# Majorana Quasi-Particles Protected by $\mathbb{Z}_2$ Angular Momentum Conservation

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# Outline/Motivation – search for Majoranas

- Past decades have witnessed lots of effort in order to generate/observe **edge Majorana fermions** (potential application in Quantum Computation)

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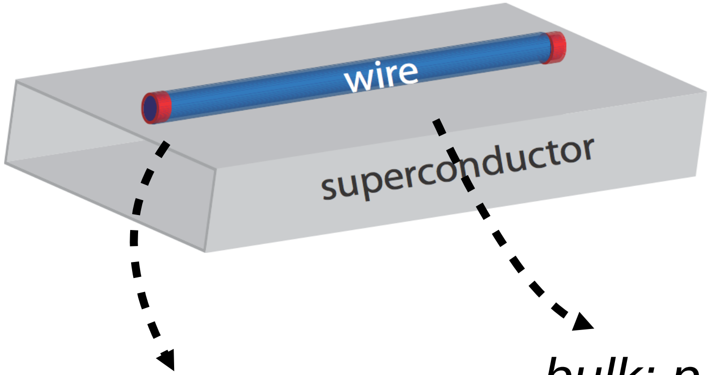
- Paradigmatic model: **Kitaev model**

***(mean-field level)***



*bulk: p-wave superconductor*

*localized edge Majoranas*

*- Many proposals for its implementation + signatures of zero energy edge Majoranas...*

*A. Y. Kitaev, Physics Uspekhi 44, 131 (2001).*

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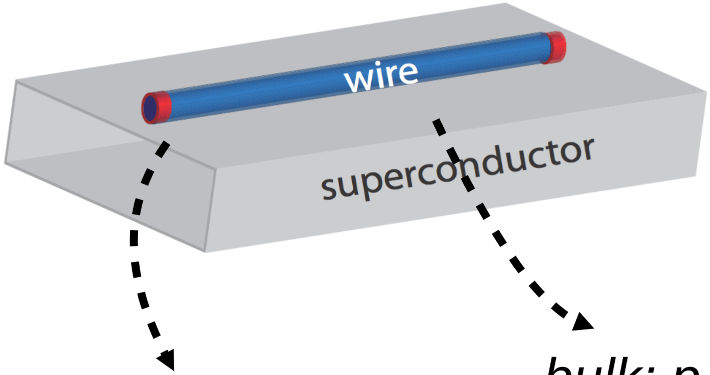

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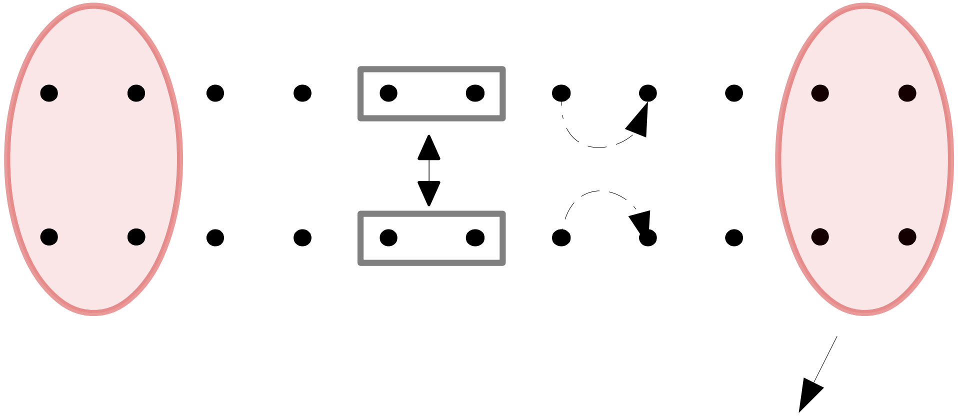
- *Distinct proposals beyond mean-field**: canonical settings***

$$\left[\hat{H}, \hat{N}\right] = 0$$

*("ladder chains")*



*localized edge Majorana*

*M. Cheng et al., Phys. Rev. B 84, 094503 (2011)*
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*N. Lang et al., Phys. Rev. B 92, 041118 (2015R)*

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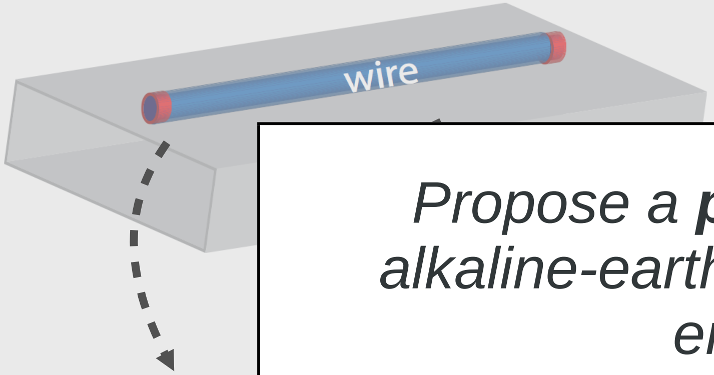

*localize*
*Majo*

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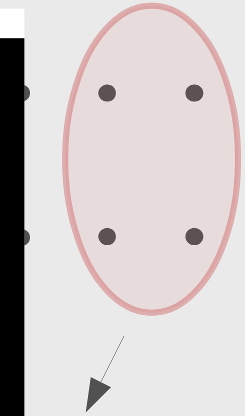

*ed edge*
*Majorana*

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*Propose a **physical canonical setting**- with alkaline-earth-like fermion atoms - hosting zero energy edge Majoranas.*

*(*ingredients **already implemented** recently in cold atoms: spin-orbit + spin-exchange)*

# Outline

- *Introduction on Kitaev model + edge Majoranas in canonical settings*

- *Our proposal for edge MF's in cold atoms settings*

- *Conclusions*

# Majorana formalism

**- “half” fermions;**
**- particles are their own anti particles;**

**(complex) fermion**

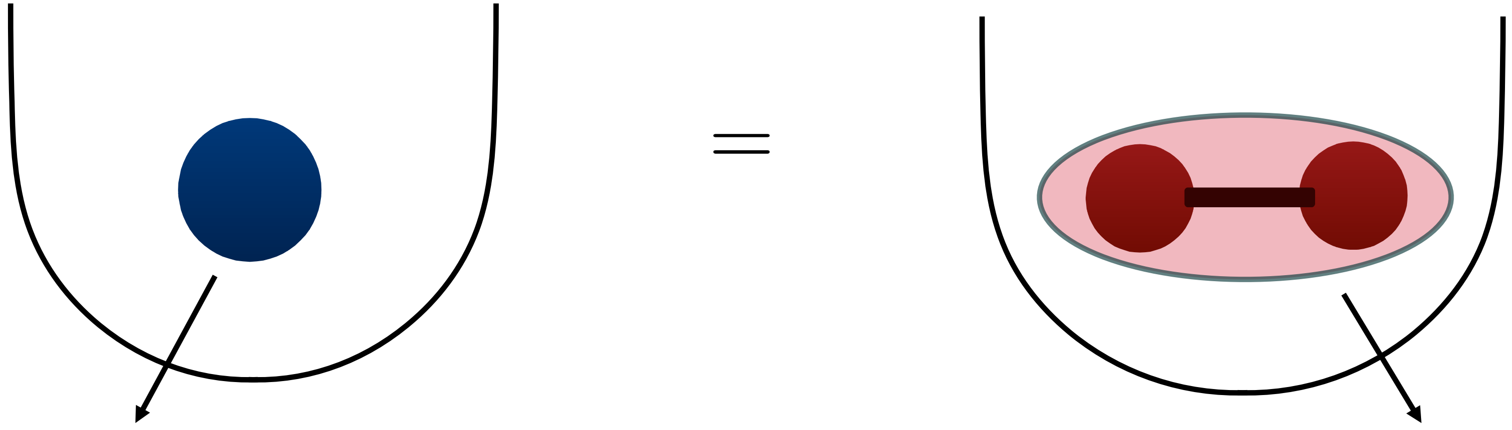
$$\{\hat{a}_j\,,\hat{a}_\ell^\dagger\} = \delta_{j,\ell}$$

$$\hat{a}_j = \hat{\gamma}_{2j-1} + i\hat{\gamma}_{2j}$$

**(real) Majorana fermion**

$$\gamma_j^\dagger = \gamma_j$$
$$\{\gamma_i, \gamma_j^\dagger\} = 2\delta_{i,j}$$

$$\hat{H} = \left(\hat{a}_1^\dagger \hat{a}_1 - \frac{1}{2}\right) = \frac{i}{2}\hat{\gamma}_1\hat{\gamma}_2$$



