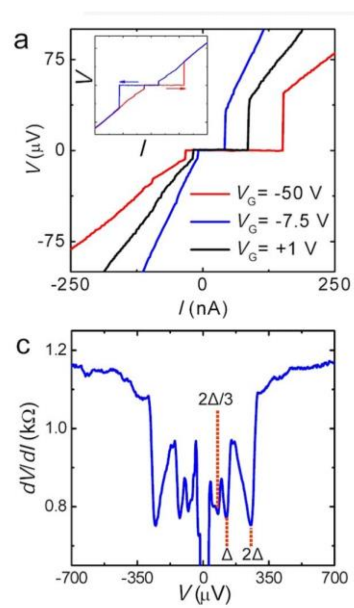




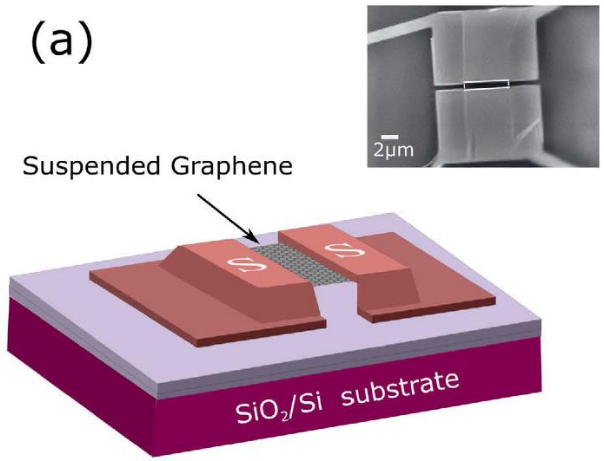


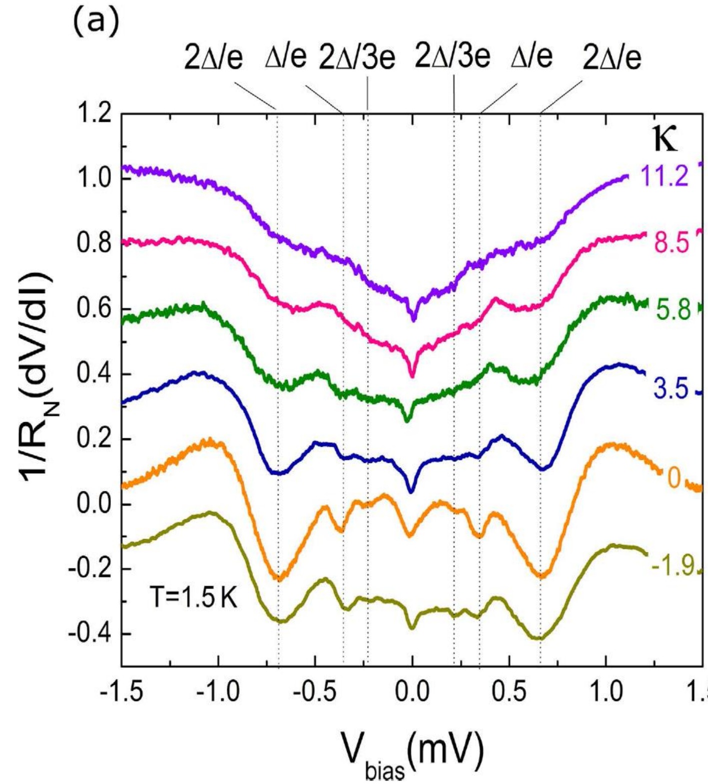
H.B. Heersche, et. al. Nature (2006).





Kumaravadiel & Du, Sci. Reps. (16)





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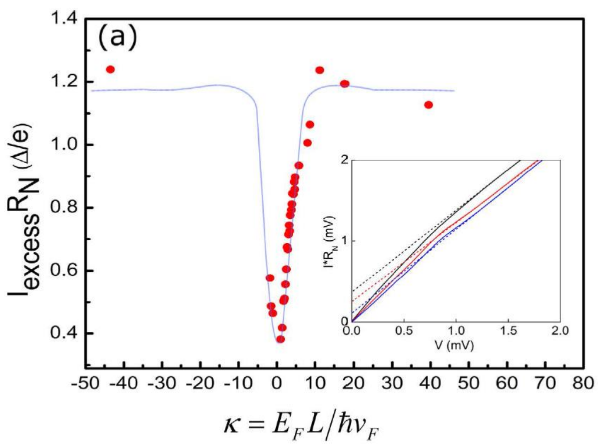
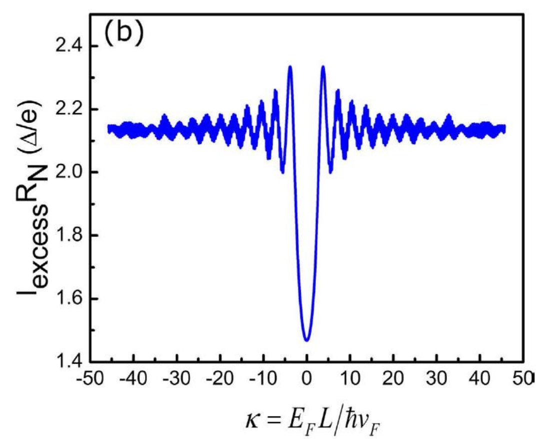


