III. From Majorana to Parafermions

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In collaboration with Jelena Klinovaja, Basel

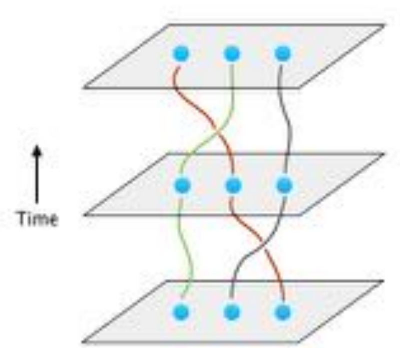
$$: Swiss NSF, Nano Basel, Quantum ETH/Basel, EU

Outline

- Topological quantum computing
- Majorana fermions in ‘super-semi’ nanowires and chains
- Majorana and spin qubits → 2D surface code
- Next generation: Parafermions in double nanowires
  → can get nearly universal TQC including CNOT

Motivation: Topological Quantum Computing

Kitaev 2003



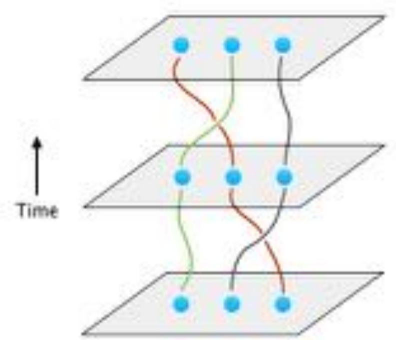
braiding of quasiparticles
with non-Abelian statistics

Majoranas, parafermions,
Fibonacci fermions,…

At T=0: TQC protected against all errors by gap

Motivation: Topological Quantum Computing

Kitaev 2003



braiding of quasiparticles with non-Abelian statistics

Majoranas, parafermions, Fibonacci fermions,…

At T=0: TQC protected against all errors by gap

At T>0: errors occur with finite probability

➔ need continuous quantum error correction for TQC

Wootton, Burri, Iblisdir, and Loss, PRX 4, 011051 (2014)
Brell, Burton, Dauphinais, Flammia, and Poulin, PRX 4, 031058 (2014)
Pedrocchi, Bonesteel, and DiVincenzo, PRL 115, 120402 (2015); PRB 92, 115441 (2015)
Hutter, Wootton, and Loss, Phys. Rev. X 5, 041040 (2015)
Burton, Brell, and Flammia, arXiv:1506.03815 (2015)
Hutter and Wootton, PRA 93, 042327 (2016)
Brown, DL, Pachos, Self, and Wootton, Rev. Mod. Phys. 88, 045005 (2016)

PHYSICAL REVIEW B 86, 085414 (2012)

Decoherence of Majorana qubits by noisy gates

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(Received 3 June 2012; published 7 August 2012)

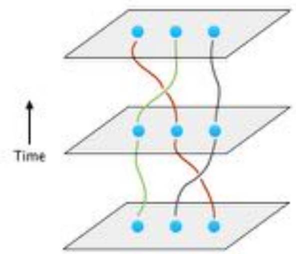
decay of ground state $b_0^\dagger|\Omega\rangle \xrightarrow{\tau} |k\rangle = b_k^\dagger|\Omega\rangle$ due to local gate fluctuations μ_{loc} on Majorana chain

$$\frac{\hbar}{\tau} \simeq \frac{\pi}{2}\lambda^2 B_D T \exp(-\Delta/T) \simeq \begin{cases} \frac{10.7}{\epsilon^2\epsilon_F[\mathrm{eV}]}\frac{m^*}{m_0}\left(\frac{l}{d}\right)^2 T\exp(-\Delta/T) & \text{for} \quad D=1, \\ \frac{281.6}{\epsilon^2}\left(\frac{m^*}{m_0}\right)^2\frac{l[\mathrm{nm}]^4}{d[\mathrm{nm}]^2} T\exp(-\Delta/T) & \text{for} \quad D=3. \end{cases}$$

exponential suppression but large prefactor

For typical metallic gates one needs $\Delta > 20\,T$ for $\tau > 1\ \mu s$

Majorana Fermions in Nanowires



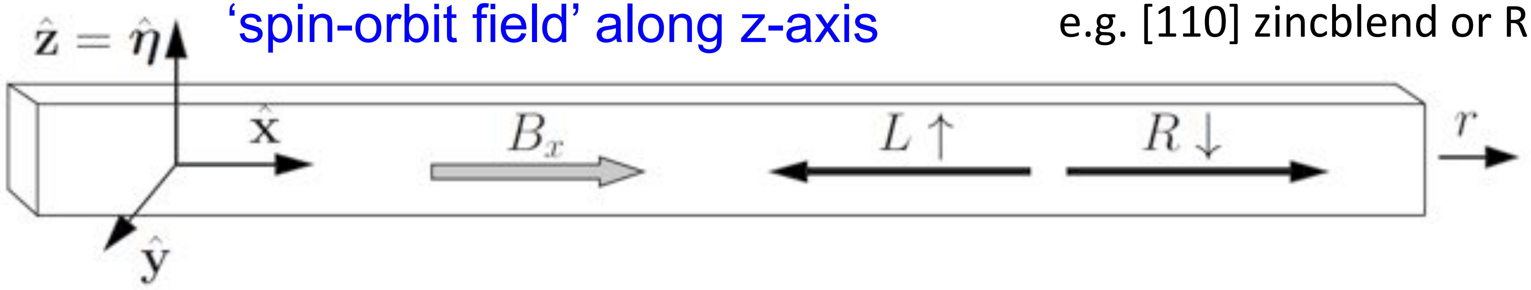
Basic ingredients:
s-wave superconductivity
& spin texture due to:

- Spin orbit interaction (SOI)
- Rotating Zeeman field (synthetic SOI, spin helix,…)
- RKKY interaction

Helical spectrum is crucial

'spin-orbit field' along z-axis

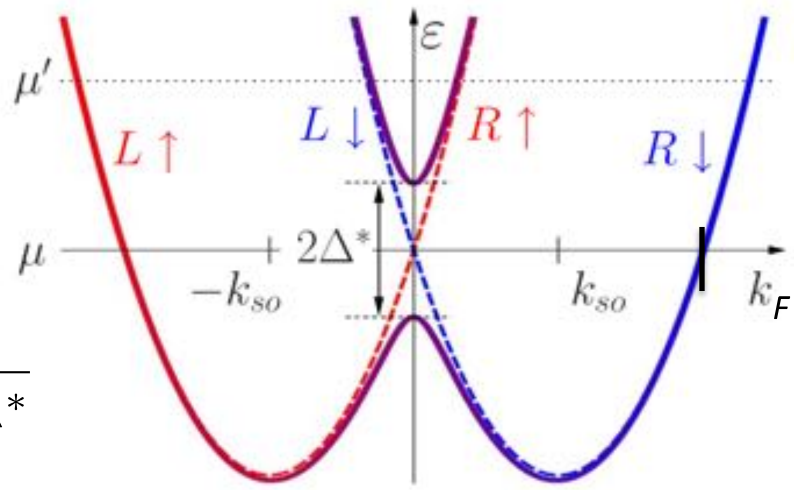
e.g. [110] zincblend or Rashba



$$H_R = \alpha p_x \sigma_z$$

Rashba (1960)

$$\epsilon_{\pm}(k) = \frac{k^2}{2m} - \mu \mp \sqrt{\alpha^2 k^2 + \Delta^*}$$

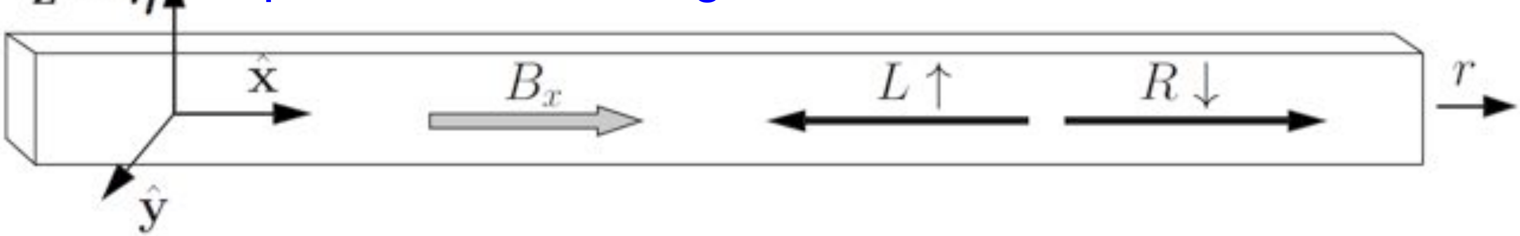


'Helical spectrum'

Helical spectrum is crucial

'spin-orbit field' along z-axis

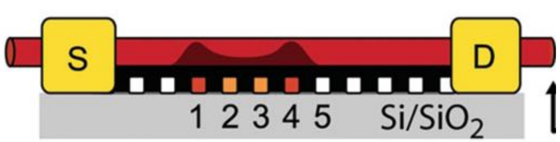
e.g. [110] zincblend or Rashba

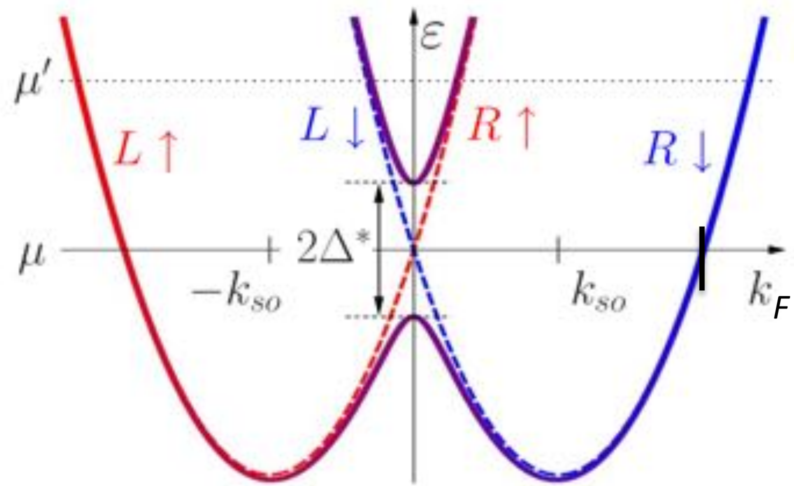


$$H_R = \alpha p_x \sigma_z$$

Rashba (1960)

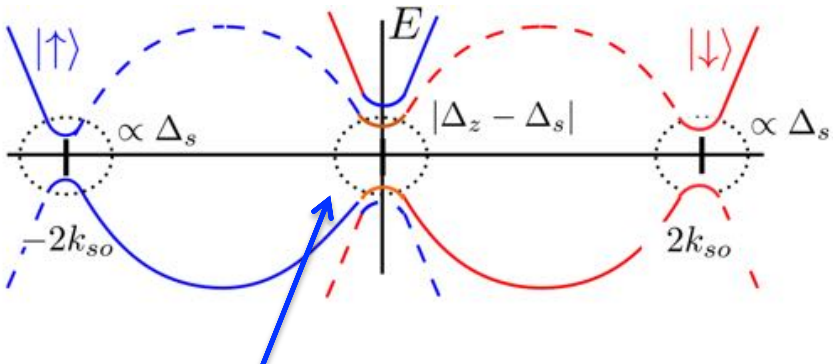
SOI measured
in quantum dots





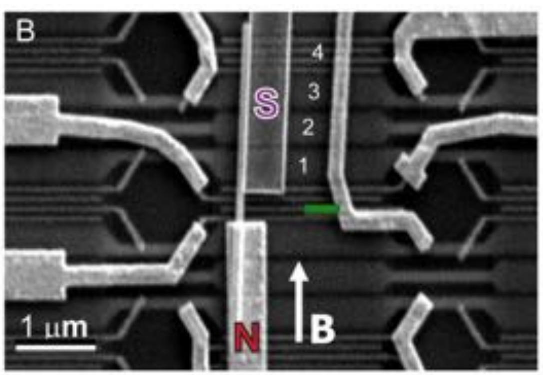
InAs nanowire λ_{SO} ~ 100 nm,
Fasth, Fuhrer, Samuelson, Golovach & DL,
PRL 98, 266801 (2007)

Competing gaps: magnetic field vs. superconductivity



Volovik, JETP Lett. 70, 609 (1999)
Sato and Fujimoto, PRB 79, 094504 (2009)
nanowires:
Lutchyn et al., PRL 105, 077001 (2010)
Oreg et al., PRL 105, 177002 (2010)

Competition between gaps; $\Delta_z > \Delta_s$: topological phase with Majorana bound state



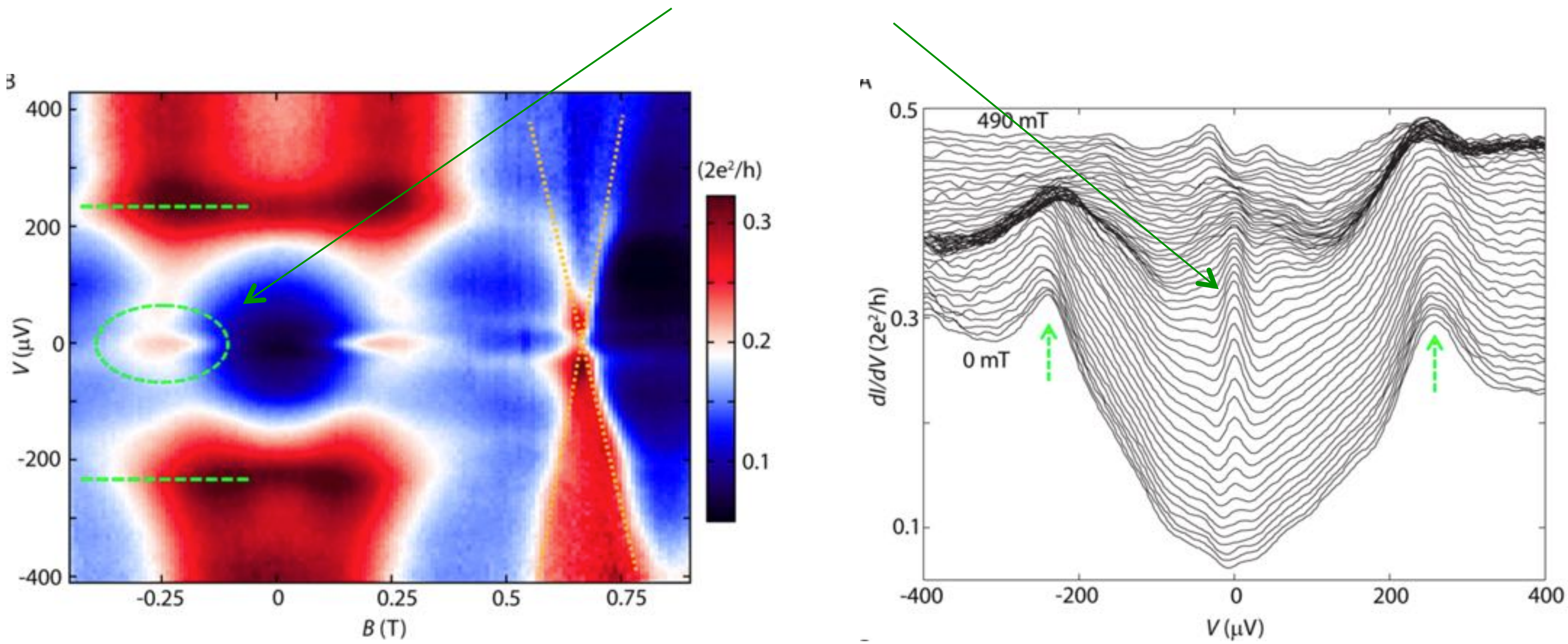
Mourik et al., Science 336, 1003 (2012)

