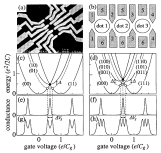
Spin Qubits in Semiconducting Nanostructures II

Daniel Loss
Department of Physics
University of Basel

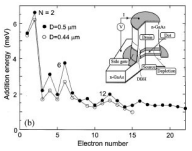
$$: Swiss NSF, Nano Basel, Quantum ETH/Basel, EU

Spin qubits in **GaAs** quantum dots

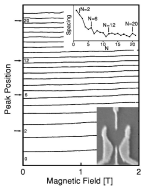
Kloeffel & DL, Annu. Rev. Condens. Matter Phys. 4, 51 (2013)



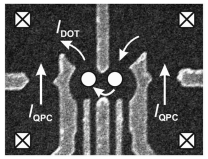
Westervelt
Gossard 1995



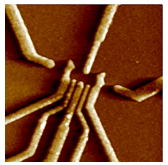
Kouwenhoven
Tarucha 1996



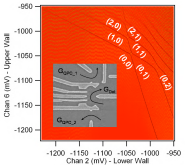
Sachrajda
2000



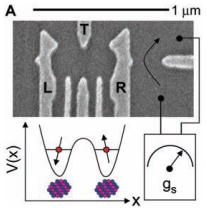
Kouwenhoven
Tarucha 2003-13



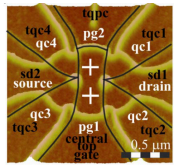
Vandersypen,
Koppens, 2003



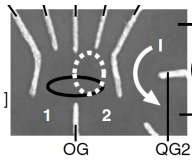
Marcus 2004



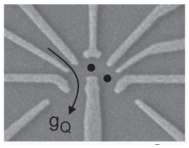
**Petta, Marcus,
Yacoby 2005**



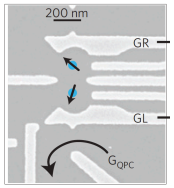
Ensslin, Ihn
2006



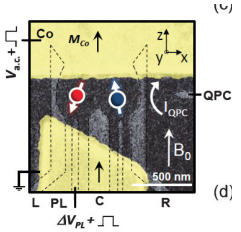
Zumbuhl,
Kastner
2008



Petta 2010



Bluhm, Dial,
Yacoby 2010-13
T_2 ~ 300 μs



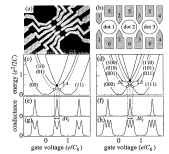
Brunner,
Pioro-Ladriere,
Tarucha 2011

… and many more …

Spin qubits in **GaAs** quantum dots

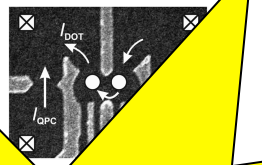
& DL, Annu. Rev. Condens. Matter Phys. 4, 51 (2013)

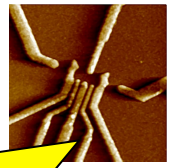
GaAs: Nuclear Spin 'Problem'?



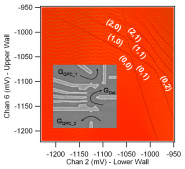
Westervelt
Gossard 1995

Ku
Taruch

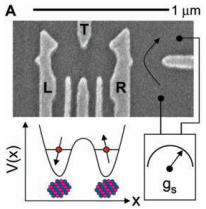




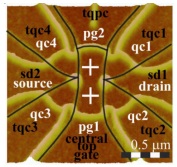
ns, 2003



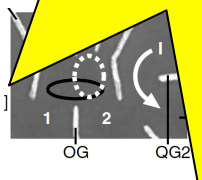
Marcus 2004



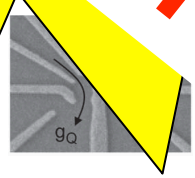
Petta, Marcus, Yacoby 2005



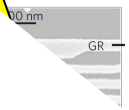
Ensslin, Ihn
2006



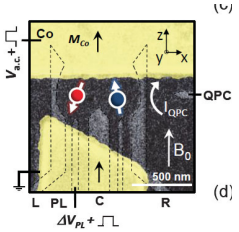
Zumbuhl,
Kastner
2008



Petta 2010



Bluhm, Dial
Yacoby 2010-
T_2 ~ 300 µs



er,
riere,
011

… and many more …

Strategies:

1. Use GaAs (still 'best' material for electrical control) and deal with nuclear spins

   ➔ echo techniques and/or dynamical narrowing

   Bluhm & Yacoby et al., Nat. Phys. 7, 109 (2011)

   Kuemmeth & Marcus et al., Nat. Nano 12, 16 (2017): T_2 ~1ms

   ➔ ordered nuclear state ➔ evidence in GaAs wire

   Zumbuhl & Yacoby, PRL 112, 066801 (2014)

2. Avoid nuclear spin problem: use holes and/or other materials such as isotopically purified C, Si, Ge,...

Electron *vs.* Hole Spin Qubits

Band Diagram Near Γ Point (k = 0)

Typical bulk spectrum of a semiconductor:

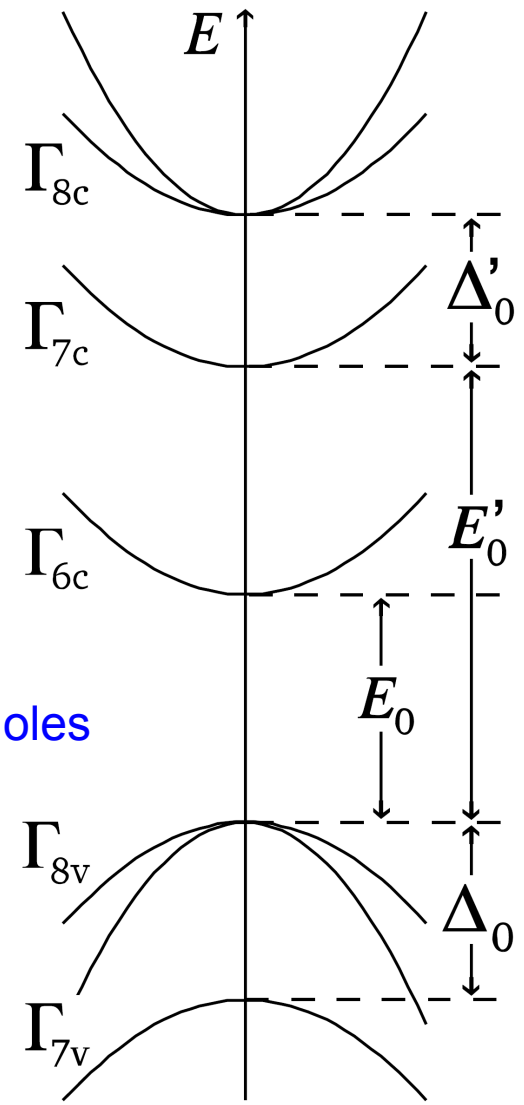
Higher bands →

Conduction band →
s-type Bloch functions

Spin 3/2, Heavy holes and light holes →

Valence band →
p-type Bloch functions

Split-Off Band →



R. Winkler (Springer, Berlin, 2003)

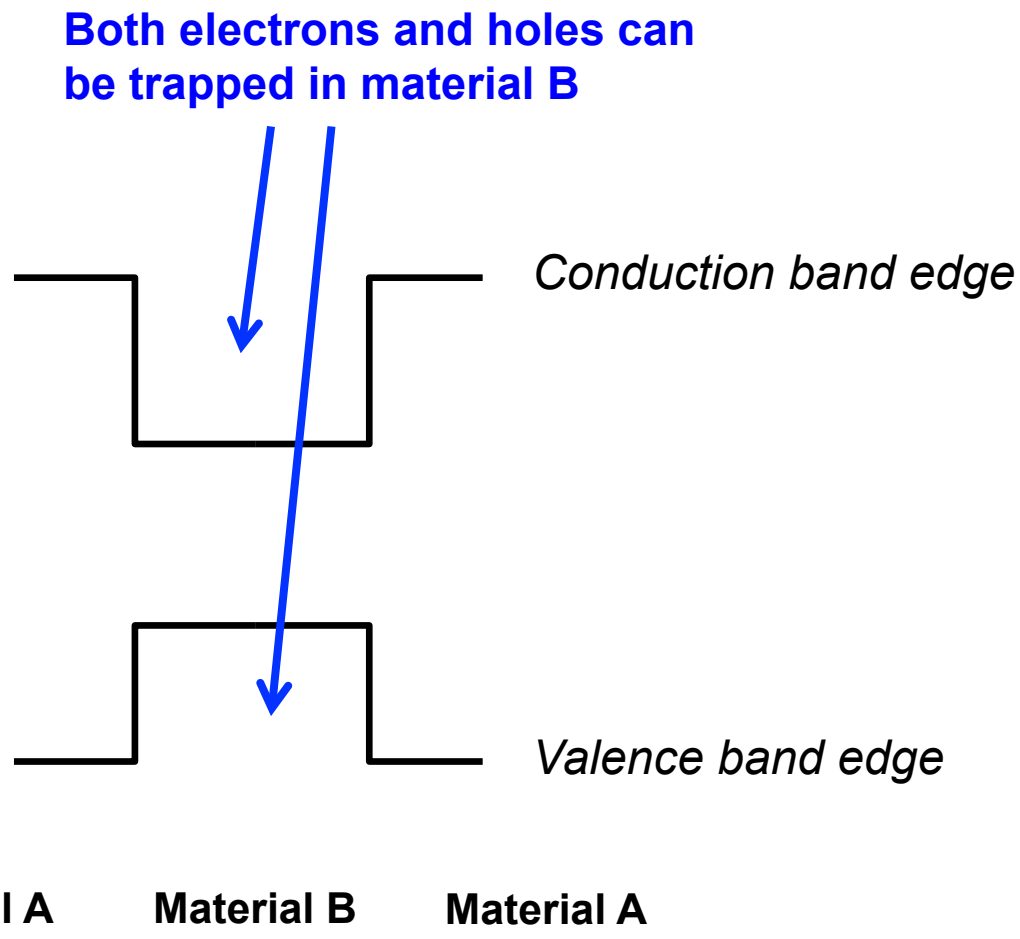
GaAs:

band gap
1.5 eV

0.3 eV

Quantum Well: Examples

Type I

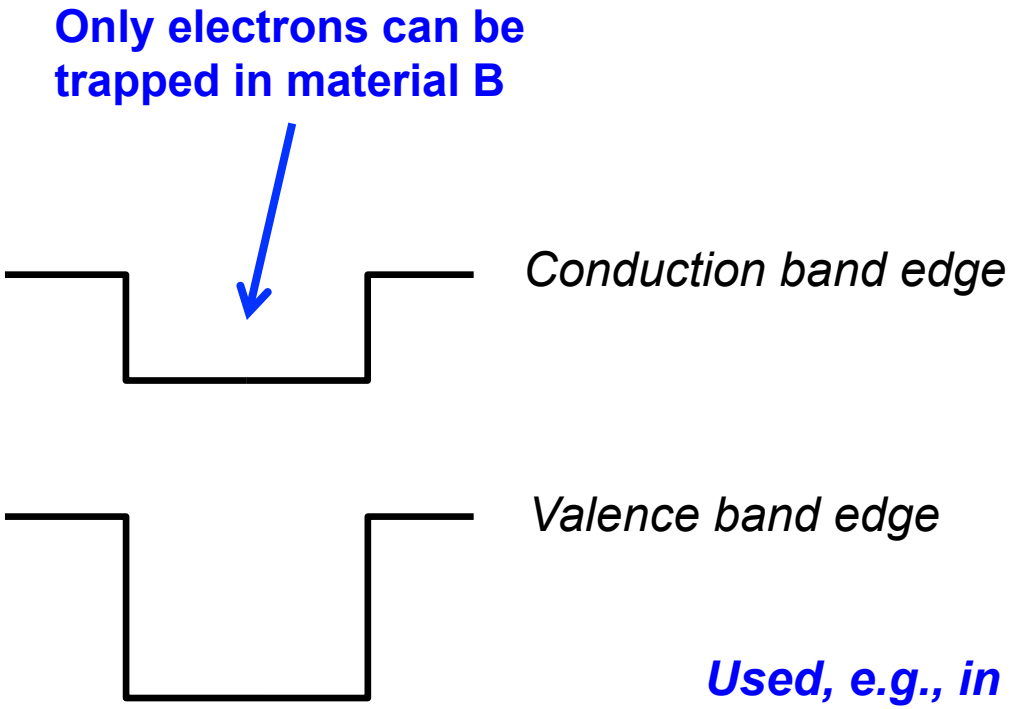


Example: **GaAs** **InGaAs** **GaAs**

Quantum Well: Examples

Type II

for electrons



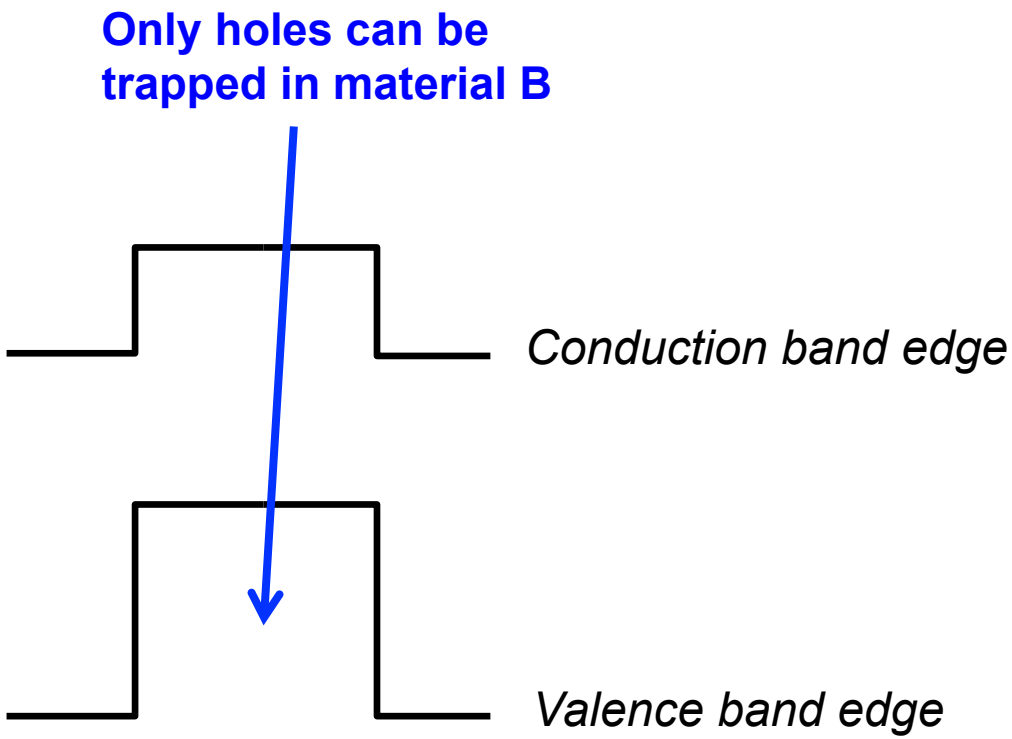
Used, e.g., in Si-SiGe heterostructures to form a 2DEG in Si

