Course: Hybrid Devices for Quantum Information Processing

David DiVincenzo

JARA Institute for Quantum Information, RWTH Aachen University & Forschungszentrum Juelich

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Hybrid devices for Quantum Information Processing

Lectures 1 and 2:   with Fabio Pedrocchi
Thermal Quasiparticles and Majorana Braiding

I discuss a model calculation of the decohence of Majorana qubits during braiding in a trijunction, due to thermally generated quasiparticles (bosonic environment). The limitations to coherence are significant.

Lecture 3:
Semiconductor Hall-effect Gyrators and Circulators

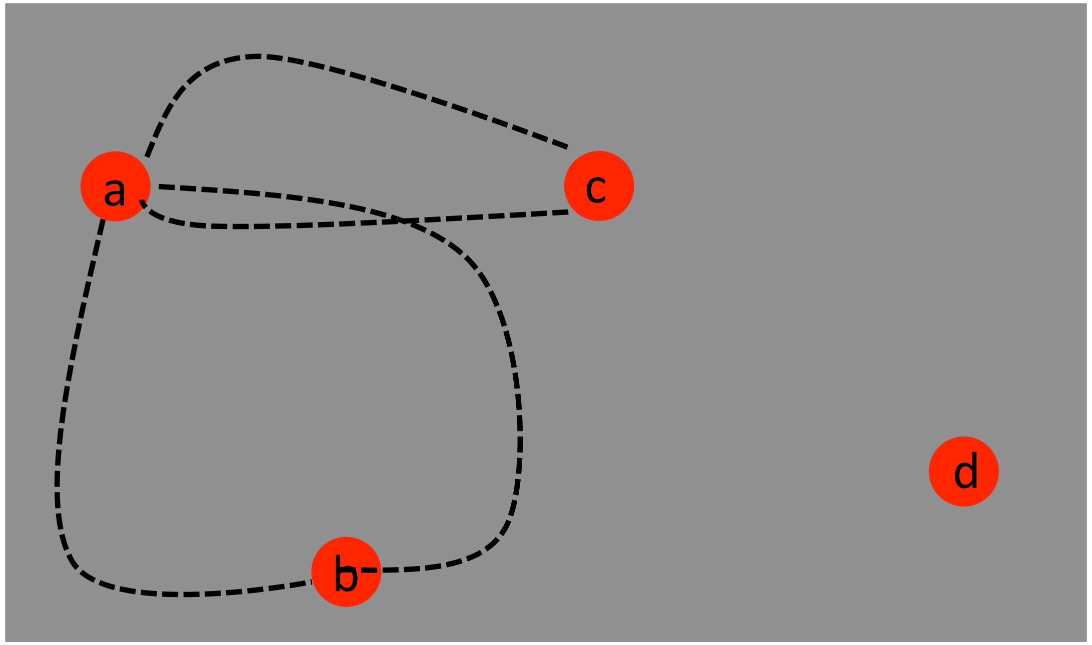
Calculations of driven propagation of chiral edge magnetoplasmons in the integer quantum Hall effect indicate a promising route for these devices in current experiments. They are very important for the miniaturization of multi-qubit quantum computers.

For today:

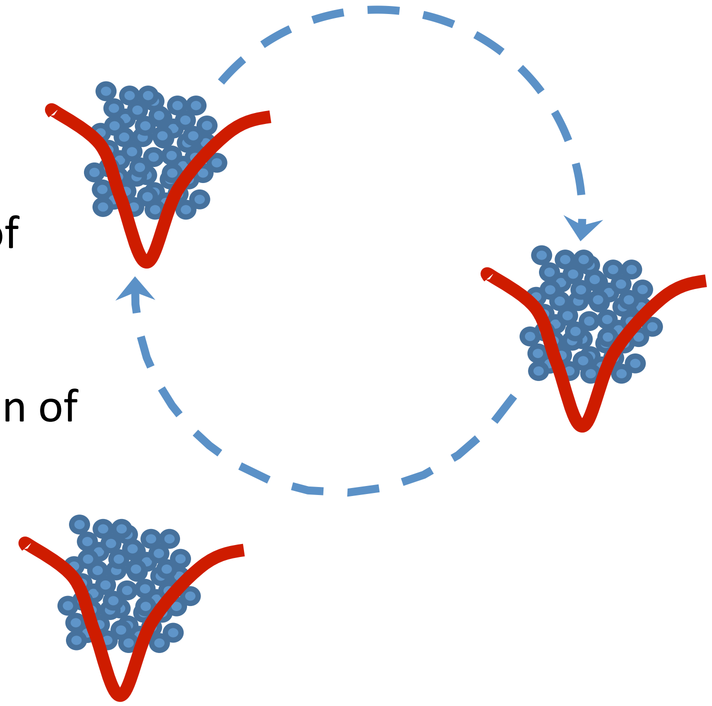
- What are anyons in general? Compute by braiding!
- Our anyons: Majorana modes
- Canonical model: “Kitaev” wire
- Diagonalize using Majorana-operator representation
- Our first qubit − a ground-state degeneracy
- Moving and braiding Majoranas − the T junction
- Why are Majoranas non-abelian?
- The problem for lecture 2:
    - *Does “topological” really make Majorana qubits fault tolerant?*

Topological Quantum Computing

Topological Quantum Computation A. Kitaev, 1997



Another interpretation of anyons as "particles": Adiabatic motion of one-particle potentials



Topological Order

Degeneracy $|\psi_1\rangle, |\psi_2\rangle, \ldots$

$$\begin{pmatrix} |\psi_1\rangle \\ |\psi_2\rangle \\ \ldots \end{pmatrix} \longrightarrow U_{ab} \cdot \begin{pmatrix} |\psi_1\rangle \\ |\psi_2\rangle \\ \ldots \end{pmatrix}$$

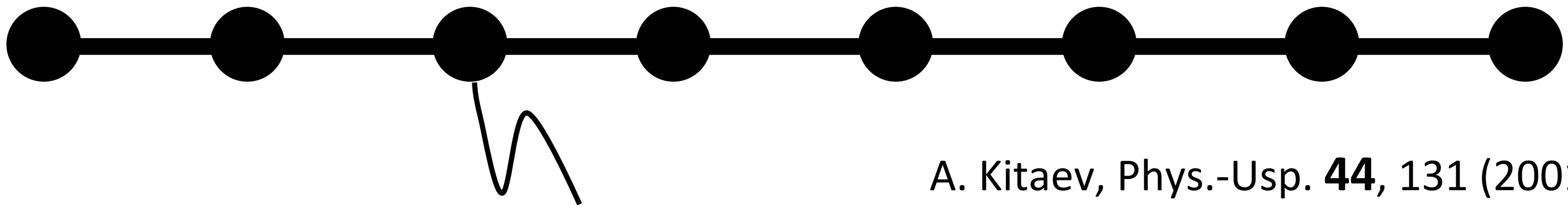
depends only on topology

$$[U_{ab}, U_{bc}] \neq 0$$

$\longrightarrow$ non-abelian anyons

“Kitaev” Wire

Archetypical (1D) model with anyons
(simple model for a nanowire)



fermionic site

A. Kitaev, Phys.-Usp. **44**, 131 (2001)
Lieb, Mattis, Schultz, Ann. Phys. **16** (1961)

$$H = -\sum_{j=1}^{L} \mu_j a_j^\dagger a_j - \sum_{j=1}^{L-1} (t a_j^\dagger a_{j+1} - \Delta a_j a_{j+1} + \text{h.c.})$$

chemical potential

hopping

superconducting pairing (proximity effect, Mourik et al. Science **336**, 1003 (2012))