Experiments:
Kouwenhoven group, Science 336, 1003 (2012)
Xu group, Nano Letters 12, 6414 (2012)
Heiblum group, Nat. Phys. 8, 887 (2012)
Rokhinson group, Nat Phys 8, 795 (2012)
Marcus group, PRB 2013, Nature (2016), Science (2016)
Ando group, PRL 107, 217001 (2011) (topological insulators)

Majorana zero-mode detected via ‘zero-bias peak’

Mourik et al., Science 336, 1003 (2012)

Interpretation: Zero bias peak in dI/dV = Majorana fermion



Similar experiments:
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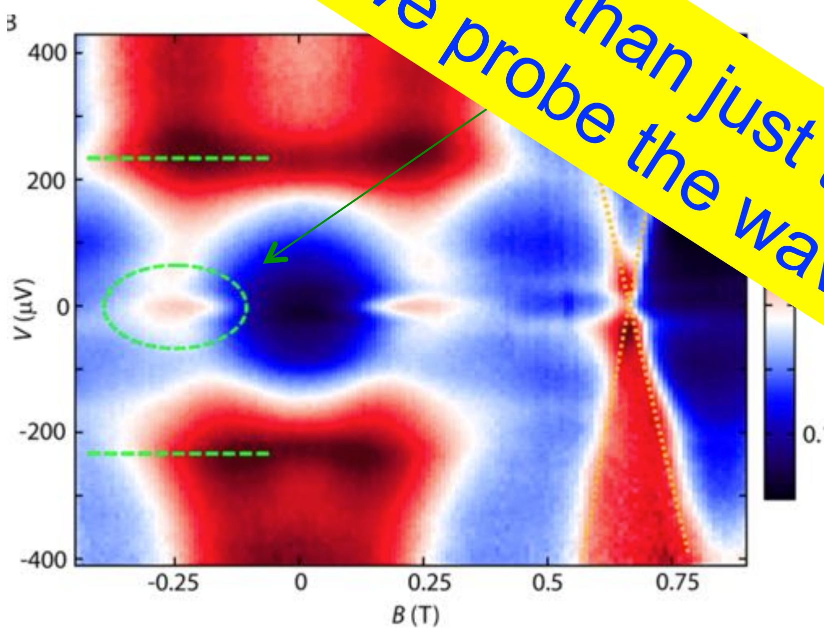
Xu group, Nano Letters 12, 6414 (2012)
Rokhinson, Nat Phys 8, 795 (2012)
Marcus group: Churchill et al., PRB 87, 241401(R) (2013); Albrecht et al., Nature 531, 206 (2016)
Ando group, PRL 2011 (topological insulators)

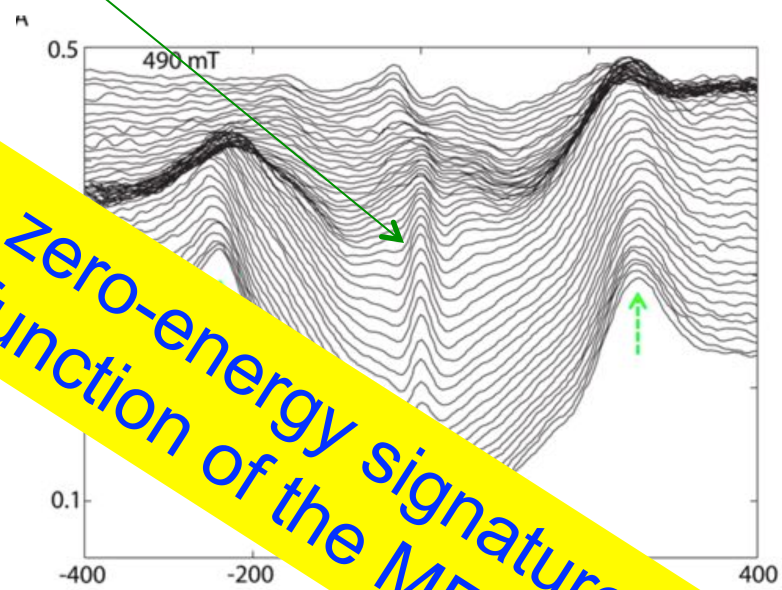
Majorana zero-mode detected via ‘zero-bias peak’

Mourik et al., Science 336, 1003 (2012)

Is there more than just the zero-energy signature?
Can we probe the wavefunction of the MF ?

: Zero bias peak in dI/dV = Majorana fermion





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Marcus group: Churchill et al., PRB 87, 241401(R) (2013); Albrecht et al., Nature 531, 206 (2016)
Ando group, PRL 2011 (topological insulators)

Composite Structure of Majorana Wavefunction

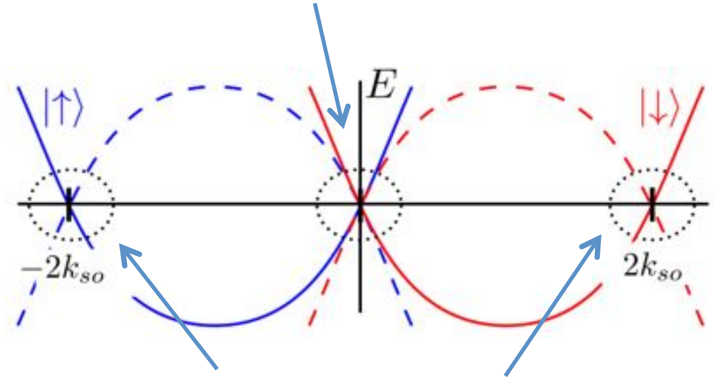
Klinovaja and DL, PRB 86, 085408 (2012)

interior branches

Linearization (strong SOI limit) :

$$\Psi(x) = R_\uparrow + L_\downarrow + L_\uparrow e^{-2ik_{so}x} + R_\downarrow e^{2ik_{so}x}$$

➔ emergent Dirac theory



exterior branches

Exterior and interior branches are decoupled:

interior branches:

$$\mathcal{H}^{(i)} = -i\hbar v_F \sigma_3 \partial_x + \Delta_z \sigma_1 \eta_3 + \Delta_{sc} \sigma_2 \eta_2$$

$$\phi^{(i)} = (R_\uparrow, L_\downarrow, R_\uparrow^\dagger, L_\downarrow^\dagger)$$

exterior branches:

$$\mathcal{H}^{(e)} = i\hbar v_F \sigma_3 \partial_x + \Delta_{sc} \sigma_2 \eta_2$$

$$\phi^{(e)} = (L_\uparrow, R_\downarrow, L_\uparrow^\dagger, R_\downarrow^\dagger)$$

Definition of a Majorana fermion

$$\gamma = \gamma^\dagger \qquad \gamma^2 = 1$$

Majorana: 'particle that is its own antiparticle'

Fermion basis:

$$\Psi = \begin{pmatrix} \Psi_\uparrow \\ \Psi_\downarrow \\ \Psi_\uparrow^\dagger \\ \Psi_\downarrow^\dagger \end{pmatrix}$$

then

Wavefunction:

$$\Phi_{MF} = \begin{pmatrix} f(x) \\ g(x) \\ f^*(x) \\ g^*(x) \end{pmatrix}$$

MF operator: $\gamma = \int dx \; \Phi_{MF}(x) \cdot \Psi = \gamma^\dagger$

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charge is zero
spin is zero
bound state

Typical Majorana wavefunction

$$\Phi_M(x) = \begin{pmatrix} i \\ 1 \\ -i \\ 1 \end{pmatrix} e^{-k_-^{(i)}x} - \begin{pmatrix} i\ e^{2ik_{so}x} \\ e^{-2ik_{so}x} \\ -i\ e^{-2ik_{so}x} \\ e^{2ik_{so}x} \end{pmatrix} e^{-k^{(e)}x}$$

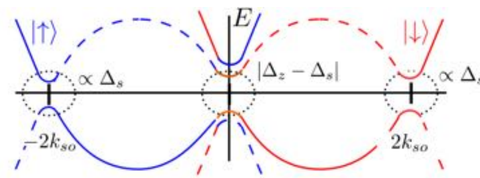
from interior branch

from exterior branch

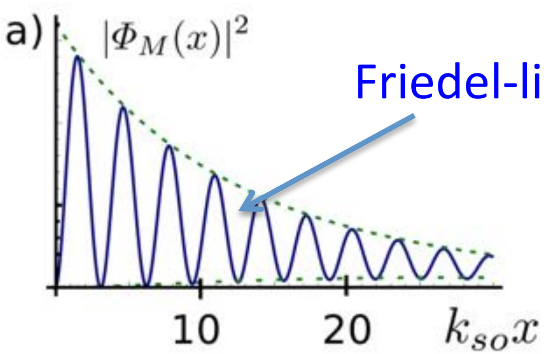
two localization lengths, given by inner and outer gap:

$$\xi_-^{(i)} = \alpha/|\Delta_s - \Delta_z|$$

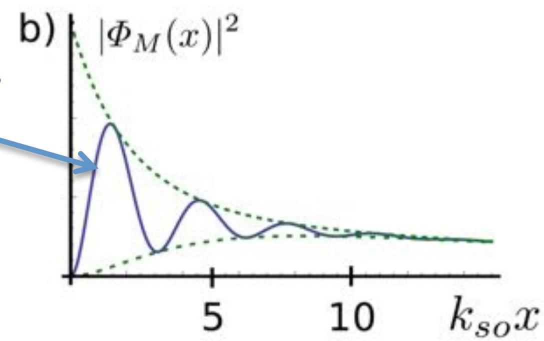
$$\xi^{(e)} = \alpha/\Delta_s$$



Friedel-like oscillations



$\Delta_z = 2\Delta_s$



$\Delta_z = 7\Delta_s$

two branches contribute equally

exterior branch dominates

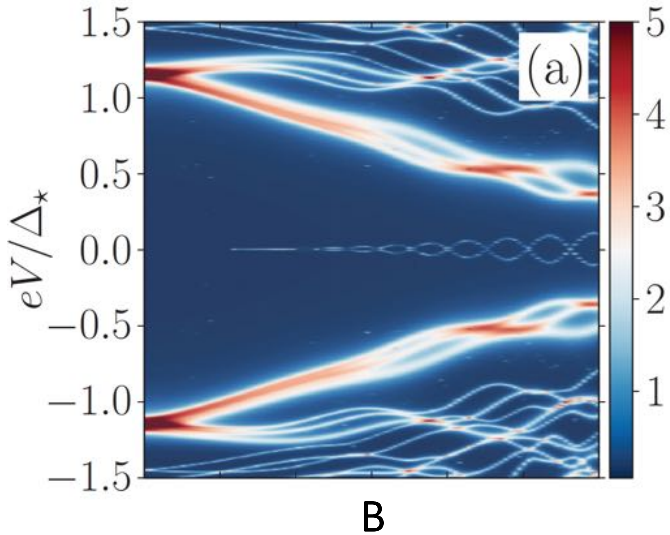
Klinovaja and Loss, PRB 86, 085408 (2012)

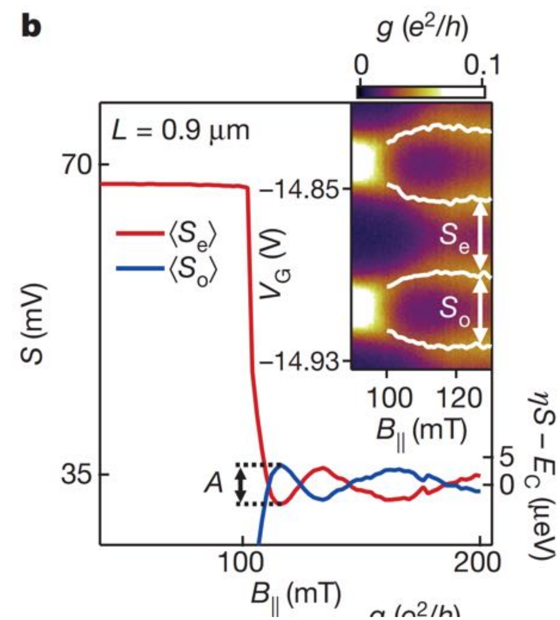
Oscillations of Majorana Splitting

Rainis et al., PRB 87, 024515 (2013)

Marcus group: Albrecht et al., Nature 531, 206 (2016)

weak SOI





Note: amplitude of oscillation increases!

Oscillations of Majorana Splitting

Rainis et al., PRB 87, 024515 (2013)

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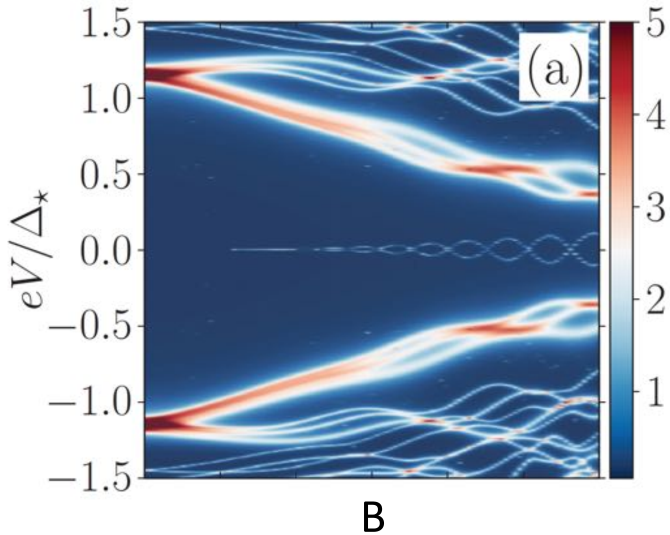

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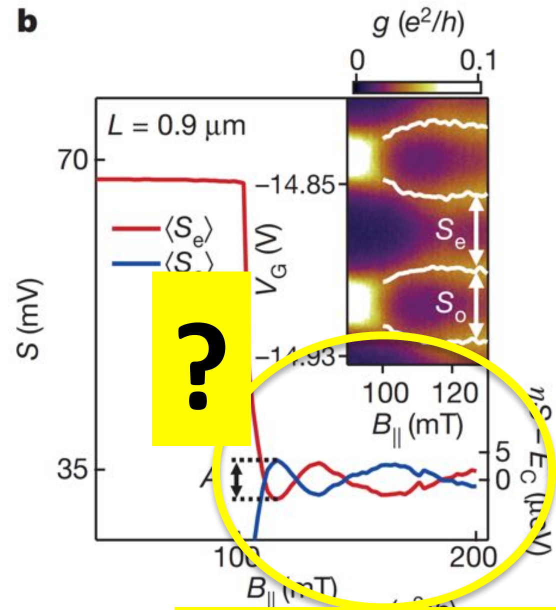
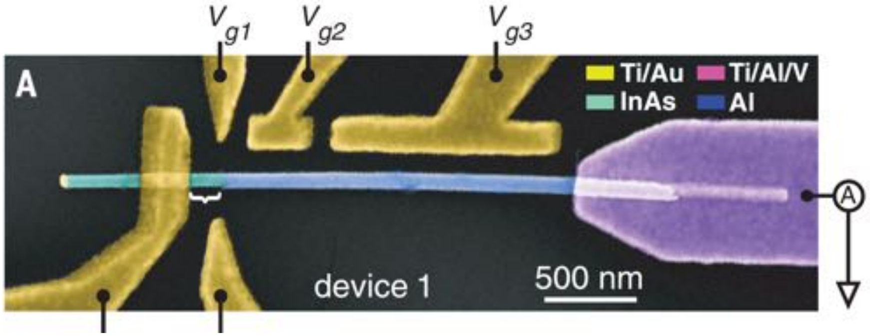
Marcus group: Albrecht et al., Nature 531, 206 (2016)

