Topological Topological Quantum Computing
Majoranas and Color Codes

Combining Topological Hardware and Topological Software: Color Code Quantum Computing with Topological Superconductor Networks

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arXiv:1704.01589

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Two flavors of topological quantum computing

Condensed matter physics

Build qubits from topological phases of matter

Majorana fermions

- Protected against local noise
- Noise-free gates by braiding

Quantum information theory

Use many physical qubits for logical qubits

Topological codes

- Logical information protected
- Measure only local operators

Two flavors of topological quantum computing


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?

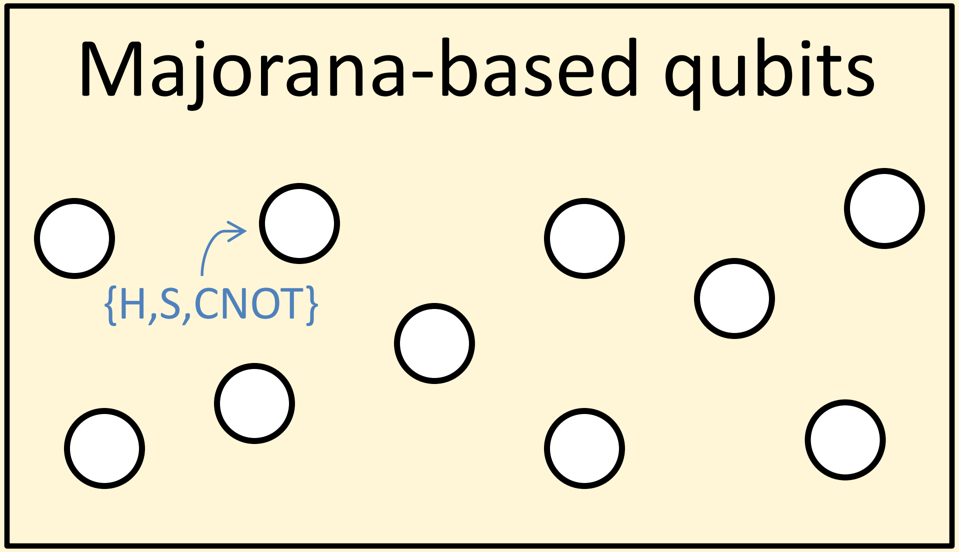
How can we add error correction to Majoranas without losing any of their nice properties?

Majorana fermions

- Protected against local noise
- Noise-free gates by braiding

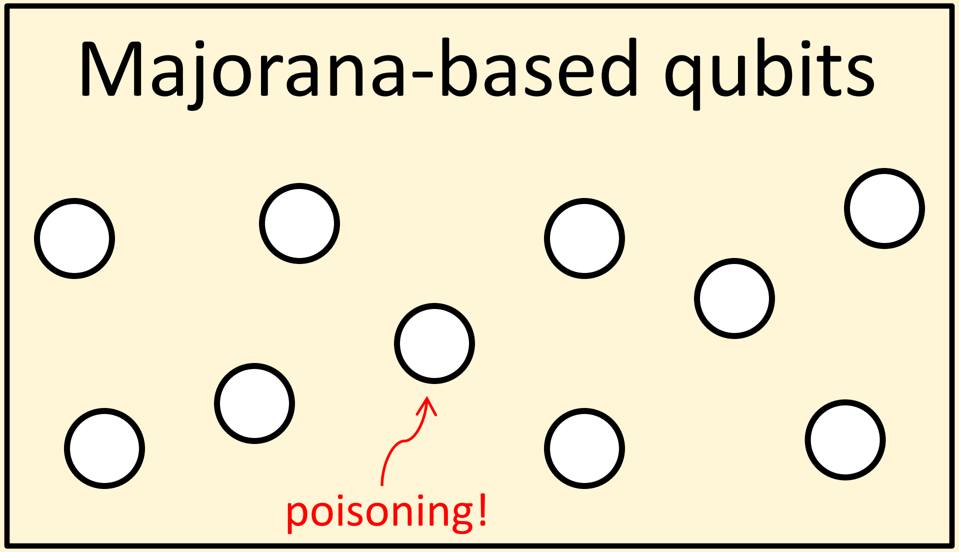
Topological codes

- Logical information protected
- Measure only local operators



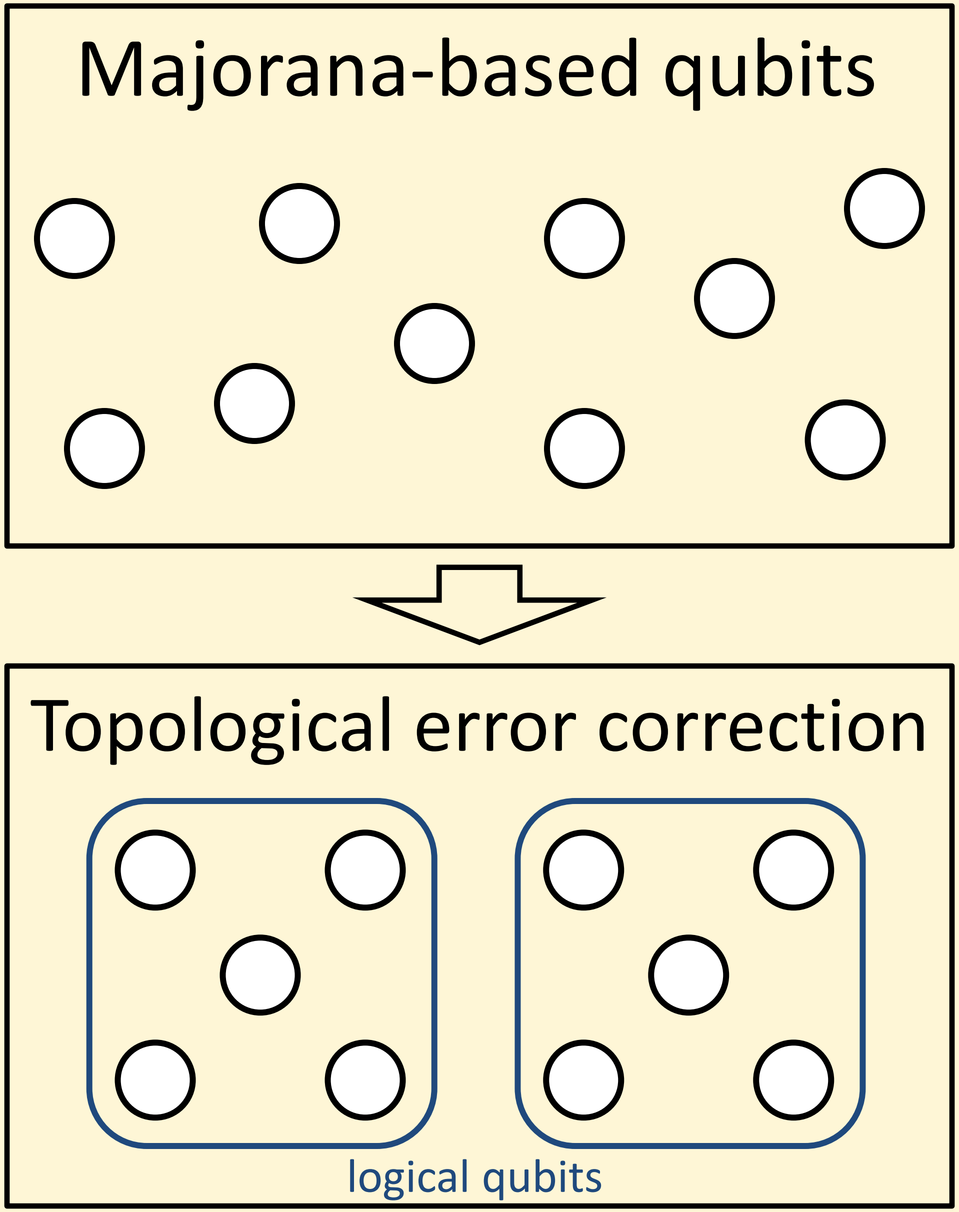
+ Topological protection against local noise

+ Topologically protected gates



+ Topological protection against local noise (up to 10ms)

+ Topologically protected gates



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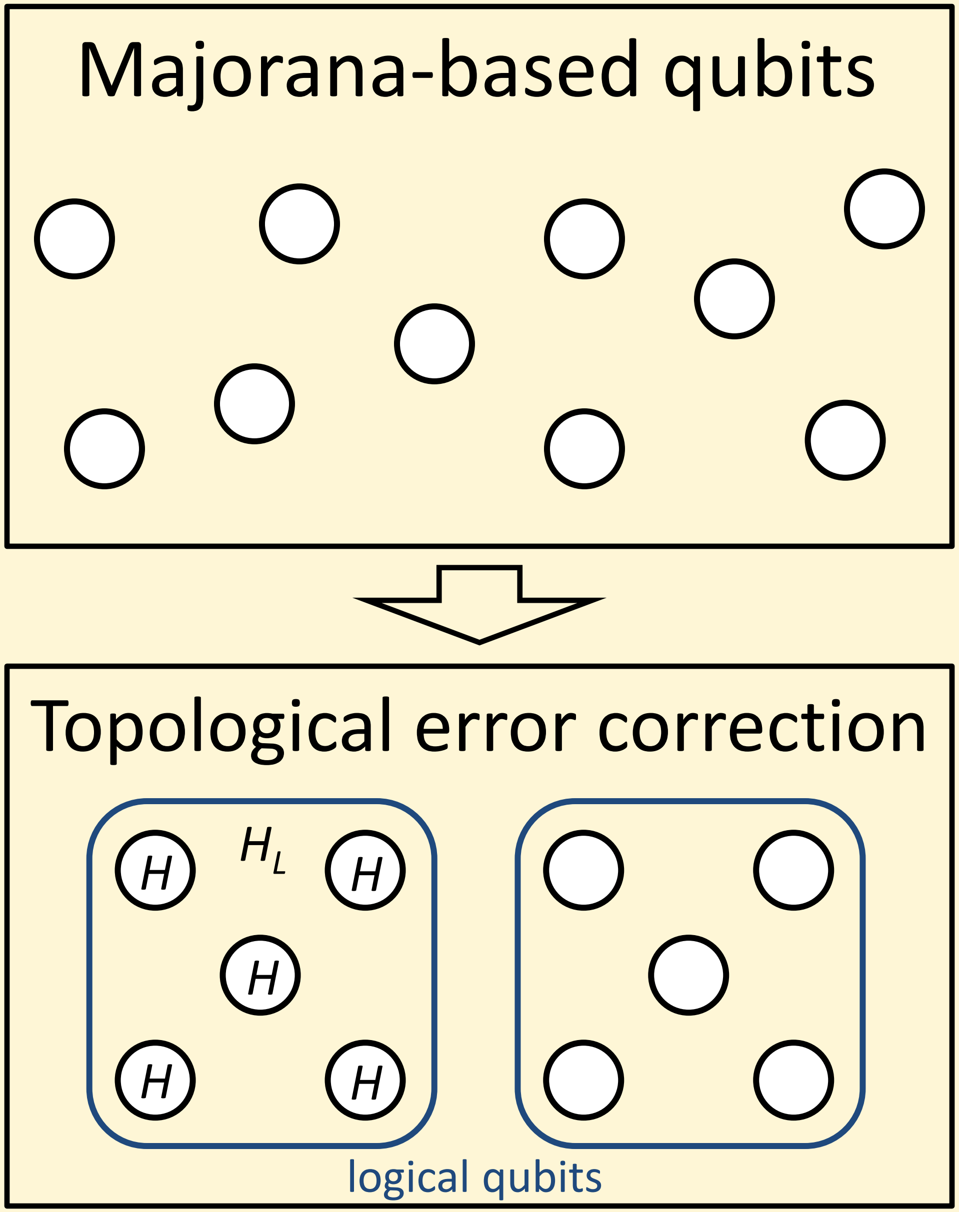
– Physical gates replaced by logical gates



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– Physical gates replaced by logical gates

desirable: $U_L = U^{\otimes n}$ (transversal gates)

Topologically protected gates of Majoranas

→ Clifford gates

Transversal gates of topological color codes

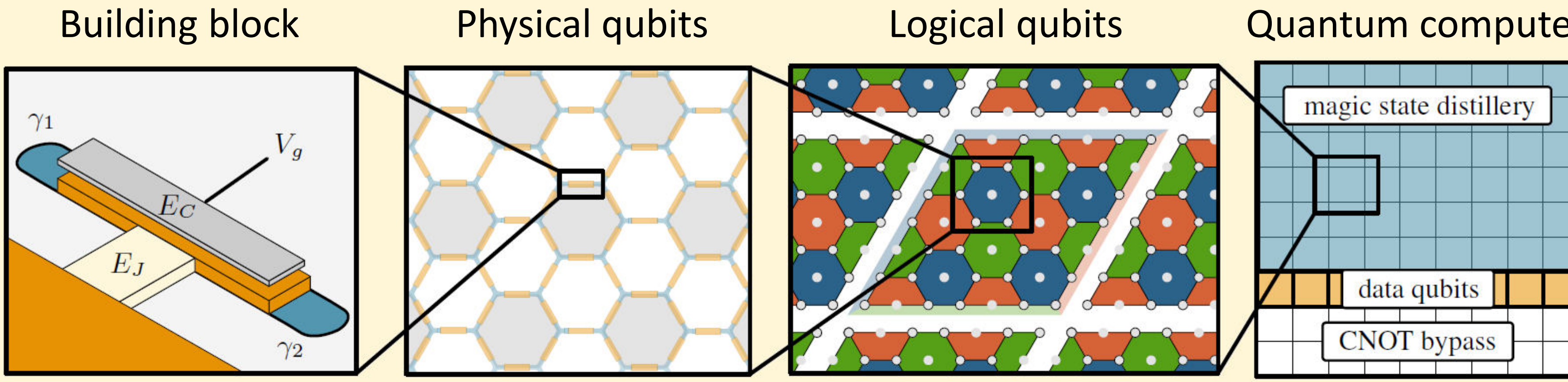
→ Clifford gates

(Clifford gates: {H,S,CNOT})

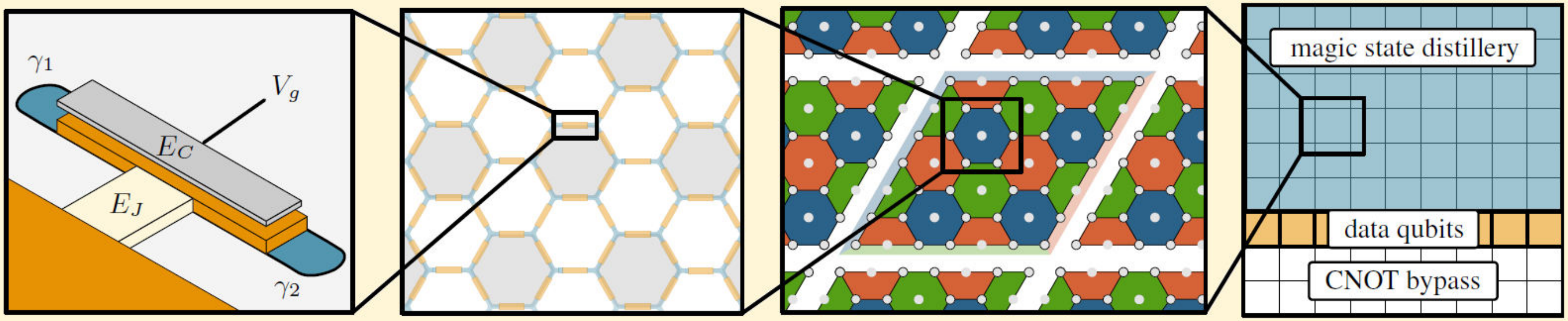
This is the main message.

↓

Majoranas and color codes are a perfect match!

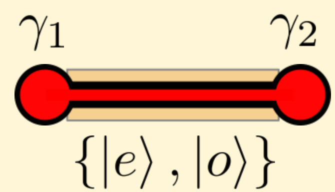

Building block
Physical qubits
Logical qubits
Quantum computer
γ_1
V_g
E_C
E_J
γ_2
magic state distillery
data qubits
CNOT bypass

Building block | Physical qubits | Logical qubits | Quantum computer



Topological Superconductor Network

<u>Operations:</u>



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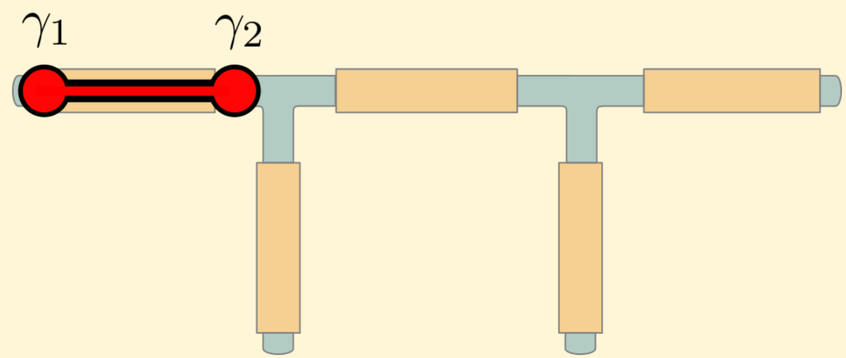
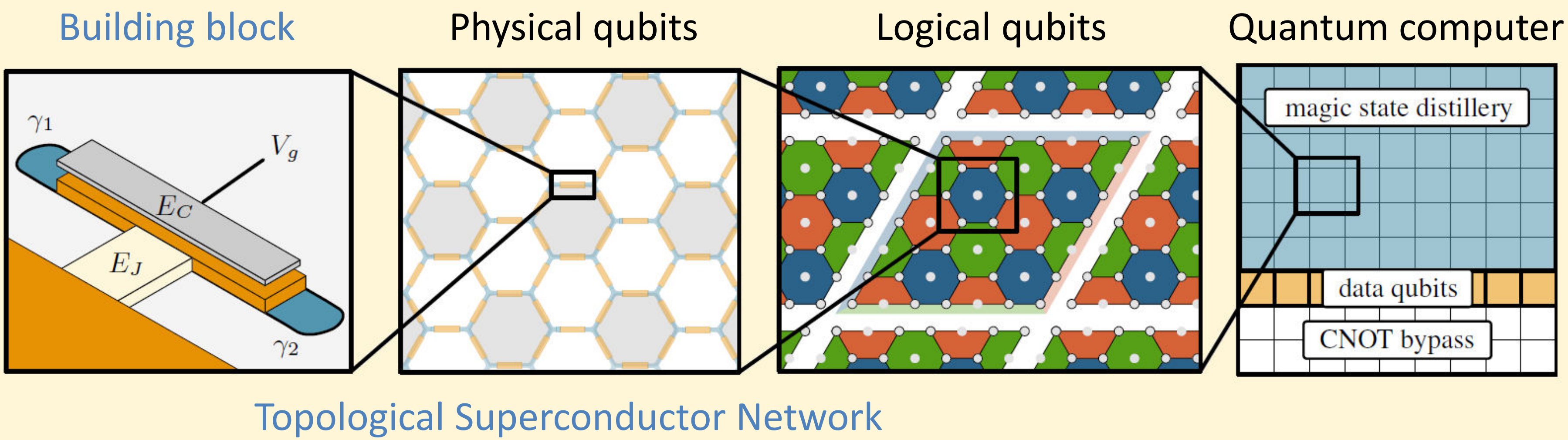


2. Measure fermion parity

3. Split degeneracy



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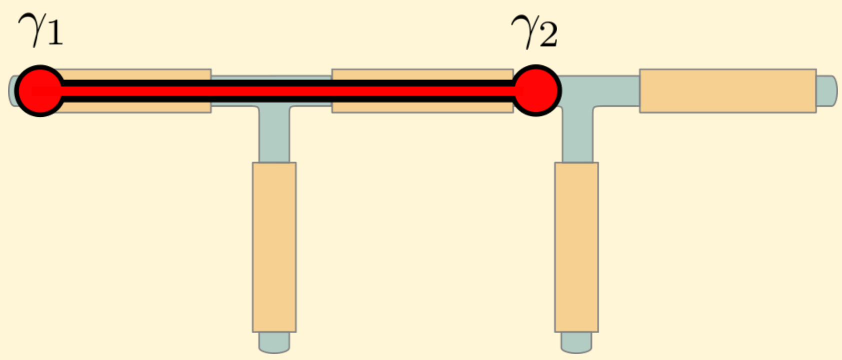
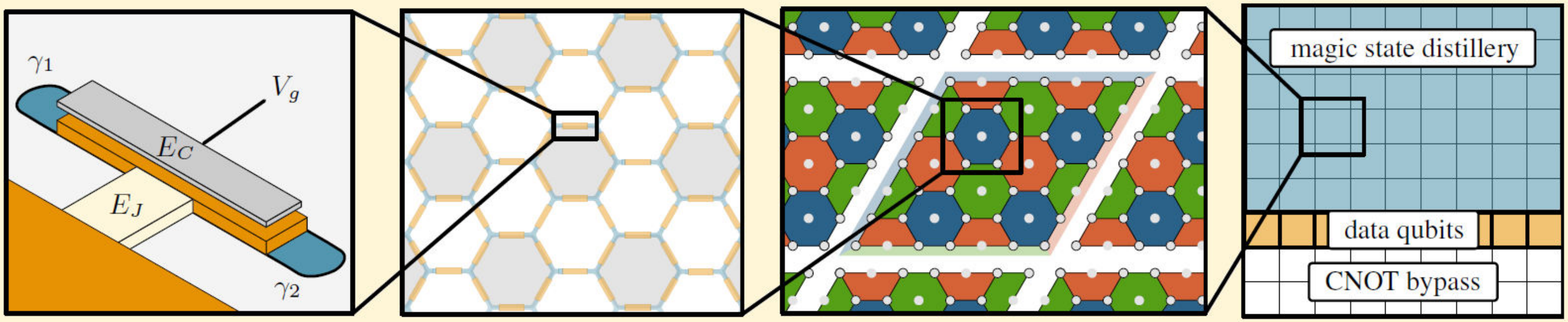


Building block

Physical qubits

Logical qubits

Quantum computer



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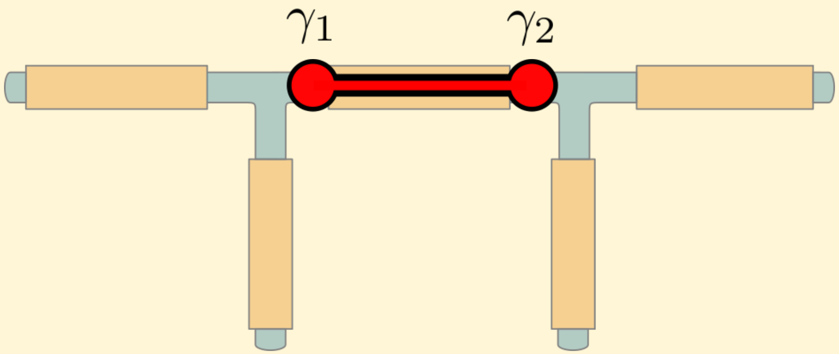

