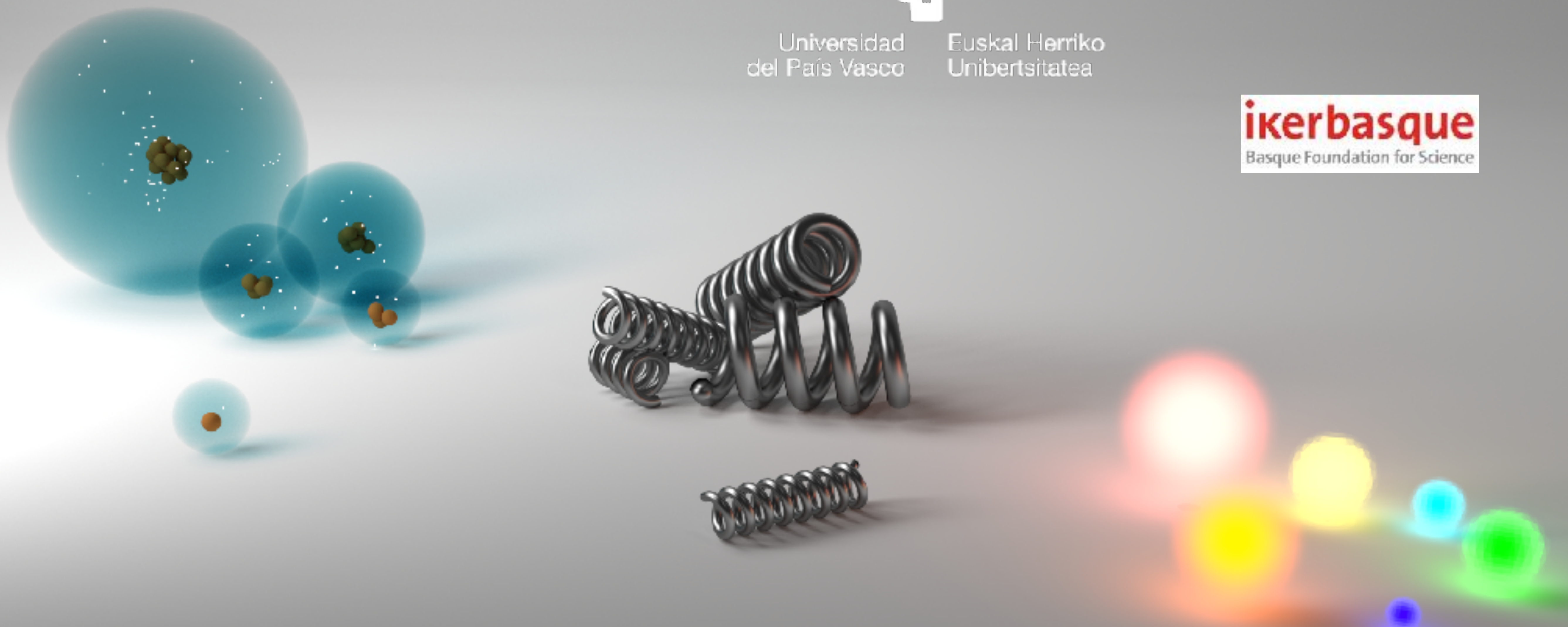

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Novel paradigms for quantum simulations

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Lectures on

"Novel paradigms for quantum simulations"

Lecture I: Analog quantum simulations

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I.2 The Jaynes-Cummings model in circuit QED and trapped ions
I.3 Analog quantum simulation of the quantum Rabi model in circuit QED
I.4 Analog quantum simulation of the quantum Rabi model in trapped ions
I.5 Analog quantum simulation of the Dirac equation in trapped ions

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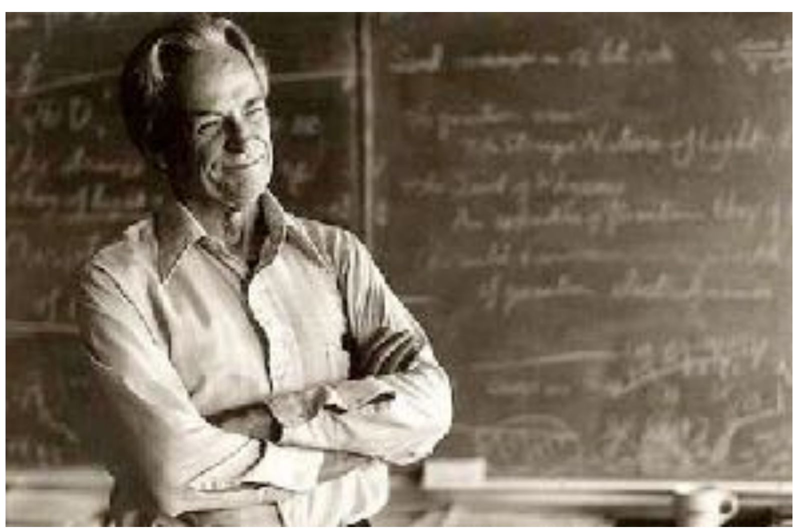
I.1 Introduction to quantum simulations

What is a quantum simulation?

Definition

Quantum simulation is the intentional reproduction of the quantum aspects of a physical or unphysical model onto a typically more controllable quantum system.

Richard Feynman



Let nature calculate for us

<=>

Greek theatre

Mimesis or imitation is always partial,
this is the origin of creativity in science and arts

Quantum simulation <=> Quantum theatre

Why are quantum simulations relevant?

a) Because we can discover analogies between unconnected fields, producing a flood of knowledge in both directions, e.g. black hole physics and Bose-Einstein condensates.

b) Because we can study phenomena that are difficult to access or even absent in nature, e.g. Dirac equation: *Zitterbewegung* & Klein Paradox, unphysical operations.

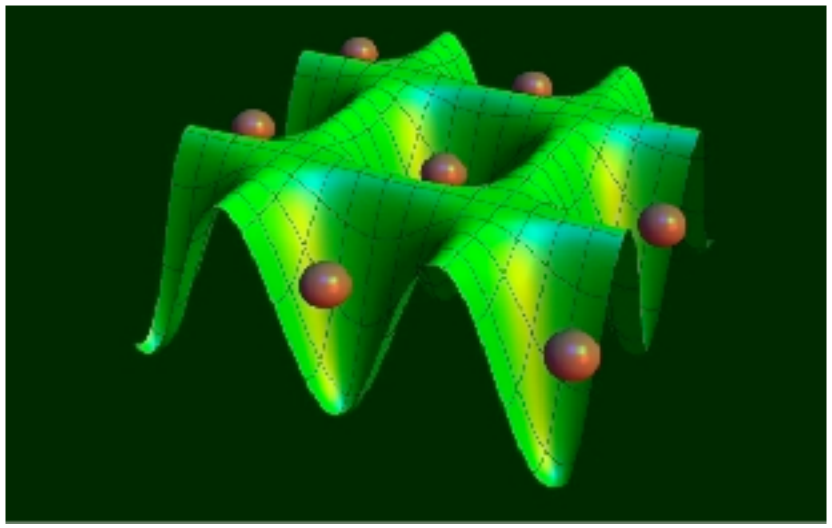
c) Because we can predict novel physics without manipulating the original systems, some experiments may reach quantum supremacy: CM, QChem, QFT, ML, AI & AL.

d) Because we can contribute to the development of novel quantum technologies via scalable quantum simulators and their merge with quantum computing.

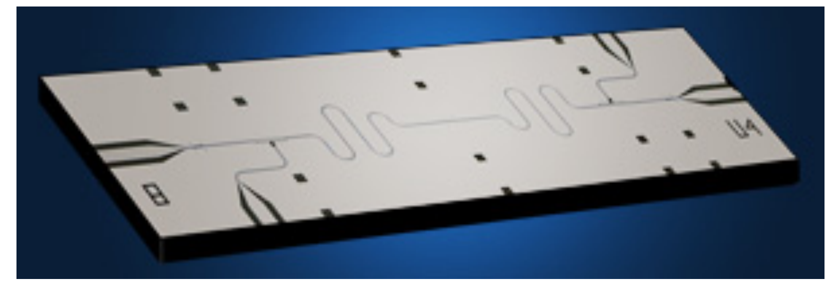
e) Because we are unhappy with reality, we enjoy arts and fiction in all its forms: literature, music, theatre, painting, quantum simulations.

