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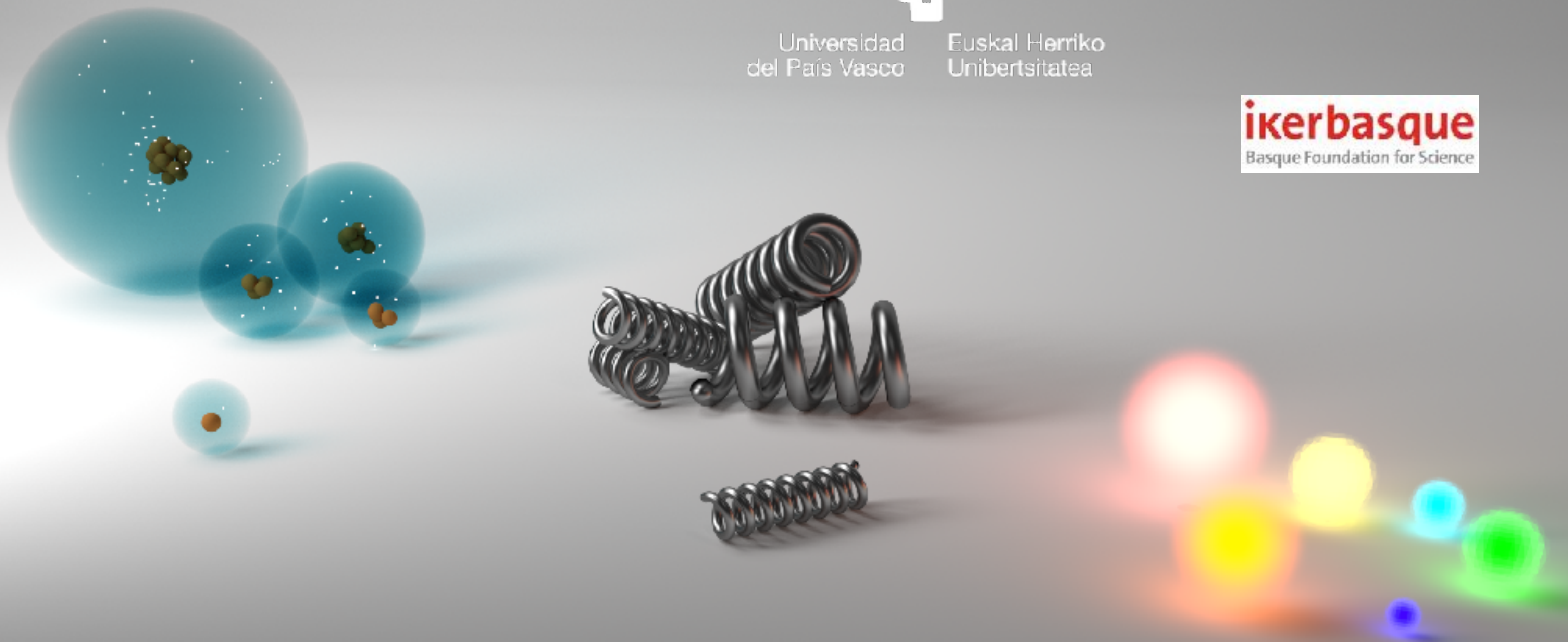


Novel paradigms for quantum simulations

E. Solano

University of the Basque Country, Bilbao, Spain

Capri, April 2017

Lectures on
“Novel paradigms for quantum simulations”

Lecture I: Analog quantum simulations

I.1 Introduction to analog quantum simulations
I.2 The Jaynes-Cummings model in circuit QED and trapped ions
I.3 Analog quantum simulation of the quantum Rabi model in circuit QED
I.4 Analog quantum simulation of the quantum Rabi model in trapped ions
I.5 Analog quantum simulation of the Dirac equation in trapped ions

Lecture II: Digital and digital-analog quantum simulations

II.1 Introduction to digital quantum simulations
II.2 Digital quantum simulation of spin models
II.3 Digital-analog quantum simulations of the quantum Rabi model
II.4 Digital quantum simulation of fermion and fermion-boson models

Lecture III: Embedding quantum simulators

III.1 Introduction to embedding quantum simulators
III.2 Quantum simulation of antilinear operations and the Majorana equation
III.3 Measurement of entanglement monotones without full tomography
III.4 Further scope on quantum simulations

II.1 Introduction to digital quantum simulations

Digital Quantum Simulations (DQS)

Goal: obtain dynamical evolution associated with H and perform measurements.

Problem: We cannot directly implement $H = H_1 + ... + H_k$ in our controllable system.
But we can individually implement each Hamiltonian of the set of interactions:

$$H_1 \quad H_2 \quad H_3 \quad H_4 \quad H_5 \quad ...$$

Solution: Stepwise implementation with Lie-Suzuki-Trotter formula.

$$e^{-iHt} \sim (e^{-iH_1 t/l} \cdots e^{-iH_k t/l})_l + \sum_{i>j} [H_i, H_j] t^2/2l \qquad H = H_1 + ... + H_k$$

Advantages: DQS can profit from quantum error correction protocols that were designed for digital quantum computers. It is scalable and universal.

Disadvantages: the number of qubits and gates for quantum supremacy can be millions. It is computationally scalable and universal but it looks, up to now, technically unreachable.

S. Lloyd, Science **273**, 1073 (1996)

II.2 Digital quantum simulation of spin models

Heisenberg and Ising models

$$H^{\mathrm{H}} = \sum_{i=1}^{N-1} J\left(\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y + \sigma_i^z \sigma_{i+1}^z\right) \qquad H^{\mathrm{I}} = \sum_{i=1}^{N-1} J \sigma_i^x \sigma_{i+1}^x$$

Dispersive transmon interaction mediated by resonator

$$H_{12}^{xy} = J\left(\sigma_1^+ \sigma_2^- + \sigma_1^- \sigma_2^+\right) = J/2\left(\sigma_1^x \sigma_2^x + \sigma_1^y \sigma_2^y\right)$$

Adding single-qubit rotations to our controllable XY interaction

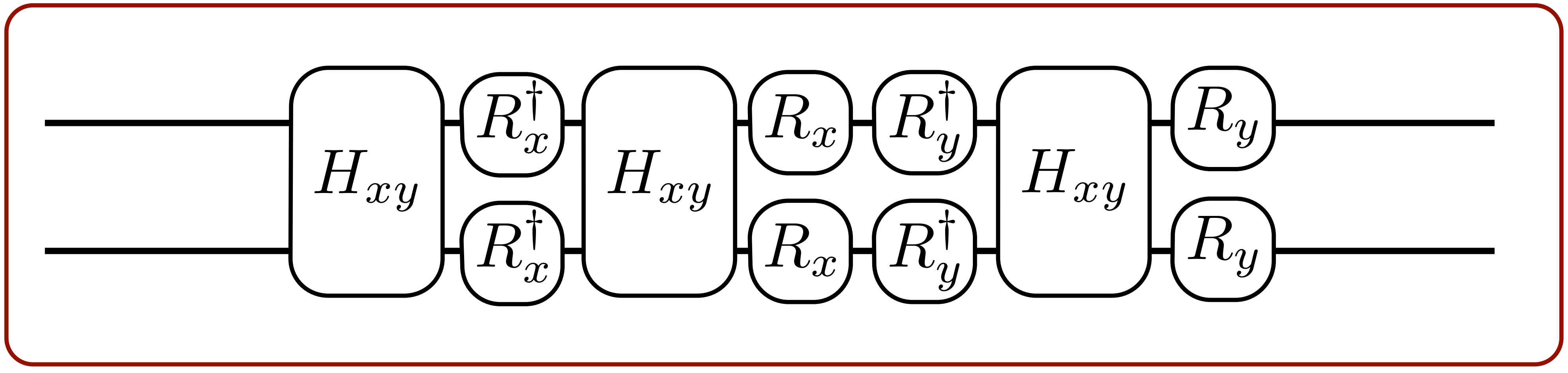
$$H_{12}^{xz} = R_{12}^x(\pi/4) H_{12}^{xy} R_{12}^{x\dagger}(\pi/4) = J/2\left(\sigma_1^x \sigma_2^x + \sigma_1^z \sigma_2^z\right)$$

$$H_{12}^{yz} = R_{12}^y(\pi/4) H_{12}^{xy} R_{12}^{y\dagger}(\pi/4) = J/2\left(\sigma_1^y \sigma_2^y + \sigma_1^z \sigma_2^z\right)$$

$$H_{12}^{x,-y} = R_1^x(\pi/2) H_{12}^{xy} R_1^{x\dagger}(\pi/2) = J/2\left(\sigma_1^x \sigma_2^x - \sigma_1^y \sigma_2^y\right)$$