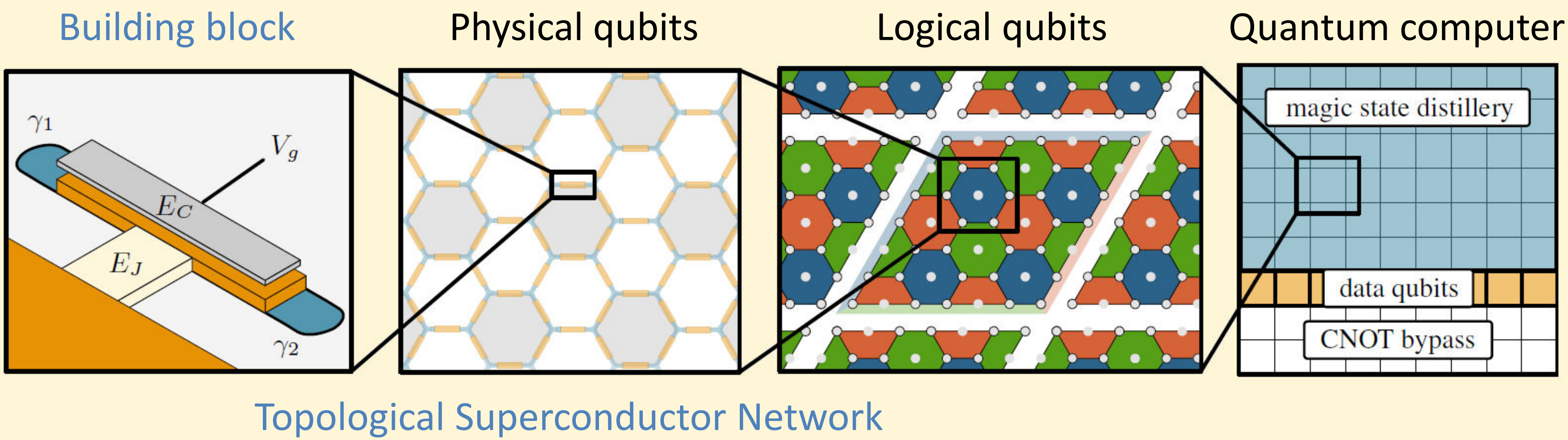

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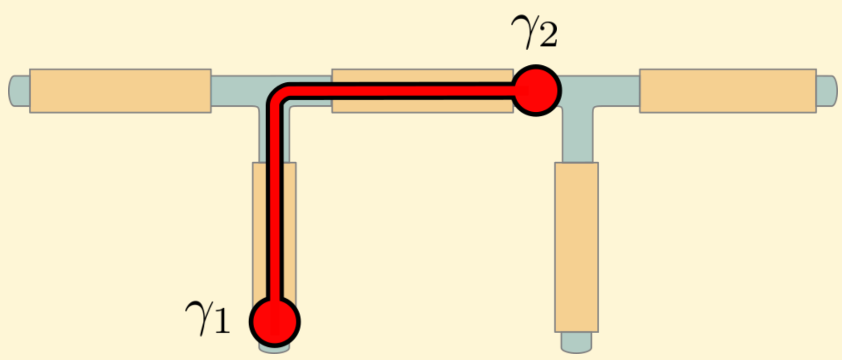

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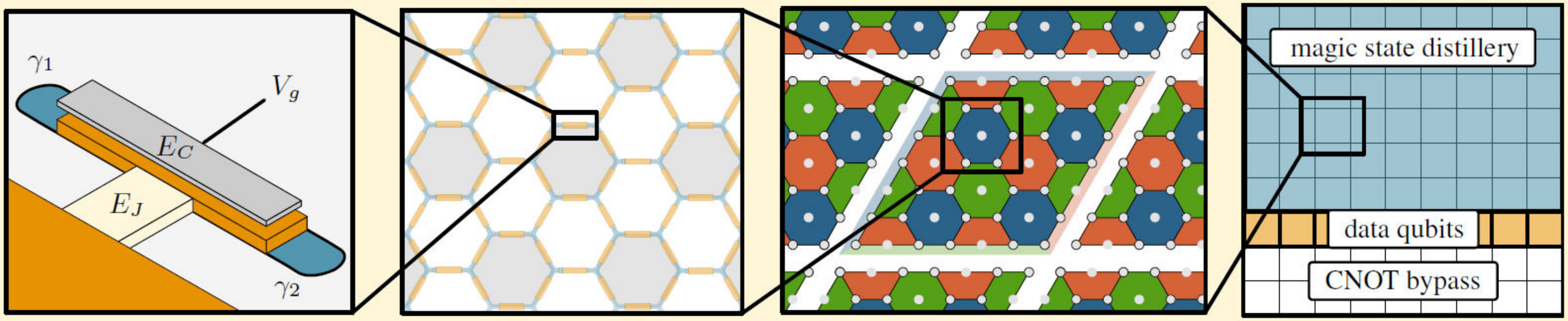

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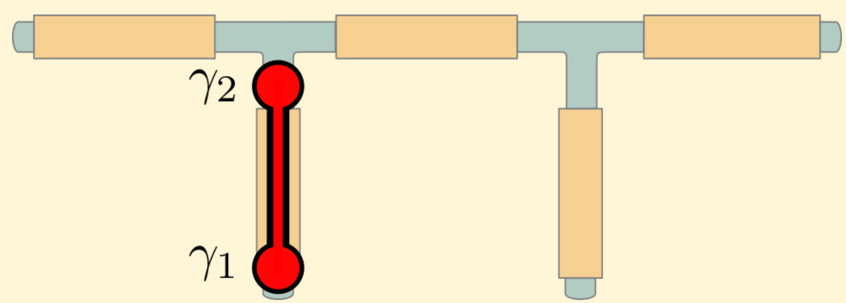

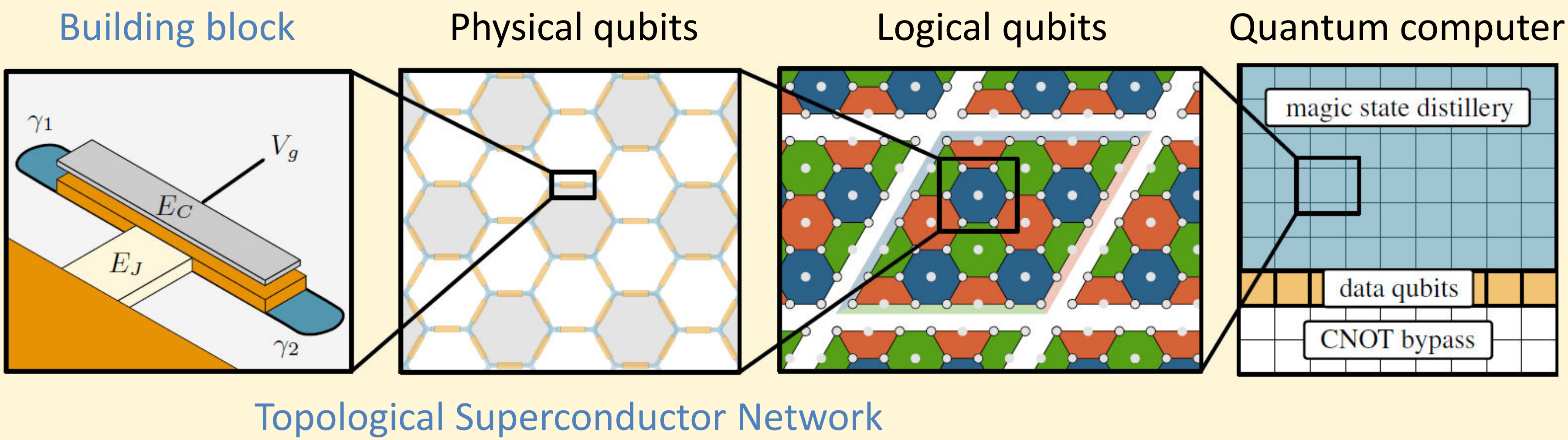

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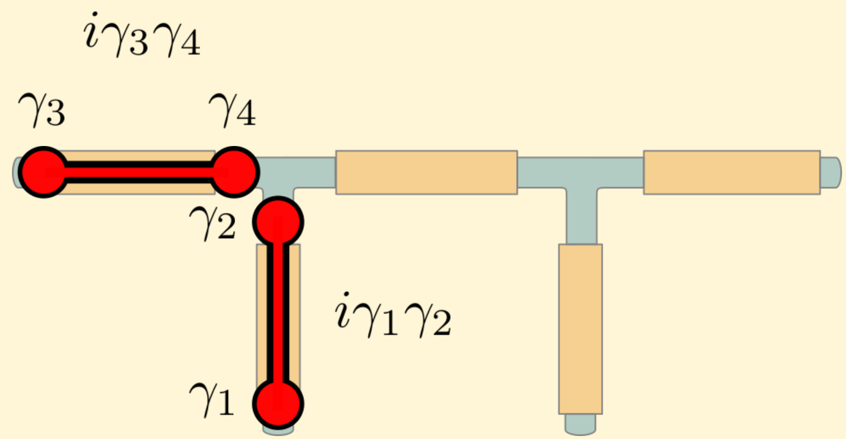

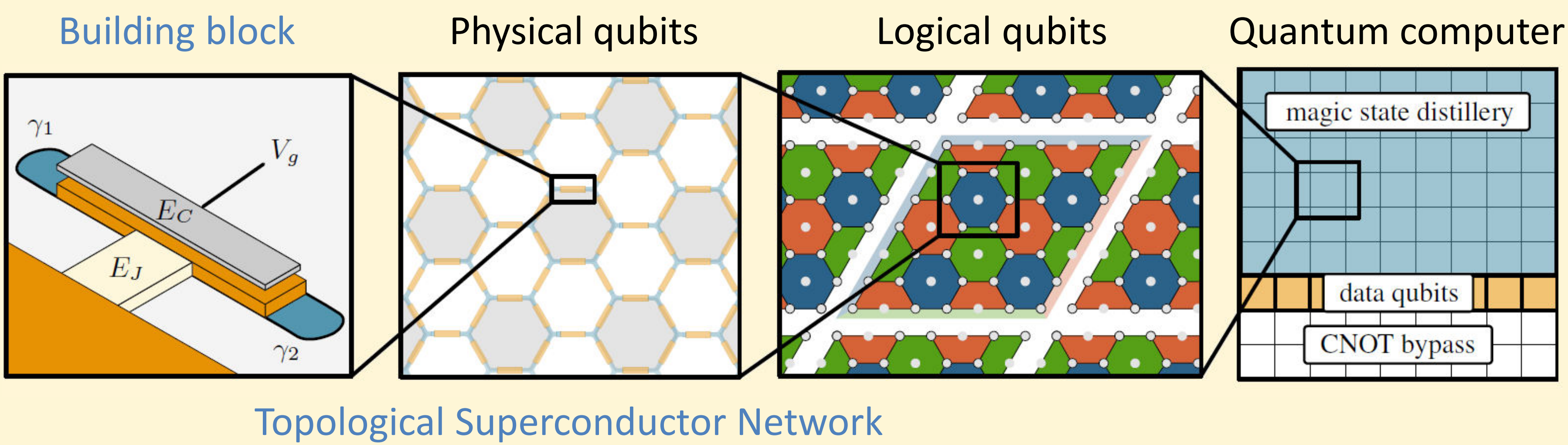

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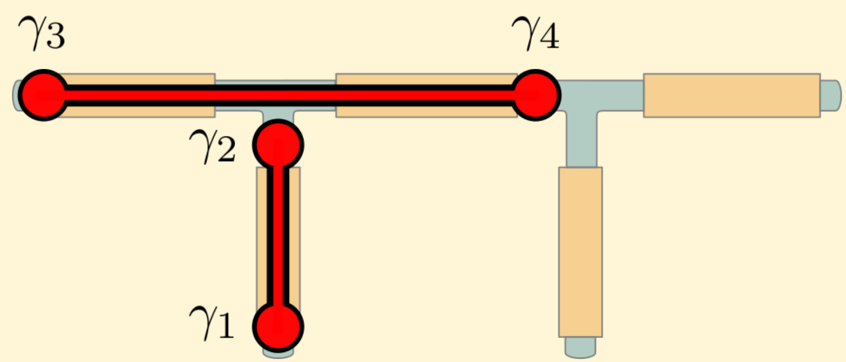

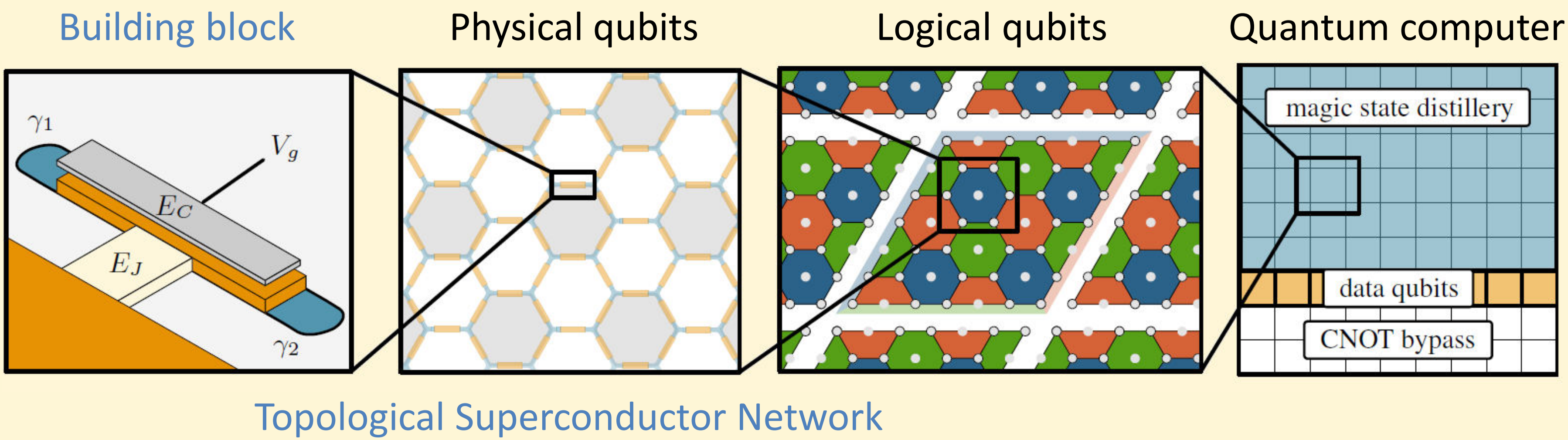

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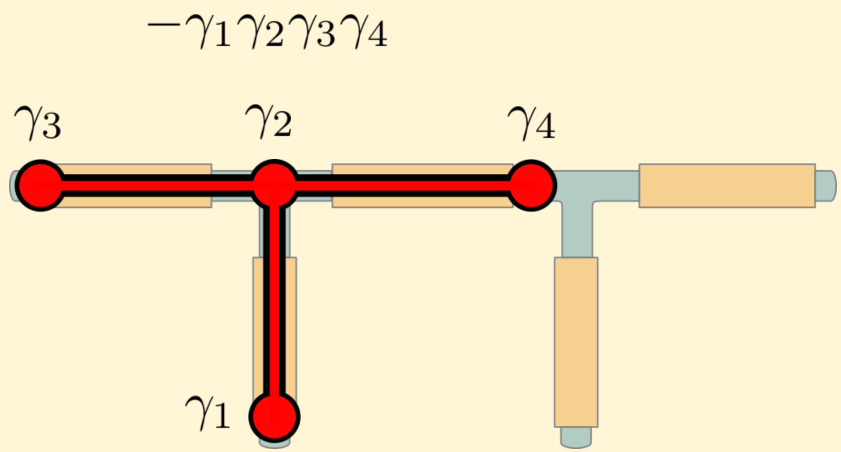

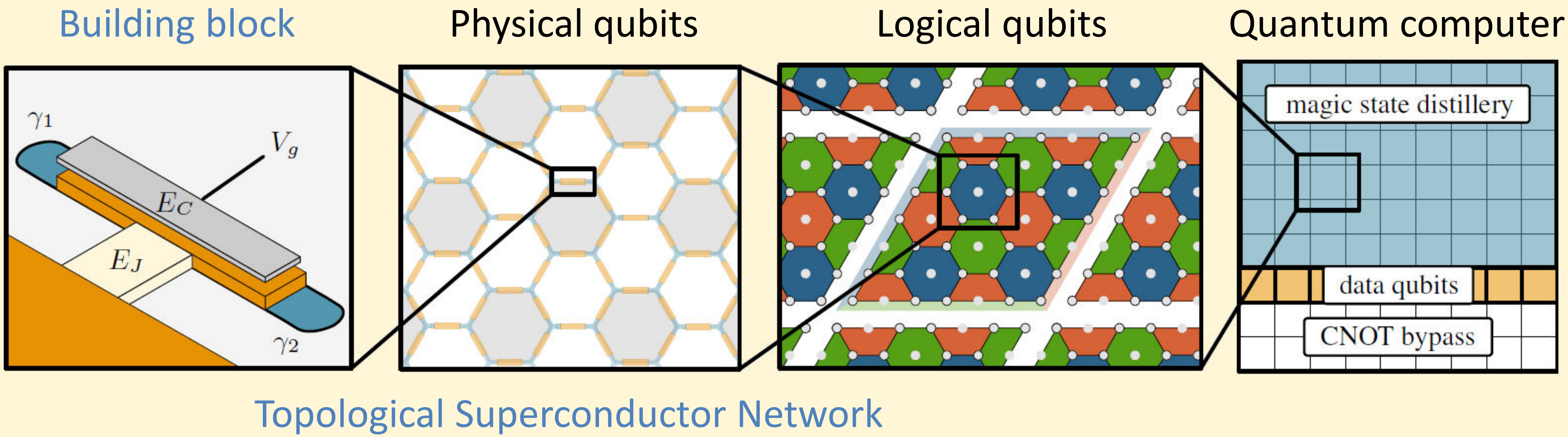

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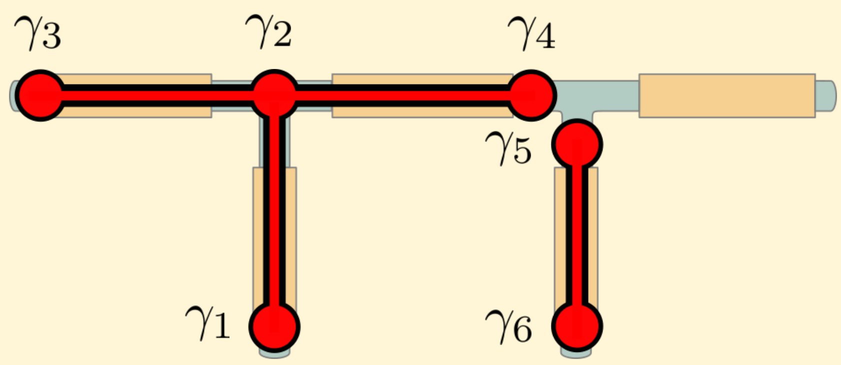

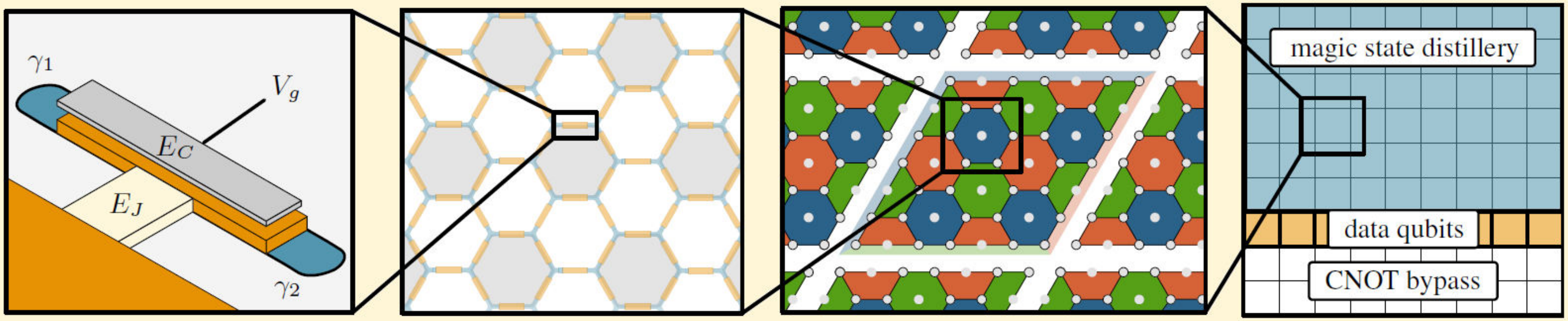

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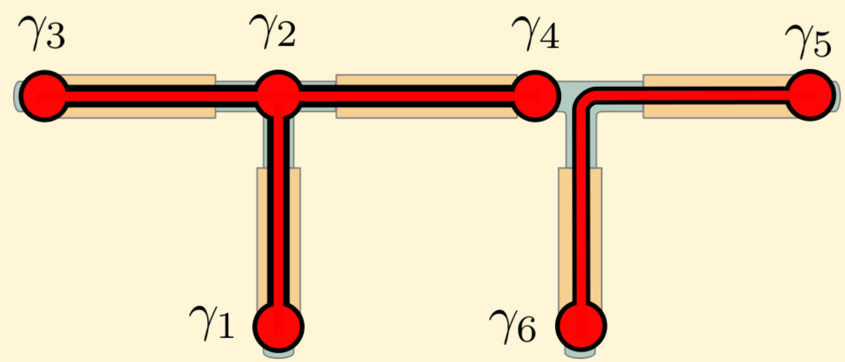

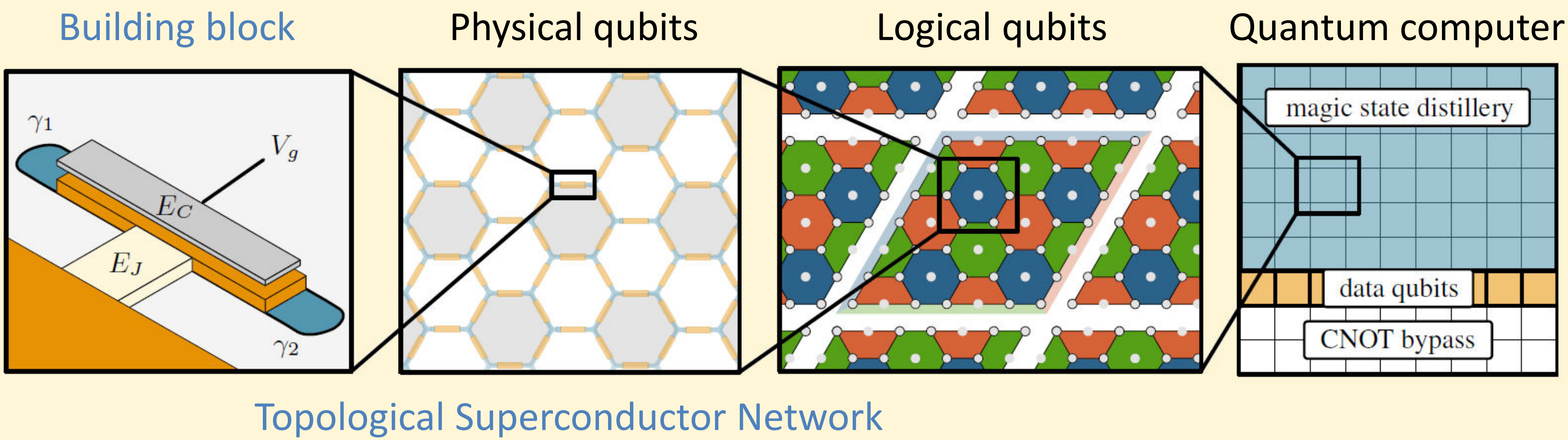

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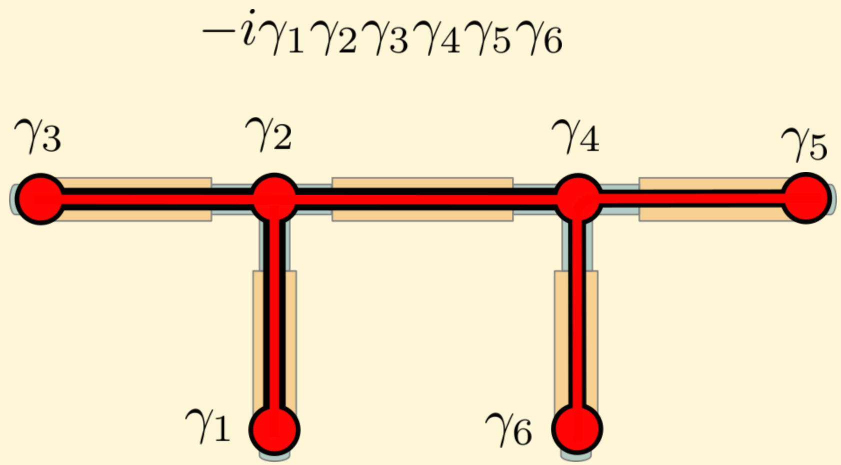