*fermion in a lattice site*

*paired Majoranas*

# Kitaev model

$$\hat{H}_{Kitaev} = \sum_j \left[ \underset{\substack{\downarrow \\ \textit{hopping}}}{-J}\hat{a}_j^\dagger \hat{a}_{j+1} + \underset{\substack{\downarrow \\ \textit{pairing} \\ \textit{correlation}}}{\Delta}\hat{a}_j^\dagger \hat{a}_{j+1}^\dagger + H.c. - \underset{\substack{\downarrow \\ \textit{chemical potential}}}{\mu} \left( \hat{a}_j^\dagger \hat{a}_j - \frac{1}{2} \right) \right]$$

$$\text{Parity symmetry:} \quad \left[ \hat{H}, (-1)^{\hat{N}} \right] = 0$$

$$\text{Topological with zero energy edge Majoranas:} \; |\frac{\Delta}{\mu}| \geq 2 \quad (J = 1)$$

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# Kitaev model

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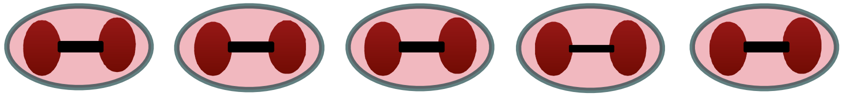
hopping

pairing correlation

chemical potential

$$\text{“Trivial phase”:} \quad J = |\Delta| = 0$$

$$\hat{H} = -\frac{i}{2}\mu \sum_{j=1}^{L} \hat{\gamma}_{2j-1}\hat{\gamma}_{2j}$$



chain completely full or empty

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# Kitaev model

$$\hat{H}_{Kitaev} = \sum_j \left[ -J\hat{a}_j^\dagger \hat{a}_{j+1} + \Delta \hat{a}_j^\dagger \hat{a}_{j+1}^\dagger + H.c. - \mu \left( \hat{a}_j^\dagger \hat{a}_j - \frac{1}{2} \right) \right]$$

*hopping*

*pairing correlation*

*chemical potential*

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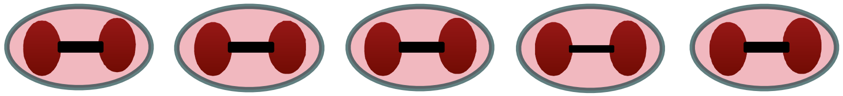
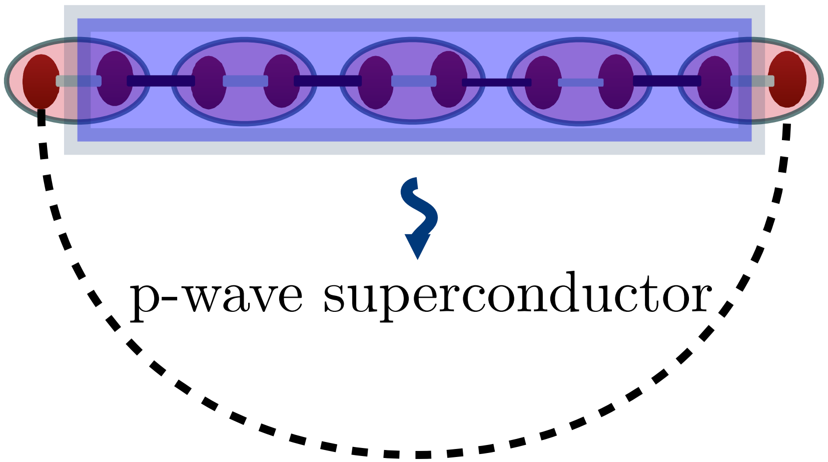


*chain completely full or empty*

$$\text{“}\mathit{Top.\ phase}\text{”}: \quad \mu = 0,\ |\Delta| = J$$

$$\hat{H} = iJ \sum_{j=1}^{L-1} \hat{\gamma}_{2j}\hat{\gamma}_{2j+1} + 0\hat{\gamma}_{2L}\hat{\gamma}_1$$



p-wave superconductor

unpaired $\{\hat{\gamma}_1, \hat{\gamma}_{2L}\}$

zero energy Majorana fermions

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# Search for zero energy edge Majoranas

$$\hat{H}_{Kitaev} = \sum_j \left[ -J a_j^\dagger a_{j+1} + \boxed{\Delta a_j^\dagger a_{j+1}^\dagger} + H.c. - \mu \left( a_j^\dagger a_j - \frac{1}{2} \right) \right]$$

*pairing correlations*

***<u>mean-field level</u>***

*- proximity-induced superconductivity (solid state):*
*zero bias voltage modes,*
*exponential suppression of energy splitting with increasing wire length*

*- coupling to a BEC reservoir (cold atoms)*

*presence and control of a reservoir...*

V. Mourik et al., Science 336, 1003 (2012);
M. T. Deng et al., Nano Lett. 12, 6414 (2012);
A. Das, Y. Ronen et al., Nat. Phys. 8, 887 (2012);
L. P. Rokhinson et al., Nat. Phys. 8,795 (2012);
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*pairing correlations*

- is it possible to stabilize MF's beyond mean field?

- *Edge Majoranas in* ***canonical settings****?*

$$\left[ \hat{H}, \hat{N} \right] = 0$$

V. Mourik et al., Science 336, 1003 (2012);
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*- proximity-induced superconductivity (solid state):*
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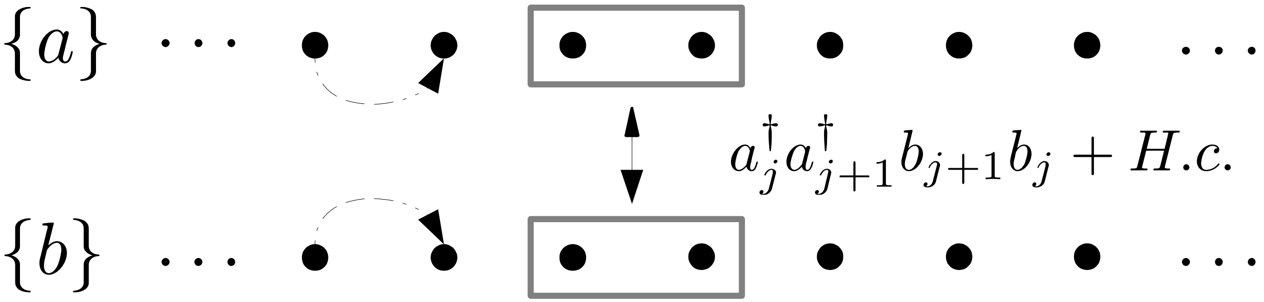
# *Edge Majoranas in Number Conserving Systems*