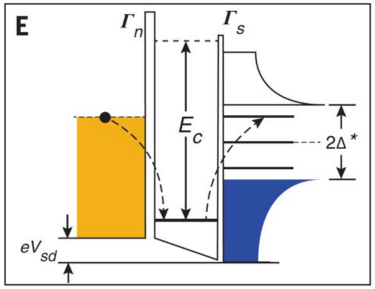


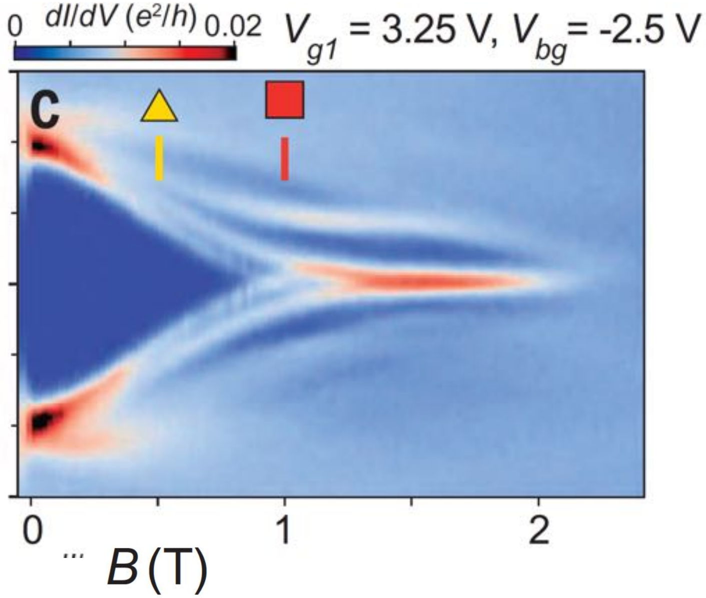
Amplitude decreases
Why?

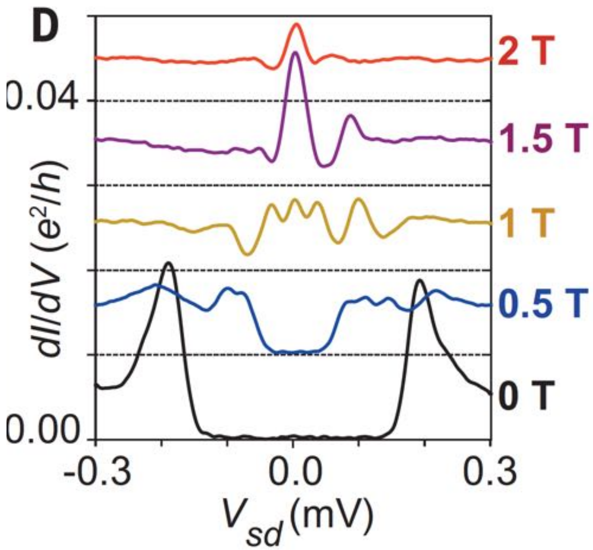
Majorana wire and quantum dot

Marcus group: Deng et al., Science 354, 1557 (2016)



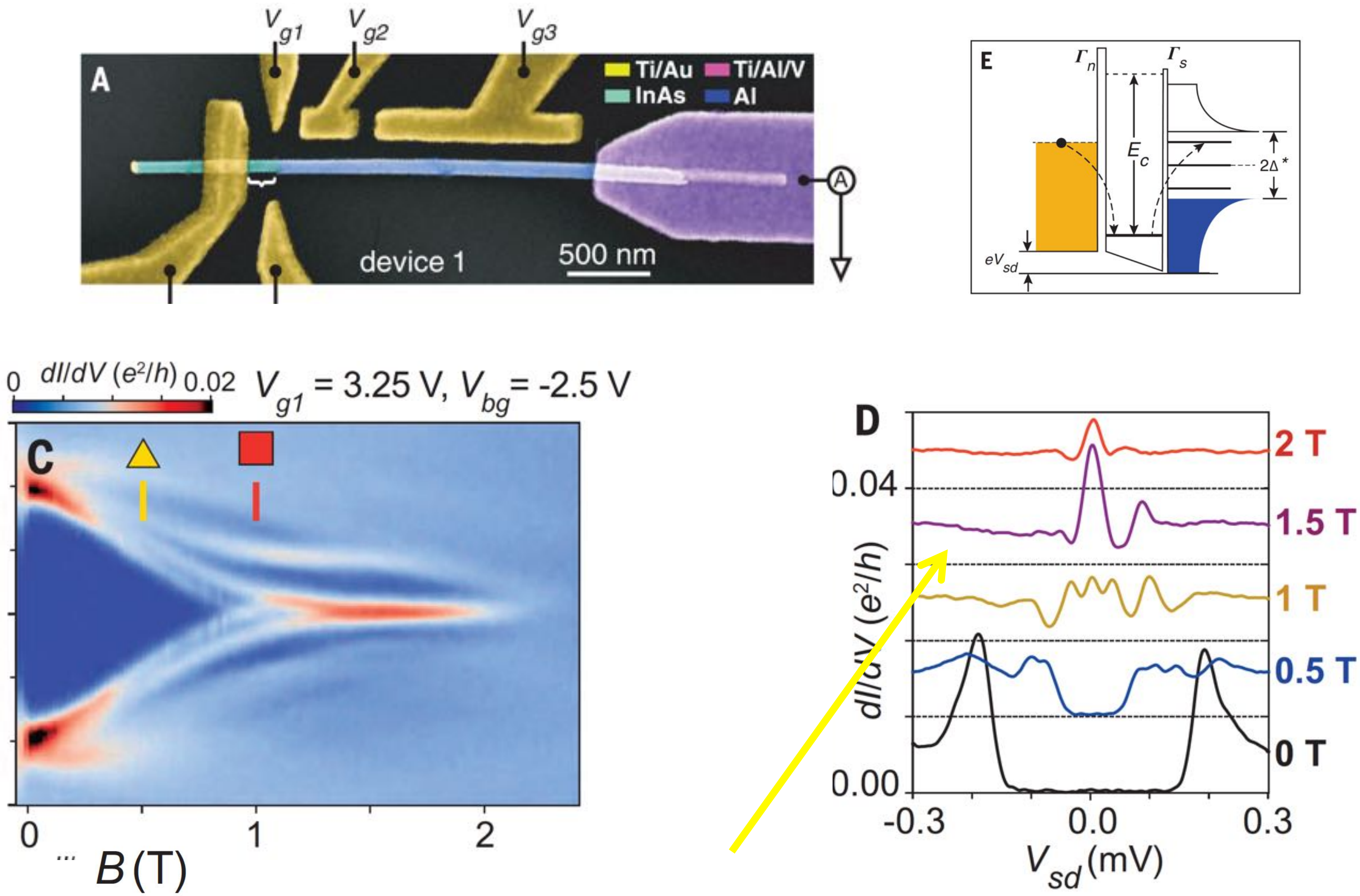






Majorana wire and quantum dot

Marcus group: Deng et al., Science 354, 1557 (2016)



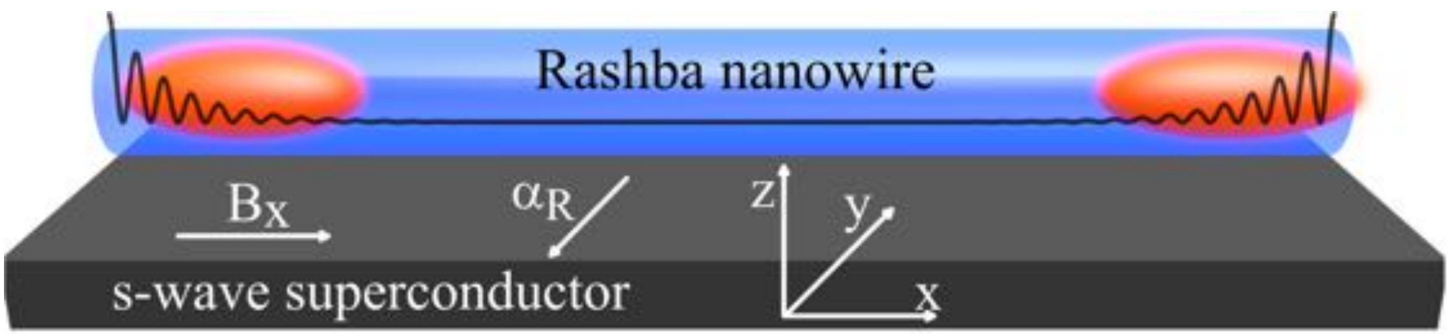
Two issues: 1) gap almost closed when ZBP appears
2) ZBP much higher conductance than bulk

Spin and Charge Signatures of Topological Phases

Szumniak, Chevallier, Loss, and Klinovaja, arXiv:1703.00265

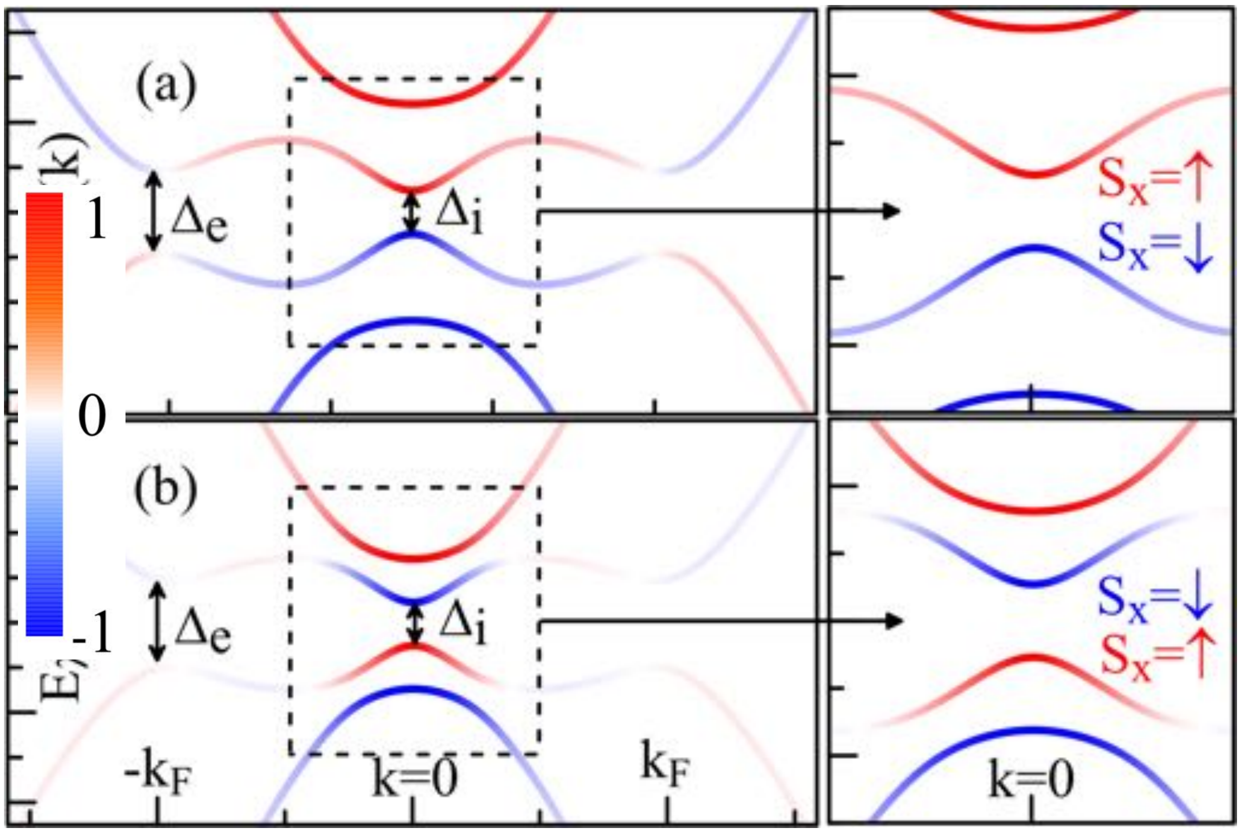
$$\vec{S}(E_n) = \frac{\hbar}{2}\sum_{j=1}^{N} \Phi_n^{\dagger}(j)\vec{\sigma}\,\Phi_n(j)$$ Quasiparticle spin

$$Q(E_n) = -|e|\sum_{j=1}^{N} \Phi_n^{\dagger}(j)\tau_Z\Phi_n(j)$$ Quasiparticle charge



Band structure: spin polarization

Inversion of bulk spin around topological gap $S_X(k) \| B_X$



topological phase

$$\Delta_Z > \Delta_{sc}$$

trivial phase

$$\Delta_Z < \Delta_{sc}$$

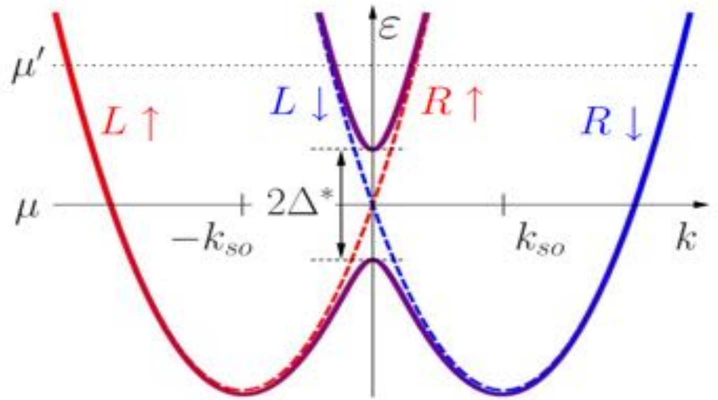
$\Delta_{sc} = 0.02,\ \alpha = 0.3$

$\mu = 0,\ |E_g| = \Delta_i$

Szumniak, Chevallier, Loss, and Klinovaja, arXiv:1703.00265

Note: SOI is given by material property

➔ need to tune chemical potential inside gap such that $k_F \approx 2k_{SO}$



Are there *self-tuning* schemes for MFs? Yes!