Quantum Platforms for Quantum Simulations

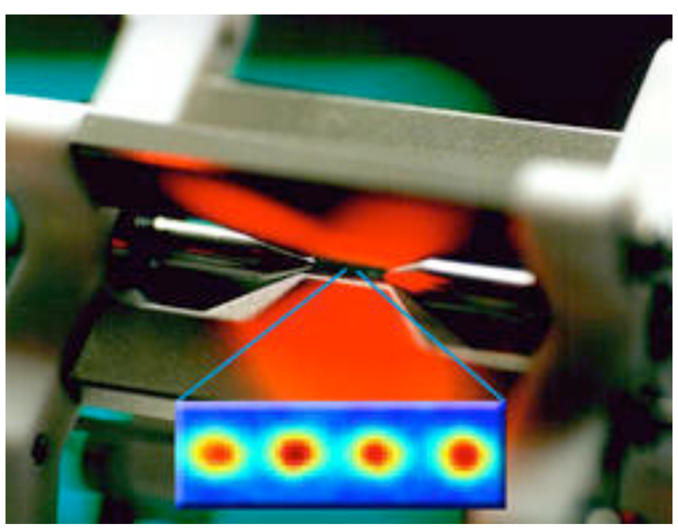
Optical lattices



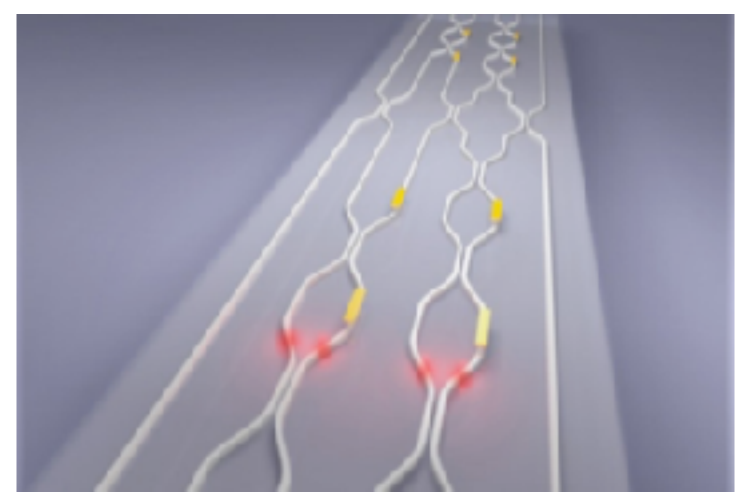
Superconducting circuits



Trapped ions



Quantum photonics



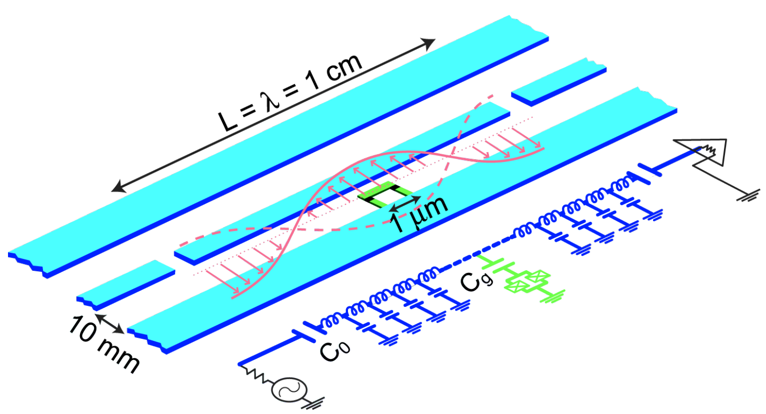
… among several others, including some attractive hybrid ones!

I.2 The Jaynes-Cummings model in circuit QED and trapped ions

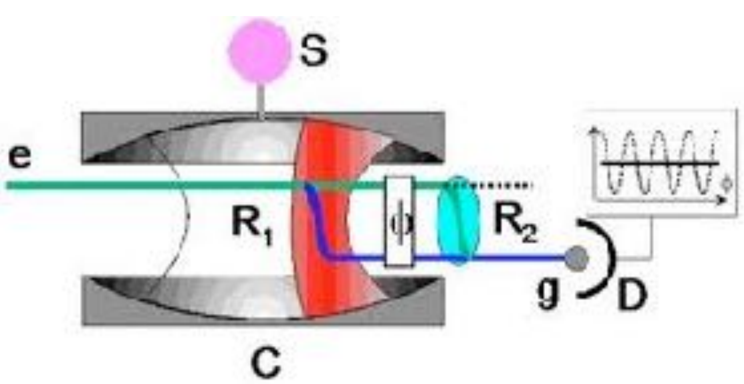
Quantum simulation of the Jaynes-Cummings model in circuit QED

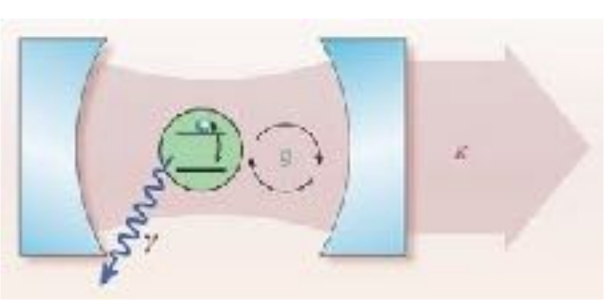
We could also see the JC model in circuit QED as a quantum simulation:
the two-level atom is replaced by a superconducting qubit, called artificial atom.

$$H_{JC} = \frac{\hbar\omega_0}{2}\sigma_z + \hbar\omega a^\dagger a + \hbar g\left(\sigma^+ a + \sigma^- a^\dagger\right)$$



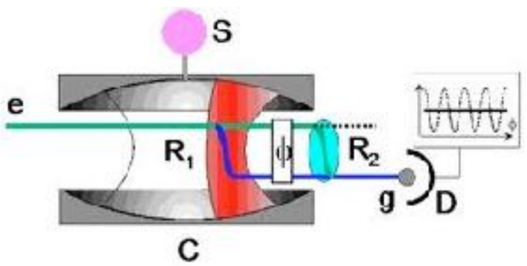
Quantum simulations are never a plain analogy, cQED has advantages in qubit control as in microwave CQED, but also longitudinal and transversal driving as in optical CQED.



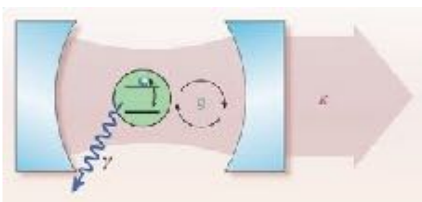


Quantum simulation of the Jaynes-Cummings model in ion traps

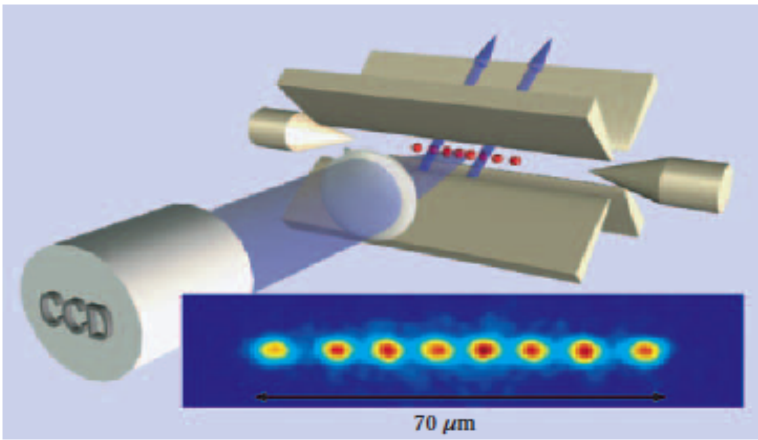
The simplest and most fundamental model describing the coupling between light and matter is the Jaynes-Cummings (JC) model in cavity QED.



$$H_{JC} = \frac{\hbar\omega_0}{2}\sigma_z + \hbar\omega a^\dagger a + \hbar g\left(\sigma^+ a + \sigma^- a^\dagger\right)$$



We could consider the implementation of the JC model in trapped ions as (one of) the first nontrivial quantum simulation(s).



$$H_0 = \hbar\nu(a^\dagger a + \frac{1}{2})$$

$$H_r = \hbar\eta\tilde{\Omega}_r\left(\sigma^+ a e^{i\phi_r} + \sigma^- a^\dagger e^{-i\phi_r}\right)$$

Red sideband excitation of the ion = JC interaction

$$H_b = \hbar\eta\tilde{\Omega}_b\left(\sigma^+ a^\dagger e^{i\phi_b} + \sigma^- a e^{-i\phi_b}\right)$$

Blue sideband excitation of the ion = anti-JC interaction

The quantized electromagnetic field is replaced by quantized ion motion

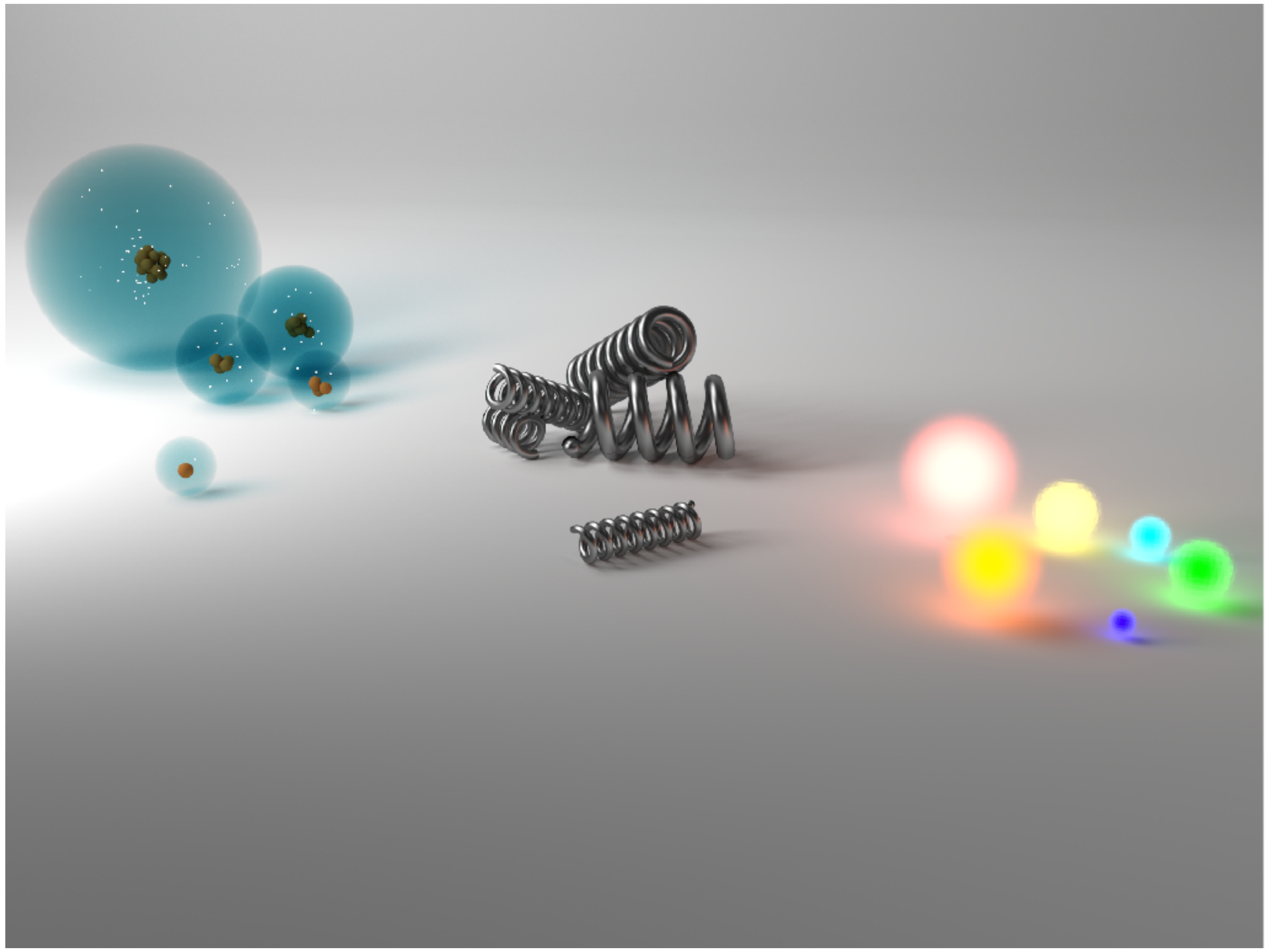
I.3 Analog quantum simulation of the quantum Rabi model in circuit QED

The quantum Rabi model: USC and DSC regimes

The quantum Rabi model (QRM) describes the dipolar light-matter coupling.
The JC model is the QRM after RWA, it is the SC regime of cavity/circuit QED.