- *Edge Majoranas in* ***canonical settings***

$$\left[\hat{H}, \hat{N}\right] = 0$$

- simple proposals: ladder chains



$$\left[\hat{H}_N, (-1)^{\hat{N}_{a(b)}}\right] = 0$$

**"local parity symmetry"**

$$\hat{H}_N = \sum_j \left[-J(a_j^\dagger a_{j+1} + b_j^\dagger b_{j+1}) + \Delta \underline{a_j^\dagger a_{j+1}^\dagger b_{j+1} b_j} + H.c.\right] + \ldots$$

*"pairing correlation"*

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# Edge Majoranas in Number Conserving Systems

- *Edge Majoranas in* ***canonical settings***

$$\left[\hat{H},\hat{N}\right]=0$$

- simple proposals: ladder chains

*(ladder is a simple example, but difficult to physically realize...)*

***Search for a naturally occurring model where a parity symmetry / pairing correlations is realized in a number-conserving fashion...***

$$\hat{H}_N=\sum_j\left[-J(a_j^\dagger a_{j+1}+b_j^\dagger b_{j+1})+\underset{\text{“pairing correlation”}}{\underline{\Delta a_j^\dagger a_{j+1}^\dagger b_{j+1}b_j}}+H.c.\right]+\dots$$

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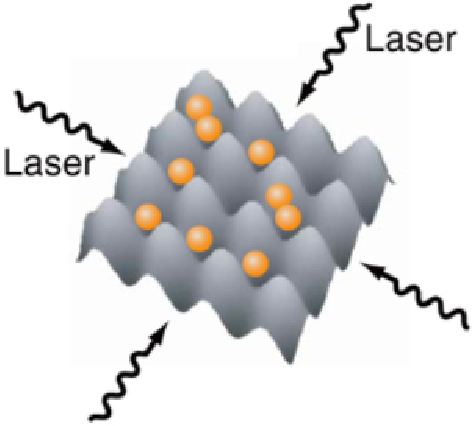
# *Cold atoms setting*

Use alkaline-earth-like fermion atoms (AEAs) in optical lattices with local **orbital + spin** degrees of freedom:

*(i)* combining **spin-orbit coupling + spin-exchange interactions** a local parity symmetry occurs naturally in the system;

*(ii)* symmetry is protected by **Angular Momentum Conservation**

*(iii)* spin-exchange acts effectively as **“pairing correlations”**



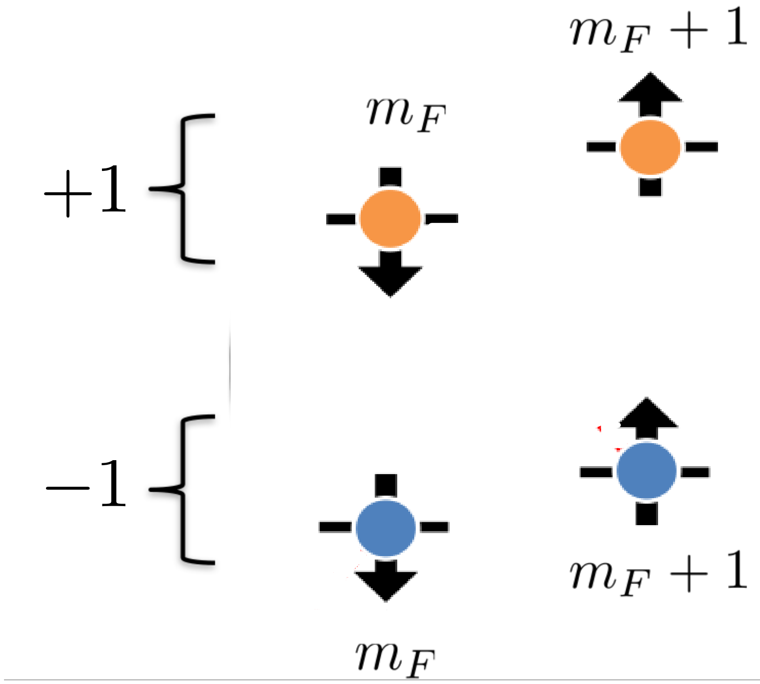
**spin-exchange interactions:**

G. Cappellini et al., Phys. Rev. Lett. 113, 120402 (2014)
F. Scazza et al., Nat. Phys. 10, 779 (2014)

**spin-orbit coupling:**

M. L. Wall et al., Phys. Rev. Lett. 116, 035301 (2016)
L. F. Livi et al., Phys. Rev. Lett. 117, 220401 (2016)
S. Kolkowitz et al., Nature 542, 66 (2017)

# Cold atoms setting

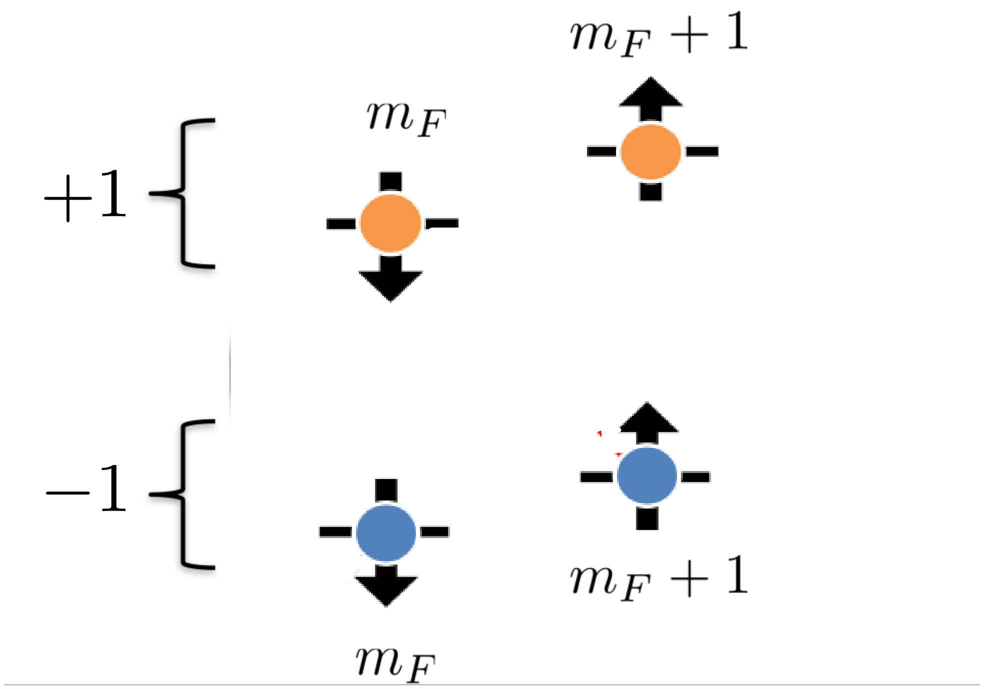


- Nuclear spin: $\{m_F, m_{F+1}\}$
- Orbital states: $\{-1, +1\}$

$$\hat{H} = \sum_j (\hat{H}_{t,j} + \hat{H}_{W,j} + \hat{H}_{\mathrm{so},j})$$

$$\left[ \begin{array}{c} \text{single-particle hopping} \\ \text{spin-exchange interactions} \\ \text{spin-orbit coupling} \end{array} \right]$$

# Cold atoms setting



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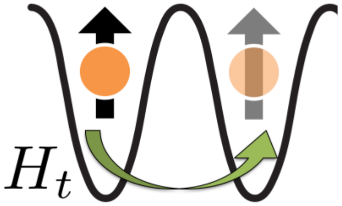

single particle hopping: $$\hat{H}_{t,j} = \sum_{\alpha,p} t(c^\dagger_{j,\alpha,p} c_{j+1,\alpha,p} + \mathrm{h.c.})$$