U. Las Heras et al., Phys. Rev. Lett. **112**, 200501 (2014)

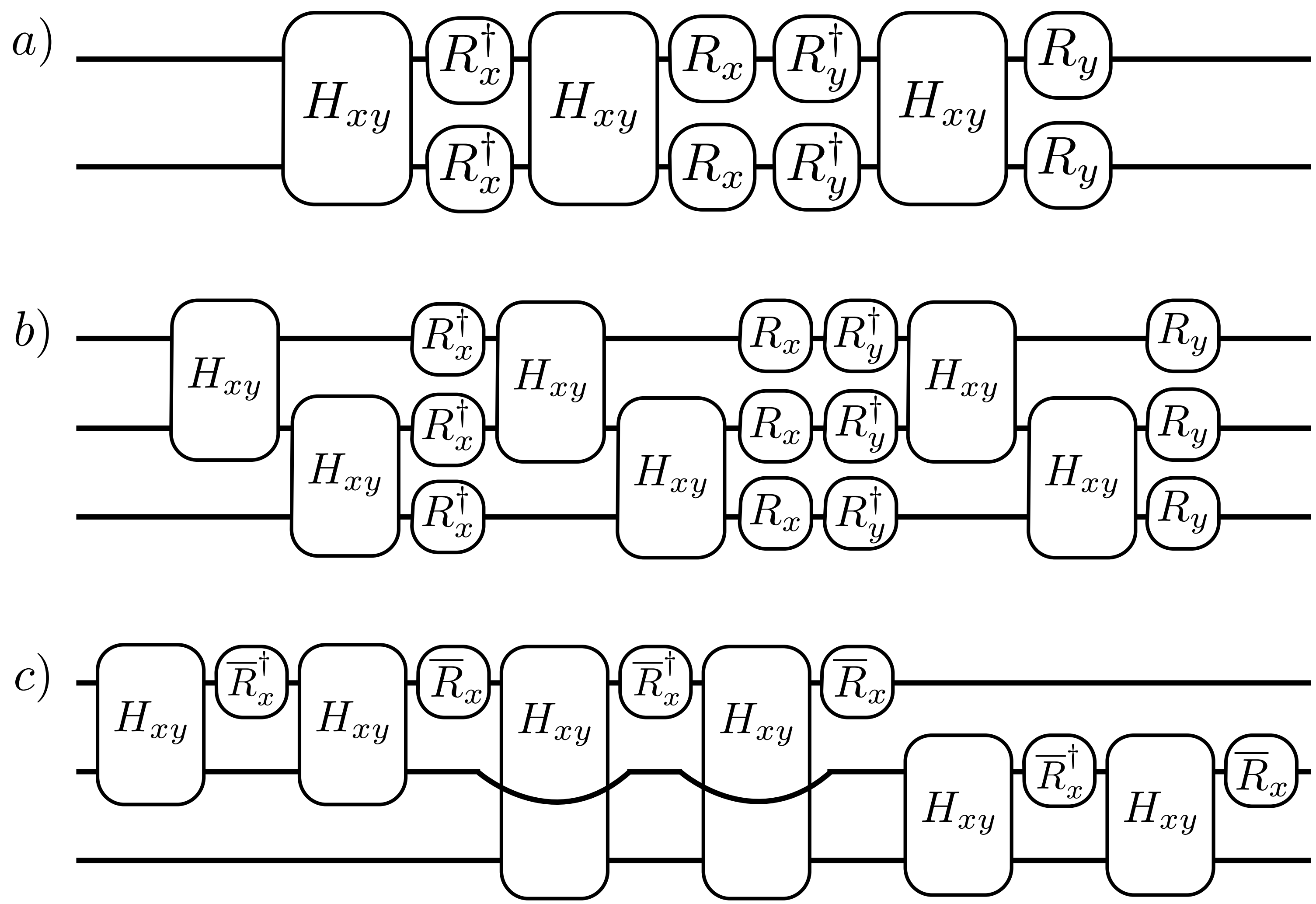
Digital Heisenberg interaction for two qubits



No digital error for this protocol!

U. Las Heras et al., Phys. Rev. Lett. **112**, 200501 (2014)

DQS of different spin models



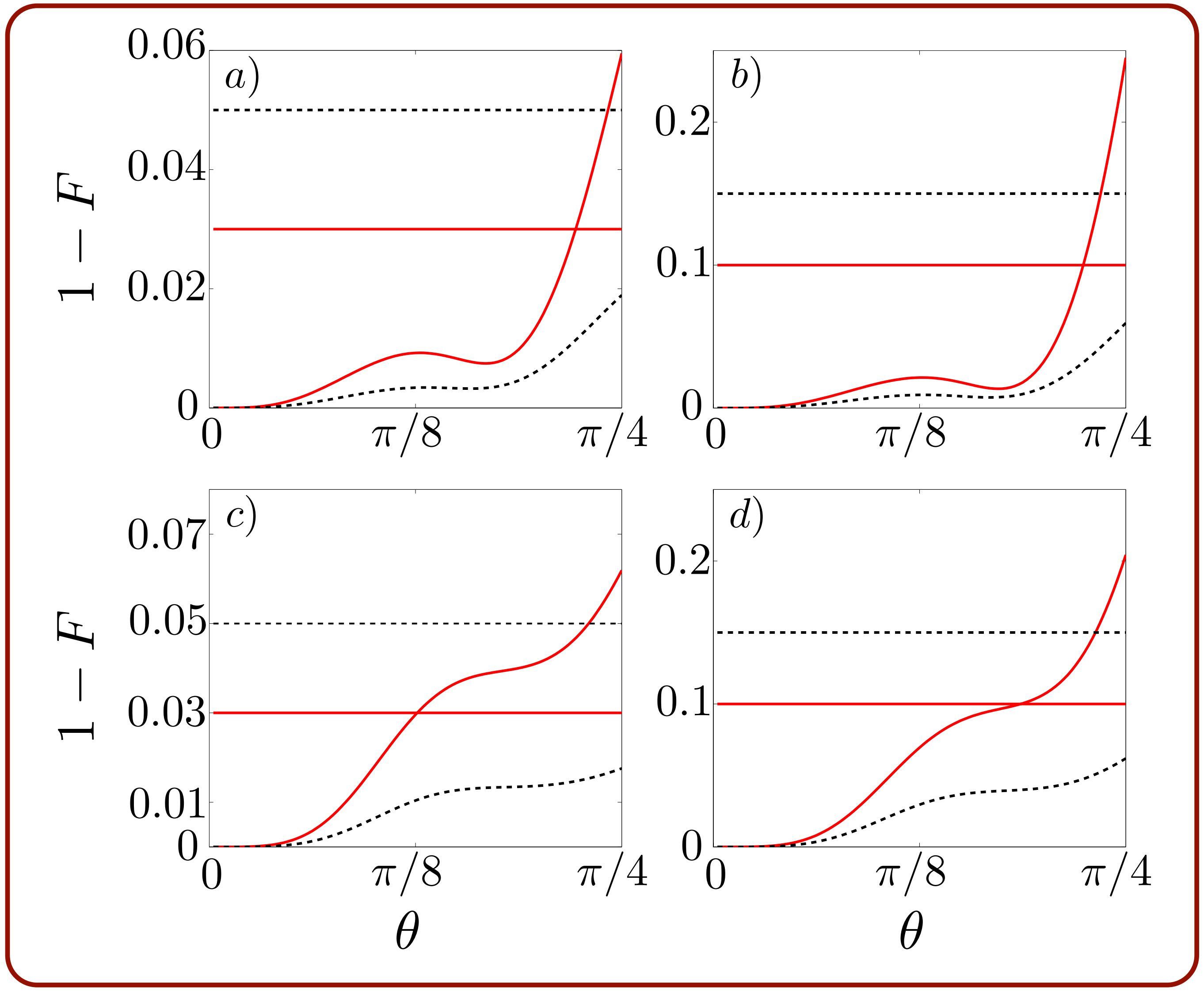
a) Heisenberg, two sites

b) Heisenberg, three sites

c) Frustrated Ising, three sites

U. Las Heras et al., Phys. Rev. Lett. **112**, 200501 (2014)

Digital error versus accumulated experimental error



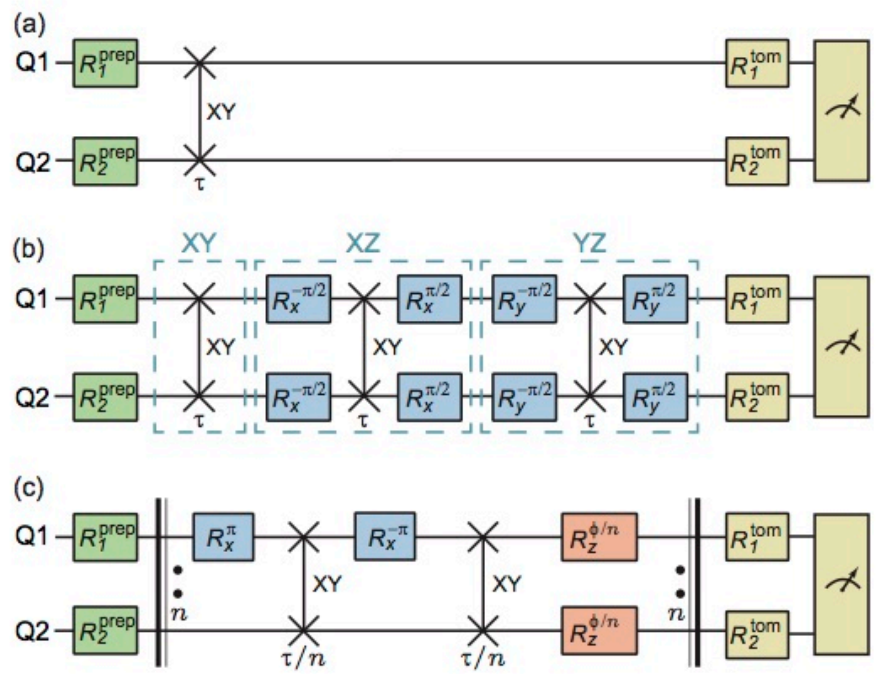
a) Heisenberg model, with ε = 10^{-2}, l = 3, 5; and b) ε = 5 × 10^{-2}, l = 2, 3.

b) Transverse field Ising model, with ε = 10^{-2}, l = 3, 5; and b) ε = 5 × 10^{-2}, l = 2, 3.

