QUANTUM PARAMETRIC PHENOMENA IN CIRCUIT-QED

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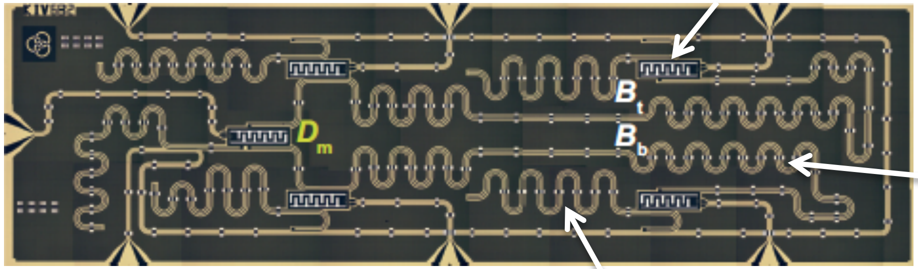
Superconducting electrical circuits

Architecture of SC quantum processors = network of Josephson elements (qubits) + resonators (coupling) + resonators (readout) + parametric amplifiers

5-qbit processors (Delft)



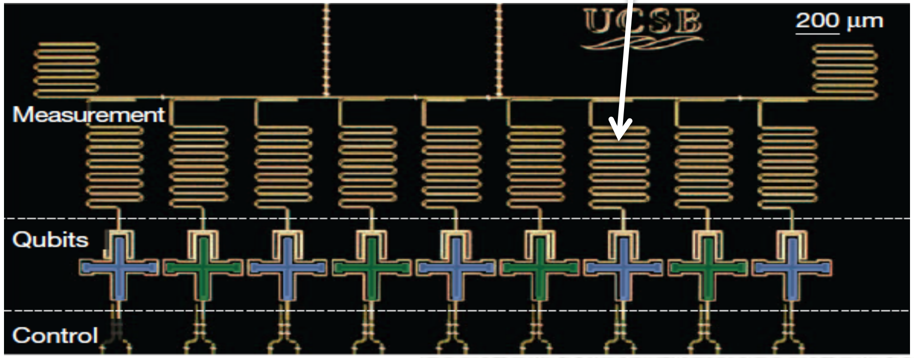
qubit

coupling

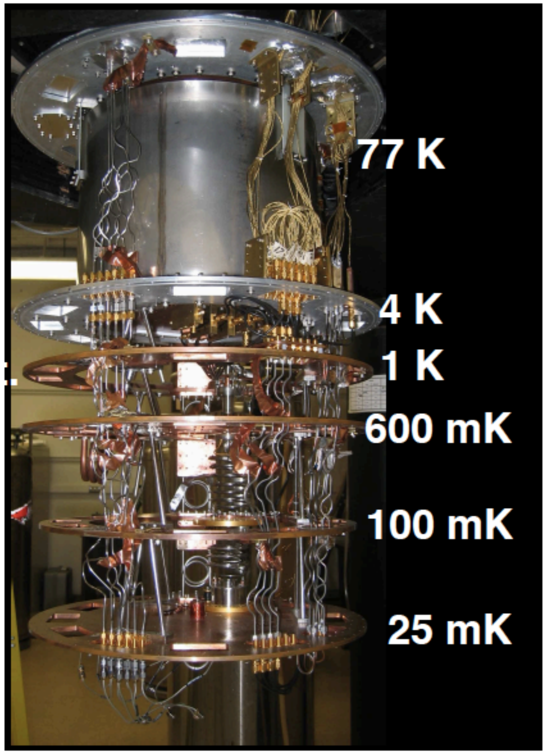
to paramp

readout

9-qbit processors (UCSB)



Ultra-low T setup



Typical qubit characteristics

- Qubit frequencies 2-10 GHz
- Currents 10 nA – 1 mkA
- Temperature < 50mK
- Decoherence time > 50 mksec

Content

❖ **Parametric resonance in electrical circuits: linear theory**
- parametric amplification
- quadrature squeezing

❖ **Quantum parametric resonance**
- amplification of quantum noise
- quantum squeezing
- BCS analogy
- Dynamical Casimir Effect

❖ **Nonlinear parametric resonance in tunable cavity**
- parametric oscillator
- photon condensation

❖ **Non-degenerate parametric resonance**
- frequency conversion
- optomechanics

PARAMETRIC RESONANCE

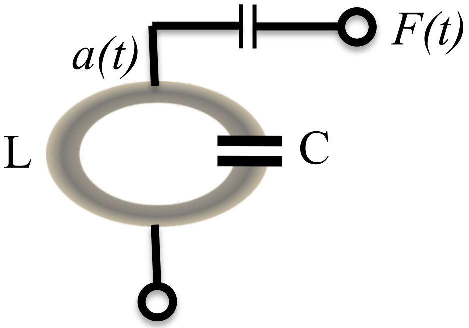
- Michael Faraday (1831) was the first to notice oscillations of one frequency being excited by forces of double the frequency, in the crispations (ruffled surface waves) observed in a wine glass excited to "sing".
  Phil. Trans. Royal Soc. (London), vol. 121, pages 299-318
- Melde (1859) generated parametric oscillations in a string by employing a tuning fork to periodically vary the tension at twice the resonance frequency of the string.
  Ann. Phys. Chem. vol. 109, pages 193-215

WIKIPEDIA
The Free Encyclopedia

- Parametric oscillation was first treated as a general phenomenon by Rayleigh (1883-87).
  Phil. Mag. vol.24, pages 145-159.
- Floquet theory 1883.
- One of the first to apply the concept to electric circuits was George Francis FitzGerald, who in 1892 tried to excite oscillations in an **LC circuit** by pumping it with a varying inductance provided by a dynamo.
- Parametric amplifiers (**paramps**) were first used in 1913-1915 for radio telephony from Berlin to Vienna and Moscow, and were predicted to have a useful future.
- The early paramps: varied inductances, varactor diodes, klystron tubes, **Josephson junctions**.

LINEAR PARAMETRIC RESONANCE

LC oscillator



$$\ddot{\varphi} + \omega_0^2\varphi + 2\gamma\dot{\varphi} = F(t)$$

$$\varphi = \frac{2e}{\hbar}\Phi$$ superconducting phase

$$F(t) = f(t)e^{-i\omega_0 t} + f^*(t)e^{i\omega_0 t}$$

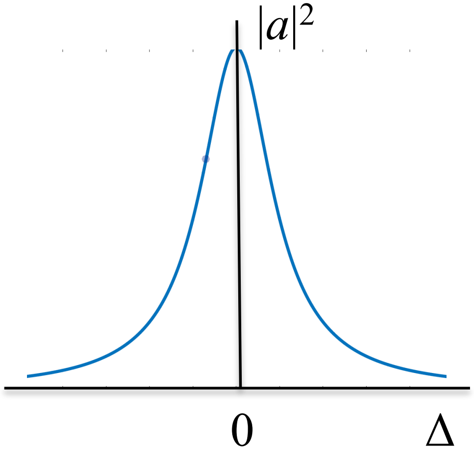
$$\varphi(t) = a(t)e^{-i\omega_0 t} + a^*(t)e^{i\omega_0 t}$$

slow varying
complex amplitudes

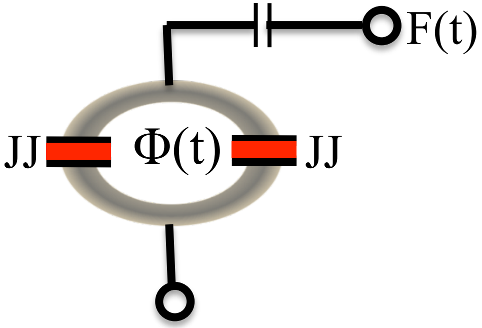
$$-i\dot{a} - i\gamma a = f(t)$$

$$f(t) = fe^{-i\Delta t}$$

$$a = -\frac{f}{\Delta + i\gamma}$$ oscillator response



Parametric LC oscillator



$$\omega_0^2 \to \omega_0^2\left(1 - \frac{4\epsilon}{\omega_0}\cos\Omega t\right) \qquad \varepsilon << \omega_0$$

$$\Omega = 2(\omega_0 + \delta) \qquad \delta << \omega_0$$

$\omega_{\mathrm{SQUID}} >> \Omega$:

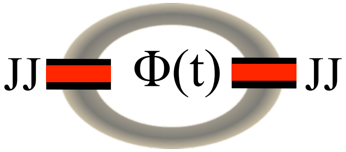
SQUID is variable inductance

Resonance approximation

$$2\cos\Omega t\,\varphi \to a^* e^{-i(\Omega-\omega_0)t}$$

$$i\dot{a} + (\delta + i\gamma)a + \boxed{\epsilon a^*} = -f(t)$$

Parametric instability



$$a \propto e^{\lambda t}$$

$$i\lambda a + (\delta + i\gamma)a + \epsilon a^* = 0$$

$$Det \quad = \quad (i\lambda + \delta + i\gamma)(-i\lambda + \delta - i\gamma) - \epsilon^2 = 0$$

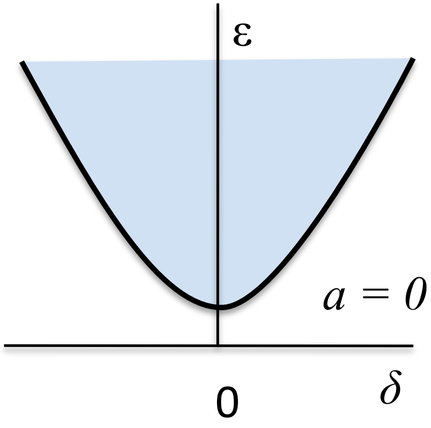
$$\lambda = -\gamma \pm \sqrt{\epsilon^2 - \delta^2}$$

