Spin Qubits in Semiconducting Nanostructures

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$$: Swiss NSF, Nano Basel, Quantum ETH/Basel, EU

Outline

I. & II: Quantum dots, spin qubits, quantum gates, decoherence, hole spins in Si/Ge nanowires, scalable systems, surface code, long-distance coupler...

III: Topological quantum computing in nanowires with Majorana fermions, parafermions,..., hybrid spin-Majorana qubits;

Prospects for Spin-Based Quantum Computing in Quantum Dots
Kloeffel and Loss, Annu. Rev. Condens. Matter Phys. 4, 51 (2013)

Quantum Memories at Finite Temperature
Brown, Loss, Pachos, Self, and Wootton, Rev. Mod. Phys. 88, 045005 (2016)

Front-Runners for Quantum Computers

- spin qubits in semiconductors ‘small & fast’
- superconducting devices
- trapped ions

} more advanced but not so ‘small & fast’

- topological quantum computing?
  ‘semi-superconductor hybrids’

‘exotic’
Majorana
Para- or
Fibonacci
fermions?

Front-Runners for Quantum Computers

- spin qubits in semiconductors 'small & fast'

semiconducting nanostructures

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A bit of the action

In the race to build a quantum computer, companies are pursuing many types of quantum bits, or qubits, each with its own strengths and weaknesses.

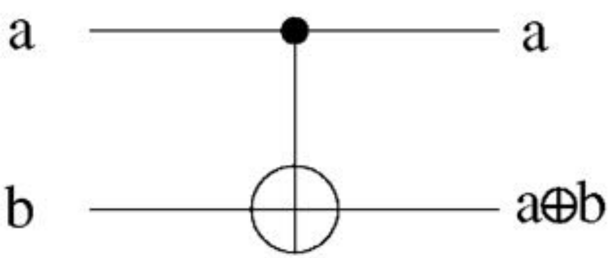
	Superconducting loops	Trapped ions	Silicon quantum dots	Topological qubits	Diamond vacancies
Description	A resistance-free current oscillates back and forth around a circuit loop. An injected microwave signal excites the current into super-position states.	Electrically charged atoms, or ions, have quantum energies that depend on the location of electrons. Tuned lasers cool and trap the ions, and put them in superposition states.	These "artificial atoms" are made by adding an electron to a small piece of pure silicon. Microwaves control the electron's quantum state.	Quasiparticles can be seen in the behavior of electrons channeled through semi-conductor structures. Their braided paths can encode quantum information.	A nitrogen atom and a vacancy add an electron to a diamond lattice. Its quantum spin state, along with those of nearby carbon nuclei, can be controlled with light.
Longevity (seconds)	0.00005	>1000	0.03 → 30-60 s	N/A ?	10
Logic success rate	99.4%	99.9%	~99%	N/A	99.2%
Number entangled	9	14	2	N/A	6
Company support	Google, IBM, Quantum Circuits	ionQ	Intel	Microsoft, Bell Labs	Quantum Diamond Technologies
Pros	Fast working. Build on existing semiconductor industry.	Very stable. Highest achieved gate fidelities.	Stable. Build on existing semiconductor industry.	Greatly reduce errors.	Can operate at room temperature.
Cons	Collapse easily and must be kept cold.	Slow operation. Many lasers are needed.	Only a few entangled. Must be kept cold.	Existence not yet confirmed.	Difficult to entangle.

Note: Longevity is the record coherence time for a single qubit superposition state, logic success rate is the highest reported gate fidelity for logic operations on two qubits, and number entangled is the maximum number of qubits entangled and capable of performing two-qubit operations.

Quantum Information

Classical digital computer

network of ‘Boolean logic gates‘, e.g. XOR (CNOT)



b \ a	0	1
0	0	1
1	1	0

- bits: $a, b = 0,1$
- physical implementation: e.g. 2 voltage levels
- ‘gate‘: electronic circuit

Quantum computer

- qubits $|a\rangle, |b\rangle \hat{=} \alpha|0\rangle + \beta|1\rangle, \quad |\alpha|^2 + |\beta|^2 = 1$
- physical implementation:
  quantum 2-level-system: $|\uparrow\rangle \equiv |0\rangle, \quad |\downarrow\rangle \equiv |1\rangle$
- ‘quantum gate‘: unitary transformation
  (is reversible!)

Quantum Computing (basics)

• basic unit: **qubit** → any state of a quantum two-level system

$$|\Psi\rangle = a|1\rangle + b|0\rangle$$

"natural" candidate: **electron spin**

• quantum computation:

1) prepare N qubits (input)
2) apply unitary transformation in 2^N-dim. Hilbert space
→ computation
3) measure result (output)

• quantum computation faster than classical:

- factoring algorithm (Shor 1994): exp N → N^2
- database search (Grover 1996): N → $N^{1/2}$
- quantum simulations

...

What a quantum computer could do (faster):

…search large database (→ biology, climate, physics…)

…break `RSA-Encryption' (banking, industry, military,…)

…simulate physical und chemical processes (or models*)
(→ energy, catalysts, C-capture, material science, drug design,…)

…machine learning & cloud computing

…play quantum games

…and many unforseen applications (hopefully)

Intense search for new quantum algorithms !

*) See e.g. Wecker et al., Phys. Rev. A 92, 062318 (2015)
'Solving the 2D Hubbard model on a quantum computer'

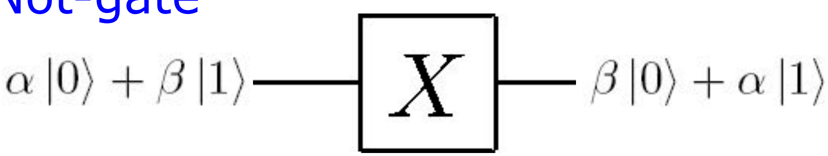
Quantum Computing with Quantum Gates

Barenco et al., PRA **52**, 3457 (1995)

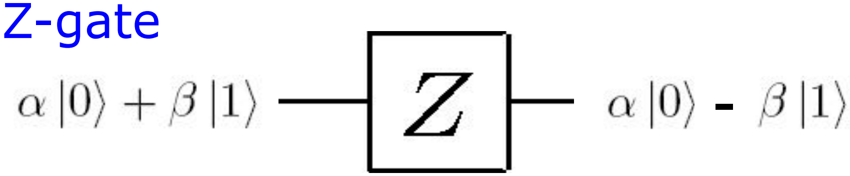
Single-qubit operations and a two-qubit gate that generates entanglement are sufficient for **universal quantum computation**:

Single-qubit gates

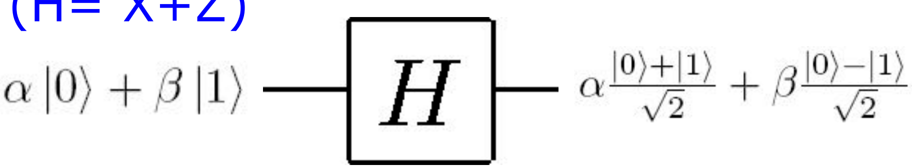
Not-gate



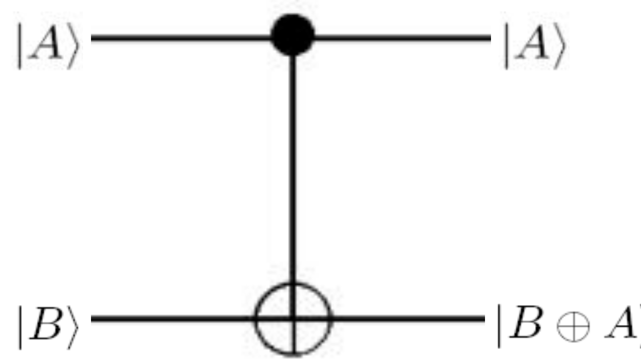
Z-gate



Hadamard-gate (H= X+Z)



2-qubit gate: CNOT (XOR) gate

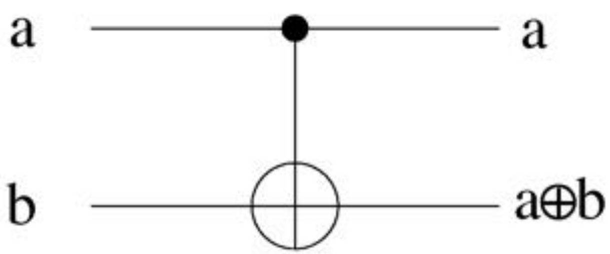


⇔ entanglement

Quantum Gates

- quantum gate: unitary transformation acting on a few qubits at a time (universal set of quantum gates: all unitary operations on n qubits [$U(2^n)$] can be expressed as a composition of these gates)

- XOR together with one-qubit gates is a universal set for quantum computation (Barenco et al. 1995)



b \ a	0	1
0	0	1
1	1	0

- action of the quantum XOR gate:

two-particle state $|\psi\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$

$$|00\rangle \mapsto |00\rangle,\ |01\rangle \mapsto |01\rangle,\ |10\rangle \mapsto |11\rangle,\ |11\rangle \mapsto |10\rangle$$

$$U_{XOR} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} = \exp\left(-\frac{i}{\hbar}\int dt H(t)\right)$$

Spin Qubits under Study (Among Others)

Quantum dots (spin and charge states)

Molecular magnets

Donor atoms

Defects in diamond

Optically trapped atoms

ion traps

...

Spin Qubits under Study (Among Others)

Quantum dots (spin and charge states)

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In this lecture, we will mostly discuss spin qubits in quantum dots

Historical remarks:
Electron qubit: `spin better than charge‘

due to longer relaxation/decoherence* times

$$\tau_\phi^{spin} >> \tau_\phi^{charge}$$

10ns -1min

1-10ns

GaAs

Awschalom et al., ’97
Tarucha et al., ‘02
Kouwenhoven/Vandersypen et al., ’03-’09
Abstreiter et al., ’04/08; Warburton et al., ‘09
Zumbuhl et al., '08-’17
Marcus, Yacoby et al.,’05- ’15 ($T_2 \sim$ 270…850 μs)

‘mesoscopics’
Fujisawa et al. ‘03
Marcus et al. ‘01

➔ natural choice for qubit: spin ½ of electron

*) theory: $T_2 \sim T_1$ for single spin in GaAs dot (‘everything optimized’)

Magnetic moment of single spin (Bohr magneton) is very weak:

Advantage: spin couples weakly to environment

➔ spin has long decoherence time (0.001-1000 μs)

Disadvantage: spin couples weakly to "observer"

➔ spin is difficult to control

Instead: control spin via charge, made possible by Pauli exclusion principle which "locks spin to charge"

➔ manipulation & detection of spin-dynamics via charge (orbital) degrees of freedom of electron

D. Loss & D. DiVincenzo, PRA **57** (1998) 120

PHYSICAL REVIEW A VOLUME 57, NUMBER 1 JANUARY 1998

Quantum computation with quantum dots

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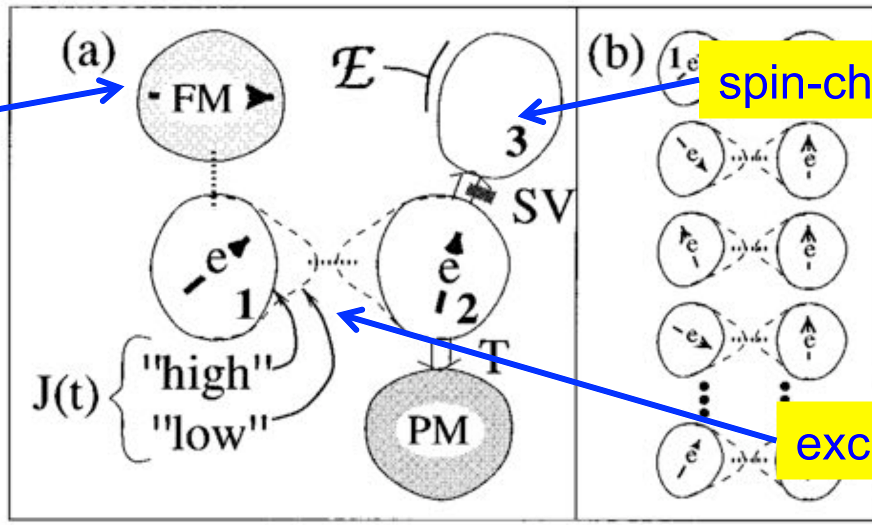
(Received 9 January 1997; revised manuscript received 22 July 1997)

spin rotation by exchange



spin-charge conversion for read-out

spintronics scheme

exchange coupling

$$U_{XOR} = e^{i\frac{\pi}{2}S_1^z}\, e^{-i\frac{\pi}{2}S_2^z}\, U_{SW}^{1/2}\, e^{i\pi S_1^z}\, U_{SW}^{1/2}$$

‘spintronics’= spin-electronics = all-electrical spin control

PHYSICAL REVIEW A VOLUME 57, NUMBER 1 JANUARY 1998

Quantum computation with quantum dots

Times Cited:
~ 6400

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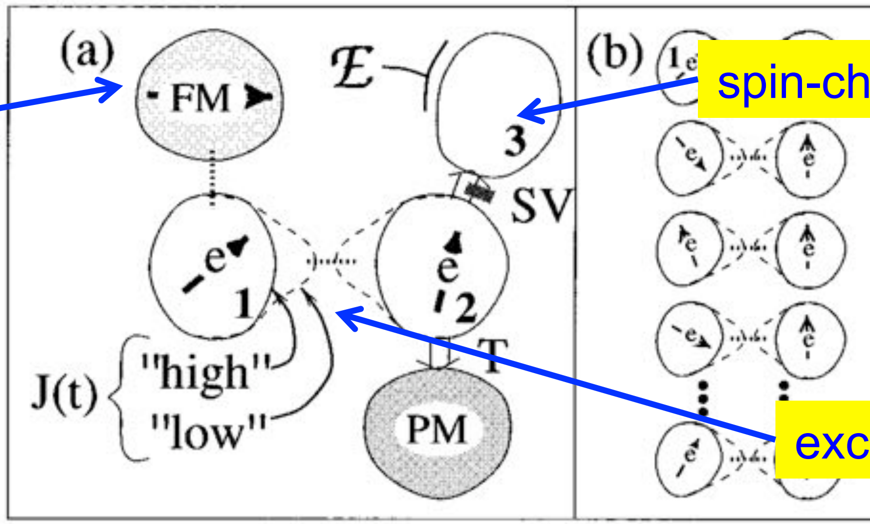