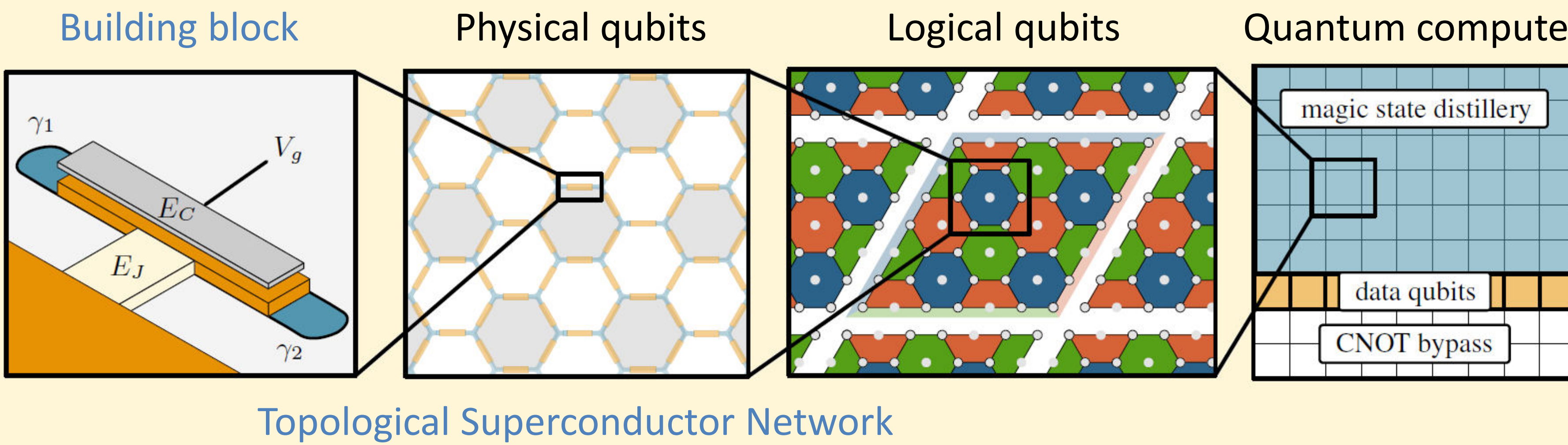

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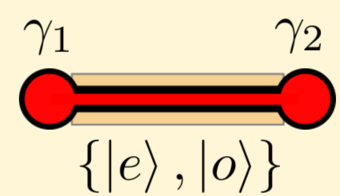

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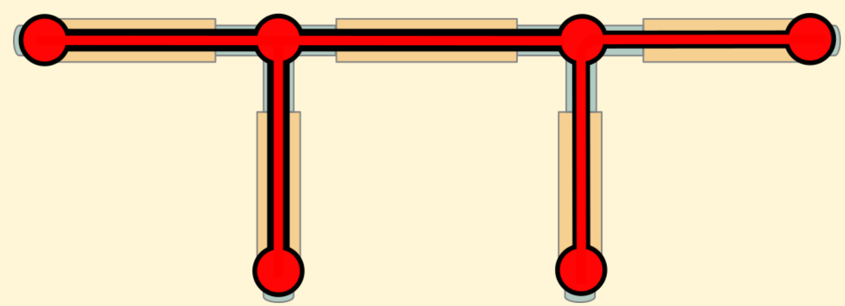

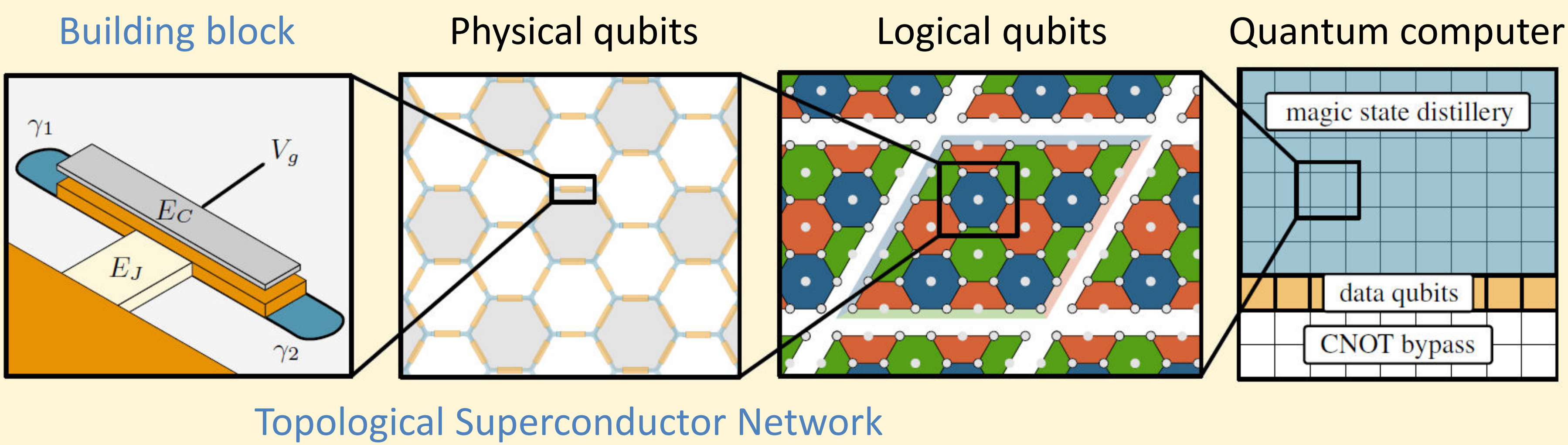

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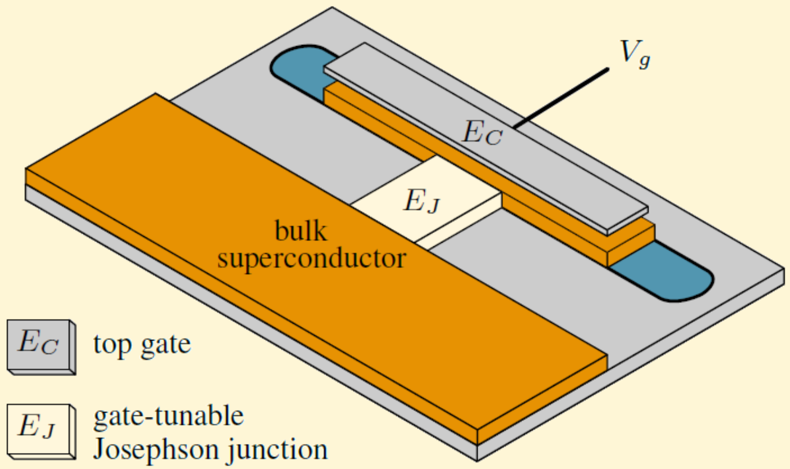
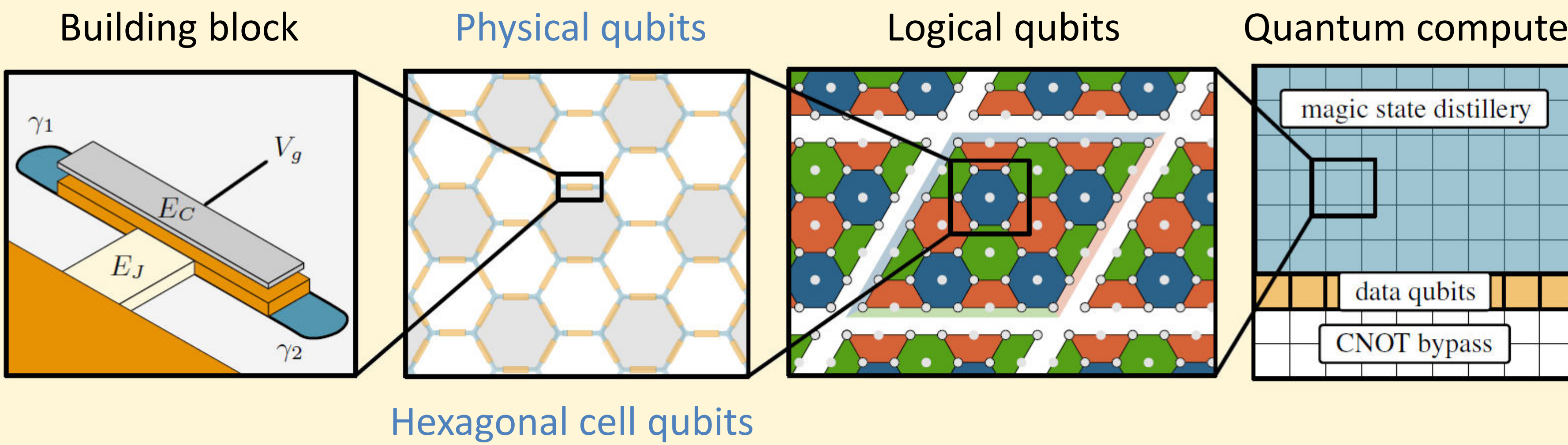
Possible implementation
Majorana Cooper pair box



D. Aasen et al., *Milestones Toward Majorana-Based Quantum Computing,* PRX **6,** 031016 (2016)



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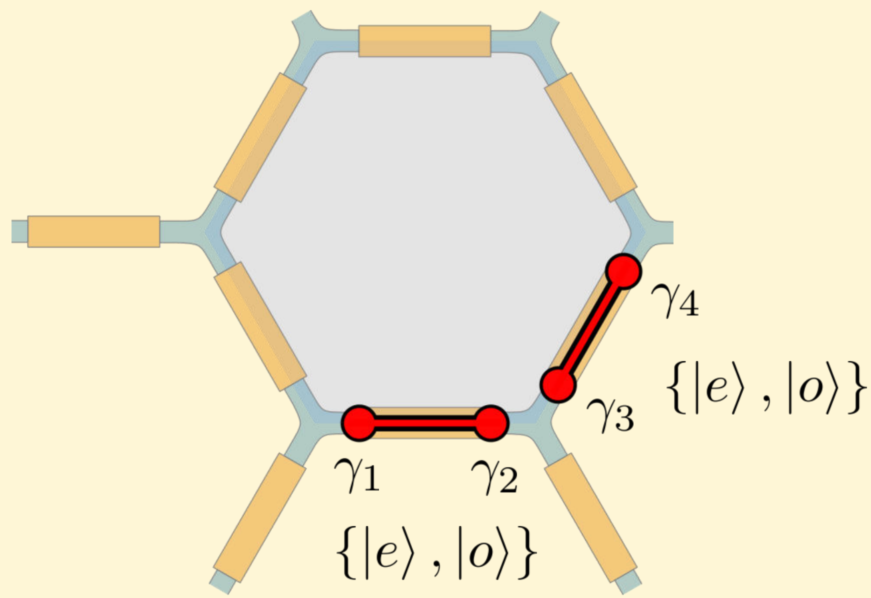
$$\sigma_z = i\gamma_1\gamma_2$$

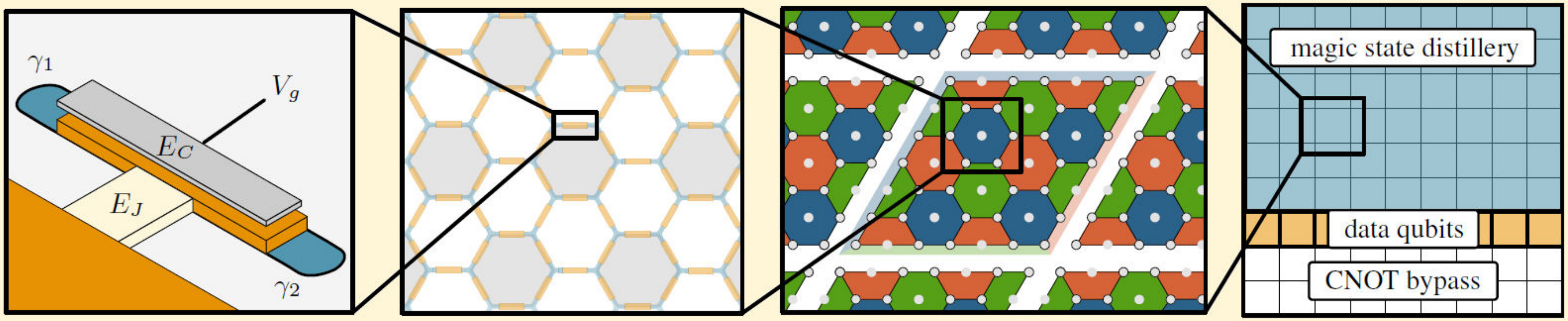
$$S = B_{1,2}$$

$$H \sim B_{1,2}B_{2,3}B_{1,2}$$

S *H* CNOT *T*



Building block Physical qubits Logical qubits Quantum computer



Hexagonal cell qubits

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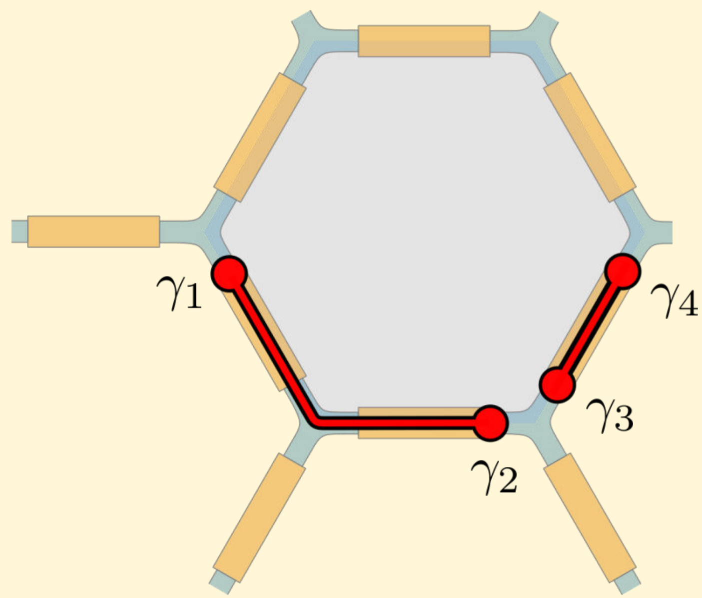

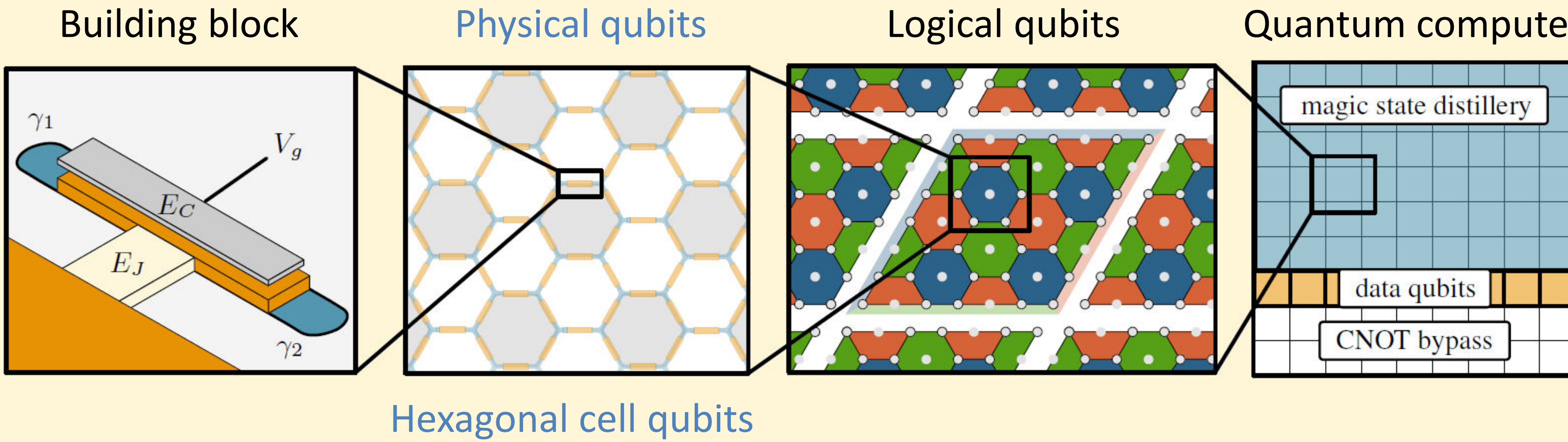

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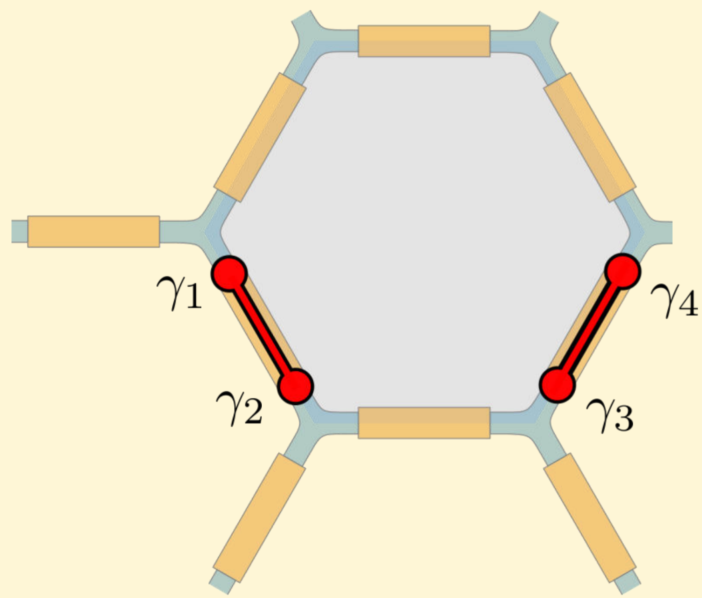

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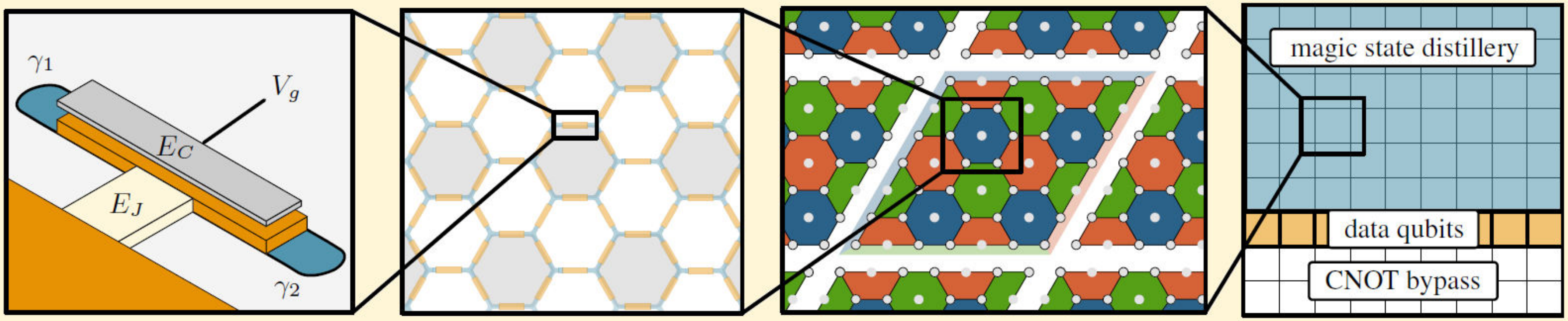

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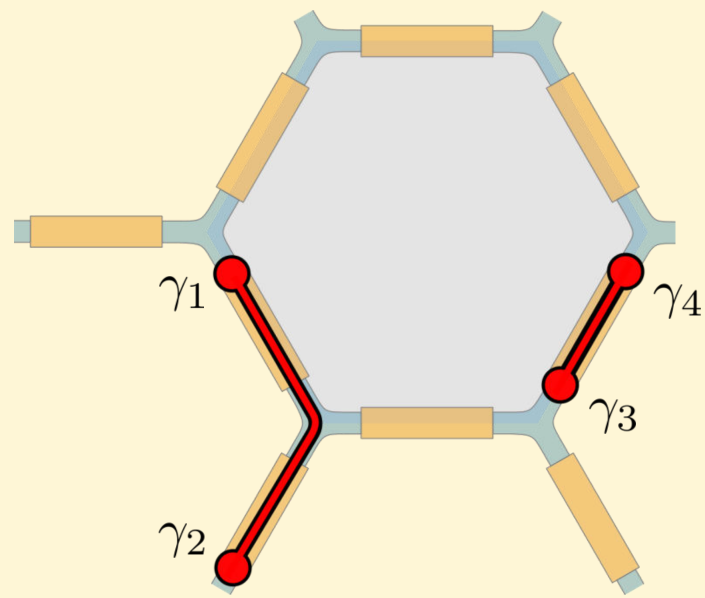

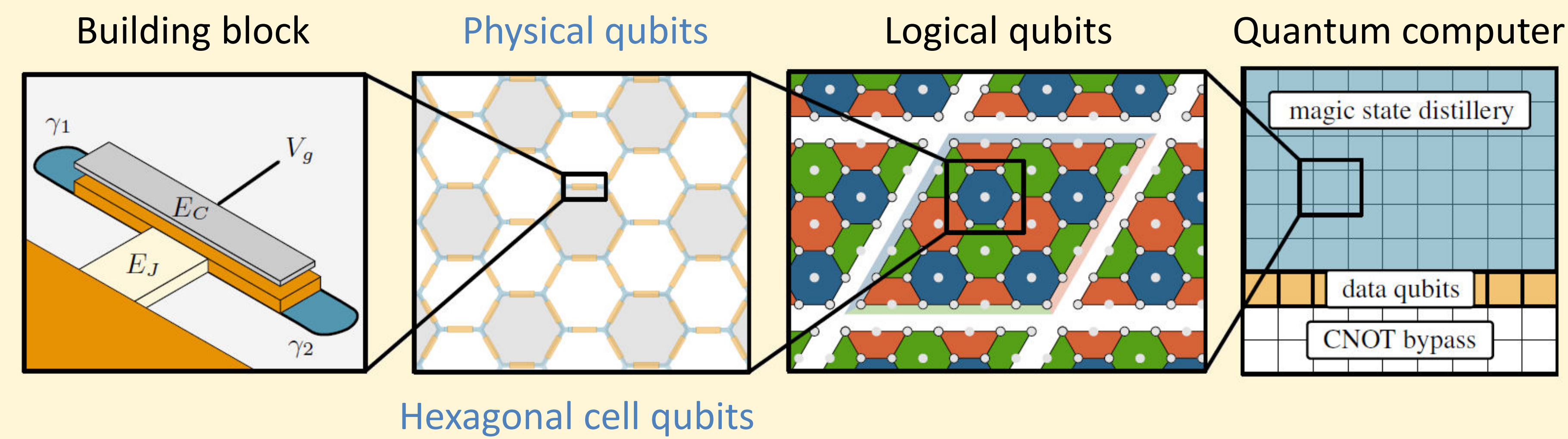

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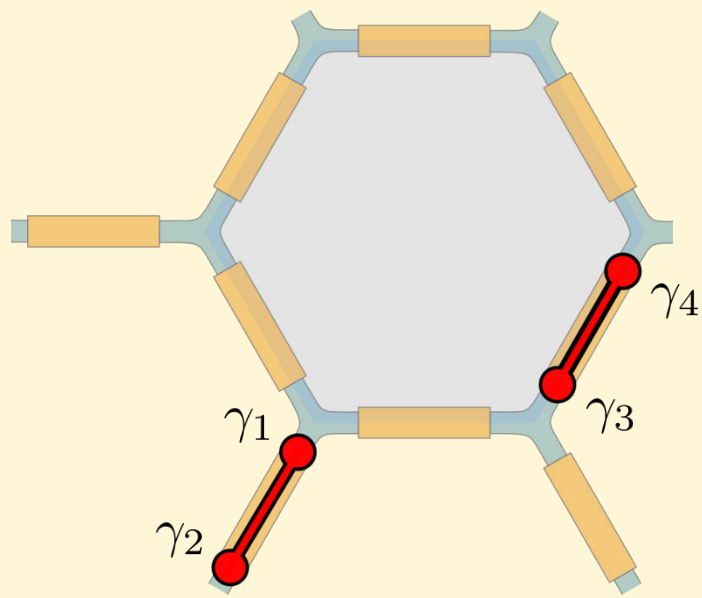