Instability region

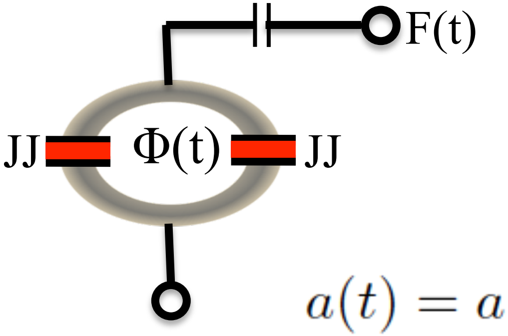
$$\delta^2 < \epsilon^2 - \gamma^2$$

Instability threshold

$$\epsilon = \sqrt{\delta^2 + \gamma^2}$$



Driven parametric oscillator

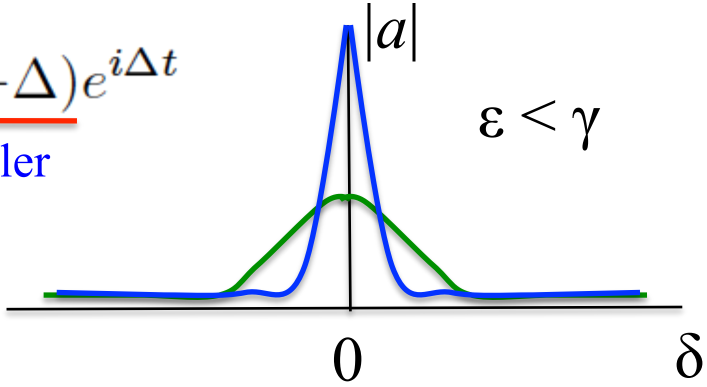


$$i\dot{a} + (\delta + i\gamma)a + \epsilon a^* = -f(t)$$

$$f(t) = f(\Delta)e^{-i\Delta t}$$

$$a(t) = a(\Delta)e^{-i\Delta t} + \underline{a(-\Delta)}e^{i\Delta t}$$

idler

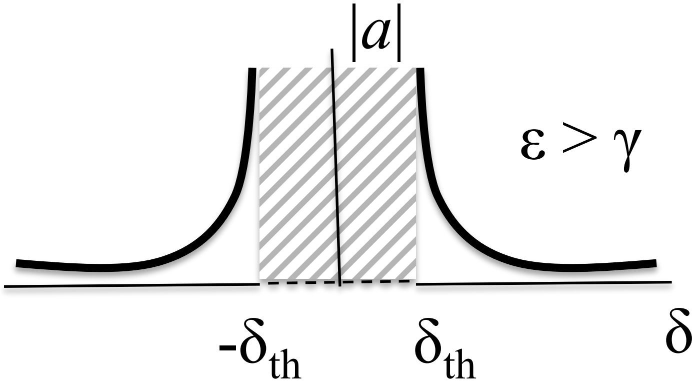


$$(\delta + \Delta + i\gamma)a(\Delta) + \epsilon a^*(-\Delta) = -f(\Delta)$$

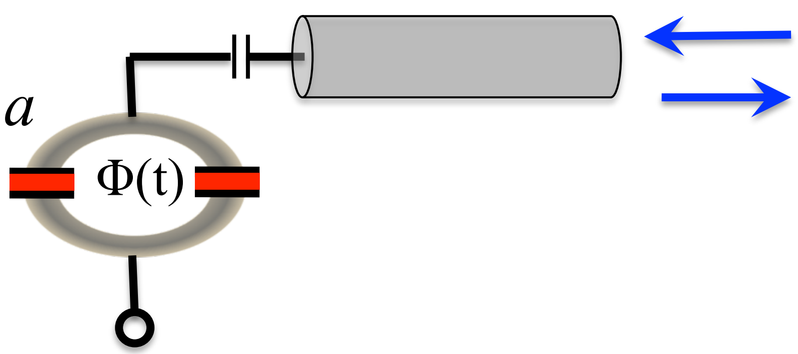
$$\Delta = 0$$

$$a = -\frac{(\delta - i\gamma)f(0) - \epsilon f^*(0)}{\delta^2 + \boxed{\gamma^2 - \epsilon^2}}$$

reduced damping
divergence at threshold



Oscillator in environment



b (input = signal + noise)

c (output)

Interesting new aspect: *external* response of the oscillator *c*

Langevin equation

$$i\dot{a} + (\delta + \underset{\text{total losses}}{i\Gamma})a + \epsilon a^* = \underset{\text{external losses}}{\sqrt{2\Gamma_0}}\,b(t)$$

Input-output relation

$$c = b - i\sqrt{2\Gamma_0}\,a$$

Walls and Milburn, *Quantum Optics*
Clerk, RMP 82, 1155 (2010)

Parametric amplification

Bogoliubov transformation

$$c(\Delta) = u(\Delta)b(\Delta) + v(\Delta)b^*(-\Delta)$$

Signal gain

$$G_s(\Delta) = \left|\frac{c(\Delta)}{b(\Delta)}\right|^2 = |u(\Delta)|^2$$

Idler gain

$$G_i(\Delta) = \left|\frac{c(-\Delta)}{b(\Delta)}\right|^2 = |v(-\Delta)|^2$$

Identities *(no internal losses)*

$$|u(\Delta)|^2 - |v(\Delta)|^2 = 1$$

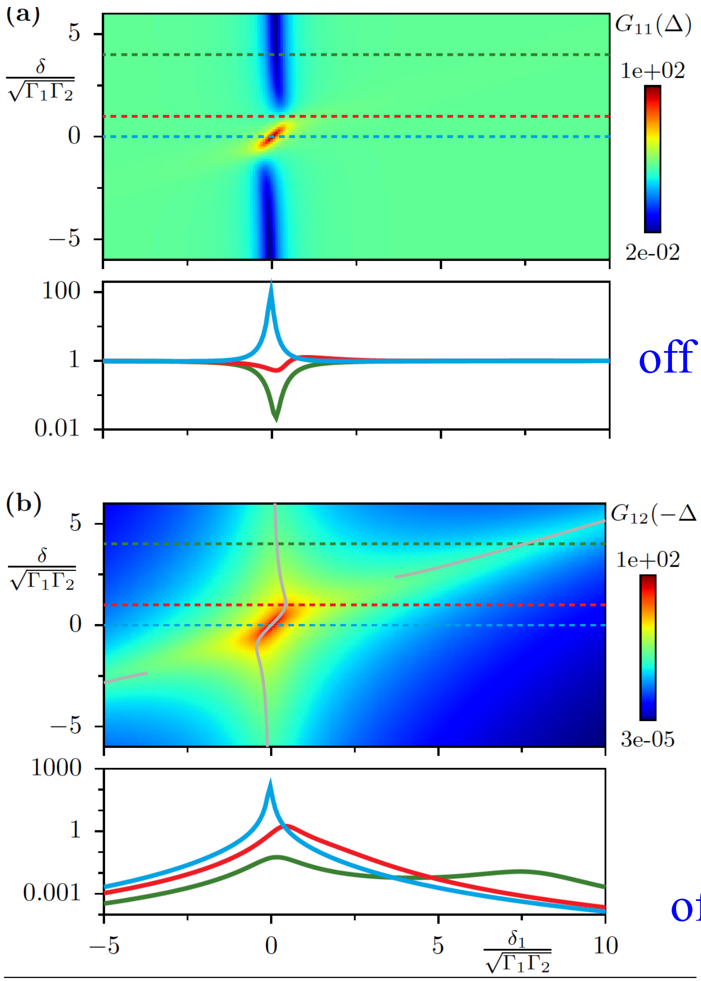
$$u(\Delta)v(-\Delta) - u(-\Delta)v(\Delta) = 0$$

Parametric amplification

$$G_s = 1 + G_i = 1 + \frac{4\epsilon^2\Gamma_0^2}{(\delta^2 + \Gamma^2 - \epsilon^2 - \Delta^2)^2 + 4\Gamma^2\Delta^2}$$

loss resonance

gain resonance



Signal

off resonance $G_S = 1$

Idler

off resonance $G_I = 0$

Quadrature squeezing (*Δ = 0)*

Parametrization

$u = \cosh r \, e^{i\chi_u} \qquad v = \sinh r \, e^{i\chi_v}$ ***r* = squeezing parameter**