- Normal phase in 1D (RKKY): Braunecker, Simon, and DL, PRL 102, 116403 (2009)
- SC phase in 1D (RKKY): Klinovaja, Stano, Yazdani, and DL, PRL 111, 186805 (2013)

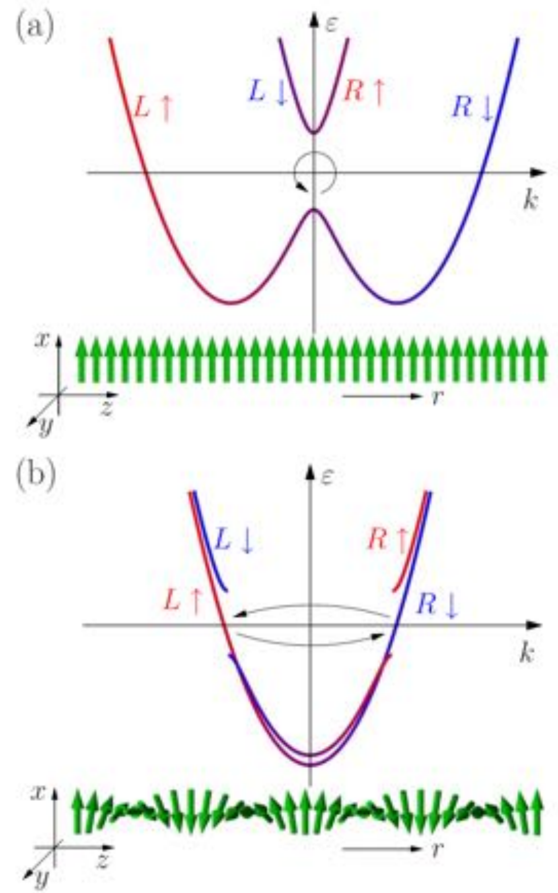
The SOI interaction can be gauged `away'!

Braunecker, Japaridze, Klinovaja, and DL, PRB 82, 045127 (2010)

Gauge trafo

$$\Psi_\sigma = e^{-i\sigma k_{so} x}\widetilde{\Psi}_\sigma$$

$B \neq 0$



Only rotating field
and **no** SOI anymore!

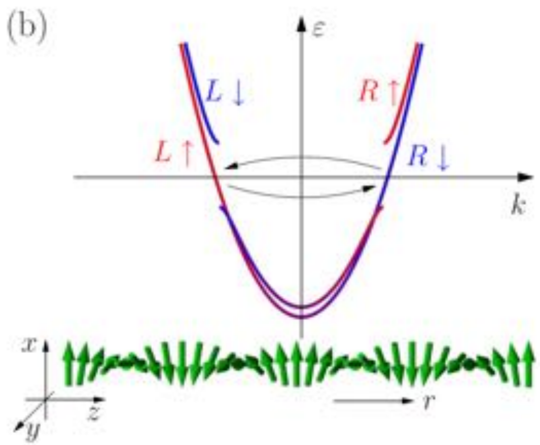
$$\widetilde{\mathbf{B}} = B(\cos(2k_{so}x)\mathbf{e}_x - \sin(2k_{so}x)\mathbf{e}_y)$$

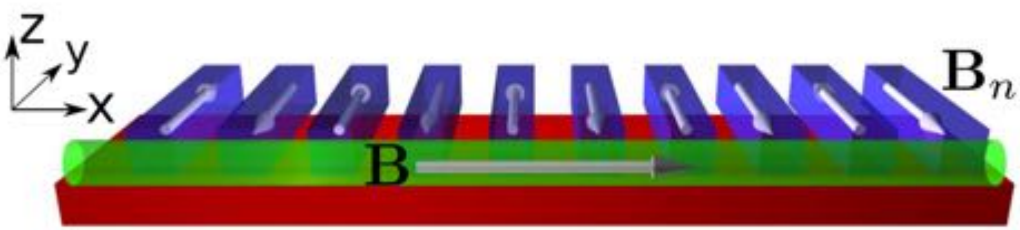
Effective SOI generated by nanomagnets ➔ Majoranas

Klinovaja, Stano, and DL, PRL 109, 236801 (2012)

rotating B-field ➔ effective SOI

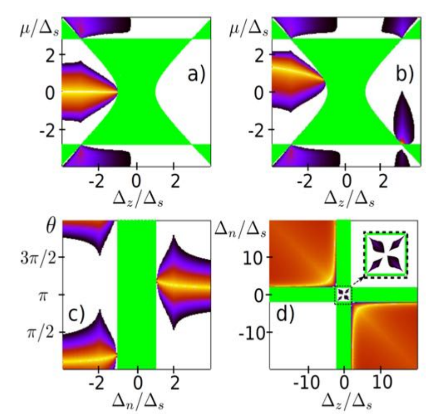
$$\widetilde{\mathbf{B}} = B(\cos(2k_{so}x)\mathbf{e}_x - \sin(2k_{so}x)\mathbf{e}$$





period of nanomagnets = spin-orbit length ~ 20 nm

rich MF phases

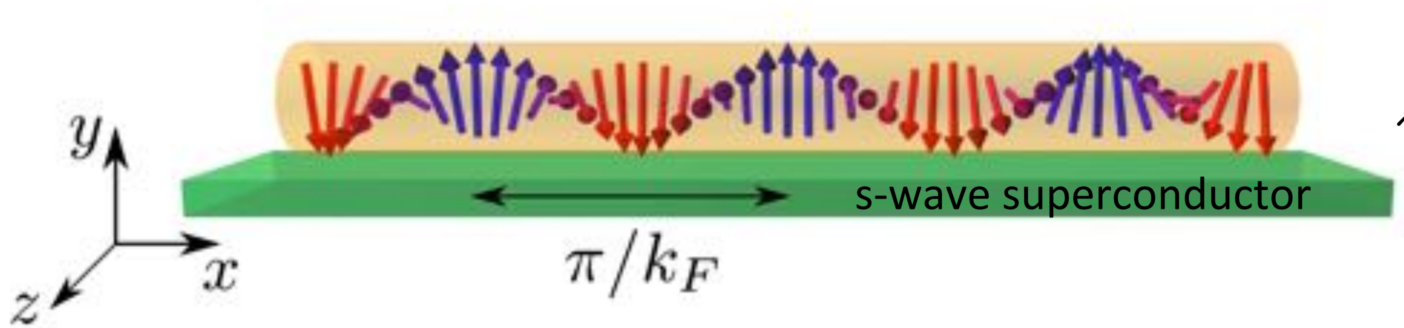


Graphene nanoribbon: Klinovaja and DL, PRX 3, 011008 (2013)

Majorana Fermions in Atom Chains

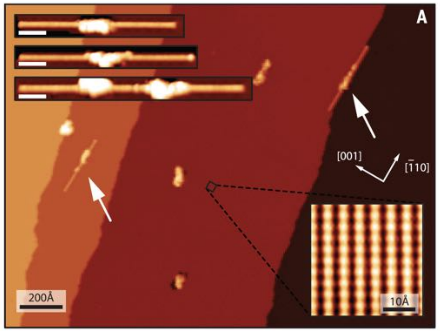
Klinovaja, Stano, Yazdani, and DL, PRL 111, 186805 (2013)

Localized moments I_j embedded in quasi 1D superconductor



$$\lambda_F \gg a$$

S. Nadj-Perge, et al., Science 346, 602 (2014)
[Princeton group]

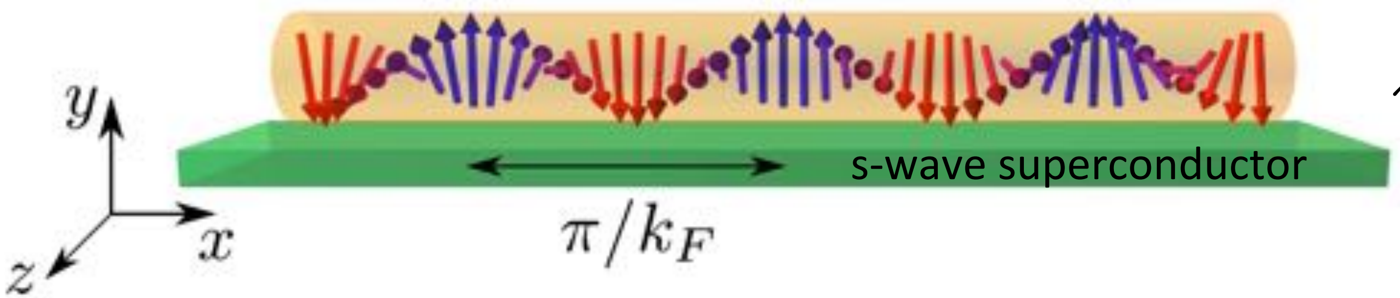


Braunecker and Simon, PRL 111, 147202 (2013)
Vazifeh and Franz, PRL 111, 206802 (2013)
Nadj-Perge, Drozdov, Bernevig, and Yazdani, PRB 88, 020407 (2013)
Pientka, Glazman, and v. Oppen, PRB 89, 180505 (2014)
Nadj-Perge, et al., Science 346, 602 (2014)

Majorana Fermions in self-tunable RKKY Systems

Klinovaja, Stano, Yazdani, and DL, PRL 111, 186805 (2013)

Localized moments ***I***$_j$ embedded in quasi 1D superconductor



$\lambda_F \gg a$

Interaction between localized moment I_i at position R_i and itinerant electron spin $\boldsymbol{\sigma}$

$$\mathcal{H}_{int}(x) = \frac{\beta}{2} \sum_i \tilde{\mathbf{I}}_i \cdot \boldsymbol{\sigma} |\psi(\mathbf{r}_{\perp,i})|^2 \delta(x - x_i), \qquad (\beta \equiv J)$$

$\beta \ll E_F$ ➔ can integrate out electrons and obtain...

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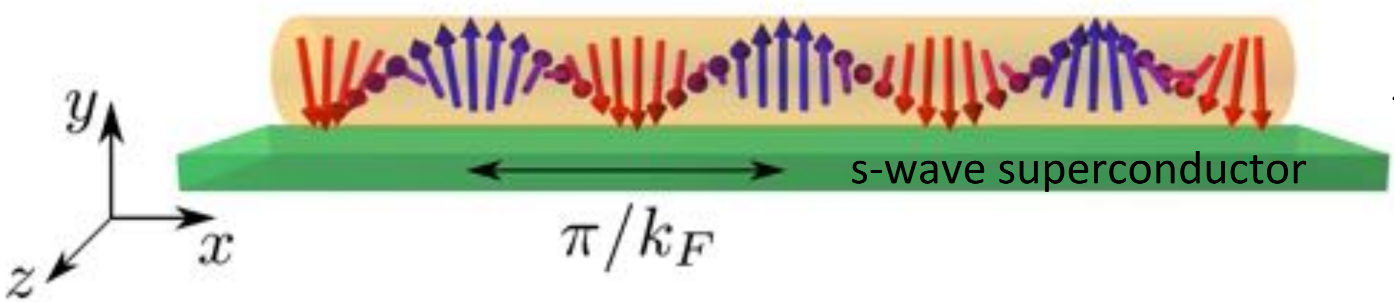


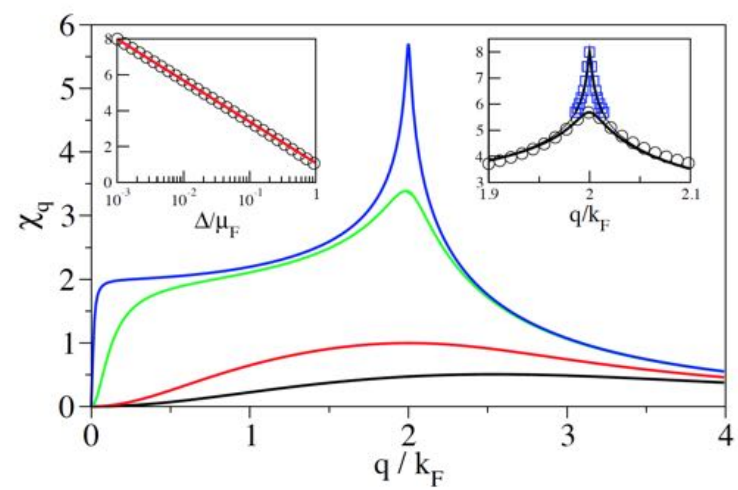
$\lambda_F \gg a$

RKKY interaction between moments $\mathbf{I}_j$ given by susceptibility of 1D superconductor :

$$H^{RKKY} = -\frac{2\beta^2}{A^2}\sum_q \chi_q \mathbf{I}_q \cdot \mathbf{I}_{-q}$$

$$\mathbf{I}_x \propto \cos(2k_F x)\hat{x} + \sin(2k_F x)\hat{y}$$

i.e. helix at **Fermi momentum** $2k_F$



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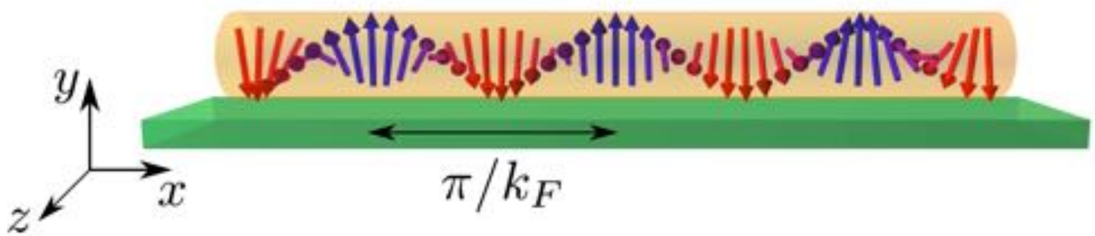
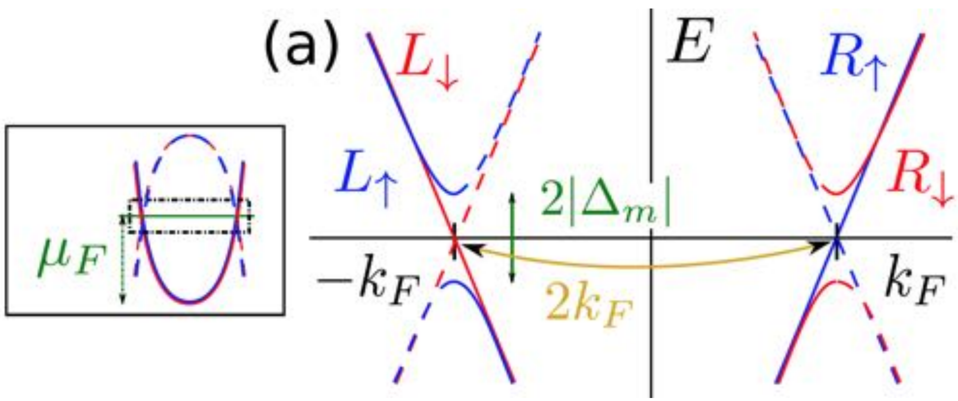


Helix has period of Fermi wave length

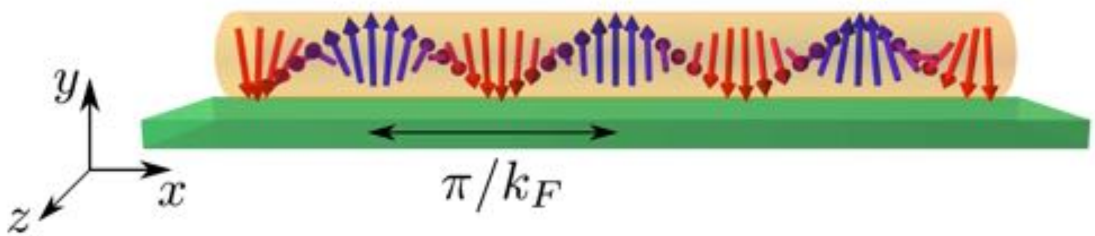
➔ partial gap ~ Δ_m opens at Fermi level μ_F without tuning!