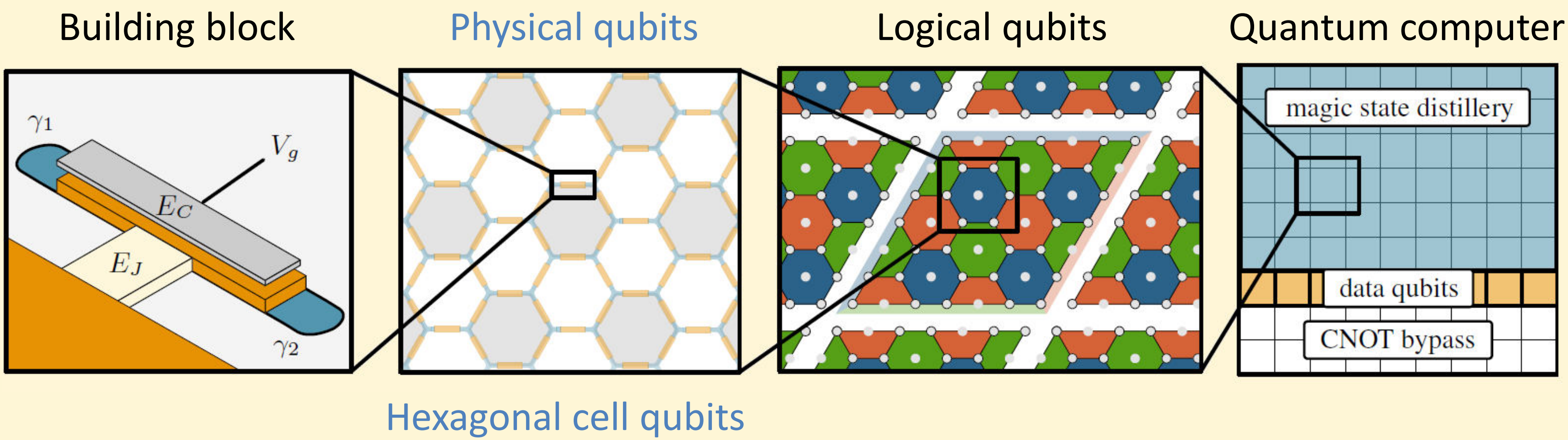

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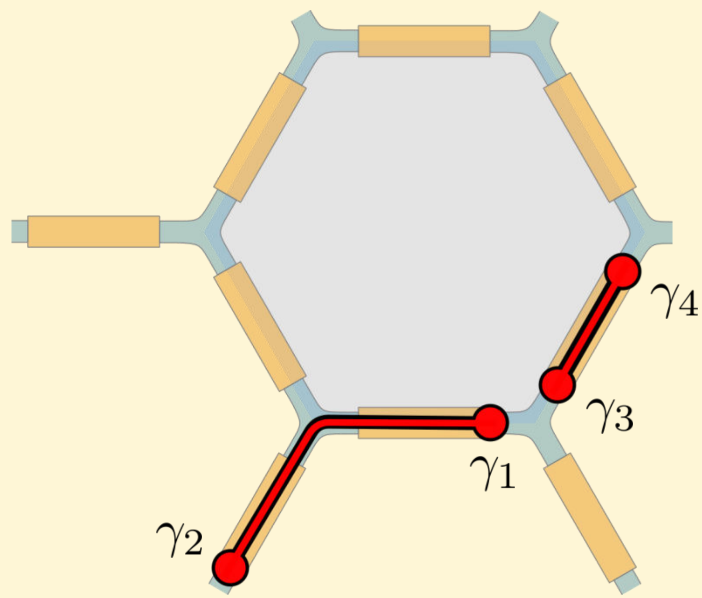

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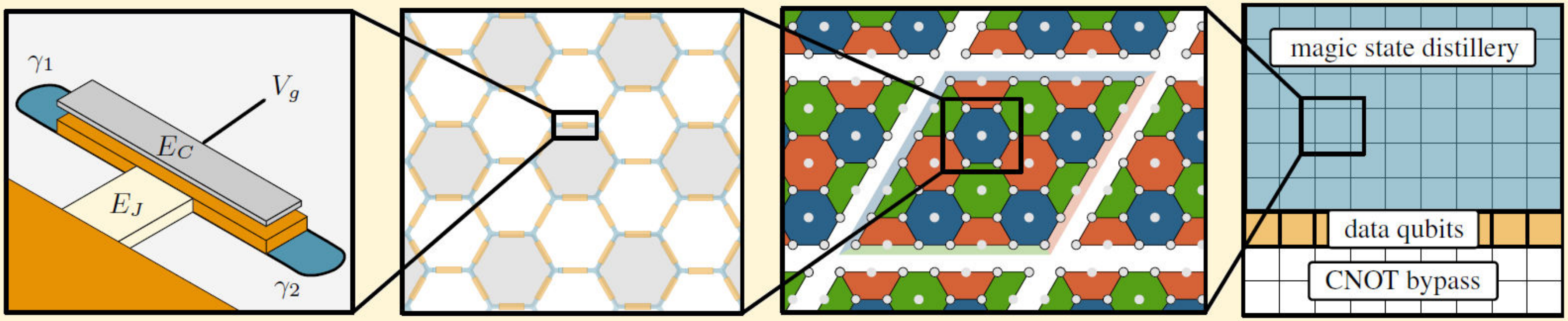

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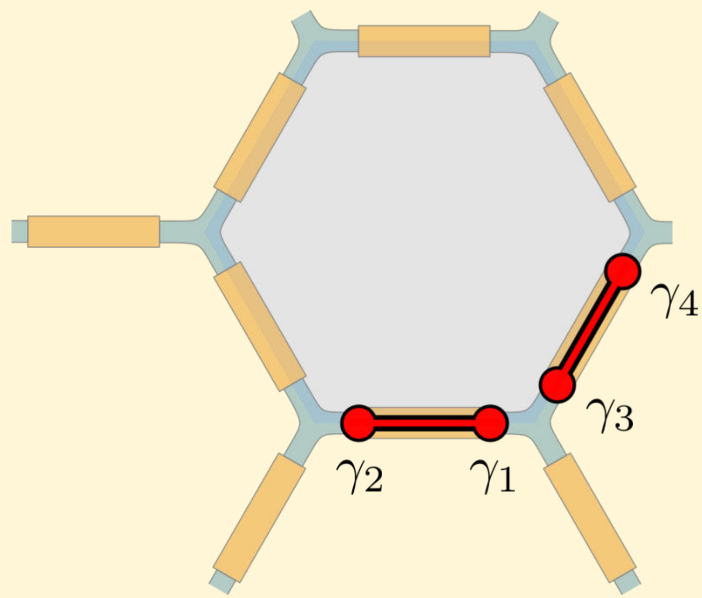

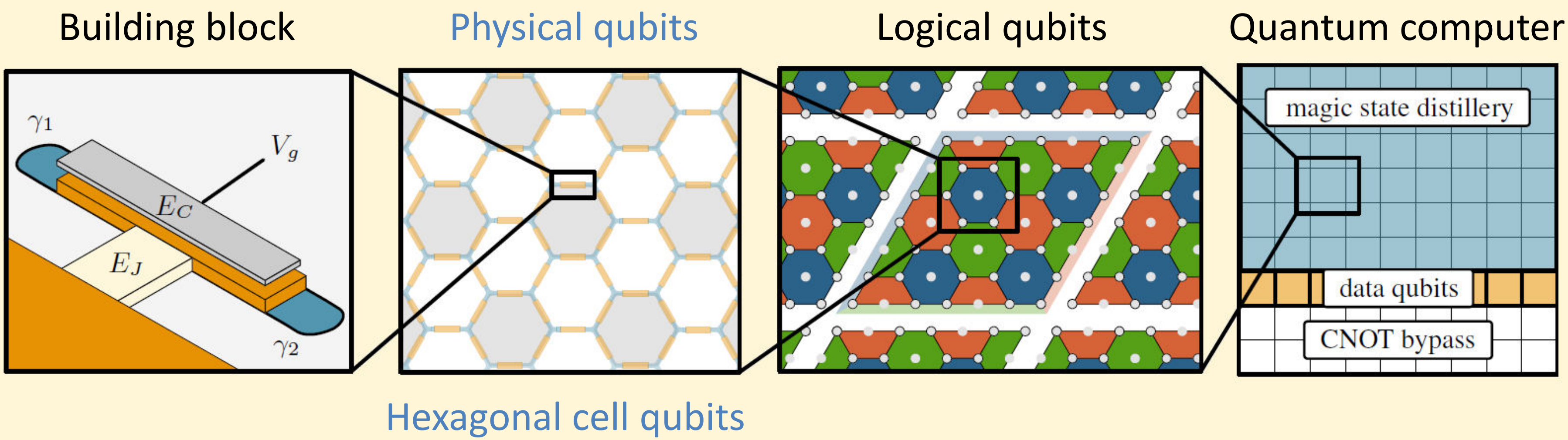

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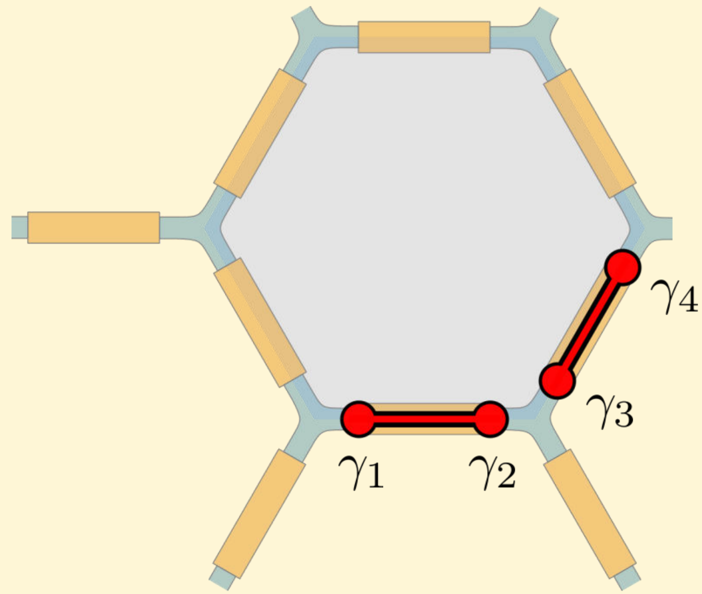

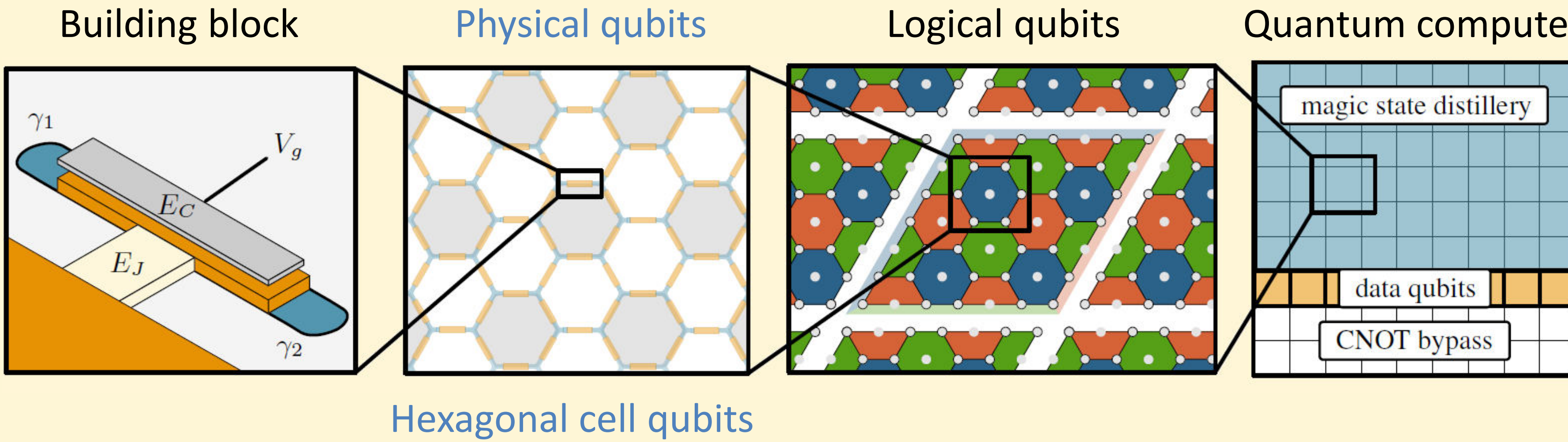

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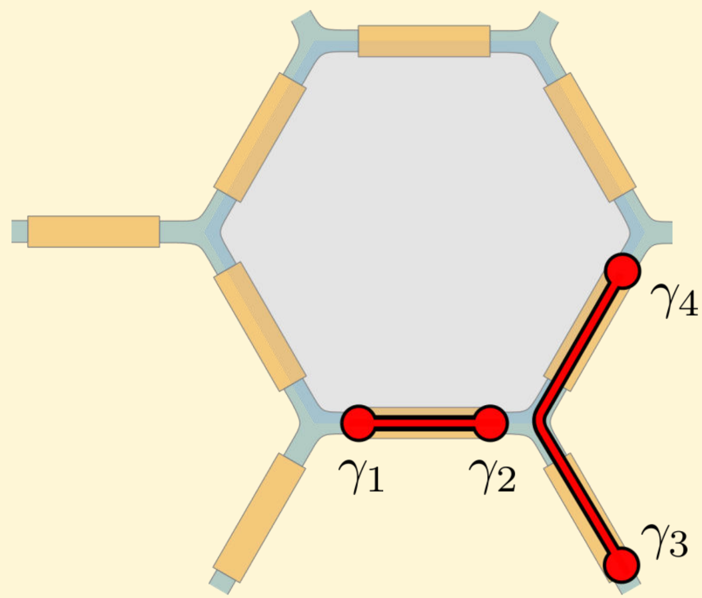

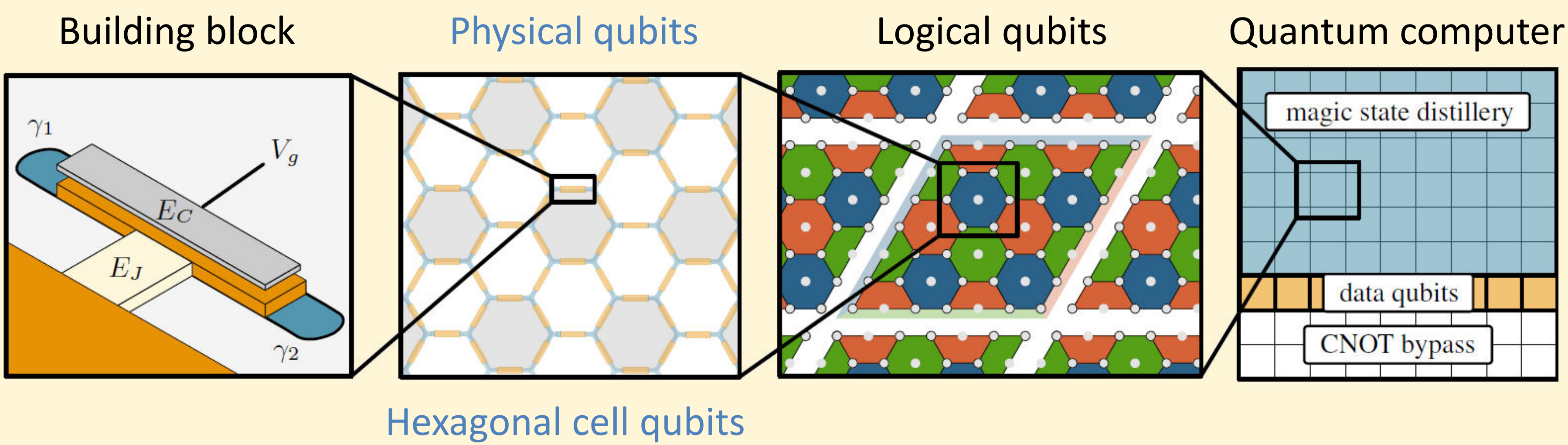

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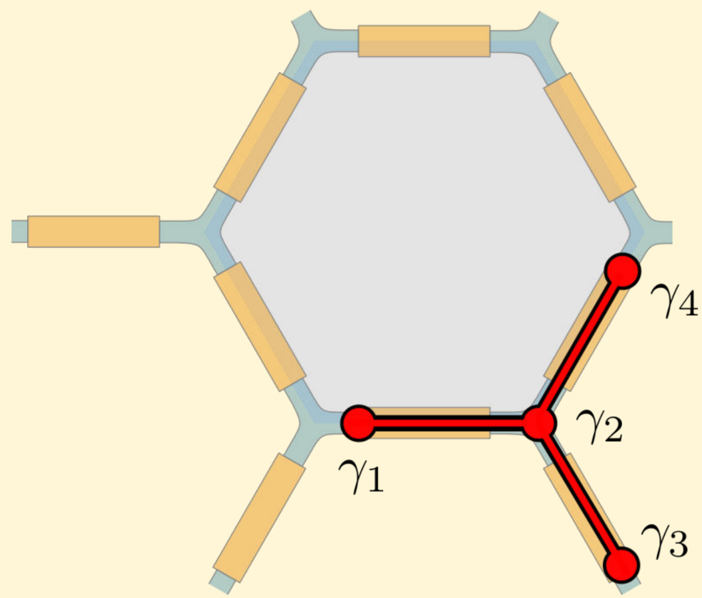

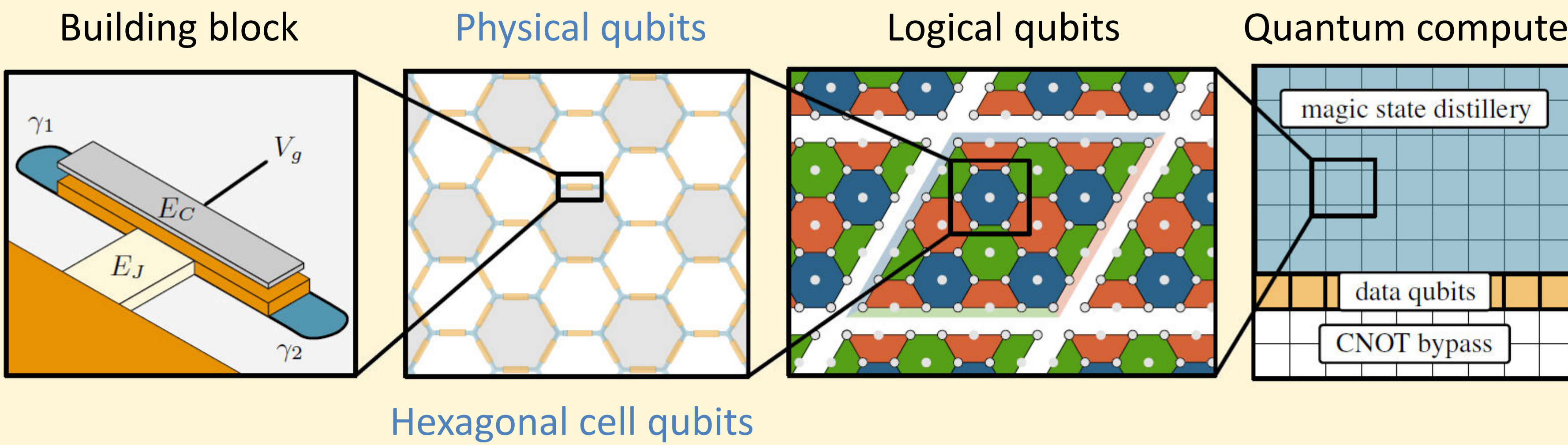

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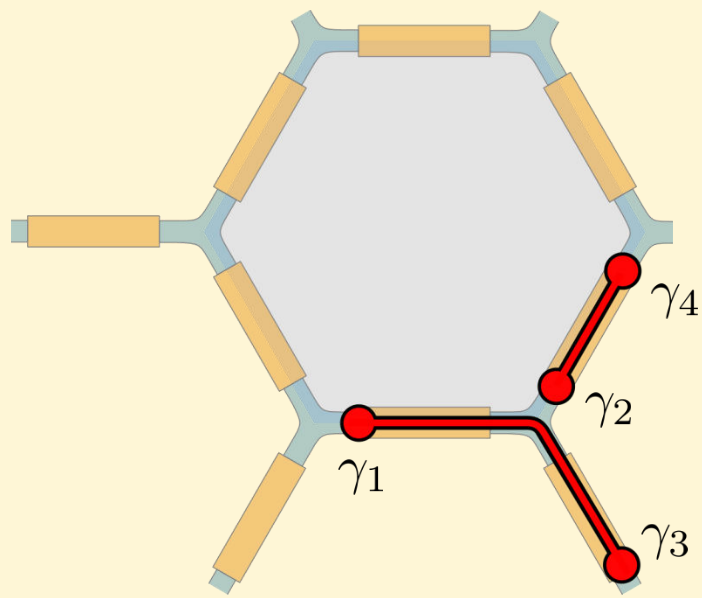
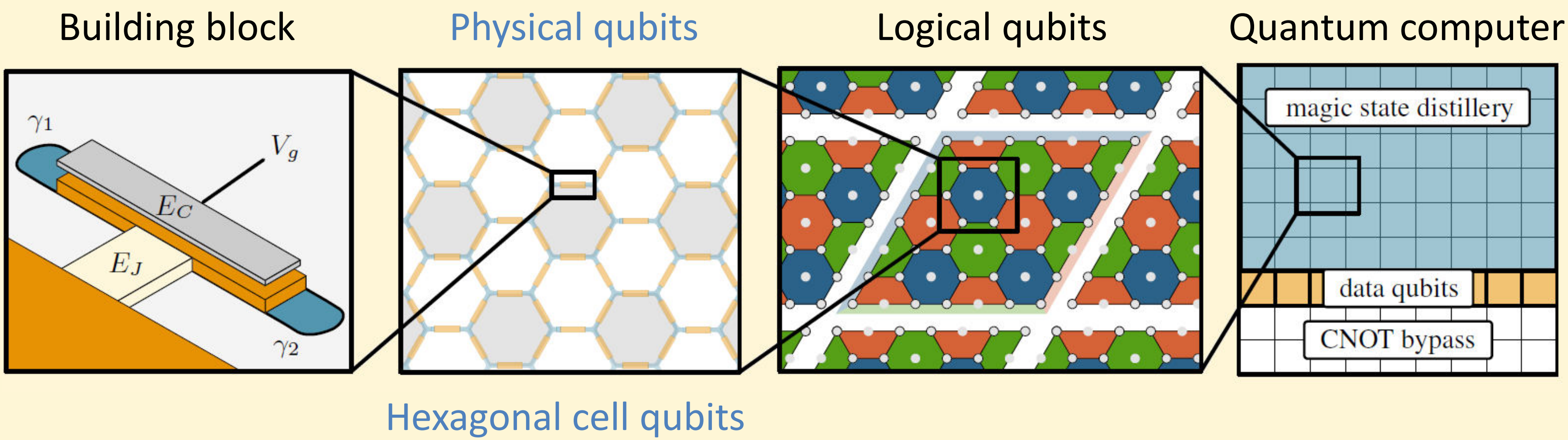




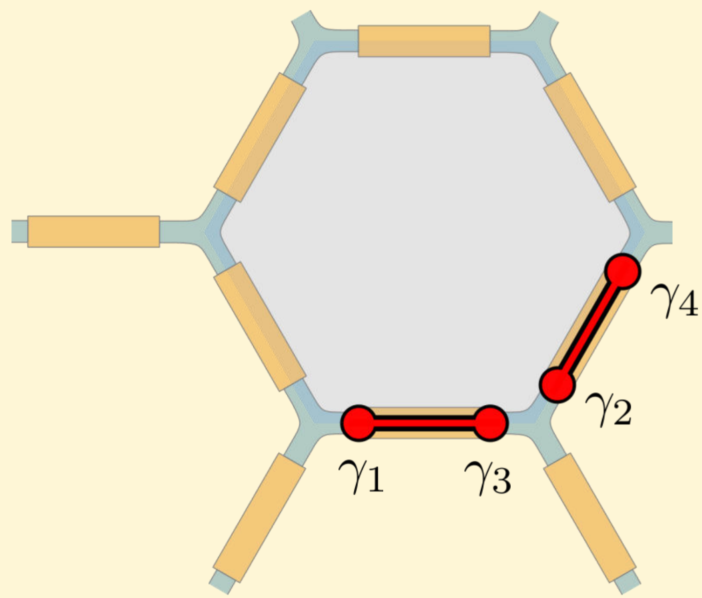
S *H* CNOT *T*

$$|0\rangle = |e,e\rangle \quad \text{or} \quad |e,o\rangle$$
$$|1\rangle = |o,o\rangle \quad \phantom{\text{or}} \quad |o,e\rangle$$