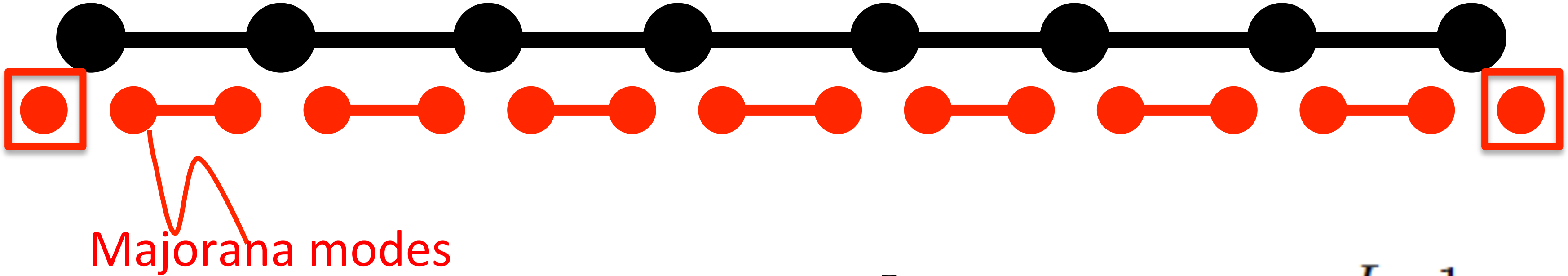
Majorana operators $\{\gamma_j, \gamma_k\} = 2\delta_{jk}$

$$\gamma^\dagger = \gamma$$

$$a_j = \frac{(\gamma_{2j-1} + i\gamma_{2j})}{2}$$

Kitaev Wire

Archetypical (1D) model with anyons
(simple model for a nanowire)



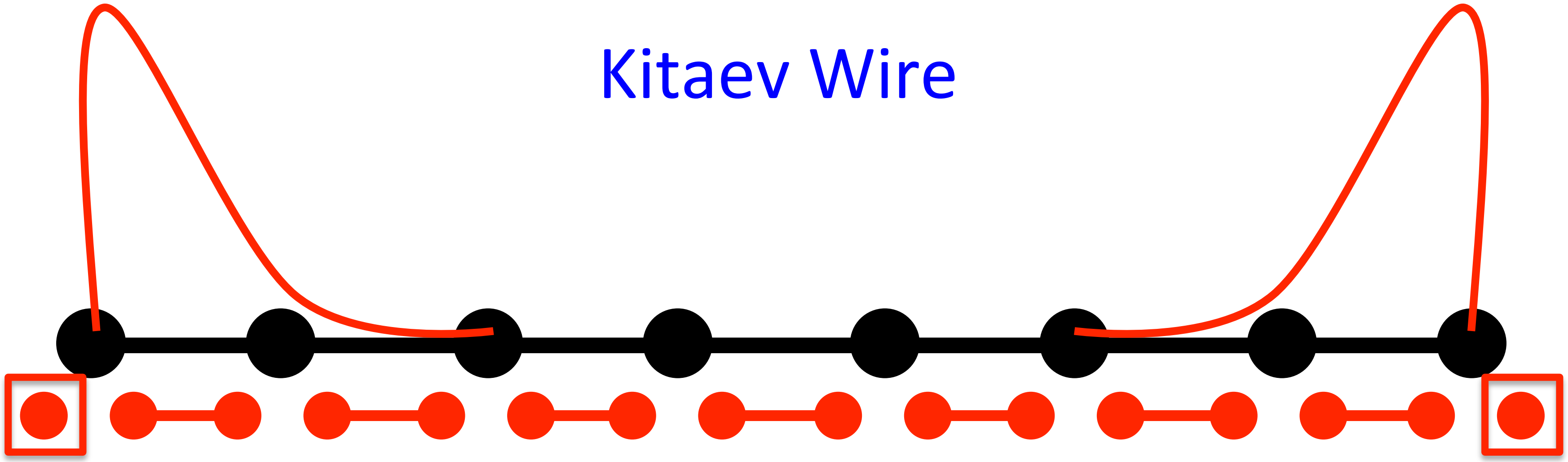
$$\mu_j = 0 \qquad t = \Delta \qquad H = -\Delta \sum_{j=1}^{L-1} i\gamma_{2j+1}\gamma_{2j} = \sum_{j=0}^{L-1} \epsilon_j \left(2d_j^\dagger d_j - 1\right)$$