$\tilde{c} = \cosh r \, \tilde{b} + \sinh r \, \tilde{b}^*$ rotated amplitudes

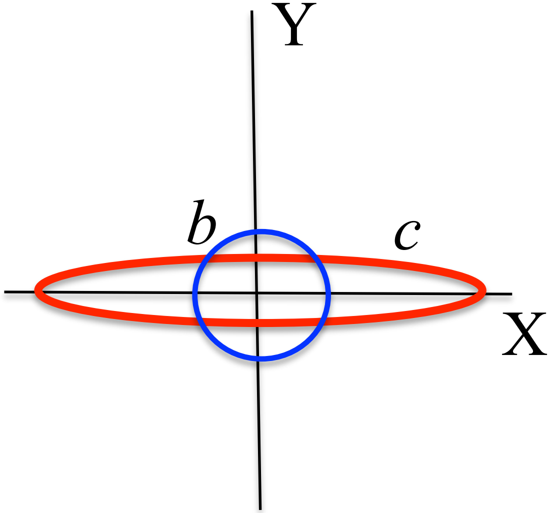
$$X = \frac{\tilde{c} + \tilde{c}^*}{\sqrt{2}}, \quad Y = \frac{\tilde{c} - \tilde{c}^*}{\sqrt{2}i}$$

quadratures

$$X = e^r x \,, \quad Y = e^{-r} y$$

amplified ***de-amplified***

$$G_{min} = \frac{1}{G_{max}}$$



Two-mode squeezing $\Delta \neq 0$

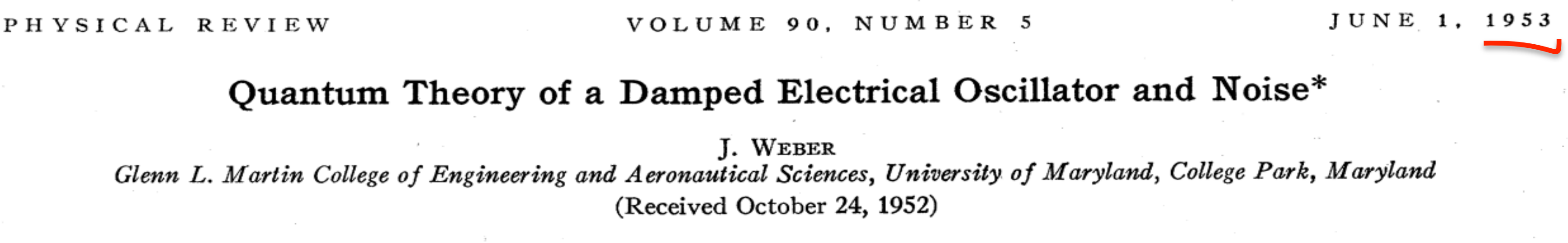
Amplification of detuned signals

$$\begin{aligned} c(\Delta) &= u(\Delta)b(\Delta) + v(\Delta)b^*(-\Delta) \\ c(-\Delta) &= u(-\Delta)b(-\Delta) + v(-\Delta)b^*(\Delta) \end{aligned}$$

$$\begin{aligned} X(\Delta) + X(-\Delta) &= e^r\left[x(\Delta) + x(-\Delta)\right] \\ Y(\Delta) - Y(-\Delta) &= e^{-r}\left[y(\Delta) - y(-\Delta)\right] \end{aligned}$$

Quantization

- Quantum Noise in electrical circuits, *Callen, Welton 1951*
- Quantum description of linear electrical circuits, *Weber 1953, Widom 1979* *Yrke & Denker 1984…Devoret 1995…*



PHYSICAL REVIEW VOLUME 90, NUMBER 5 JUNE 1, 1953

Quantum Theory of a Damped Electrical Oscillator and Noise*

J. Weber
Glenn L. Martin College of Engineering and Aeronautical Sciences, University of Maryland, College Park, Maryland

(Received October 24, 1952)

Quantum electrical circuits ?

- Kirchhoff's equations = lumped element version of Maxwell equations
- EM field is quantum $\Longrightarrow$ Quantization rule: $[\Phi , Q] = i\hbar$
- Low noise resonators are feasible at microwaves: $\omega/\Gamma = Q \sim 1000$
- Low temperature is available: $T << \omega$ ($\omega \sim 10$ GHz $T < 100$ mK)
- Nonlinear circuit is required – JJ (Leggett 1981)

Quantization

Quantization is straightforward in linear theory

$$[\, b(\Delta)\, , b^\dagger(\Delta') \,] = \delta(\Delta - \Delta')$$

bosonic input

$$c(\Delta) = u(\Delta) b(\Delta) + v(\Delta) b^\dagger(-\Delta)$$

Bogoliubov transformation

$$|u(\Delta)|^2 - |v(\Delta)|^2 = 1$$

$$u(\Delta) v(-\Delta) - u(-\Delta) v(\Delta) = 0$$

properties of coefficients
guarantee bosonic output

$$\Downarrow$$

$$[\, c(\Delta)\, , c^\dagger(\Delta') \,] = \delta(\Delta - \Delta')$$

Creation of photons

$$c(\Delta) = u(\Delta)b(\Delta) + v(\Delta)b^{\dagger}(-\Delta)$$

$$\langle 0|b^{\dagger}(\Delta)b(\Delta)|0\rangle = 0$$

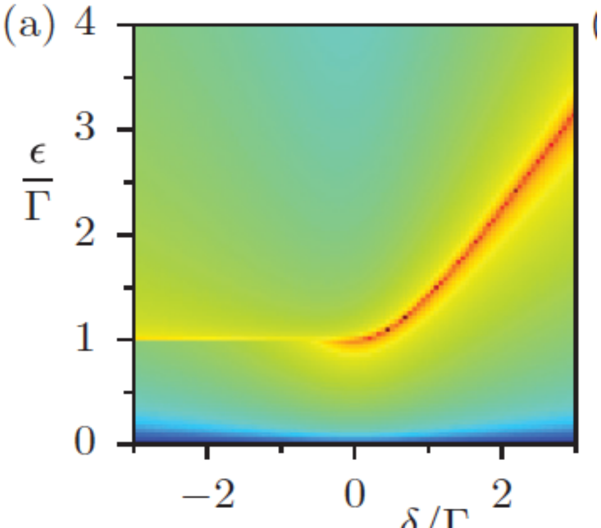
vacuum input contains no real photons

$$N(\Delta) = \langle 0|c^{\dagger}(\Delta)c(\Delta)|0\rangle = |v(\Delta)|^2$$

$$|v(\Delta)|^2 = \frac{4\epsilon^2\Gamma^2}{(\delta^2 + \Gamma^2 - \epsilon^2 - \Delta^2)^2 + 4\Gamma^2\Delta^2}$$

spectral density
of created photons

Parametric amplification in quantum regime appears as creation of real photons from vacuum



divergence at threshold

N(0;δ,ε) ⇒ ∞

Wustmann, PRB 87, 184501 (2013)

Amplification of noise

Compare amplification of signal and noise

signal

$$\begin{aligned} C(\Delta) &= u(\Delta)B(\Delta) \\ |X(\Delta)|^2 &\propto |u(\Delta)|^2 \end{aligned}$$

noise
(vacuum)

$$\begin{aligned} c(\Delta) &= u(\Delta)b(\Delta) + v(\Delta)b^\dagger(-\Delta) \\ S(\Delta) &= \int d\Delta' \, \langle x(\Delta)x(\Delta')\rangle = |u(\Delta)|^2 + |v(\Delta)|^2 \end{aligned}$$

signal to
noise ratio

$$\begin{aligned} SNR^{\mathrm{out}} &= \frac{|X|^2}{S} = \frac{G_s}{G_s + G_i} SNR^{\mathrm{in}} \\ SNR^{\mathrm{out}} &= \frac{1}{2} SNR^{\mathrm{in}}, \quad G \gg 1 \end{aligned}$$

Minimum added noise: quantum limited amplification

Clerk, RMP 82, 1155 (2010)

Squeezing operator

$$c(\Delta) = u(\Delta)b(\Delta) + v(\Delta)b^\dagger(-\Delta)$$

BT can be written through *squeezing operator*

$$c(\Delta) = S^\dagger b(\Delta) S$$

$$S = \exp\left(\int d\Delta' \, \xi(\Delta')b^\dagger(\Delta')b^\dagger(-\Delta') - \xi^*(\Delta')b(\Delta')b(-\Delta')\right)$$

$$\xi(\Delta) = r(\Delta)e^{i\rho(\Delta)} \qquad \rho = \arg\left(\frac{v}{u}\right) + \pi$$

Squeezing operator explicitly indicates correlation of photon pairs

Squeezed vacuum

$$|\Psi\rangle = S|0\rangle \quad = \quad \prod_{\Delta} \cosh^{-1} r(\Delta) \sum_{n} \tanh^{n} r(\Delta) e^{in\rho(\Delta)} |n(\Delta) n(-\Delta)\rangle$$

$$= \quad \prod_{\Delta} \cosh^{-1} r(\Delta) \sum_{n} \tanh^{n} r(\Delta) e^{in\rho(\Delta)} a^{\dagger n}(\Delta) a^{\dagger n}(-\Delta)\, |0\rangle$$

Output of parametric amplifier = squeezed vacuum is

- **pure state**
- set of photon pairs (photon pairing)

$$r \ll 1$$

$$|\Psi\rangle \quad = \quad \prod_{\Delta} \cosh^{-1} r(\Delta) [1 + \tanh r(\Delta) e^{i\rho(\Delta)} a^{\dagger}(\Delta) a^{\dagger}(-\Delta)]\, |0\rangle$$

$$= \quad \prod_{\Delta} \cosh^{-2} r(\Delta) [u(\Delta) + v(\Delta) a^{\dagger}(\Delta) a^{\dagger}(-\Delta)]\, |0\rangle$$

Weak squeezing limit: vacuum is a superposition of no photon pairs and one photon pair per mode