	Material A	Material B	Material A
Example:	**Ge**	**Si**	**Ge**

Quantum Well: Examples

Type II

for holes

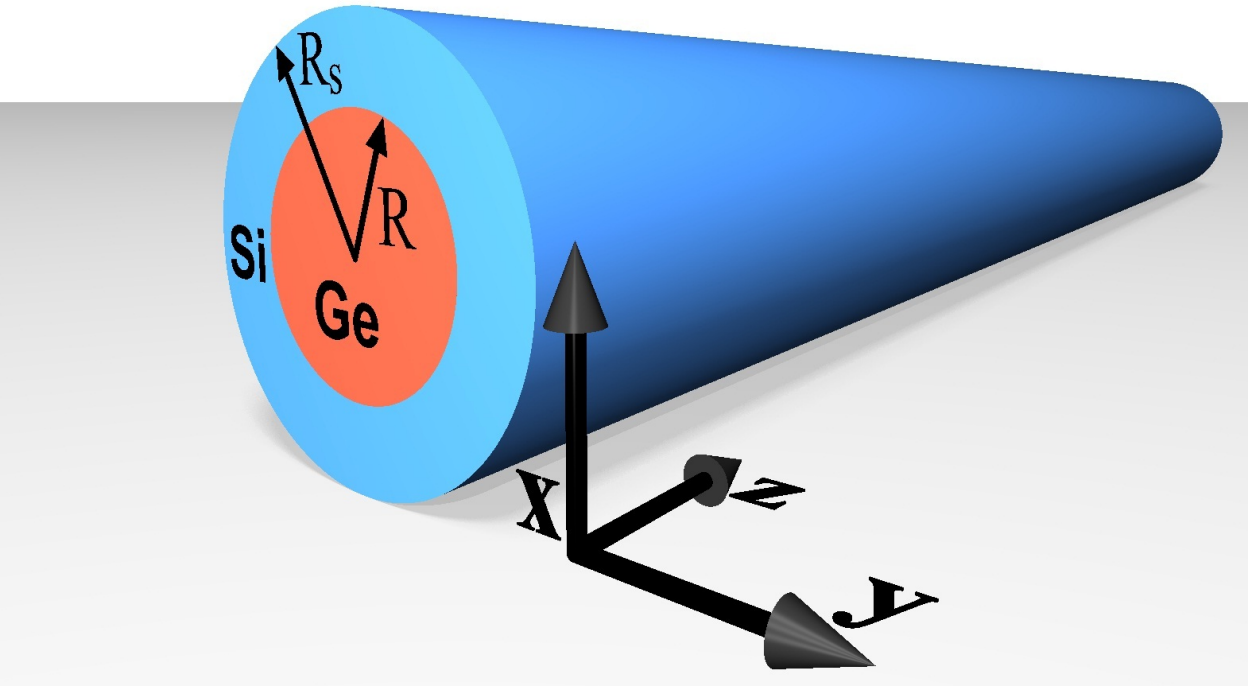


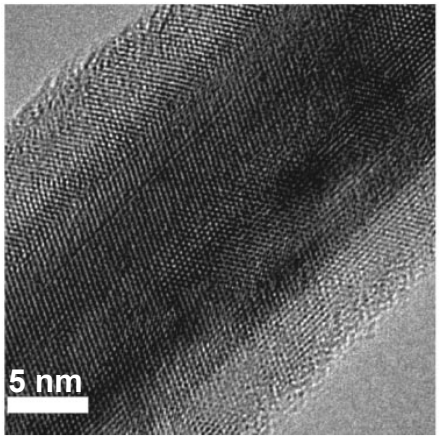
Used, e.g., in Ge-SiGe heterostructures to form a 2DHG in Ge

Material A **Material B** **Material A**

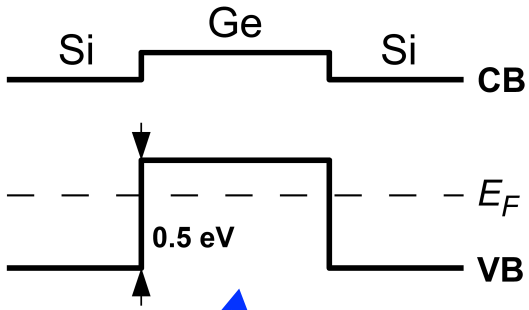
Example: **Si** **Ge** **Si**

Ge/Si Core/Shell Nanowires





HR-TEM, Lieber (2005)



Lauhon *et al.*, Nature (2002)
Lu *et al.*, PNAS USA (2005)
Kloeffel, Trif, and Loss, PRB (2011)
Hu *et al.*, Nature Nano (2012)
…

Strongly confined holes inside the Ge core

Hole Spin Qubits in GaAs and InAs

PRL **95,** 076805 (2005) PHYSICAL REVIEW LETTERS week ending 12 AUGUST 2005

Spin Relaxation and Decoherence of Holes in Quantum Dots

Denis V. Bulaev and Daniel Loss

Department of Physics and Astronomy, University of Basel, Klingelbergstrasse 82, CH-4056 Basel, Switzerland

(Received 8 March 2005; published 11 August 2005)

We investigate heavy-hole spin relaxation and decoherence in quantum dots in perpendicular magnetic fields. We show that at low temperatures the spin decoherence time is 2 times longer than the spin relaxation time. We find that the spin relaxation time for heavy holes can be comparable to or even longer than that for electrons in strongly two-dimensional quantum dots. We discuss the difference in the magnetic-field dependence of the spin relaxation rate due to Rashba or Dresselhaus spin-orbit coupling for systems with positive (i.e., GaAs quantum dots) or negative (i.e., InAs quantum dots) g factor.

$$H_{\mathrm{so}}^{\mathrm{HH}} = i\alpha(\sigma_+ P_-^3 - \sigma_- P_+^3) \qquad \text{Rashba SOI for HH}$$

$$- \beta(\sigma_+ P_- P_+ P_- + \sigma_- P_+ P_- P_+) \qquad \text{Dresselhaus SOI for HH}$$

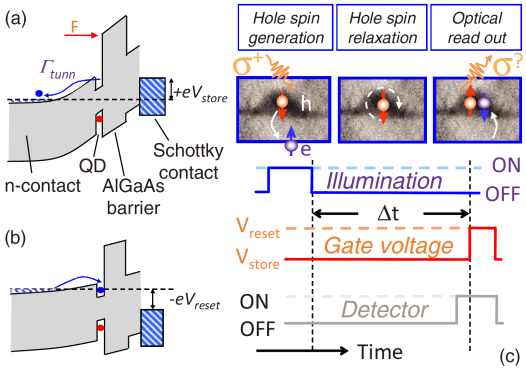
EDSR for HHs in III-V materials: Bulaev and DL, PRL 98, 097202 (2007)
ultrafast Rabi oscillations

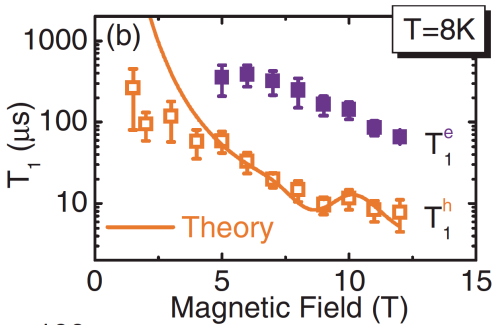
PHYSICAL REVIEW B **76**, 241306(R) (2007)

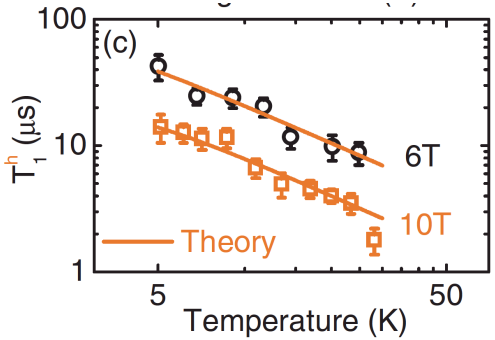
Observation of extremely slow hole spin relaxation in self-assembled quantum dots

D. Heiss, S. Schaeck, H. Huebl, M. Bichler, G. Abstreiter, and J. J. Finley*
Walter Schottky Institut, Technische Universität München, Am Coulombwall 3, D-85748 Garching, Germany

D. V. Bulaev and Daniel Loss
Department of Physics and Astronomy, University of Basel, Klingelbergstrasse 82, CH-4056 Basel, Switzerland
(Received 24 October 2007; published 21 December 2007)







PHYSICAL REVIEW B **76**, 241306(R) (2007)

Observation of extremely slow hole spin relaxation in self-assembled quantum dots

D. Heiss, S. Schaeck, H. Huebl, M. Bichler, G. Abstreiter, and J. J. Finley*
Walter Schottky Institut, Technische Universität München, Am Coulombwall 3, D-85748 Garching, Germany

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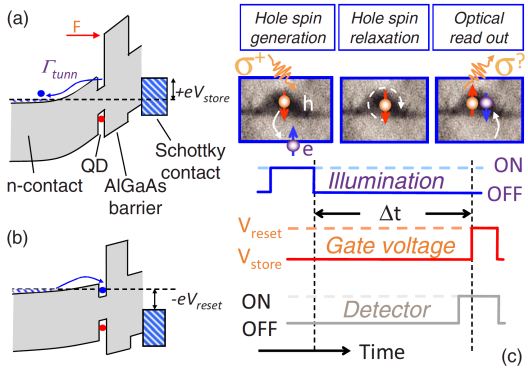
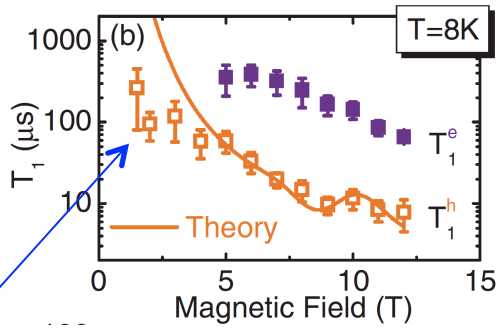
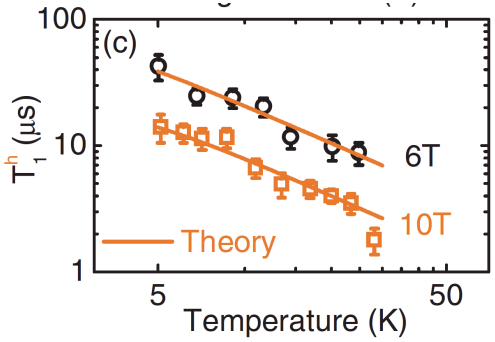
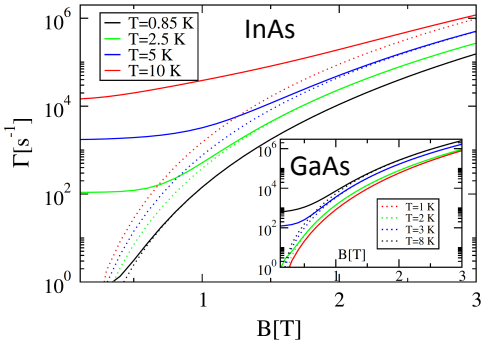
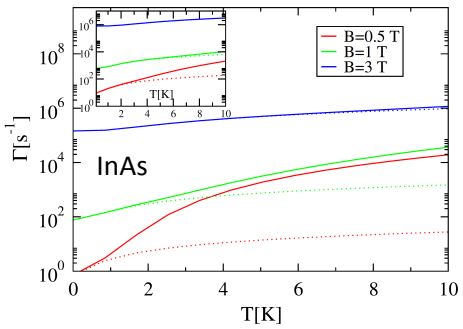






2-Phonon processes lead to saturation
M.Trif, P. Simon, and DL, PRL 103, 106601 (2009)





nature materials

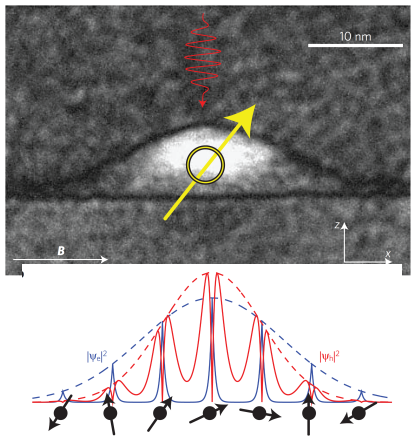
ARTICLES

PUBLISHED ONLINE: 25 JULY 2016 | DOI: 10.1038/NMAT4704

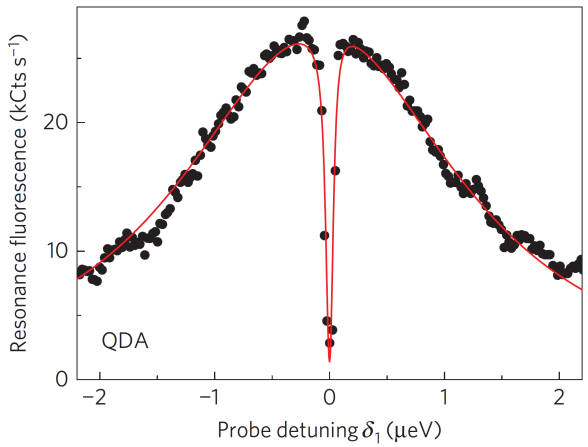
Nature Mat., 2016

Decoupling a hole spin qubit from the nuclear spins

Jonathan H. Prechtel[1], Andreas V. Kuhlmann[1], Julien Houel[1,2], Arne Ludwig[3], Sascha R. Valentin[3], Andreas D. Wieck[3] and Richard J. Warburton[1]*



InGaAs
self-assembled dots



Hyperfine interaction of holes is nearly Ising (transverse < 1%)

$$H = (b_\perp + h_z)s_z + b_\parallel s_x$$

b, $\Gamma_r = 0.68\ \mu\text{eV}$, $T_2 > 1\ \mu\text{s}$, $T_1 \gg T_2$.

