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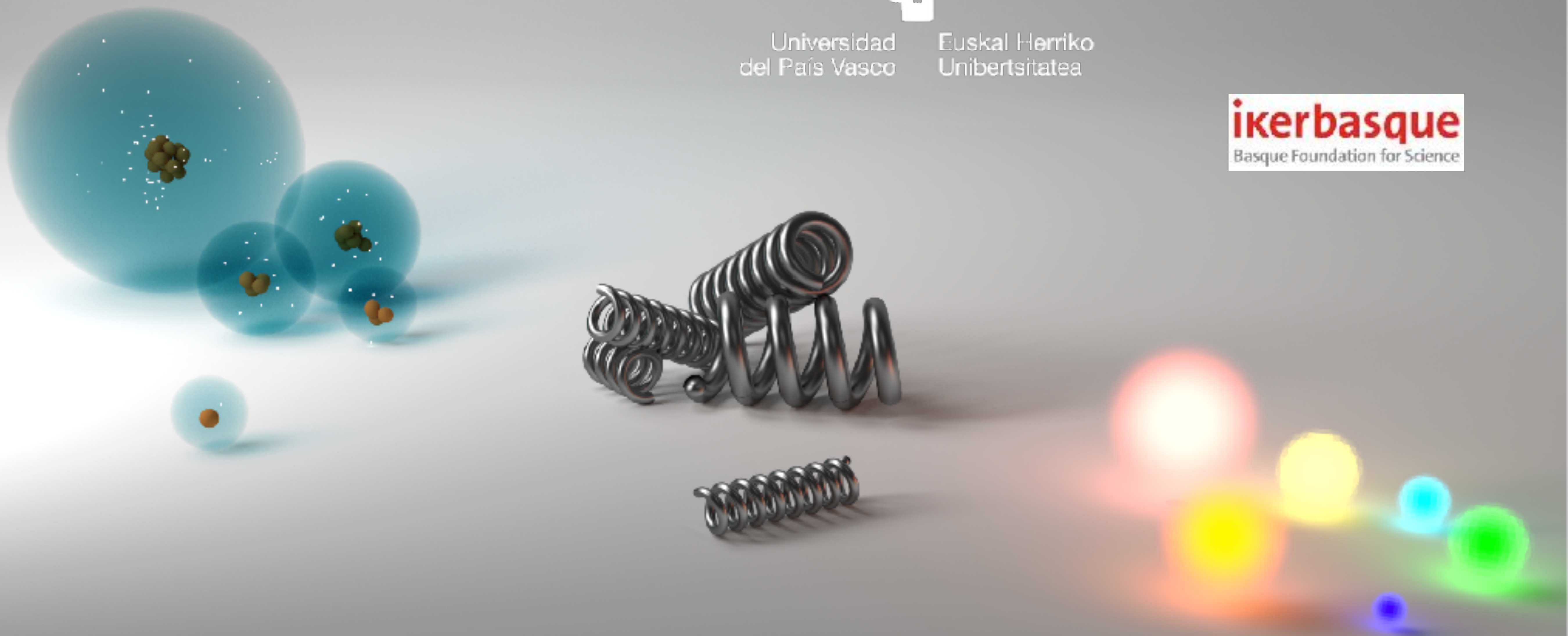

Novel paradigms for quantum simulations

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Capri, April 2017

Lectures on

“Novel paradigms for quantum simulations”

Lecture I: Analog quantum simulations

I.1 Introduction to quantum simulations
I.2 The Jaynes-Cummings model in circuit QED and trapped ions
I.3 Analog quantum simulation of the quantum Rabi model in circuit QED
I.4 Analog quantum simulation of the quantum Rabi model in trapped ions
I.5 Analog quantum simulation of the Dirac equation in trapped ions

Lecture II: Digital and digital-analog quantum simulations

II.1 Introduction to digital quantum simulations
II.2 Digital quantum simulation of spin models
II.3 Digital-analog quantum simulations of the quantum Rabi model
II.4 Digital quantum simulation of fermion and fermion-boson models

Lecture III: Embedding quantum simulators

III.1 Introduction to embedding quantum simulators
III.2 Quantum simulation of antilinear operations and the Majorana equation
III.3 Measurement of entanglement monotones without full tomography
III.4 Further scope on quantum simulations