BCS analogy

Hamiltonian

$$H = -\sum_{\Delta} \left[\delta a^{\dagger}_{\Delta} a_{\Delta} + \epsilon (a^{\dagger}_{\Delta} a^{\dagger}_{-\Delta} + a_{\Delta} a_{-\Delta}) \right]$$

$$H = \sum_{p\sigma} \left[E_p a^{\dagger}_{p\sigma} a_{p\sigma} + \Delta a^{\dagger}_{p\sigma} a^{\dagger}_{-p-\sigma} + \Delta^{*} a_{p\sigma} a_{-p-\sigma}) \right]$$

Bogoliubov transformation

$$c(\Delta) = u(\Delta) b_{\Delta} + v(\Delta) b^{\dagger}_{-\Delta}$$

$$\alpha_p = u(p) a_{p\sigma} - v(p) a^{\dagger}_{-p-\sigma}$$

Bosonic / fermionic commutation relations

$$|u(\Delta)|^2 - |v(\Delta)|^2 = 1$$

$$|u(p)|^2 + |v(p)|^2 = 1$$

Vacuum state

$$|\Psi\rangle \quad = \quad \prod_{\Delta} [\bar{u}(\Delta) + \bar{v}(\Delta) a^{\dagger}_{\Delta} a^{\dagger}_{-\Delta}] \, |0\rangle$$

$$|\Psi\rangle \quad = \quad \prod_{p} [u(p) + v(p) a^{\dagger}_{p\sigma} a^{\dagger}_{-p-\sigma})] \, |0\rangle$$

Single mode detection

Two-mode density matrix of squeezed vacuum

$$\hat{\rho}_{12} \quad = \quad \cosh^{-2} r_1 \cosh^{-2} r_2 \sum_{nm} \tanh^{2n} r_1 \; \tanh^{2m} r_2 \; |n_1 m_2\rangle\langle n_1 m_2|$$

Detection of one of the modes produces reduced density matrix

$$\begin{aligned} \rho_1 = \mathrm{Tr}_2 \rho_{12} \quad &= \quad \cosh^{-2} r_1 \sum_n \tanh^{2n} r_1 |n(1)\rangle\langle n_1| \; \cosh^{-2} r_2 \sum_m \tanh^{2m} r_2 \langle m_2|m_2\rangle \\ &= \quad \cosh^{-2} r_1 \sum_n \tanh^{2n} r_1 |n_1\rangle\langle n_1| \end{aligned}$$

Reduced density matrix describes the *Gibbs state*

$$\tanh^2 r = e^{-\hbar\omega/k_B \underline{T}}$$

In large squeezing limit (at threshold) ***T*** = ∞

Two-photon entanglement

Photon entanglement in squeezed vacuum is quantified with entropy

$$E(\Delta) = -\mathrm{Tr}\left[\rho_1(\Delta)\ln\rho_1(\Delta)\right]$$

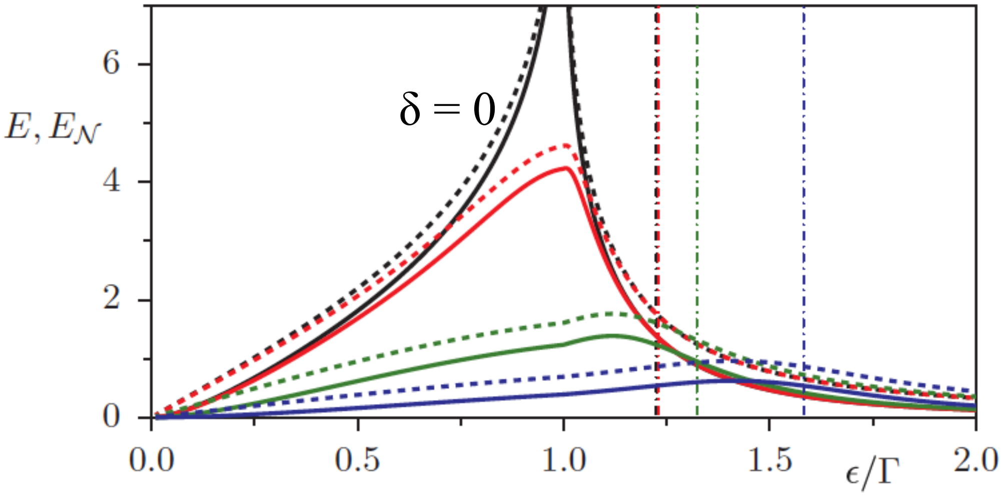
$$E = \cosh^2 r \ln(\cosh^2 r) - \sinh^2 r \ln(\sinh^2 r)$$

At threshold entanglement increases without bound in linear theory

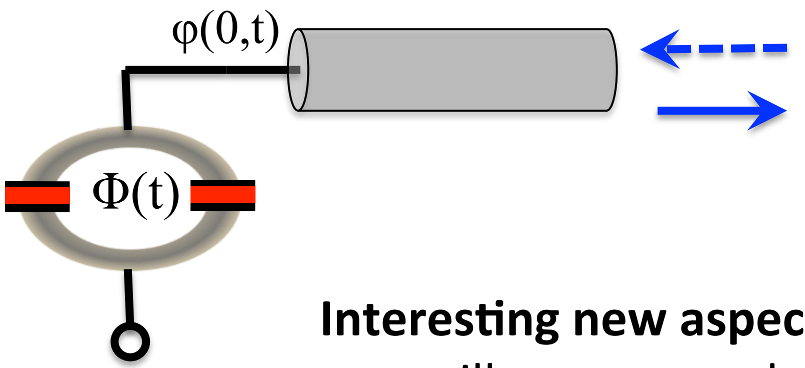
In fact it is bounded
due to nonlinear effect

$$E_{max} \sim \underline{\ln\frac{\Gamma}{\alpha}}$$

Entanglement entropy for different pump detuning



Dynamical Casimir Effect (MW)



b (vacuum modes)

c (real photons)

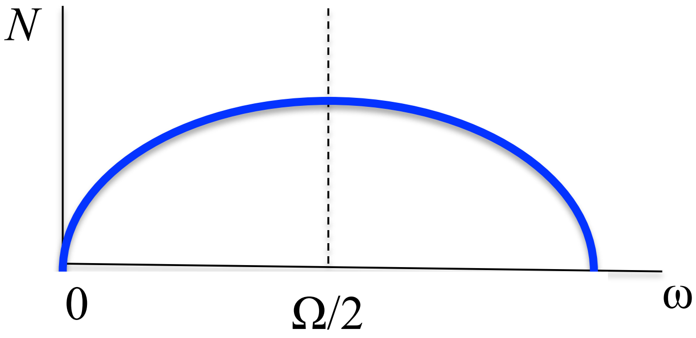
Interesting new aspect: *SQUID directly coupled to TL:*
no oscillator, no modulation of resonance

$$\varphi(0,t) + \frac{L_J(t)}{L_0}\frac{\partial\varphi}{\partial x} = 0$$

Temporal modulation of boundary condition (reflection phase)

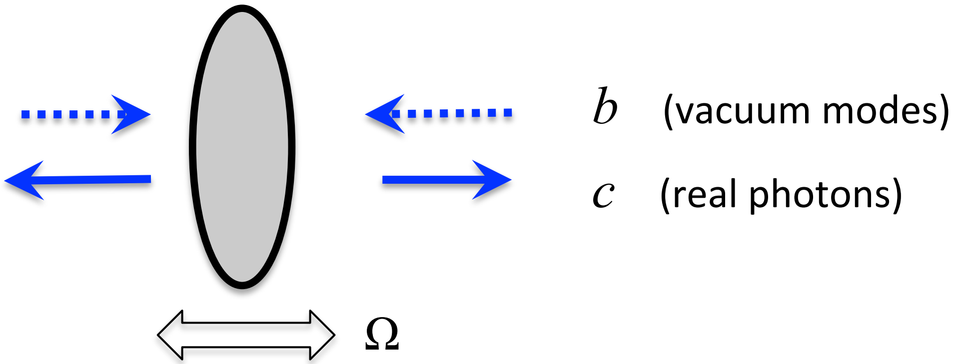
$$c(\omega) = \frac{i\delta L_J}{L_0 v}\left[\sqrt{\omega(\omega+\Omega)}\,b(\omega+\Omega) + \sqrt{\omega(\omega-\Omega)}\,b(\omega-\Omega) + \sqrt{\omega(\Omega-\omega)}\,b(\Omega-\omega)\right]$$

$$N(\omega) = \langle 0|c^\dagger(\omega)c(\omega)|0\rangle = \left(\frac{\delta L_J}{L_0 v}\right)^2 \omega(\Omega-\omega)$$



Thy: Johansson, PRL 103, 147003 (2009)
Exp: Wilson, Nature 479, 376 (2011)

Dynamical Casimir Effect (original)



b (vacuum modes)

c (real photons)

Ω

DCE: parametric pump - moving mirror

$$N(\omega) = \langle 0|c^\dagger(\omega)c(\omega)|0\rangle = \left(\frac{\delta L_J}{L_0 v}\right)^2 \omega(\Omega - \omega)$$

$$N(\omega) = \frac{8a^2\Omega}{c^2}\omega(\Omega - \omega)$$

Moore, Math. Phys. 11, 2679 (1970); Lambrecht, PRL 77, 615 (1996)