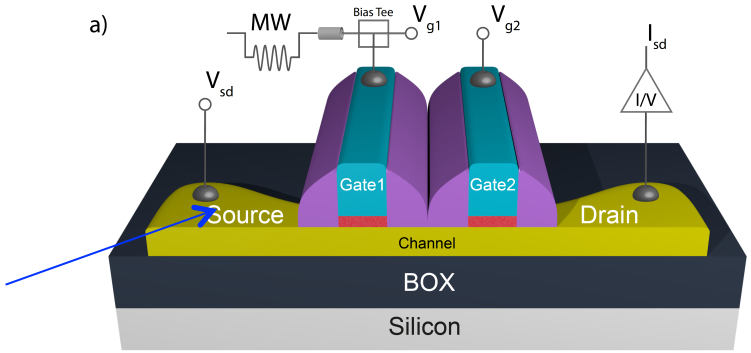
Theory: Fischer et al., PRB 78, 155329 (2008); Fischer and DL, PRL 105, 266603 (2010)

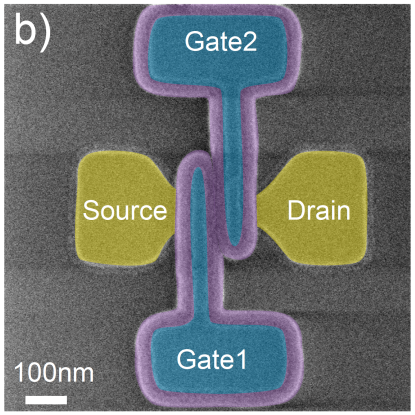
A CMOS silicon **hole** spin qubit

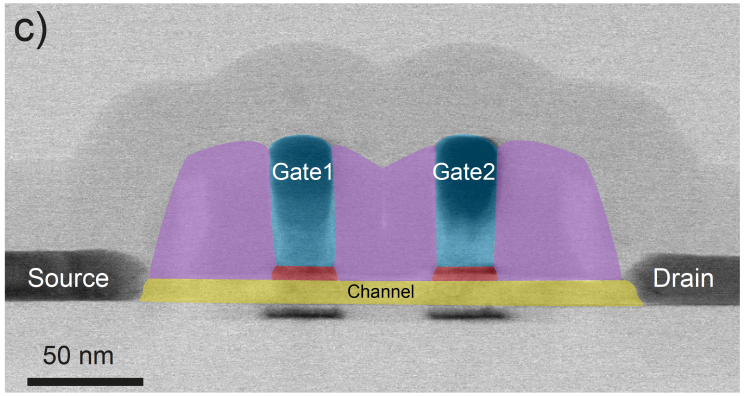
Maurand et al., Nat. Comm. 7, 13575 (2016)

'quantumize' integrated bits:

➔ heavy hole spin in p-doped Si-on-Insulator nanowire







~10 holes per dot

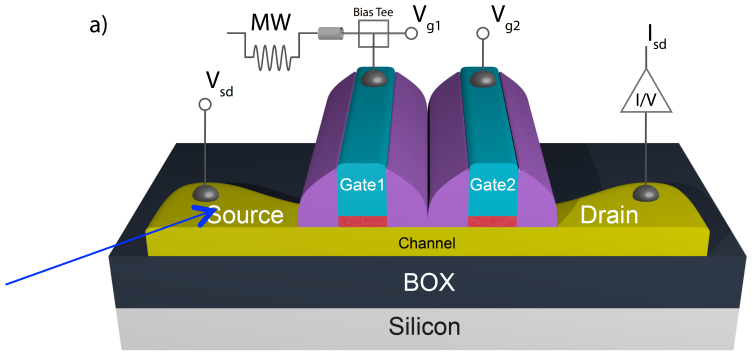
FIG. 1. CMOS qubit device. a, Simplified 3-dimensional schematic of a SOI nanowire field-effect transistor with two gates Gate 1 and Gate 2. Using a bias-T, Gate 1 is connected to a low-pass-filtered line, used to apply a static gate voltage Vg1, and to a 20-GHz bandwidth line, used to apply the high-frequency modulation necessary for qubit initialization, manipulation and readout. b, Colorized device top view obtained by scanning electron microscopy just after the fabrication of gates and spacers. c, Colorized transmission-electron-microscopy image of the device along a longitudinal cross-sectional plane.

CMOS silicon **hole** spin qubit: Fast EDSR

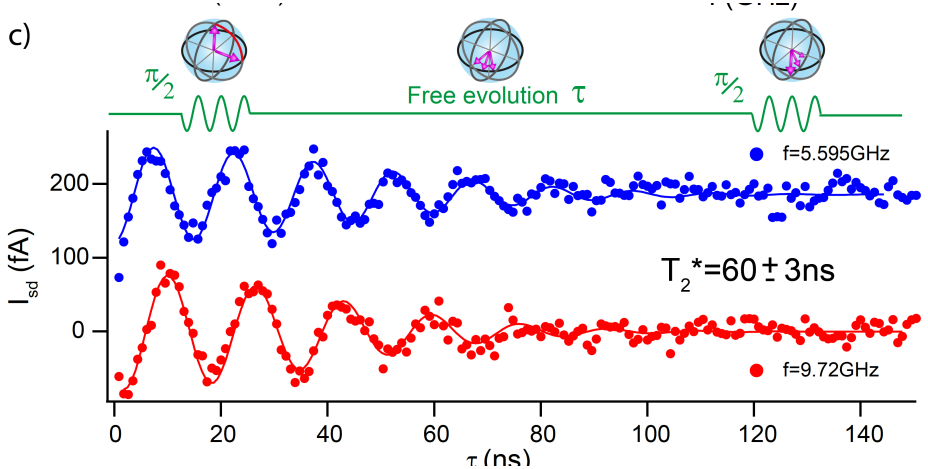
Maurand et al., Nat. Comm. 7, 13575 (2016)

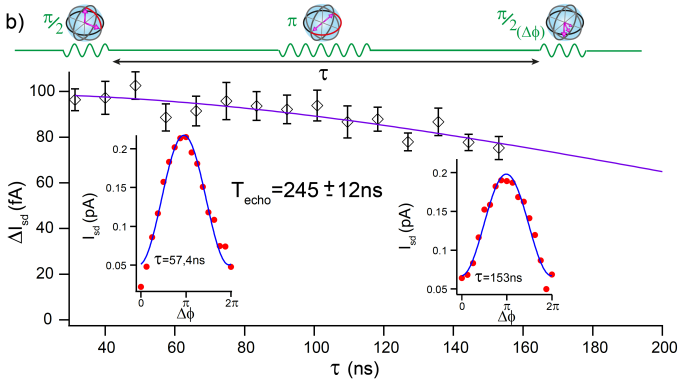
'quantumize' integrated bits:

➔ heavy hole spin in p-doped Si-on-Insulator nanowire



~10 holes per dot



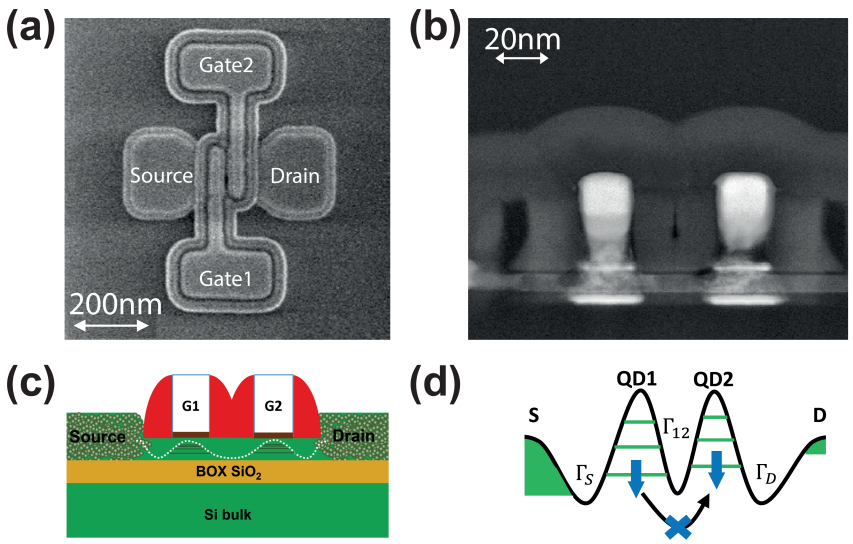


Fast Rabi oscillations ~85 MHz
and Ramsey fringe via EDSR of holes

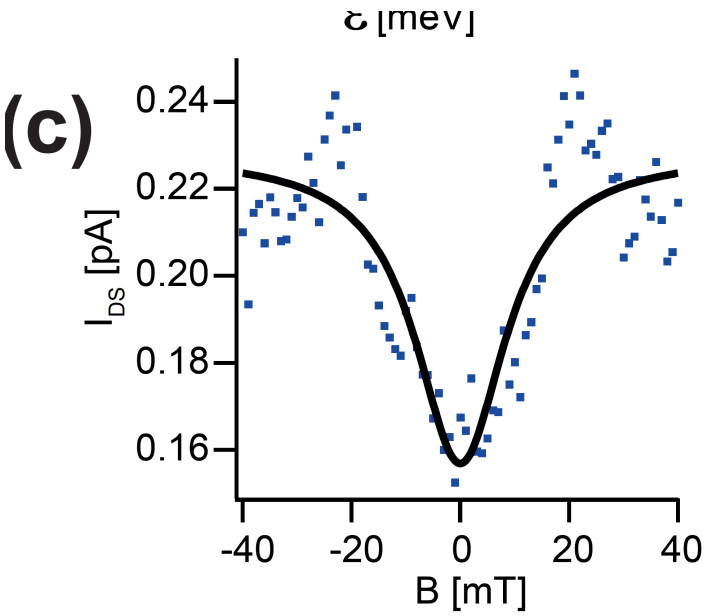
Two-axis qubit control and spin coherence times.

CMOS double dot and PSB

Sanquer & De Franceschi group, Bohuslavskyi et al., arXiv:1607.00287

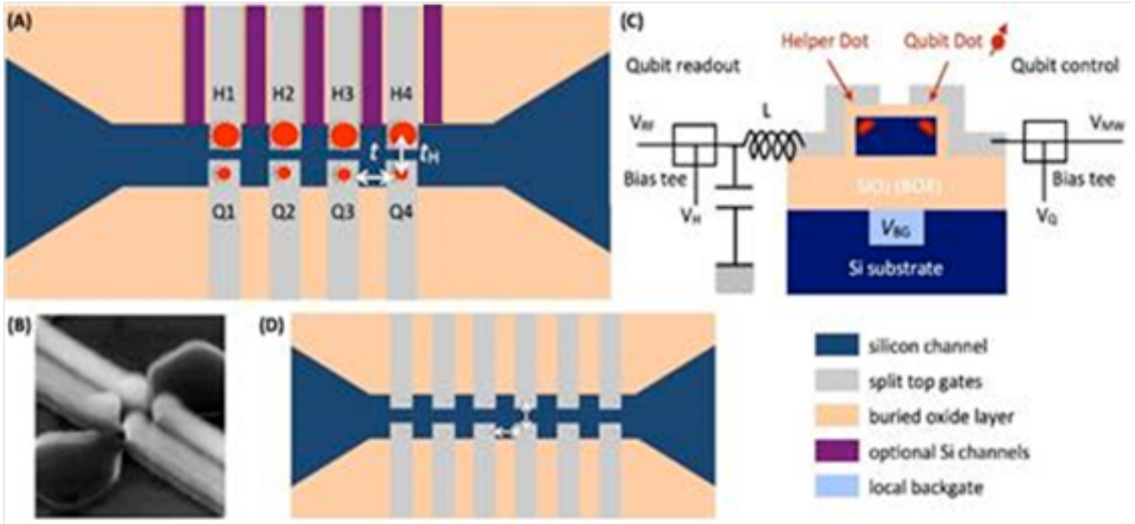


Pauli spin blockade due to spin orbit interaction



Next Step:
Spin Qubits in Industry-Standard SOI Technology

Marc Sanquer et al., 2017



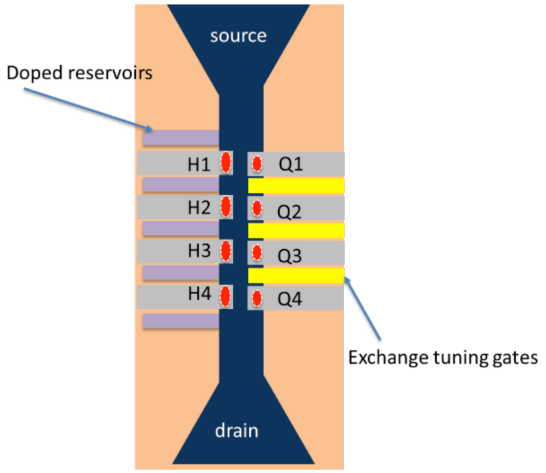


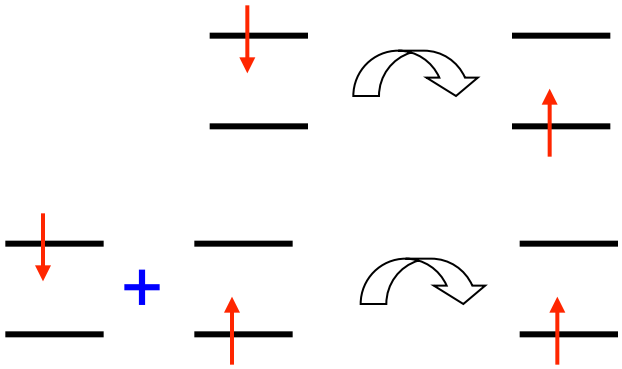
Figure 4: top view of the 4 qubits linear array.

Fig. 1. a) Schematic of the targeted 4x2 linear array of SOI quantum dots. b) SEM top view of current 2x2 split gate with gate spacing of 30 nm. c) Schematic cross-section showing the corner dots located under the slit gates. Their coupling can be tuned by using local back gate voltage (V_{BG}). The qubit dot is operated using microwaves (V_{MW}), whereas the helper dot is probed by rf signals (V_{RF}) for qubit readout. d) Scalable architecture envisioned based on our approach.

Spin decoherence in GaAs quantum dots

Two important sources of spin decay in GaAs:

1) Spin-orbit interaction (Dresselhaus & Rashba)
➔ interaction between spin and charge fluctuations



Relaxation with rate $1/T_1$

Decoherence with rate $1/T_2$
= decay of coherent superposition

2) Hyperfine interaction between electron spin and nuclear spins, can lead to non-exponential decay

General spin Hamiltonian:

$$H = g\mu_B \mathbf{S}\cdot\mathbf{B} + \mathbf{S}\cdot\mathbf{h}(t)$$

where **h**(t) is a fluctuating (internal) field with < **h**(t) >=0

Relaxation (T_1) and decoherence (T_2) times in weak coupling approx.:

$$\frac{1}{T_1} = \int_{-\infty}^{\infty} dt\,\mathrm{Re}\left[\left\langle h_X(0)h_X(t)\right\rangle + \left\langle h_Y(0)h_Y(t)\right\rangle\right]e^{-iE_Z t/\hbar}$$

$$\frac{1}{T_2} = \frac{1}{2T_1} + \int_{-\infty}^{\infty} dt\,\mathrm{Re}\left\langle h_Z(0)h_Z(t)\right\rangle$$

relaxation contribution

<< 'typically'

dephasing contribution

[if < $h_i(t)\, h_j(t')$ >~δ_{ij}, i,j=(X,Y,Z)]

See e.g. Abragam

General spin Hamiltonian:

$$H = g\mu_B \mathbf{S}\cdot\mathbf{B} + \mathbf{S}\cdot\mathbf{h}(t)$$

where **h**(t) is a fluctuating (internal) field with < **h**(t) >=0

For SOI **linear** in momentum: $\mathbf{h}(t)\cdot\mathbf{B} = 0$ (unlike spin-boson model!)