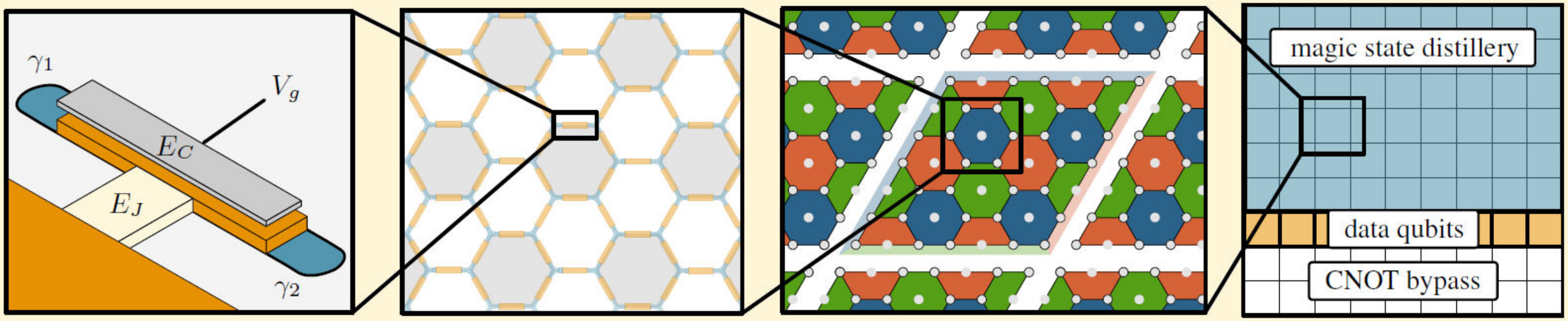
$$\sigma_z = i\gamma_1\gamma_2$$

$$S = B_{1,2}$$

$$H \sim B_{1,2}B_{2,3}B_{1,2}$$

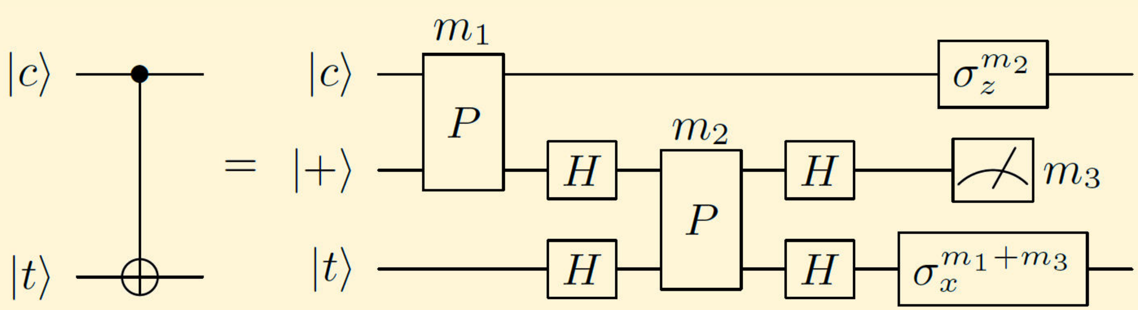


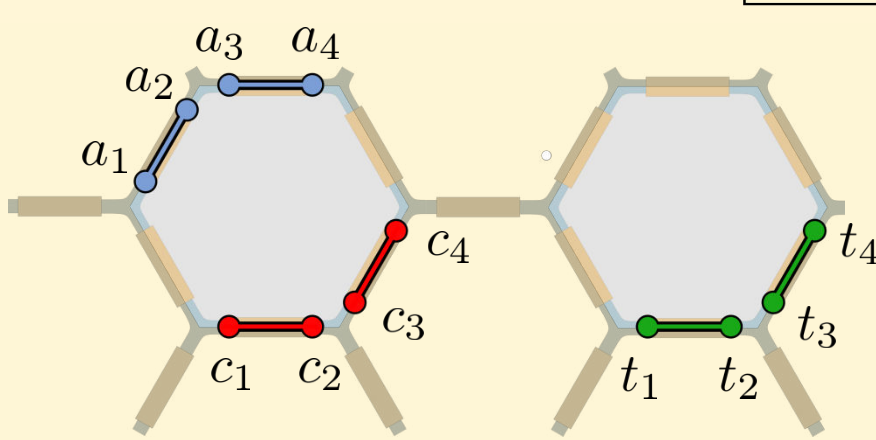
Building block | Physical qubits | Logical qubits | Quantum computer



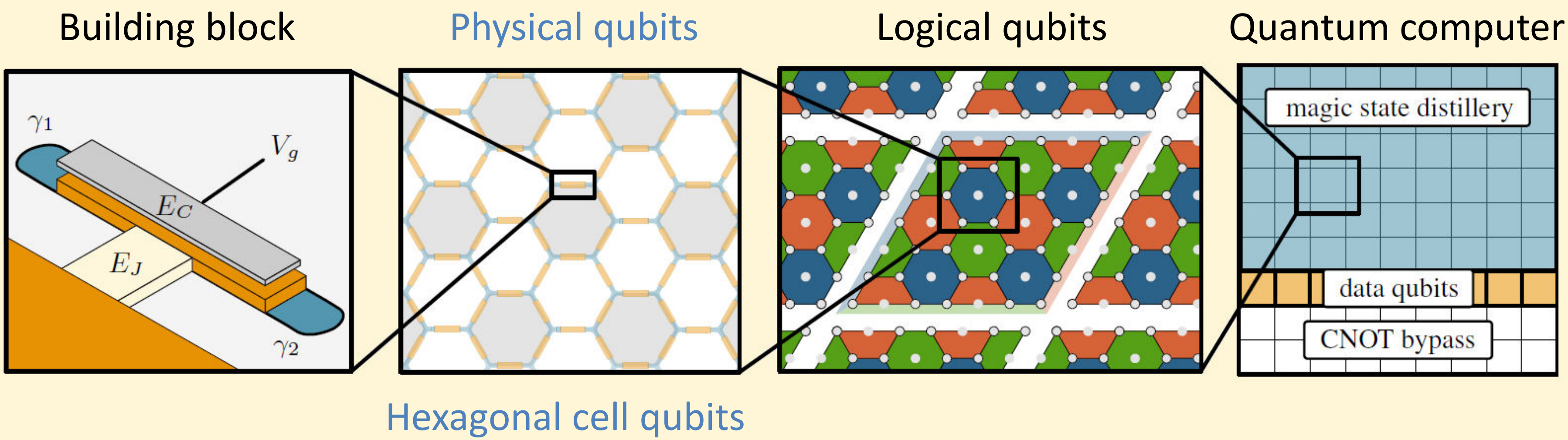
Hexagonal cell qubits

S H CNOT T





$$P = \sigma_z \otimes \sigma_z$$
$$= -c_1 c_2 a_1 a_2$$
$$(= -a_1 a_2 t_1 t_2)$$



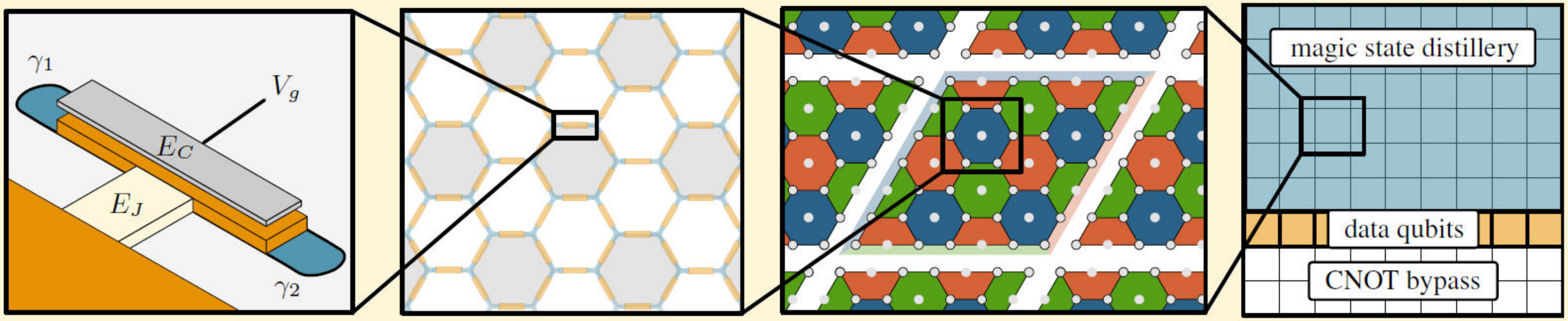
S *H* CNOT *T*

Split degeneracy (requires fine-tuning)

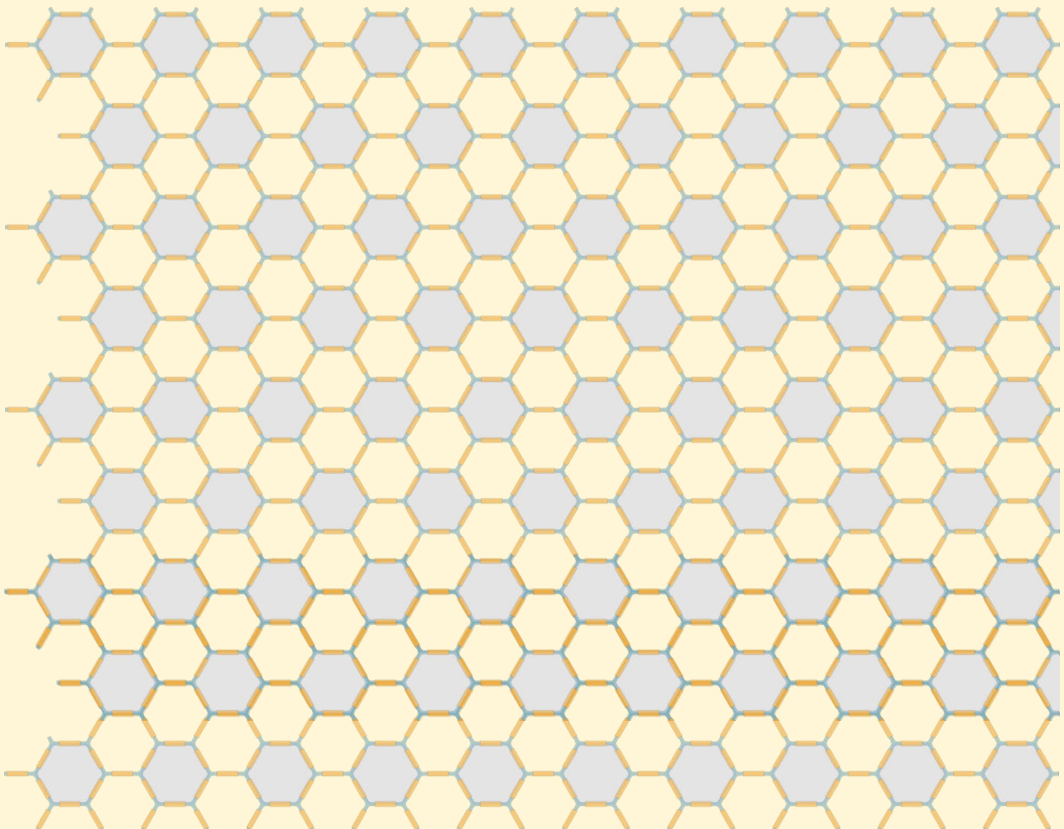
$$|\psi\rangle (t) = \alpha |0\rangle + \beta e^{i\Delta E t} |1\rangle$$

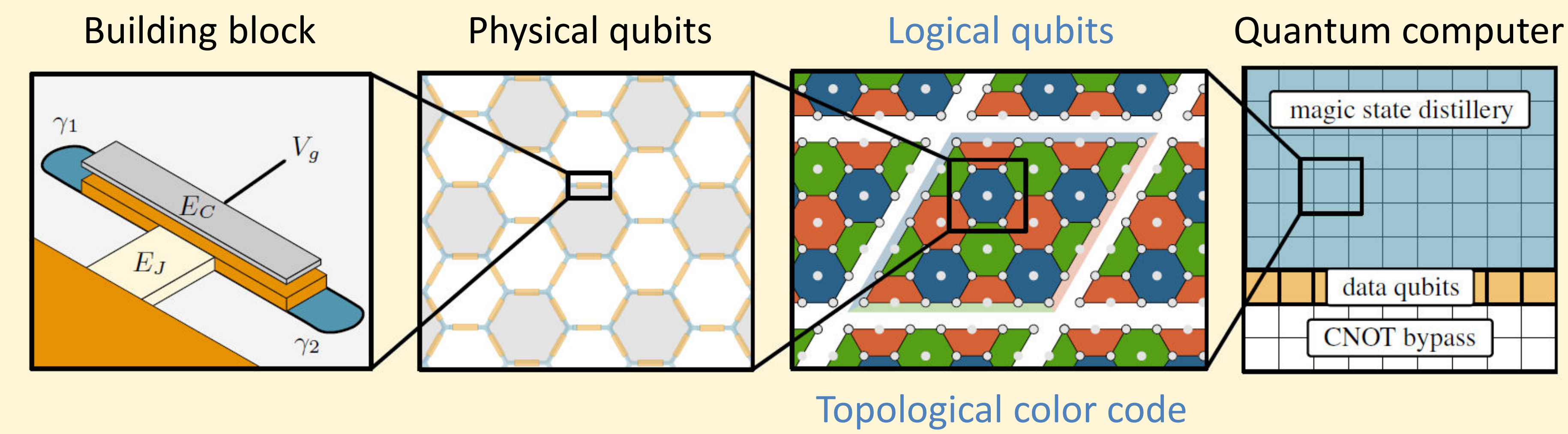
$$\Delta E t \stackrel{!}{=} \pi/4$$

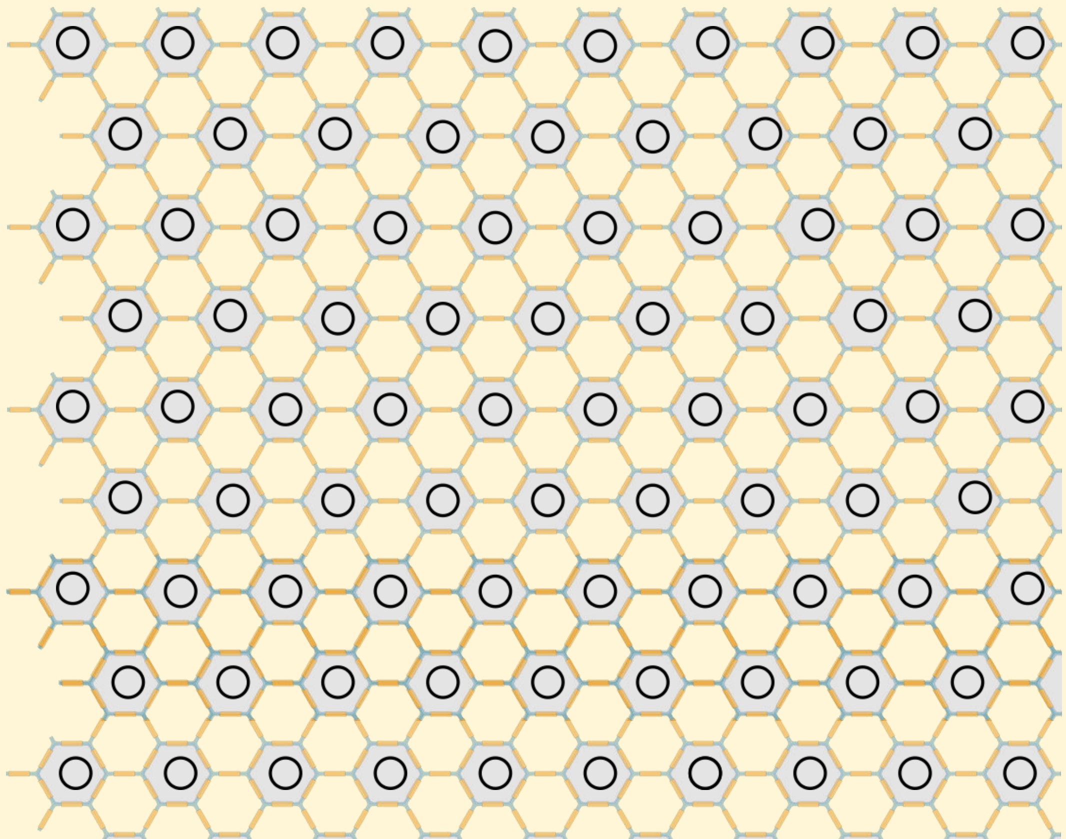
Building block | Physical qubits | Logical qubits | Quantum computer

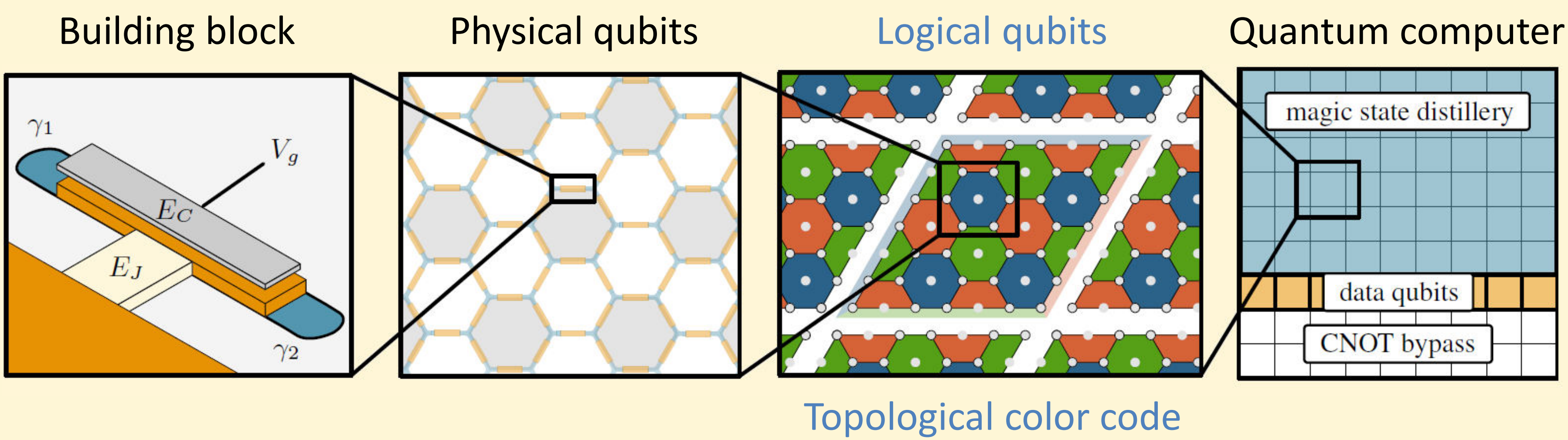


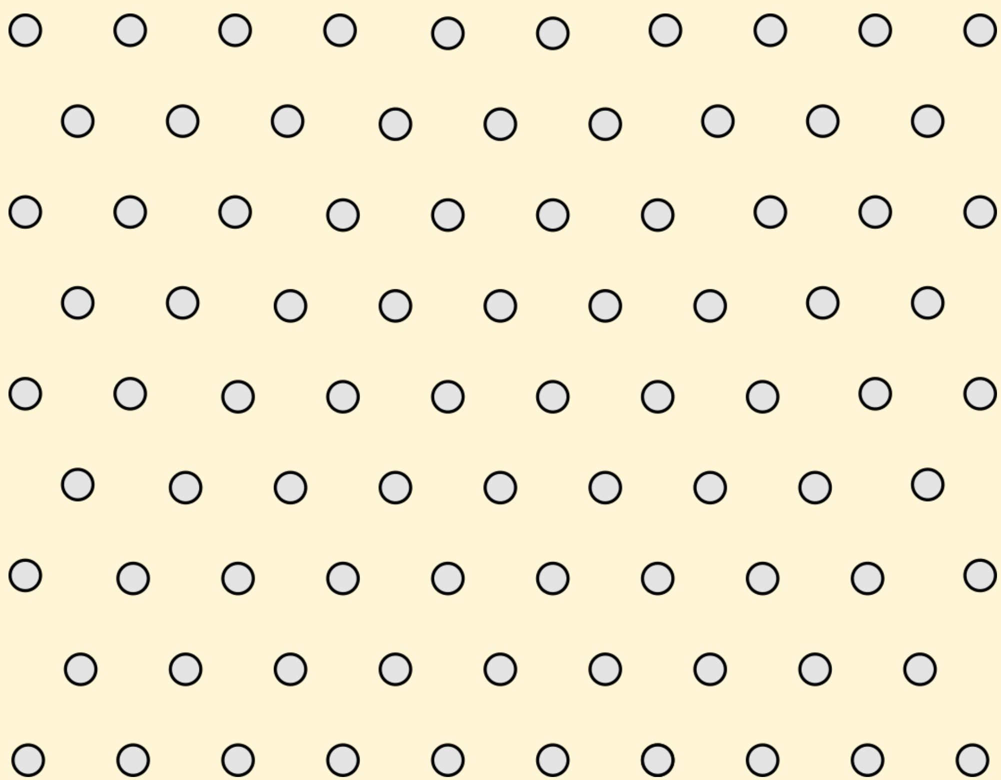
Topological color code

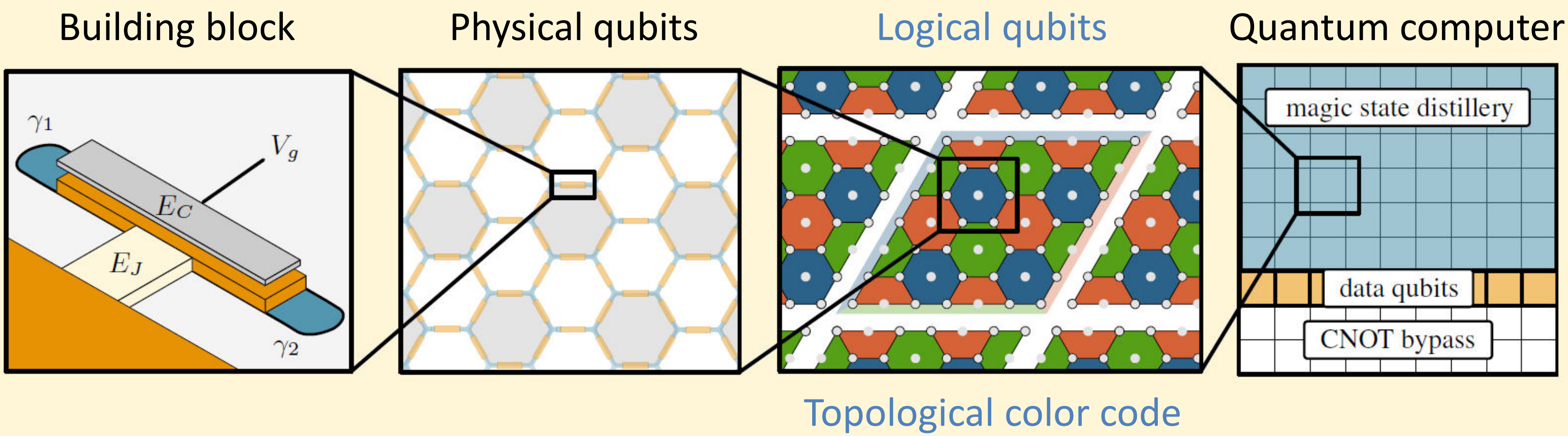



Building block
Physical qubits
Logical qubits
Quantum computer
γ_1
V_g
E_C
E_J
γ_2
magic state distillery
data qubits
CNOT bypass
Topological color code




Building block
Physical qubits
Logical qubits
Quantum computer
γ1
Vg
EC
EJ
γ2
magic state distillery
data qubits
CNOT bypass
Topological color code