Related relativistic effects

Unruh radiation: effective pump – accelerated reference frame (horizon)

$$\tanh^2 r = e^{-2\pi\omega c/a}$$

$$T = \frac{\hbar a}{2\pi k_B c}$$

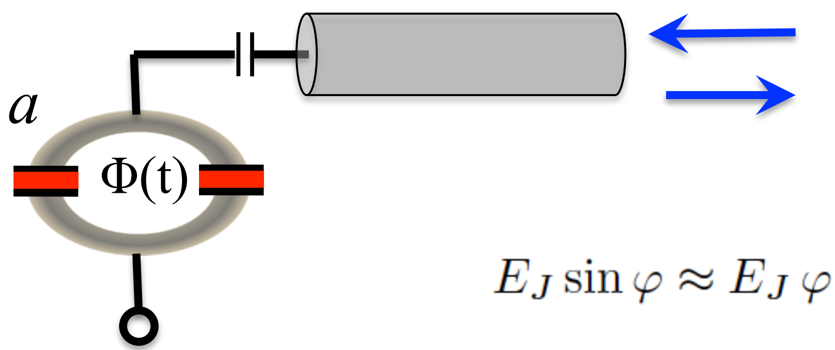
Hawking radiation: effective pump – gravitational field of black hole

$$T = \frac{\hbar c^3}{8\pi k_B G M}$$

Nation, RMP 84, 1 (2012) Hawking, Nature 248, 30 (1975)

NONLINEAR PARAMETRIC EFFECTS

JJ = Nonlinear element

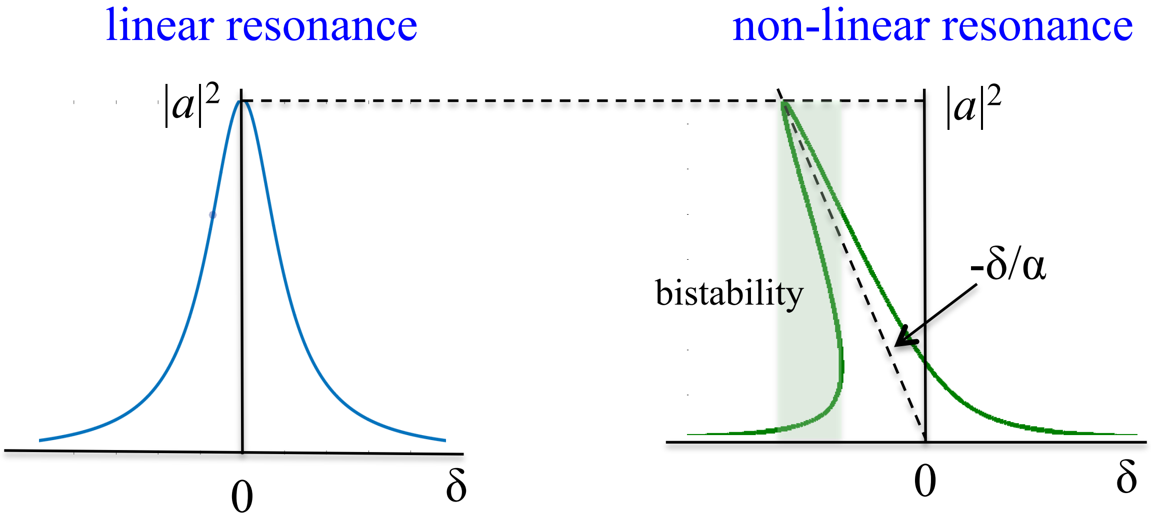


$$E_J \sin\varphi \approx E_J\,\varphi - (E_J/6)\,\varphi^3$$

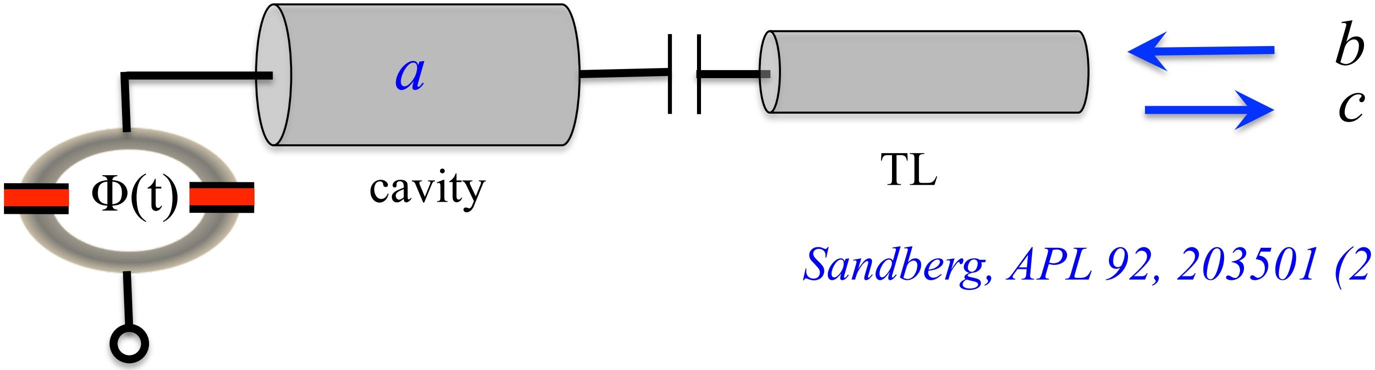
Resonance approximation $\varphi^3 \to |a|^2 a$

$$i\dot{a} + (\boxed{\delta + \alpha|a|^2} + i\Gamma)a + \epsilon a^* = \sqrt{2\Gamma_0}\, b(t)$$

nonlinear frequency shift



Tunable cavity



Sandberg, APL 92, 203501 (2008)

Langevin equation

$$i\dot{a} + \delta a + i\Gamma a + \alpha(\underline{a^\dagger a} + 1)a + \epsilon a^\dagger = \sqrt{2\Gamma}\, b$$

Effective Hamiltonian

$$H_{\mathrm{cav}}/\hbar = -\delta a^\dagger a - \frac{\alpha}{2}\left(a^\dagger a + \frac{1}{2}\right)^2 - \frac{\epsilon}{2}(a^{\dagger 2} + a^2) + \sqrt{2\Gamma}\,(a^\dagger b + b^\dagger a)$$

$$H_{\mathrm{cav}}(q,p) = \frac{\epsilon - \delta}{2}p^2 - \frac{\epsilon + \delta}{2}q^2 - \frac{\alpha}{8\hbar}(q^2 + p^2)^2$$

Derivation procedure preserves commutation relation

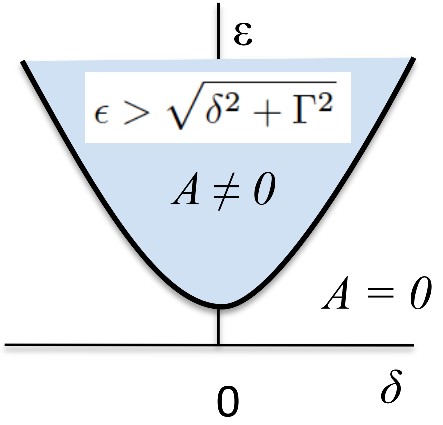
Dykman, Sow. Phys. JETP ***50, 30*** *1979;* *Wustmann, PRB 87, 184501 (2013)*

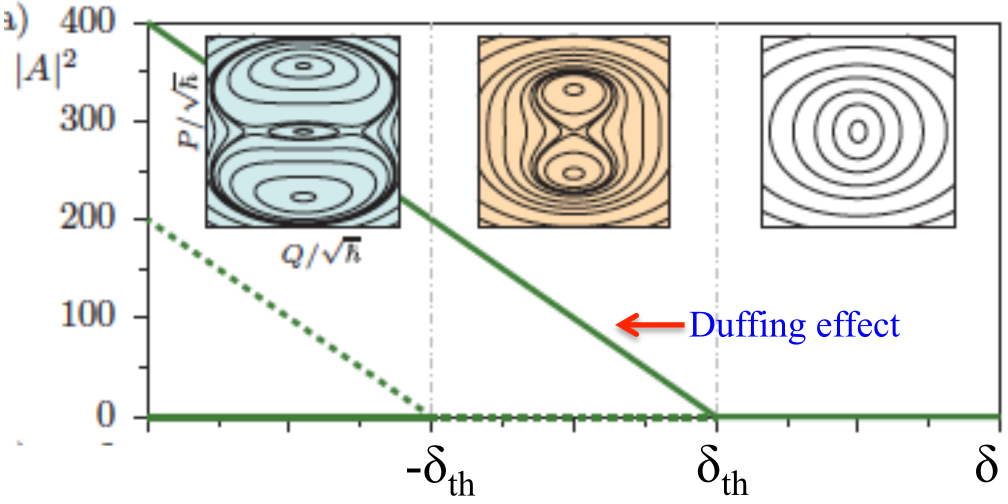
Parametric oscillation

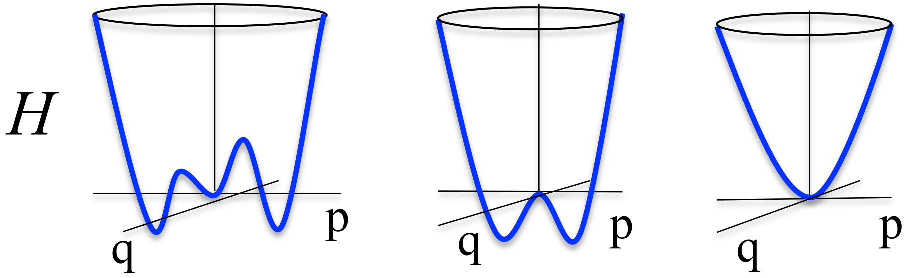
Classical solution $A = |A| e^{i\theta} e^{-i(\Omega/2)t}$

$$|A|^2 = \frac{1}{\alpha}(-\delta \pm \sqrt{\epsilon^2 - \Gamma^2})$$

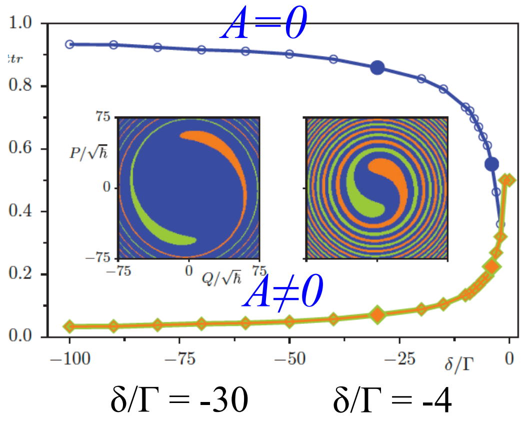
$$\theta = \theta_0,\ \theta_0 + \pi\,; \quad \theta_0 = \arcsin \frac{\Gamma}{\epsilon}$$