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Electric fields vs. Magnetic fields

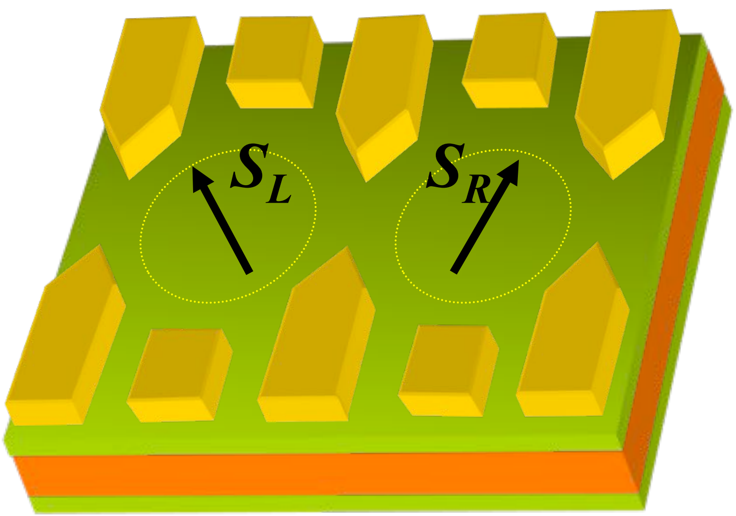
- Strong electric fields easy to produce (gates, STM-tips, etc)
- Fast switching of electric fields (picoseconds)
- Easy to apply electric fields locally and on nanoscale

- Strong magnetic (ac) fields hard to produce
- Slow switching of magnetic fields (nanoseconds)
- Hard to apply magnetic fields locally and on nanoscale

spinstronics

Quantum Processor for Spin-Qubits

DL & DiVincenzo, PRA **57** (1998)



2 quantum dots, each with
1 electron-spin (= qubit)

Key idea:
all-electrical control of spins
➔ scalable nanotechnology

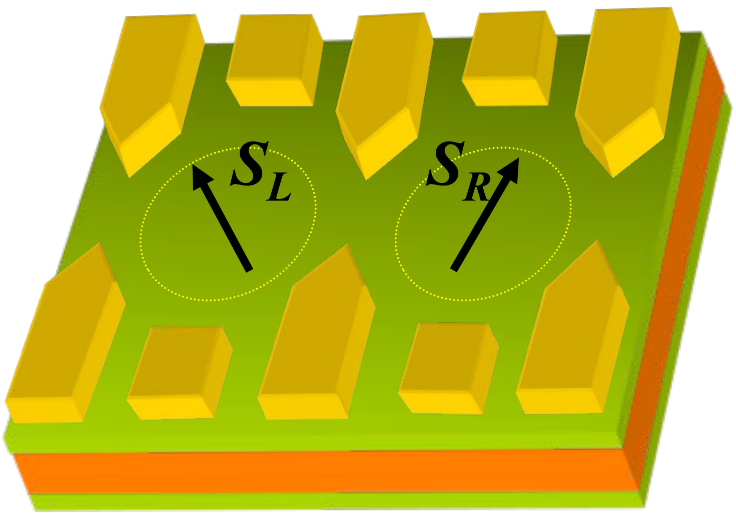
Simple effective Hamiltonian:

$$H(t) = J(t)\mathbf{S}_1 \cdot \mathbf{S}_2 + \mathbf{b}_1(t) \cdot \mathbf{S}_1 + \mathbf{b}_2(t) \cdot \mathbf{S}_2$$

Exchange coupling **Zeeman couplings**

Quantum Processor for Spin-Qubits

DL & DiVincenzo, PRA **57** (1998)



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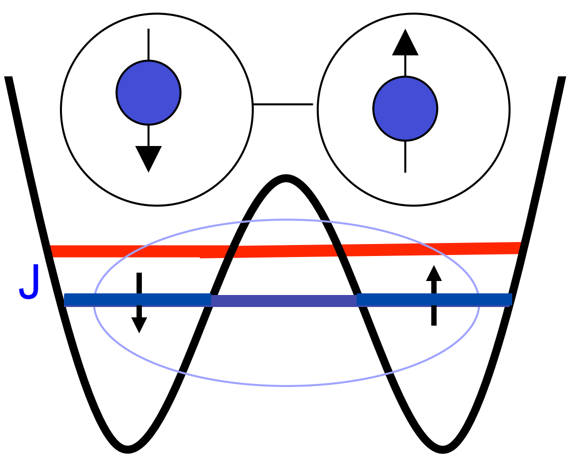


artificial hydrogen molecule ➔ exchange splitting $J \sim t^2/U$

➔ 'CNOT quantum gate'

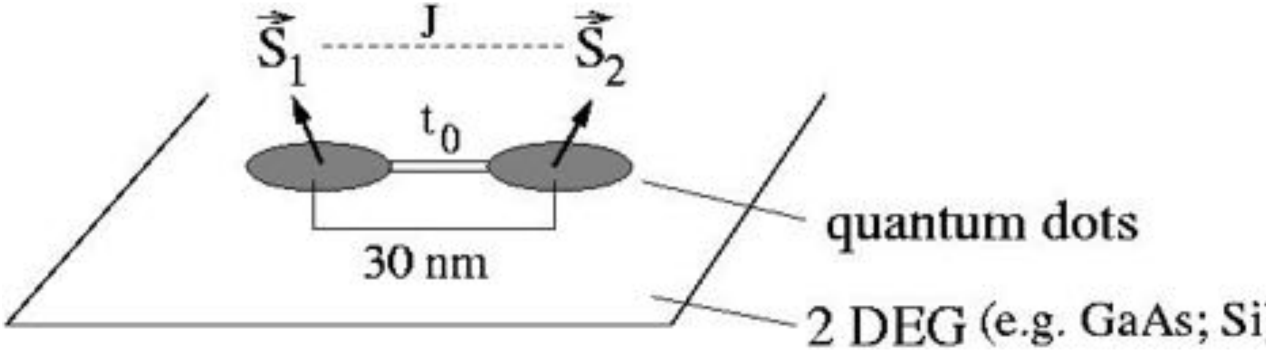
Quantum Dot Molecular Physics

- two coupled dots = artificial ´´H_2 – molecule''
- use approximative methods from molecular physics:
  → Heitler-London (valence orbits) Burkard ea, PRB 59,`99
  → Hund-Mullikan (molecular orbits) Burkard ea, PRB 59,`99
  → large scale numerics: Das Sarma & Hu ´01, Leburton ea ´01, Landman ea `01
- scale:

	artificial atom	real atom
energy	$\approx 1\,\mathrm{meV}$	$1\,\mathrm{Ry} = 13.6\,\mathrm{eV}$
length a_B	$\approx 100 - 500\,\bar{\mathrm{A}}$	$0.5\,\bar{\mathrm{A}}$

- magnetic length $l_B \approx 100$ Å at $B \approx 1$ T → molecular properties of quantum dots are very **sensitive to magnetic fields B**
- time dependent *B* field: → $J(t) = J(B(t))$
  electrical gate V(t): → $J(t) = J(V(t))$

quantum gate = two coupled dots



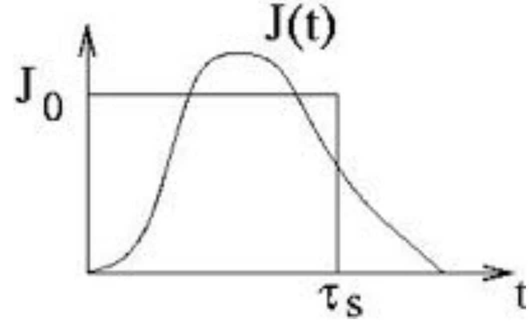
- idea: Hubbard physics: $J(t) \approx 4\, t_0(t)^2/U$ exchange

  $t_0 = t_0(t)$: tunable tunneling barrier

- e.g. swap and square-root-of-swap $U_{SW}^{1/2}$:

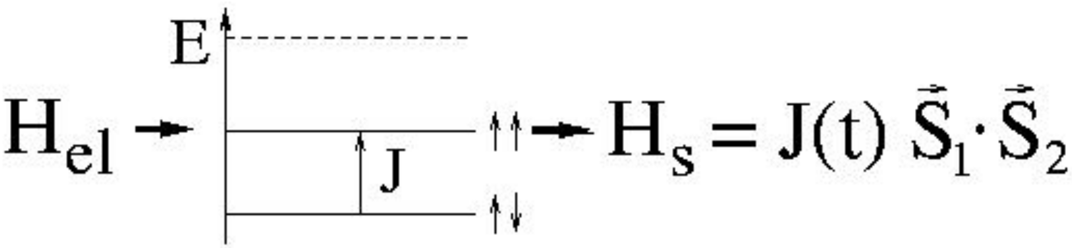
$$\int_0^t J(t')dt'/\hbar \approx J_0\tau_S/\hbar = \pi/2 (\mathrm{mod}\,\pi)$$

note: τ_s = 50 ps << T_2 = 1 ms (GaAs)

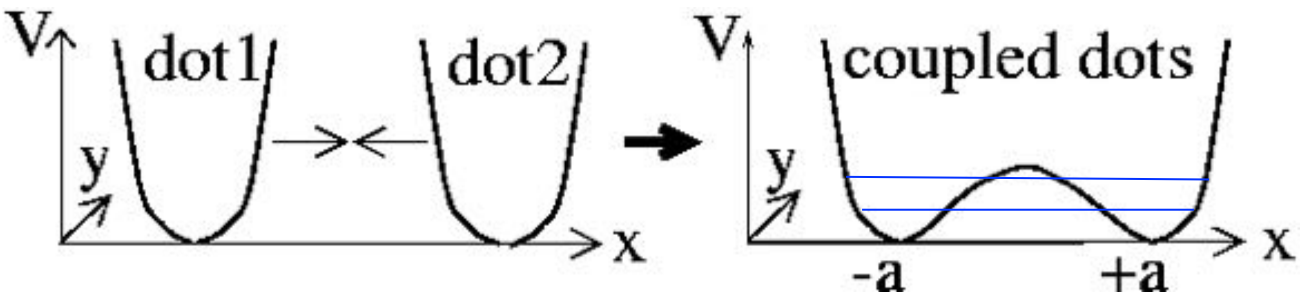


Electronic Model

- how do we find the exchange coupling *J*?



- 2D potential for electrons:



- Hamiltonian for one electron per dot:

$$
\begin{aligned}
H &= \sum_{i=1,2} h_i + \frac{e^2}{\epsilon\,|\vec{r}_1 - \vec{r}_2|} \\
h_i &= \frac{1}{2m}\left(\vec{p}_i - \frac{e}{c}\vec{A}(\vec{r}_i)\right)^2 + e x_i E + \frac{m\omega^2}{2}\left(\frac{1}{4a^2}\left(x_i^2 - a^2\right)^2 + y_i^2\right) \\
\vec{A}(\vec{r}) &= \frac{B}{2}(-y, x, 0); \qquad \vec{r} = (x, y, 0)
\end{aligned}
$$

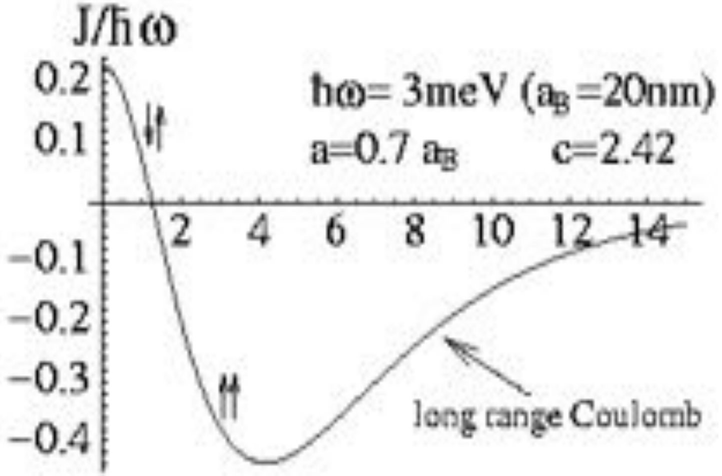
I. Heitler-London Method

- single-dot problem in a magnetic field has exact solution (Fock '28,Darwin '30) $\rightarrow \quad \varphi(\vec{r})$
  two-particle trial wavefunction (Heitler-London)

$$\begin{aligned} \Psi_\pm &= N\left[\varphi_{-a}(\vec{r}_1)\varphi_{+a}(\vec{r}_2) \pm \varphi_{-a}(\vec{r}_2)\varphi_{+a}(\vec{r}_1)\right] \\ J &= \langle \Psi_-|H_{el}|\Psi_-\rangle - \langle \Psi_+|H_{el}|\Psi_+\rangle \end{aligned}$$

- results: $d = a/a_B,\ b^2 = 1 + \omega_L^2/\omega_0^2,\ c \sim (e^2/\epsilon a_B)/\hbar\omega_0,\ \omega_L = eB/2m$
  (Burkard,Loss,DiVincenzo '99)

$$\begin{aligned} J &= \frac{\hbar\omega_0}{\sinh\left(2d^2(2b-1/b)\right)}\left[c\sqrt{b}\left(e^{-bd^2}\mathrm{I}_0\left(bd^2\right)\right.\right. \\ &- \left.\left. e^{d^2(b-1/b)}\mathrm{I}_0(d^2(b-1/b))\right) + \frac{3}{4b}\left(1+bd^2\right)\right] \end{aligned}$$



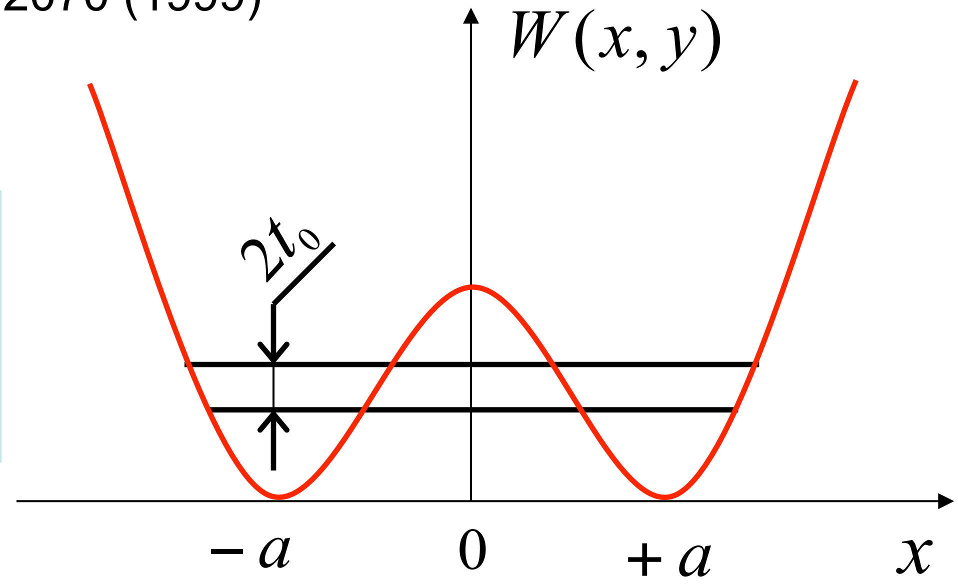
- Theorem: $J > 0$ for 2 electrons and $B = 0$.