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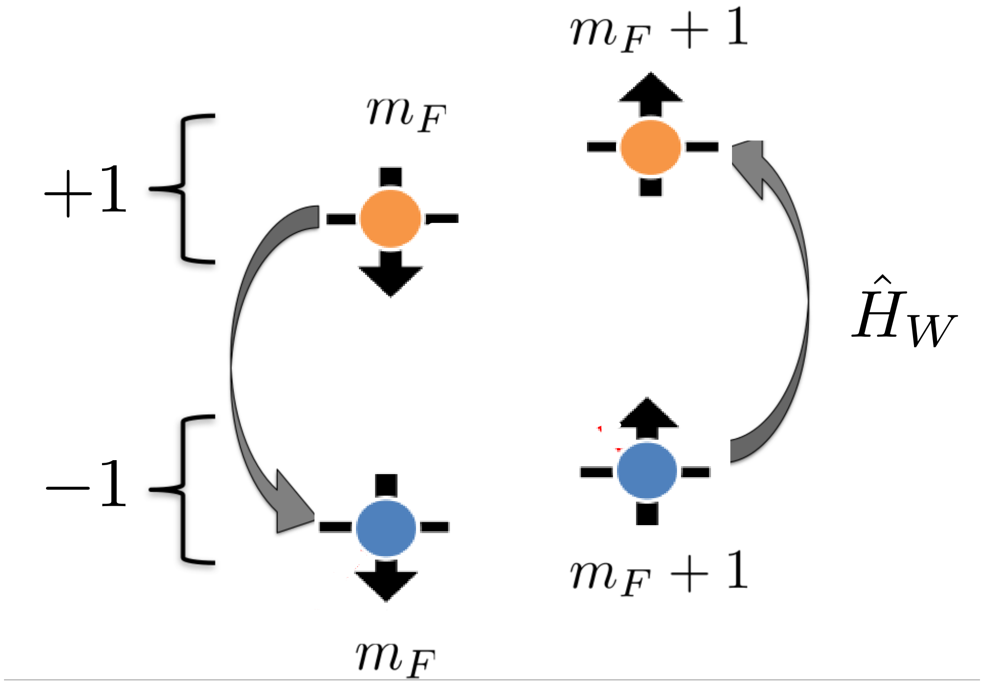

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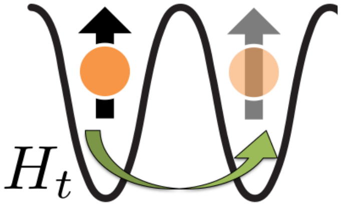
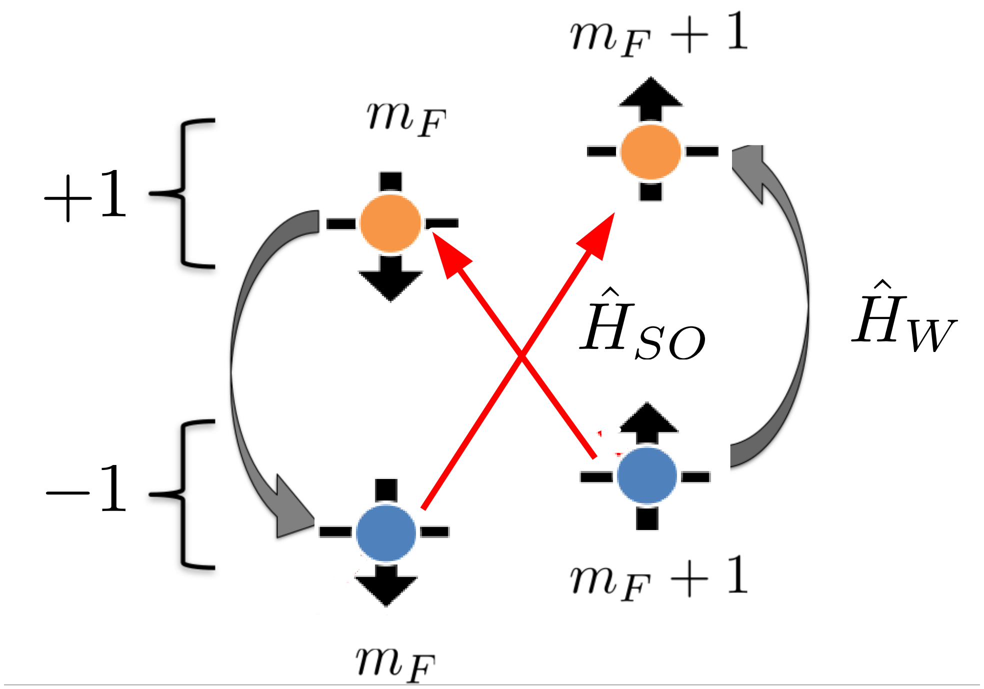


single particle hopping: $\hat{H}_{t,j} = \sum_{\alpha,p} t(c^\dagger_{j,\alpha,p} c_{j+1,\alpha,p} + \mathrm{h.c.})$

spin-exchange interaction: $\hat{H}_{W,j} = W(c^\dagger_{j,\uparrow,-1} c^\dagger_{j,\downarrow,1} c_{j,\downarrow,-1} c_{j,\uparrow,1} + \mathrm{h.c.})$

# Cold atoms setting



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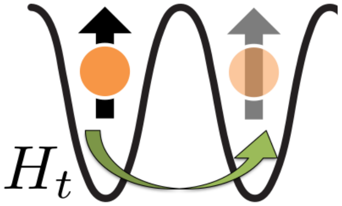

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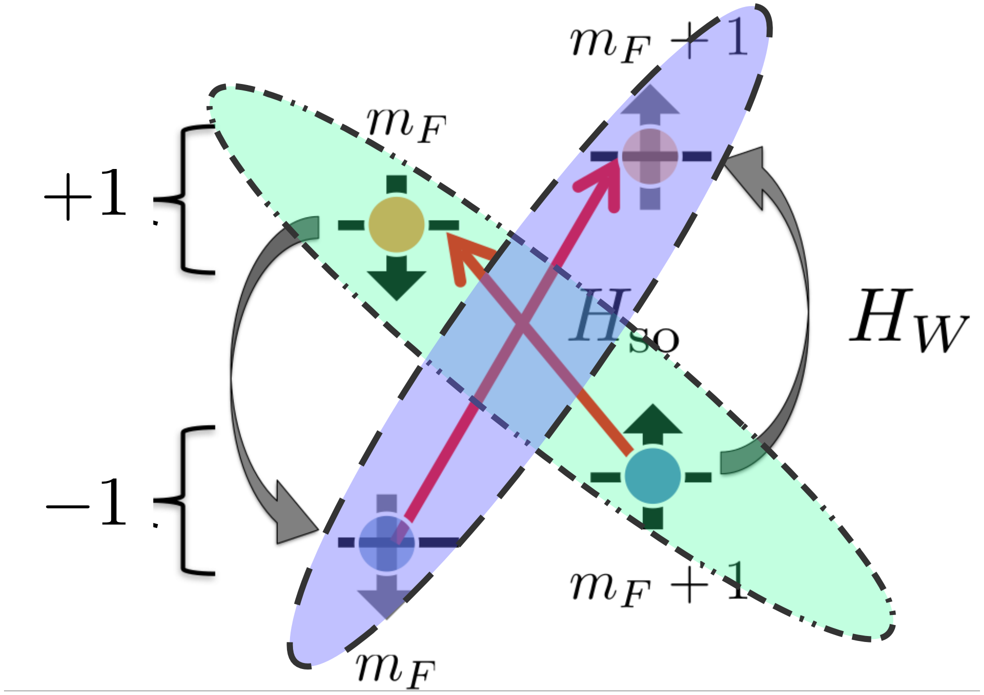
spin-orbit coupling:

$$\hat{H}_{\text{so,j}} \quad = \quad \sum_p \left\{ (\alpha_R + b) c^\dagger_{j,\uparrow,p} c_{j+1,\downarrow,-p} + \ (b - \alpha_R) c^\dagger_{j+1,\uparrow,p} c_{j,\downarrow,-p} + \text{h.c.} \right\}$$

# Cold atoms setting



## Symmetries:

- the number of fermions in each pair of states coupled by spin-orbit coupling is **conserved modulo 2**, due to the presence of the spin-exchange interactions.