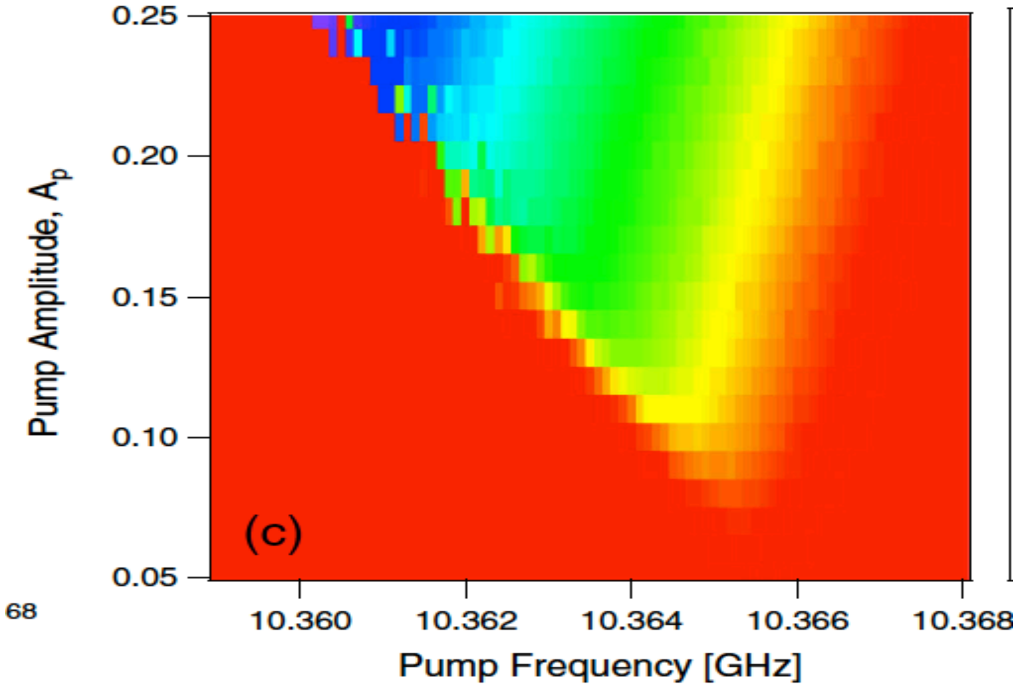


Basins of attraction

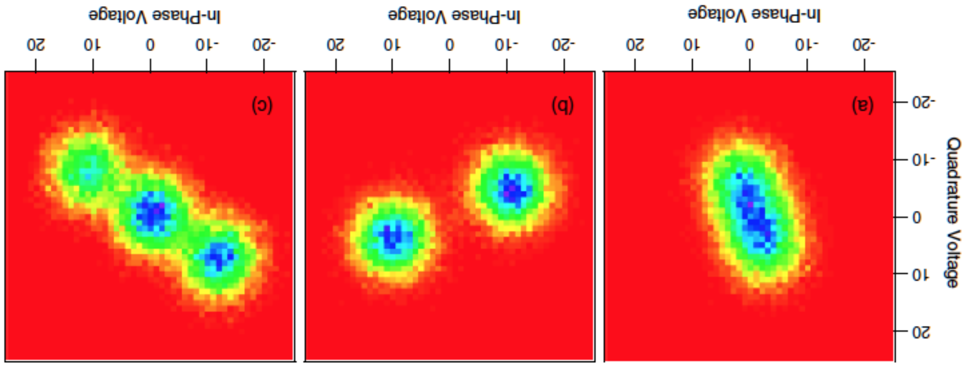


Wustmann, PRB 87, 184501 (2013)

Parametric radiation



Parametric radiation from tunable cavity vs δ and ε



Quadrature histograms for one- bi- and tri-stability regions

Wilson, PRL 105, 233907 (2010)

Oscillator quantum state

Closed cavity: CS are eigen states of parametric oscillator

$$H_{\text{cav}} = -\frac{\hbar\alpha}{2}\left(a^{\dagger 2} + \frac{\epsilon}{\alpha}\right)\left(a^2 + \frac{\epsilon}{\alpha}\right) + \frac{\hbar\epsilon^2}{2\alpha}$$

$\delta = 0$

$$a|\beta\rangle = \beta|\beta\rangle$$

coherent state

$$\left(a^2 + \frac{\epsilon}{\alpha}\right)|\beta\rangle = \left(\beta^2 + \frac{\epsilon}{\alpha}\right)|\beta\rangle$$

$$H_{\text{cav}}|\beta\rangle = \frac{\hbar\epsilon^2}{2\alpha}|\beta\rangle \qquad \boxed{\beta = \pm i\,\frac{\epsilon}{\alpha}}$$

eigen state

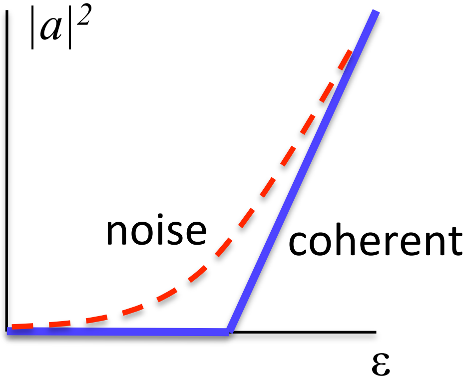
Average amplitude = classical amplitude ≠ 0

$$\langle\beta|\, a\, |\beta\rangle = \beta = |A|e^{\pm i\pi/2}$$

Puri, arXiv:1605.09408

Photonic condensation

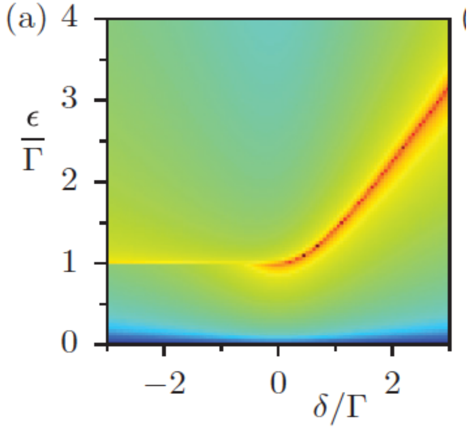
What is the parametric oscillator regime from the condensed matter physics view point?



- Below threshold no coherent field, only noise, $\langle a \rangle = 0$
- Above threshold coherent, classical field emerges $\langle a \rangle = A$
- This classical field obeys equation that resembles Gross-Pitaevsky equation for BEC, and (nonlinear) BdG equation for superconductor

$$i\dot{A} + \delta A + i\Gamma A + \alpha\,|A|^2 A + \epsilon A^* = 0$$

- Critical fluctuations at threshold

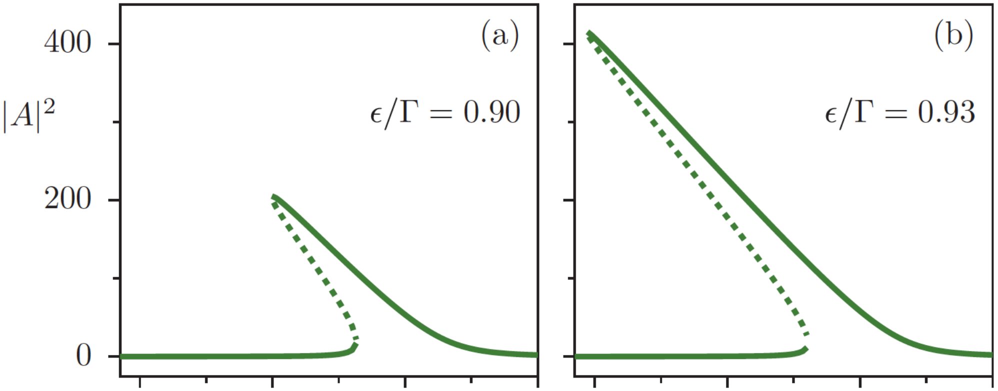


Transition to oscillator regime = photonic condensation, a quantum phase transition from disordered phase (noise) to ordered phase (oscillation) whose complex amplitude A plays role of order parameter.

Physics here is different from BEC: although we are dealing with bosons, two-mode squeezing introduces pair wise correlation quantitatively similar to those for Cooper pairs.