Building block
Physical qubits
Logical qubits
Quantum computer
γ_1
V_g
E_C
E_J
γ_2
magic state distillery
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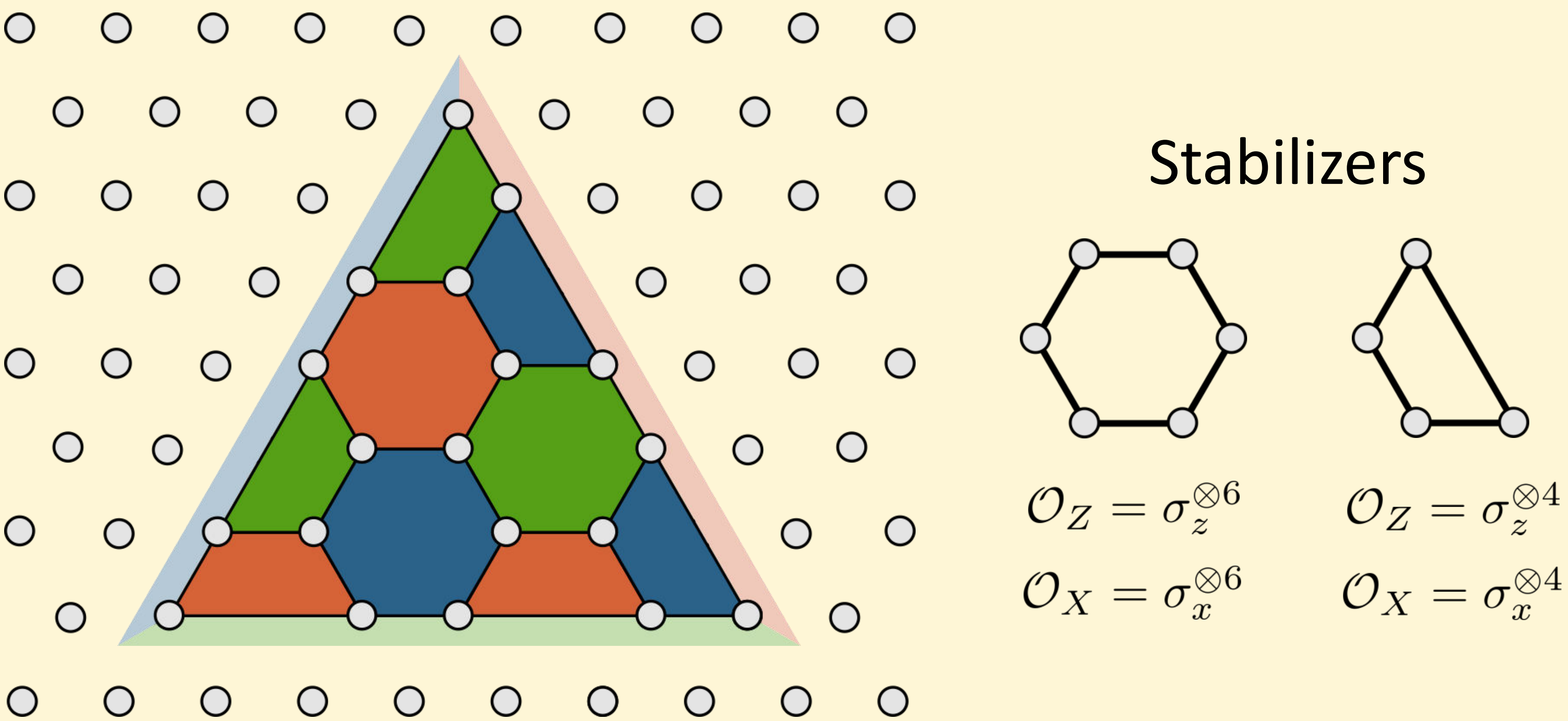
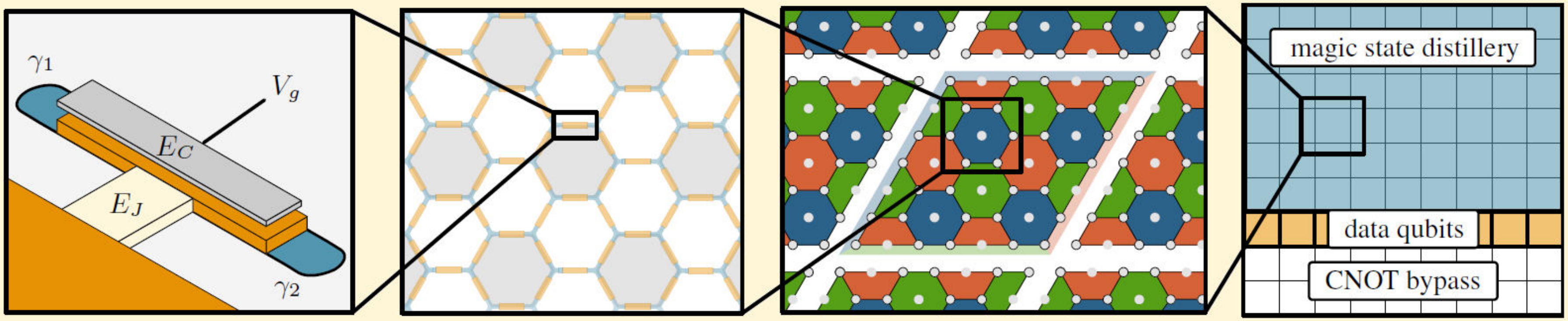
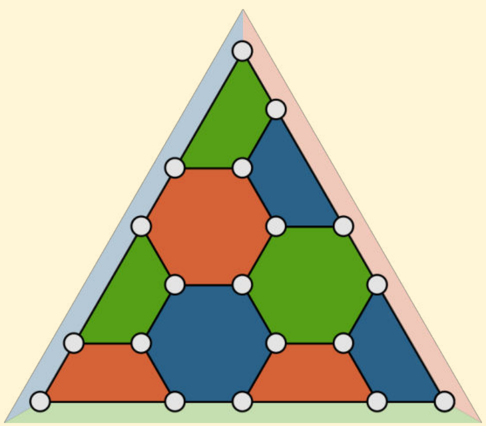
CNOT bypass
Topological color code


Stabilizers
$\mathcal{O}_Z = \sigma_z^{\otimes 6}$
$\mathcal{O}_X = \sigma_x^{\otimes 6}$
$\mathcal{O}_Z = \sigma_z^{\otimes 4}$
$\mathcal{O}_X = \sigma_x^{\otimes 4}$

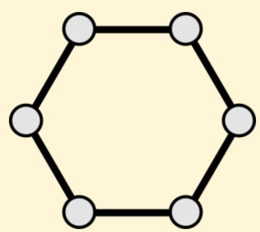
Building block | Physical qubits | Logical qubits | Quantum computer



Topological color code



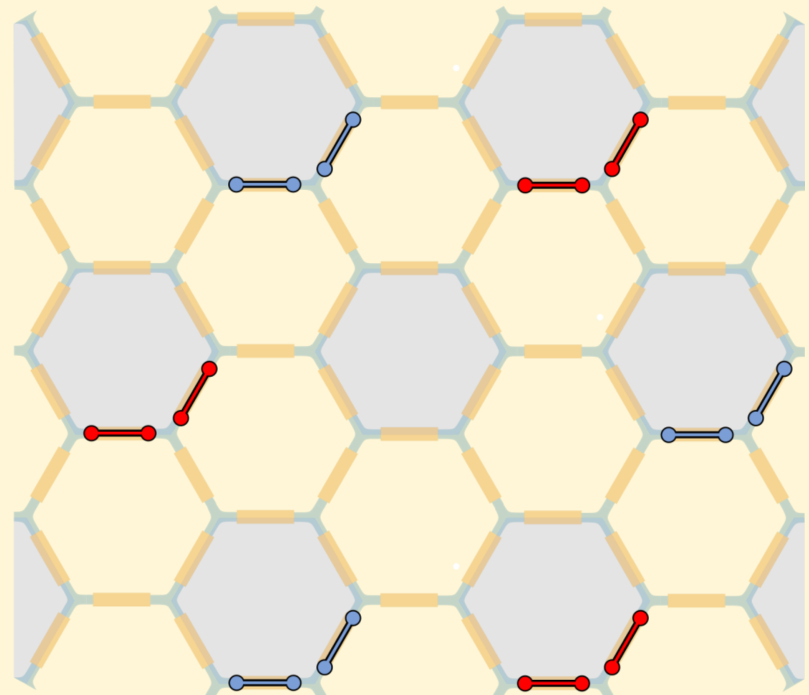
Stabilizers



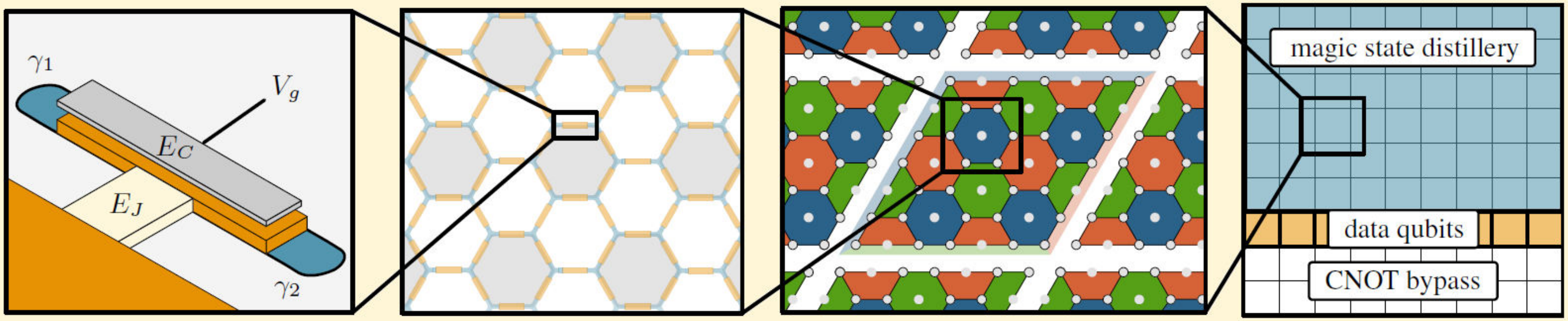
$$\mathcal{O}_Z = \sigma_z^{\otimes 6}$$

$$\mathcal{O}_X = \sigma_x^{\otimes 6}$$

Ancilla-free readout

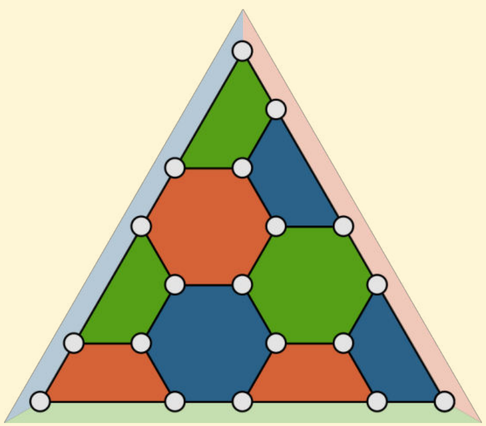


Building block | Physical qubits | Logical qubits | Quantum computer

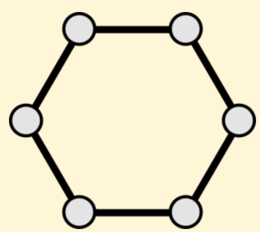


Topological color code

Ancilla-free readout



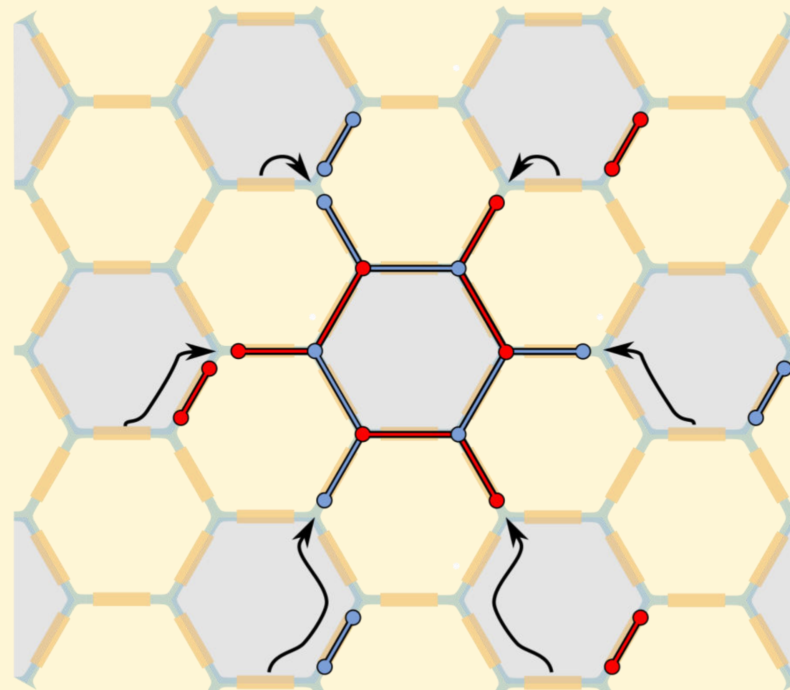
Stabilizers

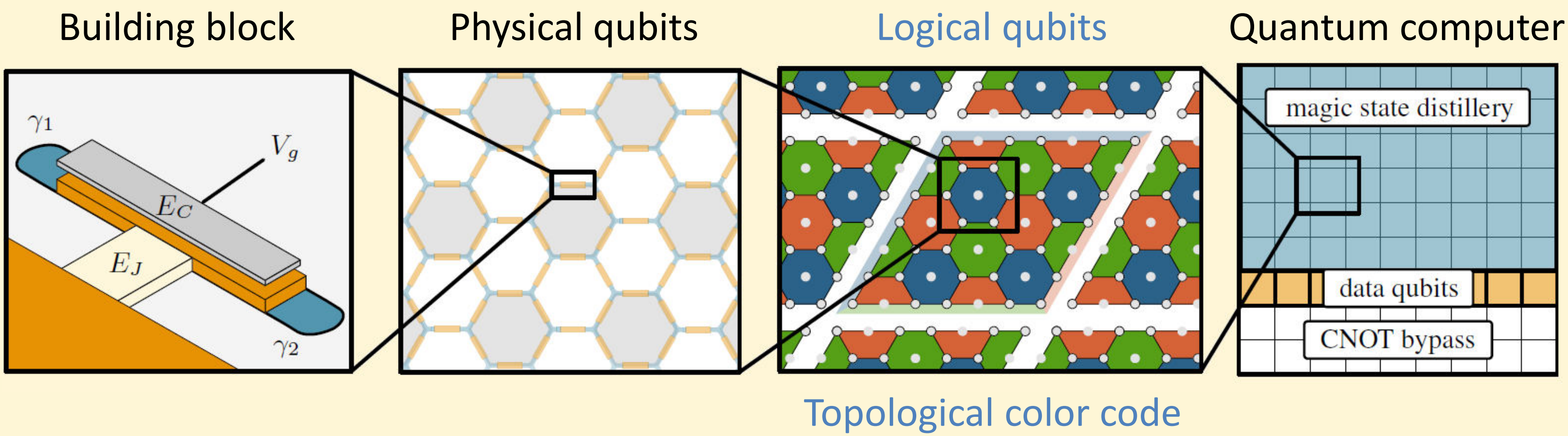


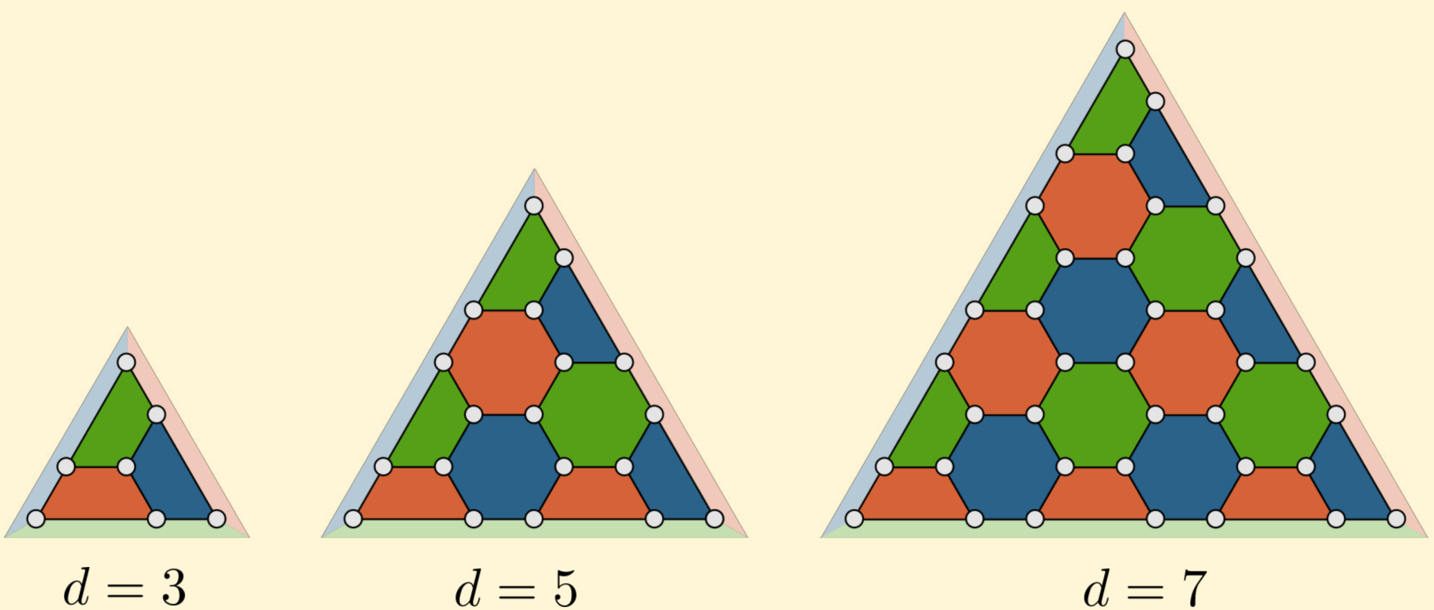
$$\mathcal{O}_Z = \sigma_z^{\otimes 6}$$

$$\mathcal{O}_X = \sigma_x^{\otimes 6}$$

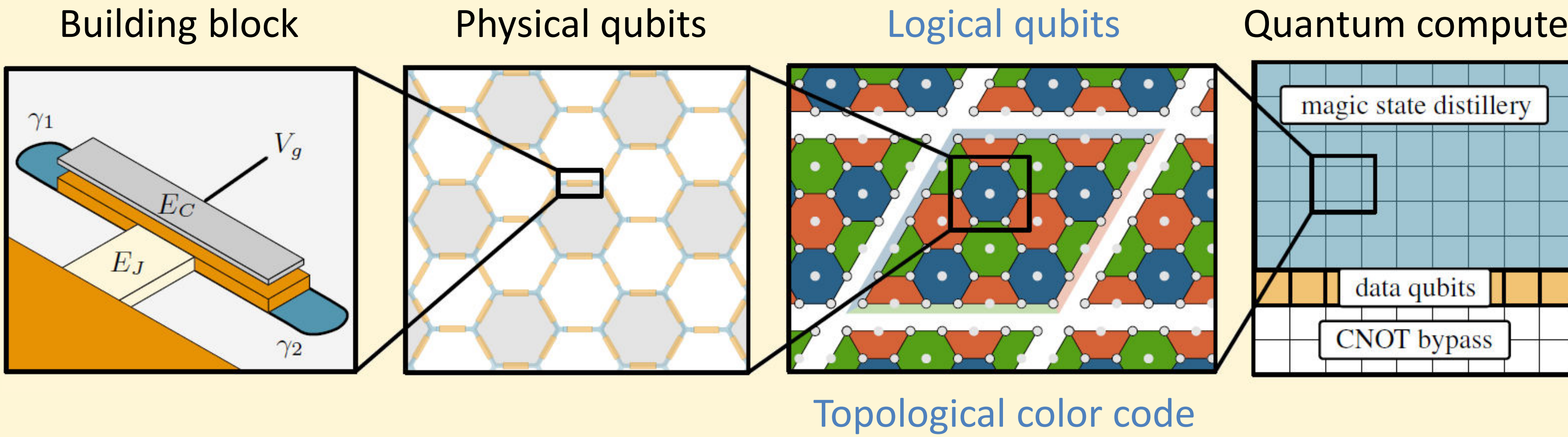
$$\sigma_z^{\otimes 6} \sim \gamma_1\gamma_2\gamma_3 \cdots \gamma_{12}$$