$\longrightarrow$ γ_1 and γ_{2L} are decoupled from H $\qquad \epsilon_0 = 0$ and $\epsilon_j = |\Delta|$

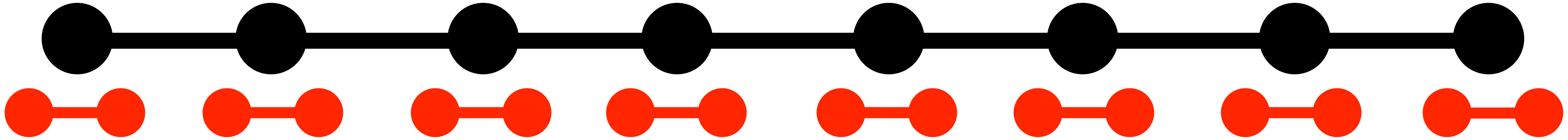
$\longrightarrow$ Non-local mode $\qquad d_0 = \dfrac{\gamma_1 + i\gamma_{2L}}{2} \qquad d_j = \frac{1}{2}(\gamma_{2j} + i\gamma_{2j+1})$

$$d_0|0\rangle = 0 \qquad d_0^\dagger|0\rangle = |1\rangle$$

Kitaev Wire



$|\mu_j| \leqslant 2t$ **Topological phase (Majorana bound states)**