(see also numerics by X. Hu et al., PRB ’00, include higher orbitals)

II. Hund-Mullikan calculation

Burkard, Loss, DiVincenzo, PRB **59**, 2070 (1999)

- confinement is approximated by a quartic potential, with typically $\hbar\omega_0$=3 meV



$$W(x,y) = \frac{m\omega_0^2}{2}\left(\frac{\left(x^2 - a^2\right)^2}{4a^2} + y^2\right)$$

- separates into two harmonic wells, if

$$a >> a_B \equiv \sqrt{\hbar/m\omega_0}$$

- Hamiltonian (neglecting the Zeeman splitting for GaAs):

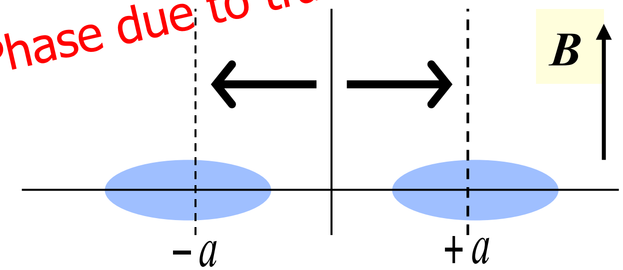
$$H_d = \sum_{i=1,2}\left[\frac{1}{2m}\left(\mathbf{p}_i - \frac{e}{c}\mathbf{A}(\mathbf{r}_i)\right)^2 + W(\mathbf{r}_i)\right] + \frac{e^2}{\kappa\left|\mathbf{r}_1 - \mathbf{r}_2\right|}$$

- gauge: $\mathbf{A} = \left(-yB/2,\ xB/2, 0\right) \quad \Rightarrow \quad B \| z$

• Fock-Darwin states (Fock '28;Darwin '30), translated by $\pm a$ in presence of magnetic field B (Burkard,Loss,DiVincenzo '99):

Phase due to translation

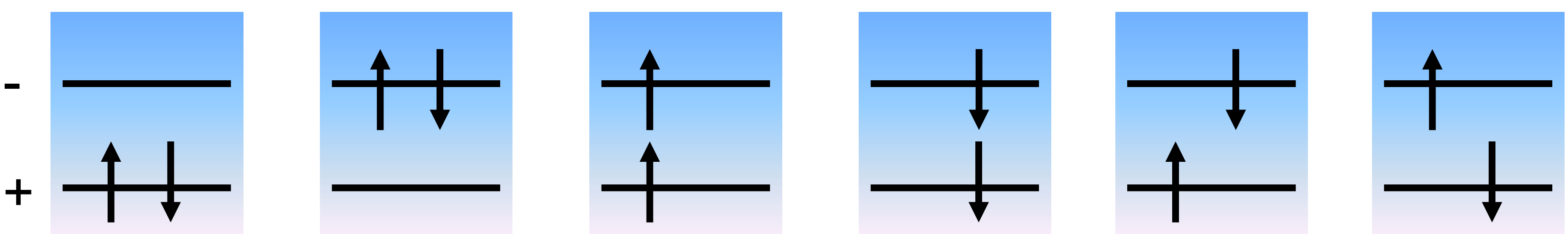
$$\varphi_{\pm a}(x,y)=\frac{1}{\lambda\sqrt{\pi}}\exp\left[-\frac{(x\mp a)^2+y^2}{2\lambda^2}\mp\frac{iya}{2l^2}\right]$$



$$l=\sqrt{\frac{\hbar c}{|e|B}}\qquad \lambda=\sqrt{\frac{\hbar}{m\omega}}\qquad \omega=\sqrt{\omega_0^2+\omega_L^2}\qquad \omega_L=\frac{|e|B}{2mc}$$

$$d_{\pm,\sigma}:\quad \psi_{\pm,\sigma}=\chi_\sigma\left(\varphi_{-a}\pm\varphi_{+a}\right)\Big/\sqrt{2(1\pm S)}\qquad S=\langle\varphi_{\pm a}|\varphi_{\mp a}\rangle$$

• two-particle states: ➔ *6 possible configurations*



J. Schliemann, D. Loss, and A. H. MacDonald, Phys. Rev. B **63**, 085311 (2001)
V. Golovach and D. Loss, Europhys. Lett. 62, 83 (2003)

Lowest energy eigenstates of DD:

$$|S1\rangle = \frac{1}{\sqrt{2}}\left(d^\dagger_{-\uparrow}d^\dagger_{+\downarrow} - d^\dagger_{-\downarrow}d^\dagger_{+\uparrow}\right)|0\rangle$$

$$|S2\rangle = \frac{1}{\sqrt{1+\phi^2}}\left(\phi\, d^\dagger_{+\uparrow}d^\dagger_{+\downarrow} + d^\dagger_{-\uparrow}d^\dagger_{-\downarrow}\right)|0\rangle$$

$$|T_+\rangle = d^\dagger_{-\uparrow}d^\dagger_{+\uparrow}|0\rangle, \quad |T_-\rangle = d^\dagger_{-\downarrow}d^\dagger_{+\downarrow}|0\rangle$$

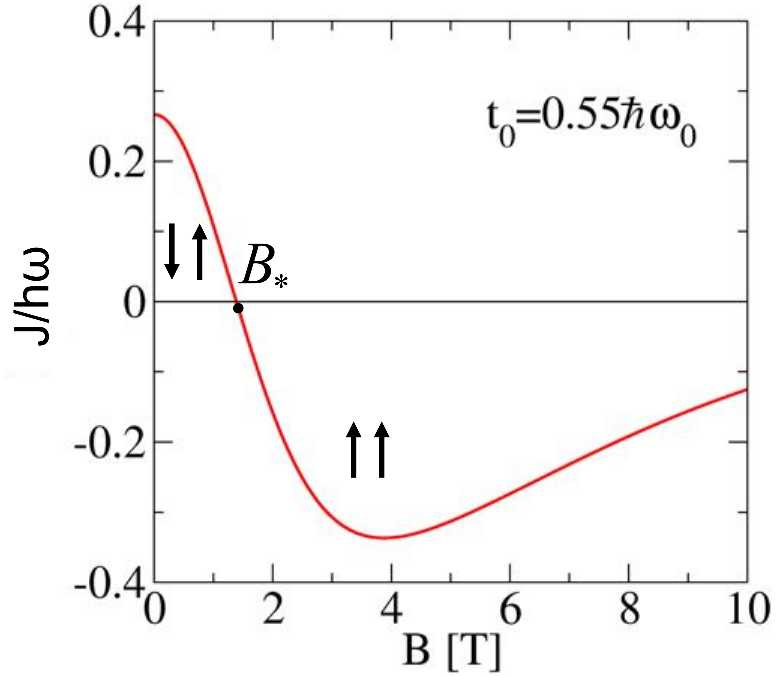
$$|T_0\rangle = \frac{1}{\sqrt{2}}\left(d^\dagger_{-\uparrow}d^\dagger_{+\downarrow} + d^\dagger_{-\downarrow}d^\dagger_{+\uparrow}\right)|0\rangle$$

$$|S\rangle = \frac{1}{\sqrt{1+\phi^2}}\left(d^\dagger_{+\uparrow}d^\dagger_{+\downarrow} - \phi\, d^\dagger_{-\uparrow}d^\dagger_{-\downarrow}\right)|0\rangle$$

$$\phi = \sqrt{1+\left(\frac{4t_H}{U_H}\right)^2} - \frac{4t_H}{U_H}$$

Exchange:

$$J = \mathrm{v} - \frac{U_H}{2} + \frac{1}{2}\sqrt{U_H^2 + 16t_H^2}$$



concurrence: $c[|S\rangle] = \frac{2\phi}{1+\phi^2}$

double occupancy: $D[|S\rangle] = \frac{(1-\phi)^2}{2(1+\phi^2)^2}$

Burkard et al. '99; Schliemann et al. '00, Golovach et al. '03

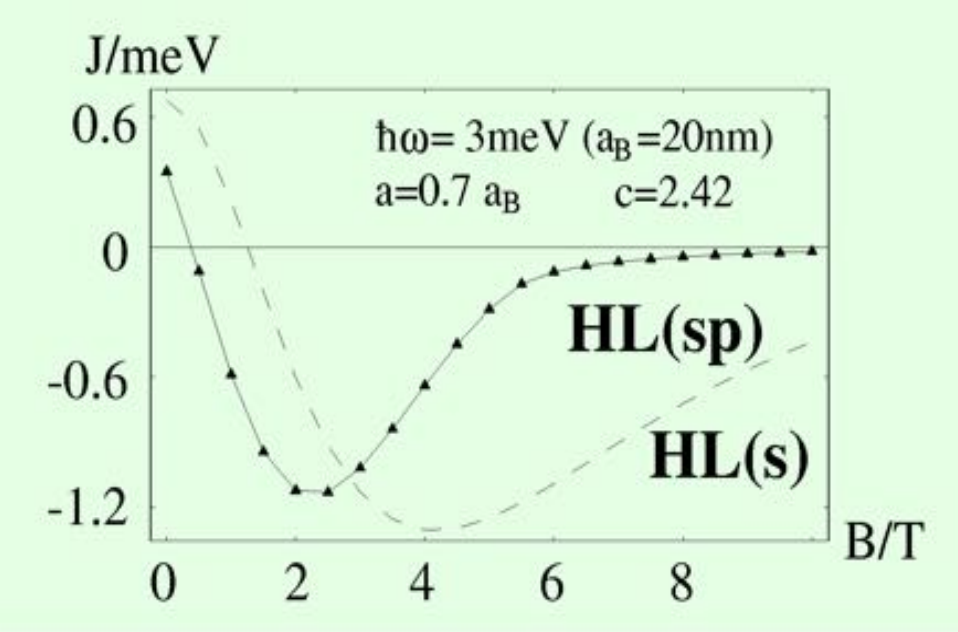
Lateral Coupling (GaAs dots)

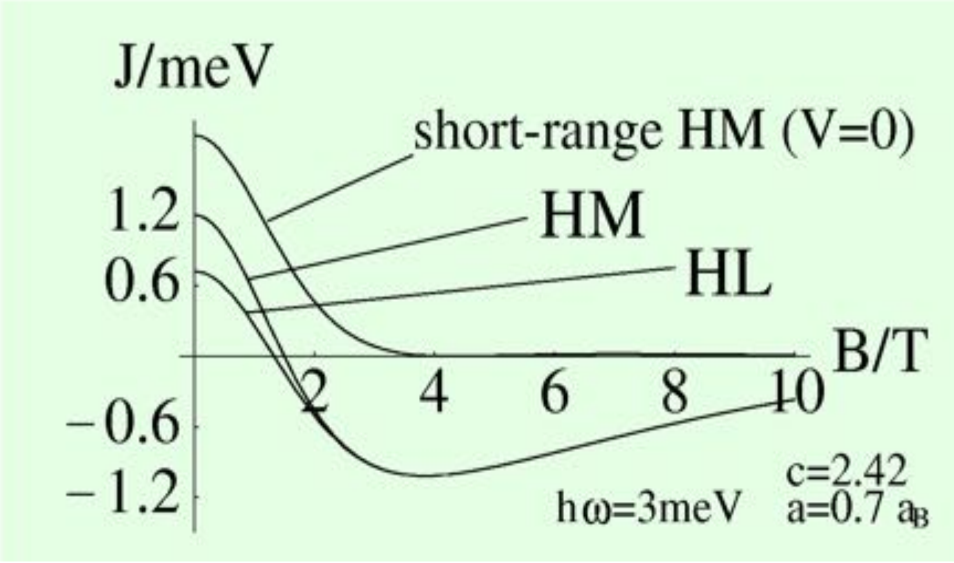
- **extended** Hubbard physics:

$$J = \frac{4t^2}{U} + V(B)$$

- note: HM→HL for increasing on-site Coulomb repulsion, i.e.