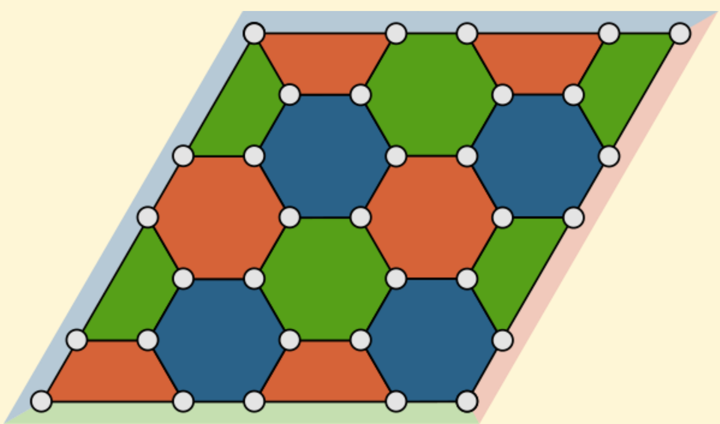




H. Bombin and M. A. Martin-Delgado, *Topological quantum distillation*, PRL **97**, 1 (2006)

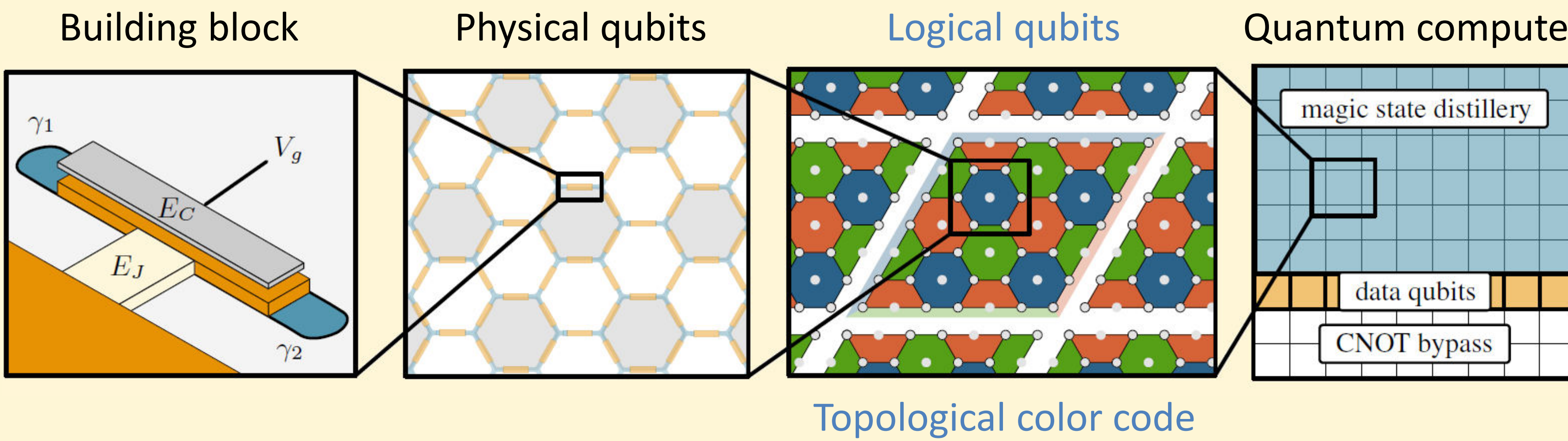


S_L H_L $CNOT_L$ T_L

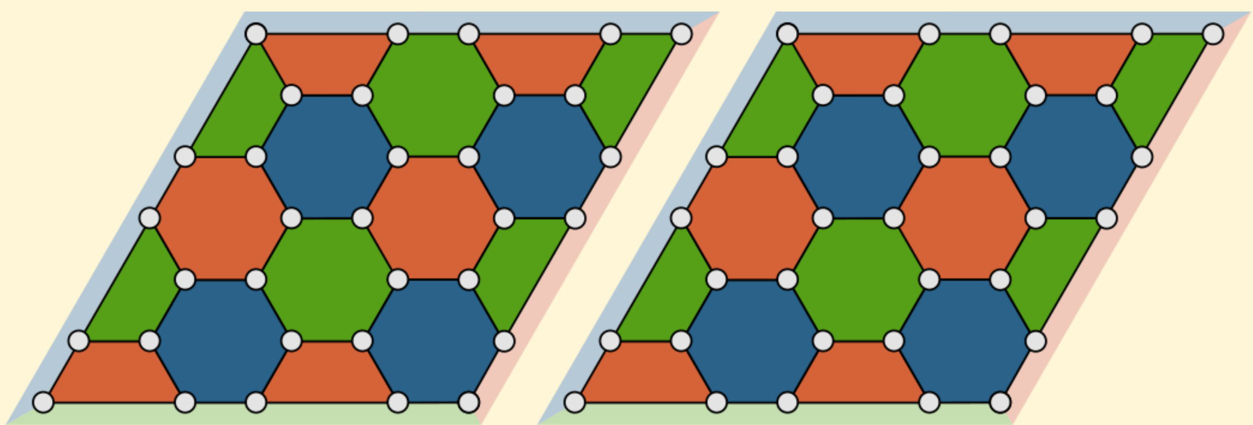


$$H_L = H^{\otimes n}$$

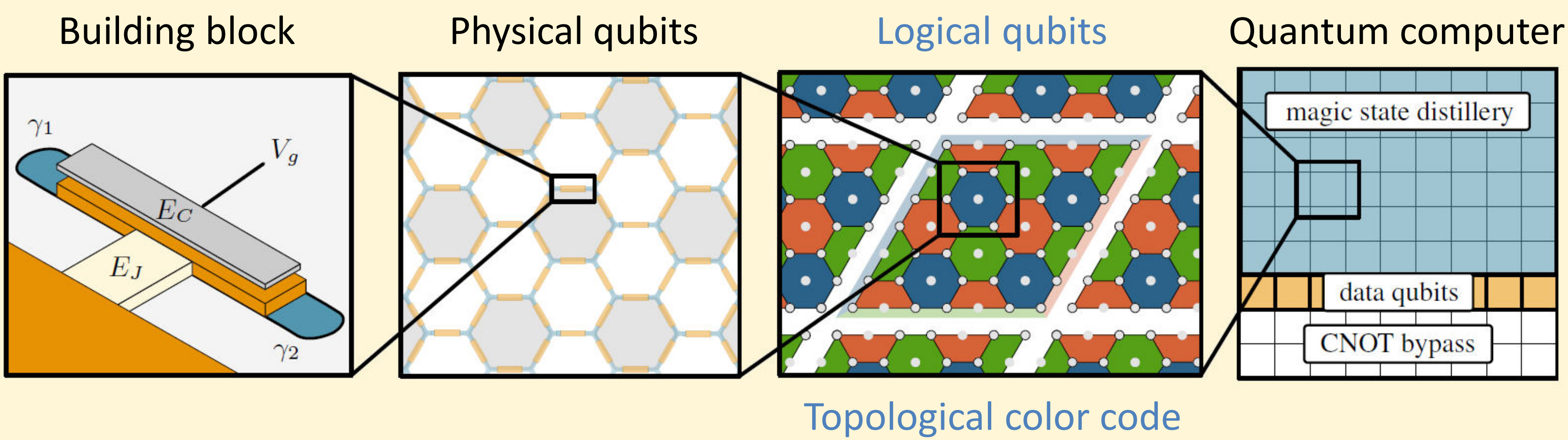
$$S_L = S^{(\dagger)\otimes n}$$



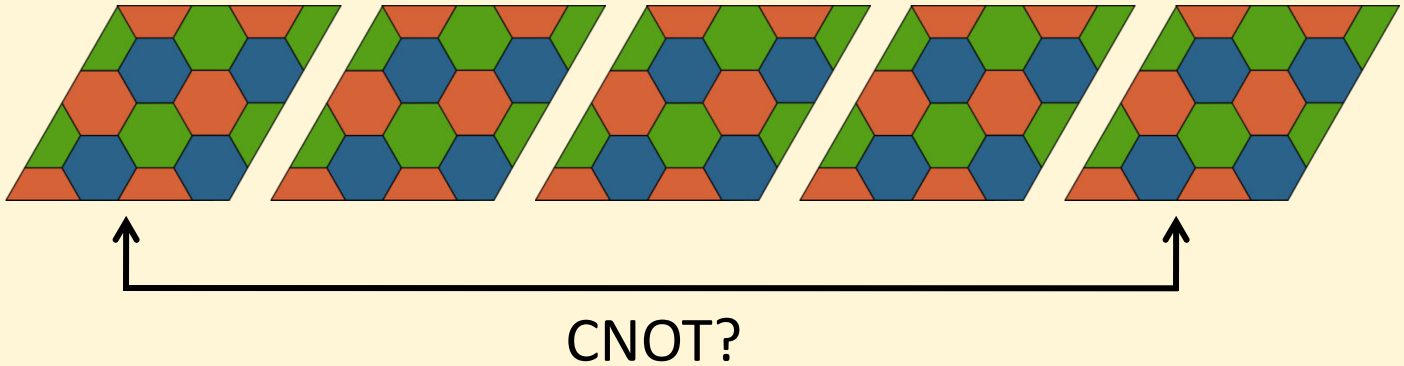
S_L H_L $CNOT_L$ T_L

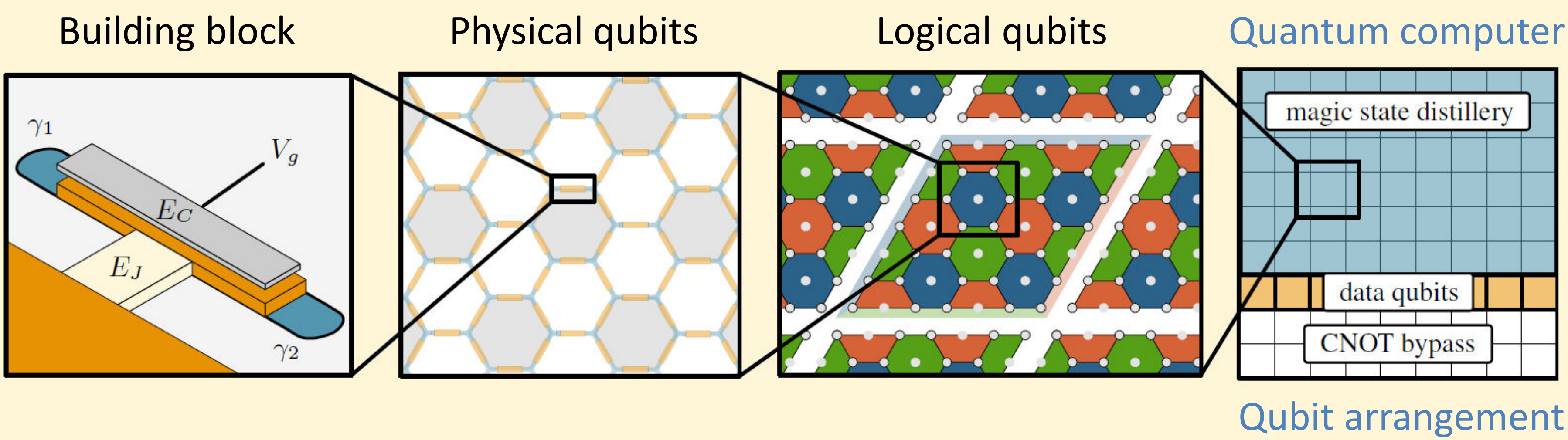


$$\mathrm{CNOT}_L = \mathrm{CNOT}^{\otimes n}$$

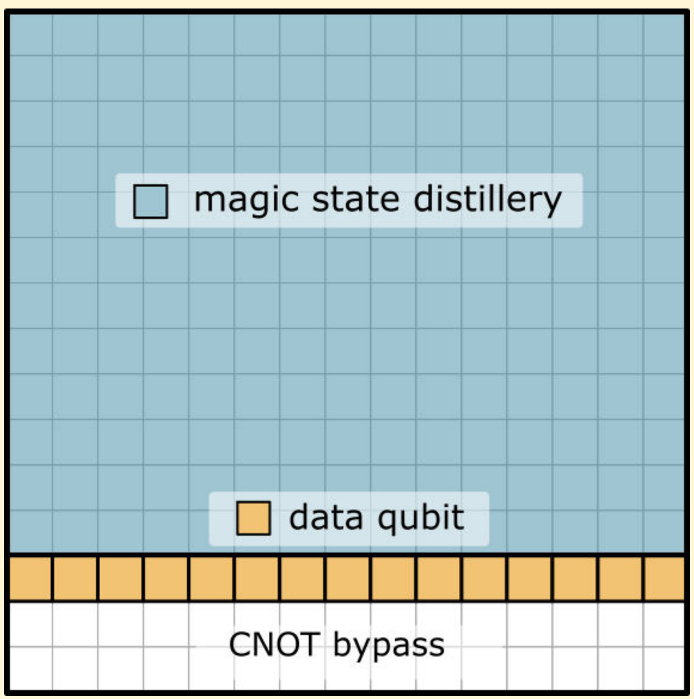


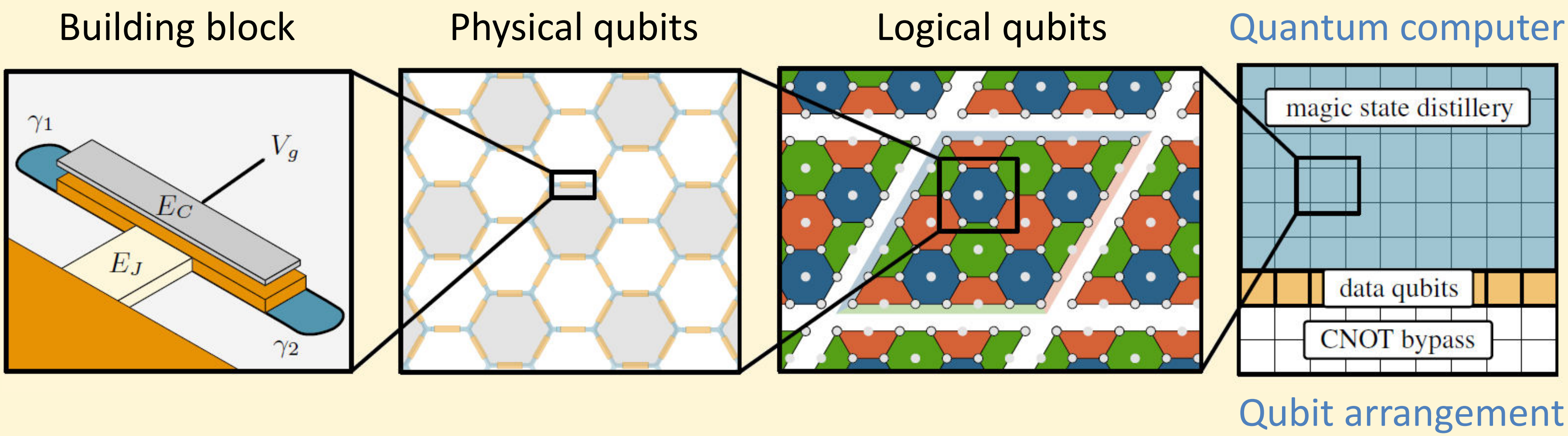
S_L H_L $CNOT_L$ T_L



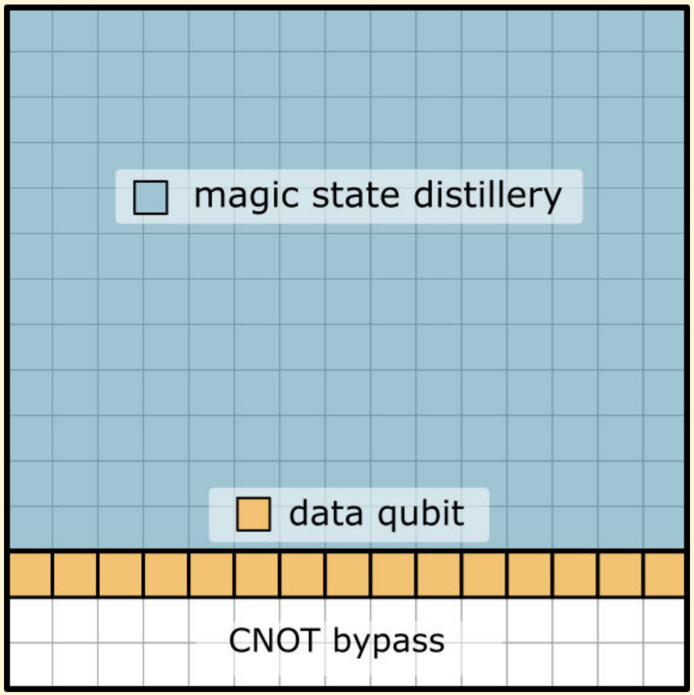


S_L H_L $CNOT_L$ T_L

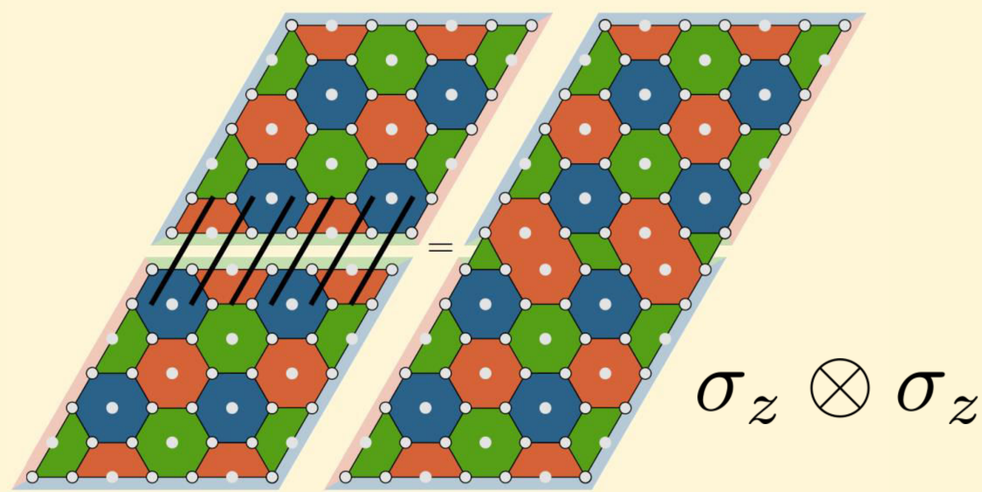




S_L H_L $CNOT_L$ T_L

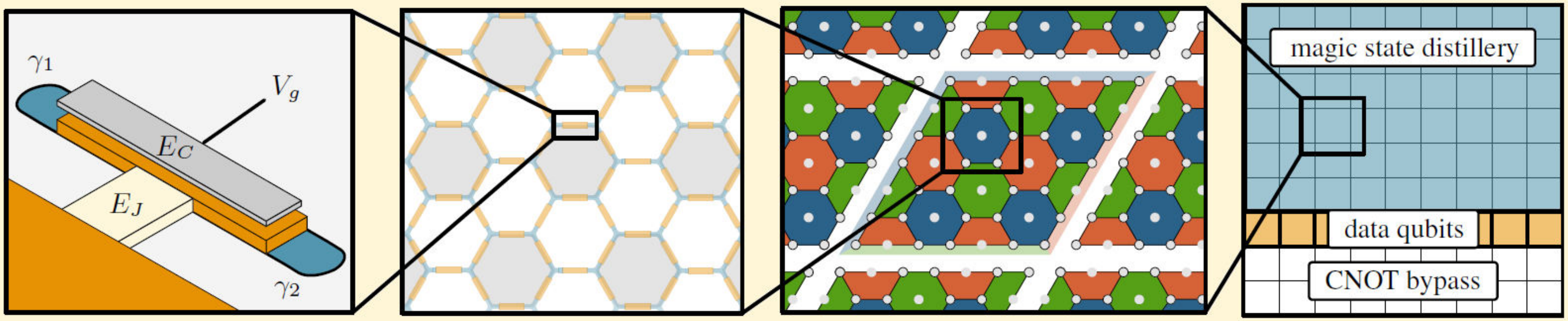


Lattice surgery: Protocol for fault-tolerant logical parity measurement



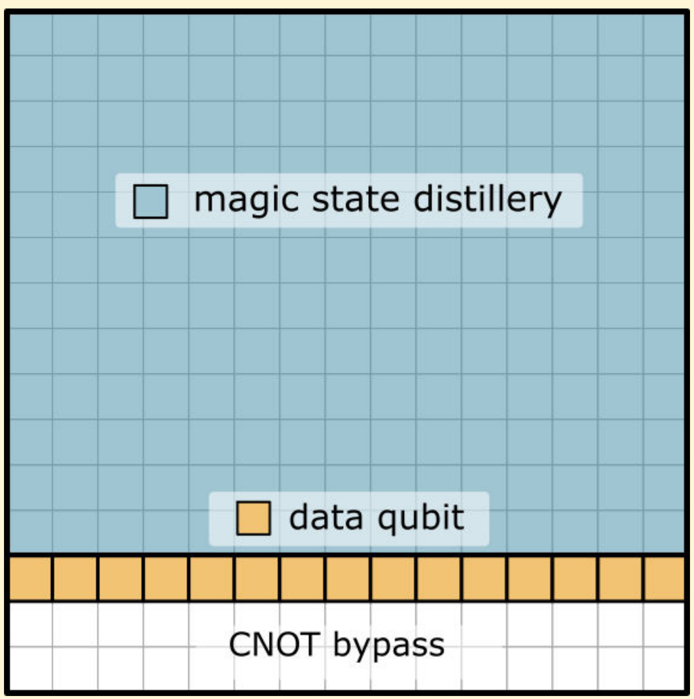
A. J. Landahl and C. Ryan-Anderson, *Quantum computing by color-code lattice surgery*, arXiv:1407.5103