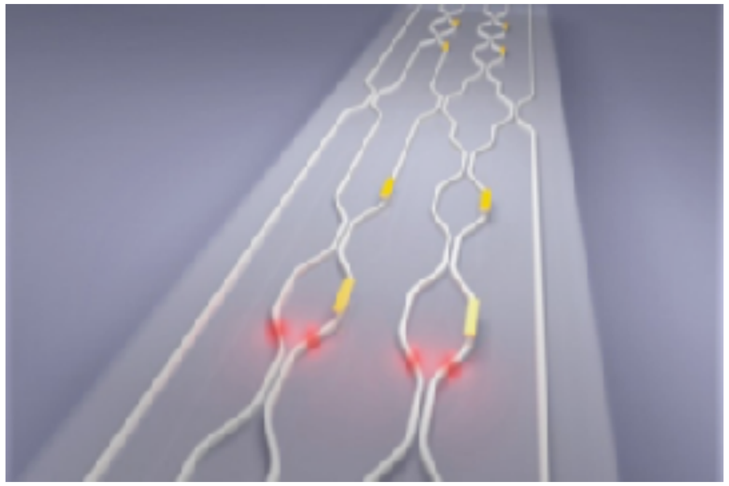
III.1 Introduction to embedding quantum simulators

Whats is an embedding quantum simulator?

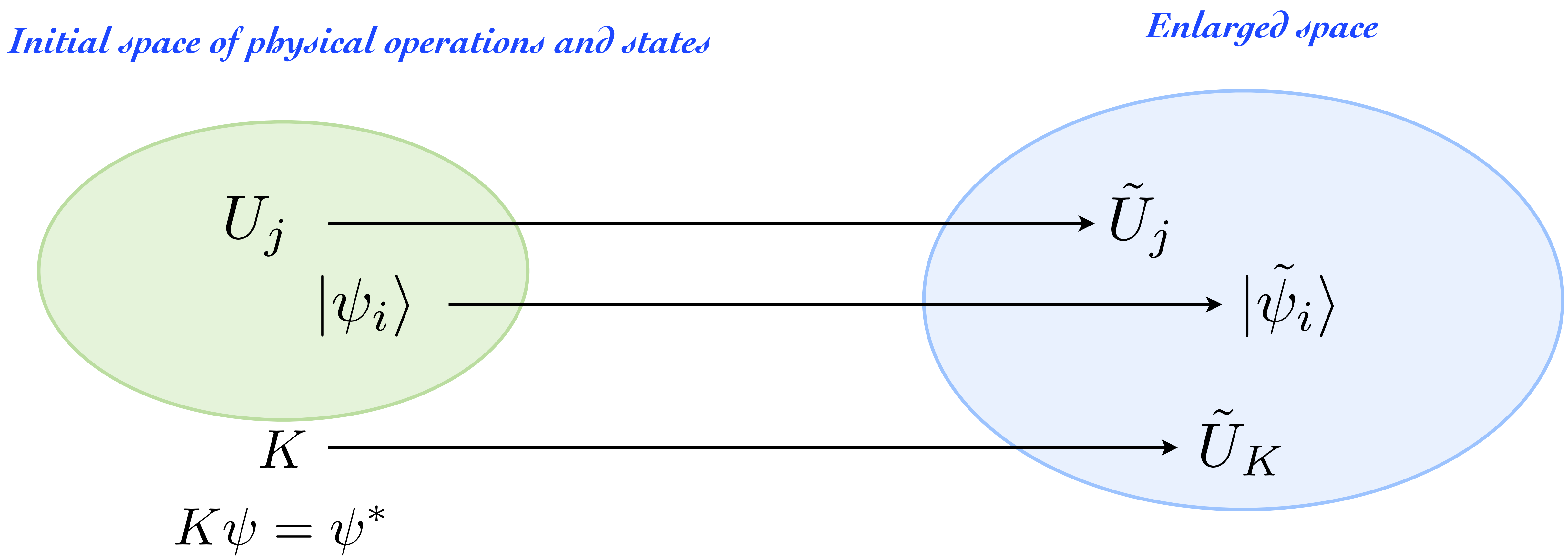
a) A one-to-one quantum simulator is a device that uses a two-level system to mimic a two-level system and a harmonic oscillator to mimic a harmonic oscillator. This may not be the clever approach when scaling up quantum simulations.

b) An embedding quantum simulator (EQS) is a device that embeds the original dynamics into an enlarged Hilbert space to enhance and optimize the extraction of information.