U. Las Heras et al., Phys. Rev. Lett. **112**, 200501 (2014)

Experimental DQS of spins models at ETH Zurich

First DQS of spin models in cQED



PHYSICAL REVIEW X **5,** 021027 (2015)

Digital Quantum Simulation of Spin Models with Circuit Quantum Electrodynamics

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U. Las Heras,[2] L. Lamata,[2] E. Solano,[2,3] S. Filipp,[1,†] and A. Wallraff[1]

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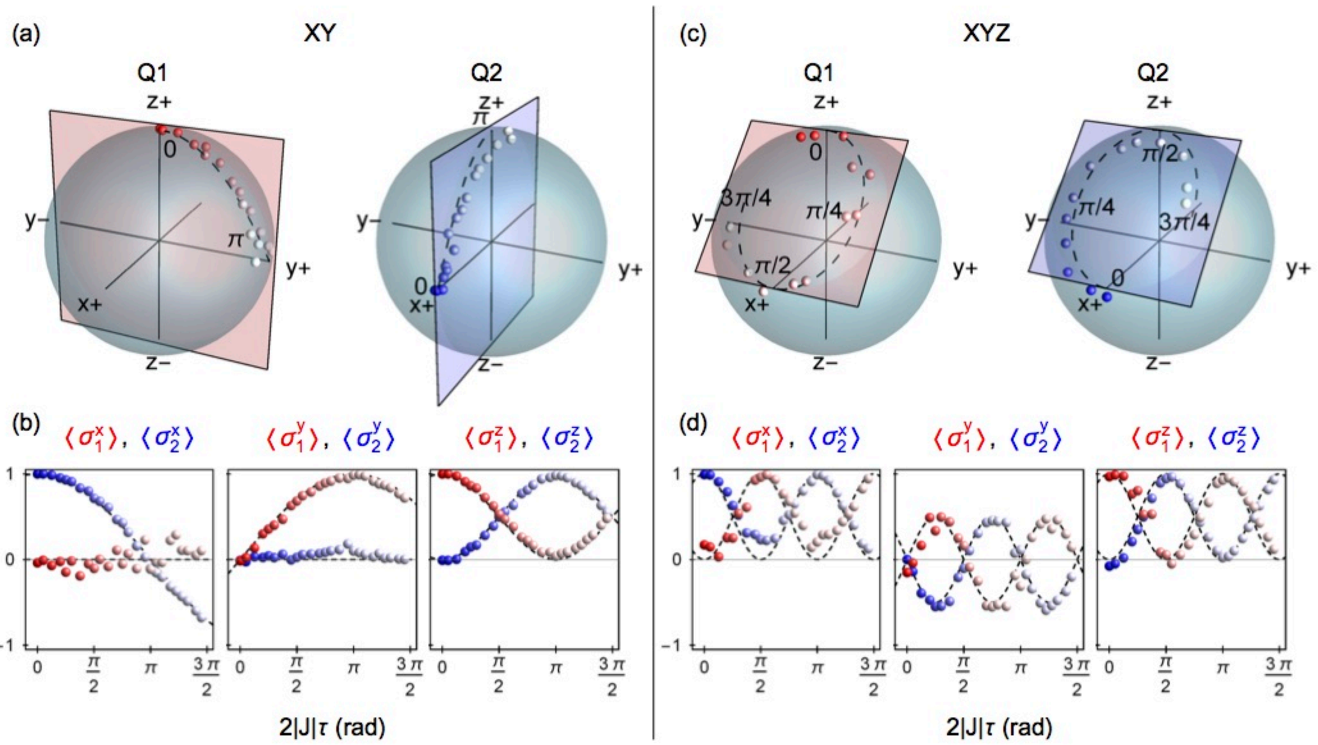
(Received 6 March 2015; published 17 June 2015)

Experimental DQS of spins models at ETH Zurich

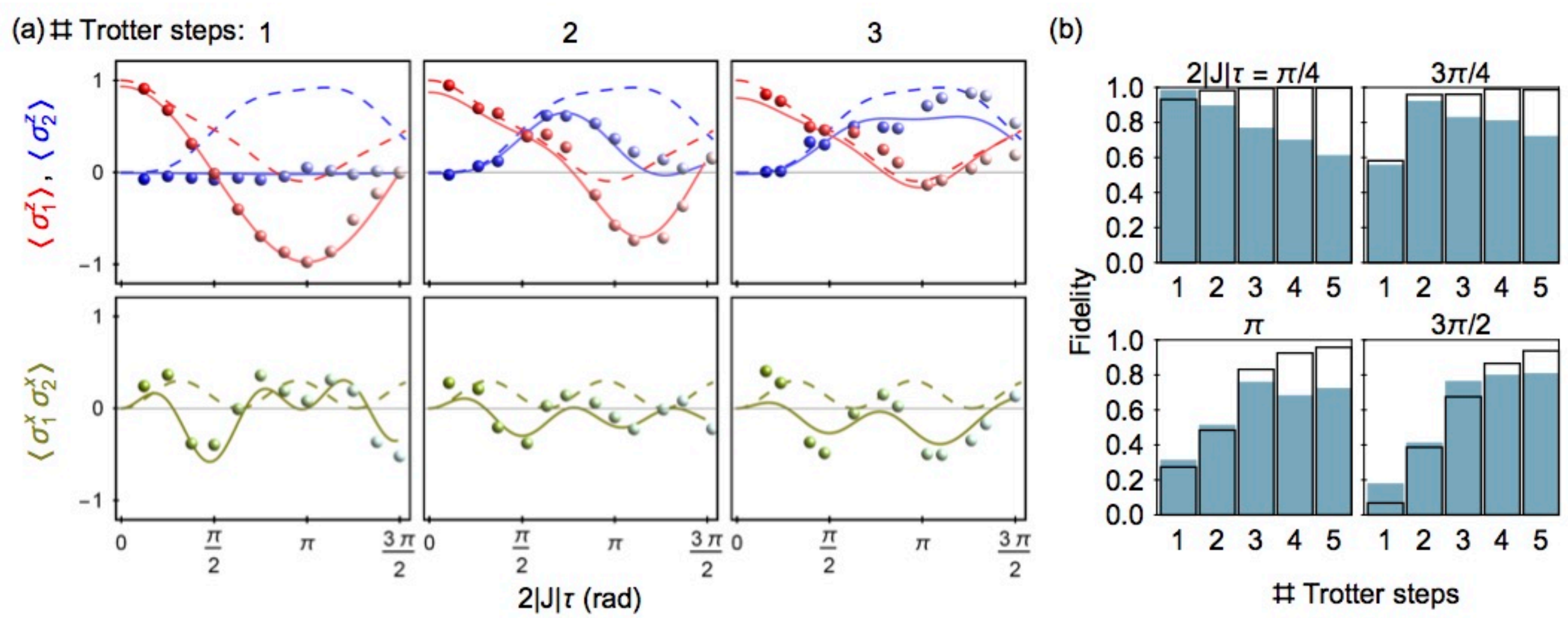
Heisenberg model



Y. Salathé et al., PRX'15

Experimental DQS of spins models at ETH Zurich

Ising model



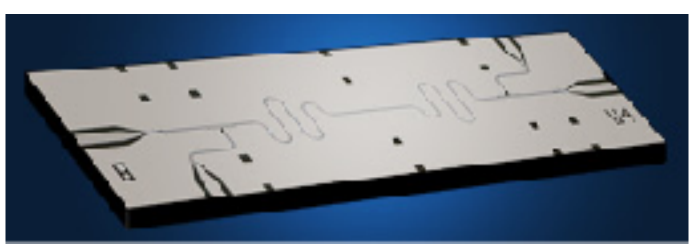
Y. Salathé et al., PRX'15

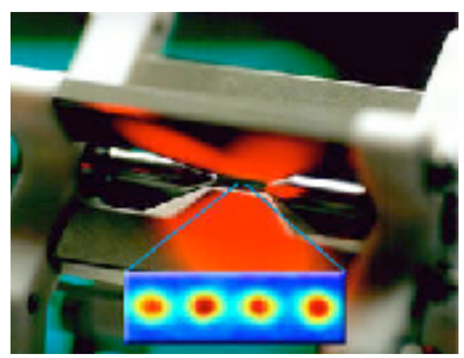
II.3 Digital-analog quantum simulation of the quantum Rabi model

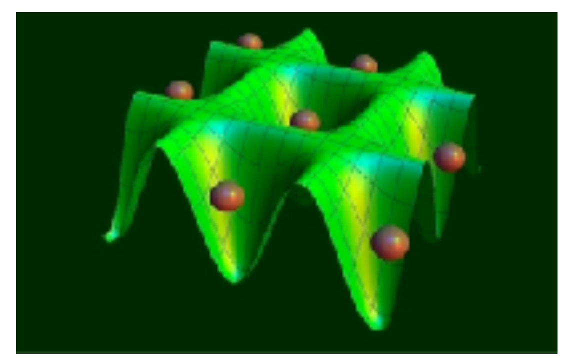
Analog or Digital Quantum Simulations?

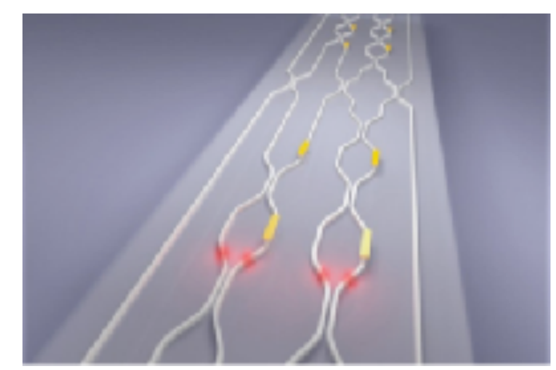
a) Analog quantum simulators (AQS) map qubits onto qubits, bosonic modes onto bosonic modes, involving always-on interactions and accumulating tiny errors that are not easy to correct.

b) Digital quantum simulators (DQS) discretize the time evolution with single/multiqubit gates. They are considered as universal quantum simulators allowing for error correction protocols.









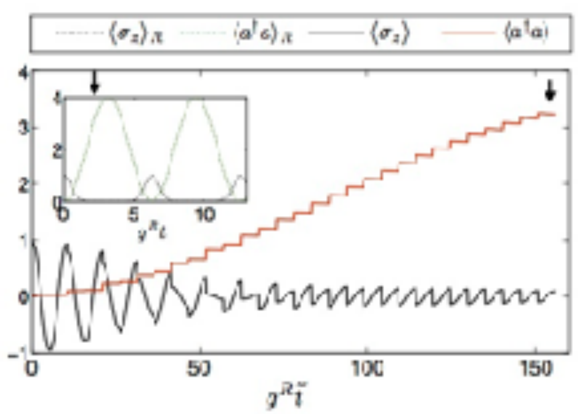
c) We propose to integrate DQS & AQS into **digital-analog quantum simulators (DAQS)** to develop a modular approach of analog blocks combined with digital techniques.