➔

$$\frac{1}{T_2} = \frac{1}{2T_1} + \int_{-\infty}^{\infty} dt\,\mathrm{Re}\langle h_z(0)h_z(t)\rangle$$

0

relaxation contribution

<< 'typically'

dephasing contribution

Golovach, Khaetskii, and DL, PRL 93, 016601 (2004)

Spin-Orbit Interaction in GaAs Quantum Dots (2DEG):

$$H_{SO} = \alpha\left(p_x\sigma_y - p_y\sigma_x\right) \quad \leftarrow \text{Rashba SOI}$$

$$-\beta\left(p_x\sigma_x - p_y\sigma_y\right) \quad \leftarrow \text{Dresselhaus SOI}$$

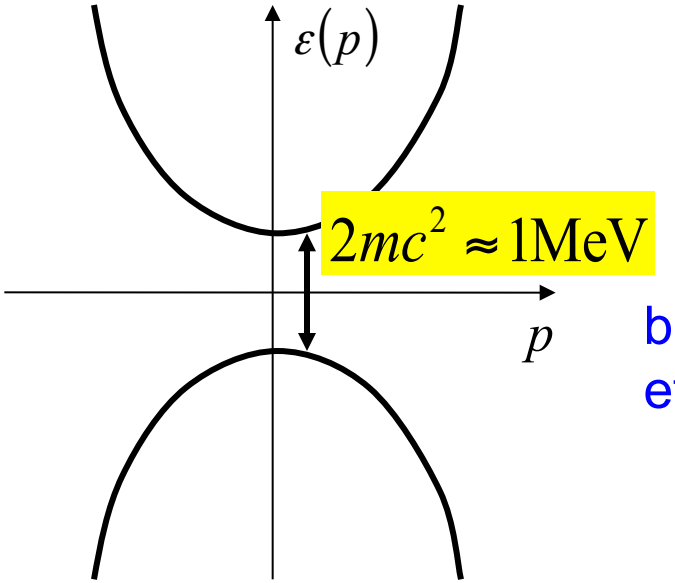
Basics on Spin-Orbit Interaction

Relativistic (Einstein) correction from Dirac equation:

$$H_{so} = \frac{1}{2mc^2}\vec{s}\cdot\left(\nabla V \times \frac{\vec{p}}{m}\right)$$

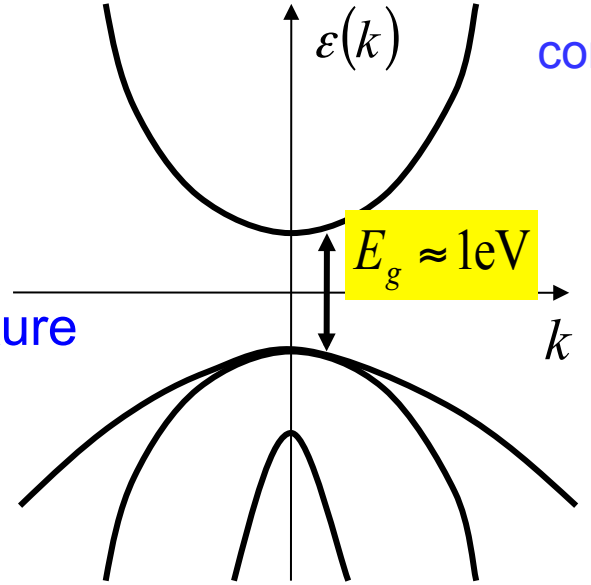
Thomas term (→ Rashba SOI)

free electrons



bandstructure effects

semiconductor (zinc-blende)



conduction band

heavy holes

light holes

Spin-Orbit Interaction in GaAs Quantum Dots (2DEG):

$$H_{SO} = \alpha\left(p_x\sigma_y - p_y\sigma_x\right) \quad \leftarrow \text{Rashba SOI}$$

$$-\beta\left(p_x\sigma_x - p_y\sigma_y\right) \quad \leftarrow \text{Dresselhaus SOI}$$

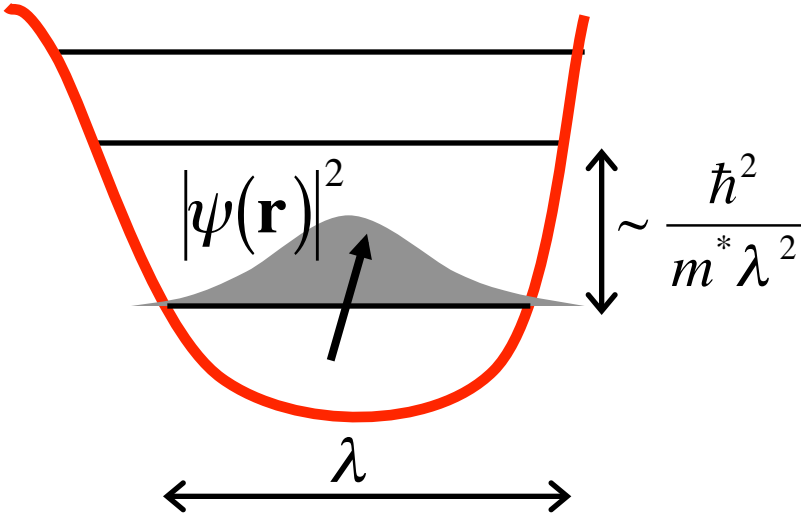
Model Hamiltonian:

$$H = H_{dot} + H_Z + H_{SO} + U_{el-ph}(t)$$

piezoelectric & deformation acoustic

$$H_{dot} = \frac{p^2}{2m^*} + U(\mathbf{r}/\lambda)$$

$$H_Z = \frac{1}{2} g\mu_B \mathbf{B}\cdot\sigma$$



$$U_{el-ph}(t) = \ldots \quad \leftarrow \text{any potential fluctuation, } \textit{e.g.}\text{, phonons}$$

Electron-phonon interaction (quasi-2D)

$$U_{el-ph} = \sum_{\mathbf{q},j} \frac{F(q_z) e^{i\mathbf{q}_\parallel \mathbf{r}}}{\sqrt{2\rho_c \omega_{qj}/\hbar}} \left(e\beta_{\mathbf{q}j} - iq\Xi_{\mathbf{q}j}\right)\left(b^+_{-\mathbf{q}j} + b_{\mathbf{q}j}\right)$$

- piezo-electric interaction:

$$\beta_{\mathbf{q}j} = \frac{2\pi}{q^2\kappa}\beta^{\mu\nu\varpi} q_\mu \left(q_\nu e^{(j)}_\varpi(\mathbf{q}) + q_\varpi e^{(j)}_\nu(\mathbf{q})\right) \qquad \mathbf{q} = \left(\mathbf{q}_\parallel, q_z\right)$$

- deformation potential interaction:

$$\Xi_{\mathbf{q}j} = \frac{1}{2q}\Xi^{\mu\nu}\left(q_\mu e^{(j)}_\nu(\mathbf{q}) + q_\nu e^{(j)}_\mu(\mathbf{q})\right)$$

for GaAs: $\Xi_{\mathbf{q}j} = \Xi_0 \delta_{j,1}$ and $\beta^{\mu\nu\varpi} = \begin{cases} h_{14}, & \mu\nu\varpi = xyz\,(cyclic) \\ 0, & \text{otherwise} \end{cases}$

Quantum well form-factor $F(q_z)$:

$$F(q_z) = \int dz\, e^{iq_z z} \left|\psi(z)\right|^2$$

- parabolic quantum well:

$$\psi(z) = \pi^{-1/4} d^{-1/2} e^{-z^2/2d^2} \quad \Rightarrow \quad F(q_z) = e^{-q_z^2 d^2/4}$$

- rectangular quantum well (0<z<d):

$$\psi(z) = \sqrt{\frac{2}{d}} \sin\frac{\pi z}{d} \quad \Rightarrow \quad F(q_z) = \frac{e^{iq_z d} - 1}{iq_z d} \frac{1}{1 - \left(q_z d/2\pi\right)^2}$$

- triangular quantum well (Fang-Howard approx.):

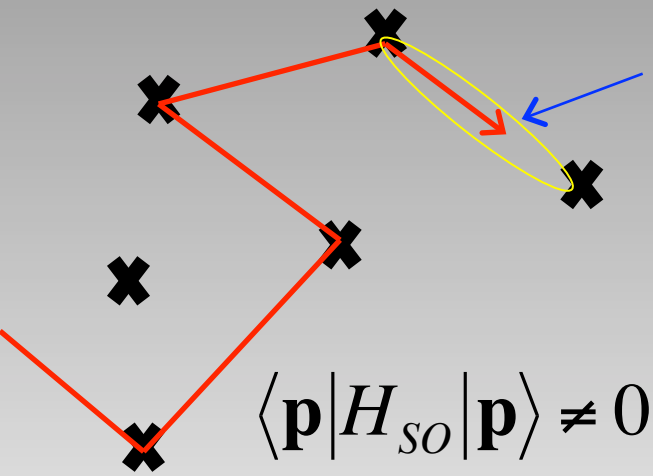
$$\psi(z) = \sqrt{\frac{b^3}{2}} z e^{-zb/2}, \quad b = \left(\frac{33 e^2 m^* n_0}{8\hbar^2 \varepsilon\varepsilon_0}\right)^{1/3}, \Rightarrow \quad F(q_z) = \frac{1}{\left(1 - iq_z/b\right)^3}$$

Parameter regime:

1. $\lambda << \lambda_{SO}, \quad \lambda_{SO} = \hbar / m^* \beta$ (typically: λ~100 nm, and λ_{SO}~1-10 μm)

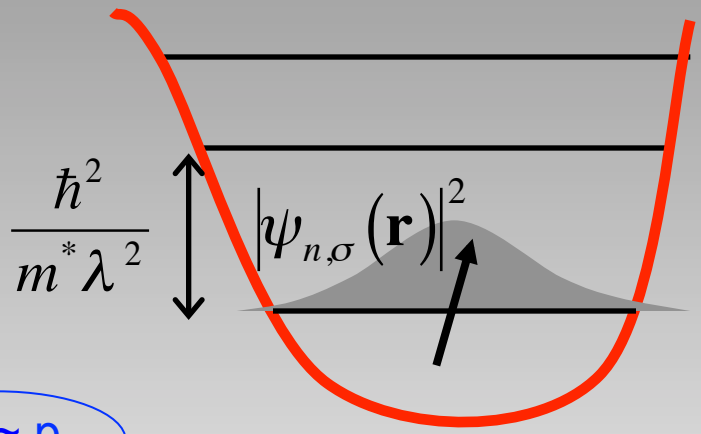
2. spin-orbit interaction in quantum dot is *weak*

3. $g\mu_B B << \hbar^2 / m^* \lambda^2$



$$\langle \mathbf{p}|H_{SO}|\mathbf{p}\rangle \neq 0$$
$$\langle \mathbf{p}|H_{SO}|\mathbf{p}'\rangle = 0$$

quantum dot:



$H_{SO} \sim p_{x,y}$

$$\langle n|H_{SO}|n\rangle \propto \langle n|[H_0, x]|n\rangle = 0$$

$$\langle n|H_{SO}|m\rangle \neq 0: \quad \frac{\langle n|H_{SO}|m\rangle}{E_n - E_m} \sim \frac{\lambda}{\lambda_{SO}} << 1$$

Parameter regime:

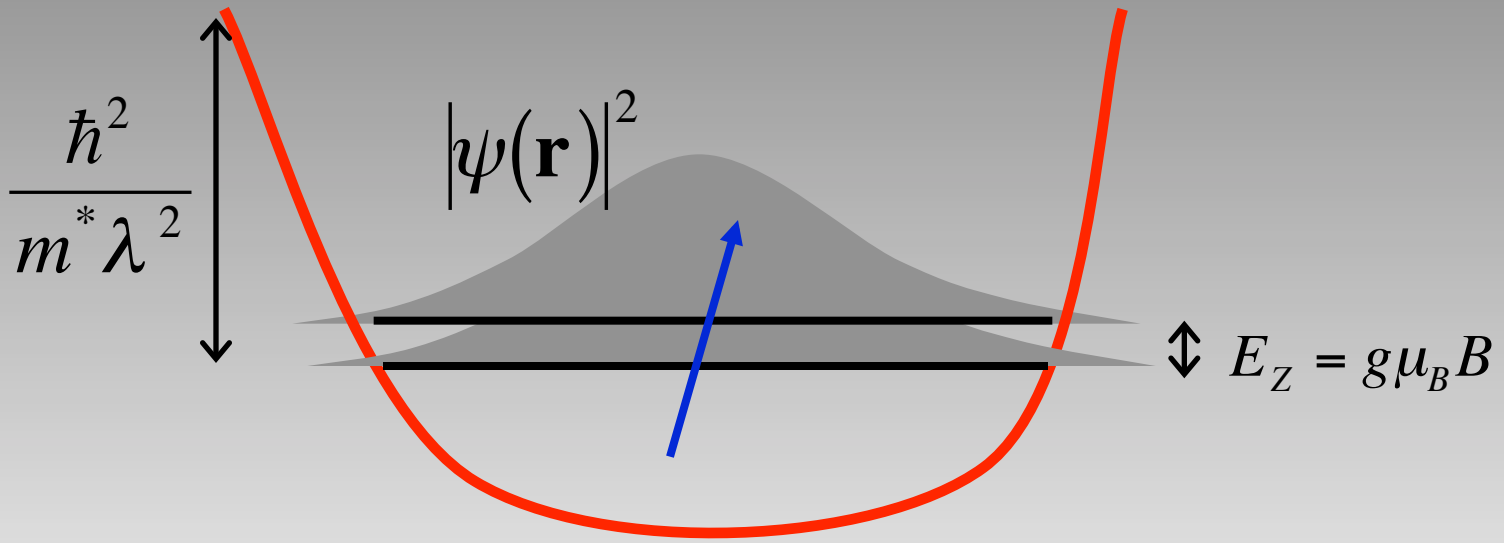
$$\lambda << \lambda_{SO}, \quad \lambda_{SO} = \hbar / m^* \beta$$

(typically: λ~100 nm, and λ_{SO}~1-10 μm)

$$k_B T << \hbar^2 / m^* \lambda^2$$

(typically: $h^2/m^*\lambda^2 \sim 1$ meV ≈ 10 K)

the dot stays in its orbital ground state



Spin = Kramers *doublet* of ground state

Derivation via Schrieffer-Wolff transformation *):

$$H = H_{dot} + H_Z + H_{SO} + U_{el-ph}(t)$$

$$\tilde{H} = e^S H e^{-S} \approx H_d + H_Z + U_{el-ph}(t) + \left[S, U_{el-ph}(t)\right]$$

unitary trafo removes 1st order in H_{SO}

with S defined by $\left[H_d + H_Z, S\right] \equiv (\hat{L}_d + \hat{L}_Z)S = H_{SO}$

Liouville superoperators

with $p_x = im^*[H_d, x]$, get $H_{SO} = i\left[H_d, \sigma \cdot \xi\right] \equiv i\hat{L}_d \sigma \cdot \xi$

thus: $$S = \frac{1}{\hat{L}_d + \hat{L}_Z} i\hat{L}_d \sigma \cdot \xi = \left(\frac{1}{\hat{L}_d} - \frac{\hat{L}_Z}{\hat{L}_d^2} + \ldots\right) i\hat{L}_d \sigma \cdot \xi = S^{(0)} + S^{(1)} + \ldots$$

$$S^{(0)} = i\sigma \cdot \xi, \qquad H_d \left|n\right\rangle = E_n \left|n\right\rangle$$

no orbital B-effect to O(H_{so}): $\left[S^{(0)}, U_{el-ph}(t)\right]_{nn} = \left[i\sigma \cdot \xi, U_{el-ph}(t)\right]_{nn} = 0$