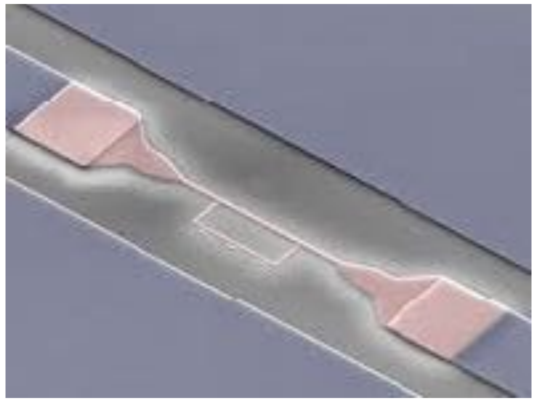
$$H_{Rabi} = \frac{\hbar\omega_0}{2}\sigma_z + \hbar\omega a^\dagger a + \hbar g\left(\sigma^+ + \sigma^-\right)\left(a + a^\dagger\right)$$

The QRM is not used for describing usual experiments because the RWA is valid in the microwave and optical regimes in quantum optics, where the JC model is enough.



Ultrastrong coupling regime of the QRM

We have recently seen the advent of the ultrastrong coupling (USC) regime of light-matter interactions in cQED, where $0.1 < g/w < 1$, and RWA is not valid.



T. Niemczyk et al., Nature Phys. **6**, 772 (2010)

P. Forn-Díaz et al., PRL **105**, 237001 (2010)

- Current experimental efforts reach perturbative and nonperturbative USC regimes where g/w ~ 0.1-1.0
- Recently, the analytical solutions of the QRM were presented: D. Braak, PRL **107**, 100401 (2011).

There are interesting and novel physical phenomena in the USC regime of the QRM:

a) Physics beyond RWA: Bloch-Siegert shifts, entangled ground states, among others.

$$\sigma^\dagger a + \sigma a^\dagger + \sigma^\dagger a^\dagger + \sigma a$$

b) Faster and stronger quantum operations

b.1) Ultrafast quantum gates (CPHASE) that may work at the subnanosecond scale

b.2) New regimes of light-matter coupling: Deep strong coupling (DSC) regime of QRM.

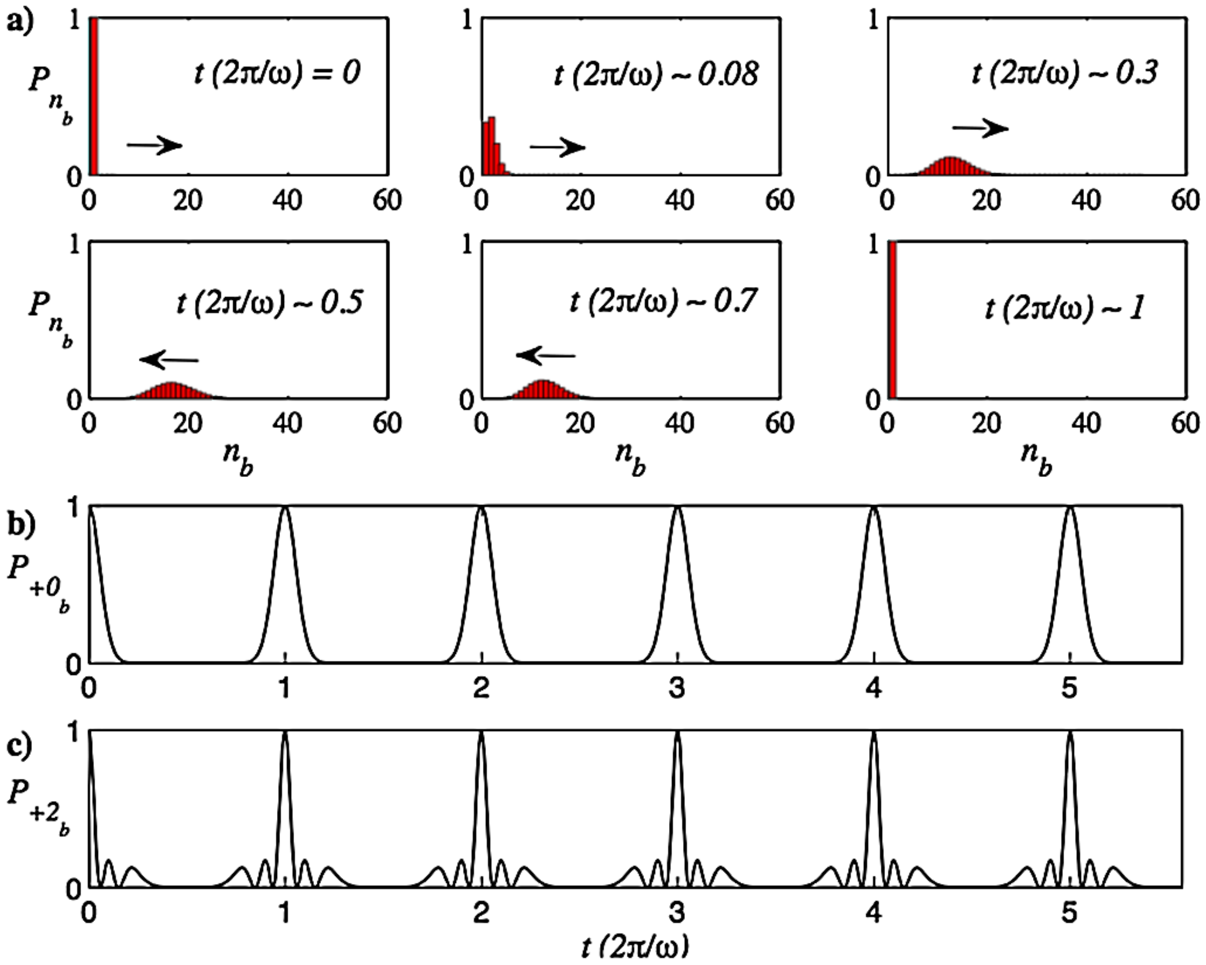
Deep strong coupling regime of the QRM

The DSC regime of the JC model happens when g/w > 1.0, and we can ask whether such a regime could be experimentally reached or ever exist in nature.

$$\Pi = -\sigma_z(-1)^{n_a} = -(|e\rangle\langle e| - |g\rangle\langle g|)(-1)^{a^\dagger a}$$

$$|g0_a\rangle \leftrightarrow |e1_a\rangle \leftrightarrow |g2_a\rangle \leftrightarrow |e3_a\rangle \leftrightarrow \ldots (p = +1)$$
$$|e0_a\rangle \leftrightarrow |g1_a\rangle \leftrightarrow |e2_a\rangle \leftrightarrow |g3_a\rangle \leftrightarrow \ldots (p = -1)$$

Forget about Rabi oscillations or perturbation theory:
parity chains and photon number wavepackets
define the physics of the DSC regime.



J. Casanova, G. Romero, et al., PRL **105**, 263603 (2010)

Is it possible to cheat technology or nature?

We may reach USC/DSC regimes in the lab but be unable to observe predictions, mainly due to the difficulty in ultrafast on/off coupling switching.

What can we do then? Here, we propose two options:

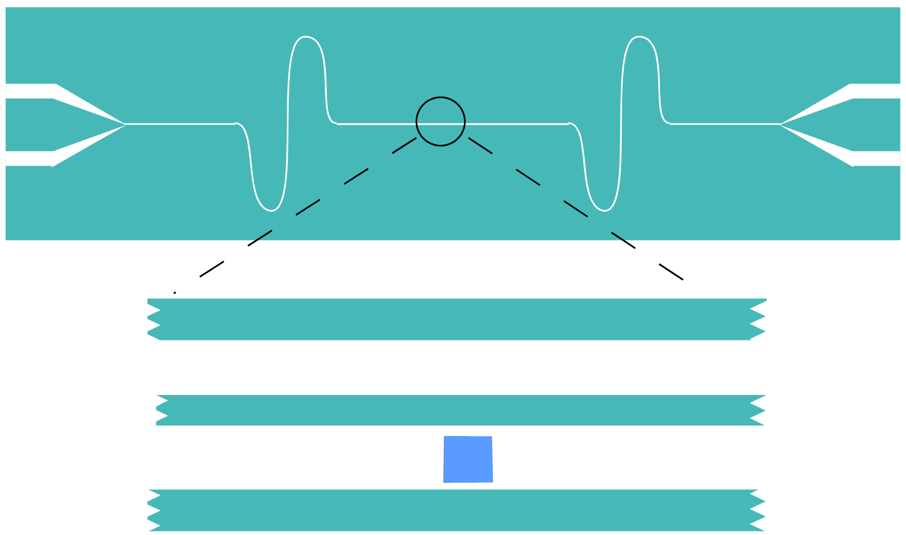
a) We go brute force and try to design ultrafast switching techniques that allow us to design a quantum measurement of relevant observables.

b) We could also reveal these regimes via quantum simulations.

b.1) Recently appeared several experiments realizing the quantum Rabi model and light-matter coupling in USC/DSC regimes

b.2) Is it possible a quantum simulation of the QRM with access to all regimes?