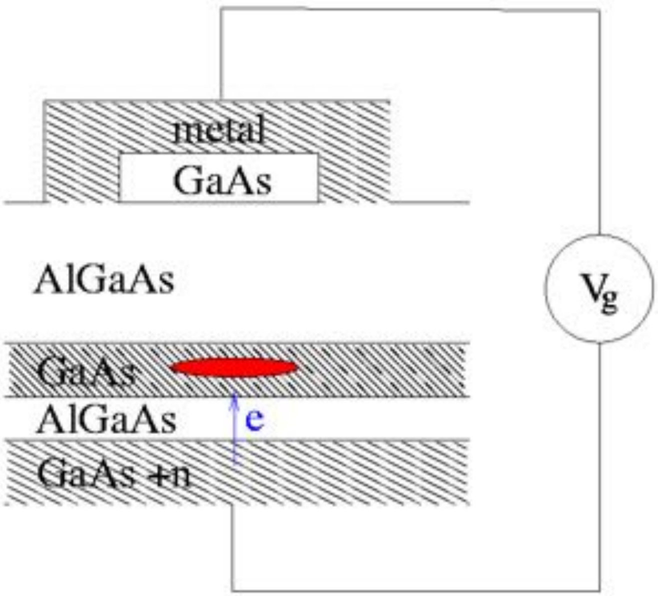
$$U(B,c) \sim c\sqrt{B} \nearrow$$

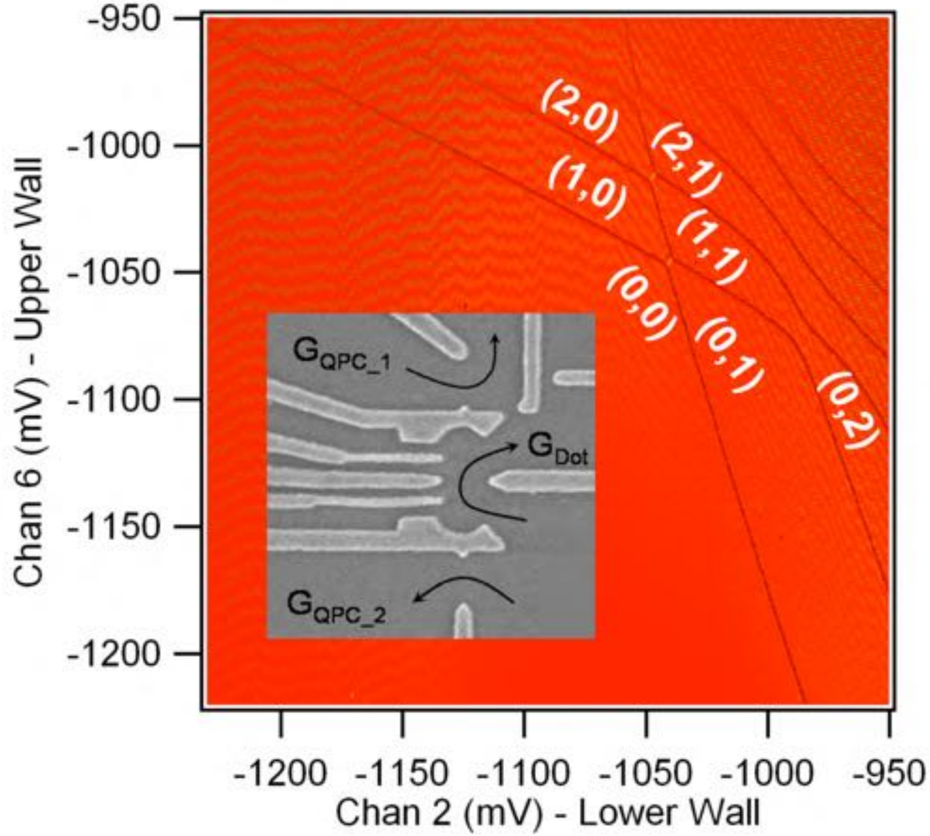




GaAs/AlGaAs Heterostruktur
2DEG 90 nm depth, $n_s = 2.9 \times 10^{11}$ cm^{-2}

Temp.: 100 mK

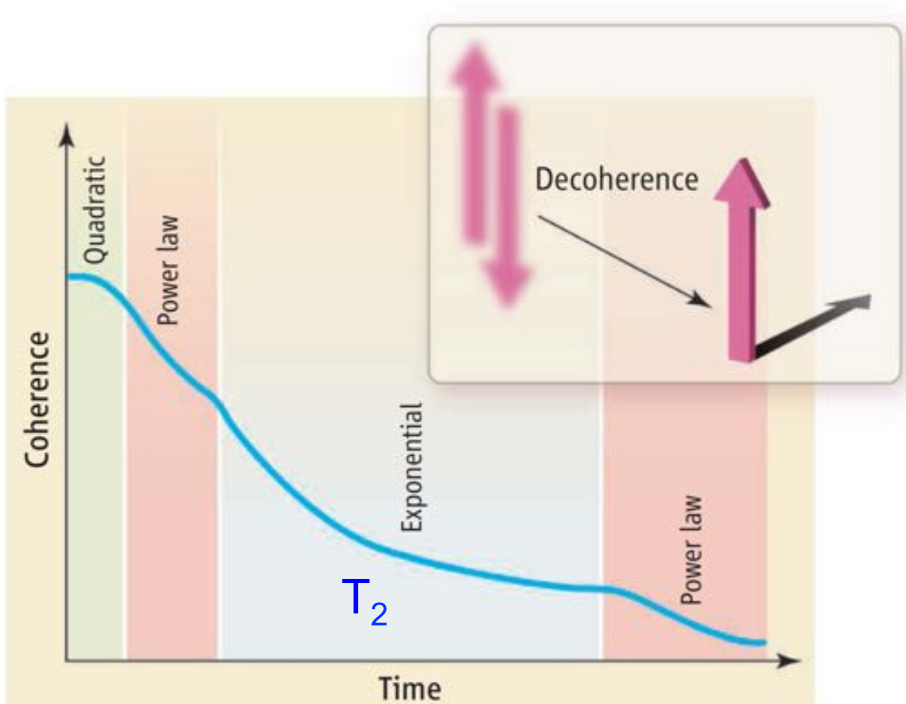




C. Marcus *et al.*, PRL 2004

Many sources cause **decoherence** of spin qubit:

Fischer and DL, Science 324, 1277 (2009)



time scale: T_2

The goal is to reach long decoherence times T_2
and short gate times τ such that $T_2 / \tau > 10^3$!

Sources of spin decoherence in GaAs quantum dots:

• spin-orbit interaction (band structure effects):
couples lattice vibrations with spin ➔ spin-phonon interaction, but weak in quantum dots due to 1. low momentum, 2. no 1st order s-o terms due to confinement (Khaetskii&Nazarov, '00; Golovach et al., '04-'10)

• spin-orbit intercation ➔ gate errors (XOR); but they can be minimized (Bonesteel et al., Burkard et al., '02, '03)

• dipole-dipole interaction: weak

• hyperfine interaction with nuclear spins: dominant decoherence source (Burkard, DL, DiVincenzo, PRB '99; Coish &DL, 2004-10, Das Sarma 2006…, Erlingson&Nazarov 2002,…), but absent e.g. in Si/Ge based dots!

Swichting Rate

Determine $N_{Op} \approx \tau_{\phi} / \tau_{s}$ for GaAs

- calculate *J*(*v*) **statically** and then take *J*(*t*) = *J*(*v*(*t*)) for time-dependent *v*(*t*), where *v* = *V*, *B*, *a*, *E* is control parameter
- sufficient criterion for this to work [$\bar{J} = (1/\tau_S) \int_0^{\tau_S} dt\, J(t)$]

$$1/\tau_S \approx |\dot{v}/v| \ll \bar{J}/\hbar$$ **adiabaticity condition**

- compatible with $J\tau_S = n\pi$, $n = 1, 3, 5, \ldots$ (needed for XOR)
- self-consistency of calculation of *J*: $J \ll \Delta\epsilon$
- thus: $1/\tau_S \ll \bar{J}/\hbar \ll \Delta\epsilon/\hbar$ $, \pi U^2 / 8t_0$ (no double occupancy)
- numbers: $J \approx 0.2\,\mathrm{meV} \rightarrow$ $\tau_S \gtrsim 50\,\mathrm{ps}$ ➔ very fast gate
- decoherence of spin ca. 100 μs

➡ $N_{Op} \approx \tau_{\varphi} / \tau_{s} \approx 10^6$ sufficient for upscaling

Quantum XOR (CNOT) via Hamiltonian

$$U(t) = T \exp\left\{-\frac{i}{\hbar}\int_0^t H(t')\,dt'\right\},\ H \neq 0 \text{ during } \tau_s$$

can show that: (Loss+DiVincenzo, PRA **57** (120), 1998)

$$U_{\text{XOR}} \leftrightarrow \int^t H_{\text{XOR}} = \pi S_1^z S_2^z - \frac{\pi}{2}(S_1^z + S_2^z) \qquad s = \frac{1}{2}$$

H_{XOR} is **pure** Ising: not very physical (for **real** spin) !
instead use **Heisenberg** $H = J\mathbf{S}_1 \cdot \mathbf{S}_2$ for U_{XOR}
and **Zeeman** $H_B = \mathbf{B}_1 \cdot \mathbf{S}_1 + \mathbf{B}_2 \cdot \mathbf{S}_2$ for single-qubit operations

e.g. swap gate: qubit 1 $\leftrightarrow$ qubit 2, choose $\int^t J(t)/\hbar \approx J_0\tau_s/\hbar = \pi \,(\text{mod} 2\pi)$

$$\Rightarrow U(t) = e^{i\pi/4} U_{\text{sw}} = e^{i\pi/4}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

basis: $\{|\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle\}$

$$U_{\text{XOR}} = e^{i\frac{\pi}{2}S_1^z}\, e^{-i\frac{\pi}{2}S_2^z}\, U_{\text{sw}}^{\frac{1}{2}}\, e^{i\pi S_1^z}\, U_{\text{sw}}^{\frac{1}{2}}$$

(DL+DDV '97)

Entanglement with 'sqrt-of-swap'

$$U_{XOR} = e^{i(\pi/2)S_1^z} e^{-i(\pi/2)S_2^z} U_{SW}^{1/2} e^{+i\pi S_1^z} U_{SW}^{1/2},$$

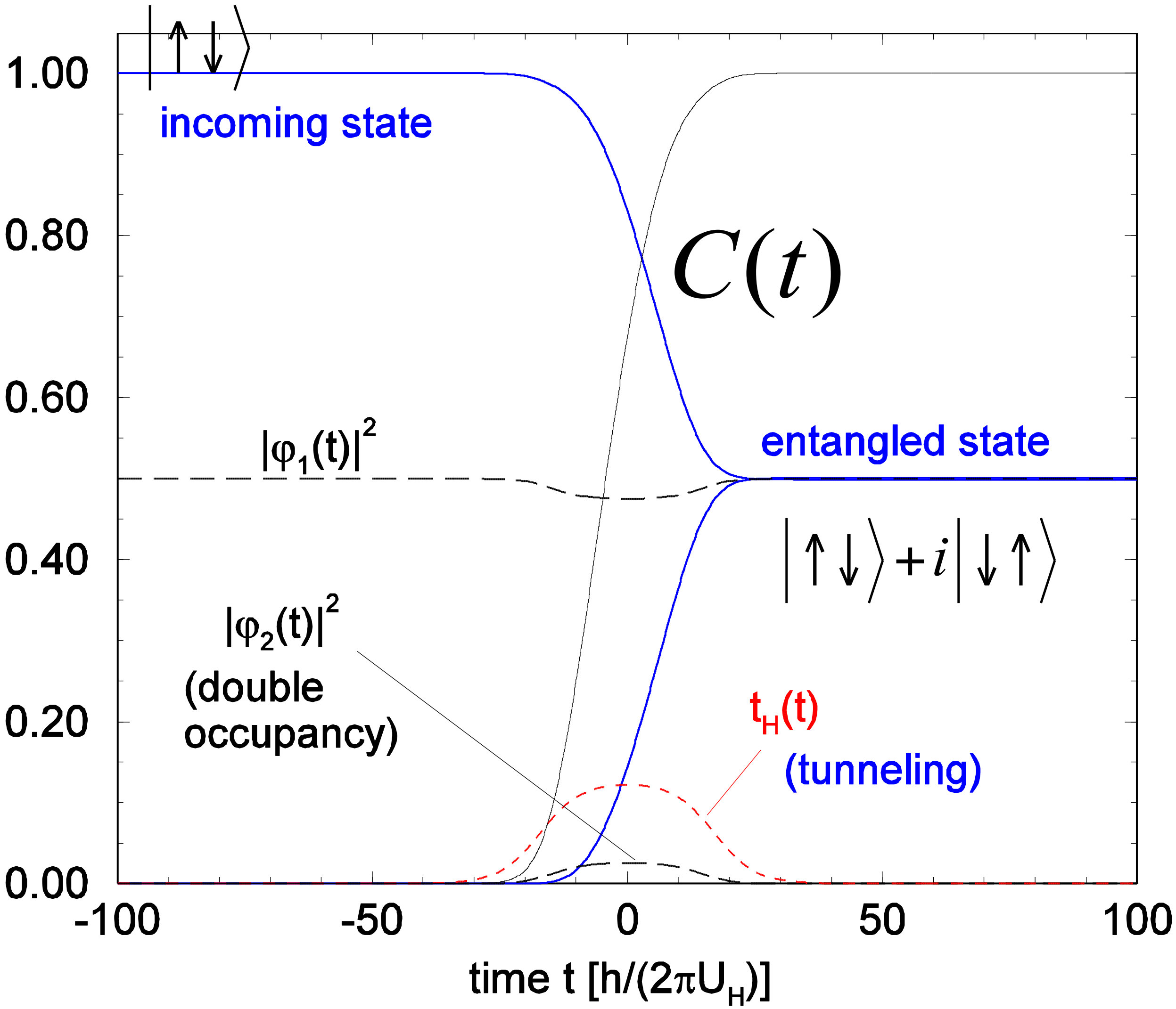
Square-root-of-swap: $|\uparrow\downarrow\rangle \xrightarrow{U_{SW}^{1/2}} |\uparrow\downarrow\rangle + i\,|\downarrow\uparrow\rangle$

$= |\uparrow\rangle_1 \times |\downarrow\rangle_2$

= entangler: product state ➔ entangled state

➔ Entanglement is crucial for quantum computing!

Dynamics of Entanglement for square-root-of-swap



The square-root of a swap is obtained by halfing the duration of the tunneling pulse.

The result is a fully entangled two-qubit state having only a vanishingly small amplitude for double-occupancies of one of the dots.

During the process the indistinguishability of electrons and their fermionic statistics are essential.