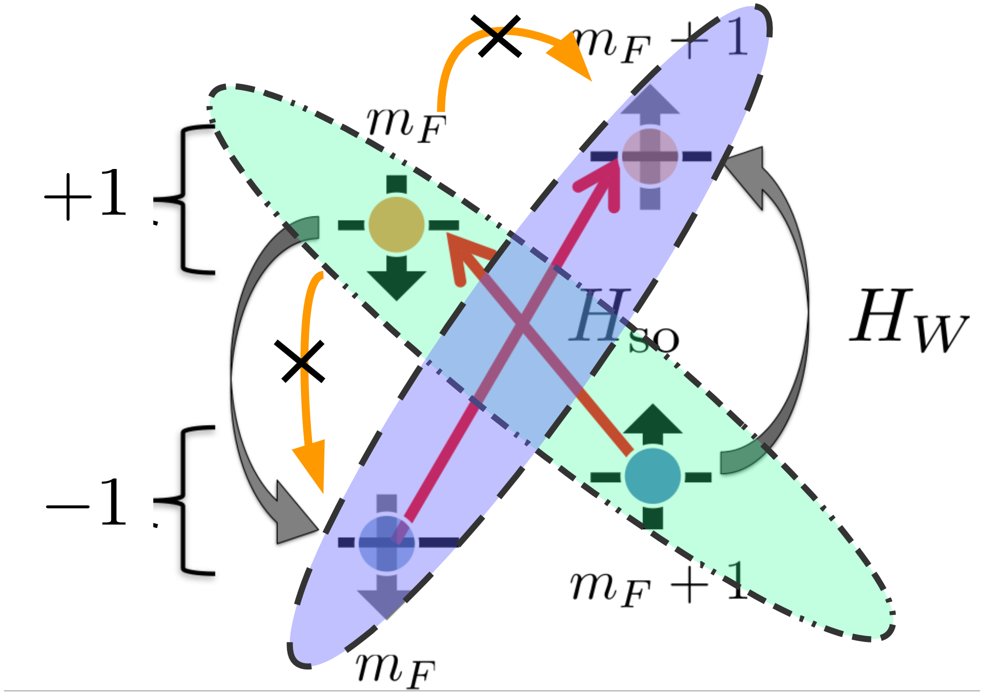
$$\mathbb{Z}_2 \text{ sectors: } \begin{array}{l} [(\downarrow, -1), (\uparrow, 1)] \\ [(\downarrow, 1), (\uparrow, -1)] \end{array}$$

"local" parity symmetry

$$\left[\hat{H}, \hat{P}\right] = 0$$

$$\hat{P} = (-1)^{\left(\sum_j (n_{j,\uparrow,1} + n_{j,\downarrow,-1})\right)}$$

# Cold atoms setting



## Symmetries:

*- the number of fermions in each pair of states coupled by spin-orbit coupling is* ***conserved modulo 2****, due to the presence of the spin-exchange interactions.*

***terms breaking the symmetry are not present in the microscopic dynamics: violate Angular Momentum Conservation***

$$\textbf{e.g.,}\quad \hat{c}^{\dagger}_{j,\downarrow,\underline{-1}}\hat{c}_{j,\downarrow,\underline{+1}}\,, \quad \hat{c}^{\dagger}_{j,\underline{\downarrow},-1}\hat{c}_{j,\underline{\uparrow},-1}\,, \quad \ldots$$

$$\hat{P}=(-1)^{\left(\sum_j (n_{j,\uparrow,1}+n_{j,\downarrow,-1})\right)}$$

# Properties of the ground states

***Overall analysis:***

- ***Bosonization;***

*map to “ladder chains” + braiding: non-abelian statistic*

- ***Lower Band Projected Hamiltonian***
- ***DMRG (density matrix renormalization group);***

*Check list of ground state topological properties*

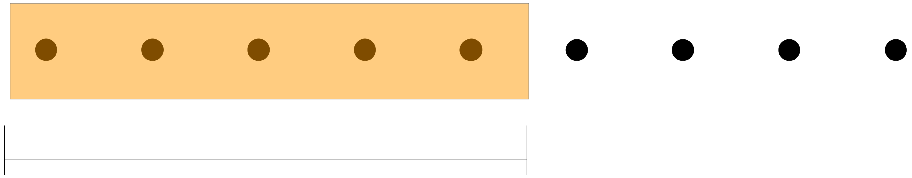
✓ ***entanglement spectrum degeneracy***

✓ ***edge-to-edge correlations***

✓ ***spectral analysis:***
***. ground state degeneracy***
***. single particle excitation gap***