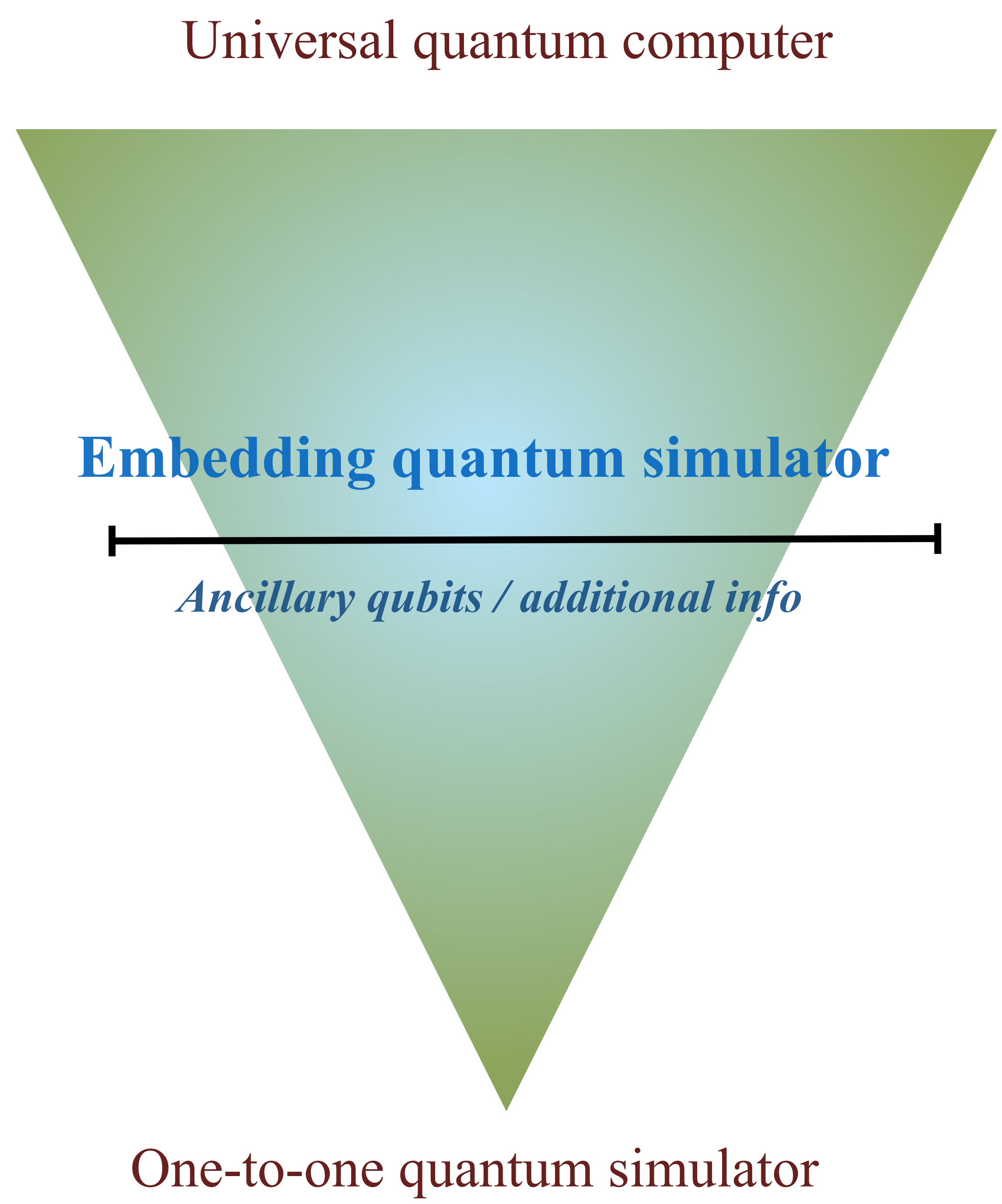
EQS merge the concepts of quantum simulation with quantum computing.



III.1 Quantum simulation of antilinear operations and the Majorana equation



An enlarged space is a Hilbert space where forbidden operations are encoded in physical operations.


Universal quantum computer
Embedding quantum simulator
Ancillary qubits / additional info
One-to-one quantum simulator

III.2 Quantum simulation of antilinear operations and the Majorana equation

Quantum simulation of unphysical operations

PHYSICAL REVIEW X **1**, 021018 (2011)

Quantum Simulation of the Majorana Equation and Unphysical Operations

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We design a quantum simulator for the Majorana equation, a non-Hamiltonian relativistic wave equation that might describe neutrinos and other exotic particles beyond the standard model. Driven by the need of the simulation, we devise a general method for implementing a number of mathematical operations that are unphysical, including charge conjugation, complex conjugation, and time reversal. Furthermore, we describe how to realize the general method in a system of trapped ions. The work opens a new front in quantum simulations.