*) Bravyi, DiVincenzo, and DL, Annals of Physics 326, 2793-2826 (2011)

leading order is due to Zeeman term (no orbital):

$$S^{(1)} = -\frac{1}{\hat{L}_d}\hat{L}_Z i\sigma\cdot\xi = g\mu_B\sigma\cdot\left[\mathbf{B}\times\left(\frac{1}{\hat{L}_d}\xi\right)\right],$$

giving $H_{\text{eff}} = \langle\psi|\widetilde{H}|\psi\rangle + \text{spin}-\text{independent constant}$

$$H_{\text{eff}} = \frac{1}{2}g\mu_B\left(\mathbf{B}+\delta\mathbf{B}(t)\right)\cdot\boldsymbol{\sigma}, \qquad \delta\mathbf{B}(t) = 2\mathbf{B}\times\boldsymbol{\Omega}(t),$$

where $\Omega(t) = \langle\psi|\left[\left(\hat{L}_d^{-1}\xi\right), U_{el-ph}(t)\right]|\psi\rangle \propto \lambda/\lambda_{SO},$

$$\xi = \left(y'/\lambda_-, x'/\lambda_+, 0\right), \quad 1/\lambda_\pm = m^*(\beta\pm\alpha)/\hbar, \quad \begin{cases} x' = (x+y)/\sqrt{2} \\ y' = -(x-y)/\sqrt{2} \end{cases}$$

Parameter regime:

1. $\lambda << \lambda_{SO}, \quad \lambda_{SO} = \hbar / m^* \beta$ (typically: λ~100 nm, and λ_{SO}~1-10 μm)
2. $k_B T << \hbar^2 / m^* \lambda^2$ (typically: $h^2/m^*\lambda^2 \sim 1$ meV ≈ 10 K)
3. $g\mu_B B << \hbar^2 / m^* \lambda^2$

In this regime, we find effective spin Hamiltonian (~ H_{SO}, $U_{e\text{-}ph}$):

$$H_{\text{eff}} = \frac{1}{2} g\mu_B \left(\mathbf{B} + \delta\mathbf{B}(t)\right) \cdot \sigma,$$

$$\delta\mathbf{B}(t) = 2\mathbf{B} \times \Omega(t),$$

Golovach, Khaetskii & DL, PRL 93 (2004)

➔ no dephasing!
i.e. $T_2 = 2T_1$

where $\Omega(t) = \langle\psi| \left[\left(\hat{L}_d^{-1}\xi\right), U_{el-ph}(t) \right] |\psi\rangle,$

$\xi = (y'/\lambda_-, x'/\lambda_+, 0),$ $\quad 1/\lambda_\pm = m^*(\beta \pm \alpha)/\hbar,$ $\quad \begin{cases} x' = (x+y)/\sqrt{2} \\ y' = -(x-y)/\sqrt{2} \end{cases}$

Bloch Equations (Born approx. in δB):

$$\left\langle \dot{\mathbf{S}} \right\rangle = g\mu_B \mathbf{B} \times \left\langle \mathbf{S} \right\rangle - \Gamma \left\langle \mathbf{S} \right\rangle + \mathbf{Y}$$

(spin: Kramers doublet)

$\tau_c = \lambda / s = 100$ ps $\ll T_{1,2}$
& super-Ohmic spectrum

➔ Born-Markov approx. ok

Decay tensor:

$$\Gamma_{ij} \propto J_{ij}(w) = \frac{g^2 \mu_B^2}{2\hbar^2} \int_0^\infty \left\langle \delta B_i(0) \delta B_j(t) \right\rangle e^{-iwt} dt$$

spectral function

decay: $\Gamma = \Gamma^r + \Gamma^d,$

relaxation: $\Gamma^r_{ij} = \delta_{ij}\left(\delta_{pq} - l_p l_q\right) J^+_{pq}(\omega) - \left(\delta_{ip} - l_i l_p\right) J^+_{pj}(\omega) - \delta_{ij} \varepsilon_{kpq} l_k I^-_{pq}(\omega) + \varepsilon_{ipq} l_p I^-_{qj}(\omega),$

dephasing: $\Gamma^d_{ij} = \delta_{ij} l_p l_q J^+_{pq}(0) - l_i l_p J^+_{pj}(0)$ ➔ **0**

$$J^{\pm}_{ij}(w) = \mathrm{Re}\left[J_{ij}(w) \pm J_{ij}(-w)\right], \quad I^{\pm}_{ij}(w) = \mathrm{Im}\left[J_{ij}(w) \pm J_{ij}(-w)\right]$$

Relaxation rate:

Bose function

super-Ohmic: ~z^3

$$\frac{1}{T_1} \propto \mathrm{Re}\, J_{XX}(z) = \frac{\omega^2 z^3 (2n_z + 1)}{\left(2\Lambda_+ m^* \omega_0^2\right)^2} \sum_j \frac{\hbar}{\pi \rho_c s_j^5} \int_0^{\pi/2} d\theta \sin^3\theta$$

$$\times e^{-(z\lambda \sin\theta)^2 / 2 s_j^2} \left| F\left(\frac{|z|}{s_j} \cos\theta \right) \right|^2 \left(e^2 \overline{\beta_{j\theta}^2} + \frac{z^2}{s_j^2} \overline{\Xi_j^2} \right) \propto \lambda^2 / \lambda_{SO}^2$$

$z \to \omega = g\mu_B B$

quantum well

piezo

deformation

Golovach, Khaetskii, Loss, PRL 93 (2004)

$$\frac{2}{\Lambda_\pm^2} = \frac{1 - l_{x'}^2}{\lambda_-^2} + \frac{1 - l_{y'}^2}{\lambda_+^2} \pm \sqrt{\left(\frac{1 - l_{x'}^2}{\lambda_-^2} + \frac{1 - l_{y'}^2}{\lambda_+^2} \right)^2 - \frac{4 l_z^2}{\lambda_+^2 \lambda_-^2}}$$

effective SO length

Relaxation rate:

Bose function

super-Ohmic: ~z^3

$$\frac{1}{T_1} \propto \mathrm{Re}\, J_{XX}(z) = \frac{\omega^2 z^3 (2n_z + 1)}{\left(2\Lambda_+ m^* \omega_0^2\right)^2} \sum_j \frac{\hbar}{\pi \rho_c s_j^5} \int_0^{\pi/2} d\theta \sin^3\theta$$

$$\times e^{-(z\lambda \sin\theta)^2 / 2 s_j^2} \left| F\left(\frac{|z|}{s_j} \cos\theta \right) \right|^2 \left(e^2 \overline{\beta_{j\theta}^2} + \frac{z^2}{s_j^2} \overline{\Xi_j^2} \right) \propto \lambda^2 / \lambda_{SO}^2$$

$z \to \omega = g\mu_B B$

quantum well

piezo

deformation

$s_1 \approx 4.7 \times 10^3 m/s,\ s_2 = s_3 \approx 3.37 \times 10^3 m/s$ speed of sound

$$\sqrt{\overline{\Xi_j^2}} = \delta_{j,1} \Xi_0,\ \Xi_0 \approx 7 eV,\ \sqrt{\overline{\beta_{1,\vartheta}^2}} = 3\sqrt{2}\pi h_{14} \kappa^{-1} \sin^2\vartheta \cos\vartheta,\ \sqrt{\overline{\beta_{2,\vartheta}^2}} = \sqrt{2}\pi h_{14} \kappa^{-1} \sin 2\vartheta,$$

$$\sqrt{\overline{\beta_{3,\vartheta}^2}} = 3\sqrt{2}\pi h_{14} \kappa^{-1} (3\cos^2\vartheta - 1) \sin\vartheta,\quad h_{14} \approx 0.16 C/m^2,\ \kappa \approx 13$$

$$\frac{2}{\Lambda_\pm^2} = \frac{1 - l_{x'}^2}{\lambda_-^2} + \frac{1 - l_{y'}^2}{\lambda_+^2} \pm \sqrt{\left(\frac{1 - l_{x'}^2}{\lambda_-^2} + \frac{1 - l_{y'}^2}{\lambda_+^2} \right)^2 - \frac{4 l_z^2}{\lambda_+^2 \lambda_-^2}}$$

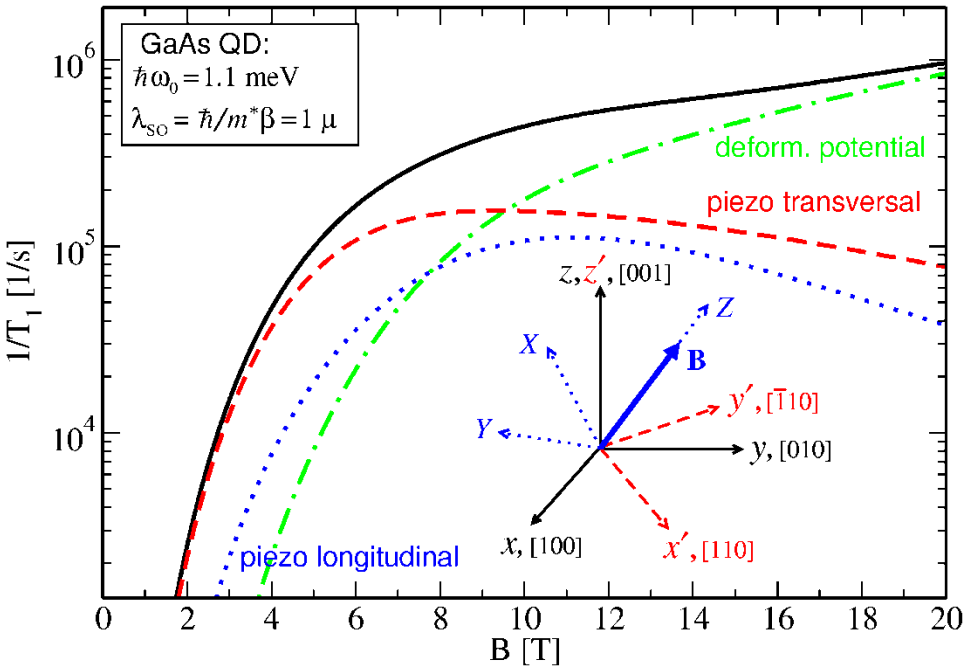
effective SO length

Spin relaxation rate $1/T_1$ for GaAs quantum dot

$$\frac{1}{T_1} \propto (g\mu_B B)^{2+2+1} \times \int_0^{\pi/2} d\theta \sin^k\theta\, e^{-(g\mu_B B\lambda\sin\theta)^2/2s_j^2}$$

$$\delta B^2 \propto B^2 \qquad \nu_{ph}(\omega) \propto \omega^2 \qquad H_{SO} \propto p_\alpha$$

power-5 law for B< 3T (GaAs)



Golovach et al., PRL 93 (2004)

Numerical value of T_1 for GaAs parameters (13!):

$$\left(\hbar\omega_0, \lambda, d, \lambda_{SO} = \hbar/m^*\beta, \alpha, \kappa, \Xi_0, h_{14}, s_1, s_2 = s_3, \rho_c, m^*\right) =$$

$$\left(\begin{array}{l} 1.1\,\text{meV}\ ,\ 32\,\text{nm}, 5\,\text{nm}\ ,\ 9\,\mu\text{m},\ 0, 13.1, 6.7\,\text{eV}, 0.16\ \text{C/m}^2\ , \\ 4.73\times10^5\ \text{cm/s}, 3.35\times10^5\ \text{cm/s}, 5.3\times10^3\ \text{kg/m}^3\ , 0.067 m_e \end{array}\right)$$

Zumbuhl ea PRL 89 (276803) 2003

$$|g| = 0.43 \pm 0.04 - (0.0077 \pm 0.0020) B(T)$$

or with linear fit: $|g| = 0.29$ Hanson ea PRL 91 (196802) 2003

Theory: $T_1 \approx 750\ \mu\text{s}, \quad \text{for B} = \ 8\text{T}$ $\propto \lambda_{SO}{}^2 / \lambda^2$

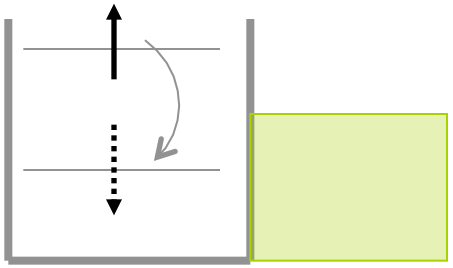
$T_1 = 550 - 1100\ \mu\text{s}$ due to uncertainties in *g* factor

$T_1 = 2.7\ \text{ms}$ for λ_{SO}=17μm [Huibers ea PRL 83, 5090 (1999)]

Experiment: $T_1^{\text{exp.}} = 800\,\mu s @ 8T$ Elzerman *et al.*, Nature 430, 431 (2004)

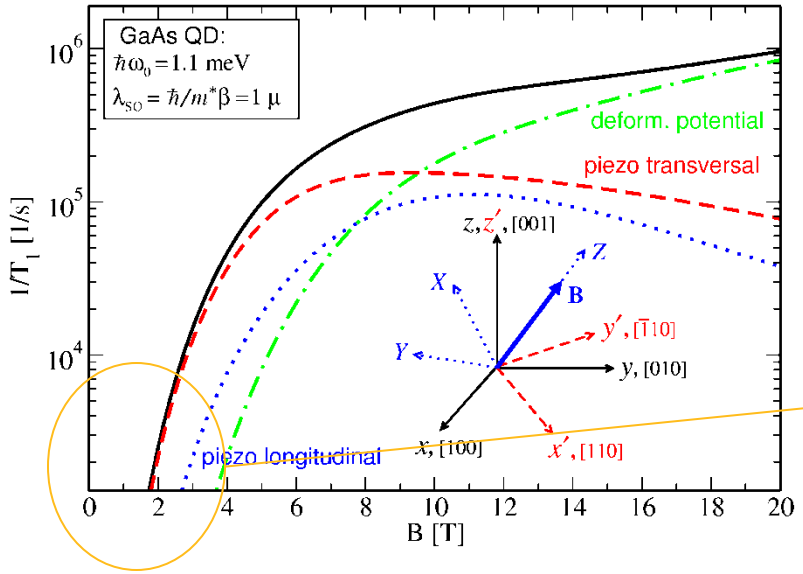
Read-out via spin-charge conversion:

DL&DiVincenzo 1998



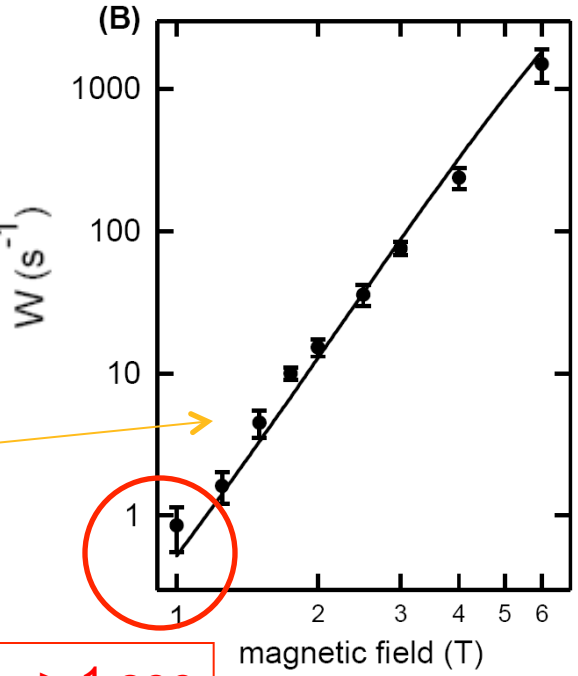
spin relaxation rates $1/T_1$:

Theory: Basel, 2004



➔ prediction confirmed

Experiment: MIT 2008



> 1 sec

Golovach, Khaetskii, DL, PRL `04

Amasha, Zumbuhl, et al., PRL `08

$1/T_{1,2}$ depends strongly on B-field direction ("magic angles"):

$$\frac{1}{T_1} = \frac{f(\varphi,\theta,\alpha)}{T_1(\theta=\pi/2,\alpha=0)}$$

Golovach, Khaetskii, Loss
PRL 93, 016601 (2004);
PRB 77, 045328 (2008)

"ellipsoid"

$$f(\varphi,\theta,\alpha) = \frac{1}{\beta^2}\left[(\alpha^2+\beta^2)(1+\cos^2\theta) + 2\alpha\beta\sin^2\theta\sin 2\varphi\right]$$

Rashba and Dresselhaus interfere! *)

Special case:

$$\alpha = \beta,\ \ \theta = \pi/2,\ \ \varphi = 3\pi/4 \quad \rightarrow \quad T_1 \rightarrow \infty$$

exact!