$\Delta_m = \alpha\beta\rho_0\tilde{I}/2$ helical Zeeman field seen by electrons (backaction)

Majorana Fermions in self-tunable RKKY Systems

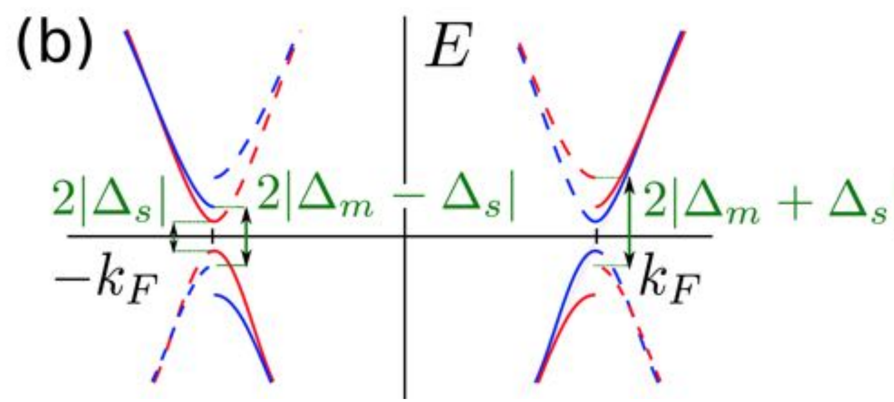
Klinovaja, Stano, Yazdani, and DL, PRL 111, 186805 (2013)



Helix opens partial gap *automatically* at Fermi energy μ_F :

Wire in topological phase if

$$\Delta_s < \Delta_m$$

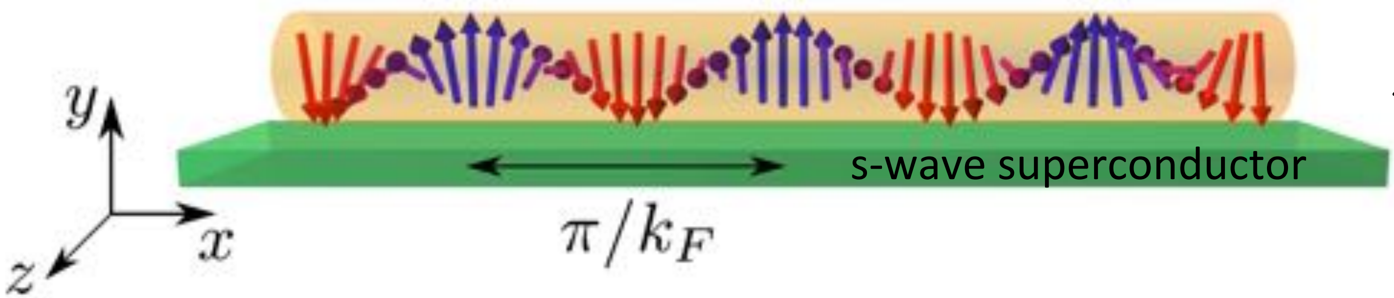


See also, Braunecker and Simon, PRL 111, 147202 (2013)
Vazifeh and Franz, PRL 111, 206802 (2013)

Majorana Fermions in self-tunable RKKY Systems

Klinovaja, Stano, Yazdani, and DL, PRL 111, 186805 (2013)

Localized moments I_j embedded in quasi 1D superconductor



$\lambda_F \gg a$

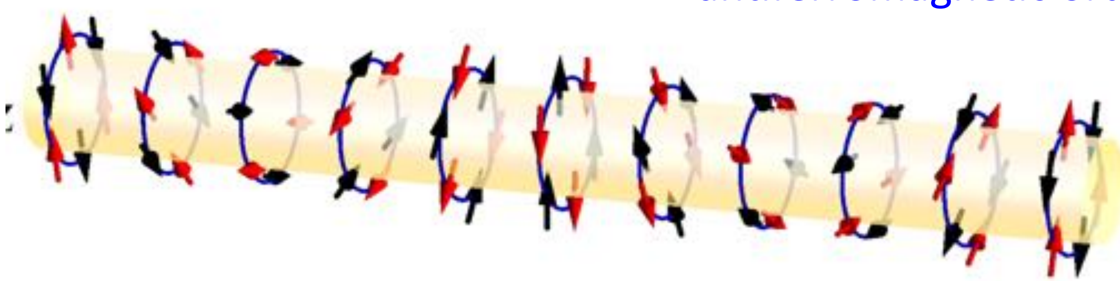
RKKY interaction between moments I_j given by susceptibility of 1D superconductor :

	chain	magnetic wire	nuclear wire
material	Fe	GaMnAs	InAs
a	0.3 nm	0.565 nm	0.605 nm
g_e	2	-0.44	-8
$\tilde{I} \cdot g_s \cdot \mu$	$2 \cdot 2 \cdot \mu_B$	$5/2 \cdot 2 \cdot \mu_B$	$9/2 \cdot 1.2 \cdot \mu_N$
β	1.6 meV nm^3	9 meV nm^3	4.7 μeV nm^3
α	1	0.02	1
μ_F	10 meV	20 meV	1 meV
Δ_s	1 meV	0.5 meV	0.1 meV
T_c	14 K	2 K	7.4 mK
B_c	5 T	0.7 T	4 T
Δ_m	6 meV	5 meV	0.2 meV
ξ	4 nm	0.4 μm	0.5 μm

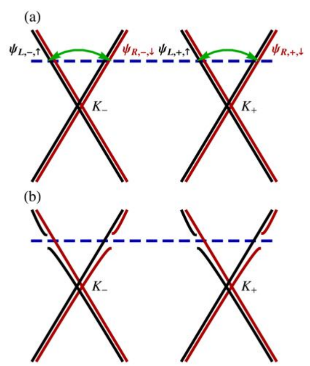
Nuclear spin helix & Majoranas in ^{13}C nanotubes

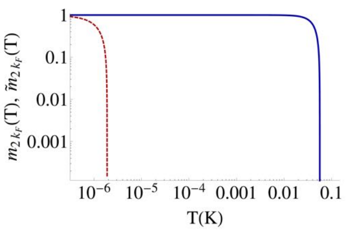
Chen-Hsuan Hsu, Stano, Klinovaja, and DL, PRB B 92, 235435 (2015)

antiferromagnetic order of nuclear spins

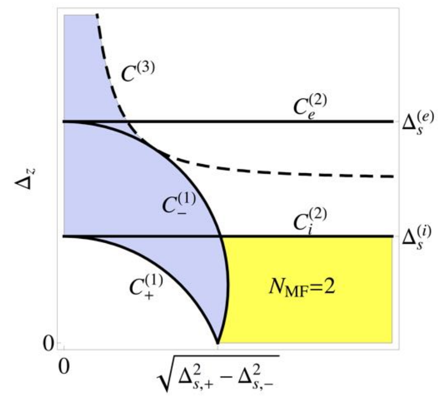


nuclear spin helix opens gaps at E_F ,
strongly renormalized by e-e interactions :





add superconductivity ➔ topological regime
with 2 Majoranas at each end of the nanotube



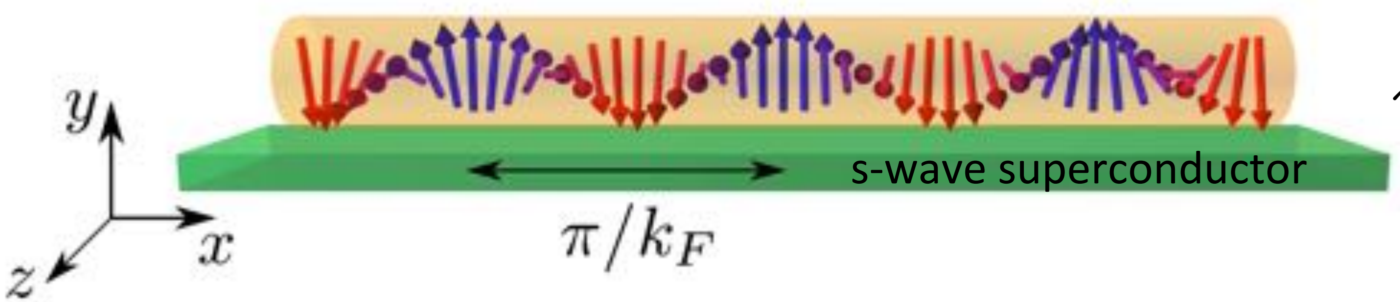
$$\Delta_m = \tilde{\Delta}_a \left(\frac{IA_0}{\tilde{\Delta}_a}\right)^{\frac{1}{2-\tilde{K}}}$$

A_{exp}~ 100 μeV ,T_0 ~ 100 mK, Δ_s ~2 K, ξ_{MF}~ 3 μm

Majorana Fermions in self-tunable RKKY Systems

Klinovaja, Stano, Yazdani, and DL, PRL 111, 186805 (2013)

Localized moments I_j embedded in quasi 1D superconductor



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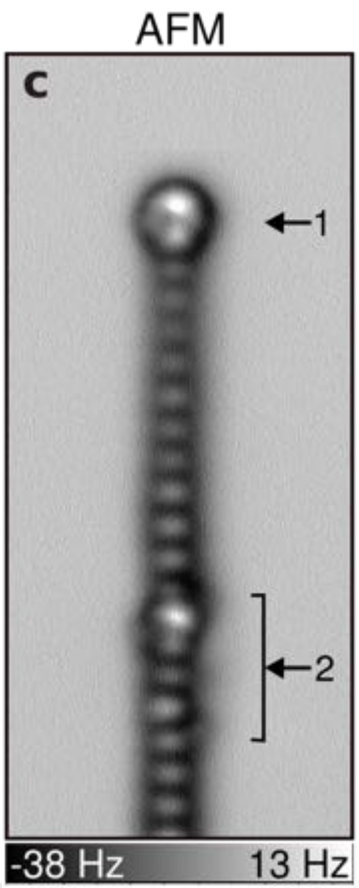
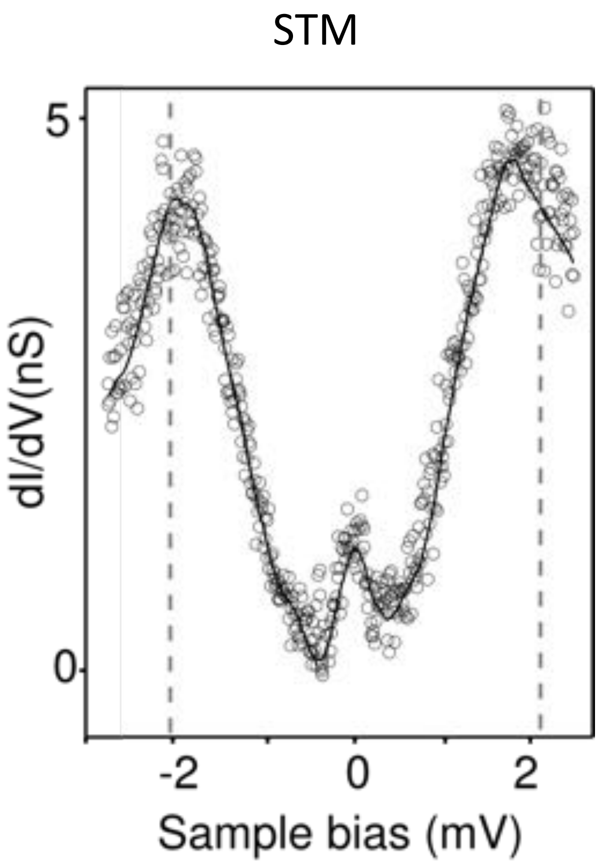
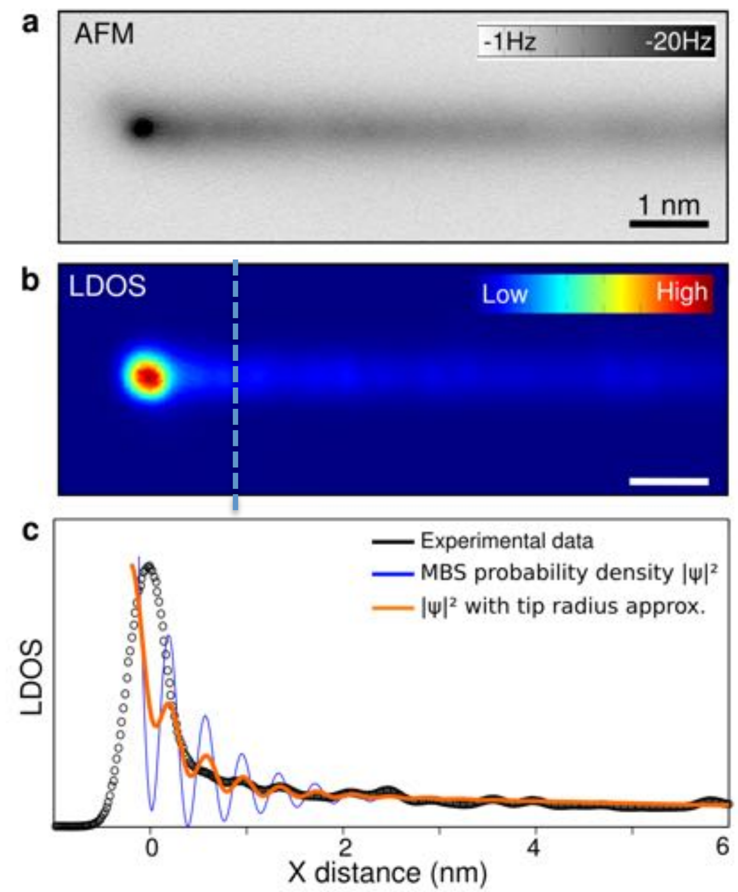
Mono-Atomic Fe chains on Superconducting Pb-Surface: Majorana signature

Pawlak et al., npj Quantum Information (2016) 2, 16035







mono-atomic

zero-energy mode

$\xi_1 = 0.8\ \mathrm{nm},\ \xi_2 = 20\ \mathrm{nm}$

Front-Runners for Quantum Computers

- spin qubits in semiconductors 'small & fast'

semiconducting nanostructures

➔ 'topological spintronics'

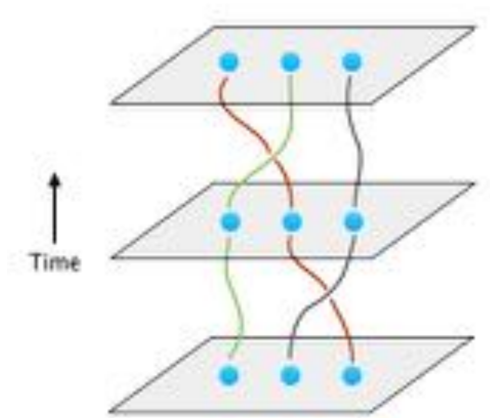
- topological quantum computing?

  'semi-super devices'

'exotic'
Majorana
Para- or
Fibonacci
fermions?