Simulating USC/DSC regimes of the QRM



$$\mathcal{H}_{\mathrm{JC}} = \frac{\hbar\omega_q}{2}\sigma_z + \hbar\omega a^\dagger a + \hbar g(\sigma^\dagger a + \sigma a^\dagger)$$

Two-tone microwave driving

$$\mathcal{H}_D = \hbar\Omega_1(e^{i\omega_1 t}\sigma + \text{H.c.}) + \hbar\Omega_2(e^{i\omega_2 t}\sigma + \text{H.c.})$$

Leads to the effective Hamiltonian: QRM in all regimes

$$\mathcal{H} = \hbar(\omega - \omega_1)a^\dagger a + \frac{\hbar\Omega_2}{2}\sigma_z + \frac{\hbar g}{2}\sigma_x(a + a^\dagger)$$

A two-tone driving in cavity QED or circuit QED can turn any JC model into a USC or DSC regime of the QRM model.

D. Ballester, G. Romero, et al., PRX **2**, 021007 (2012)

Quantum simulation of relativistic quantum mechanics

1+1 Dirac equation $$i\hbar\frac{d\psi}{dt} = (c\sigma_x p + mc^2\sigma_z)\psi$$

$$\omega_{\rm eff} = \omega - \omega_1 = 0 \longrightarrow \mathcal{H}_{\rm D} = \frac{\hbar\Omega_2}{2}\sigma_z + \frac{\hbar g}{\sqrt{2}}\sigma_x p$$

$$\mathcal{H}_D = \hbar\sum_j \Omega_j(e^{i(\omega_j t+\phi)}\sigma + \text{H.c.}) \qquad \phi = \pi/2$$

Zitterbewegung, via measuring $\langle X\rangle(t)$

R. Gerritsma et al., Nature **463**, 68 (2010)

1+1 Dirac particle + Potential

Add a classical driving to the cavity

$$\mathcal{H} = \mathcal{H}_{JC} + \hbar\sum_{j=1,2}(\Omega_j e^{-i(\omega_j t+\phi_j)}\sigma^\dagger + \text{H.c.}) + \hbar\xi(e^{-i\omega_1 t}a^\dagger + \text{H.c.})$$

$$\mathcal{H}_{\rm eff} = \frac{\hbar\Omega_2}{2}\sigma_z - \frac{\hbar g}{\sqrt{2}}\sigma_y\hat{p} + \hbar\sqrt{2}\xi\hat{x}$$

Klein paradox

R. Gerritsma et al., PRL **106**, 060503 (2011)

Measuring $\langle X\rangle$ to observe these effects

Quadrature moments have been measured at ETH and WMI:

E. Menzel et al., PRL **105**, 100401(2010); C. Eichler et al., PRL **106**, 220503 (2011)

Experimental AQS of QRM: KIT

Simulation scheme

Ballester PRX **2** (2012)

$$\hat{H}/\hbar = \frac{\omega_q}{2}\hat{\sigma}_z + \omega_r \hat{b}^\dagger \hat{b} + g\left(\hat{\sigma}_- \hat{b}^\dagger + \hat{\sigma}_+ \hat{b}\right) + \hat{\sigma}_x \left(\eta_1 \cos\omega_1 t + \eta_2 \cos\omega_2 t\right)$$

transversal microwave drives

- rotating frame with respect to ω_1

$$\hat{H}_1/\hbar = (\omega_q - \omega_1)\frac{\hat{\sigma}_z}{2} + (\omega_r - \omega_1)\hat{b}^\dagger \hat{b} + g\left(\hat{\sigma}_- \hat{b}^\dagger + \hat{\sigma}_+ \hat{b}\right) + \frac{\eta_1}{2}\hat{\sigma}_x + \frac{\eta_2}{2}\left(\hat{\sigma}_+ e^{i(\omega_1 - \omega_2)t} + \hat{\sigma}_- e^{-i(\omega_1 - \omega_2)t}\right)$$

- interaction picture in $\eta_1/2\, \sigma_x$, basis change via Hadamard transformation, constraint: $\omega_1 - \omega_2 = \eta_1$

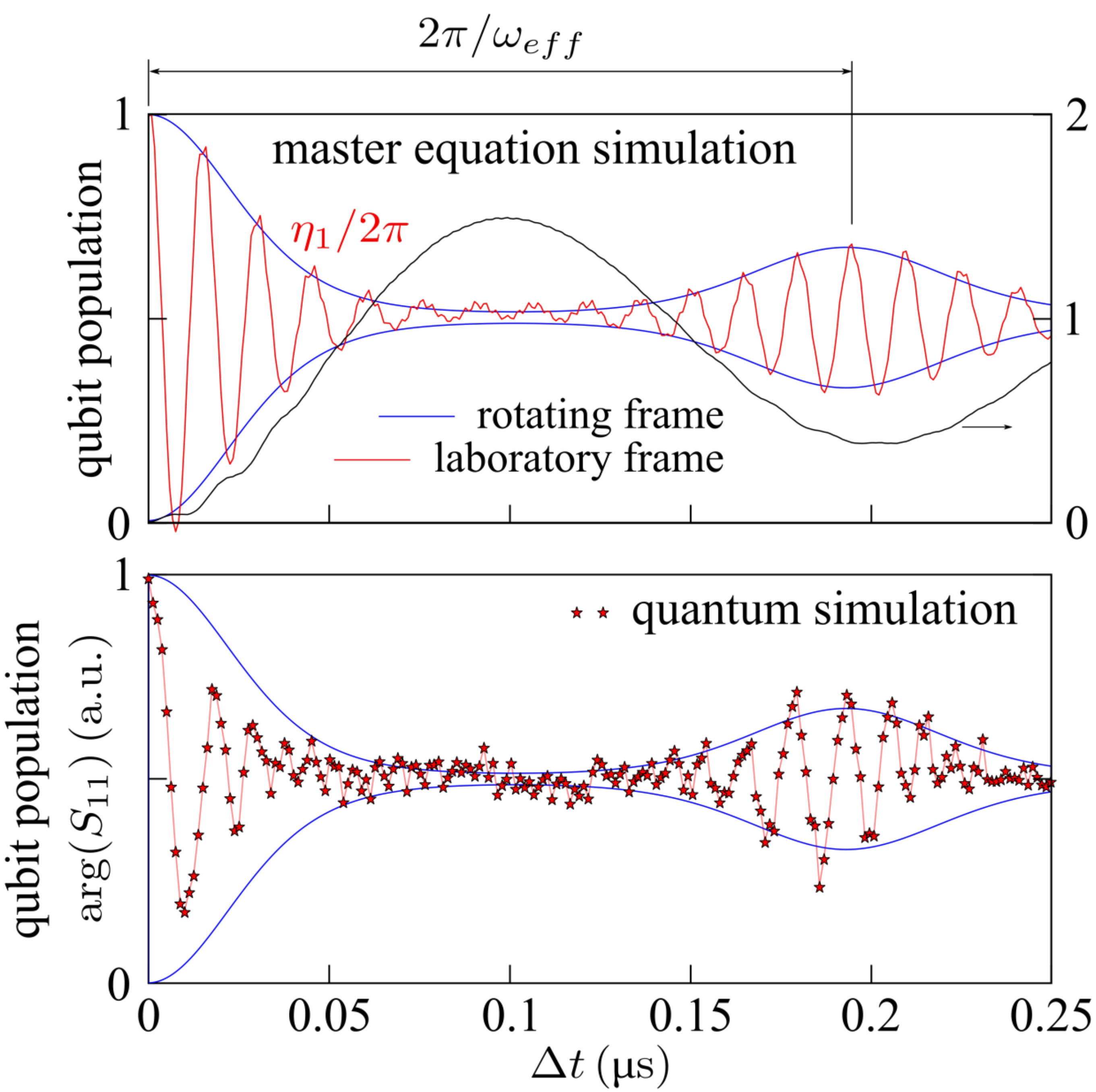
→ effective Hamiltonian with $\omega_{eff} \equiv \omega_r - \omega_1 \approx MHz$

$$\hat{H}_{eff}/\hbar = \frac{\eta_2}{2}\frac{\hat{\sigma}_z}{2} + \omega_{eff}\hat{b}^\dagger \hat{b} + \frac{g}{2}\sigma_x\left(\hat{b}^\dagger + \hat{b}\right)$$

~ MHz ~ MHz 5 MHz

Quantum simulation of relativistic quantum mechanics

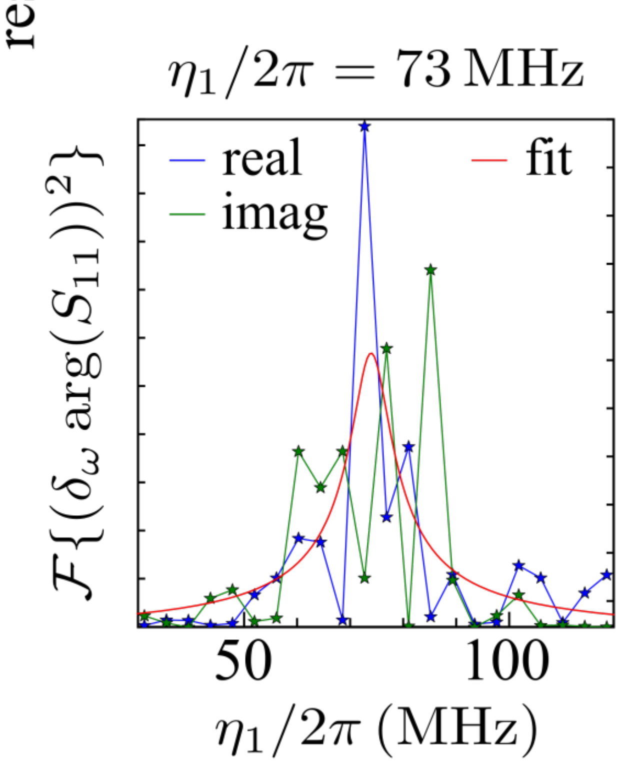
Quantum state collapse and revival



$$\hat{H}_{eff}/\hbar = \omega_{eff}\hat{b}^{\dagger}\hat{b} + \frac{g}{2}\sigma_x\left(\hat{b}^{\dagger} + \hat{b}\right)$$