Breaking the rules quantum physics

The Majorana equation

$$i\hbar\partial\!\!\!/\psi = mc\psi_c$$

The Majorana equation is a relativistic wave equation where the mass term contains the charge conjugate of the spinor, ψ_c

The simultaneous presence of ψ and ψ_c makes impossible to factorize and produce a Hamiltonian equation $i\hbar\partial_t\psi = H\psi$

Ettore Majorana

The Majorana equation at 1+1 dimensions

$$i\hbar\partial_t\psi = c\sigma_x p_x\psi - imc^2\sigma_y\psi^* \longrightarrow H = c\sigma_x p_x - imc^2\sigma_y K \qquad H^\dagger \neq H$$

The Majorana equation violates one of the axioms of Quantum Mechanics

$$\psi \neq e^{i\varphi}\psi$$

Building the dynamics in an enlarged space

$$i\hbar\partial_t\psi = c\sigma_x p_x\psi - imc^2\sigma_y\psi^*$$

Initial space

Enlarged space

$$\psi = \begin{pmatrix} \psi_1^r + i\psi_1^{im} \\ \psi_2^r + i\psi_2^{im} \end{pmatrix}$$

$$\psi = M\Psi$$

$$M = \begin{pmatrix} 1 & 0 & i & 0 \\ 0 & 1 & 0 & i \end{pmatrix}$$

$$\Psi = \begin{pmatrix} \psi_1^r \\ \psi_2^r \\ \psi_1^{im} \\ \psi_2^{im} \end{pmatrix}$$

$\downarrow K$ $\qquad\qquad$ $\downarrow \sigma_z \otimes I$

$$\psi^* = \begin{pmatrix} \psi_1^r - i\psi_1^{im} \\ \psi_2^r - i\psi_2^{im} \end{pmatrix}$$

$$\psi^* = M\Psi^*$$

$$M = \begin{pmatrix} 1 & 0 & i & 0 \\ 0 & 1 & 0 & i \end{pmatrix}$$

$$\Psi^* = \begin{pmatrix} \psi_1^r \\ \psi_2^r \\ -\psi_1^{im} \\ -\psi_2^{im} \end{pmatrix}$$

$$i\hbar\partial_t\psi = c\sigma_x p_x\psi - imc^2\sigma_y\psi^* \longleftrightarrow i\hbar\partial_t\Psi = \left[c(I\otimes\sigma_x)p_x - mc^2\sigma_x\otimes\sigma_y\right]\Psi$$

Implementable in a quantum setup

Measuring observables in the enlarged space

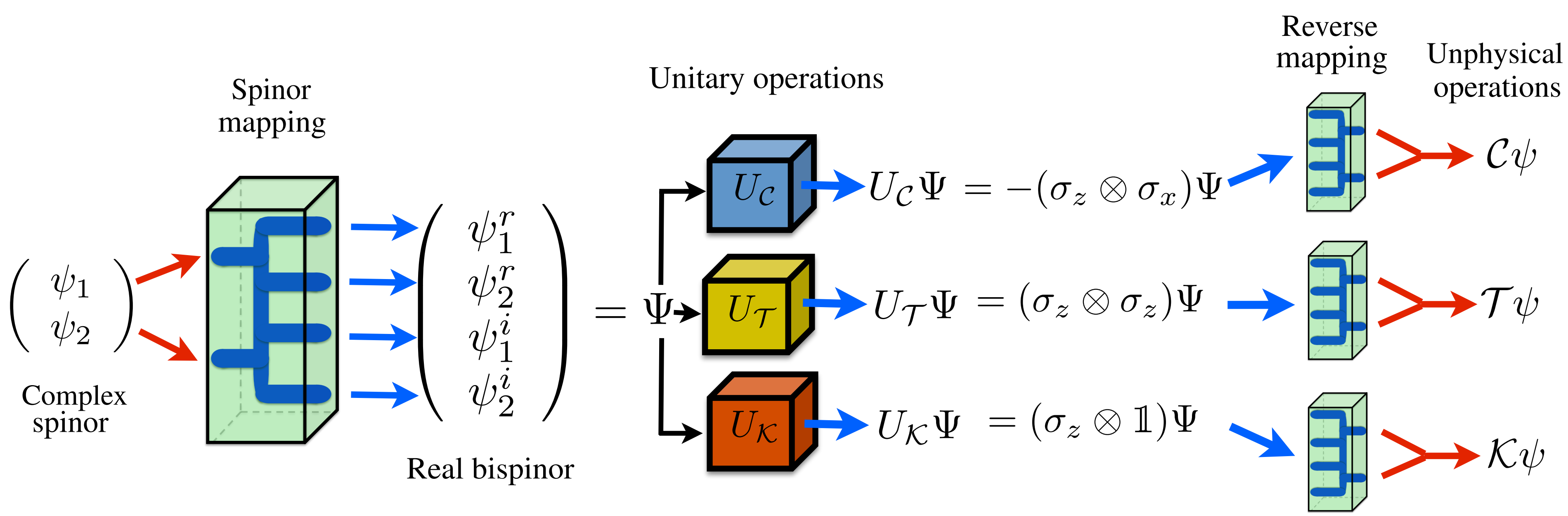
$$\psi = M\Psi \qquad M = \begin{pmatrix} 1 & 0 & i & 0 \\ 0 & 1 & 0 & i \end{pmatrix}$$