Complexity Simulating Complexity

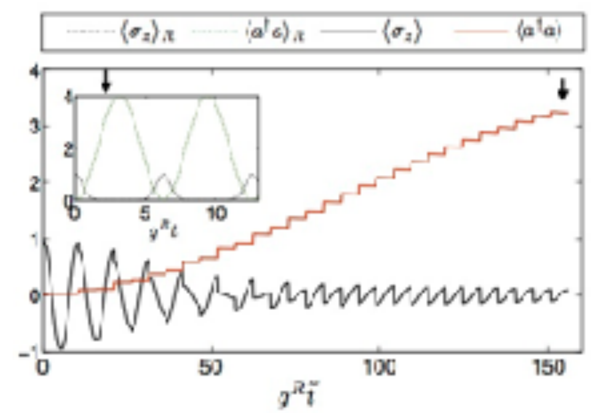
A fist experiment in DAQS for superconducting circuits

Bilbao theory + Delft experiment

Digital quantum Rabi and Dicke models
Mezzacapo et al., Sci. Rep. 2014



Experiment at TU Delft



In DAQS, **analog blocks** are combined sequentially with **digital steps**.

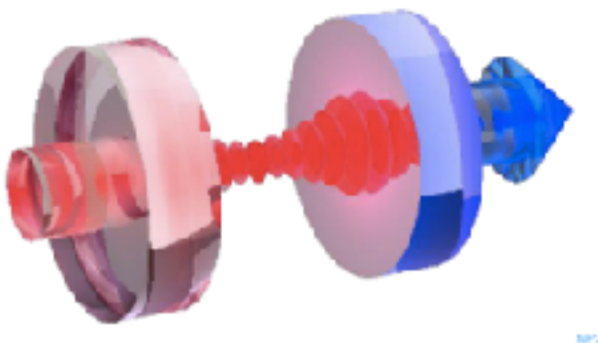
Analog blocks are made of collective quantum gates, that is, in-built complex operations.
Digital steps are local quantum operations that may act also in a global manner.

Analog blocks provide the complexity of the simulated model, **digital steps provide flexibility**.
Similar spirit can be followed by introducing digital-adiabatic quantum computers (DAQC).

How DAQS works in superconducting circuits?

Digital quantum Rabi and Dicke models
Mezzacapo et al., Sci. Rep. 2014

Quantum Rabi model: most fundamental light-matter interaction



$$H_R = \omega_r^R a^\dagger a + \frac{\omega_q^R}{2}\sigma^z + g^R \sigma^x (a^\dagger + a)$$

Small coupling as compared to mode & qubit frequencies: Jaynes-Cummings model

$$H = \omega_r a^\dagger a + \frac{\omega_q}{2}\sigma^z + g(a^\dagger \sigma^- + a\sigma^+)$$

Digital quantum Rabi and Dicke models

Mezzacapo et al., Sci. Rep. 2014

Interaction available in cQED: Jaynes-Cummings model

$$H = \omega_r a^\dagger a + \frac{\omega_q}{2}\sigma^z + g(a^\dagger \sigma^- + a\sigma^+)$$

↓

Digital decomposition: JC + local rotations

$$H_R = H_1 + H_2,$$

$$H_1 = \frac{\omega_r^R}{2} a^\dagger a + \frac{\omega_q^1}{2}\sigma^z + g(a^\dagger \sigma^- + a\sigma^+),$$

$$H_2 = \frac{\omega_r^R}{2} a^\dagger a - \frac{\omega_q^2}{2}\sigma^z + g(a^\dagger \sigma^+ + a\sigma^-),$$

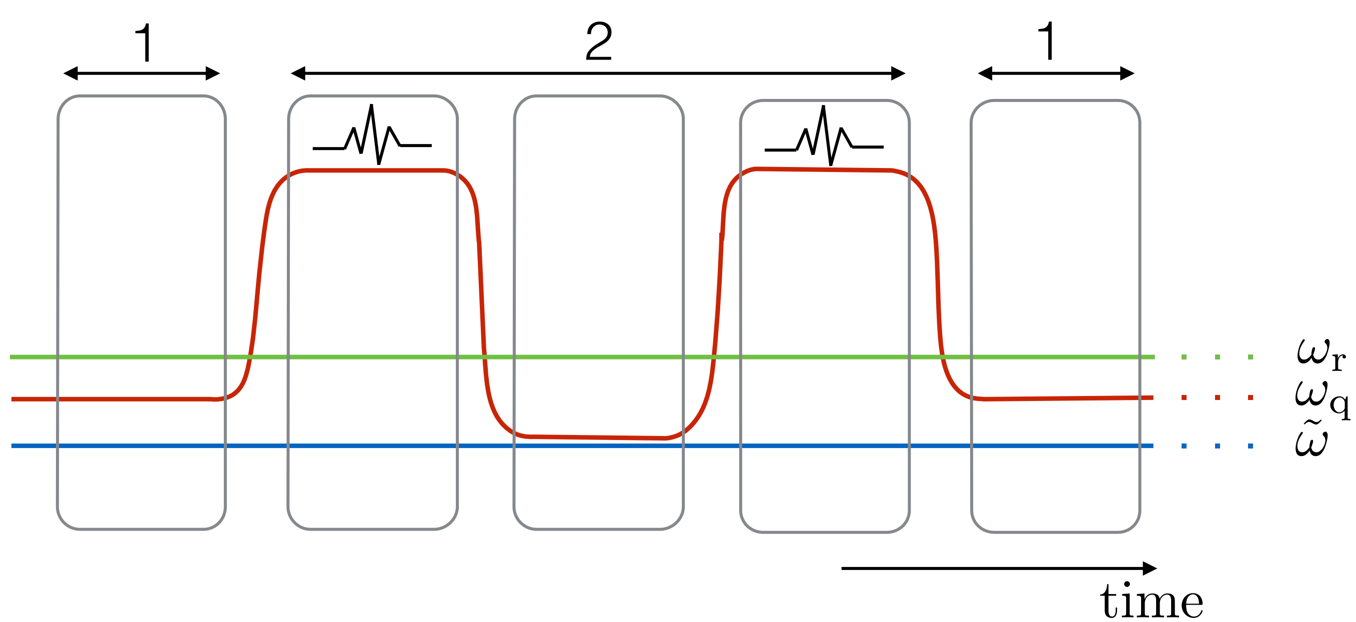
Digital quantum Rabi and Dicke models

Mezzacapo et al., Sci. Rep. 2014

JC in interaction picture $\tilde{H} = \tilde{\Delta}_r a^\dagger a + \tilde{\Delta}_q \sigma^z + g(a^\dagger \sigma^- + a\sigma^+),$

and we get AJC $e^{-i\pi\sigma^x/2}\tilde{H}e^{i\pi\sigma^x/2} = \tilde{\Delta}_r a^\dagger a - \tilde{\Delta}_q \sigma^z + g(a^\dagger \sigma^+ + a\sigma^-).$

→ Trotterization

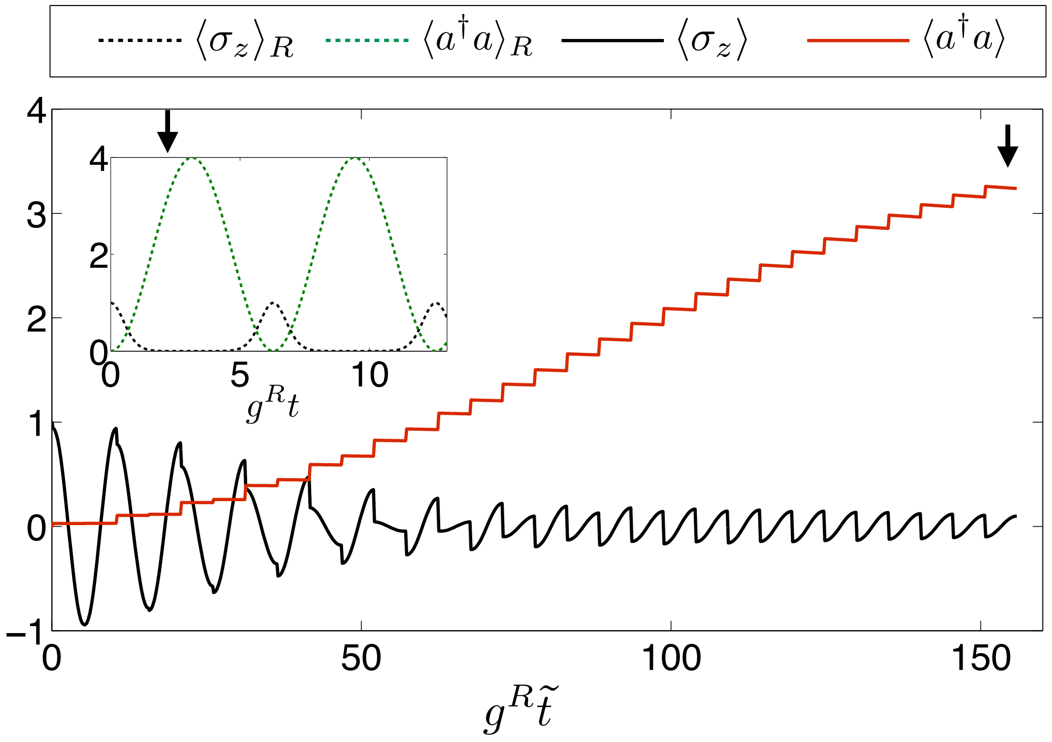


Digital quantum Rabi and Dicke models

Mezzacapo et al., Sci. Rep. 2014

$g^R = \omega_q^R/2 = \omega_r^R/2$	$\tilde{\omega} = 7.4$ GHz, $\omega_q^1 - \omega_q^2 = 200$ MHz
$g^R = \omega_q^R = \omega_r^R$	$\tilde{\omega} = 7.45$ GHz, $\omega_q^1 - \omega_q^2 = 100$ MHz
$g^R = 2\omega_q^R = \omega_r^R$	$\tilde{\omega} = 7.475$ GHz, $\omega_q^1 - \omega_q^2 = 100$ MHz

Some parameters…



$\omega_q^R = 0$, and $g^R = \omega_r^R$.