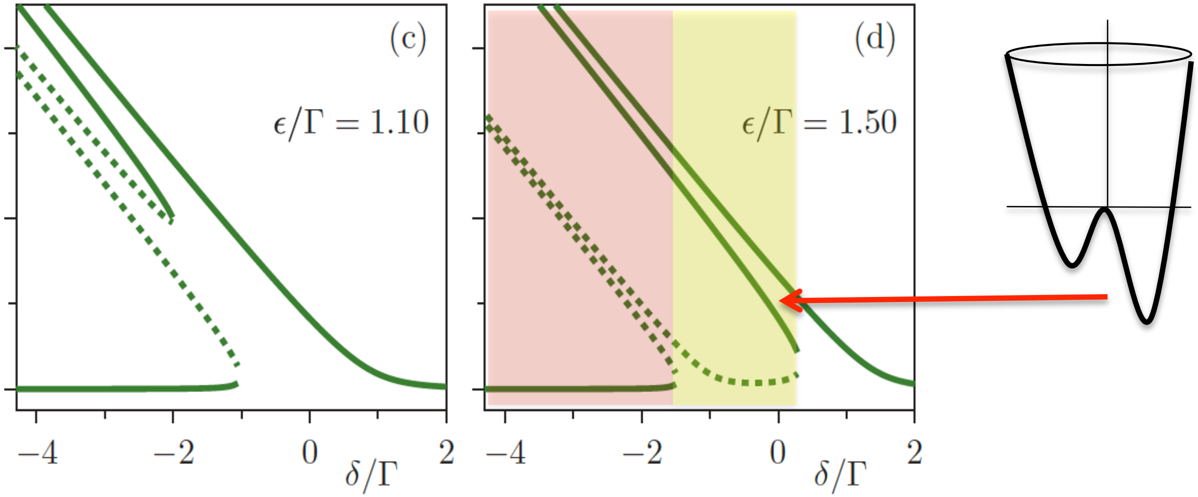
Cavity nonlinear response

Below threshold



Narrowing of resonance and strong nonlinearity close to threshold

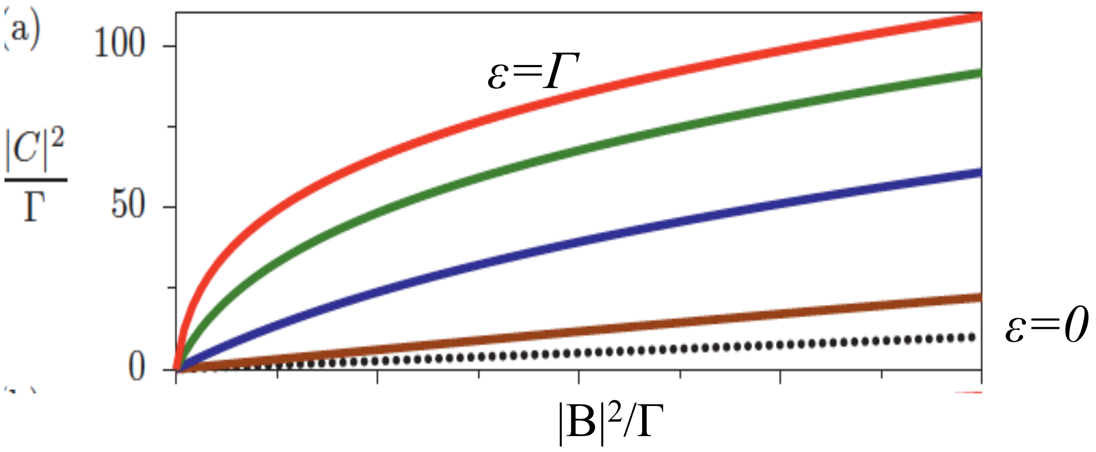
Above threshold



Driving removes degeneracy (tilts metapotential)

Wustmann, PRB 87, 184501 (2013)

Nonlinear amplification

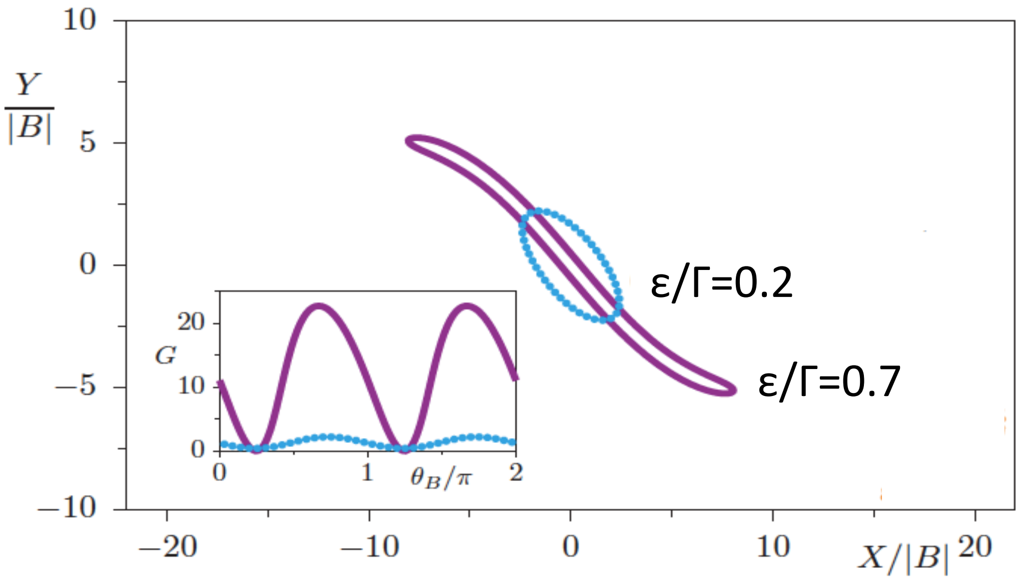


Max differential gain: at threshold for small input ***ε → Γ , B → 0***
Linear gain diverges, nonlinear gain is bound at threshold

$$G_{\max} = (4\Gamma/\alpha)^{4/5}(|B|^2/\Gamma)^{-4/5}$$

- Maximum amplification is controlled by parameter **Γ/α**
- Same refers to squeezing parameter hence noise squeezing and entanglement
- Signal and noise are differently amplified and squeezed **⇒ SNR > 1**

Squeezing

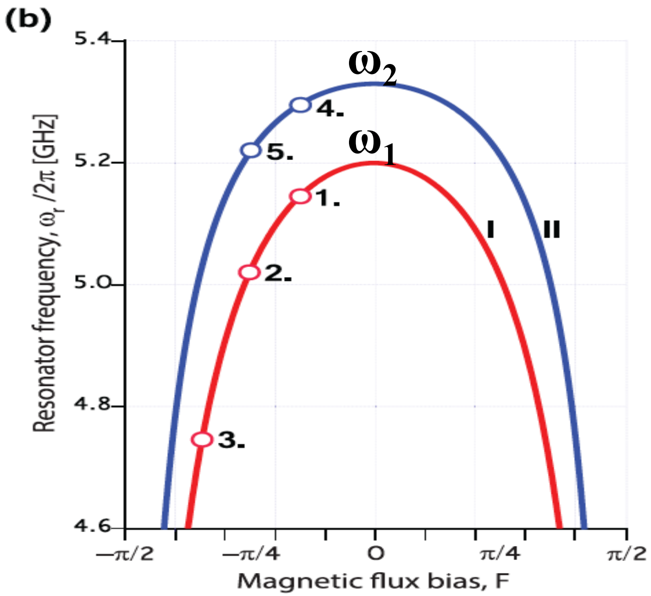


Nonlinear gain for phase sensitive amplification

$$G_S(B) = \cosh 2r(B) + \sinh 2r(B)\sin(2\theta_B + \nu(B))$$

$$G_{max}G_{min} \approx (\alpha/4\Gamma)^{1/2}\,(|B|^2/\Gamma)^{1/2} < 1 \qquad @\ \ \varepsilon = \Gamma$$

Non-degenerate resonance



Cavity contains many modes

$$(k_n d)\tan k_n d = \frac{2E_J \cos F}{E_{L,\mathrm{cav}}}$$

Non-equidistant spectrum

Two distinctly different regimes

$$\Omega = \omega_1 + \omega_2 \qquad H \propto -\hbar\epsilon(a_1^\dagger a_2^\dagger + a_1 a_2)$$