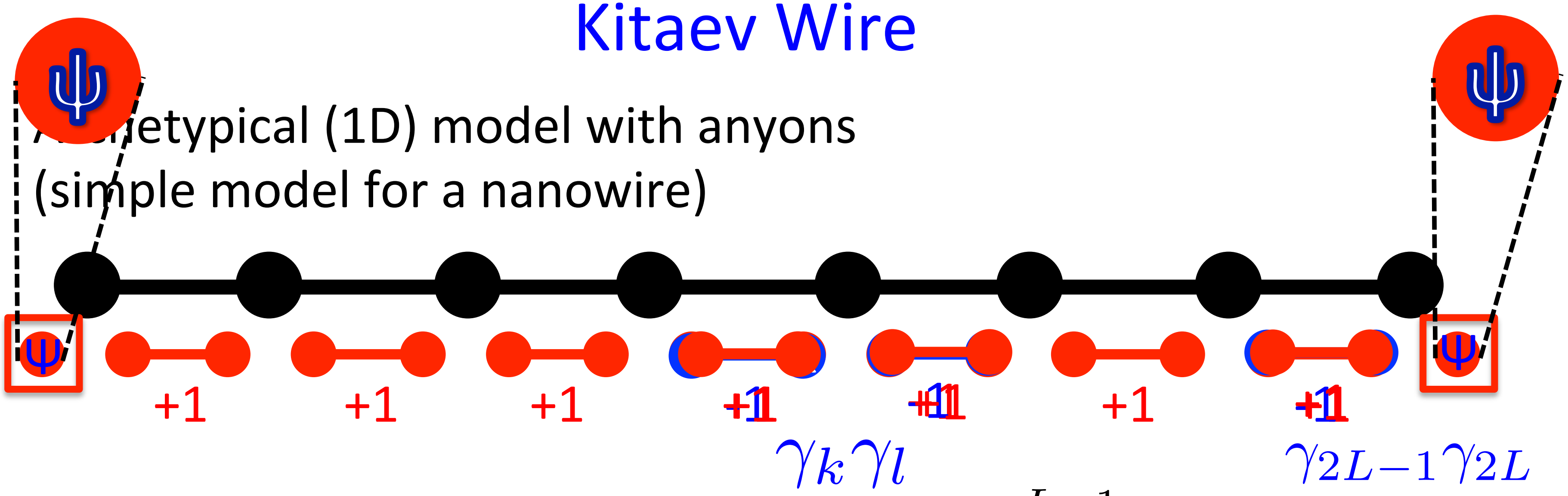
$|\mu_j| \gg 2t$

$$H = -i\frac{\mu}{2}\sum_{j=1}^{L}\gamma_{2j-1}\gamma_{2j}$$

Non Topological phase (No Majorana bound states)

Kitaev Wire

Archetypical (1D) model with anyons
(simple model for a nanowire)



$\mu_j = 0 \qquad t = \Delta \qquad H = -\Delta \sum_{j=1}^{L-1} i\gamma_{2j+1}\gamma_{2j}$

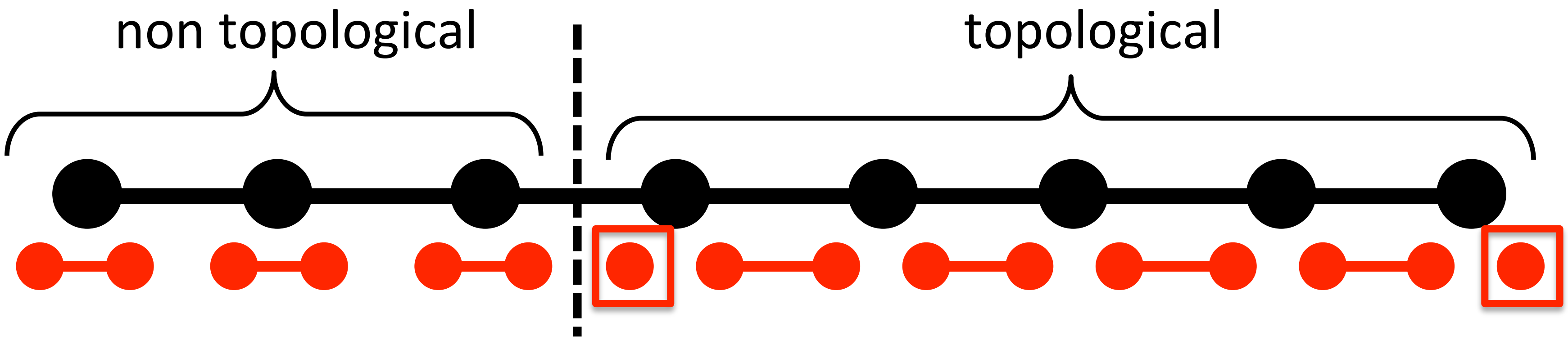
In the ground-state subspace ⟶ $i\gamma_{2j+1}\gamma_{2j} = +1$