Braiding of Majoranas for Topological QC

Kitaev 2003



topologically protected: Hadamard and π/4 gate (Clifford gates)
noisy: CNOT (entangling) and π/8 gate (non-Clifford)

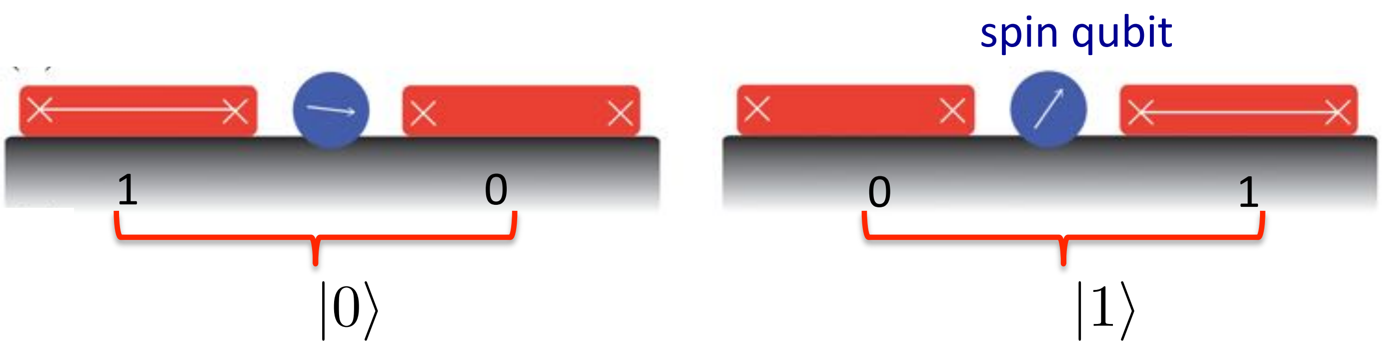
Universal QC with Majoranas via *noisy gates*, Bravyi, PRA 2008; Landau et al., PRL 2016; Vijay & Fu, arXiv:1609.00950; Karzig et al., arXiv:1610.05289 (Station Q)

Are there alternatives (non-braiding) to generate CNOT and π/8 gates ?

Universal Quantum Computation with Hybrid Spin-Majorana Qubits

Hoffman, Schrade, Klinovaja, and DL, PRB 94, 045316 (2016)

Majorana Spin hybrid (`MaSh’) qubit



odd parity MF qubit

delocalized fermion:

$$f_\nu = (\gamma'_\nu + i\gamma_\nu)/2 \quad \left\{ \begin{array}{l} 0 \\ 1 \end{array} \right.$$

degenerate states
‘topological protection’

But: parity fluctuates at MHz-rate ➔ ‘Majorana box qubit’

Rainis & DL, PRB 85, 174533 (2012); Marcus et al., arxiv:1612.05748

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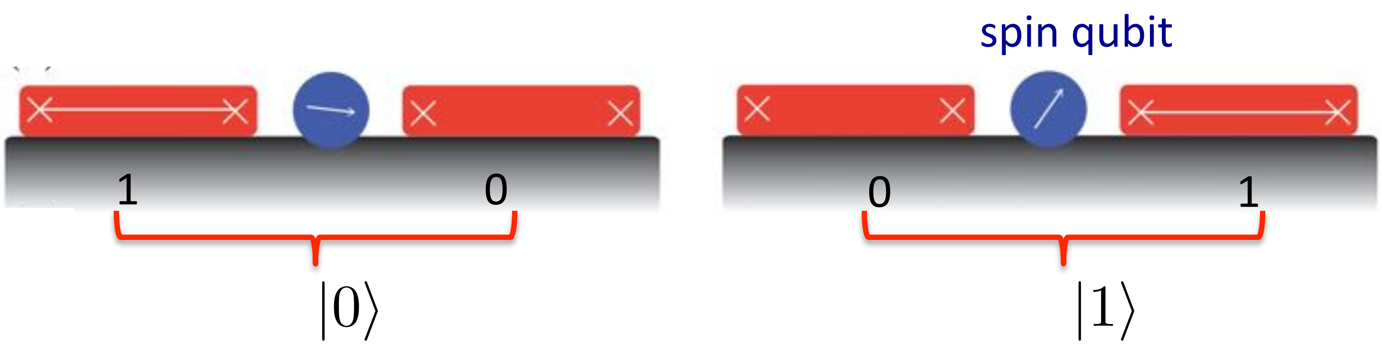


odd parity
MF qubit

delocalized fermion:

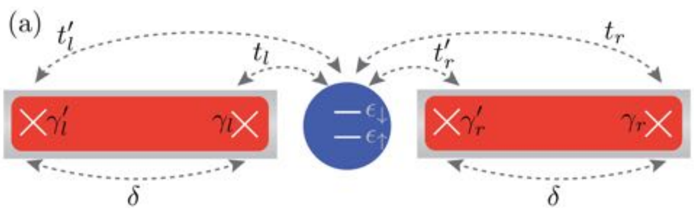
$$f_\nu = (\gamma'_\nu + i\gamma_\nu)/2 \quad \left\{ \begin{array}{l} 0 \\ 1 \end{array} \right.$$

degenerate states
‘topological protection’

perform perturbation expansion in tunneling t between dot and wires…

Universal Quantum Computation with Hybrid Spin-Majorana Qubits

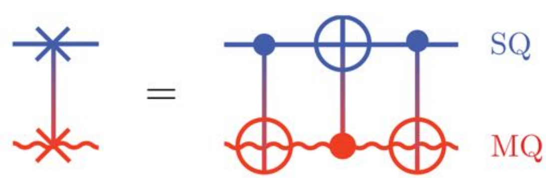
Hoffman, Schrade, Klinovaja, and DL, PRB 94, 045316 (2016)



Majorana Spin hybrid (\`MaSh’) qubit

$$\mathcal{H}_{\mathrm{hCP}} = 2|t|^2(\mathbb{1} + \sigma_1)(\mathbb{1} + \eta_1)/\epsilon_0$$

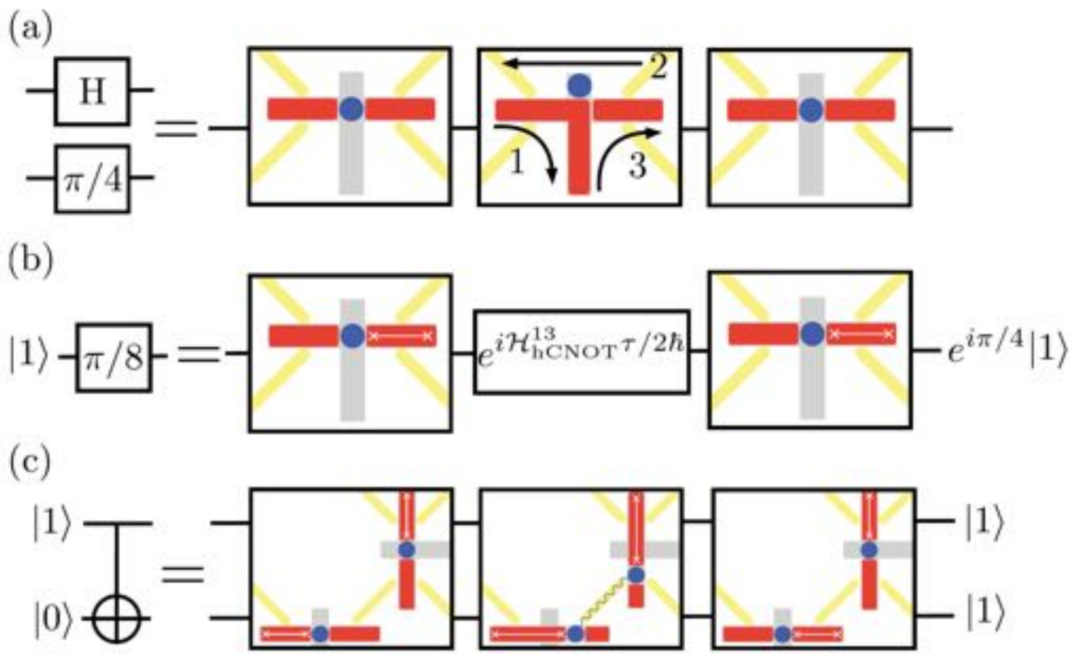
hybrid SWAP gate:



➔ coherent ‘quantum state transfer’ (faster than projective measurement)
but gate is noisy ➔ need to control it down to < 1% for surface code

Universal set of gates for Majorana Qubits via Spin Qubit

MF qubit



CNOT

braiding (topological) gates

noisy gates

(need: noise < 1%)

$$\mathcal{H}_{\mathrm{MQ}}^{(12)} = \frac{|t|^4}{\epsilon_0^2 \mathcal{J}}[\sigma_2^{(1)}\sigma_2^{(2)} + \sigma_3^{(1)}\sigma_3^{(2)}][1-\eta_1^{(1)}\eta_1^{(2)}].$$

qubit-qubit coupling via floating gates

Clifford gates: {H, π/4, CNOT}

Hoffman et al., PRB 94, 045316 (2016)

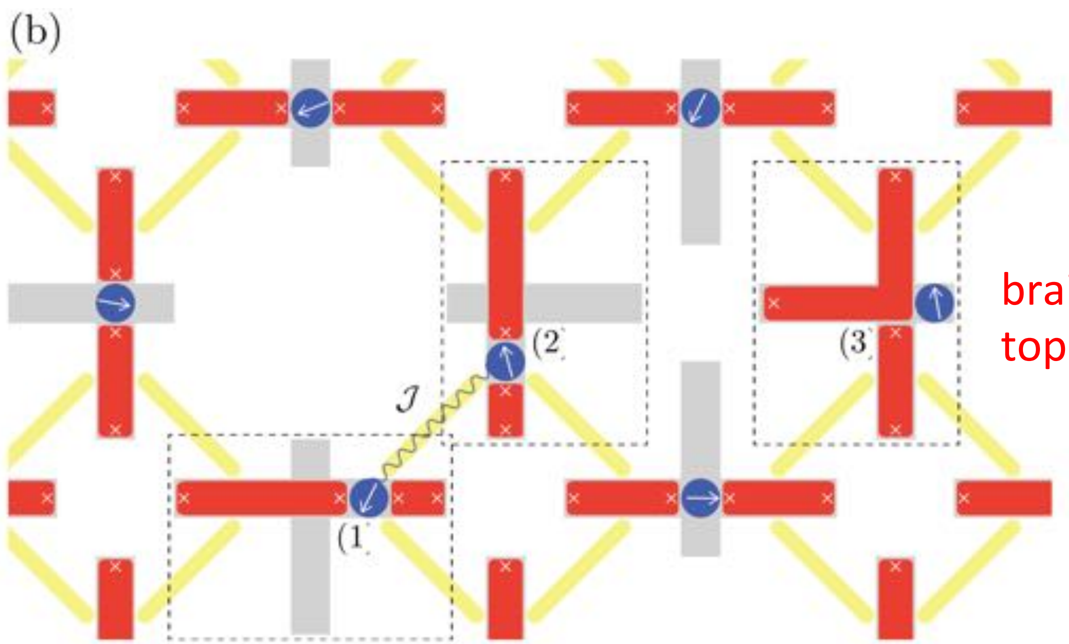
2D Surface code built from topological T-junctions

Hoffman, Schrade, Klinovaja, and DL, PRB 94, 045316 (2016)

Majorana Spin hybrid (MaSh) qubit = T-junction nanowire with spin ½ dot

spin rotation (π/8)
(non-Clifford gate)

noisy



braiding (H & π/4),
topological

qubit-qubit coupling via floating gates
and spin exchange *J* (CNOT)

$$\mathcal{H}_{MQ}^{(12)} = \frac{|t|^4}{\epsilon_0^2 \mathcal{J}} \left[\sigma_2^{(1)}\sigma_2^{(2)} + \sigma_3^{(1)}\sigma_3^{(2)}\right] \left[1 - \eta_1^{(1)}\eta_1^{(2)}\right]$$

What is beyond Majorana femions?

`Parafermion’

(= *fractional Majorana fermion* zero-energy bound state)

$$\gamma^n = 1$$

$$\gamma\gamma' = \gamma'\gamma e^{-2i\pi/n}, \quad \mathrm{n}{=}2{,}3{,}\ldots$$

$$[H, \gamma] = 0$$ zero mode

Majorana (n=2): can get 2 out of 4 universal quantum gates by braiding

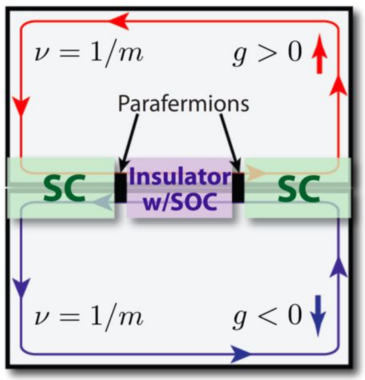
Parafermion (n>2): 2 & weak entanglement*. But: CNOT? (no phase gate)

Fibonacci anyon: 4 universal

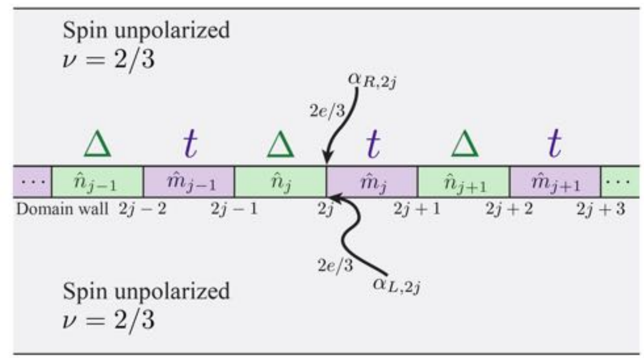
*) D. Clarke, J. Alicea, and K. Shtengel, Nat. Commun. 4, 1348 (2013)

Parafermions from QHE edge states

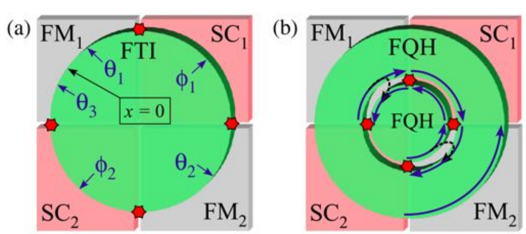
hybrid between QHE and SC



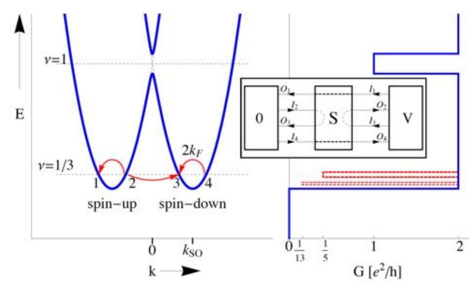
D. Clarke, J. Alicea, and K. Shtengel, Nat. Commun. 4, 1348 (2013)



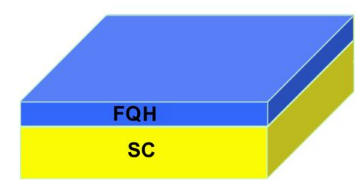
R. Mong, D. Clarke, J. Alicea, N. Lindner, P. Fendley, C. Nayak, Y. Oreg, A. Stern, E. Berg, K. Shtengel, and M. P. A. Fisher, Phys. Rev. X 4, 011036 (2014).



N. Lindner, E. Berg, G. Refael, and A. Stern, Phys. Rev. X 2, 041002 (2012)



Y. Oreg, E. Sela, and A. Stern, Phys. Rev. B 89, 115402 (2014).

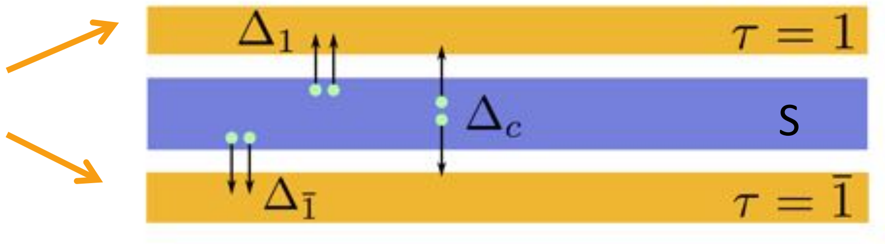


A. Vaezi, PRX 4, 031009 (2014)

Double Rashba wire + superconductor

Klinovaja and DL, PRL 112, 246403 (2014) & PRB 90, 045118 (2014)

unequal SOI



no B-field !