$$\langle O\rangle_\psi = \langle\psi|O|\psi\rangle = \langle\Psi|M^\dagger O\, M|\Psi\rangle =: \langle\tilde{O}\rangle_\Psi$$

One example: the pseudohelicity

$$\Sigma = \sigma_x p_x \qquad \tilde{\Sigma} = M^\dagger \sigma_x p_x\ M = (\mathbb{1}\otimes\sigma_x - \sigma_y\otimes\sigma_x)\otimes p_x$$

General scheme to implement unphysical operations



III.3 Measurement of entanglement monotones without full tomography

Measurement of entanglement monotones with EQS

PRL **111**, 240502 (2013) PHYSICAL REVIEW LETTERS week ending 13 DECEMBER 2013

Embedding Quantum Simulators for Quantum Computation of Entanglement

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We introduce the concept of embedding quantum simulators, a paradigm allowing the efficient quantum computation of a class of bipartite and multipartite entanglement monotones. It consists in the suitable encoding of a simulated quantum dynamics in the enlarged Hilbert space of an embedding quantum simulator. In this manner, entanglement monotones are conveniently mapped onto physical observables, overcoming the necessity of full tomography and reducing drastically the experimental requirements. Furthermore, this method is directly applicable to pure states and, assisted by classical algorithms, to the mixed-state case. Finally, we expect that the proposed embedding framework paves the way for a general theory of enhanced one-to-one quantum simulators.

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Entanglement monotones without full tomography

PHYSICAL REVIEW A **72**, 012337 (2005)

Constructing *N*-qubit entanglement monotones from antilinear operators

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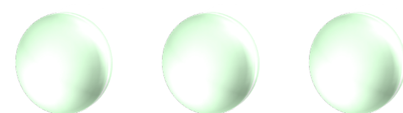


Three qubit case

$$(\sigma_\mu \otimes \sigma_y \otimes \sigma_y) \cdot (\sigma^\mu \otimes \sigma_y \otimes \sigma_y) = \langle\psi|\sigma_\mu \otimes \sigma_y \otimes \sigma_y|\psi^*\rangle g^{\mu\nu} \langle\psi|\sigma_\mu \otimes \sigma_y \otimes \sigma_y|\psi^*\rangle$$

$$g^{\mu\nu} = \mathrm{diag}\{-1, 1, 0, 1\}$$

$$(\sigma_0, \sigma_1, \sigma_2, \sigma_3) = (I, \sigma_x, \sigma_y, \sigma_z)$$

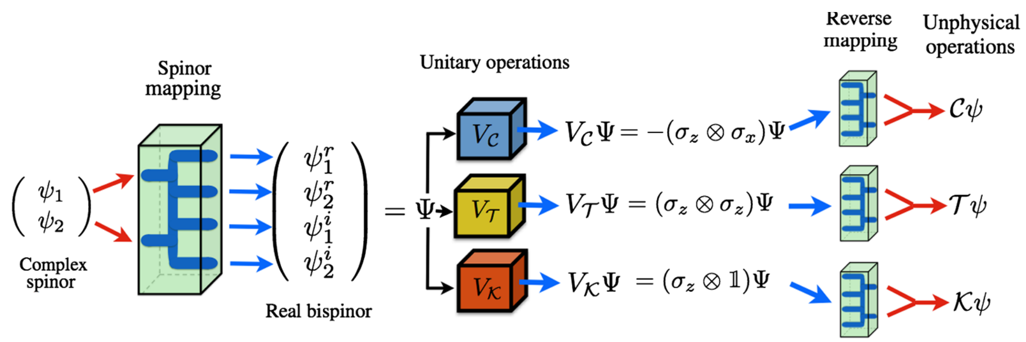
$$\langle\psi|I \otimes \sigma_y \otimes \sigma_y|\psi^*\rangle$$