Excitations ⟶ $i\gamma_{2j+1}\gamma_{2j} = -1$ Ψ quasi-particle

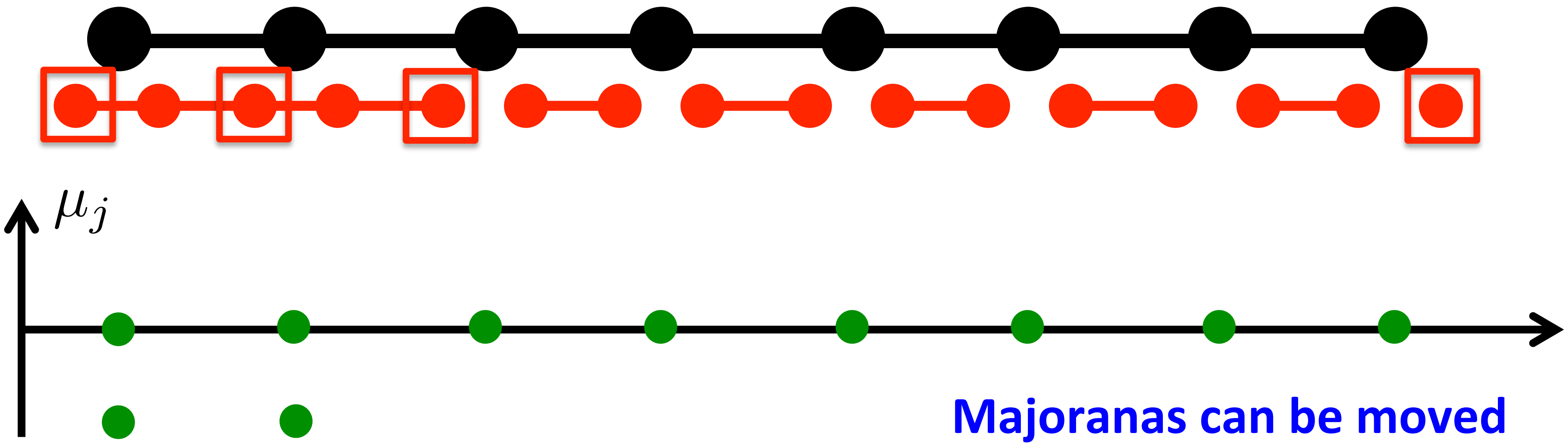
Parity flip ⟶ $i\gamma_1\gamma_{2L} \to -i\gamma_1\gamma_{2L}$

$$\gamma_1\gamma_2\cdots\gamma_{2L} \propto Z - \text{error}$$

Kitaev Wire



→ Majorana bound states appear at the **junction** between topological and non topological segments