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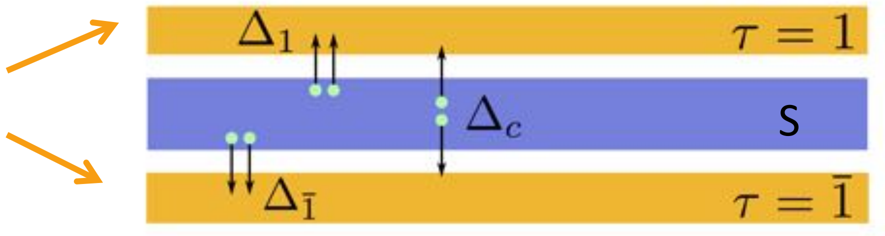
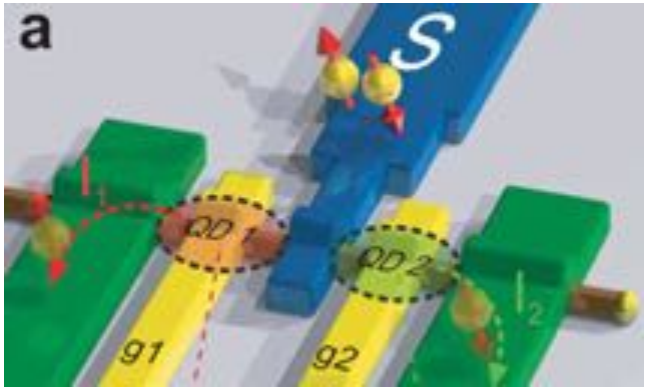


no B-field !

'Cooper pair splitter' physics

Recher, Sukhorukov & DL, PRB 63, 165314 (2001): quantum dots

Recher & DL, PRB 65, 165327 (2002) : Luttinger wires



Hofstetter et al., Nat. 461, 960 (2009)
Das et al., Nat. Comm. 3, 1165 (2012)
Deacon et al., Nat. Comm. 6, 7446 (2015)

Crossed Andreev Pairing vs. Direct Pairing

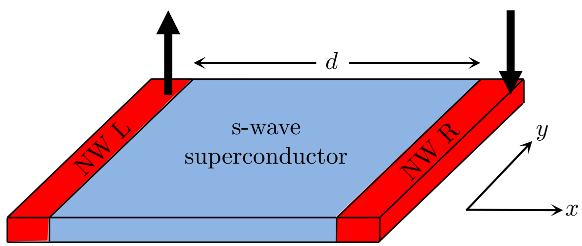
C. Reeg, J. Klinovaja and D. Loss, arXiv:1701.07107

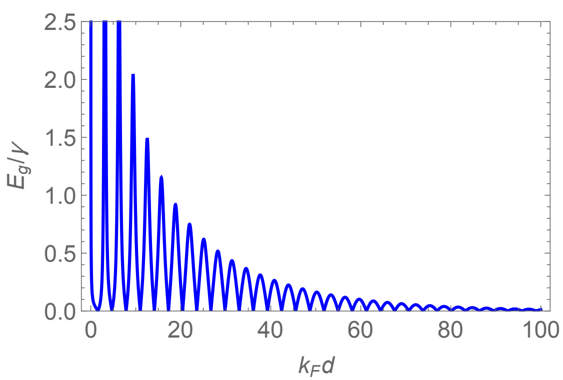
how to distinguish between direct and crossed Andreev contributions?

Direct $\propto t_i^2 \propto \gamma_i$

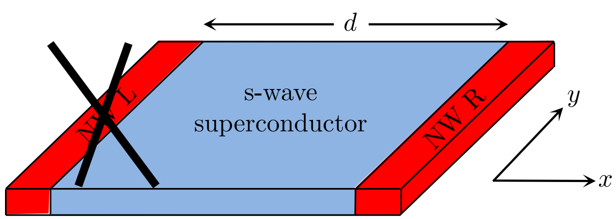
Crossed Andreev $\propto t_L t_R \propto \sqrt{\gamma_L \gamma_R}$

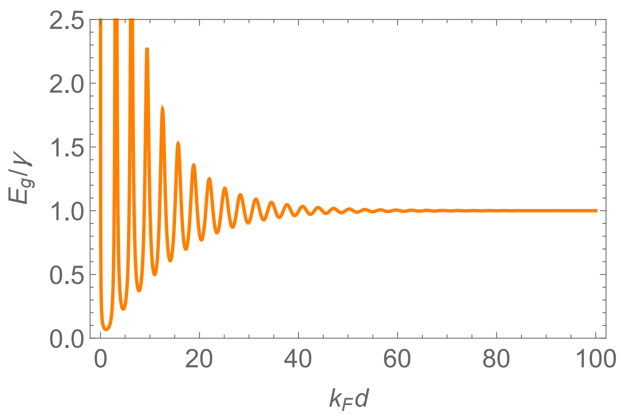
- Crossed Andreev pairing: spin-polarized wires (quantum Hall edge states)





- Direct pairing: decouple one wire





Interplay Between Direct and Crossed Andreev

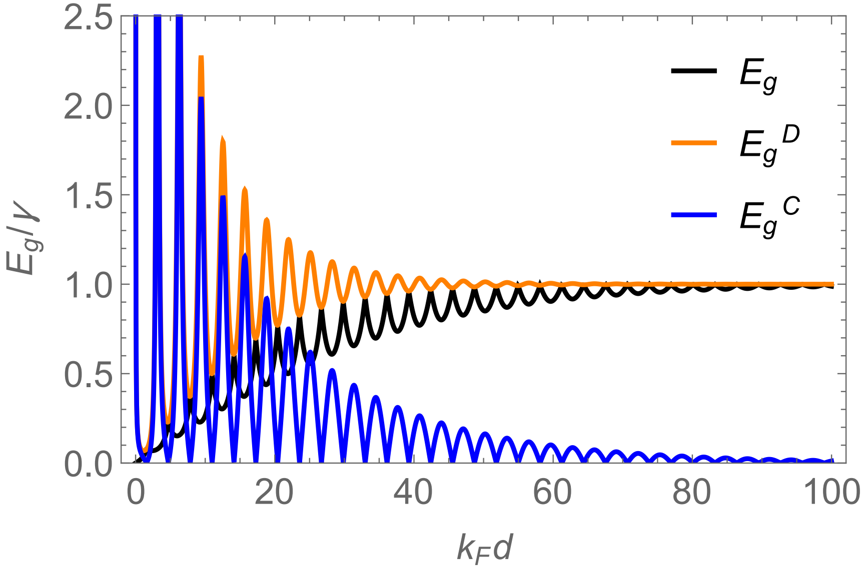
C. Reeg, J. Klinovaja and D. Loss, arXiv:1701.07107

Comparing three gaps (weak coupling):

$$E_g(d) = \frac{\gamma \sinh(d/\xi_s)}{\cosh(d/\xi_s) + |\cos k_F d|}$$

$$E_g^D(d) = \frac{\gamma \sinh(2d/\xi_s)}{\cosh(2d/\xi_s) - \cos 2k_F d}$$

$$E_g^C(d) = \frac{2\gamma \sinh(d/\xi_s)}{\cosh(2d/\xi_s) - \cos 2k_F d} |\cos k_F d|$$



$$E_g(d) = E_g^D(d) - E_g^C(d)$$

Direct and crossed Andreev pairing interfere destructively

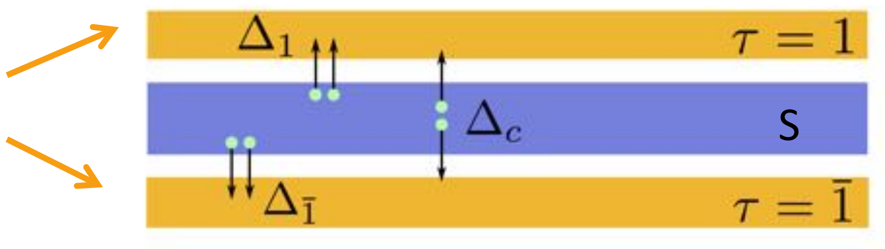
Proximity-induced gap is minimized when pairing is maximized

Caveat: the standard method of integrating-out SC gives wrong results in general!

Double Rashba wire + superconductor

Klinovaja and DL, PRL 112, 246403 (2014) & PRB 90, 045118 (2014)

unequal SOI



no B-field !

Consider inter- and intra-wire pairing

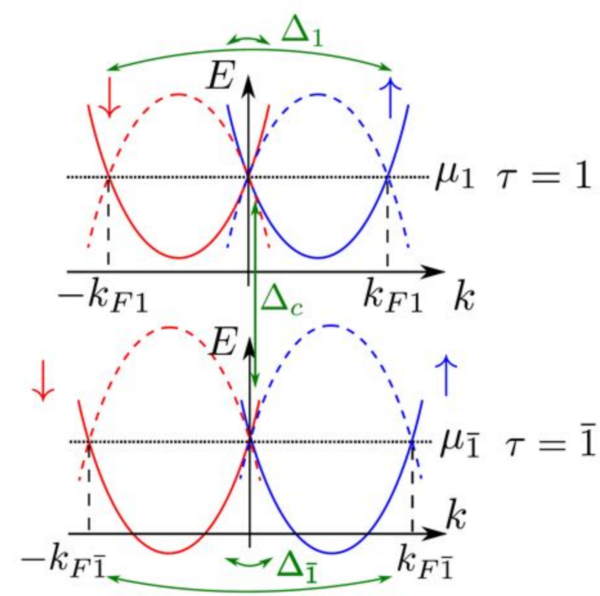
‘crossed Andreev’ term Δ_c usual Δ_τ

➔ *Two competing gap mechanisms*

$$\Delta_c^2 > \Delta_1 \Delta_{\bar{1}}$$

due to interactions

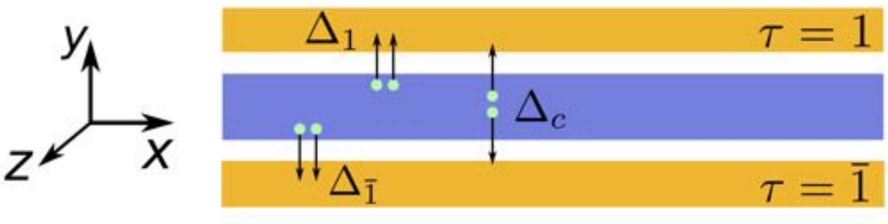
Recher & DL,
PRB 65, 165327 (2002)



➔ Kramers pairs of Majoranas in crossed Andreev dominated regime

Parafermions: need interactions!

Klinovaja and DL, PRB 90, 045118 (2014)



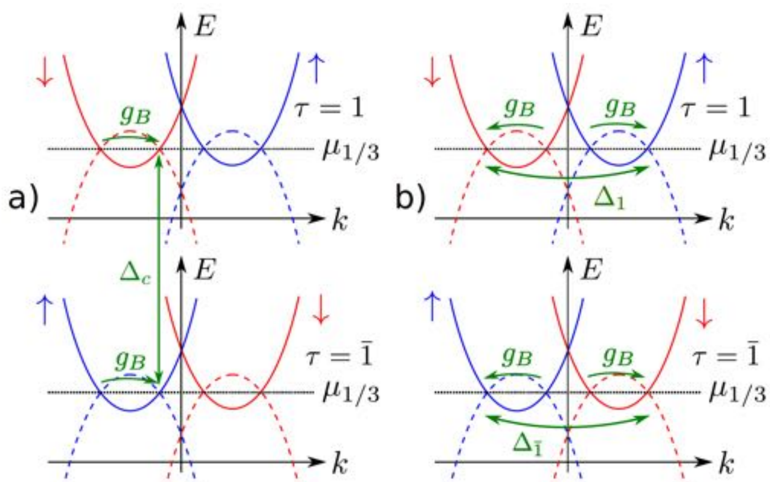
unequal SOI

no B-field !

Consider inter- and intra-wire pairing *plus* e-e interactions (g_B)

'crossed Andreev' term Δ_c

usual Δ_τ

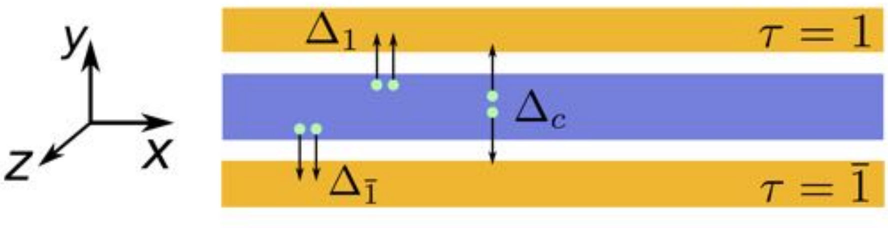


$$\mu_{1/3} = E_{so}/3^2$$

4 Fermi points: $\pm k_{so}(1 \pm 1/3)$

Parafermions: need interactions!

Klinovaja and DL, PRB 90, 045118 (2014)



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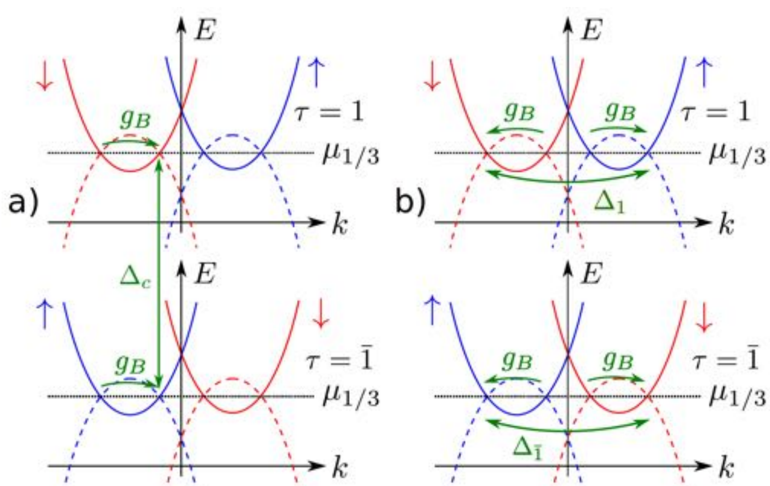


4 Fermi points: $\pm k_{so}(1 \pm 1/3)$

Note: gap opens only due to interactions

➔ fractionalized excitations

➔ parafermions at boundaries

Topological Stability of Parafermion Phase ?

- Parafermions in double wire emerge only due to strong interactions; without interactions the system is gapless.

- topological stability of ground state…?