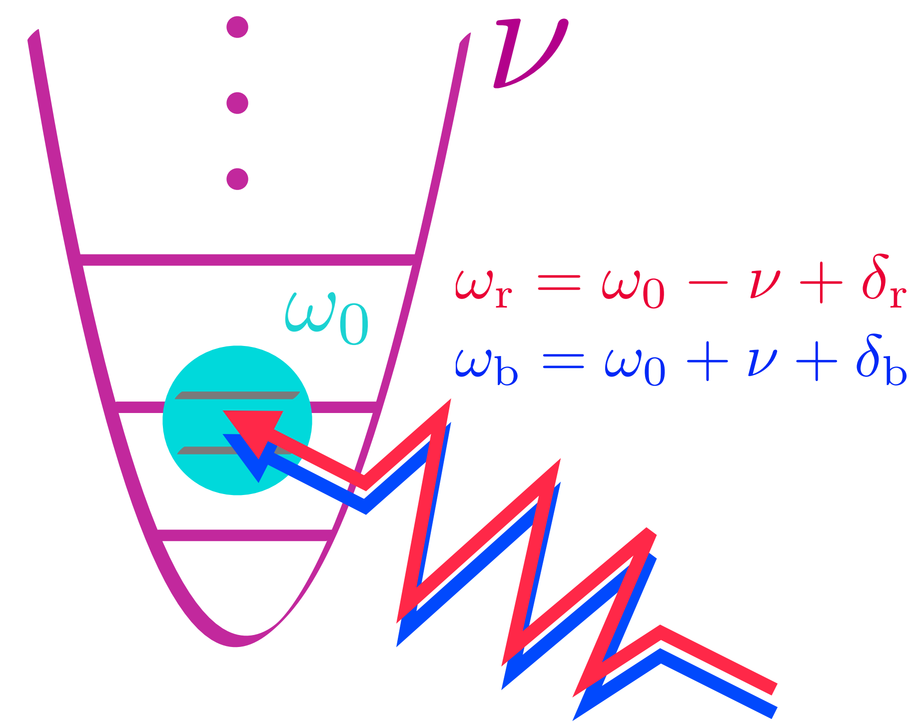
5 MHz 2.5 MHz

$T_1 = 5\ \mu s$
$Q_L = 13000$



Analog quantum simulation of QRM in trapped ions

$$H = \hbar\frac{i\eta\Omega}{2}(a\sigma^+ e^{-i\delta_\mathrm{r}t} + \mathrm{H.c.}) + \hbar\frac{i\eta\Omega}{2}(a^\dagger\sigma^+ e^{-i\delta_\mathrm{b}t} + \mathrm{H.c.})$$



Interaction picture

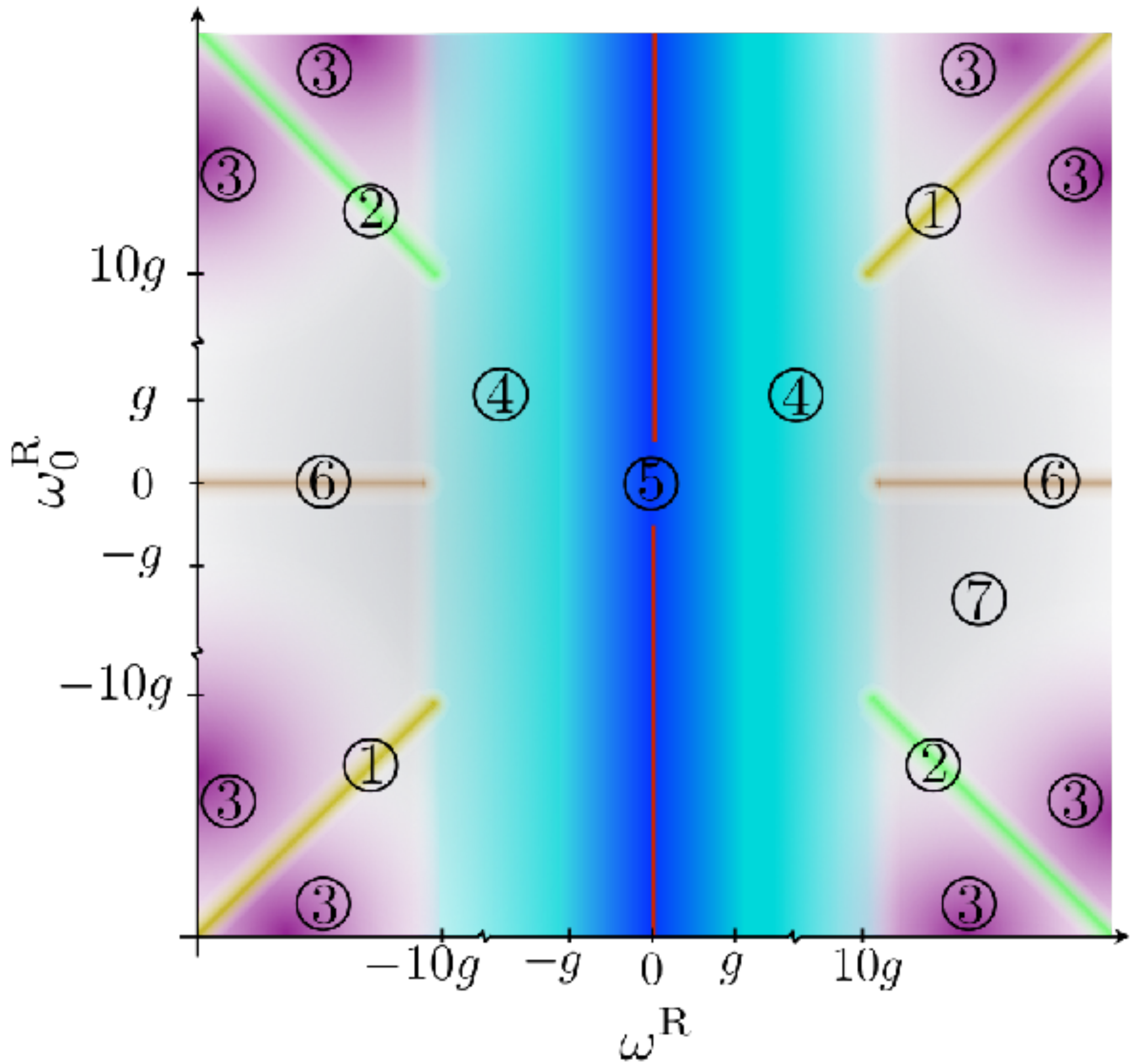
$$H = \hbar\frac{\delta_\mathrm{r} - \delta_\mathrm{b}}{2}a^\dagger a - \hbar\frac{\delta_\mathrm{r} + \delta_\mathrm{b}}{4}\sigma_z + \hbar\frac{i\eta\Omega}{2}(a + a^\dagger)(\sigma^+ - \sigma^-)$$

High tunability $\quad \omega_0^R = -\frac{1}{2}(\delta_r + \delta_b), \; \omega^R = \frac{1}{2}(\delta_r - \delta_b), \; g = \frac{\eta\Omega}{2}$

Interaction picture transformation commutes with the observables of interest $\sigma_z, \; a^\dagger a$

J. S. Pedernales et al., Sci. Rep. 5, 15472 (2015)

Coupling regimes of the QRM



1. JC $g \ll |\omega^{\rm R}|, |\omega_0^{\rm R}|$ $|\omega^{\rm R} - \omega_0^{\rm R}| \ll |\omega^{\rm R} + \omega_0^{\rm R}|$

2. AJC $g \ll |\omega^{\rm R}|, |\omega_0^{\rm R}|$ $|\omega^{\rm R} - \omega_0^{\rm R}| \gg |\omega^{\rm R} + \omega_0^{\rm R}|$

3. Dispersive regime $g < |\omega^{\rm R}|, |\omega_0^{\rm R}|, |\omega^{\rm R} - \omega_0^{\rm R}|, |\omega^{\rm R} + \omega_0^{\rm R}|$