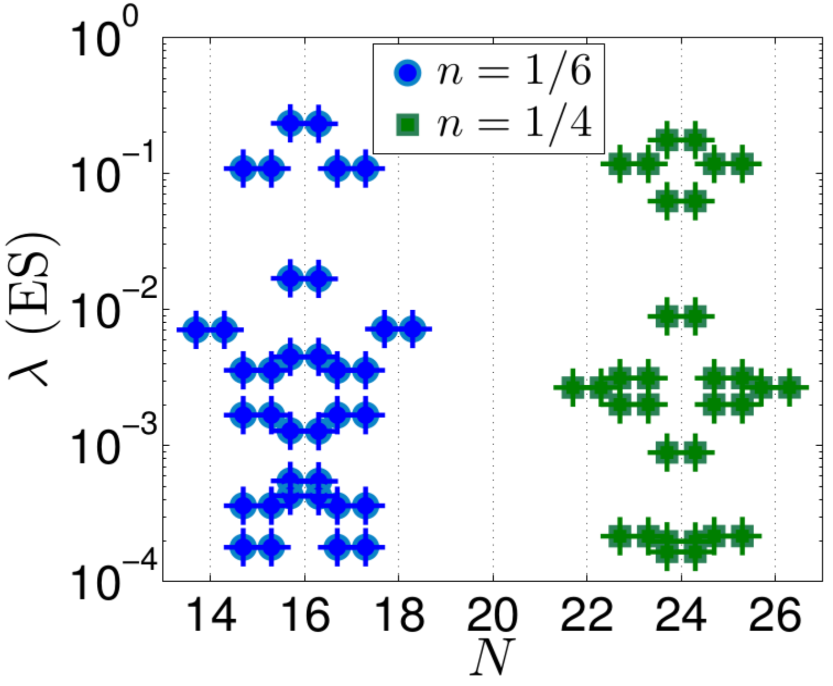
# Properties of the ground states

✓ **Double degeneracy in the entanglement spectrum:**

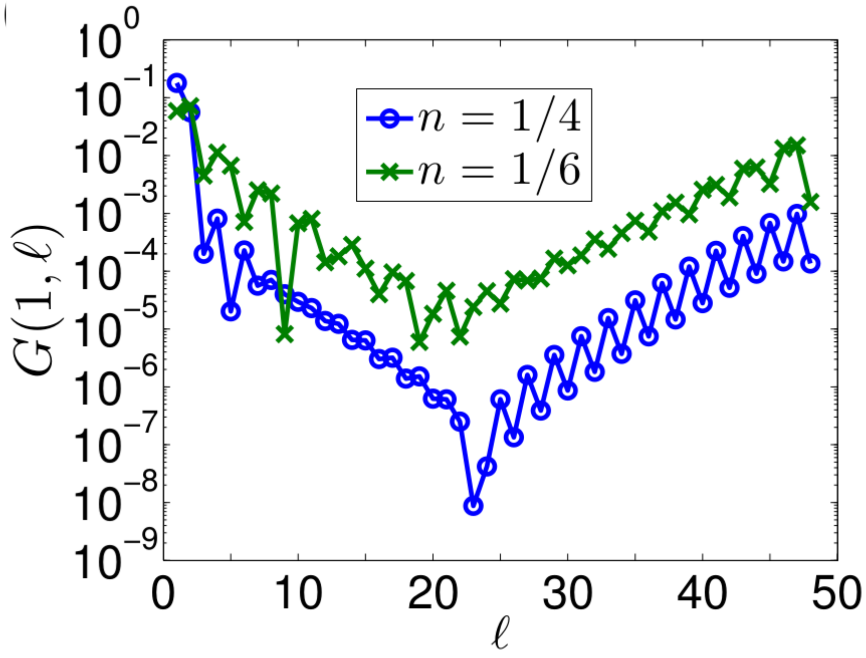


$$\rho_\ell = \sum_{N,j} \lambda_j(N)\hat{\sigma}_j(N)$$

$\mathrm{W/t} = \text{-8},\ \alpha = b = 4t$



✓ **Non-local edge-edge correlations:**

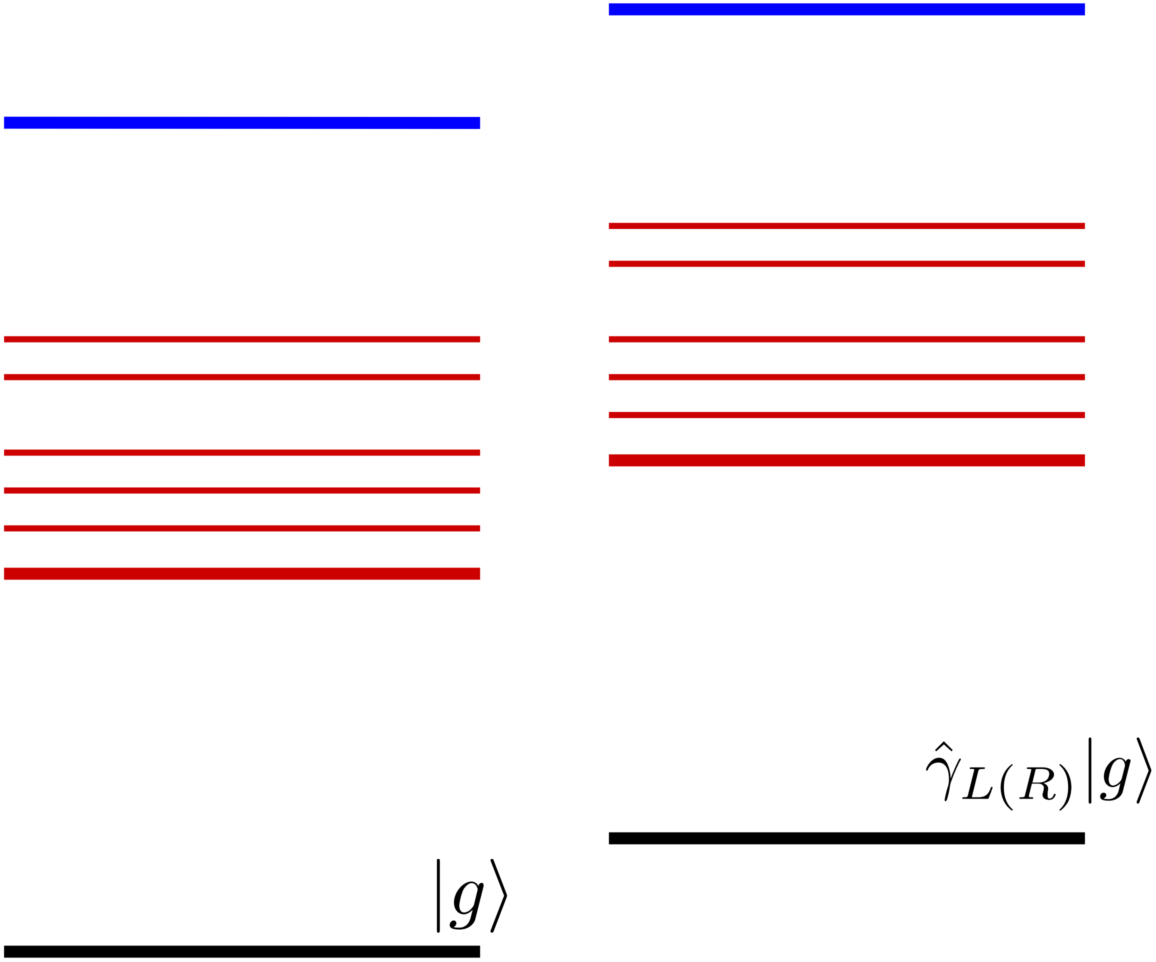


$$G(j,\ell) \equiv \langle c^\dagger_{j,\uparrow,1} c_{\ell,\uparrow,1} \rangle$$