*) Schliemann, Egues, and DL, PRL `03

$1/T_{1,2}$ depends strongly on B-field direction ("magic angles"):

$$\frac{1}{T_1} = \frac{f(\varphi,\theta,\alpha)}{T_1(\theta=\pi/2,\alpha=0)}$$

Golovach, Khaetskii, Loss
PRL 93, 016601 (2004)

"ellipsoid"

$$f(\varphi,\theta,\alpha) = \frac{1}{\beta^2}\left[\left(\alpha^2+\beta^2\right)\left(1+\cos^2\theta\right) + 2\alpha\beta\sin^2\theta\sin 2\varphi\right]$$

Rashba and Dresselhaus interfere! *)

Special case:

$$\alpha = \beta,\ \ \theta = \pi/2,\ \ \varphi = 3\pi/4 \quad \rightarrow \quad T_1 \rightarrow \infty$$

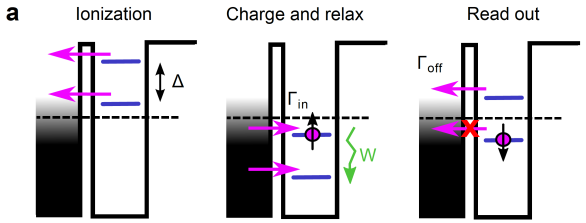
exact!

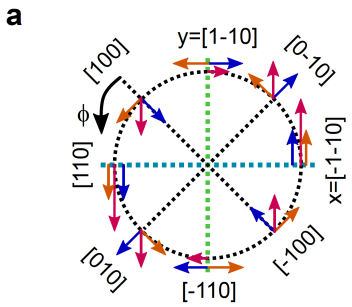
Experiment ?!

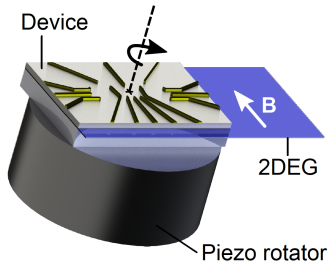
*) Schliemann, Egues, DL, PRL `03

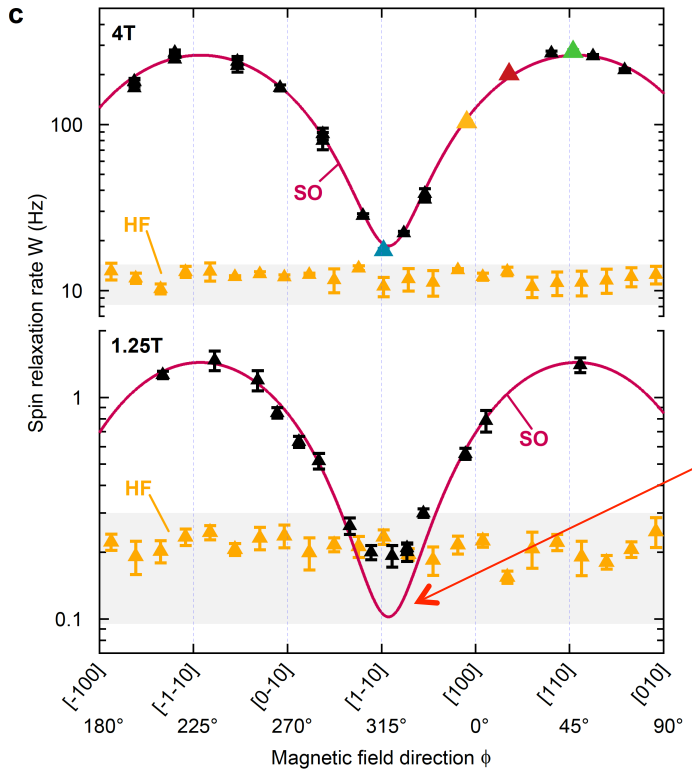
Ultra-long spin relaxation in a single electron GaAs quantum dot

Camenzind,Yu, Stano, Zimmerman, Gossard, Loss, and Zumbühl, preprint 2017









T=60 mK

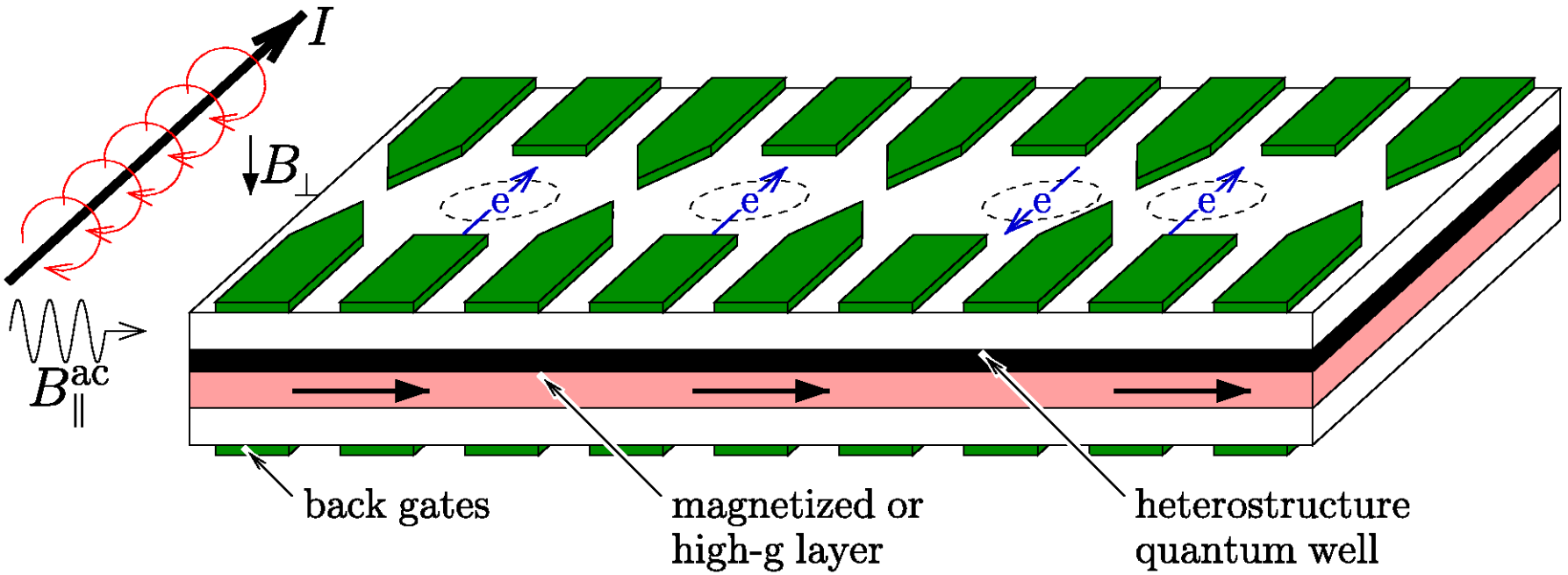
$T_1 \sim 1$ min

dominant source:
hyperfine interaction
with nuclear spins

➔ current world record
even longer than in Si: $T_1^{Si} \sim 30$ s,
M. Simmons et al., 2017

Scaling-up of spin qubits ➔ quantum dot array

DL & DiVincenzo, PRA **57** (1998) 120



$$H = \sum_{\langle ij \rangle} J_{ij}(t)\mathbf{S}_i \cdot \mathbf{S}_j + \sum_i (g_i \mu_B \mathbf{B}_i)(t) \cdot \mathbf{S}_i$$

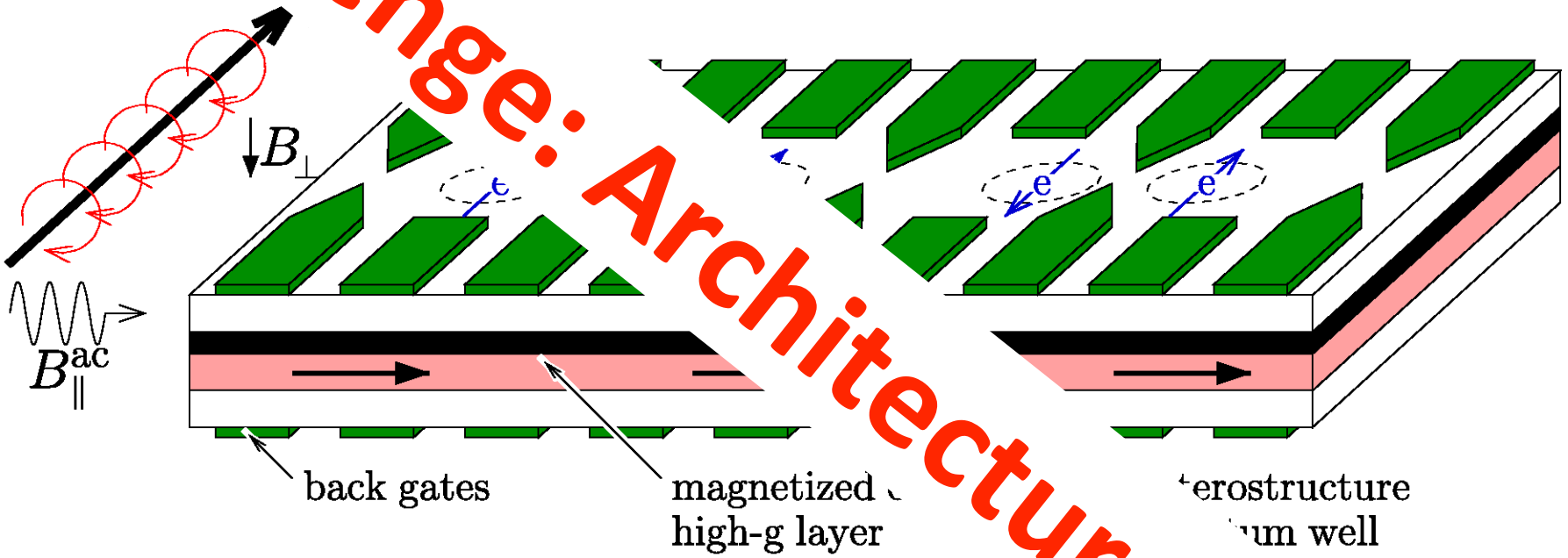
n.n. exchange

local Zeeman

ng-up of spin qubits ➔ quantum dot array

& DiVincenzo, PRA **57** (1998) 120

Challenge: Architecture !?

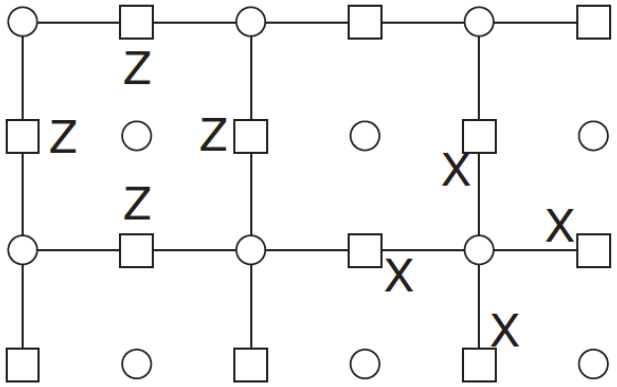


$$H = \sum_{\langle ij \rangle} J_{ij}(t)\mathbf{S}_i \cdot \mathbf{S}_j + \sum_i (g_i \mu \cdot \mathbf{S}_i$$

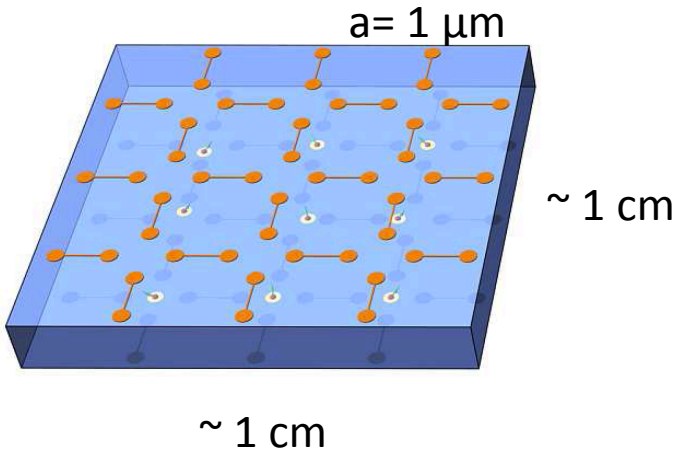
n.n. exchange

local Zeem

Quantum Chip Size in Surface Code Architecture



?