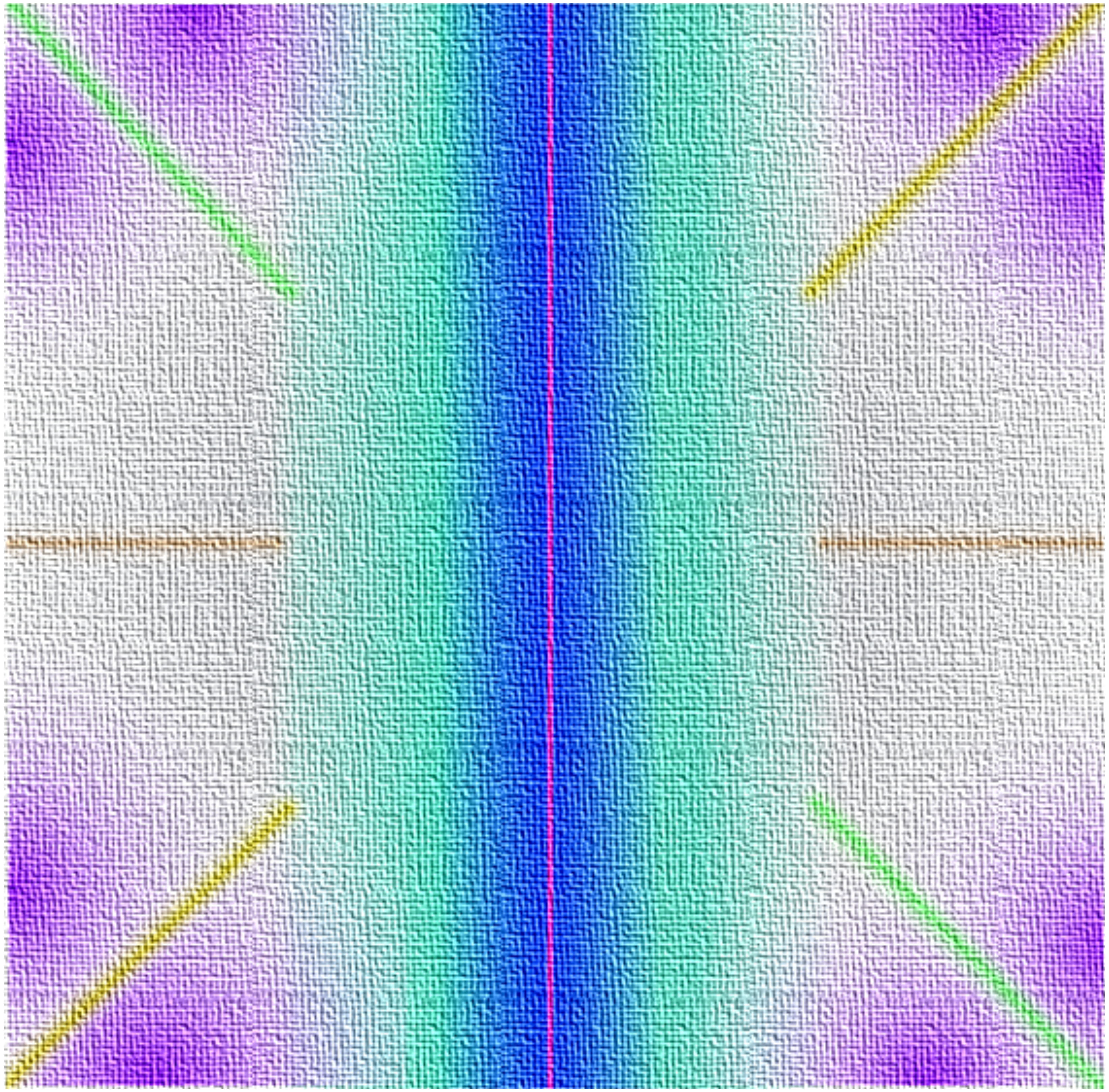
4. USC $g < |\omega^{\rm R}| < 10g$

5. DSC $|\omega^{\rm R}| < g$

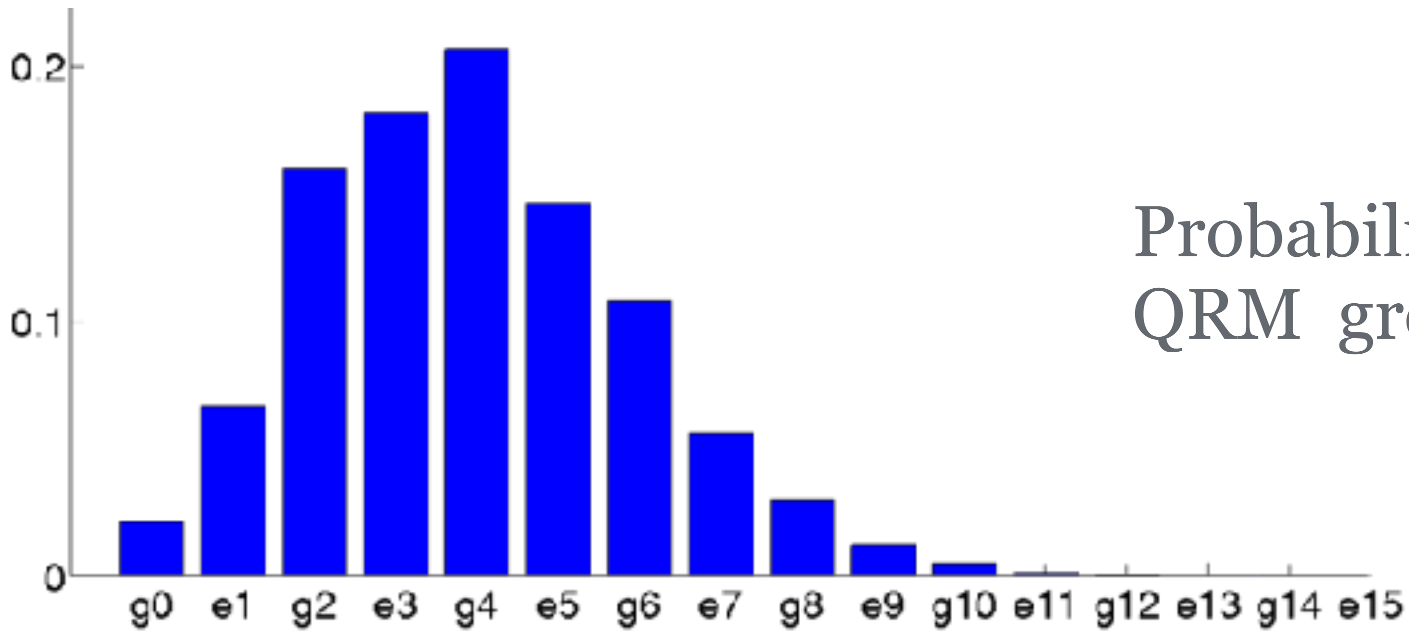
6. Decoupling regime $|\omega_0^{\rm R}| \ll g \ll |\omega^{\rm R}|$

7. Open to study $|\omega_0^{\rm R}| \sim g \ll |\omega^{\rm R}|$

Dirac equation $\omega^{\rm R} = 0$

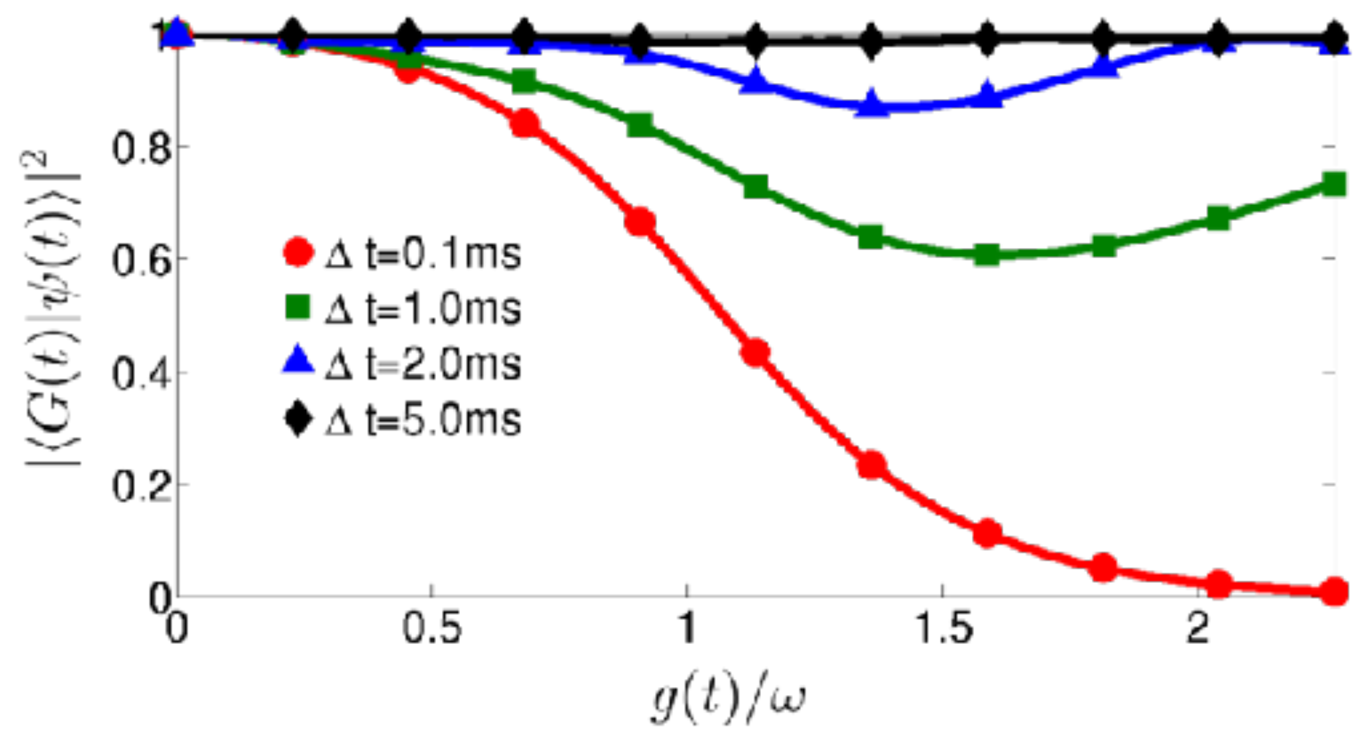


Cover of the special issue on the quantum Rabi model
in Journal of Physics A, 2016-17



Probability distribution of the QRM ground state for $g/\omega = 2$

Adiabatic generation of entangled ground state of QRM



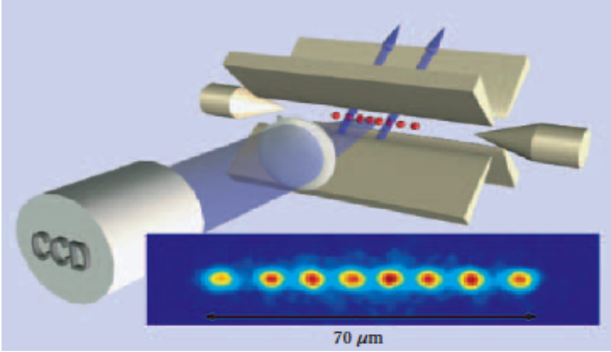
I.5 Analog quantum simulation of the Dirac equation in trapped ions

Quantum simulation of the Dirac equation with trapped ions

Basic interactions in trapped ions

a) The carrier excitation:

$$H_{\sigma_\phi} = \hbar\Omega\sigma_\phi = \hbar\Omega\left(\sigma^+ e^{i\phi} + \sigma^- e^{-i\phi}\right) \quad \begin{cases} \phi = 0 \rightarrow H_{\sigma_x} = \hbar\Omega\sigma_x \\ \phi = -\dfrac{\pi}{2} \rightarrow H_{\sigma_y} = \hbar\Omega\sigma_y \end{cases}$$