J. Schliemann, D. Loss, and A. H. MacDonald, Phys. Rev. B **63**, 085311 (2001)

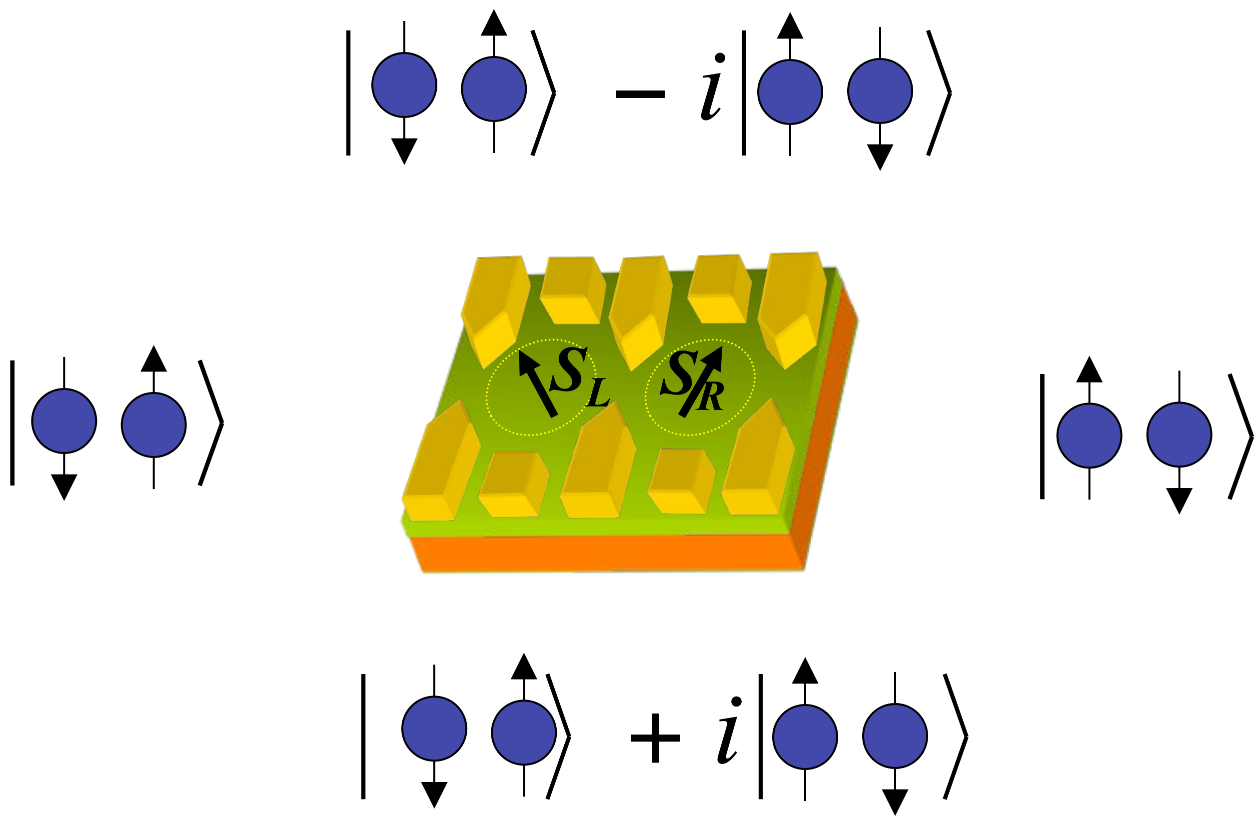
Quantum XOR gate (DL & DDV `97)

$$U_{XOR} = e^{i\frac{\pi}{2}S_1^z} e^{-i\frac{\pi}{2}S_2^z} U_{SW}^{1/2} e^{+i\pi S_1^z} U_{SW}^{1/2},$$

	$\lvert\uparrow\uparrow\rangle$	$\lvert\uparrow\downarrow\rangle$	$\lvert\downarrow\uparrow\rangle$	$\lvert\downarrow\downarrow\rangle$
$U_{SW}^{1/2}$	$\Downarrow$	$\Downarrow$	$\Downarrow$	$\Downarrow$
	$\lvert\uparrow\uparrow\rangle$	$\frac{e^{-i\pi/4}}{\sqrt{2}}(\lvert\uparrow\downarrow\rangle + i\,\lvert\downarrow\uparrow\rangle)$	$\frac{e^{-i\pi/4}}{\sqrt{2}}(i\,\lvert\uparrow\downarrow\rangle + \lvert\downarrow\uparrow\rangle)$	$\lvert\downarrow\downarrow\rangle$
$e^{i\pi S_1^z}$	$\Downarrow$	$\Downarrow$	$\Downarrow$	$\Downarrow$
	$i\,\lvert\uparrow\uparrow\rangle$	$\frac{ie^{-i\pi/4}}{\sqrt{2}}(\lvert\uparrow\downarrow\rangle - i\,\lvert\downarrow\uparrow\rangle)$	$\frac{ie^{-i\pi/4}}{\sqrt{2}}(i\,\lvert\uparrow\downarrow\rangle - \lvert\downarrow\uparrow\rangle)$	$-i\,\lvert\downarrow\downarrow\rangle$
$U_{SW}^{1/2}$	$\Downarrow$	$\Downarrow$	$\Downarrow$	$\Downarrow$
	$i\,\lvert\uparrow\uparrow\rangle$	$\lvert\uparrow\downarrow\rangle$	$-\,\lvert\downarrow\uparrow\rangle$	$-i\,\lvert\downarrow\downarrow\rangle$
$e^{-i\frac{\pi}{2}S_2^z} \times e^{i\frac{\pi}{2}S_1^z}$	$\Downarrow$	$\Downarrow$	$\Downarrow$	$\Downarrow$
	$i\,\lvert\uparrow\uparrow\rangle$	$i\,\lvert\uparrow\downarrow\rangle$	$i\,\lvert\downarrow\uparrow\rangle$	$-i\,\lvert\downarrow\downarrow\rangle$

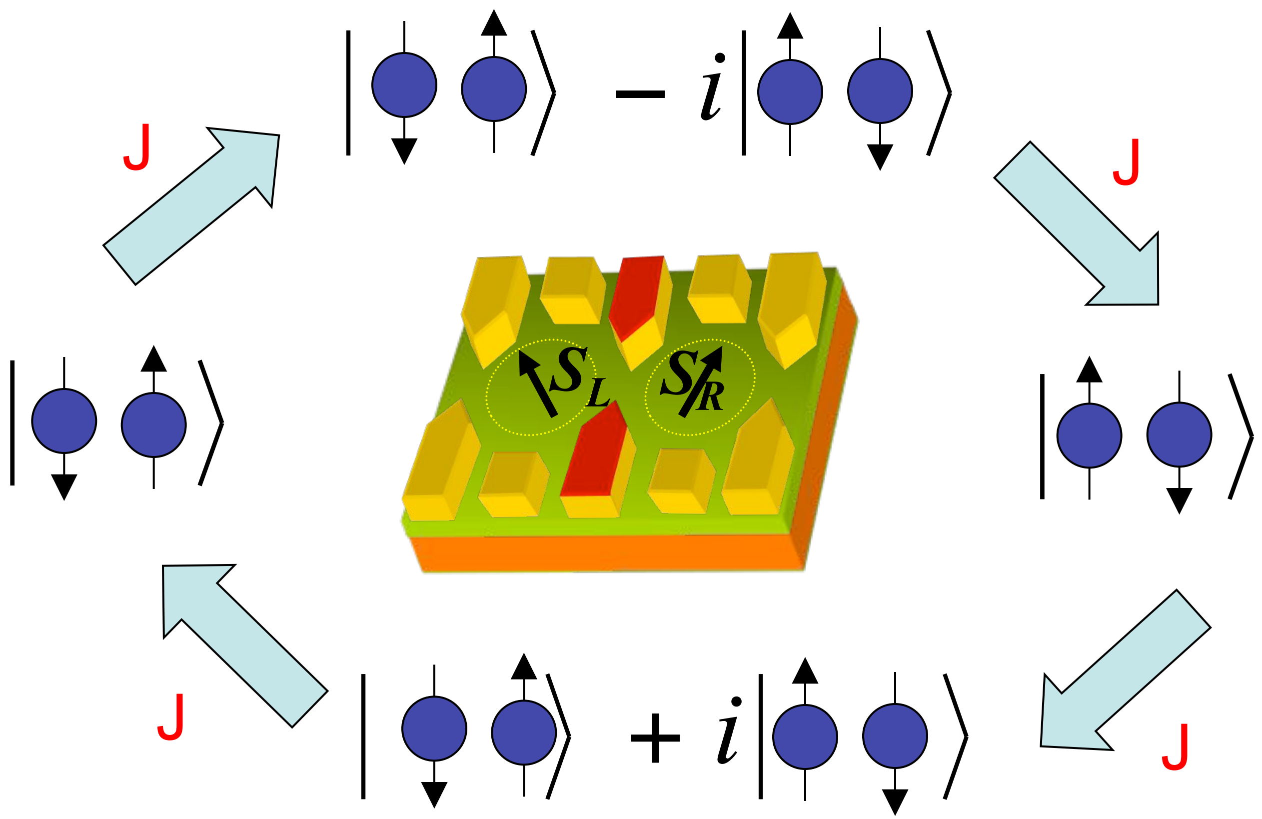
How to make entanglement 'visible'

Loss & DiVincenzo, 1998



How to make entanglement 'visible'

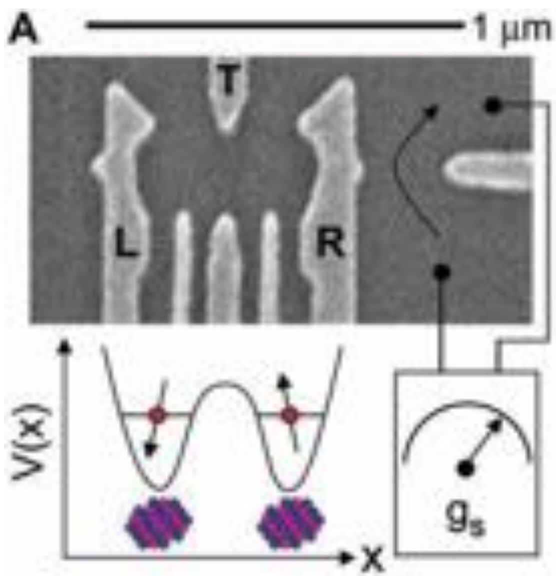
Loss & DiVincenzo, 1998

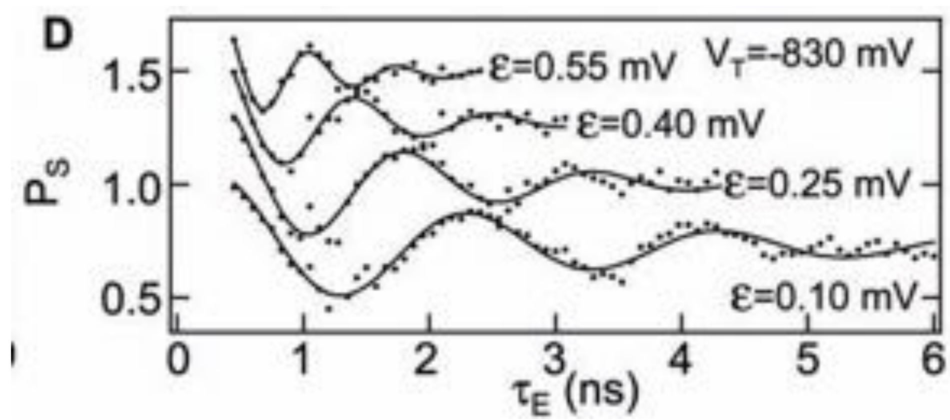


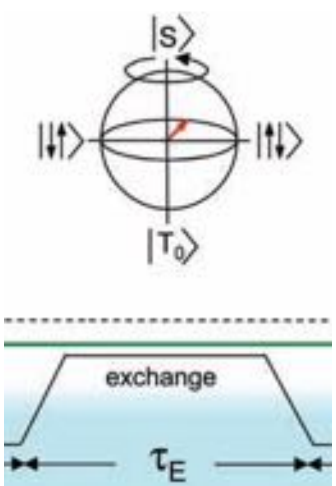
➔ entanglement oscillates !

Entanglement oscillations

Petta, Yacoby, Marcus et al., Science 2005



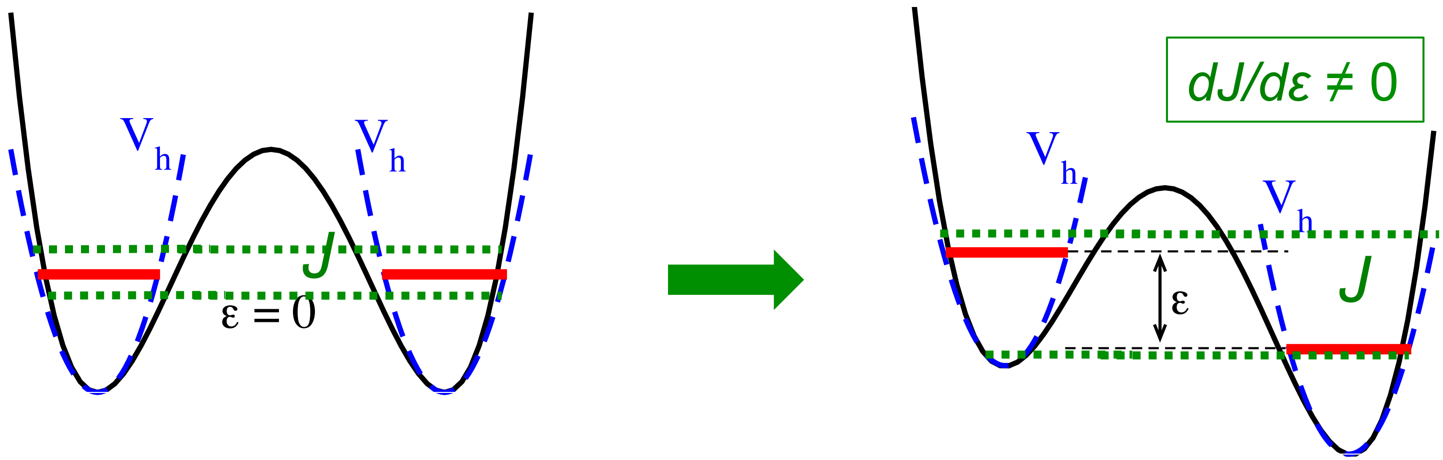




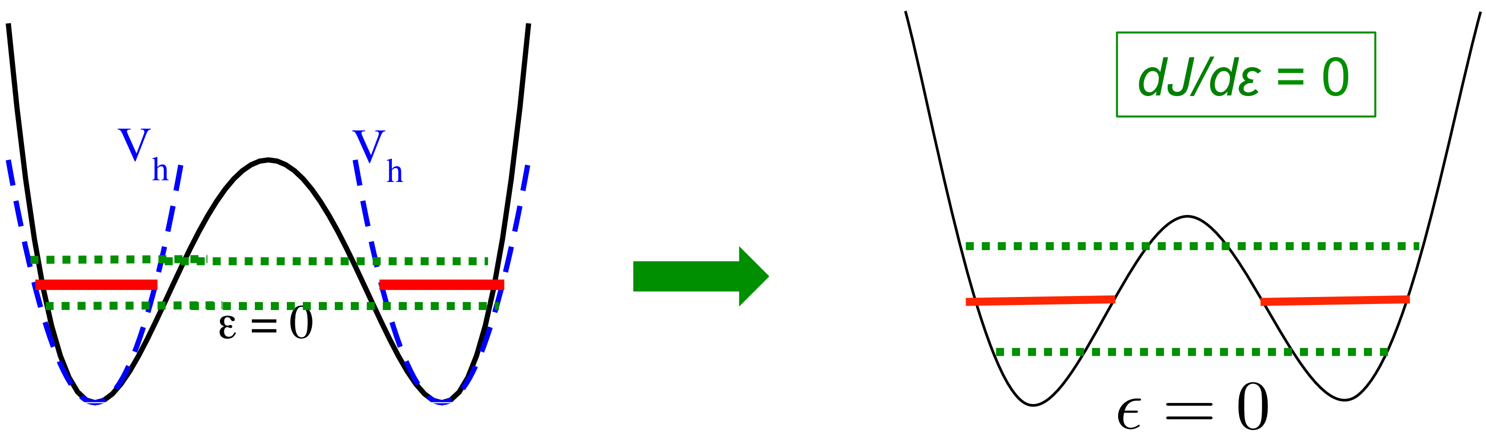
ultra-fast 'clock speed':
entanglement generated in 180 ps !