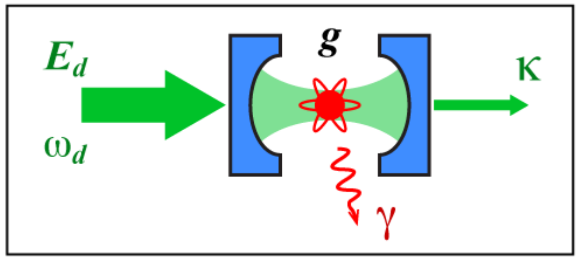
USC & DSC regimes are simulated. Move now towards to the Dicke model!

Experimental DQS of the quantum Rabi model: Delft

N K Langford *et al.*, in preparation (2016)

Trotter decomposition

Rabi, Phys Rev (1936)
Mezzacapo *et al.*, Sci Rep (2014)



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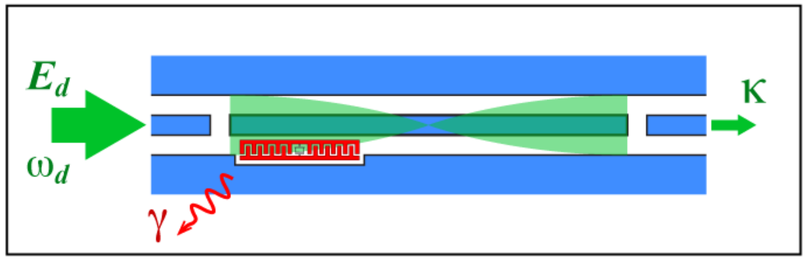
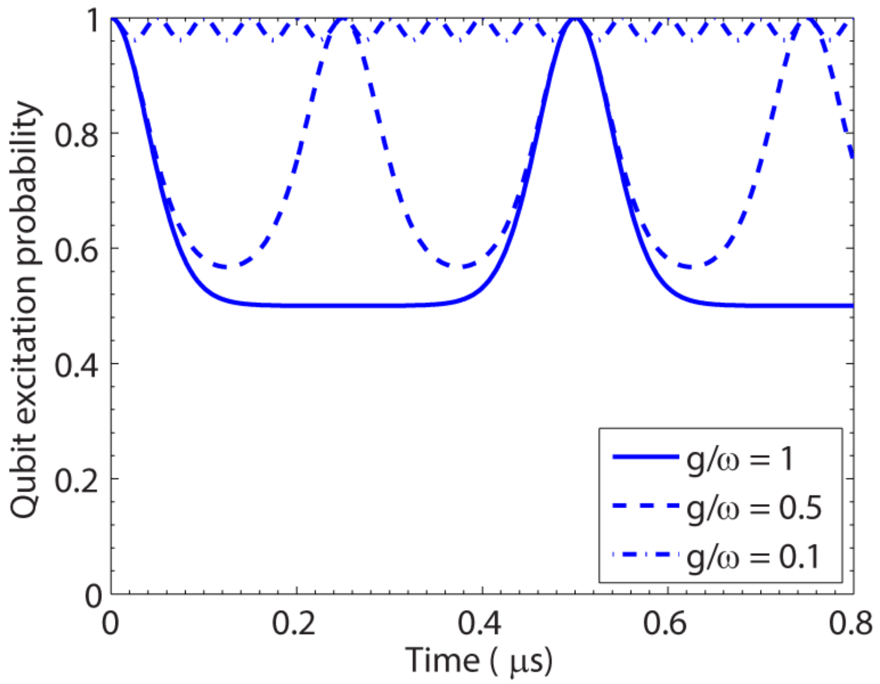


$$H = \hbar\omega_r\, a^\dagger a + \tfrac{1}{2}\hbar\omega_q\,\sigma_z + \hbar g\,(\sigma_+ + \sigma_-)\,(a + a^\dagger) = H_{\mathrm{JC}} + H_{\mathrm{AJC}}$$

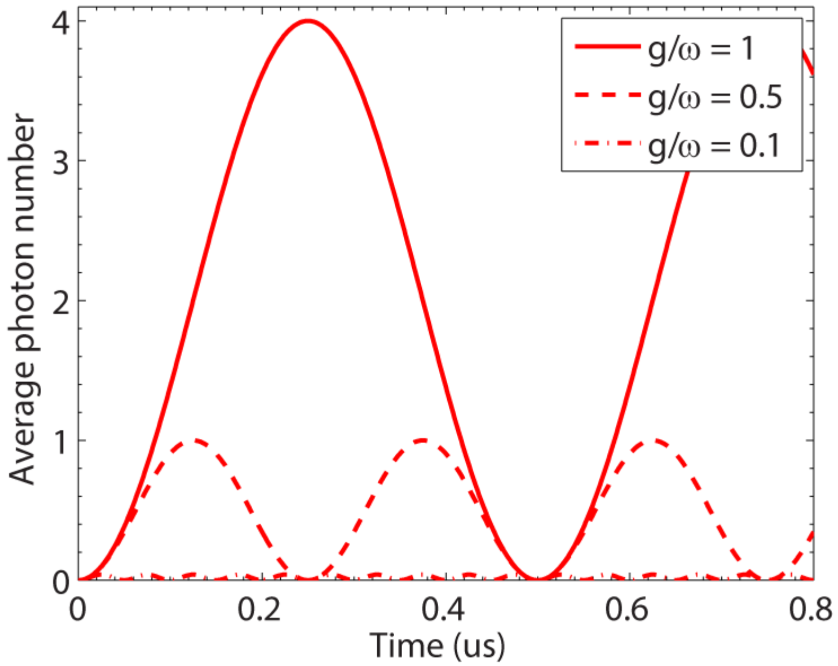
$$H_{\mathrm{JC}} = \hbar\Delta_r\, a^\dagger a + \tfrac{1}{2}\hbar\Delta_q\,\sigma_z + \hbar g\,(a\sigma_+ + a^\dagger\sigma_-)$$

$$H_{\mathrm{AJC}} = X\, H_{JC}\, X$$

Expected dynamics for g = 2 MHz



qubit



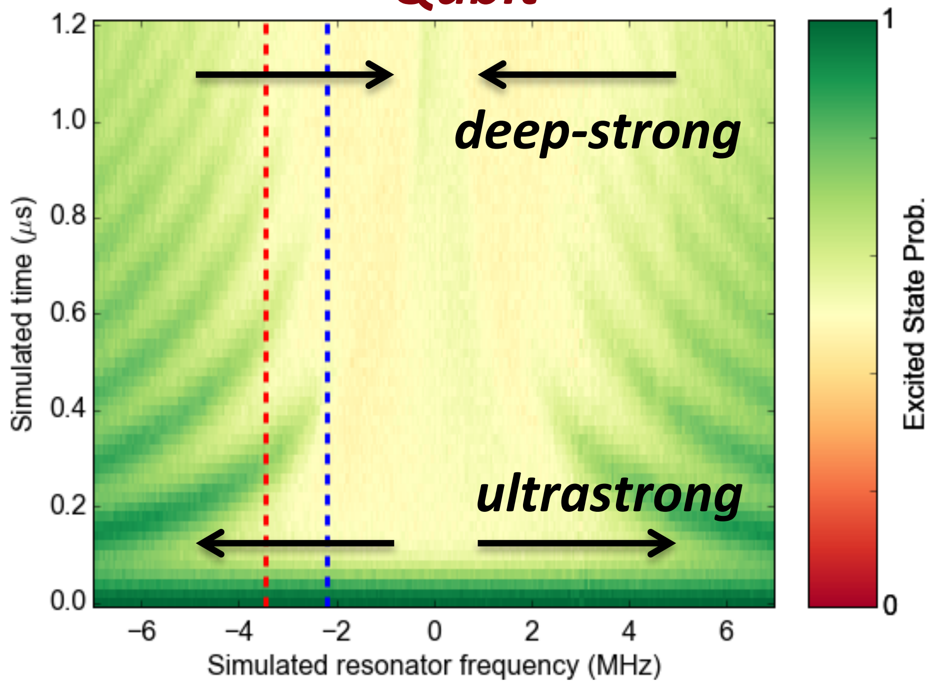
resonator

DQS of the QRM with transmons: Delft

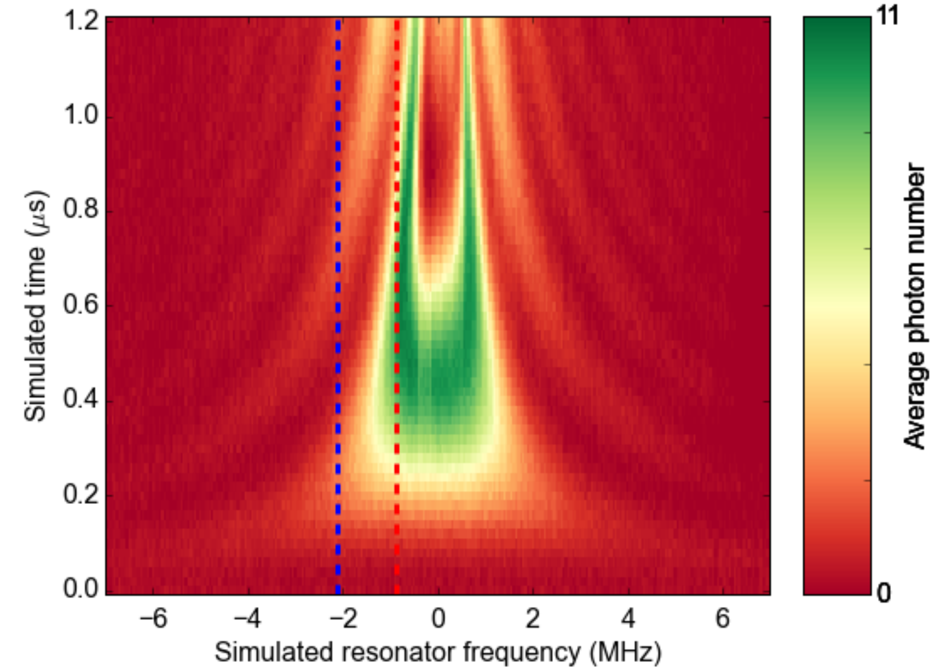
N K Langford *et al.*, in preparation (2016)