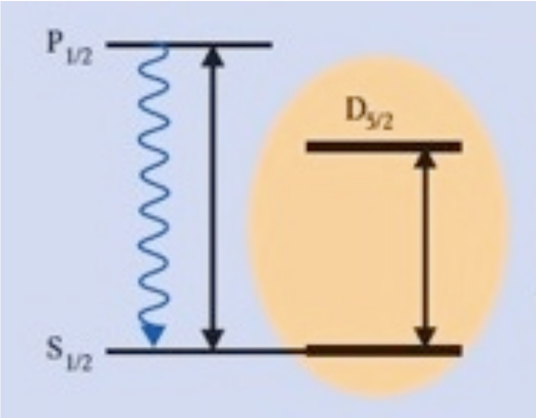


b) The red sideband excitation:

$$H_r = \hbar\eta\tilde{\Omega}_r\left(\sigma^+ a e^{i\phi_r} + \sigma^- a^\dagger e^{-i\phi_r}\right)$$

c) The blue sideband excitation:

$$H_b = \hbar\eta\tilde{\Omega}_b\left(\sigma^+ a^\dagger e^{i\phi_b} + \sigma^- a e^{-i\phi_b}\right)$$



d) The linear superposition of red and blue sideband excitations:

$$H_{r+b} = \hbar\eta\tilde{\Omega}\sigma_\phi\left(\alpha x + \beta p_x\right) \quad \text{with} \quad \begin{aligned} x &= \sqrt{\frac{\hbar}{2M\nu}}\left(a^\dagger + a\right) = \Delta\left(a^\dagger + a\right) \\ p_x &= i\sqrt{\frac{\hbar M\nu}{2}}\left(a^\dagger - a\right) = \frac{i\hbar}{2\Delta}\left(a^\dagger - a\right) \end{aligned}$$

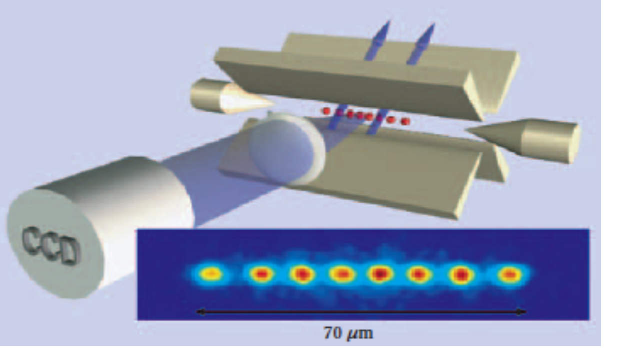
Simulating the Dirac equation

a) The linear superposition of carrier, red and blue sideband excitations, yield an effective Hamiltonian corresponding to the 1+1 Dirac Hamiltonian for a free particle:

$$i\hbar \frac{\partial}{\partial t}\phi = H_D^{ion}\phi = \left(2\eta\Delta\tilde{\Omega}\sigma_x p_x + \hbar\Omega\sigma_z\right)\phi = \begin{pmatrix} \hbar\Omega & 2\eta\Delta\tilde{\Omega}\, p_x \\ 2\eta\Delta\tilde{\Omega}\, p_x & -\hbar\Omega \end{pmatrix}\phi,$$

to be compared with the original:

$$i\hbar \frac{\partial}{\partial t}\phi = H_D\phi = \left(c\sigma_x p_x + mc^2\sigma_z\right)\phi = \begin{pmatrix} mc^2 & c\,p_x \\ c\,p_x & -mc^2 \end{pmatrix}\phi$$



producing the parameter correspondence:

$$\begin{cases} \hbar\Omega = mc^2 \\ 2\eta\Delta\tilde{\Omega} = c \end{cases}$$

b) Similar steps produce the quantum simulation of higher dimensional Dirac equations

L. Lamata, J. León, T. Schätz, and E. Solano, PRL **98**, 253005 (2007)

c) If we consider the relativistic limit, $mc^2 \ll cp_x \; (m \to 0)$, the Dirac dynamics produces constantly growing Schrödinger cats as in quantum optical systems:

$$H_D^{ion} = 2\eta\Delta\tilde{\Omega}\sigma_x p_x + \hbar\Omega\sigma_z \to H_D^{rel} = 2\eta\Delta\tilde{\Omega}\sigma_x p_x$$

See, for example, Solano et al., PRL (2001), Solano et al., PRL (2003), Haljan et al., PRL (2005), and Zähringer et al., PRL (2010).

d) If we consider now the nonrelativistic limit, $mc^2 \gg cp_x$, the Dirac dynamics would be happy to have a quantum optician calculating the second-order effective Hamiltonian:

$$H_D^I = 2\eta\Delta\tilde{\Omega}\left(\sigma^+ e^{2i\Omega t} + \sigma^- e^{-2i\Omega t}\right)p_x \to H_{\text{eff}} = \sigma_z \frac{p_x^2}{\left(\dfrac{\hbar\Omega}{2\eta^2\Delta^2\tilde{\Omega}^2}\right)} = \sigma_z \frac{p_x^2}{2m}$$

with simulated mass $m = \dfrac{\nu\Omega}{2\eta^2\tilde{\Omega}^2} M$

This is a free Schrödinger dynamics derived from the nonrelativistic limit of the Dirac equation!

e) The *Zitterbewegung* (ZB) is a jittering motion of the expectation value of the position operator $\langle x(t) \rangle$. It appears as a consequence of the superposition of positive and negative energy components.

In the Heisenberg picture, we can write the evolution of the Dirac position operator

$$x(t) = x(0) + \frac{c^2 p_x}{H_D} t + \frac{i\hbar c}{2H_D}\left(e^{2iH_D t/\hbar} - 1\right)\left(\sigma_x - \frac{cp_x}{H_D}\right)$$

f) The prediction of ZB is considered controversial, see several papers appeared in the last few years questioning existence/absence. The predicted ZB frequency/amplitude for our "relativistic" ion are

$$\omega_{ZB} \sim 2\left|\bar{E}_D\right| / \hbar = 2\sqrt{p_0^2 c^2 + m^2 c^4} / \hbar \equiv 2\sqrt{\left(2\eta\Delta\tilde{\Omega}p_0\right)^2 / \hbar + \Omega^2}$$

$$x_{ZB} \sim \frac{\hbar}{2mc}\left(\frac{mc^2}{\bar{E}_D}\right)^2 \equiv \frac{\eta\hbar^2\tilde{\Omega}\Omega\Delta}{4\eta^2\tilde{\Omega}^2\Delta^2 p_0^2 + \hbar^2\Omega^2} \sim \Delta$$

$$\omega_{ZB} \sim 0 - 10^6\,\text{Hz}$$

$$x_{ZB} \sim 0 - 10^3\,\text{Å}$$

From a theoretical point of view, the quantum simulation of the ZB looked cool!

However, the ZB amplitude was disappointing: how can one measure in the lab the ion position as a function of the interaction time with a resolution beyond the width of the motional ground state?

g) The answer to the previous question is: designing a highly precise measurement of the ion position!

We had proposed in 2006 such a method called "instantaneous" measurements for CQED and trapped ions.

If the initial state of the probe qubit and the unknown motional system is

$$\rho_{at-m}(0) = |+\rangle\langle+|\rho_m \quad \text{where} \quad |+\rangle = \frac{1}{\sqrt{2}}\left(|g\rangle + |e\rangle\right)$$

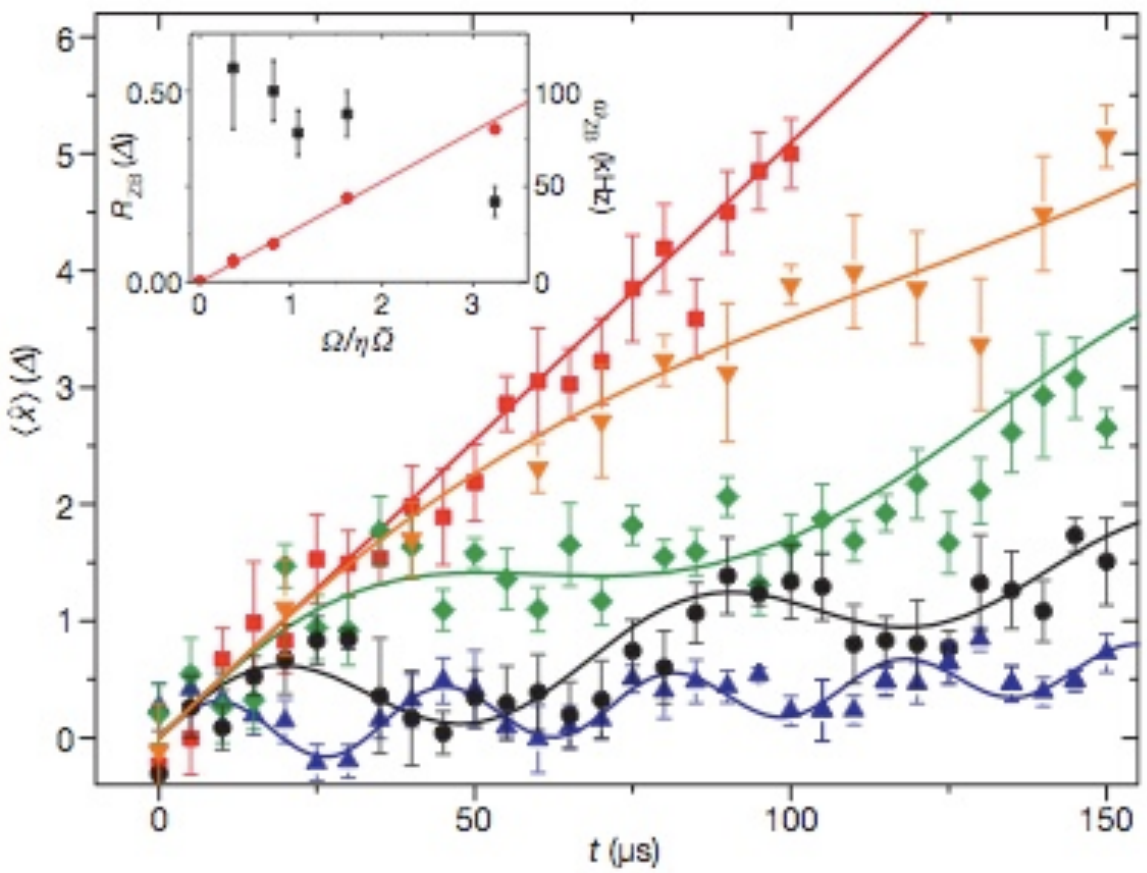
it can be proved that after a red-sideband excitation during an interaction time "t"

$$\langle x(t)\rangle = \left.\frac{dP_e(t)}{dt}\right|_{t=0} \quad \text{where} \quad P_e(t) = Tr\left[\rho_{at-m}(t)|e\rangle\langle e|\right]$$

It is possible to encode relevant motional system observables in the short-time dynamics of the probe qubit, in fact we can get the full wavefunction from the first and second derivatives at t=0!