Switching of exchange *J*

1. *Asymmetric* via bias ε



2. *Symmetric* via barrier height



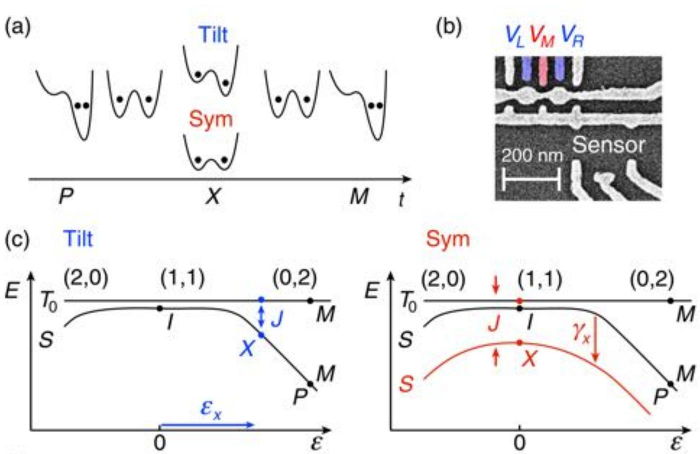
Burkard, Loss, and DiVincenzo, PRB (1999)

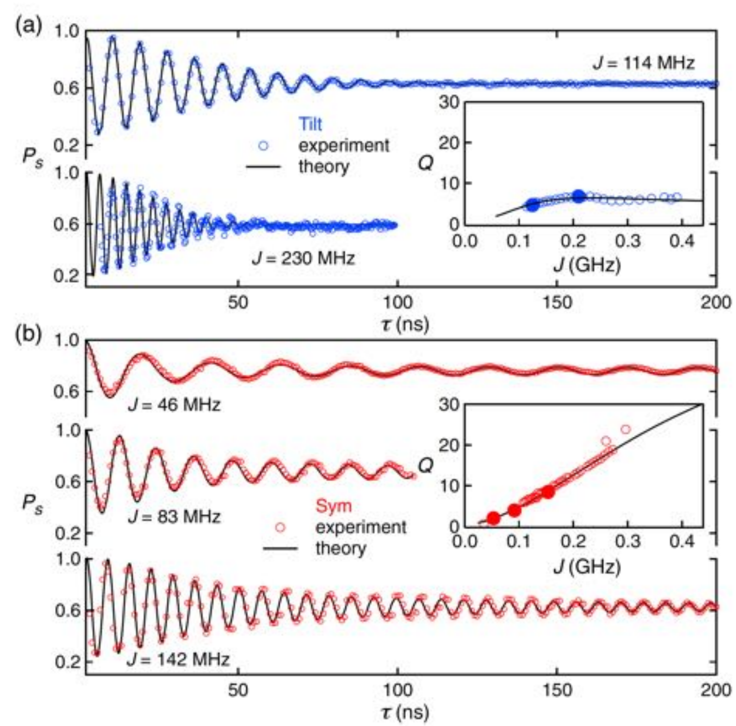
PRL 116, 116801 (2016) PHYSICAL REVIEW LETTERS week ending 18 MARCH 2016

Noise Suppression Using Symmetric Exchange Gates in Spin Qubits

Frederico Martins,[1] Filip K. Malinowski,[1] Peter D. Nissen,[1] Edwin Barnes,[2,3] Saeed Fallahi,[4] Geoffrey C. Gardner,[5] Michael J. Manfra,[4,5,6] Charles M. Marcus,[1] and Ferdinand Kuemmeth[1,*]
[1]Center for Quantum Devices, Niels Bohr Institute, University of Copenhagen, 2100 Copenhagen, Denmark

Sqrt-of-Swap gate via J





asymmetric

symmetric

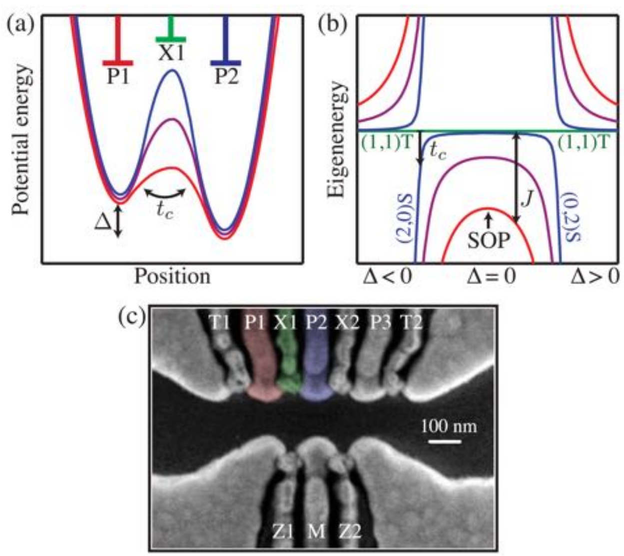
FIG. 3. (a) Tilt-induced exchange oscillations (i.e., $\gamma_x = 0$ mV) for $\varepsilon_x = 79.5$ and 82 mV, generating oscillation frequencies indicated by J. (b) Same as (a) but for the symmetric mode of operation ($\varepsilon_x = 13.5$ mV), with $\gamma_x = 100$, 120, and 140 mV.

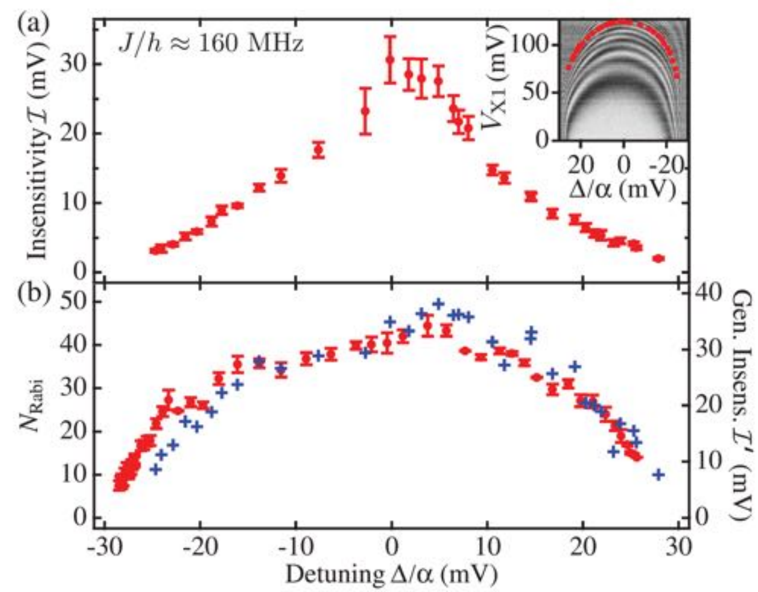
PRL 116, 110402 (2016) PHYSICAL REVIEW LETTERS week ending 18 MARCH 2016

Reduced Sensitivity to Charge Noise in Semiconductor Spin Qubits via Symmetric Operation

M. D. Reed, B. M. Maune, R. W. Andrews, M. G. Borselli, K. Eng, M. P. Jura, A. A. Kiselev, T. D. Ladd, S. T. Merkel, I. Milosavljevic, E. J. Pritchett, M. T. Rakher, R. S. Ross, A. E. Schmitz, A. Smith, J. A. Wright, M. F. Gyure, and A. T. Hunter*
HRL Laboratories, LLC, 3011 Malibu Canyon Road, Malibu, California 90265, USA
(Received 5 August 2015; published 16 March 2016)

Si/Ge quantum dots





Serial vs. Parallel gate

I. Serial gate: LD, PRA 57, 120 (1998)

$$H(t) = J(t)\mathbf{S}_1 \cdot \mathbf{S}_2 + \mathbf{b}_1(t)\cdot\mathbf{S}_1 + \mathbf{b}_2(t)\cdot\mathbf{S}_2$$ controlled such that

$$H(t) = J(t)\mathbf{S}_1 \cdot \mathbf{S}_2$$ or $$H(t) = \mathbf{b}_1(t)\cdot\mathbf{S}_1 + \mathbf{b}_1(t)\cdot\mathbf{S}_2$$

$$U_{SW}^{1/2} = \exp(i\int_0^{\tau_s} dt J(t)\mathbf{S}_1 \cdot \mathbf{S}_2) \;,\quad if \int_0^{\tau_s} dt J(t) = \pi/2 + 2\pi n$$

$$U_{XOR} = e^{-i(\pi/2)S_2^y}[e^{i(\pi/2)S_1^z} e^{-i(\pi/2)S_2^z} U_{SW}^{1/2} e^{+i\pi S_1^z} U_{SW}^{1/2}] e^{i(\pi/2)S_2^y}$$

➔ need 7 pulses (5 for CPF)

Serial vs. Parallel gate

II. Parallel gate: Burkard et al., PRB 60, 11404 (1999)

$$H(t) = J(t)\mathbf{S}_1 \cdot \mathbf{S}_2 + \mathbf{b}_1(t) \cdot \mathbf{S}_1 + \mathbf{b}_2(t) \cdot \mathbf{S}_2$$

$$U_{CPF} = \exp(i \int_0^{\tau_s} dt H(t))$$

only 1 pulse for CPF !

$$if \quad \int J = \pi/2, \quad and \quad \int b_{1/2}^z = \pi(1 \pm \sqrt{3})/4$$

$$U_{XOR} = e^{-i(\pi/2)S_2^y} U_{CPF} e^{i(\pi/2)S_2^y}$$

➔ need only 3 pulses

Implementation scheme: Meunier *et al.*, PRB 83, 121403 (2011)

Single-Qubit Operations or *How to Flip a Spin*?

1. Electron Spin Resonance (ESR)

An ac magnetic field is applied perpendicular to a static magnetic field, with a frequency that matches the Zeeman splitting

2. Electric-Dipole-Induced Spin Resonance (EDSR)

Exploits spin-orbit interaction and ac electric field

3. Electrically Driven ESR in a Slanting Magnetic Field

Exploits a magnetic field gradient and ac electric field

4. Electrically Driven ESR in an exchange field of auxilliary spin

Exploits the exchange field, magnetic field gradient, and ac electric field

Single-Qubit Operations or *How to Flip a Spin*?

1. Electron Spin Resonance (ESR)

An ac magnetic field is applied perpendicular to a static magnetic field, with a frequency that matches the Zeeman splitting

Fast

2. Electric-Dipole-Induced Spin Resonance (EDSR)

Exploits spin-orbit interaction and **ac electric field**

3. Electrically Driven ESR in a Slanting Magnetic Field

Exploits a magnetic field gradient and **ac electric field**

4. Electrically Driven ESR in an exchange field of auxilliary spin

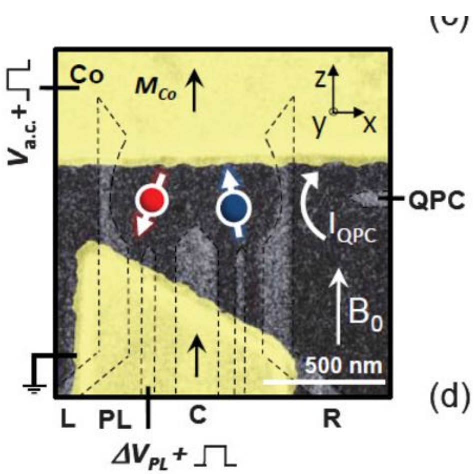
Exploits the exchange field, magnetic field gradient, and **ac electric field**

GHz

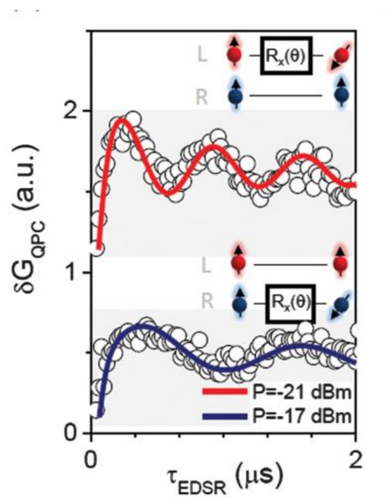
PRL 107, 146801 (2011) Selected for a Viewpoint in *Physics* PHYSICAL REVIEW LETTERS week ending 30 SEPTEMBER 2011

Two-Qubit Gate of Combined Single-Spin Rotation and Interdot Spin Exchange in a Double Quantum Dot