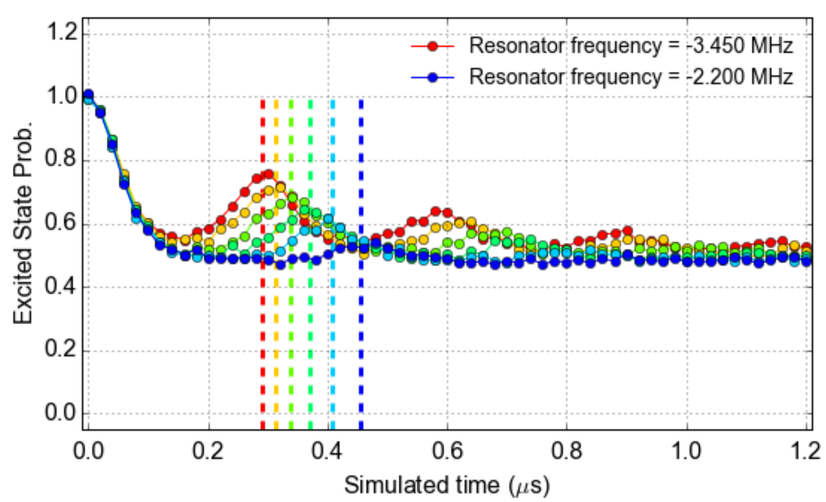
- For g ~ 1.95 MHz, g/ω = 1 gives expected qubit revival at 0.51 microseconds
- Qubit revivals beyond 0.4 us, photon number oscillations beyond 1.1 us (g/ω > 2)

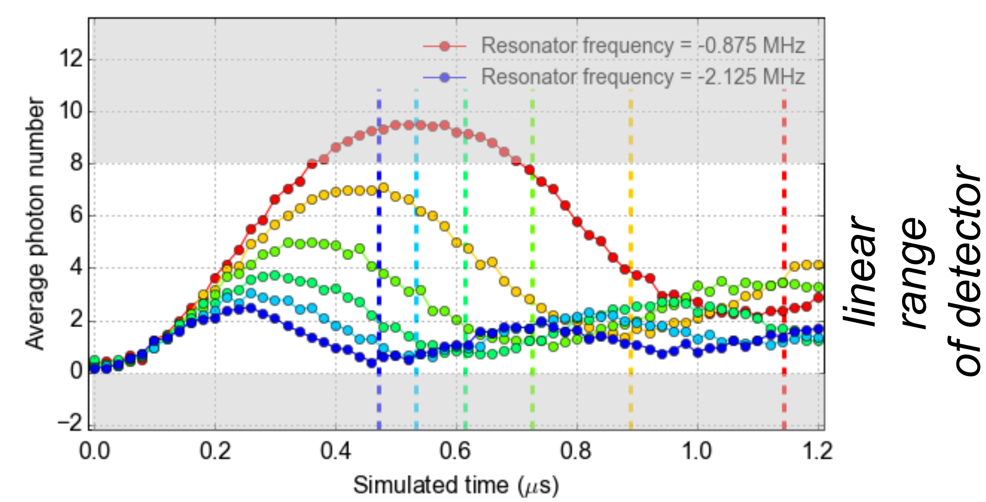
Qubit



Resonator Photon Number



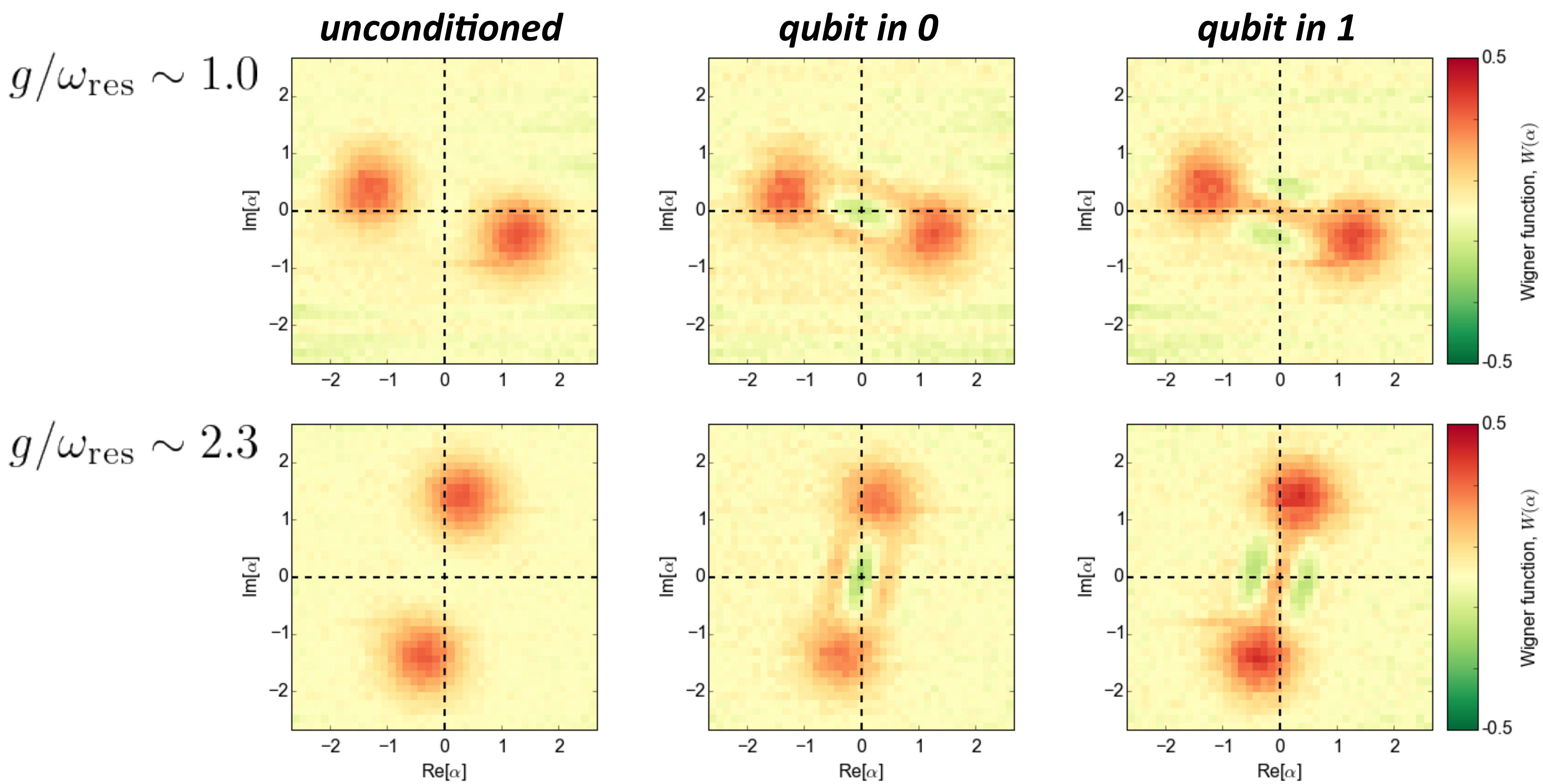




“Schrödinger cats” in DSC regime of QRM: Delft

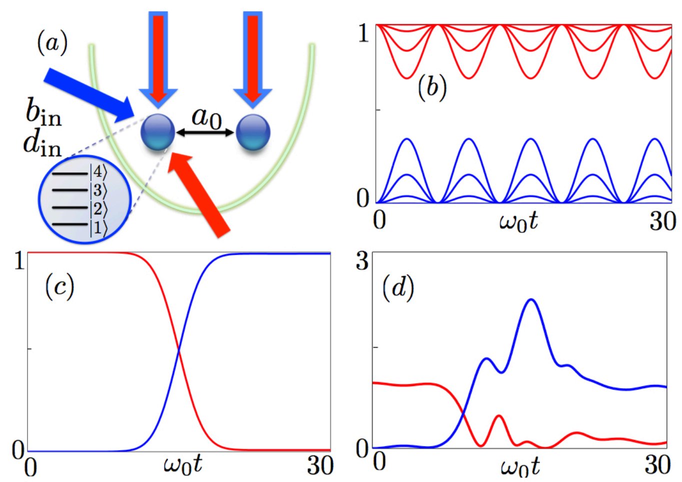
N K Langford *et al.*, in preparation (2016)

- DSC regime leads to “Schroedinger cat”-like entanglement between qubit & resonator
- Witnessed by observing negativity in both conditional cavity Wigner functions (state conditioned on measuring the qubit in 0 or 1) – smoking gun for deep-strong coupling