F. Pollmann et al., Phys. Rev. B 81, 064439 (2010)
A. M. Turner et al., Phys. Rev. B 83, 075102 (2011)

# Properties of the ground states

**Spectral analysis (OBC):**



$\hat{\gamma}_{L(R)}|g\rangle$

$|g\rangle$

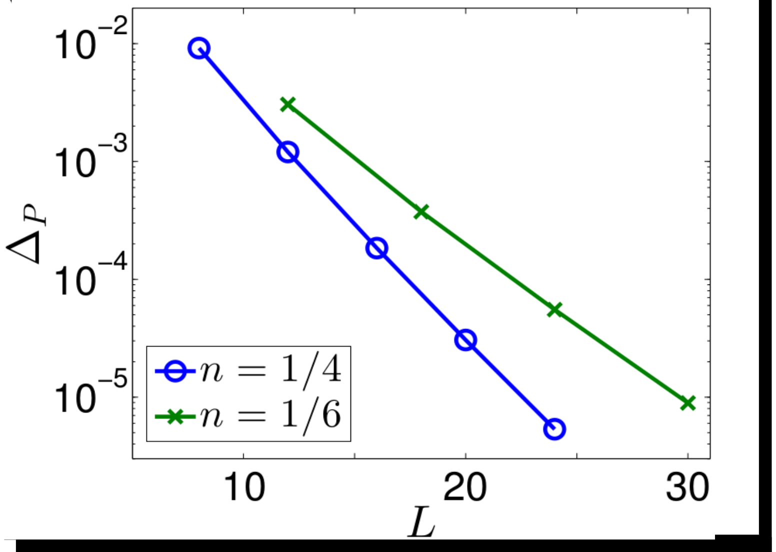
$P = +1$ $P = -1$

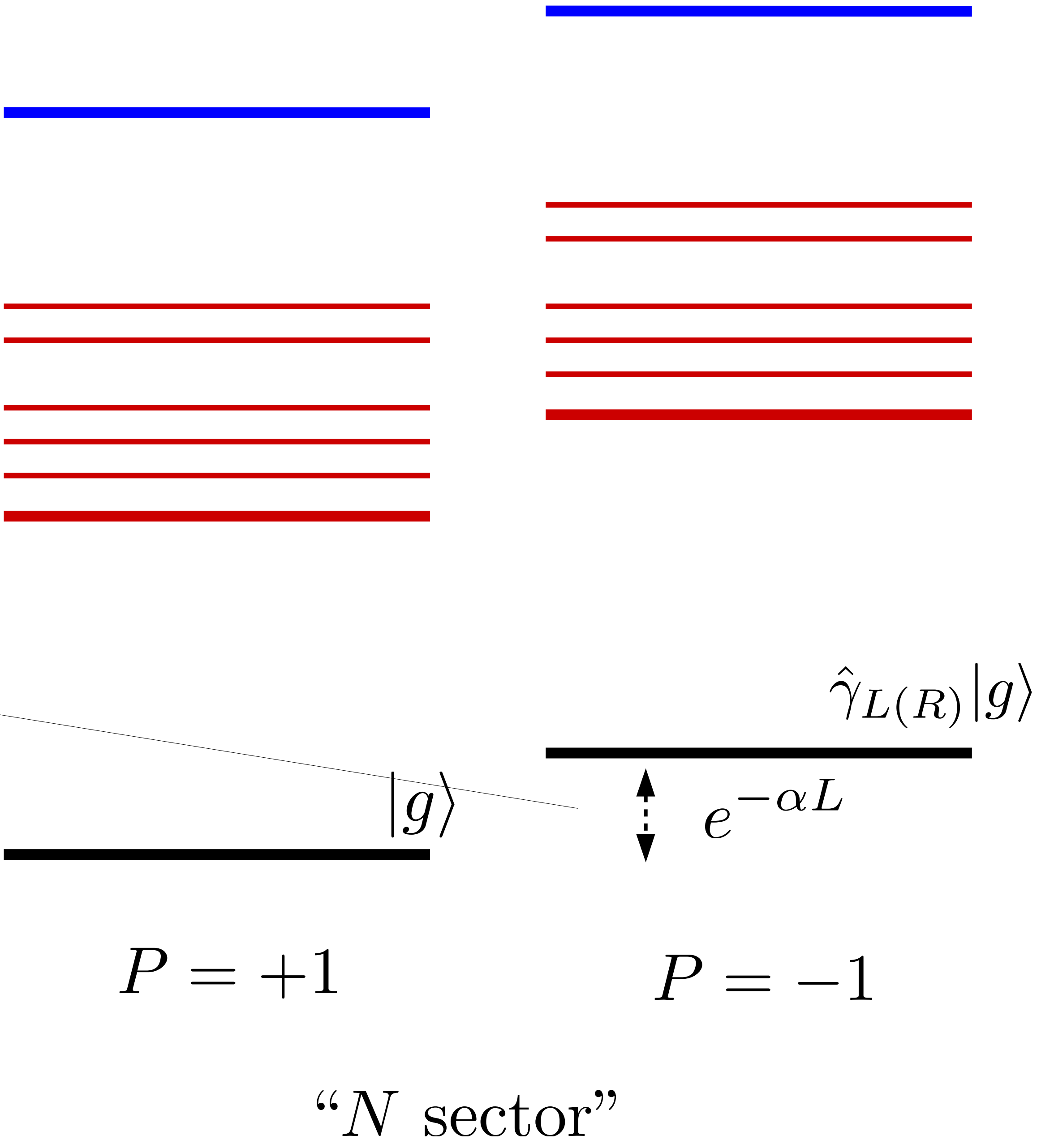
“$N$ sector”

# Properties of the ground states

## Spectral analysis (OBC):

$$\Delta_P = E^0(N,+P) - E^0(N,-P)$$





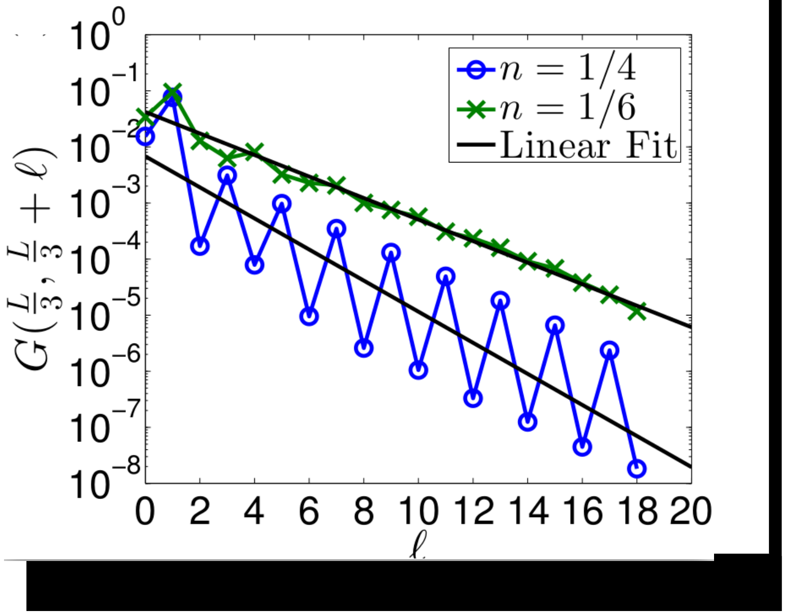
# Properties of the ground states

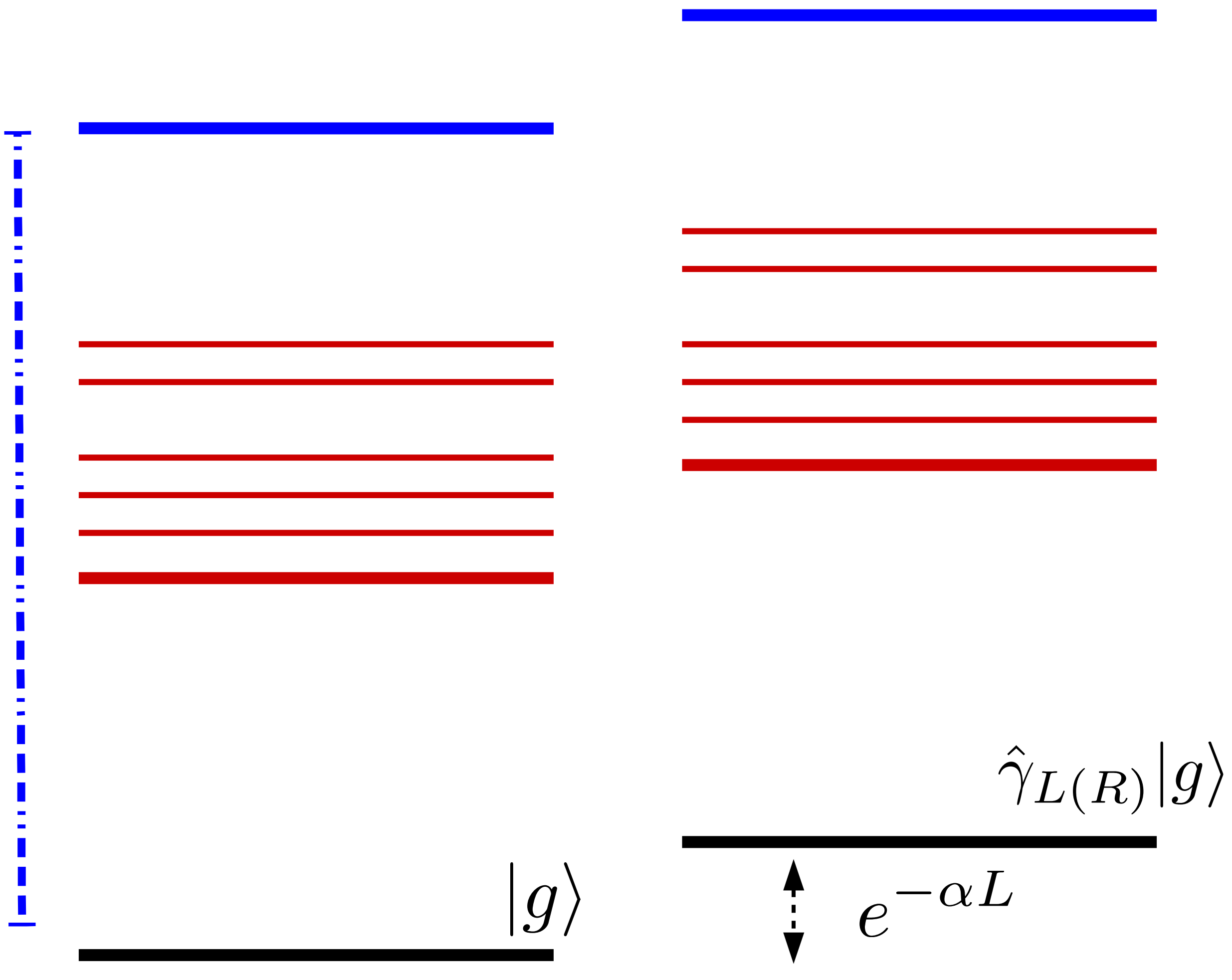
## Spectral analysis (OBC):