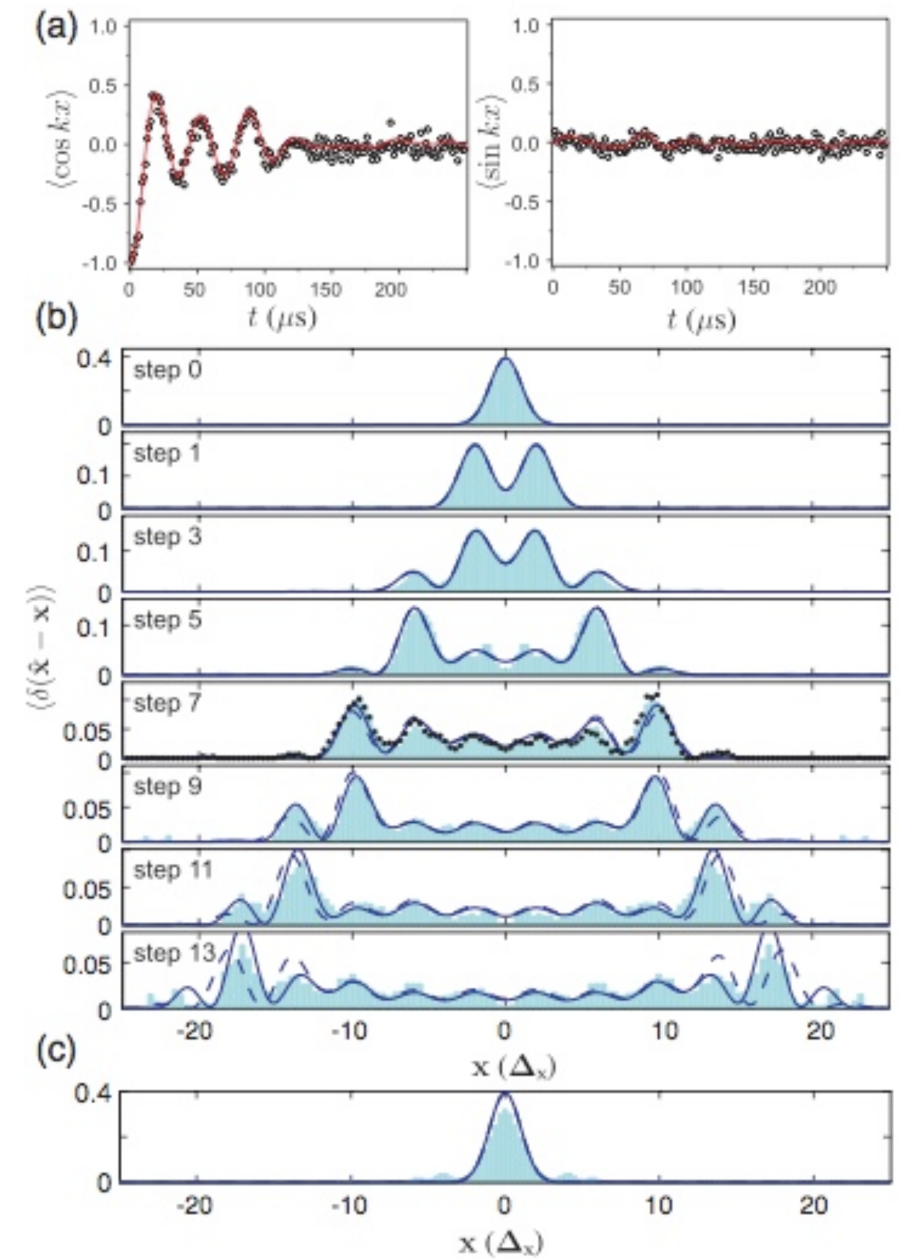
We have produced several papers studying different results for the "instantaneous" measurements. Some of them are theoretical and some of them have already seen the light of experiments.

Lougovski et al., Eur. Phys. J. D (2006); Bastin et al., J. Phys. B: At. Mol. Opt. Phys. (2006); Franca Santos et al., PRL (2006); Gerritsma et al., Nature (2010), Zähringer et al., PRL (2010); Casanova et al., PRA 81, 062126 (2010).



"Instantaneous" measurements of ZB
with sub-Δ resolution and beyond the diffraction limit.

R. Gerritsma et al., Nature (2010)

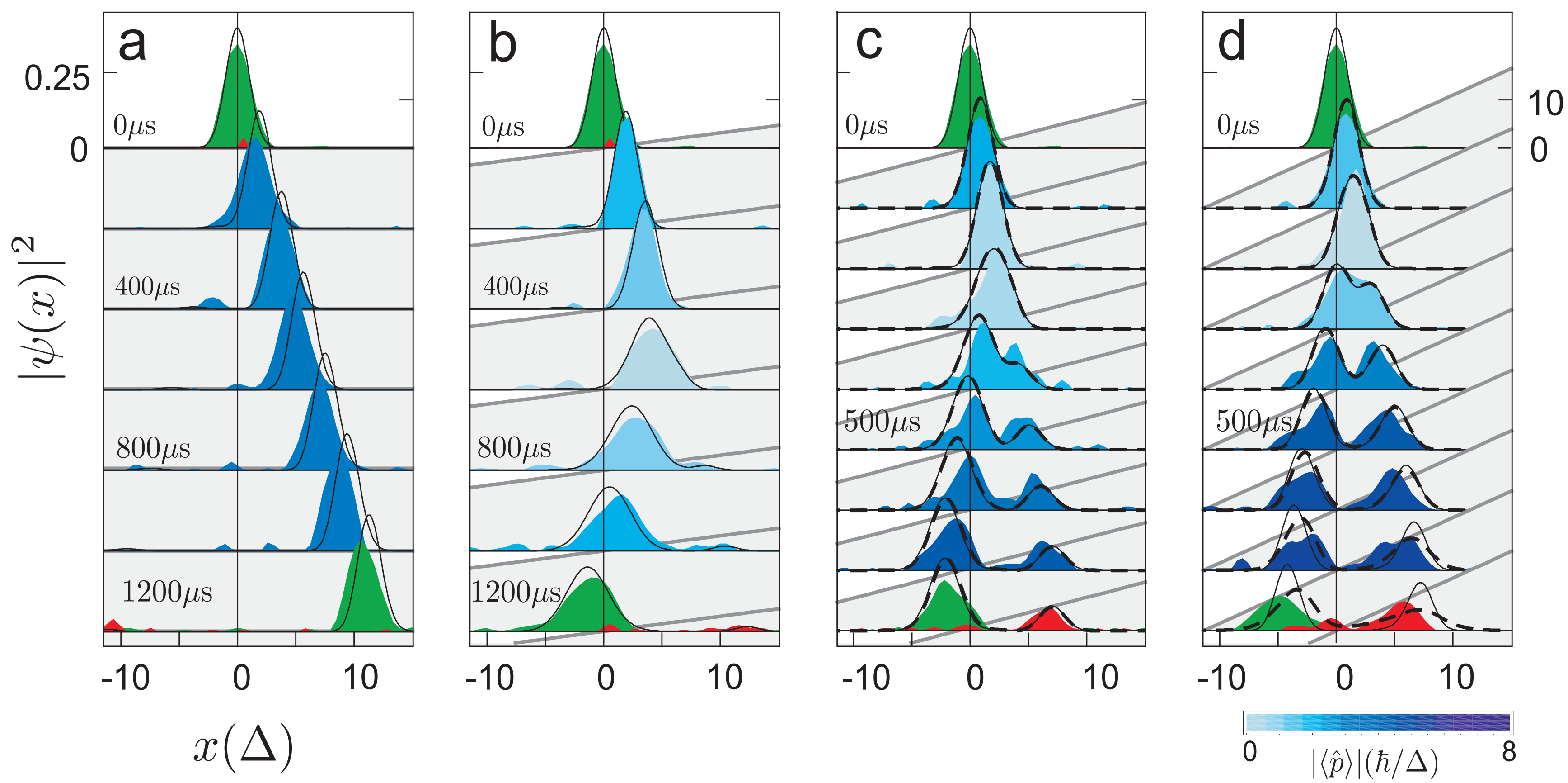


Reconstruction of absolute square wavefunction
of quantum walks in trapped ions.

F. Zähringer et al., PRL (2010)

h) We have also proposed the quantum simulation of the Klein Paradox

$$i\hbar \frac{\partial}{\partial t}\Phi = H_{DLP}\Phi = \left(c\sigma_x p_x + \alpha x + mc^2\sigma_z\right)\Phi$$



The Dirac Linear Potential is not always reflecting the particle. This amounts to a Klein Paradox behavior, where the particle can move from positive to negative energy components via tunneling.

J. Casanova et al., PRA **82**, 020101(R) (2010); R. Gerritsma et al., PRL **106**, 060503 (2011).