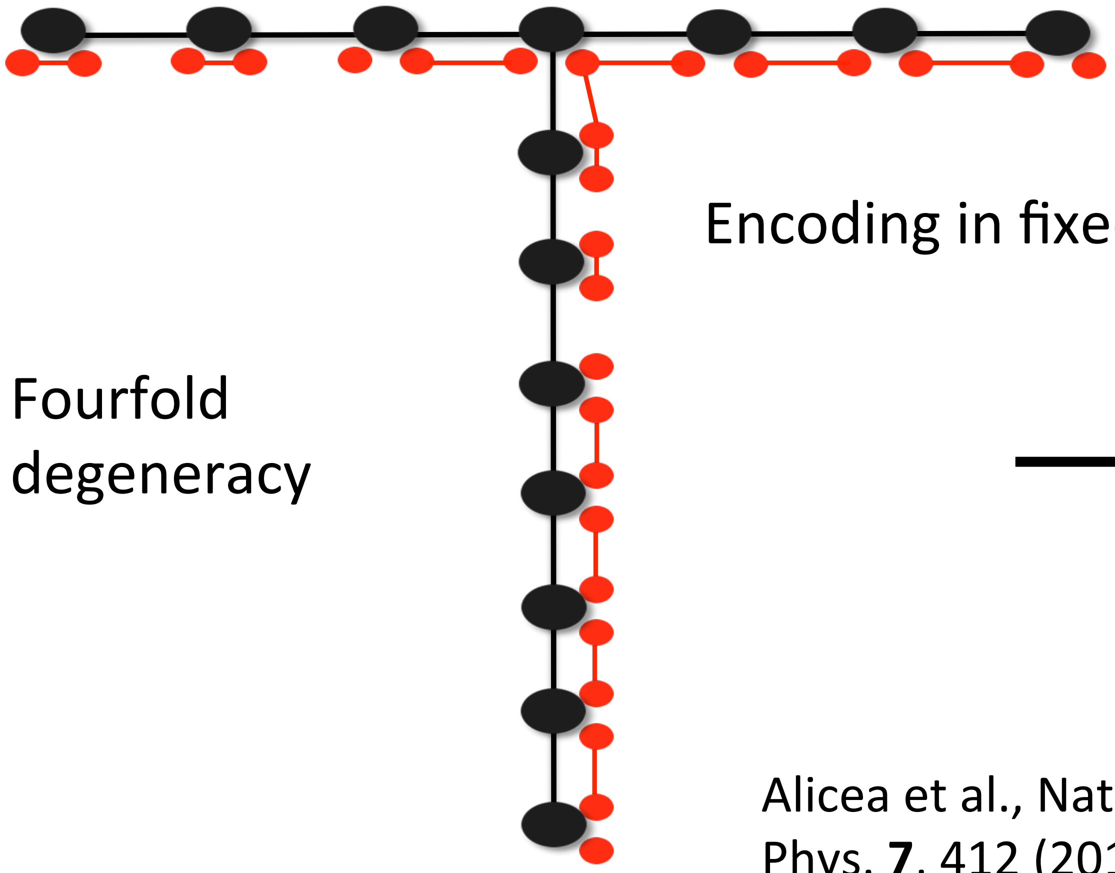


Majoranas can be moved

Trijunction

Not enough space to exchange Majoranas

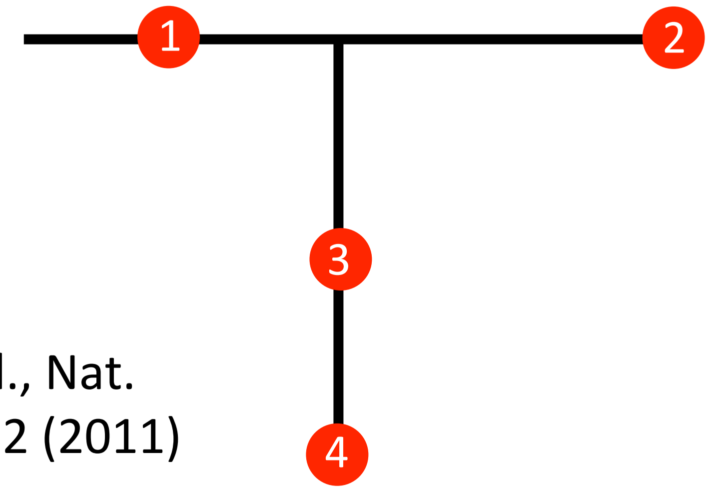
→ Trijunction $H_T(\tau) = H_H(\tau) + H_V(\tau) - (ta^\dagger_{L/2}a_{L+1} - \Delta a_{L/2}a_{L+1} + \text{h.c.})$