Building block | Physical qubits | Logical qubits | Quantum computer

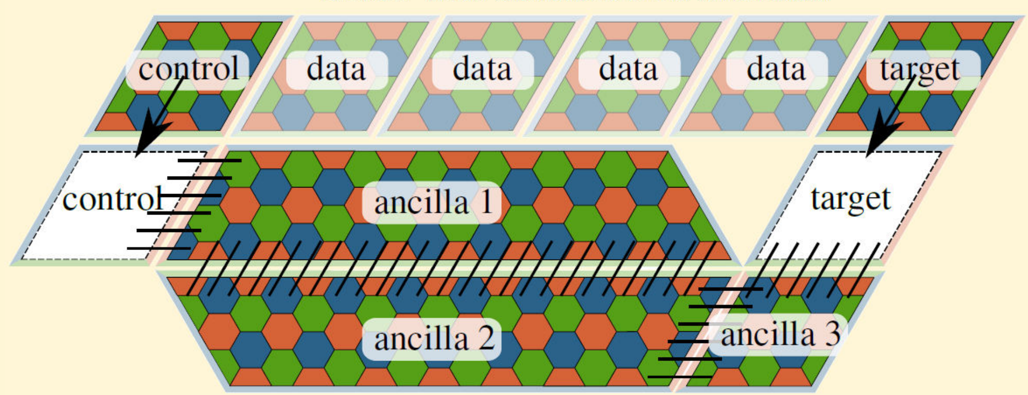


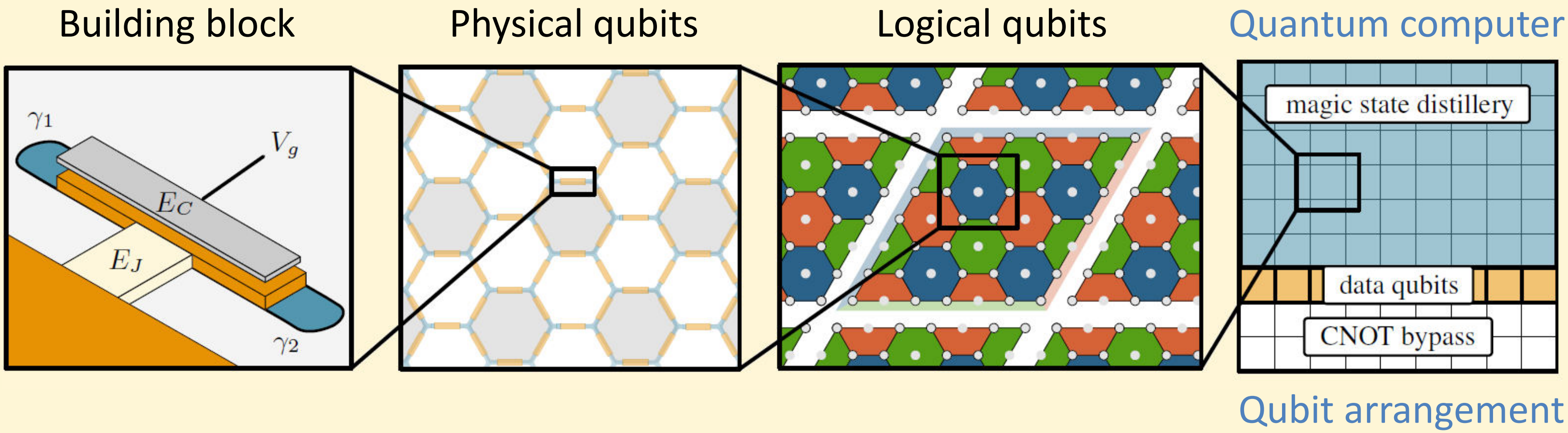
Qubit arrangement

S_L H_L $CNOT_L$ T_L

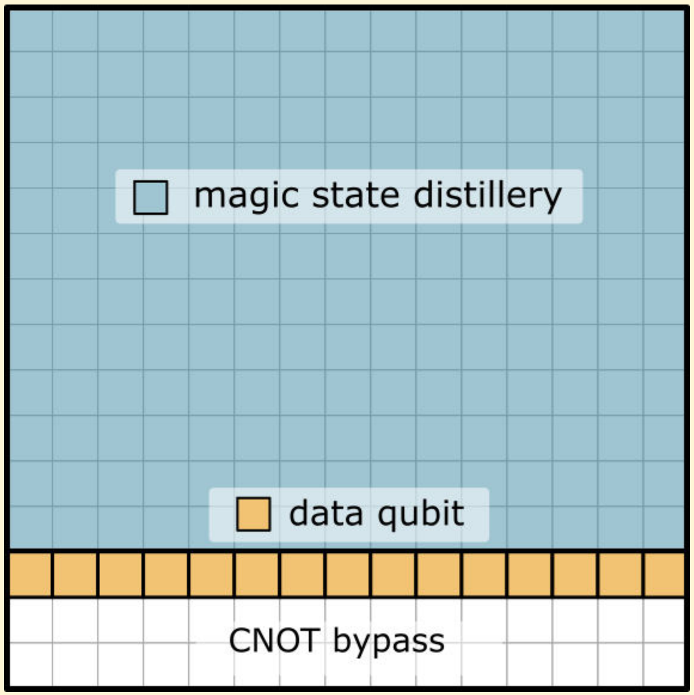


Lattice surgery: Protocol for fault-tolerant logical parity measurement



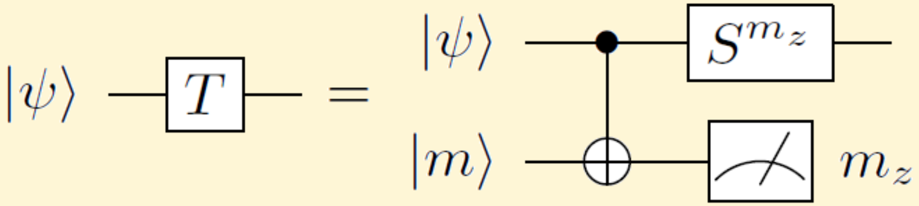


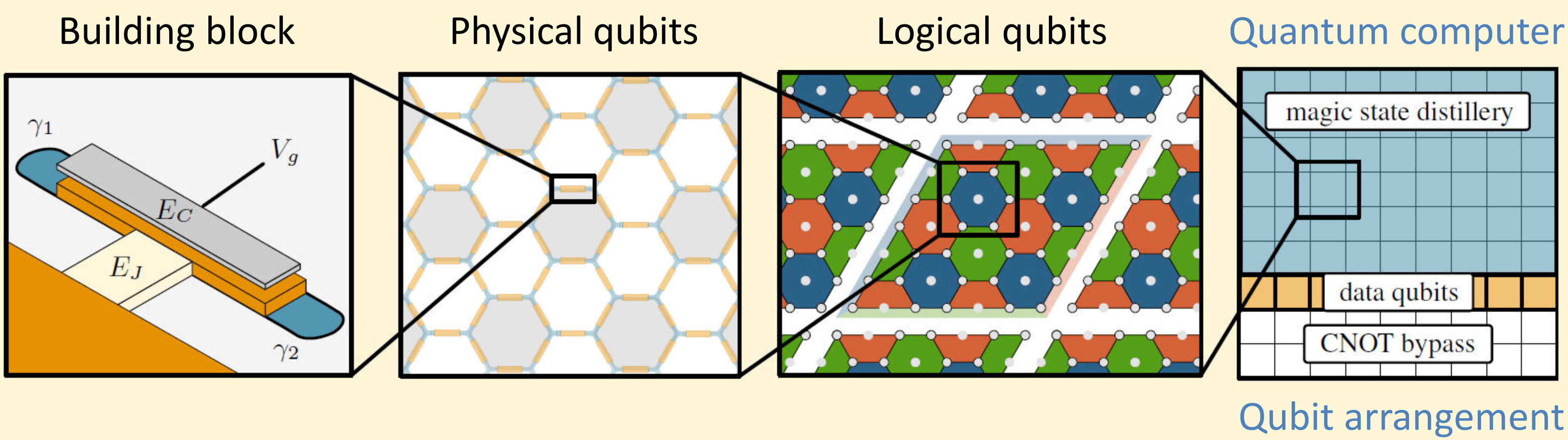
S_L H_L $CNOT_L$ T_L



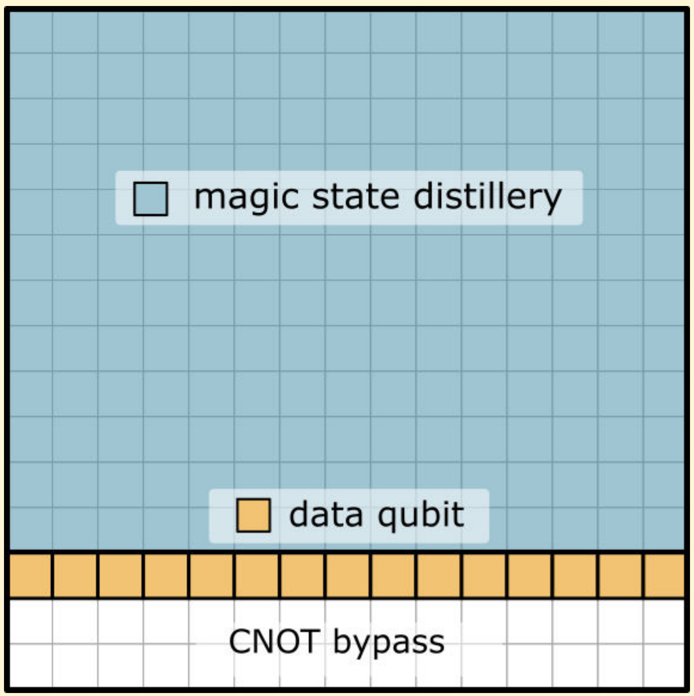
Magic state

$$|m\rangle = T\,|+\rangle = |0\rangle + e^{i\pi/4}\,|1\rangle$$



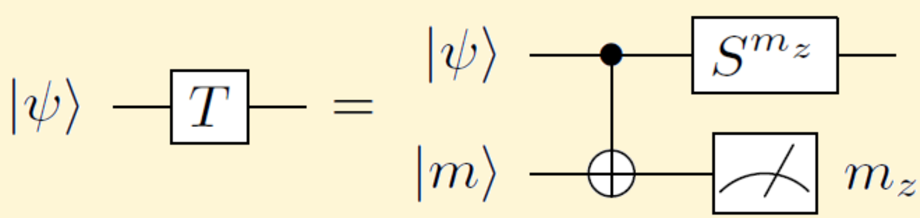


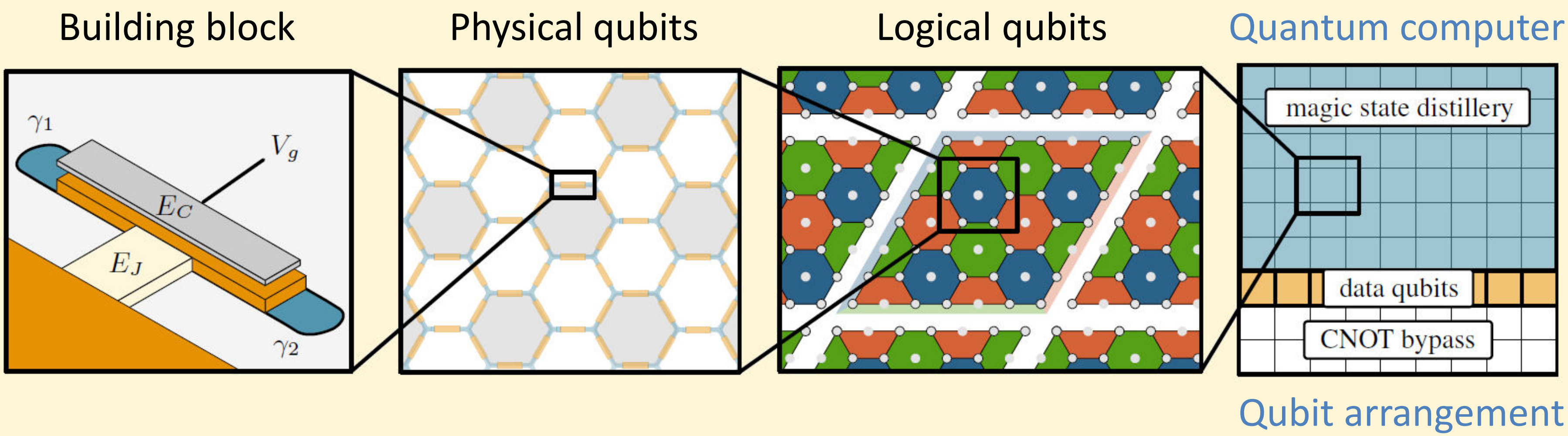
S_L H_L $CNOT_L$ T_L



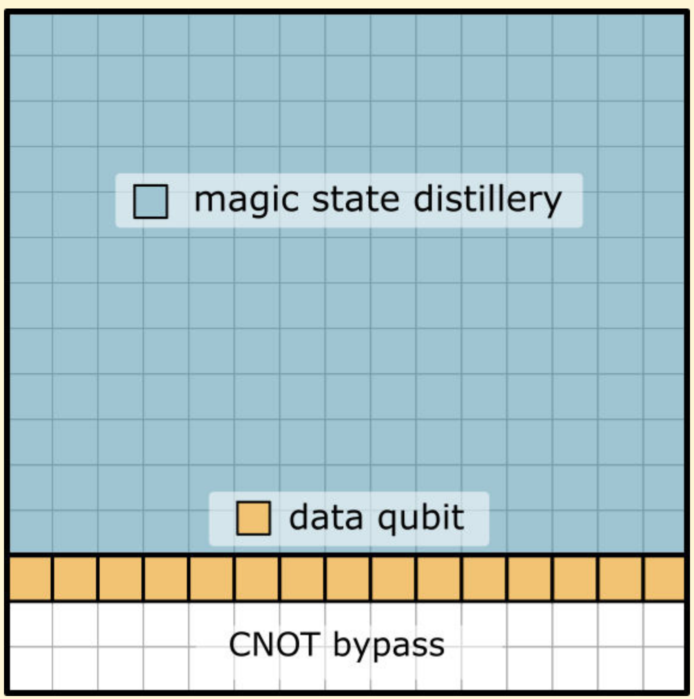
Bad magic state

$$|\widetilde{m}\rangle = T\,|+\rangle = |0\rangle + e^{i(\pi/4+\varepsilon)}\,|1\rangle$$





S_L H_L $CNOT_L$ T_L



Bad magic state

$$|\tilde{m}\rangle = T\,|+\rangle = |0\rangle + e^{i(\pi/4+\varepsilon)}\,|1\rangle$$

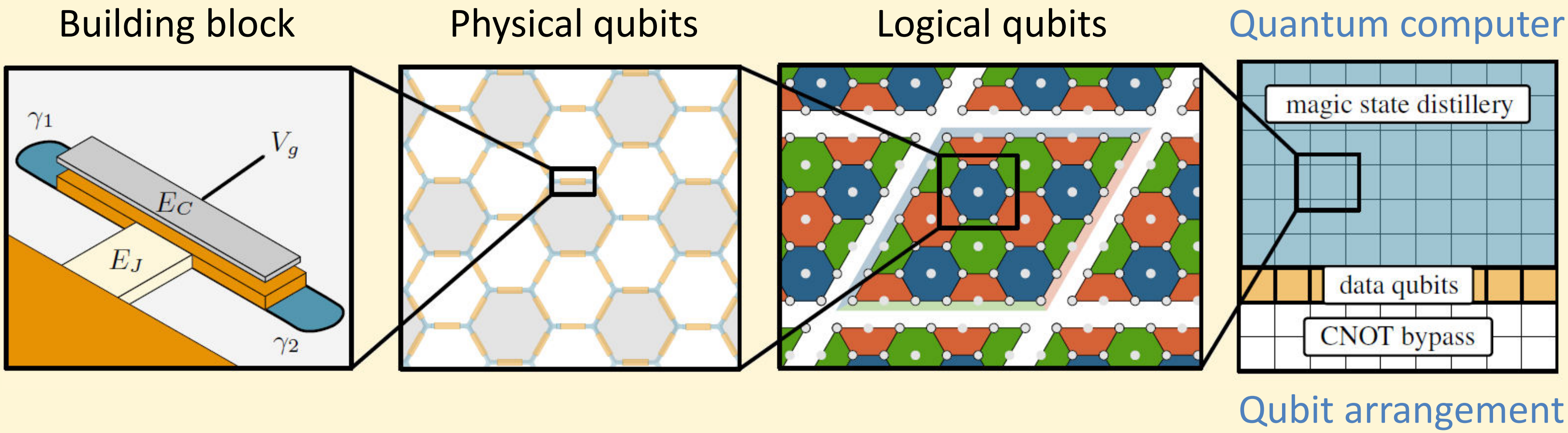
Magic state distillation

$|\tilde{m}\rangle$ $|\tilde{m}\rangle$ $|\tilde{m}\rangle$ $|\tilde{m}\rangle$ $|\tilde{m}\rangle$ $|\tilde{m}\rangle$ $|\tilde{m}\rangle$ $|\tilde{m}\rangle$ $|\tilde{m}\rangle$ $|\tilde{m}\rangle$ $|\tilde{m}\rangle$

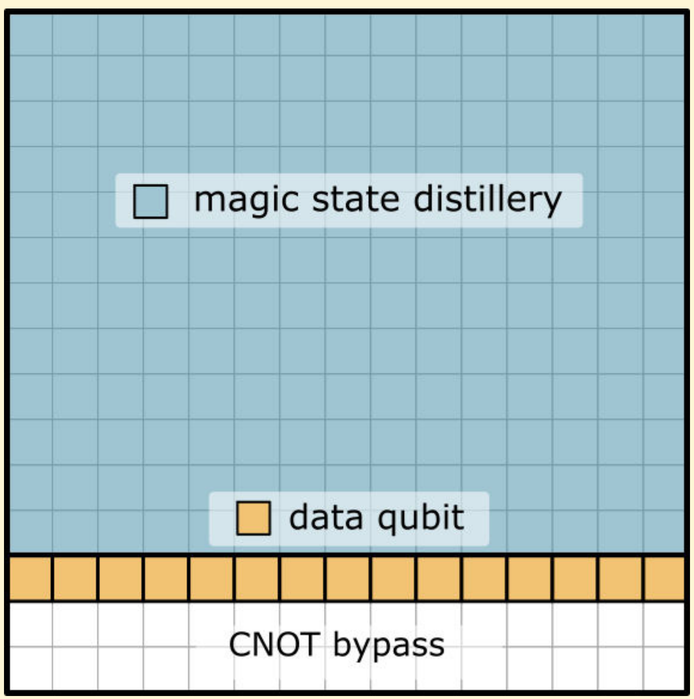
Better

$$|m\rangle$$

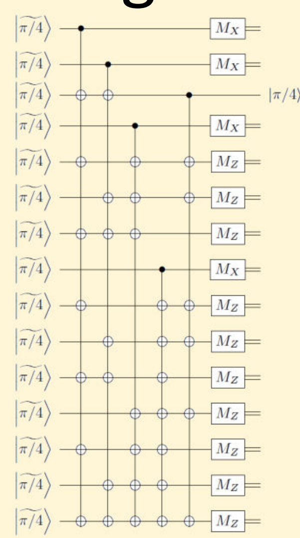
magic state



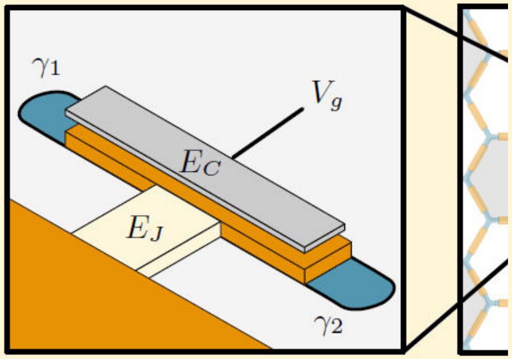
S_L H_L $CNOT_L$ T_L

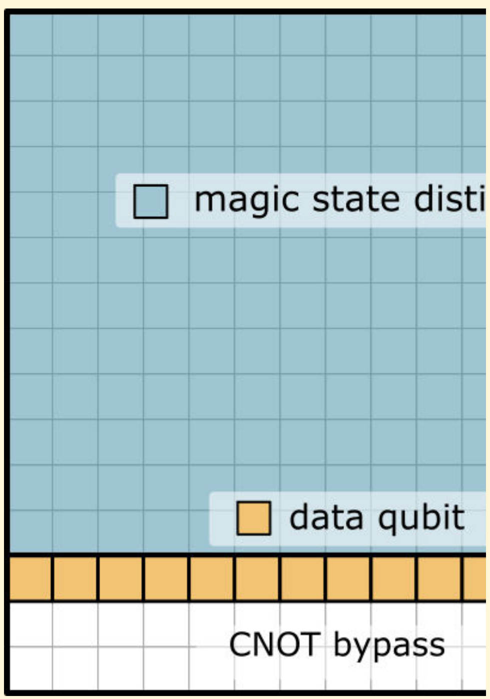


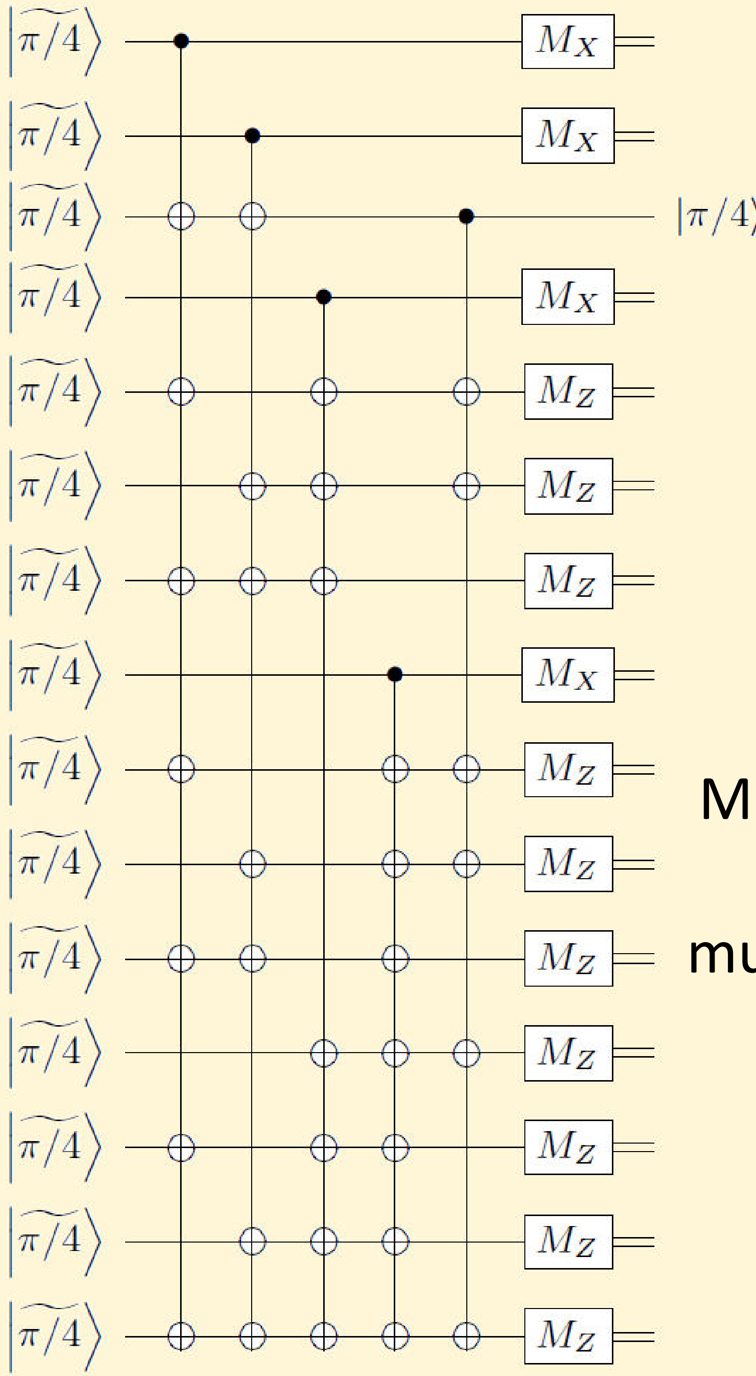
Magic state distillation requires multi-target CNOT gates



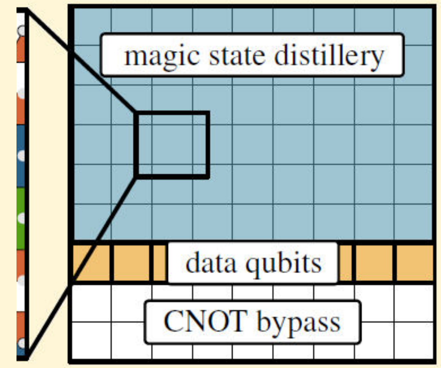
Building block







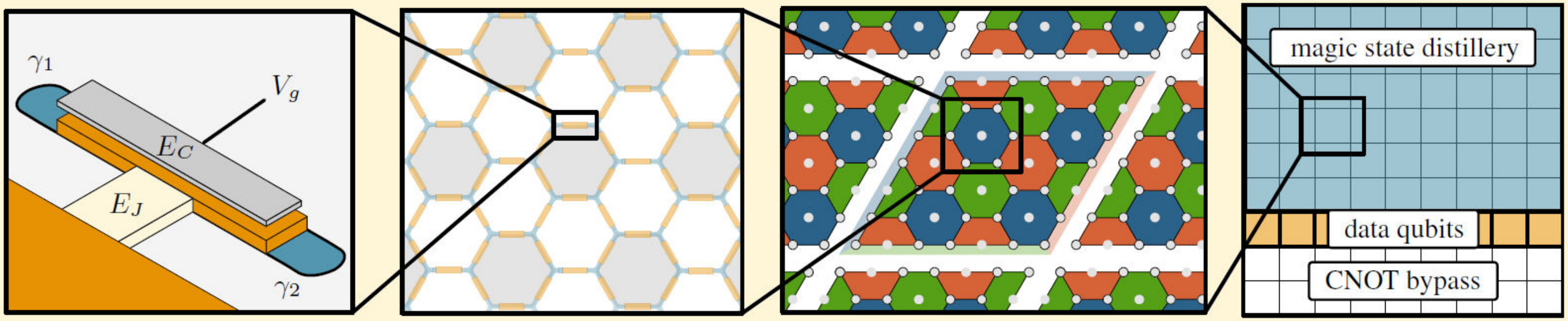
Quantum computer



Qubit arrangement

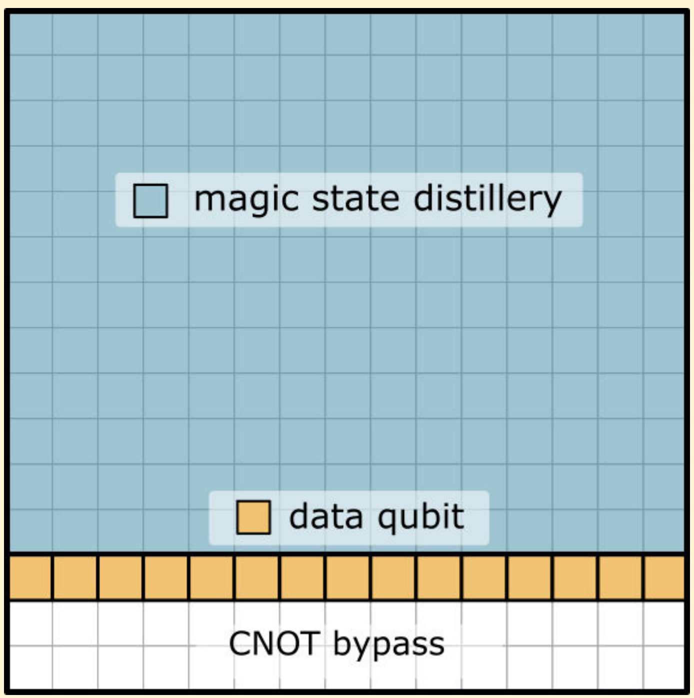
Magic state distillation requires multi-target CNOT gates

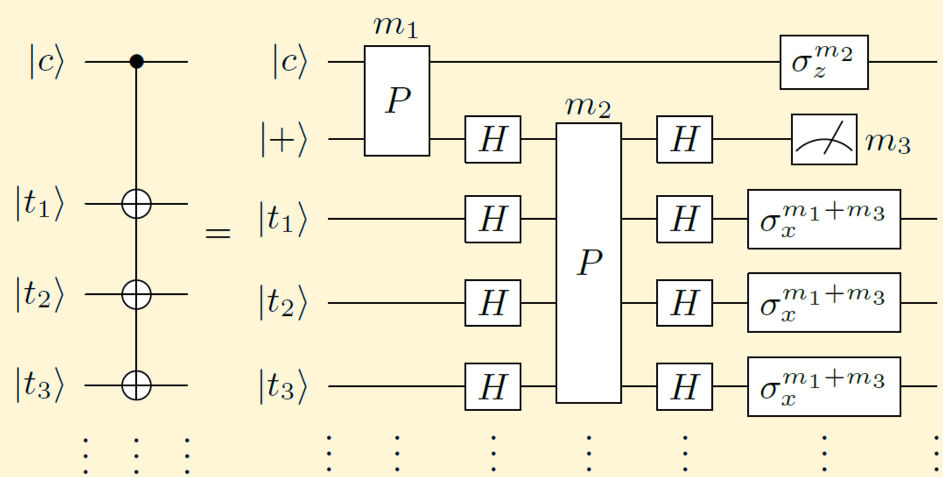
Building block Physical qubits Logical qubits Quantum computer



Qubit arrangement

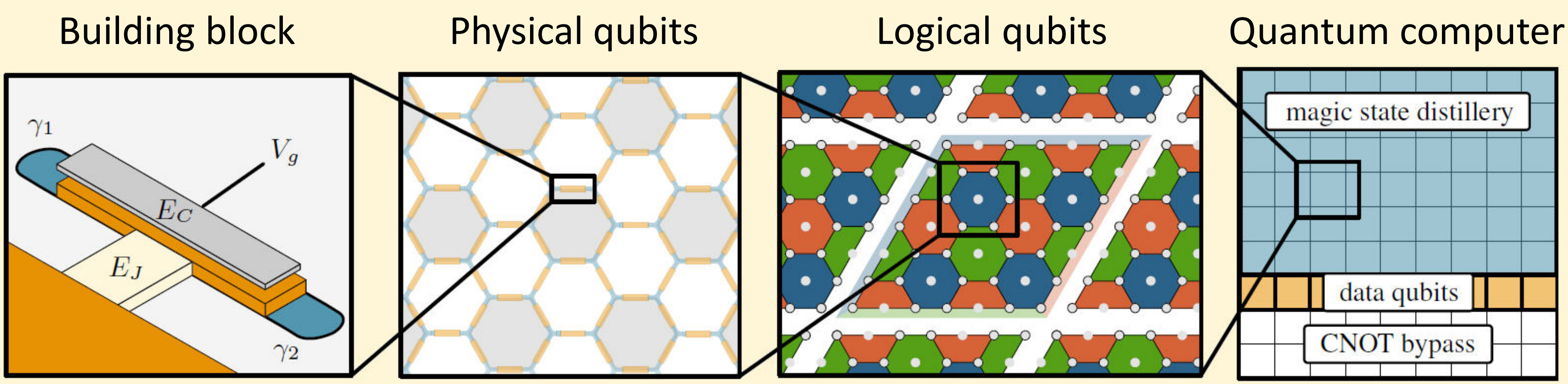
S_L H_L $CNOT_L$ T_L





Fast multi-target CNOTs

→ Fast distillation



arXiv:1704.01589

Summary

- Scalable
- Voltage-controlled
- Fault-tolerant
- Topologically protected Clifford gates
- Ancilla-free syndrome readout
- Constant-time CNOTs between distant qubits
- Fast magic state distillation