Bravyi and Kitaev, arXiv:quant-ph/9811052
Raussendorf and Harrington, PRL 98,190504 (2007)

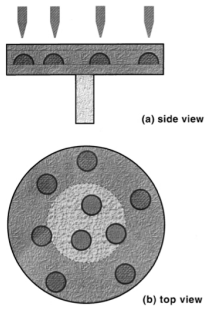
➔ powerful quantum computer of size 1 cm^2 with 10^8 spin qubits
GHz clock-speed ➔ can factor a 2000-bit number (RSA key) in 26h

error threshold of ca. 1 % – 18 % (perfect measurement)*

Wang, Fowler, and Hollenberg, PRA 83, 020302 (2011)
*Error correction: Wootton and DL, PRL 109, 160503 (2012)

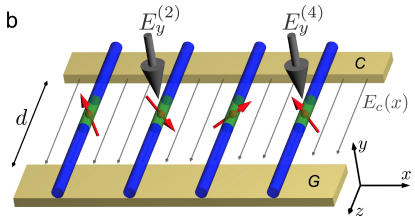
Long-distance coupling of spin qubits

Electromagnetic cavity & waves



Imamoglu et al.,
PRL 83, 4204 (1999)

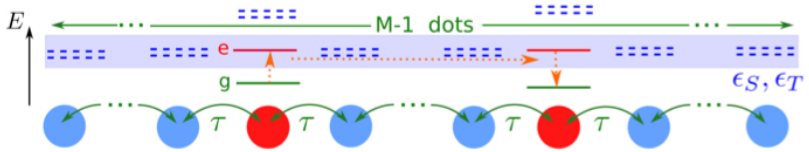
optical



microwaves

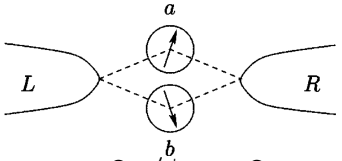
Trif, Golovach & DL, PRB 77, 045434 (2008)
Kloeffel et al., PRB 88, 241405 (2013)
Nigg, Fuhrer, and DL, PRL 2017

PACT



Stano et al., PRB 92, 075302 (2015)

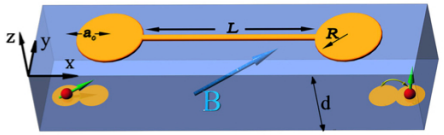
superconductor (CAR)



$$J(r) = J_0 e^{-2r/\xi} \sin^2(k_F r)/(k_F r)^2$$

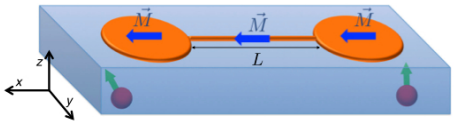
Choi, Bruder & DL, PRB 62, 13569 (2000)

Floating metallic gate



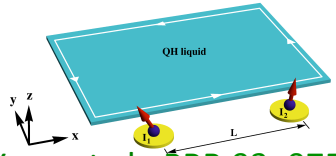
Trifunovic et al., PRX 2, 011006 (2012)

Dipolar ferromagnet



Trifunovic et al., PRX 3, 041023 (2013)

QHE edge states



Yang et al., PRB 93, 075301 (2016)

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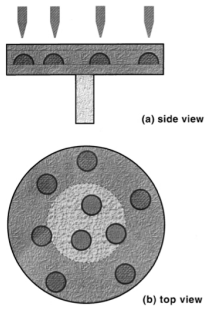
Electromagnetic cavity & waves



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PRL 83, 4204 (1999)

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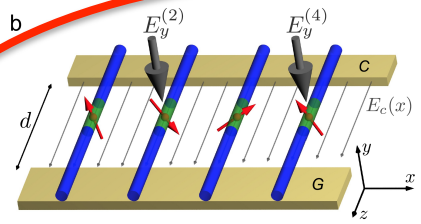
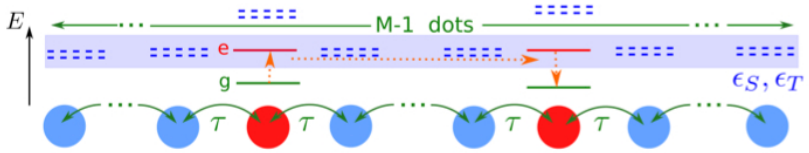


Trif, Golovach & DL, PRB 77, 045434 (2008)
Kloeffel et al., PRB 88, 241405 (2013)
Nigg, Fuhrer, and DL, PRL 2017

PACT



Stano et al., PRB 92, 075302 (2015)

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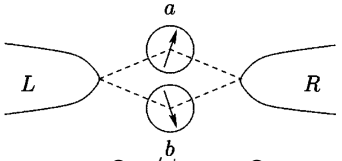

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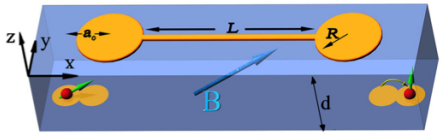
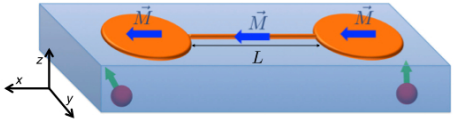


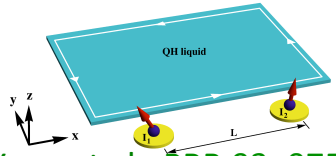
Trifunovic et al., PRX 2, 011006 (2012)

Dipolar ferromagnet



Trifunovic et al., PRX 3, 041023 (2013)

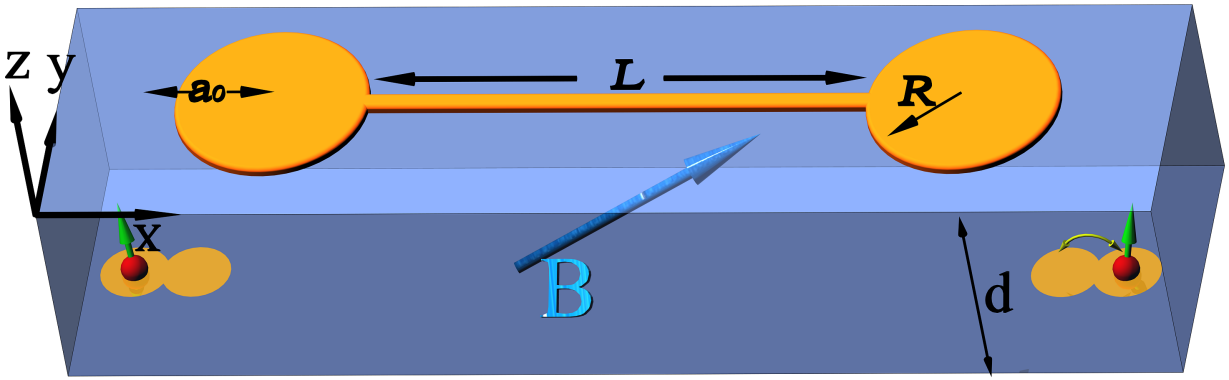
QHE edge states



Yang et al., PRB 93, 075301 (2016)

Long-distance entanglement via floating gates

Trifunovic, Dial, Trif, Wootton, Abebe, Yacoby, DL, Phys. Rev. X 2, 011006 (2012)

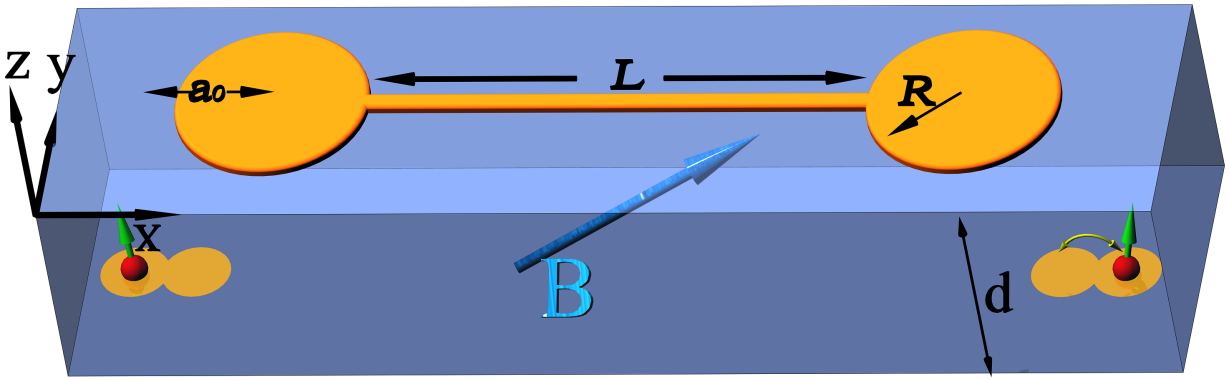


Electrostatics + spin orbit interaction ➔ effective exchange:

$$H_{s-s} = J_x \sigma_1^x \sigma_2^x + J_y \sigma_1^y \sigma_2^y$$

Long-distance entanglement via floating gates

Trifunovic, Dial, Trif, Wootton, Abebe, Yacoby, DL, Phys. Rev. X 2, 011006 (2012)

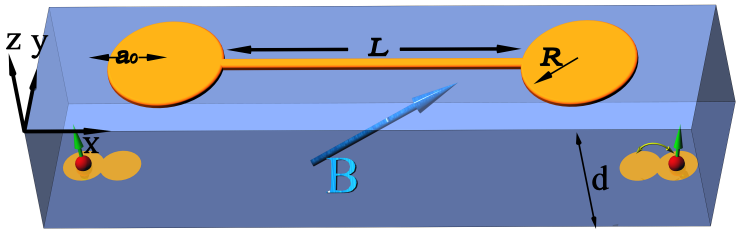


Electrostatics + spin orbit interaction ➔ effective exchange:

$$J \simeq \frac{\pi \alpha_q \alpha_C}{4} \left(\frac{\partial q_{ind}}{\partial \tilde{x}} \right)^2_{\boldsymbol{r}=0} \left(\frac{E_Z}{\omega_x} \right)^2 \left(\frac{\lambda}{\lambda_{SO}} \right)^2 \hbar\omega_x$$ ~ 1-100 μeV

$$\tau_{\text{switching}} = \hbar / J \approx 1\text{ns} \; - \; 10\text{ps}$$

Electrostatics



induced charge on gate (flat disc):

$$q_{\rm ind}(\boldsymbol{r}) = \frac{2}{\pi}\arcsin(R/\xi_{\boldsymbol{r}}),$$

Sten, J. Electrost. 64, 647 (2006)

$$2\xi_{\boldsymbol{r}}^2 = R^2 + d^2 + |\boldsymbol{a}_0 + \boldsymbol{r}|^2 + \sqrt{(R^2 + d^2 + |\boldsymbol{a}_0 + \boldsymbol{r}|^2)^2 - 4R^2|\boldsymbol{a}_0 + \boldsymbol{r}|^2},$$

Electrostatic coupling:

$$V(\boldsymbol{r}_1, \boldsymbol{r}_2) = \frac{\pi\alpha_q}{\kappa}\frac{e^2 q_{\rm ind}(\boldsymbol{r}_1) q_{\rm ind}(\boldsymbol{r}_2)}{R}$$

~ ln (L/R_w) /L , for L>>R

where $\alpha_q = \dfrac{C_{\rm d}}{C_{\rm w} + 2C_{\rm d}}$

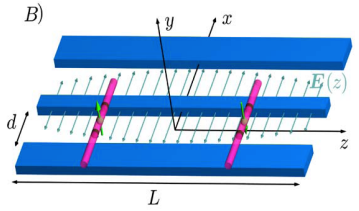
$C_{\rm d} = 2R/\pi,$ capacitance of disc

$C_{\rm w} = \dfrac{L}{2\ln(L/R_{\rm w})},$ capacitance of wire

Long-distance Spin-Spin Coupling via Photons

Spin1-photon + Spin2-photon ➔ Spin1-Spin2

$$H_{s-s} = \sum_{i=1,2} \tilde{E}^{i}_{iZ}\sigma^{j}_{Z} + J\left(\sigma^{1}_{+}\sigma^{2}_{-} + \sigma^{2}_{+}\sigma^{1}_{-}\right)$$



XY Spin-Spin Interaction – universal for quantum computing!

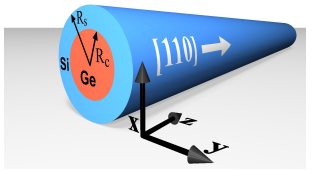
$J = \frac{\nu_{E,1}\nu_{E,2}}{2}\left(\frac{1}{\Delta_1} + \frac{1}{\Delta_2}\right)$: effective exchange coupling – ***long-range (millimeters!)***

effective Zeeman + Stark shift + Lamb shift: $\tilde{E}^{eff}_{iZ} = E^{eff}_{iZ} + \frac{\nu^2_{E,i}}{\Delta_i}\left(n + \frac{1}{2}\right)$

detuning of the spin from the cavity mode: $\Delta_i = E^{eff}_{Z} - \hbar\omega$

Trif, Golovach, and DL, PRB 77, 045434 (2008)

Setup with Ge/Si nanowire QDs



Kloeffel, Stano, Trif, & DL, PRB 88, 241405 (2013)

Ge/Si NWs typically have small core diameters ~ 10–20 nm

⟶ Elongated QDs

E_y controls SOI and g factor

B_y induces Zeeman splitting

$E_{z,0}\cos(\omega t)$

ac field sightly shifts potential

E_y

B_y

Harmonic confinement

Ge/Si NW

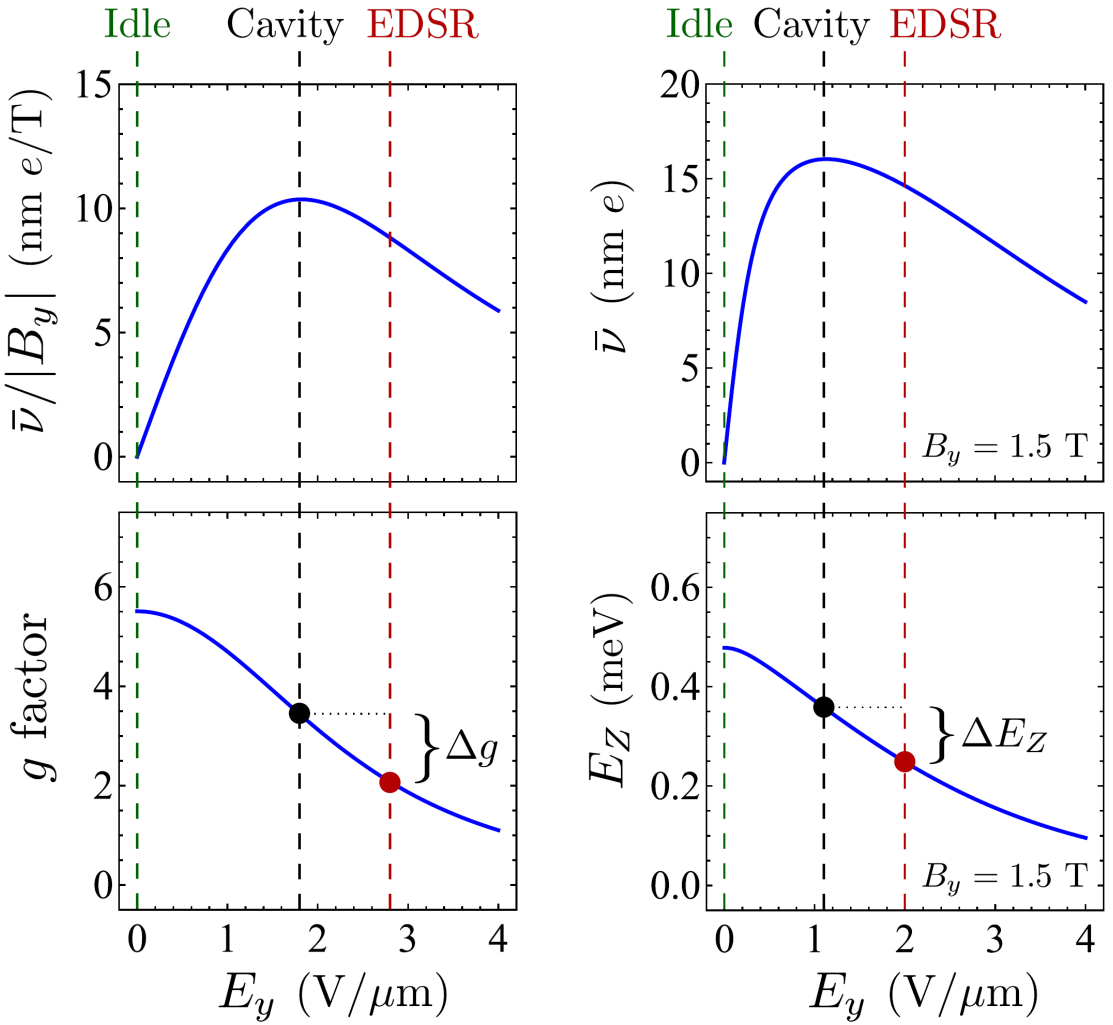
z

Electric gates

Kloeffel *et al.*, PRB
88, 241405(R) (2013)