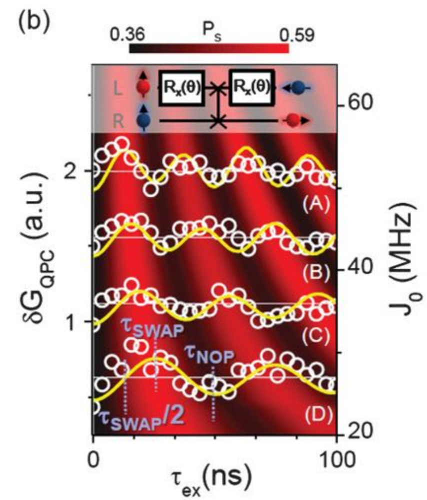
R. Brunner,[1,2,*] Y.-S. Shin,[1] T. Obata,[1,3] M. Pioro-Ladrière,[4] T. Kubo,[5] K. Yoshida,[1] T. Taniyama,[6,7] Y. Tokura,[1,5] and S. Tarucha[1,3]



double dot in
B field gradient



EDSR
~ MHz

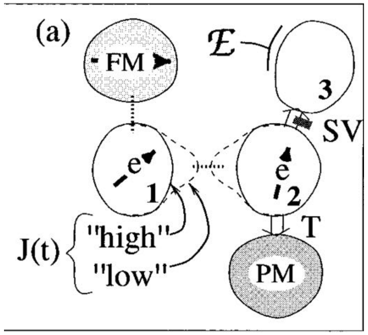


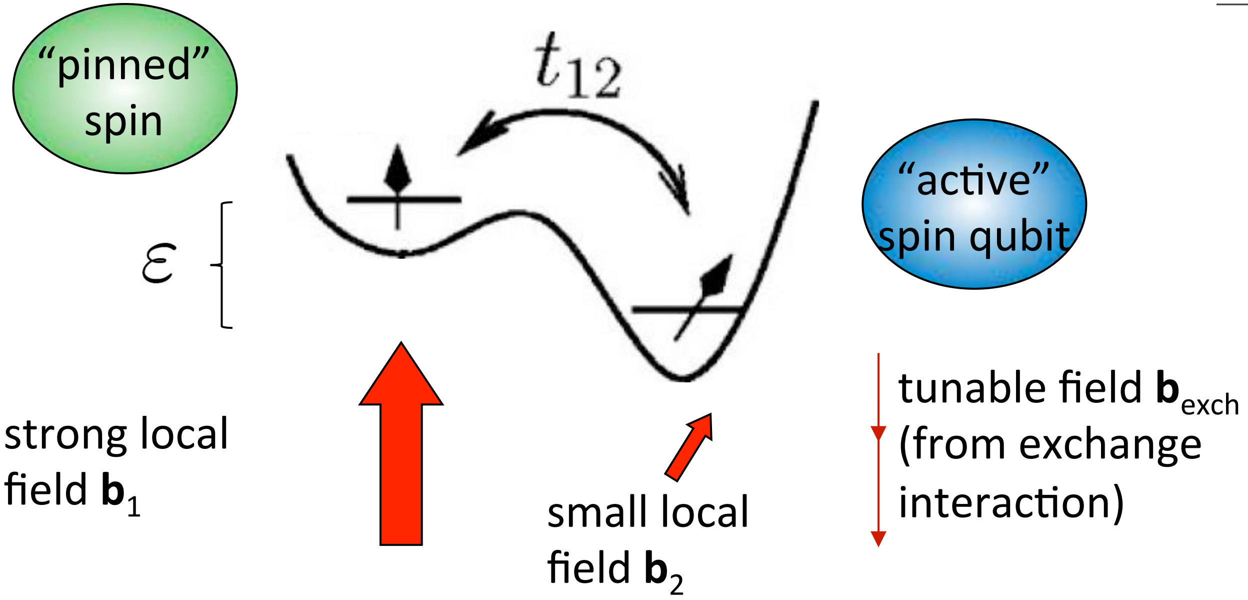
entanglement
~ GHz

Ultra-fast single-qubit gates via *exchange*

Alternative to ESR/EDSR: double dot with pulsed J-gate!

DL and DiVincenzo, PRA 57 (1998)
Coish and DL, PRB 75, 161302 (2007)
Chesi, Wang, Yoneda, Otsuka, Tarucha, and DL, PRB 90, 235311 (2014)



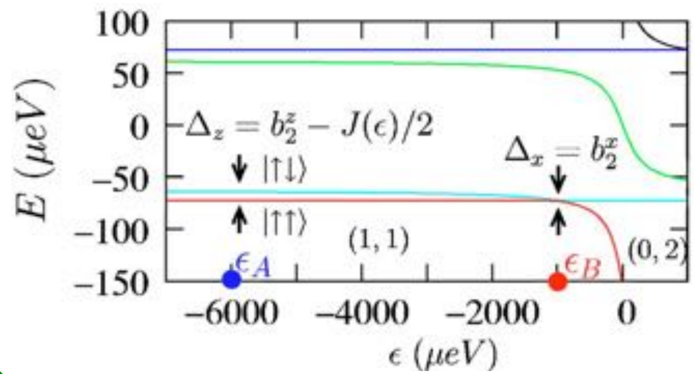


Single-qubit gates with high fidelity and ultrafast ~ 1 ns

Single-spin rotation via exchange: Two regimes

I. $b_1 \gg b_2$

Advantage: hybridization of logical states and (1,1) charge configuration can be made very small; but difficult to reach



Coish & DL, PRB 2007

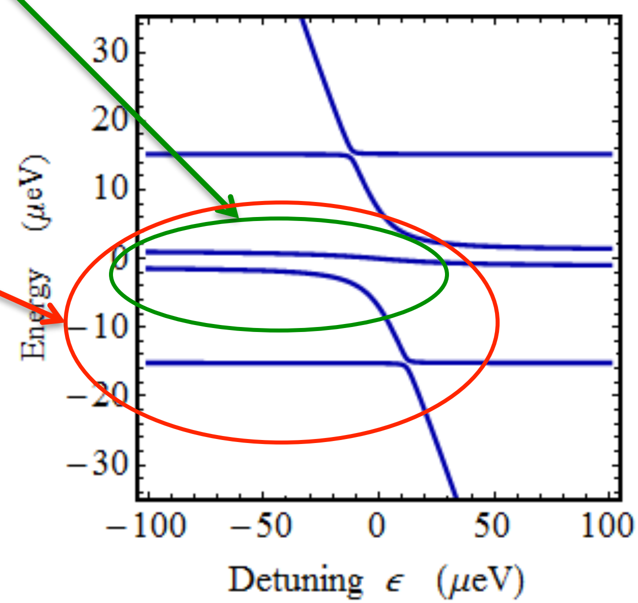
II. $b_1 \simeq b_2 \simeq B$

This is a more typical situation in exp.:

$B \gtrsim 1\ \mathrm{T}$ (due to saturation field of micromagnet)

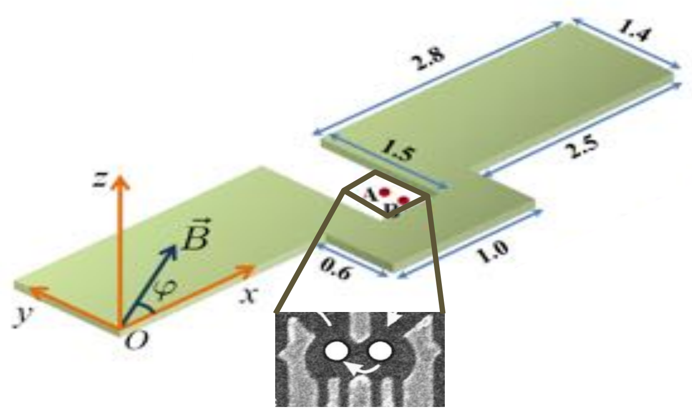
$\Delta b = b_1 - b_2 \simeq 10 - 50\ \mathrm{mT}$

Ultra-short gate times: 1 ns, with very high fidelity for GaAs double dots



Chesi et al., PRB 2014

Single-spin manipulation in double dots with micromagnet

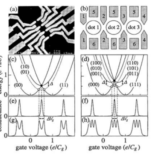


Chesi et al., PRB **90**, 235311 (2014)

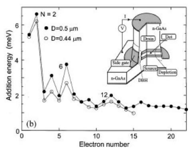
- Single-qubit gates implemented via exchange (as for two-qubit gates)
- Ultra-short gate times: 1 ns, with very high fidelity for GaAs double dots
- Noise sources: Nuclear and charge noise present but not a problem

Most Advanced: Spin qubits in **GaAs** quantum dots

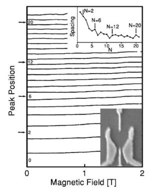
Kloeffel & DL, Annu. Rev. Condens. Matter Phys. 4, 51 (2013)



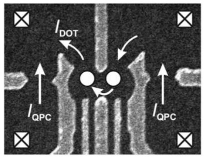
Westervelt
Gossard 1995



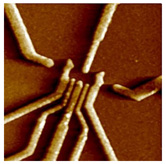
Kouwenhoven
Tarucha 1996



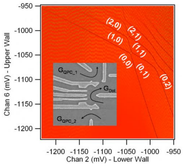
Sachrajda
2000



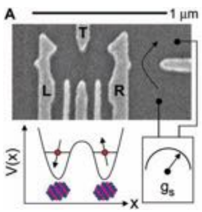
Kouwenhoven
Tarucha 2003-13



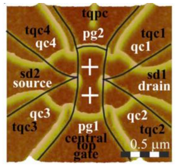
Vandersypen,
Koppens, 2003



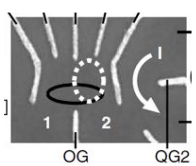
Marcus 2004



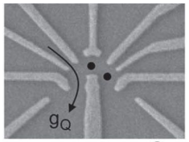
Petta, Marcus,
Yacoby 2005



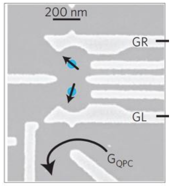
Ensslin, Ihn
2006



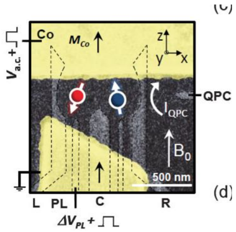
Zumbuhl,
Kastner
2008



Petta 2010



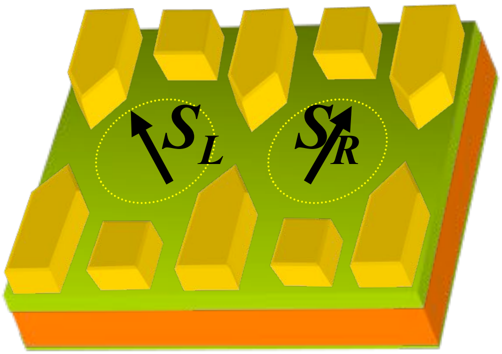
Bluhm, Dial,
Yacoby 2010-13
T_2 ~ 300 µs



Brunner,
Pioro-Ladriere,
Tarucha 2011

… and many more …

Spin-Qubits from Electrons



simplest spin-qubit:
spin-1/2 of 1 electron $|0\rangle = \uparrow, \ |1\rangle = \downarrow$

Many more choices for spin qubits:

- 'exchange-only qubits' DiVincenzo, Burkard *et al.* \`00; Sachrajda \`12; Marcus \`13;
  3 electrons: $|0\rangle = S\uparrow, \ |1\rangle = T_+\downarrow - T_0\uparrow$ Doherty \`15; Taylor \`16; Rashba/Halperin \`13

'singlet-triplet' qubits Levy \`02, Taylor *et al.* \`05, Klinovaja *et al.*\`12

2 electrons: $|0\rangle = S, \ |1\rangle = T_0$

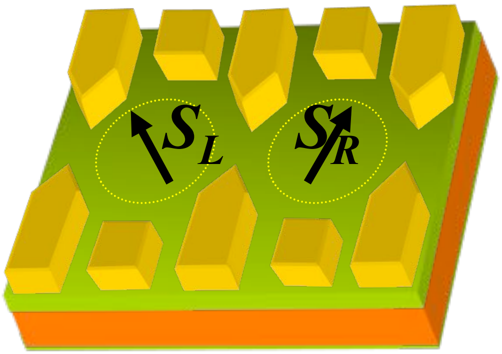
- 'spin-cluster qubits' Meier, Levy & DL, \`03
  N electrons: AF spin chains, ladders, clusters,...