$$\langle\psi|\sigma_x \otimes \sigma_y \otimes \sigma_y|\psi^*\rangle$$

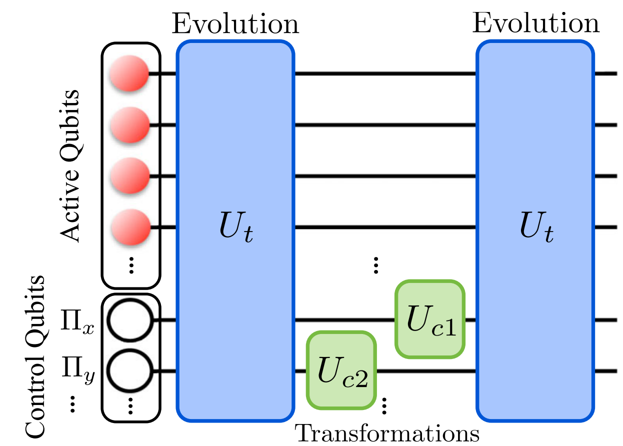
$$\langle\psi|\sigma_z \otimes \sigma_y \otimes \sigma_y|\psi^*\rangle$$

6 observables
in the enlarged space ⟷ 63 observables
needed for full tomography

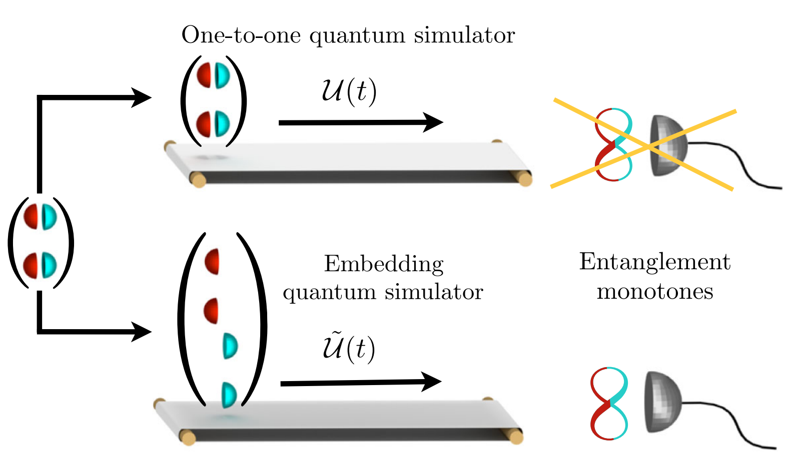
Majorana equation: theory
J. Casanova et al., PRX ’11



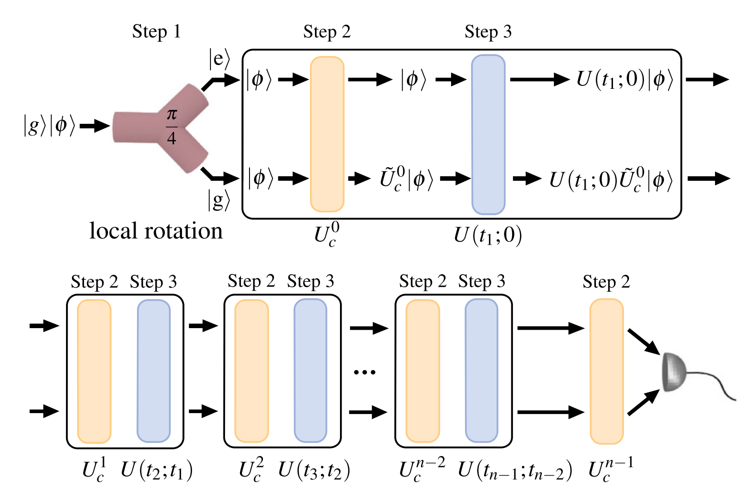
Noncausal kinematics and Galilean transformations
U. Alvarez-Rodriguez et al., PRL ’13