Single- and Two-Qubit Gates

Choosing different E_y for EDSR and cavity-mediated coupling allows independent control of single- and two-qubit operations



R = 7.5 nm

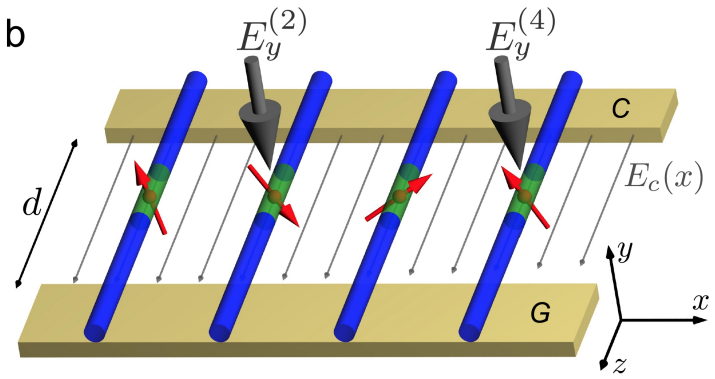
R_s = 10 nm

l_g = 50 nm

Hole-Spin Coupling via Photons

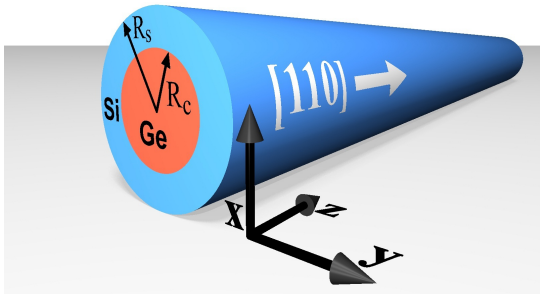
Kloeffel,Trif, Stano, and DL, PRB 88, 241405 (2013)

hole spin qubits coupled via cavity mode



Wallraff *et al*., Nature (2004)

Ge/Si nanowire

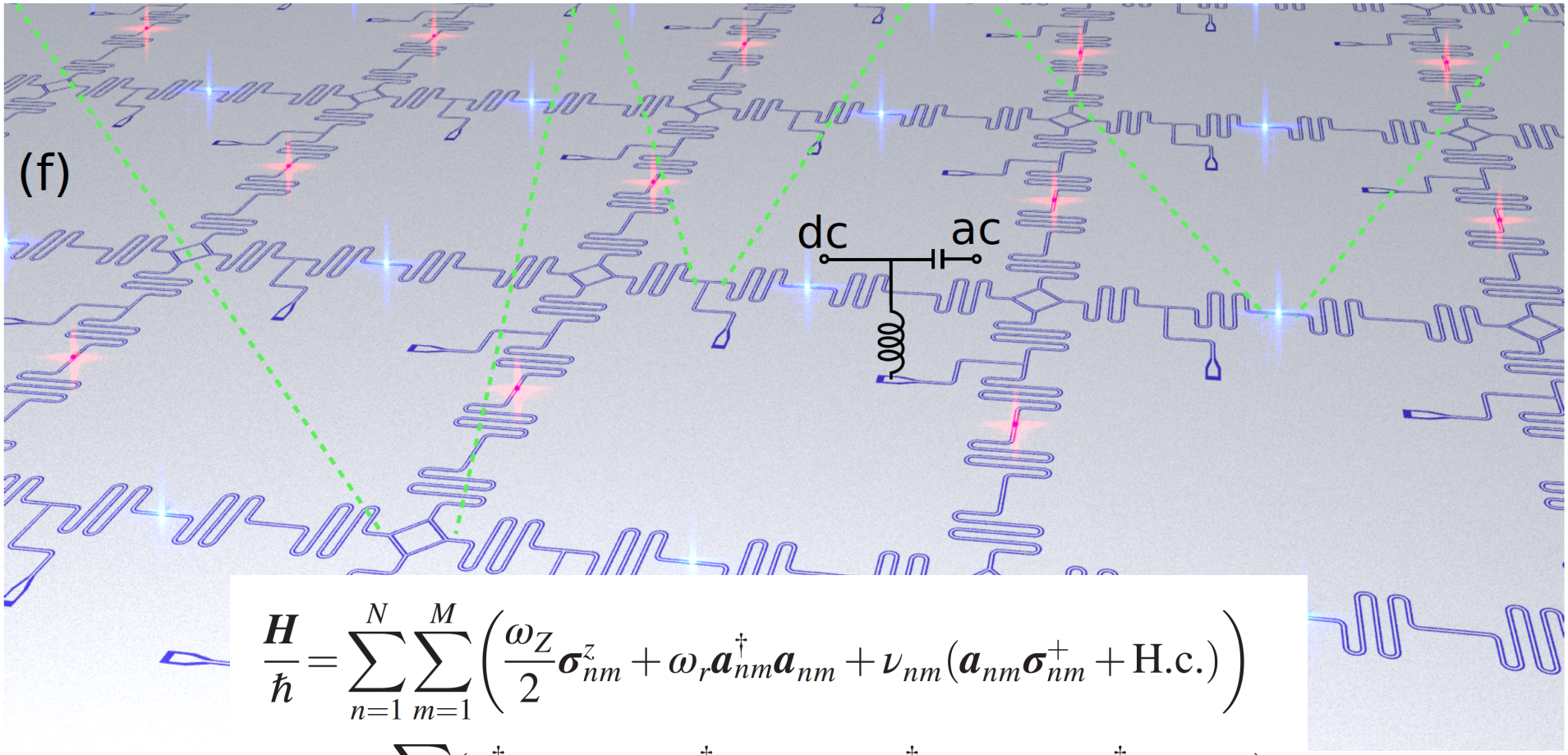


Ultra-fast gate operation times:

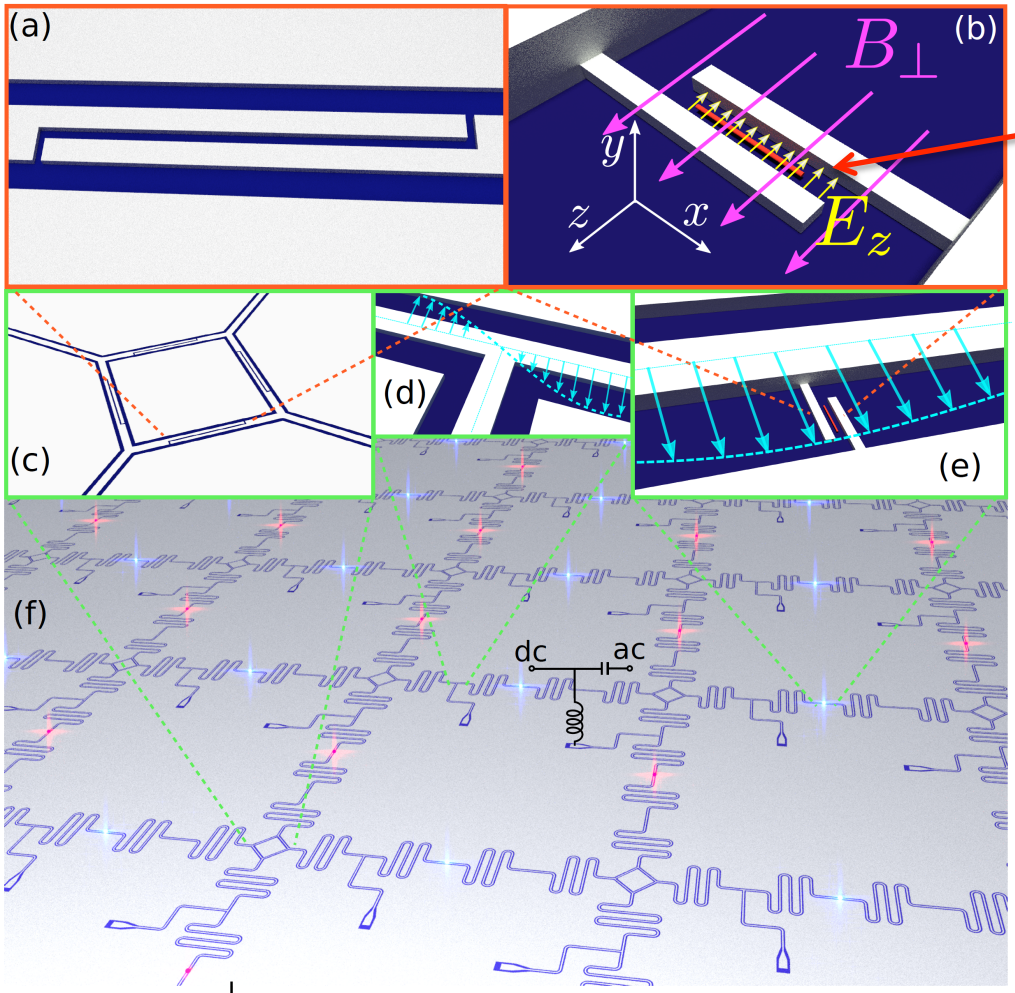
- single qubit : 40 - 100 ps
- two-qubit: 20 - 100 ns [limited by cavity field E_z ~ 3V/m]
- at reduced noise and large on/off ratio

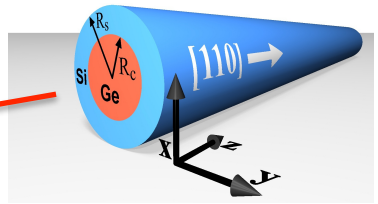
Grid-bus Surface Code with Hole Spin Qubits

Nigg, Fuhrer, and DL, PRL 118, 147701 (2017)



$$\frac{\boldsymbol{H}}{\hbar} = \sum_{n=1}^{N}\sum_{m=1}^{M}\left(\frac{\omega_Z}{2}\boldsymbol{\sigma}_{nm}^{z} + \omega_r \boldsymbol{a}_{nm}^{\dagger}\boldsymbol{a}_{nm} + \nu_{nm}\left(\boldsymbol{a}_{nm}\boldsymbol{\sigma}_{nm}^{+} + \text{H.c.}\right)\right)$$
$$+ J\sum_{n,m}\left(\boldsymbol{a}_{nm}^{\dagger}\boldsymbol{a}_{nm+1} + \boldsymbol{a}_{nm}^{\dagger}\boldsymbol{a}_{nm-1} + \boldsymbol{a}_{nm}^{\dagger}\boldsymbol{a}_{n+1m} + \boldsymbol{a}_{nm}^{\dagger}\boldsymbol{a}_{n-1m}\right).$$

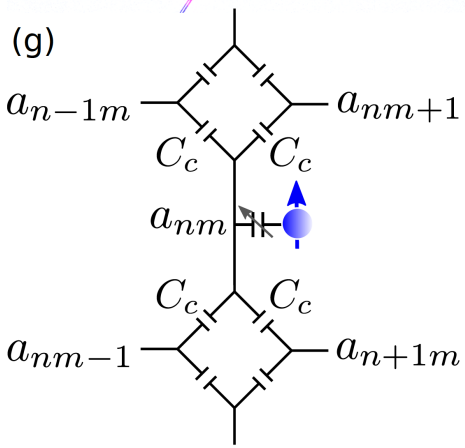


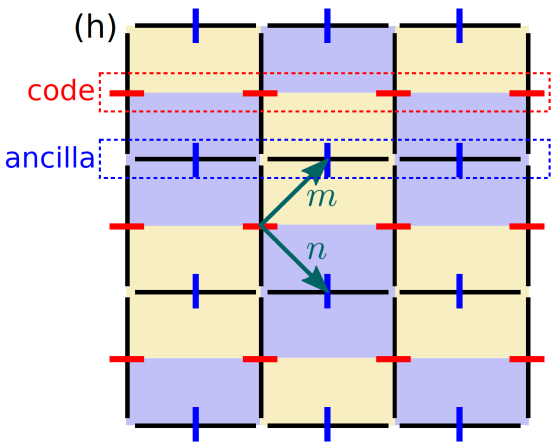


hole spin qubit in Ge/Si nanowire

$$\frac{\boldsymbol{H}_{XY}}{\hbar} = \sum_{nm,n'm'} K_{nm,n'm'} \boldsymbol{\sigma}^{+}_{nm} \boldsymbol{\sigma}^{-}_{n'm'} + \text{H.c.},$$

$$K_{nm,n'm'} = \frac{(\Delta m + \Delta n)!}{\Delta n! \Delta m!} \frac{\nu_{nm}\nu_{n'm'}}{\Delta} \left(\frac{J}{\Delta}\right)^{\Delta m + \Delta n}$$





Nigg, Fuhrer, and DL,
PRL 118, 147701 (2017)

Parallel syndrome mapping

Nigg, Fuhrer, and DL, PRL 118, 147701 (2017)

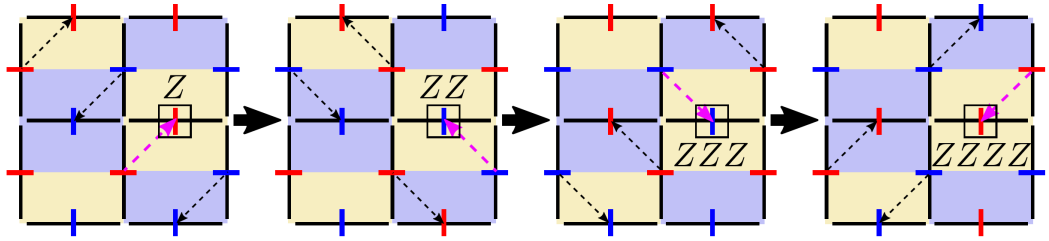


FIG. 2. Frequency layout for parallelization of syndrome mapping in four steps. Red (blue) bars denote qubits at frequency $\omega_Z^{(\mathrm{r})}$ ($\omega_Z^{(\mathrm{b})}$). The dashed black arrows indicate which couplings are resonant, i.e., active in a given configuration. The mapping of a $ZZZZ$ stabilizer is highlighted as an example (magenta arrows).

Summary

- Quantum computing with spin qubits in quantum dots
- Decoherence of spin qubits due spin-phonon effects
- Scaling-up: Surface code with floating gates or cavities

Prospects for Spin-Based Quantum Computing in Quantum Dots
Kloeffel and DL, Annu. Rev. Condens. Matter Phys. 4, 51 (2013)