- \`spin-orbit qubits‘ Golovach, Borhani &DL, \`07; Kouwenhoven *et al*., \`11;

. hole spins: Bulaev & DL, \`05; Marcus *et al*., \`11; Kloeffel, Trif & DL, \`11-‘16 (Si/Ge NW)

- molecular magnets Leuenberger & DL, ’01; Affronte *et al*., ’06,
  Lehmann *et al*., ’07; Trif *et al*., ’08, ’10, \`16

Spin-Qubits from Electrons



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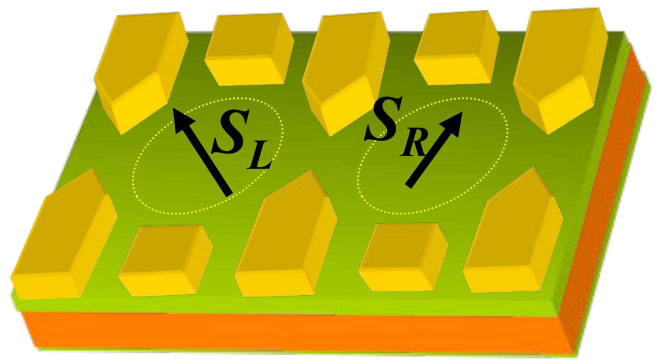
- 'exchange-only qubits' DiVincenzo, Burkard *et al.* `00; Sachrajda `12; Marcus `13;
  3 electrons: $|0\rangle = S\uparrow, |1\rangle = T_+\downarrow - T_0\uparrow$ Doherty `15; Taylor `16; Rashba/Halperin `13
- 'singlet-triplet' qubits Levy `02, Taylor *et al.* `05, Klinovaja *et al.*`12
  2 electrons: $|0\rangle = S, |1\rangle = T_0$
- 'spin-cluster qubits' Meier, Levy & DL, `03
  N electrons: AF spin chains, ladders, clusters,...
- `spin-orbit qubits' Golovach, Borhani &DL, `07; Kouwenhoven *et al.*, `11;

. hole spins: Bulaev & DL, `05; Marcus *et al.*, `11; Kloeffel, Trif & DL, `11-'16 (Si/Ge NW)

- molecular magnets Leuenberger & DL, '01; Affronte *et al.*, '06,
  Lehmann *et al.*, '07; Trif *et al.*, '08, '10, `16

Most popular spin qubits (in GaAs)



LD spin qubit:
spin-1/2 of 1 electron

Loss and DiVincenzo , *Phys. Rev. A* **57**, p120 (1998)

$$|0\rangle = \uparrow, \; |1\rangle = \downarrow$$

'singlet-triplet' qubits :

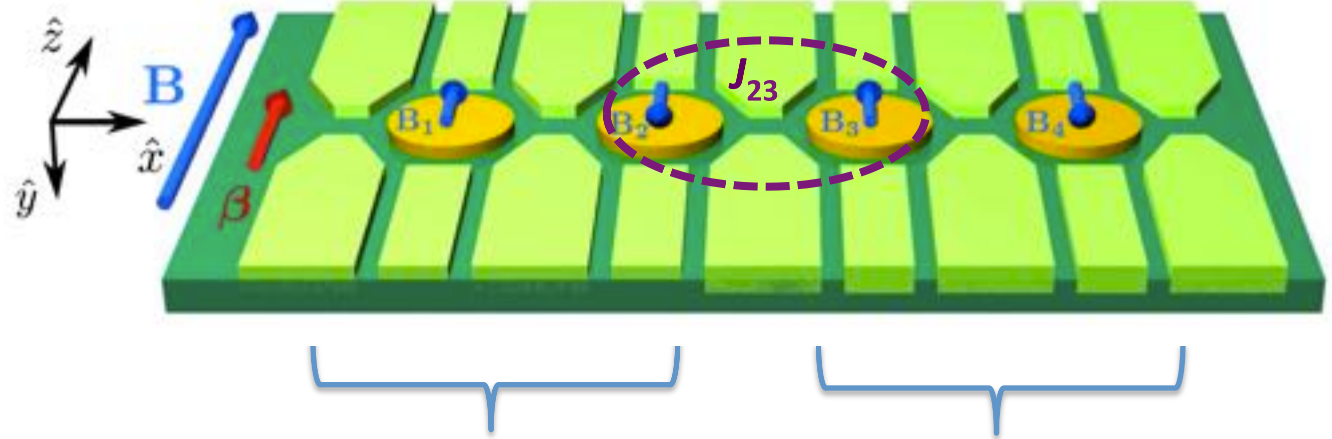
2 electrons:

Levy (2002), Taylor *et al.* (2005)

$$|0\rangle = S, \; |1\rangle = T_0$$

Prospects for Spin-Based Quantum Computing in Quantum Dots
C. Kloeffel and D. Loss, Annu. Rev. Condens. Matter Phys. 4, 51 (2013);

Singlet-Triplet (ST) Qubit



four dots = two ST-qubits

Computational basis:

$$S^z = 0$$

$$[11] = (+-+-)$$
$$[10] = (+--+)$$
$$[01] = (-++-)$$
$$[00] = (-+-+)$$

Note: Decoherence time is very long $T_2 \sim 250\ \mu s$
Bluhm et al., Nat. Phys. 7, 109 (2011)

CNOT Gate via Exchange: fast and noise-free

Klinovaja, Stepanenko, Halperin, and DL, PRB 86, 085423 (2012)

$$H^B = \sum_{i=1}^{4} (b + b_i)\hat{S}_i^z$$ Zeeman interaction

$$b = g\mu_B B \text{ and } b_i = g\mu_B B_i,$$

global magnetic field ***B*** → quantization axis
local magnetic field ***B***$_i$ → single qubit operations

$$H_{ex}(t) = J_{12}(t)\mathbf{S}_1 \cdot \mathbf{S}_2 + J_{23}(t)\mathbf{S}_2 \cdot \mathbf{S}_3 + J_{34}(t)\mathbf{S}_3 \cdot \mathbf{S}_4$$ exchange interaction

Rashba SOI leads to*) $H_{SO} = \boldsymbol{\beta}(t) \cdot (\mathbf{S}_i \times \mathbf{S}_j)$ Dzyaloshinskii-Moriya (SOI) term

*) Burkard and Loss, Phys. Rev. Lett. 88, 047903 (2002)

CNOT Gate via Exchange: fast and noise-free

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local magnetic field ***B****$_i$* ➔ single qubit operations

$$H_{ex}(t) = J_{12}(t)\mathbf{S}_1 \cdot \mathbf{S}_2 + J_{23}(t)\mathbf{S}_2 \cdot \mathbf{S}_3 +$$

ction

same time-dependence!

Rashba SOI leads to*⁾ $H_{SO} = \boldsymbol{\beta}(t) \cdot (\mathbf{S}_i \times \mathbf{S}_j)$ Dzyaloshinskii-Moriya (SOI) term

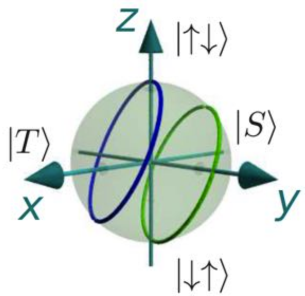
*⁾ Burkard and Loss, Phys. Rev. Lett. 88, 047903 (2002)

Computational scheme

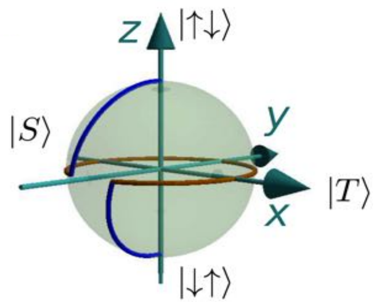
Klinovaja et al., PRB 86, 085423 (2012)

- phase gate C_{23} for spins on dots 2 and 3
- π_{12} pulses (J_{12} swaps spins 1 and 2) and π_{34} (J_{34} swaps spins 3 and 4)
- phase gate C_{23} for spins on dots 2 and 3
- π_{12} (J_{12} swaps spins 1 and 2) and π_{34} (J_{34} swaps spins 3 and 4)

C_{23} phase gate



π pulses



rotation axis: $J_{23}\mathbf{e}_x - \beta_{23}\mathbf{e}_y + \Delta b_{23}\mathbf{e}_z$

rotation axis: $J_{12}\mathbf{e}_x - \beta_{12}\mathbf{e}_y + \Delta b_{12}\mathbf{e}_z$

…from one to many quantum dots...

12 quantum dots - 4 RX spin qubits

Marcus & Kuemmeth et al., 2015/16

Qubit 1 Qubit 2 Qubit 3 Qubit 4

J_{12}^{Q1} J_{23}^{Q1} J_{eff}^{Q1-Q2} J_{12}^{Q2} J_{23}^{Q2} J_{eff}^{Q2-Q3} J_{12}^{Q3} J_{23}^{Q3} J_{eff}^{Q3-Q4} J_{12}^{Q4} J_{23}^{Q4}

Quadruple-quantum-dot

Baart, Jovanovic, Reichl, Wegscheider, and Vandersypen, arXiv:1606.00292

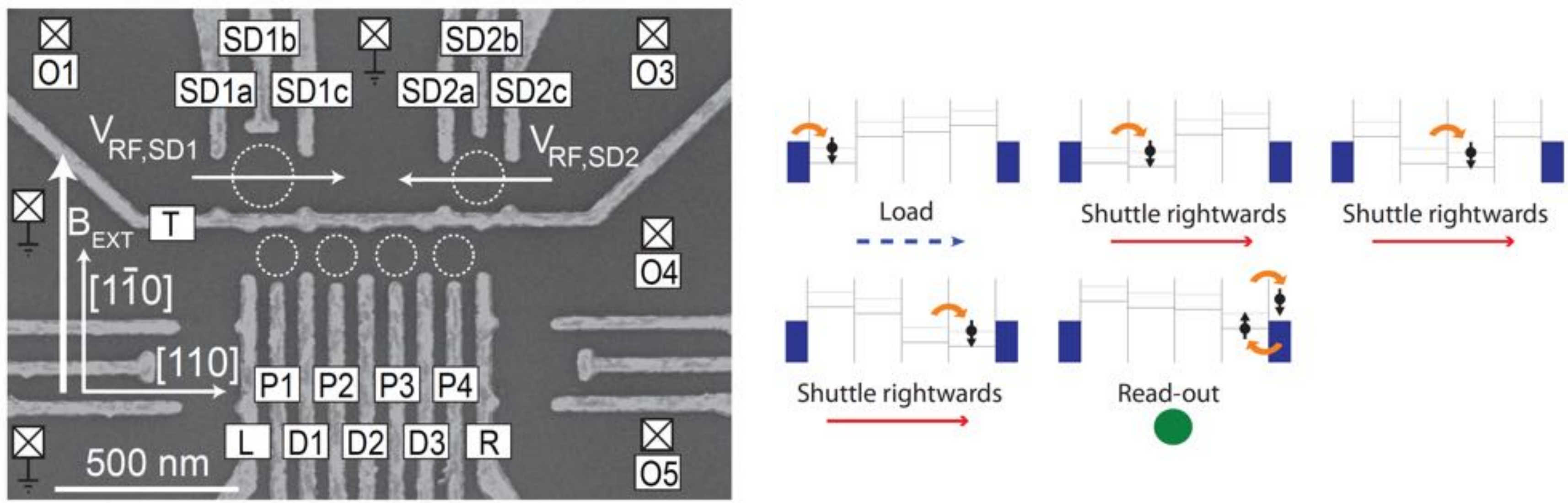


FIG. 1: (a) SEM image of a sample nominally identical to the one used for the measurements. Dotted circles indicate quantum dots, squares indicate Fermi reservoirs in the 2DEG, which are connected to ohmic contacts. The gates that are not labeled are grounded. The reflectance of SD2, $V_{RF,SD2}$, is monitored. (b) Charge stability diagram of the quadruple dot. The occupancy of each dot is denoted by (l, m, n, p) corresponding to the number of electrons in dot 1 (leftmost), 2, 3 and 4 (rightmost) respectively. The fading of charge transition lines from dot 2 and 3 can be explained in a similar way as in Ref. 17 (black dotted lines indicate their positions) and becomes less prominent for a slow scan (see Supplementary Information II). The pulse sequence for loading and read-out is indicated in the charge stability diagram via arrows, see also panel b. The black rectangle corresponds to the hot spot in dot 4 where spins relax on a sub-microsecond timescale; this hot spot is only used for the measurements of Fig. 3. (c) Read from left to right and top to bottom. The system is initialized by loading one electron from the left reservoir. Next, we shuttle the electron to dot 2, 3 and 4 sequentially and finally read out the spin state using spin-selective tunneling.

PHYSICAL REVIEW APPLIED 6, 054013 (2016)

Scalable Gate Architecture for a One-Dimensional Array of Semiconductor Spin Qubits

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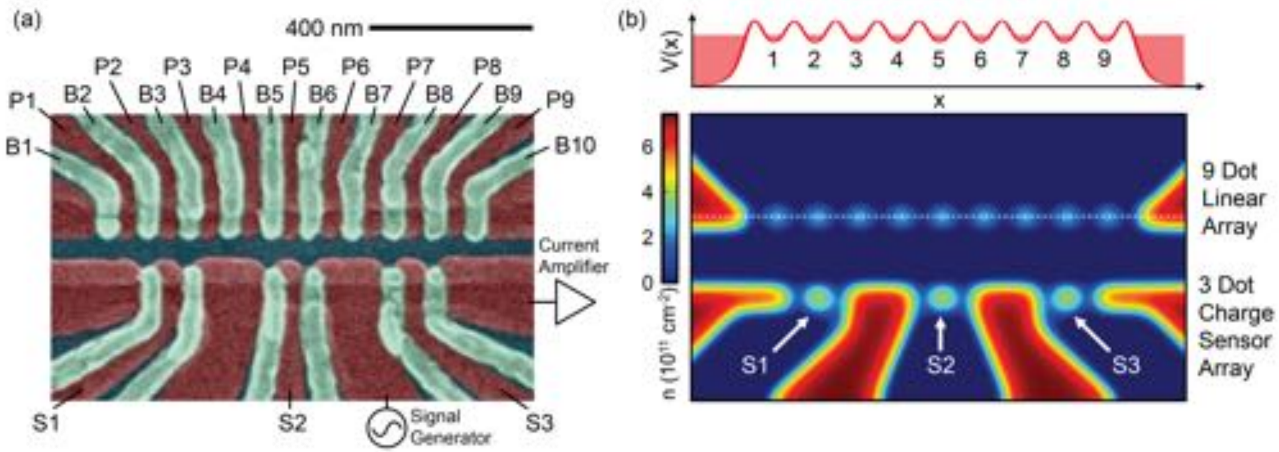
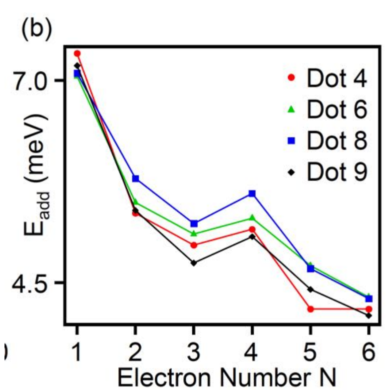
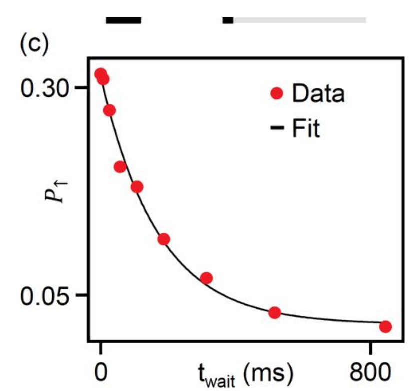
12 (=9+3) quantum dots in Si/SiGe heterostructure





Dot	α (meV/mV)	E_c (meV)	E_{orb} (meV)
1	0.14	6.6	2.7
2	0.13	6.1	2.6
3	0.11	5.6	2.1
4	0.14	7.3	3.3
5	0.14	7.2	3.3
6	0.14	7.1	3.0
7	0.14	7.7	3.5
8	0.14	7.1	3.4
9	0.13	7.2	3.4



A logical qubit in a linear array of semiconductor quantum dots

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arXiv:1608.06335