*Single particle gap: exponential decay of bulk Green function*

$$\Delta_{\mathrm{sp}} > 0$$

$$\langle c_j^\dagger c_\ell \rangle \sim e^{-\frac{|j-\ell|}{\Delta_{sp}}}$$





$P = +1$ $P = -1$

"$N$ sector"

# Properties of the ground states

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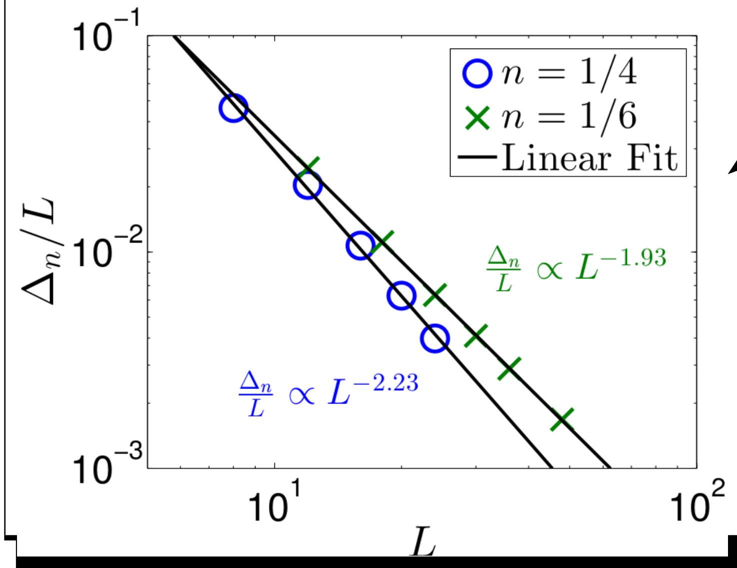
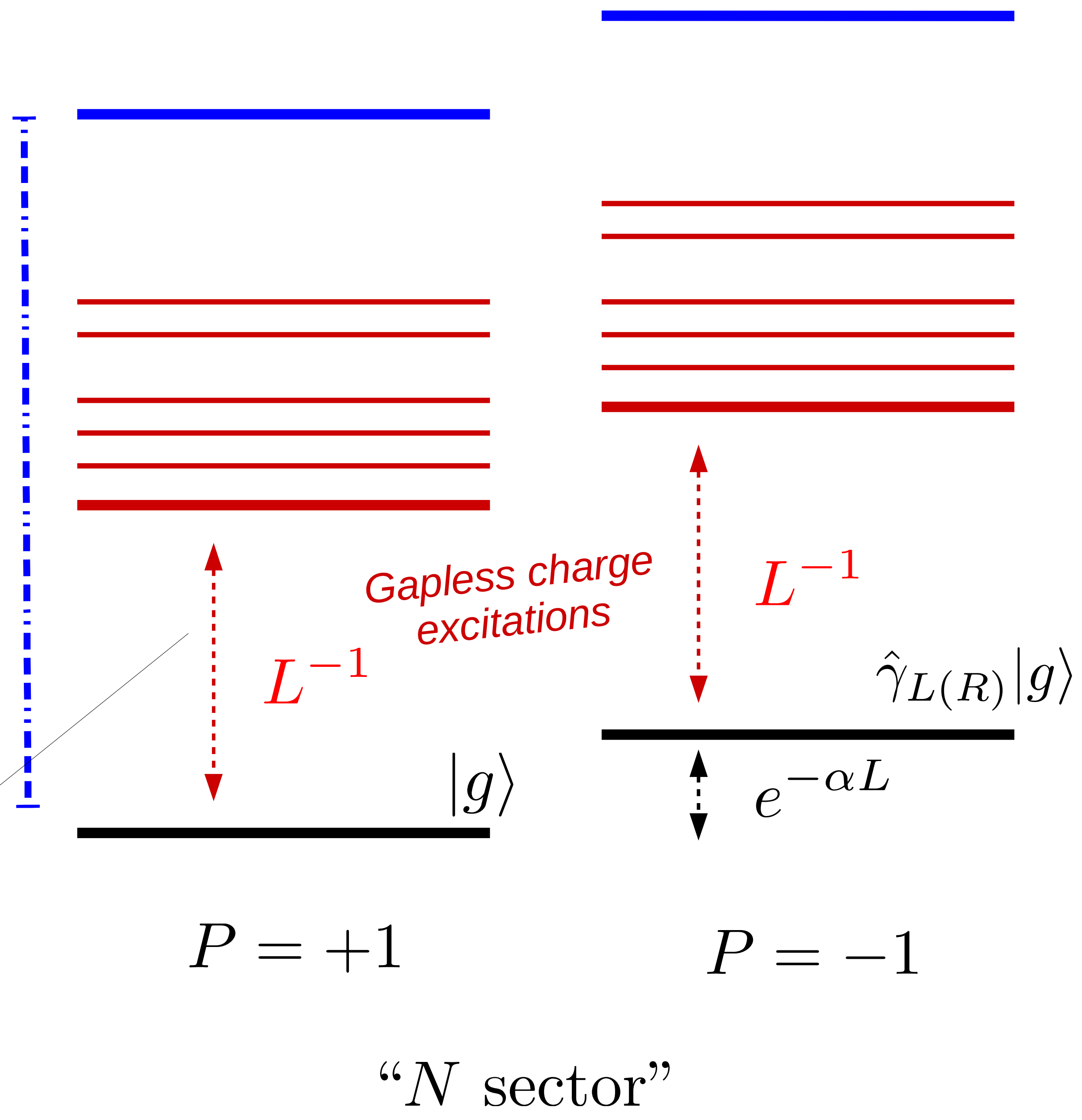
*Single particle gap: exponential decay of bulk Green function*

$$\Delta_{\rm sp} > 0$$

$$\Delta_n = E^1(N,P) - E^0(N,P)$$





*Gapless charge excitations*

$L^{-1}$

$|g\rangle$

$P = +1$

$L^{-1}$

$\hat{\gamma}_{L(R)}|g\rangle$

$e^{-\alpha L}$

$P = -1$

“$N$ sector”

# Conclusions

- Majorana quasi-particles can emerge as edge modes of AEA's in opticall lattices in the presence of **spin-orbit couplings + spin-exchange interactions**;
- The protection of the emergent Z_2 parity symmetry stems from **angular momentum conservation**;
- We hopefully expect such results could help to the observation of Majorana edge modes in the canonical setting, where all basic ingredients for our recipe have been experimentally realized over the last two years.

Thanks for the attention!