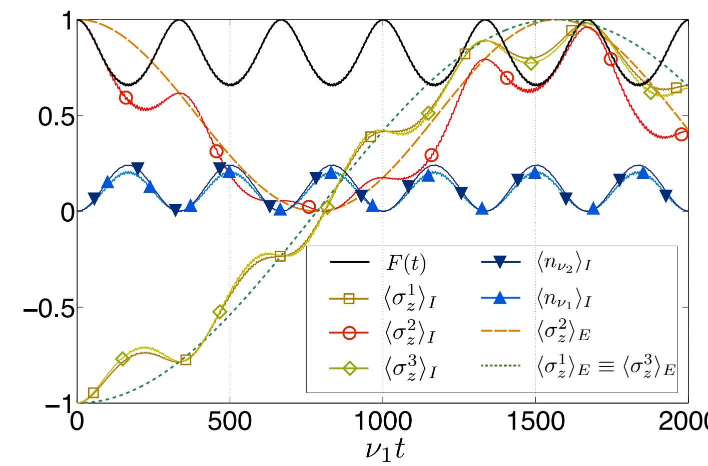
Further works involving DAQS concepts

Quantum Field Theory models
Casanova et al., PRL 2011



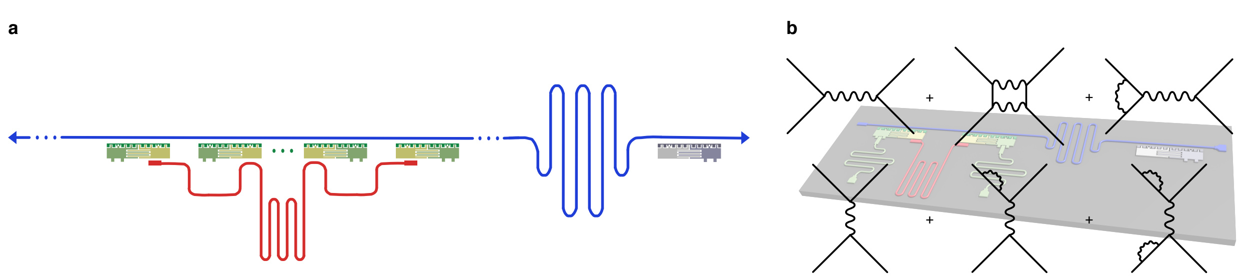
Trapped ions

Holstein Models
Mezzacapo et al., PRL 2012



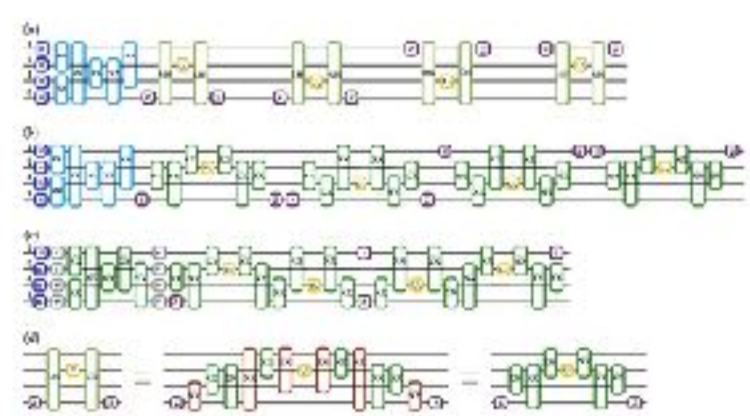
Trapped ions

Quantum field theory models
L. García-Álvarez et al., PRL 2015



Superconducting circuits

Quantum chemistry models
L. García-Álvarez et al., SciRep 2016



Superconducting circuits

II.4 Digital quantum simulation of fermion and fermion-boson models

DQS of fermionic models

i) Jordan Wigner

$$b_i^\dagger = I \otimes I \otimes \ldots \otimes \sigma_i^+ \otimes \sigma_{i-1}^z \otimes \ldots \otimes \sigma_1^z$$

$$\{b_i, b_j^\dagger\} = \delta_{i,j}$$

Local fermion interactions → Nonlocal spin interactions

J. Casanova et al., PRL '12

U. Las Heras et al., EPJ QT (2015)

ii) Trotter expansion

$$e^{-iHt} \simeq (e^{-iH_1 t/n} e^{-iH_2 t/n} \ldots e^{-iH_m t/n})^n$$

iii) Efficient implementation spin operators

$$H = \sum_{i=1}^{m} H_i \quad , H_i = g_i \sigma_{i_1} \otimes \sigma_{i_2} \otimes \ldots \otimes \sigma_{i_N}$$

M. Müller et al., NJP'11

$$H = \sum_{n=2}^{\alpha} \left[\sum_{i_1 \ldots i_n = 1}^{N} g_{i_1 \ldots i_n} c_{i_1} \cdots c_{i_n} + \text{H.c.} \right],$$ Fermionic Hamiltonian

$$\{c_{i_l}, c_{i_{l'}}^\dagger\} = \delta_{l,l'}$$

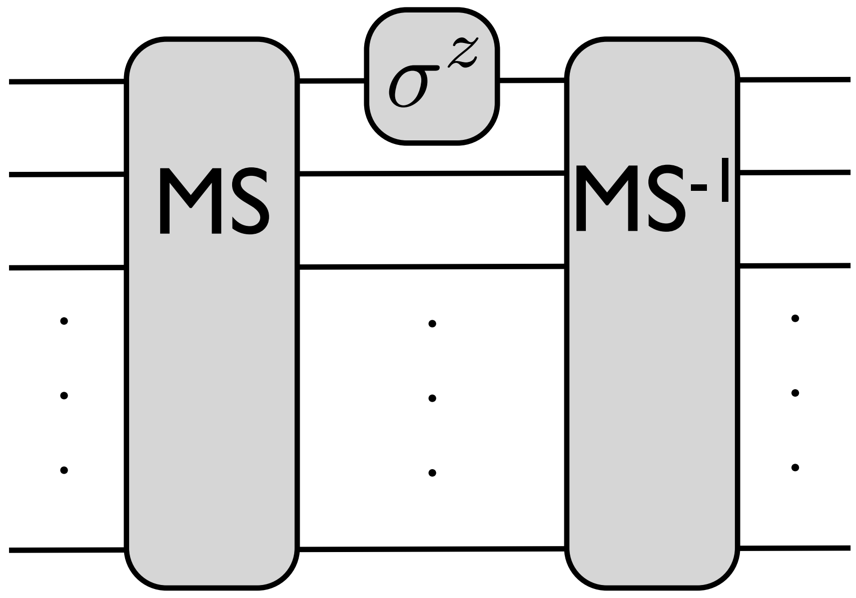
$$H = \sum_{i=1}^{m} H_i \qquad H_i = g_i \sigma_{i_1} \otimes \sigma_{i_2} \otimes \ldots \otimes \sigma_{i_N}$$

Trapped-ion implementation:
Mølmer-Sørensen gates (highly efficient)

M. Müller et al., NJP'11

J. Casanova et al., PRL '12

U. Las Heras et al., EPJ QT (2015)



Accessible fermion and fermion-boson models

i) Kondo

$$H = \sum_{p\sigma} \epsilon_p b^\dagger_{p\sigma} b_{p\sigma} - J \sum_{pp'j} e^{iR_j\cdot(p-p')} [(b^\dagger_{p\uparrow} b_{p'\uparrow} - b^\dagger_{p\downarrow} b_{p'\downarrow})\sigma^z_j + b^\dagger_{p\uparrow} b_{p'\downarrow}\sigma^-_j + b^\dagger_{p\downarrow} b_{p'\uparrow}\sigma^+_j].$$

ii) Hubbard

$$H = w \sum_{\delta i\sigma} b^\dagger_{i\sigma} b_{i+\delta\sigma} + U \sum_j b^\dagger_{j\uparrow} b_{j\uparrow} b^\dagger_{j\downarrow} b_{j\downarrow},$$

iii) Fröhlich

$$H = \sum_p \frac{p^2}{2m} b^\dagger_p b_p + \omega_0 \sum_q a^\dagger_q a_q + \sum_{qp} M(q) b^\dagger_{p+q} b_p (a_q + a^\dagger_{-q}),$$

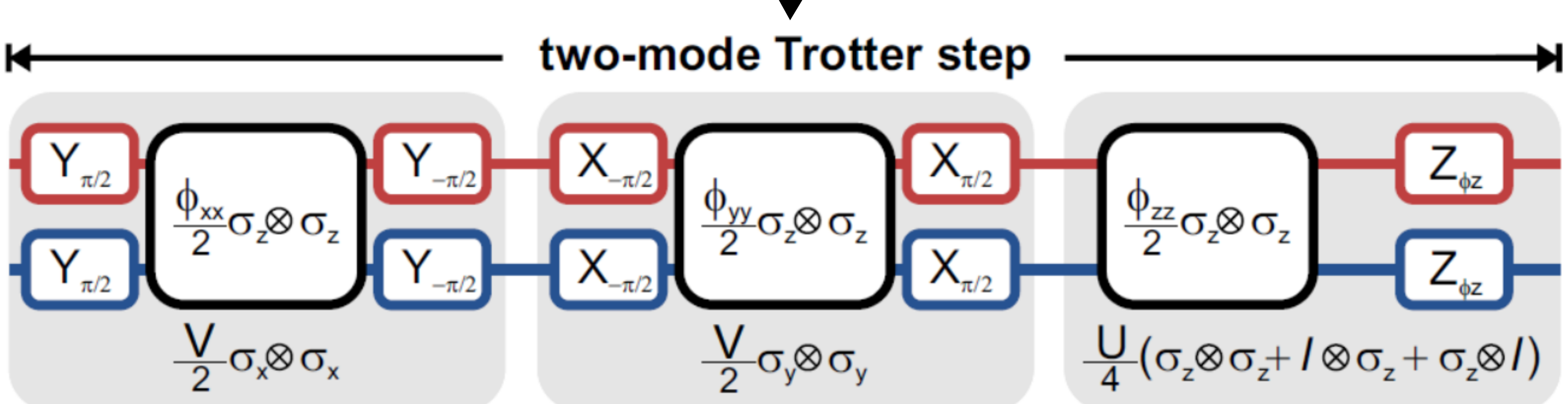
DQS of fermionic interactions in superconducting circuits

$$H = -V \sum_{i,j} \left(b_i^\dagger b_j + b_j^\dagger b_i \right) + U \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}$$

↓

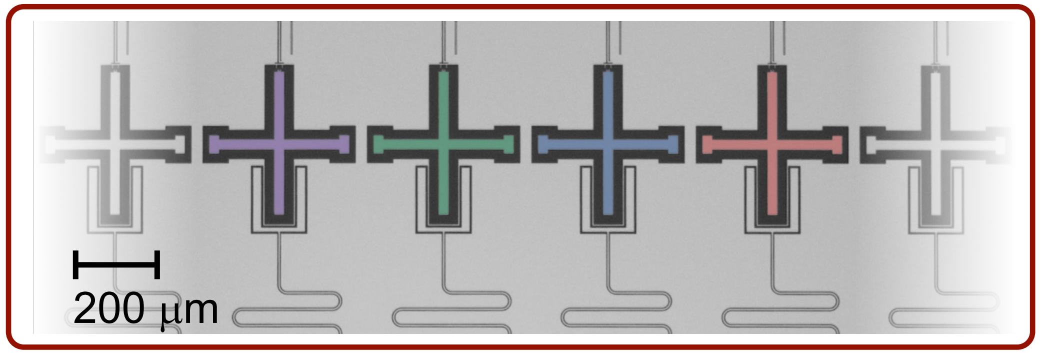
$$H = \frac{V}{2} (\sigma_x \otimes \sigma_x + \sigma_y \otimes \sigma_y) + \frac{U}{4} (\sigma_z \otimes \sigma_z + I \otimes \sigma_z + \sigma_z \otimes I),$$