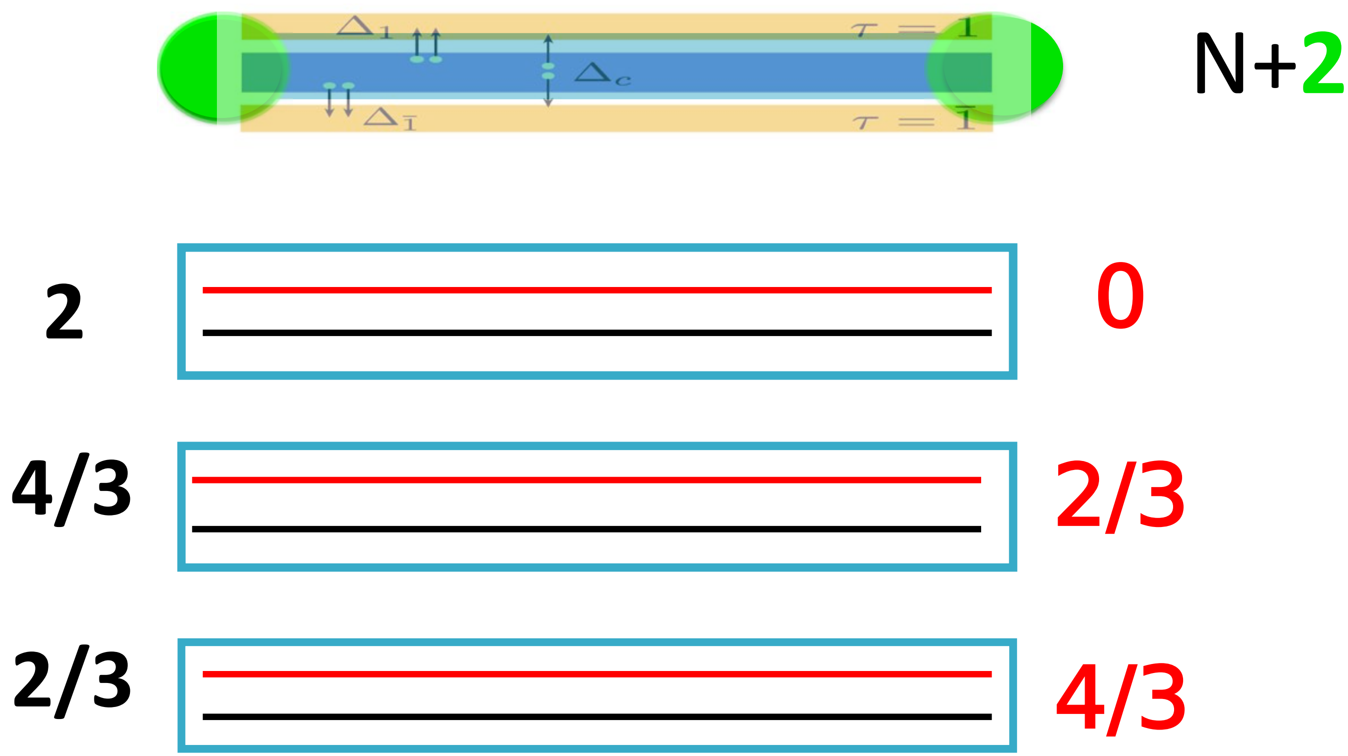
  Klinovaja, Mandal, Simon, DL

- Fundamentally different case: start from system with pairing gap Δ and then add weak interactions g such that gap does not close ($g \ll \Delta$)
  ➔ only topological phases with Majorana fermions are allowed

  Fidowsky & Kitaev 2010/11
  Turner, Berg & Pollmann 2011

Two Z_3-Parafermions

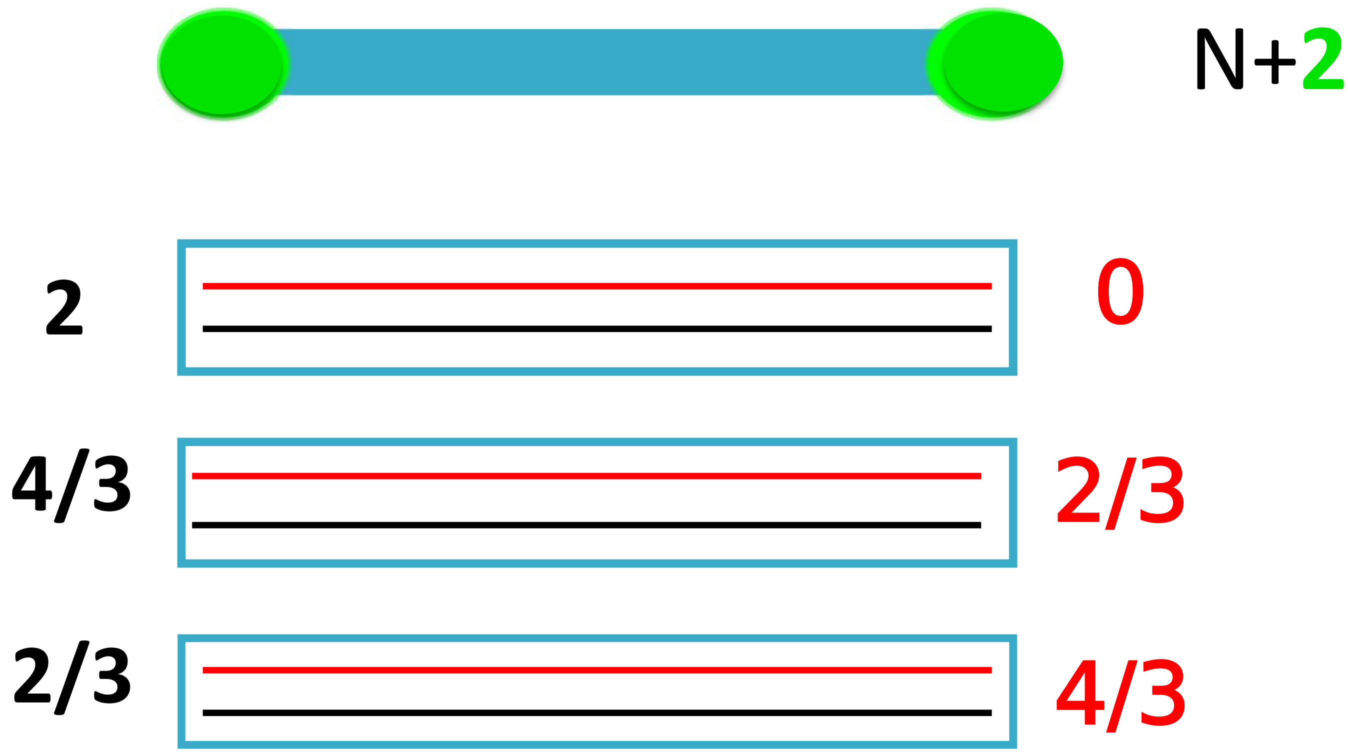
3-fold degeneracy (fractional 'charges'): 0, 2/3, 4/3 (mod 3)



Note: assume that only the spin-up Kramers partner can be occupied (and the spin-down Kramers partner is always empty)

Two Z_3-Parafermions

3-fold degeneracy (fractional 'charges'): 0, 2/3, 4/3 (mod 2)



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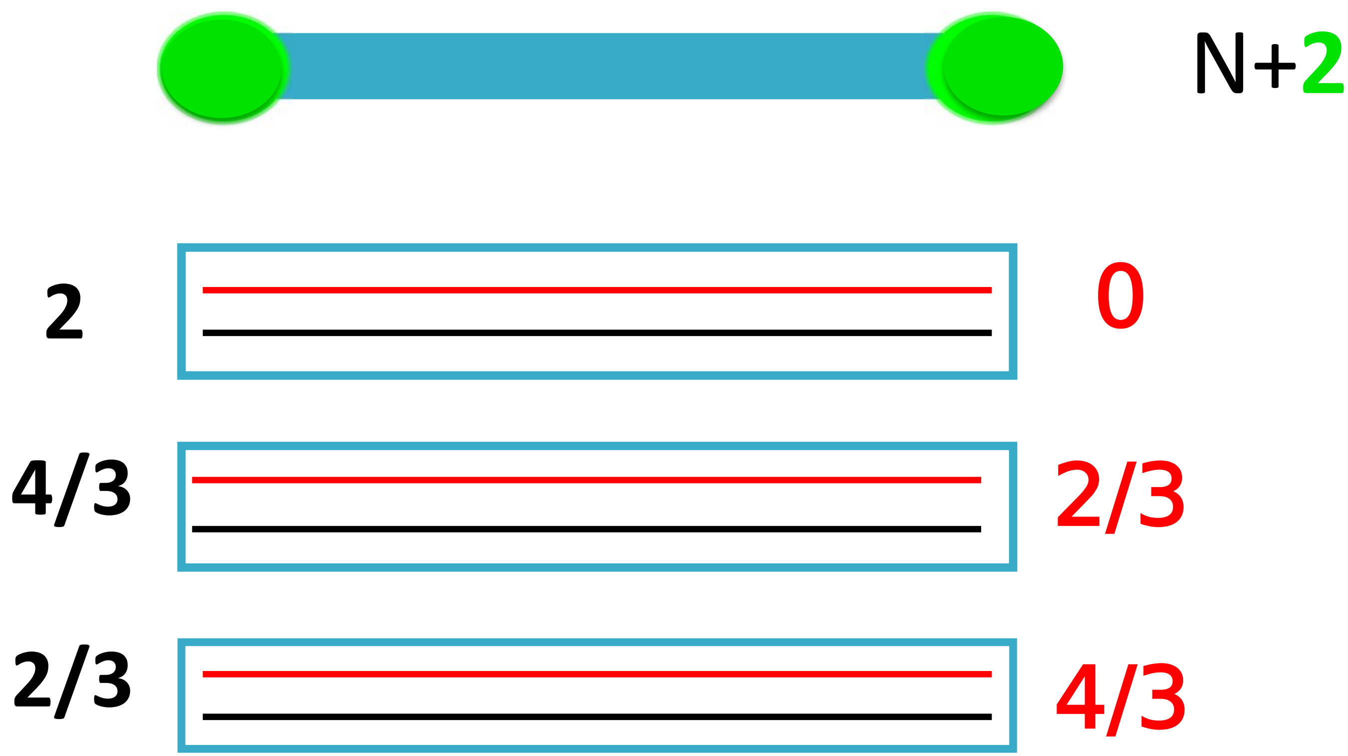
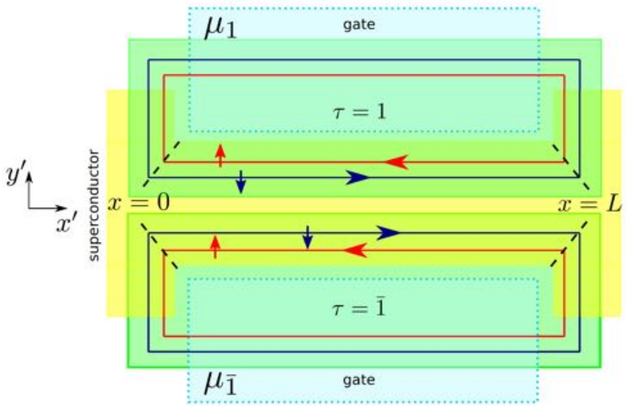


use the 3 degenerate states as a qu-trit (3-level system)

4 PFs= 1 qutrit: $|0>_L = |0;0>, |1>_L = |2/3;4/3>, |2>_L = |4/3;2/3>$

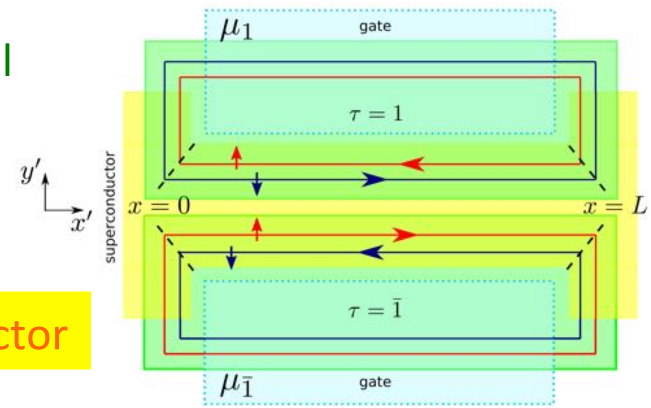
Kramers Pairs of Majorana Fermions and Parafermions *in Fractional Topological Insulators*

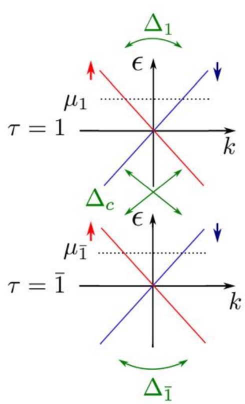
Klinovaja, Yacoby, and DL, PRB 90, 155447 (2014)

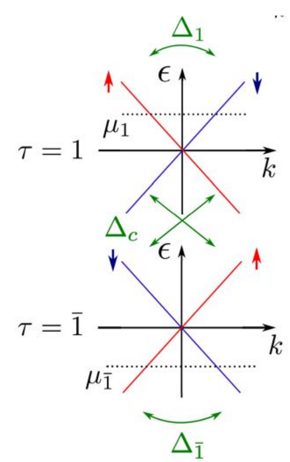


topological insulator

superconductor







Bound states at interfaces between regions of dominant crossed Andreev and usual superconducting pairing terms

Braiding of Z_d Parafermions for TQC

Hutter and DL, PRB 93, 125105 (2016)

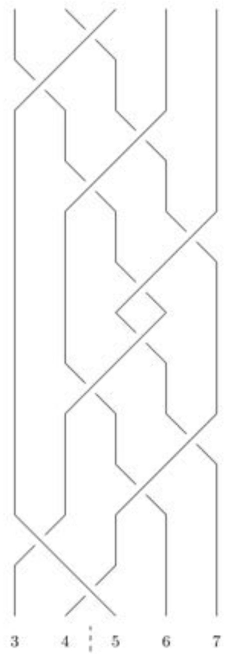


FIG. 1. Illustration of the braid S which acts like $(C_X)^{-2}$ on the computational subspace. Time flows upwards. The small dashed line shows the separation between logical qudits A and B.

Theorem 2. *If d is odd, braiding of $\mathbb{Z}_d$ parafermions allows one to generate the entire many-qudit Clifford group (up to phases).*

(missing non-Clifford T-gate for d odd well-studied)

CNOT gate C_X:

$$\mathcal{T}(S^{-(d+1)/2}) = C_X \, .$$

d odd

4 PFs = 1 qudit

e.g. d=3 in parity-0 sector, the logical qutrit becomes:

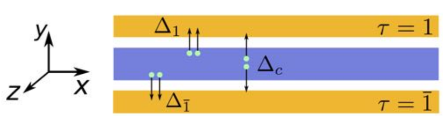
$$|0>_L = |0;0>, |1>_L = |2/3;4/3>, |2>_L = |4/3;2/3>$$

For d=3 we need S^2, i.e. 2x12=24 braidings for one CNOT gate

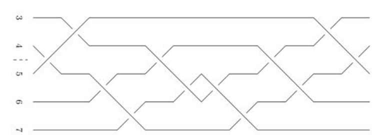
[See also proof by PF diagrammatics, Jaffe, Liu, and Wozniakowski, arXiv:1602.02671]

Summary

- Topological quantum computing
- Majorana fermions in 'super-semi' nanowires and chains
- Majorana and spin qubits ➔ 2D surface code
- Next generation: Parafermions in double nanowires
  ➔ can get nearly universal TQC including CNOT



Klinovaja and DL, PRB 90, 045118 (2014)



Hutter and DL, PRB 93, 125105 (2016)