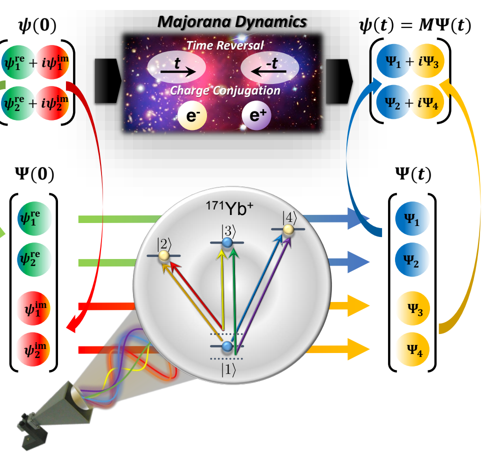
Computation of entanglement monotones
R. Di Candia et al., PRL ’13



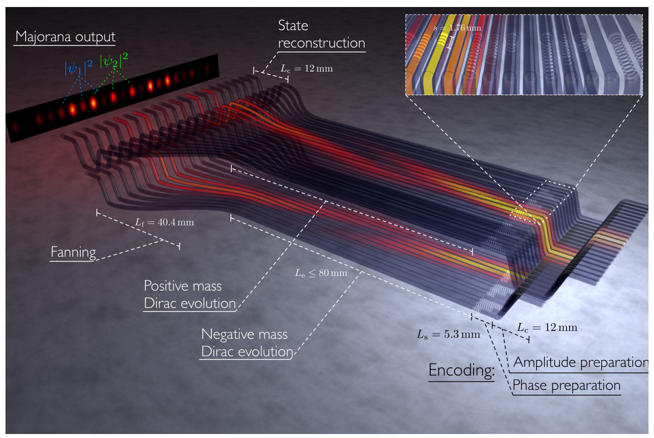
Computation of n-time correlation functions
J. S. Pedernales et al., PRL ‘14



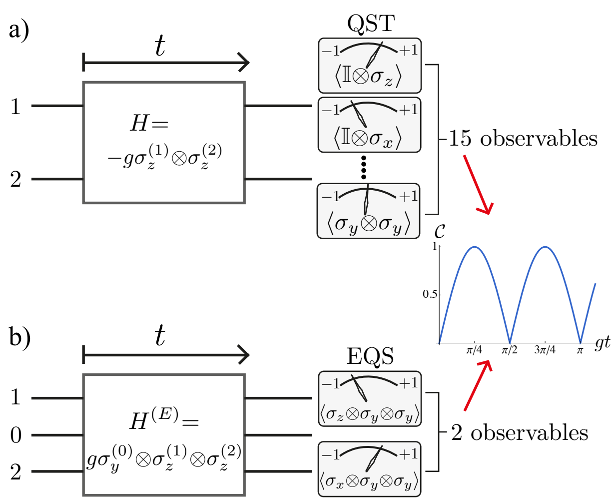
Majorana equation: Ions
X. Zhang et al., Nat. Comm. ’15



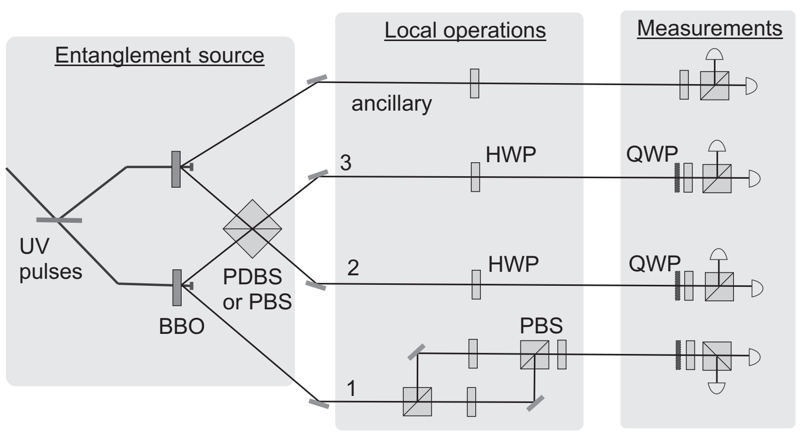
Majorana equation: photons
Szameit et al., Optica ‘15