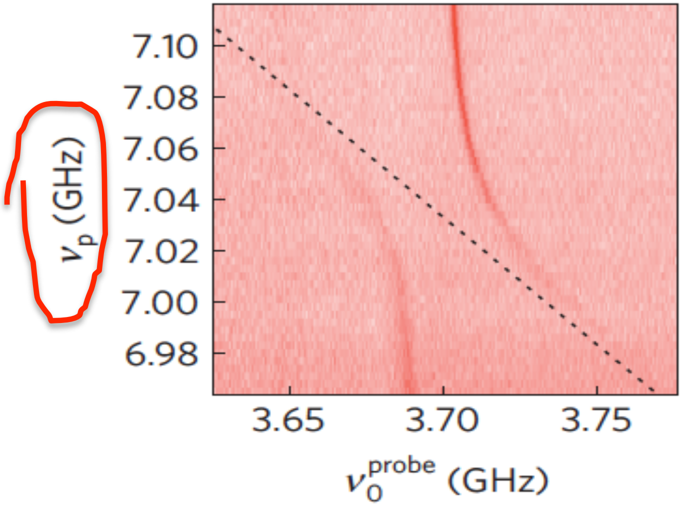
down-conversion:
amplification, oscillation

$$\Omega = \omega_2 - \omega_1 \qquad H \propto -\hbar\epsilon(a_1^\dagger a_2 + a_1 a_2^\dagger)$$

up-conversion:
frequency conversion

Frequency conversion

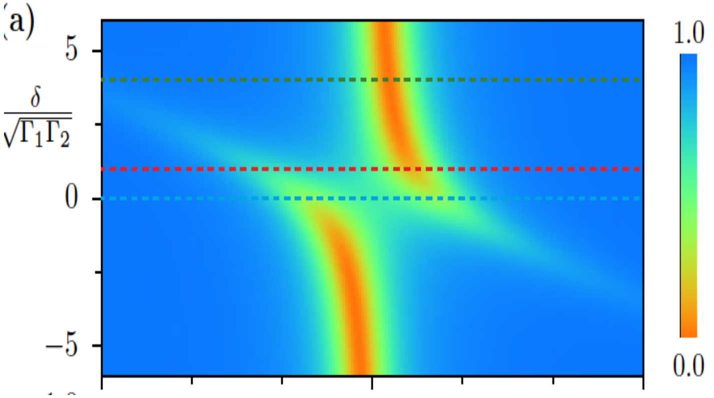
$$\Omega = \omega_2 - \omega_1$$

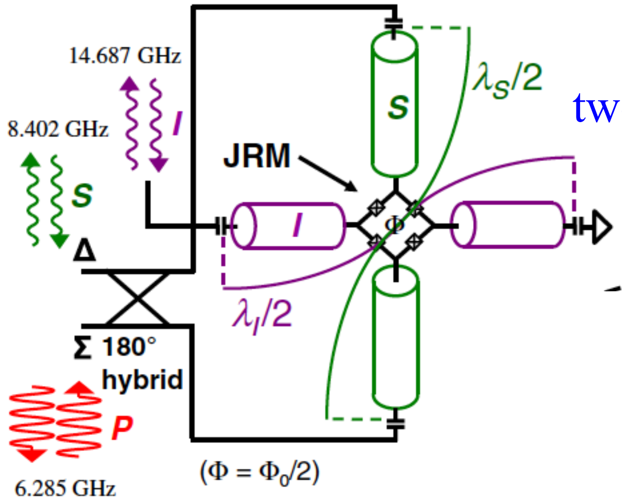


one cavity

Zakka-Bajjani, Nature Phys 7, 599 (2011)

reflection S_{11}

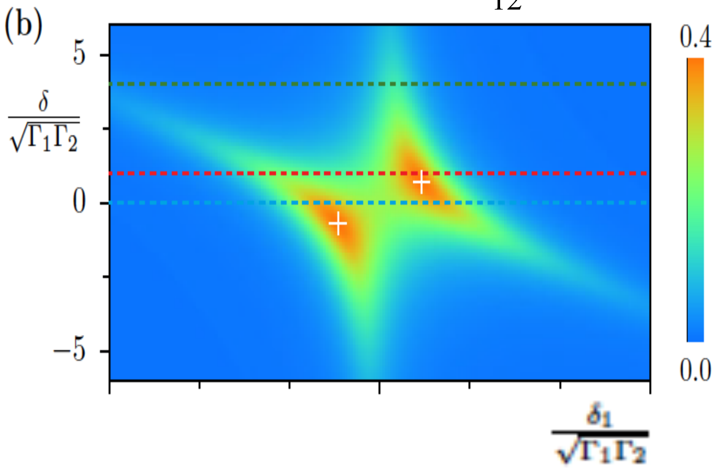




two cavities

Abdo, PRL 110, 173902 (2013)

transmission S_{12}



Wustmann, arXiv:1704.05083

Frequency conversion

$$(-\delta + \Delta + i\Gamma_1)a_1(\Delta) + \epsilon a_2^*(\Delta) = \sqrt{2\Gamma_{10}}\, b_1(\Delta)$$
$$(\delta + \Delta + i\Gamma_2)a_2(\Delta) + \epsilon a_1^*(\Delta) = \sqrt{2\Gamma_{20}}\, b_2(\Delta)$$

Linearized Langevin equations

$$|S_{12}(\Delta)|^2 = \frac{4\epsilon^2\Gamma_{01}\Gamma_{02}}{(\Delta^2 - \Gamma_1\Gamma_2 - \epsilon^2)^2 + \Delta^2(\Gamma_1 + \Gamma^2)^2}$$

Transmission coefficient

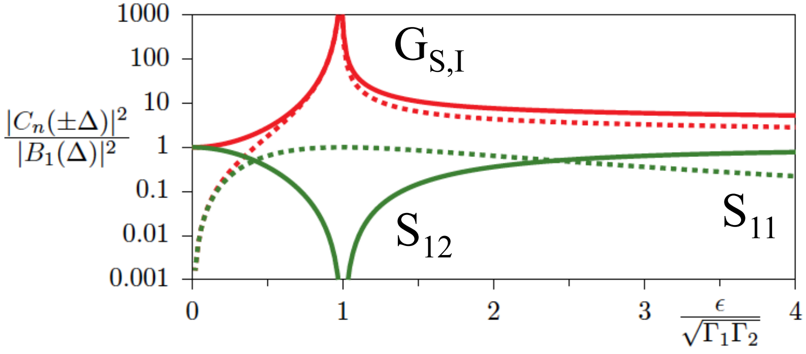
In absence of internal losses conversion is unitary

Full conversion

$$|S_{12}(\Delta)|^2 = 1$$

at $\Delta = 0\,,\;\; \epsilon^2 = \Gamma_1\Gamma_2$

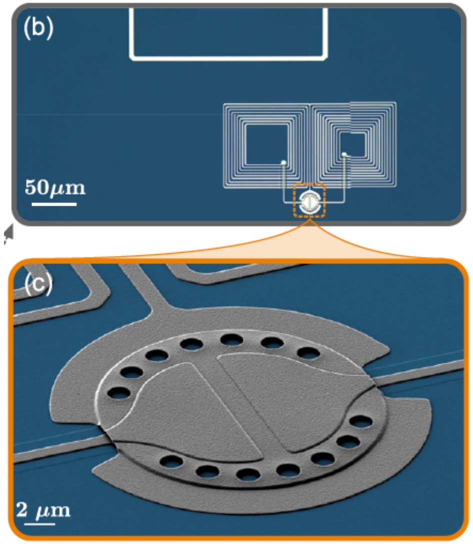
= instability threshold in amplification regime



Wustmann, arXiv:1704.05083

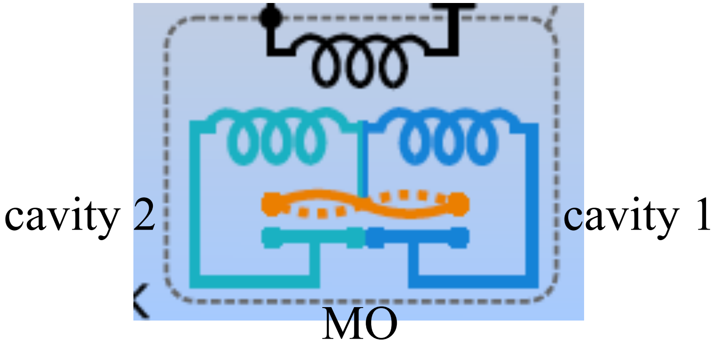
Frequency conversion:
optomechanics

- Parametric conversion is able to couple **vastly different** EM frequencies
- **Holy Grail:** MW ⇔ optic quantum convertor for long distance communication
- SQUID is not suitable for optical frequencies
- Mechanical oscillator instead of SQUID (DCE)

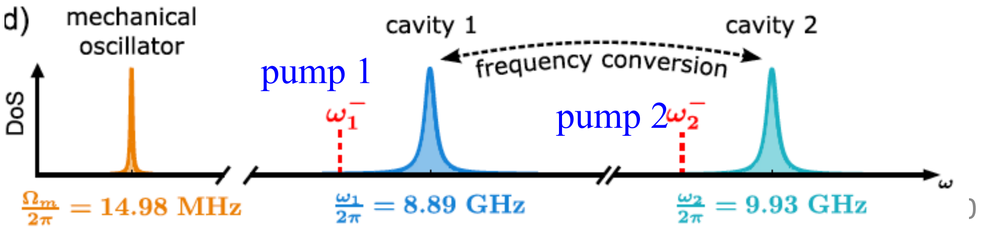


Lecocq, PRL 116, 043601 (2016)

Cavity Optomechanics 2014;
Aspelmeyer RMP 86 1391 (2014)



$$H_{\mathrm{OM}} \propto -\hbar g_{\mathrm{OM}} A_p \left(a^\dagger b + b^\dagger a\right)$$



Summary

- Parametrically driven EM oscillator
  - is a quantum limited amplifier
  - is capable to squeeze noise
  - creates entangled photon pairs out of vacuum
- Parametrically squeezed vacuum exhibits two-mode entanglement; single mode detector sees it as black body radiation
- The largest parametric effects are at the instability threshold; nonlinearity bounds amplification and squeezing at the threshold
- Parametric instability indicates dissipative phase transition of photonic pairs to parametric oscillator state
- Non-degenerate parametric resonance allows for frequency conversion
- Parametric resonance in EM-mechanical hybrids gives solution to problem of long distance quantum communication between SC qubit clusters

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