↓



DQS of fermionic interactions in superconducting circuits

Experiment in Google/UCSB in collaboration with Bilbao



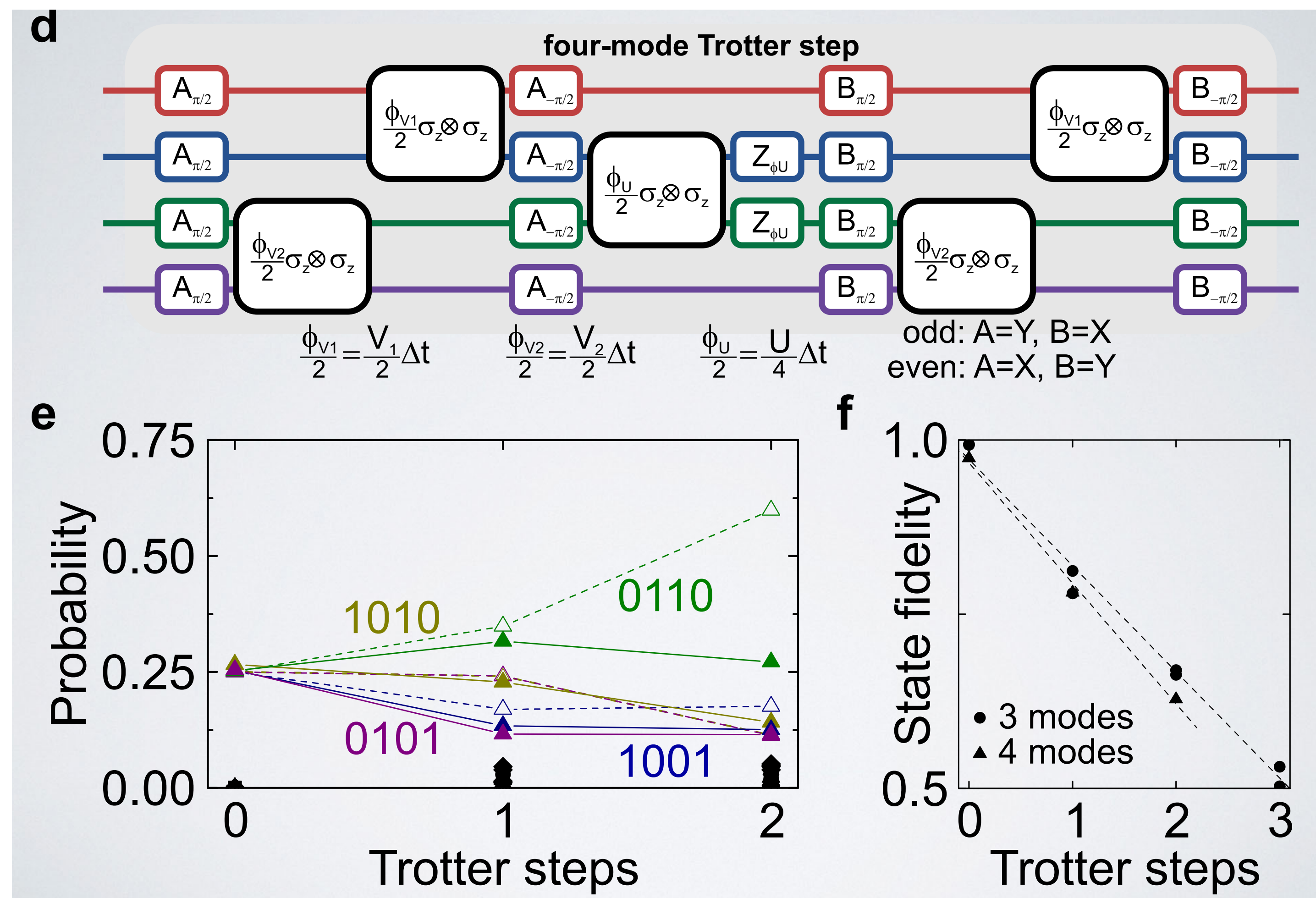
Multipurpose digital 9-qubit chip

DQS of 2-4 fermionic modes

More than 300 quantum gates

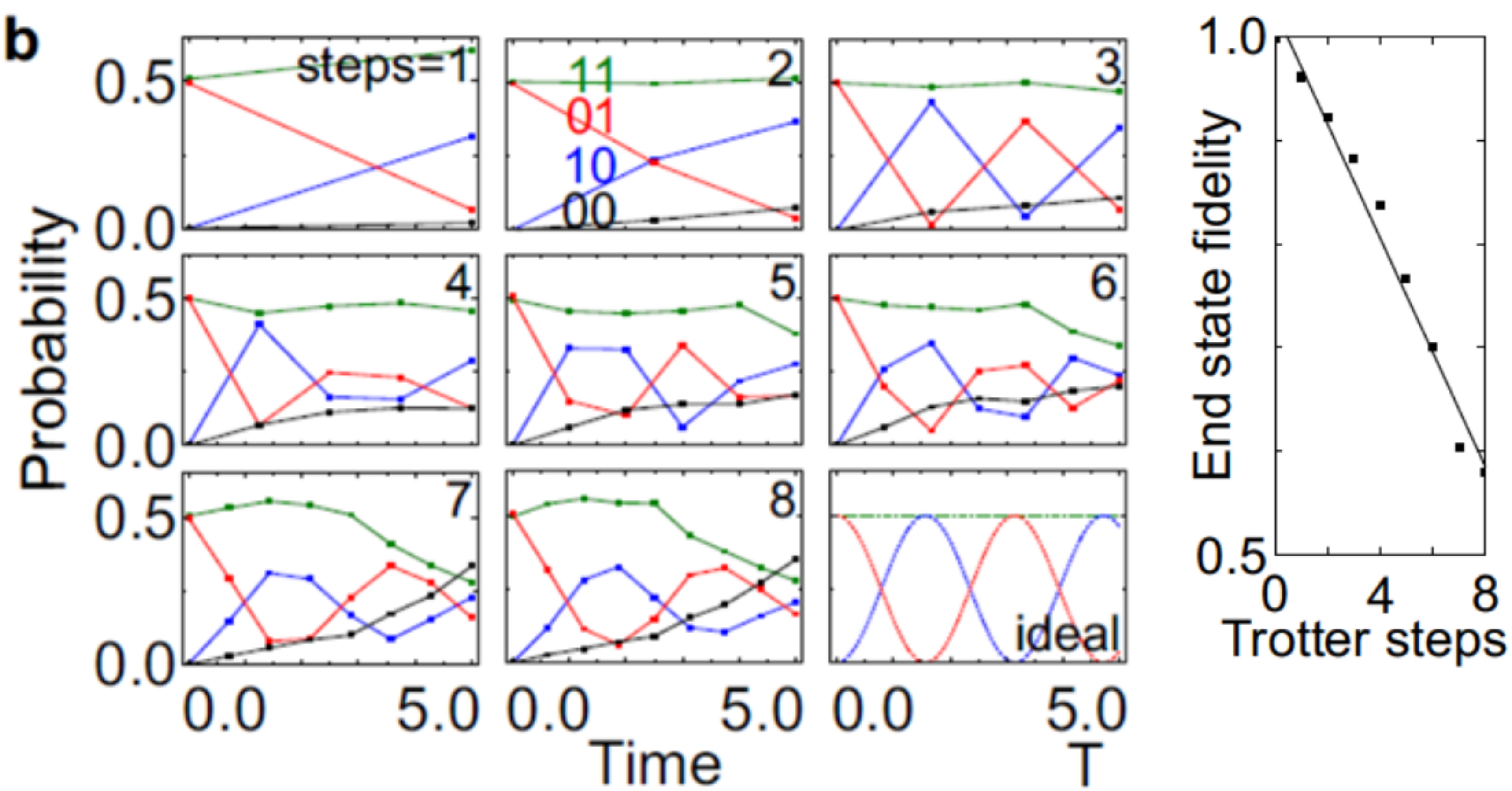
U. Las Heras et al., EPJ QT (2015)

R. Barends et al., Nat. Comm. **6**, 7654 (2015)



DQS of fermionic interactions in superconducting circuits

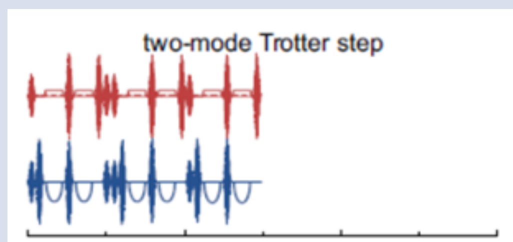
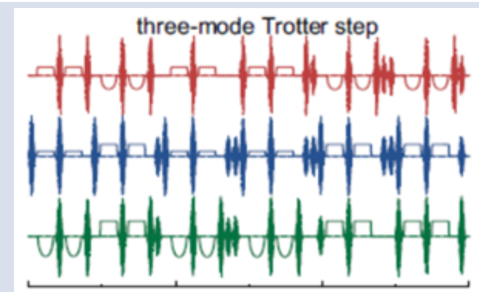
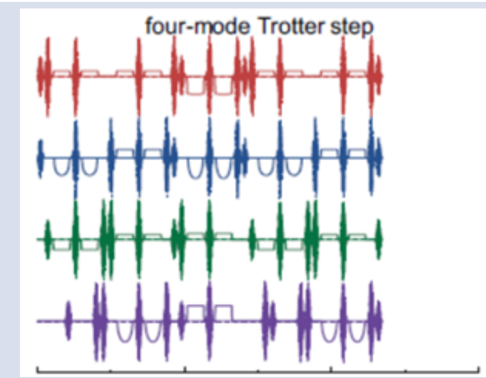
Experiment in Google/UCSB in collaboration with Bilbao



Simple two-mode example:
•hopping shows fermionic interaction
•more steps: more resolution, but more error

DQS of fermionic interactions in superconducting circuits

Experiment in Google/UCSB in collaboration with Bilbao

Two-mode	Three-mode	Four-mode
		
1Q, 2Q gates: 28, 6	87, 12	98, 10
Estimated error: 0.067	0.16	0.15
Exp. Simulation error: 0.054	0.15	0.17