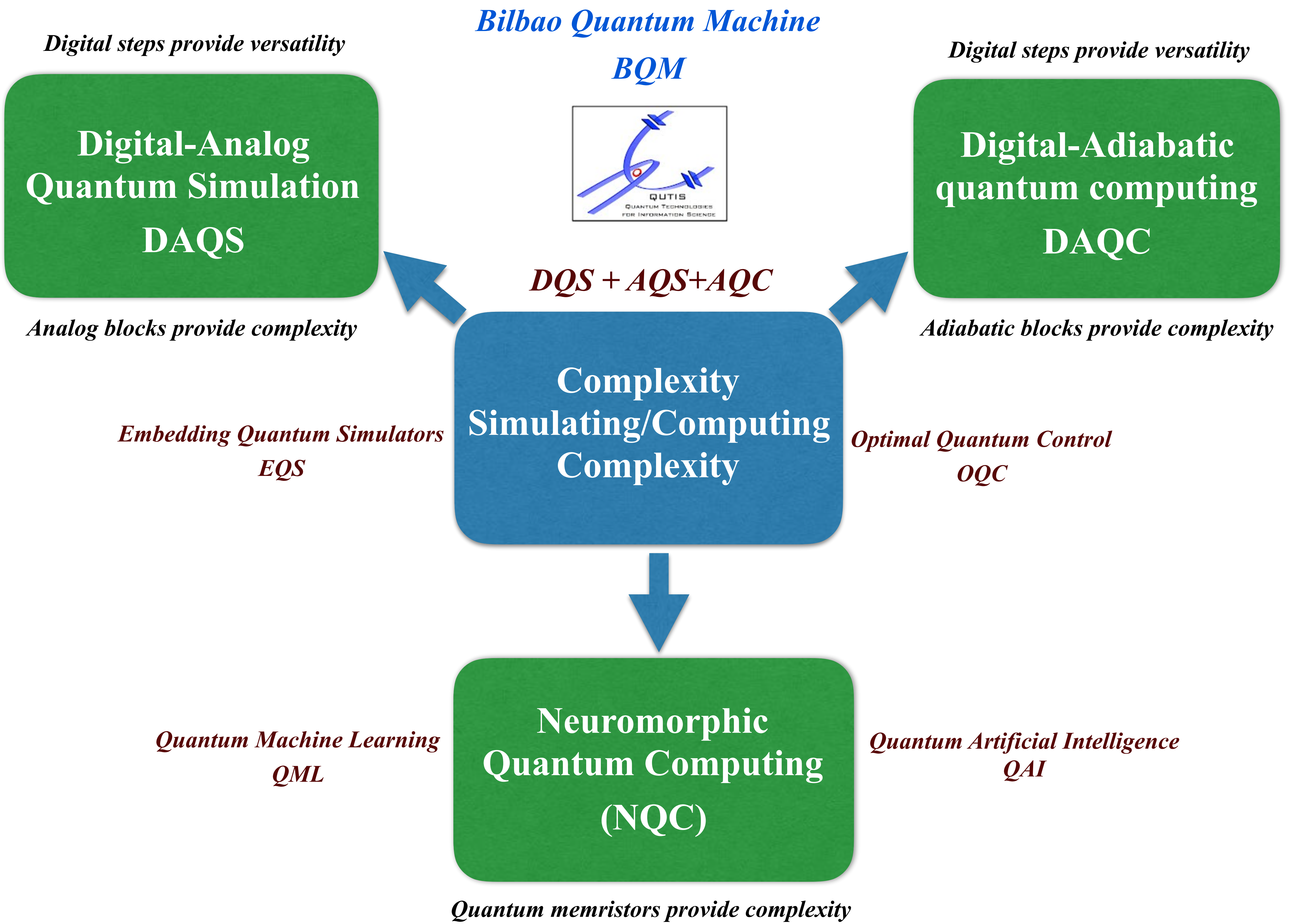
Entanglement monotones: photons I
J. Loredo et al., PRL ’15



Entanglement monotones: photons II
Jian Wei Pan et al., PRL ’15



III.4 Further scope on quantum simulations



Bilbao Quantum Machine
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Digital steps provide versatility
Digital-Analog Quantum Simulation
DAQS
Analog blocks provide complexity
Digital steps provide versatility
Digital-Adiabatic quantum computing
DAQC
Adiabatic blocks provide complexity
DQS + AQS+AQC
Complexity Simulating/Computing Complexity
Embedding Quantum Simulators
EQS
Optimal Quantum Control
OQC
Neuromorphic Quantum Computing
(NQC)
Quantum Machine Learning
QML
Quantum Artificial Intelligence
QAI
Quantum memristors provide complexity