Excess current

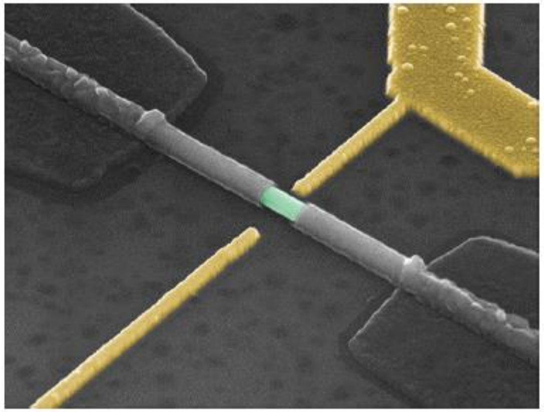
thy

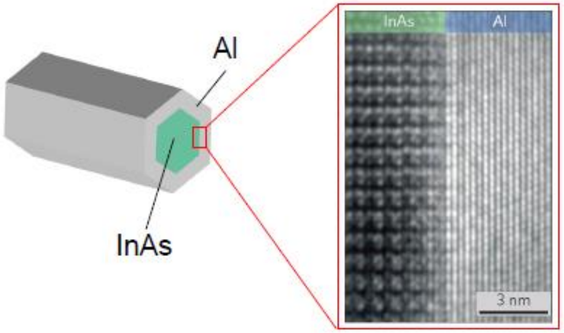


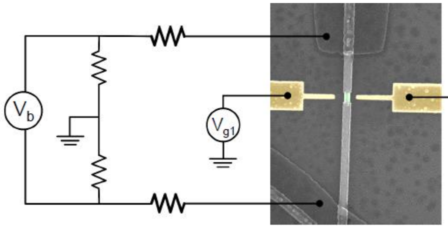
MAR in InAs/Al nanowires

Goffman et al, NJP (17)

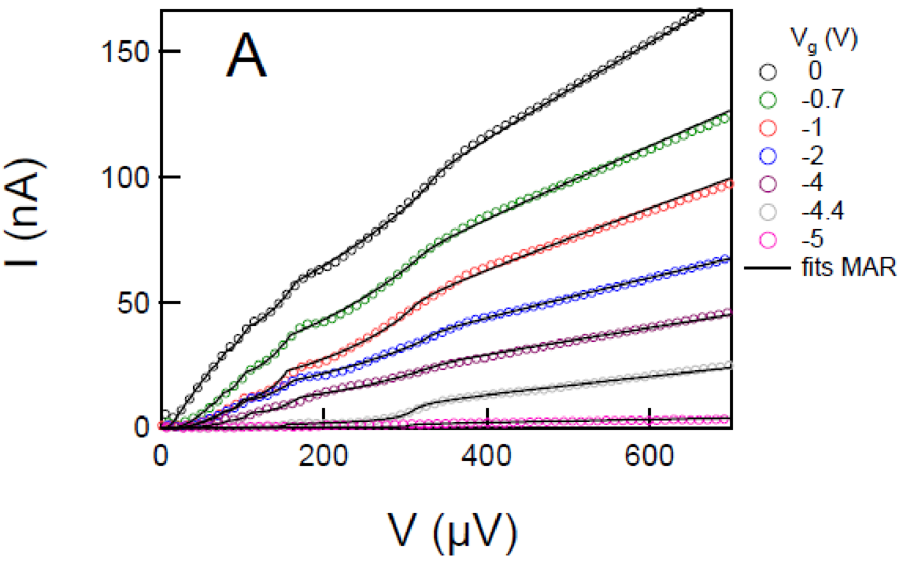
GATED InAs JOSEPHSON JUNCTION



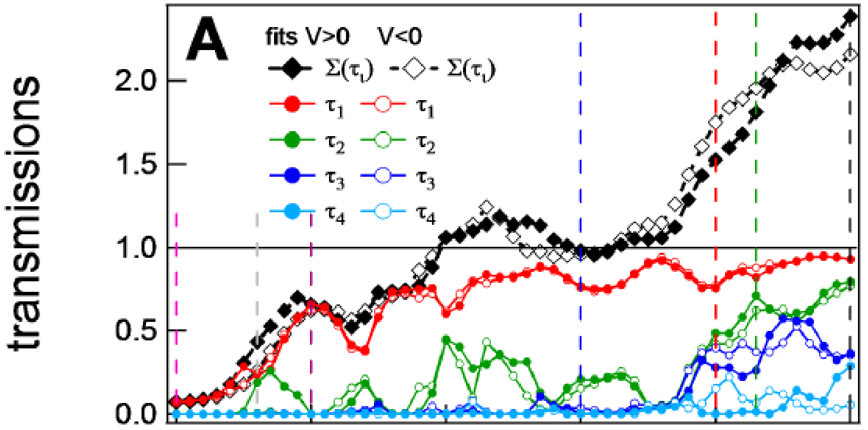




FITS WITH MAR THEORY



FITS PARAMETERS: TRANSMISSIONS



Conclusions (First lecture):

Basic introduction to Hamiltonian approach techniques

Application to superconducting atomic contacts: ideal test system for coherent MAR theory

Extension to other systems: hybrid carbon nanostructures (CNT QDs & graphene)

Need to learn more on interaction effects (tbc)