Fourfold degeneracy

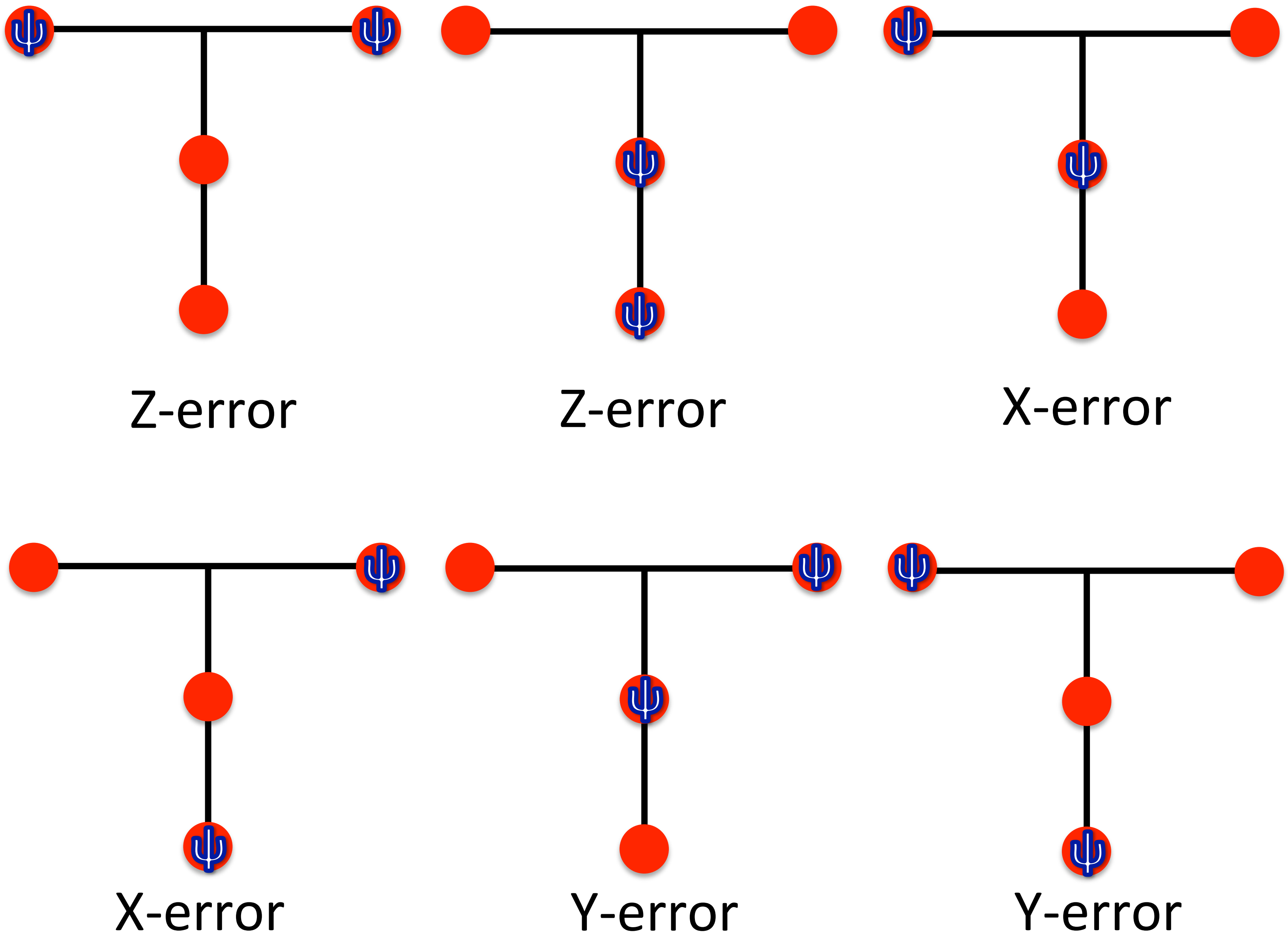
Encoding in fixed parity sector

$$i\gamma_1\gamma_2\, i\gamma_3\gamma_4 = +1$$



Alicea et al., Nat. Phys. **7**, 412 (2011)

Logical errors



PRL 115, 120402 (2015) PHYSICAL REVIEW LETTERS week ending 18 SEPTEMBER 2015

Majorana Braiding with Thermal Noise

Fabio L. Pedrocchi and David P. DiVincenzo
JARA Institute for Quantum Information, RWTH Aachen University, D-52056 Aachen, Germany
(Received 7 July 2015; published 15 September 2015)

PHYSICAL REVIEW B **92**, 115441 (2015)

Monte Carlo studies of the self-correcting properties of the Majorana quantum error correction code under braiding

Fabio L. Pedrocchi,[1] N. E. Bonesteel,[2] and David P. DiVincenzo[1]
[1] *JARA Institute for Quantum Information, RWTH Aachen University, D-52056 Aachen, Germany*
[2] *Department of Physics and National High Magnetic Field Laboratory, Florida State University, Tallahassee, Florida 32310, USA*
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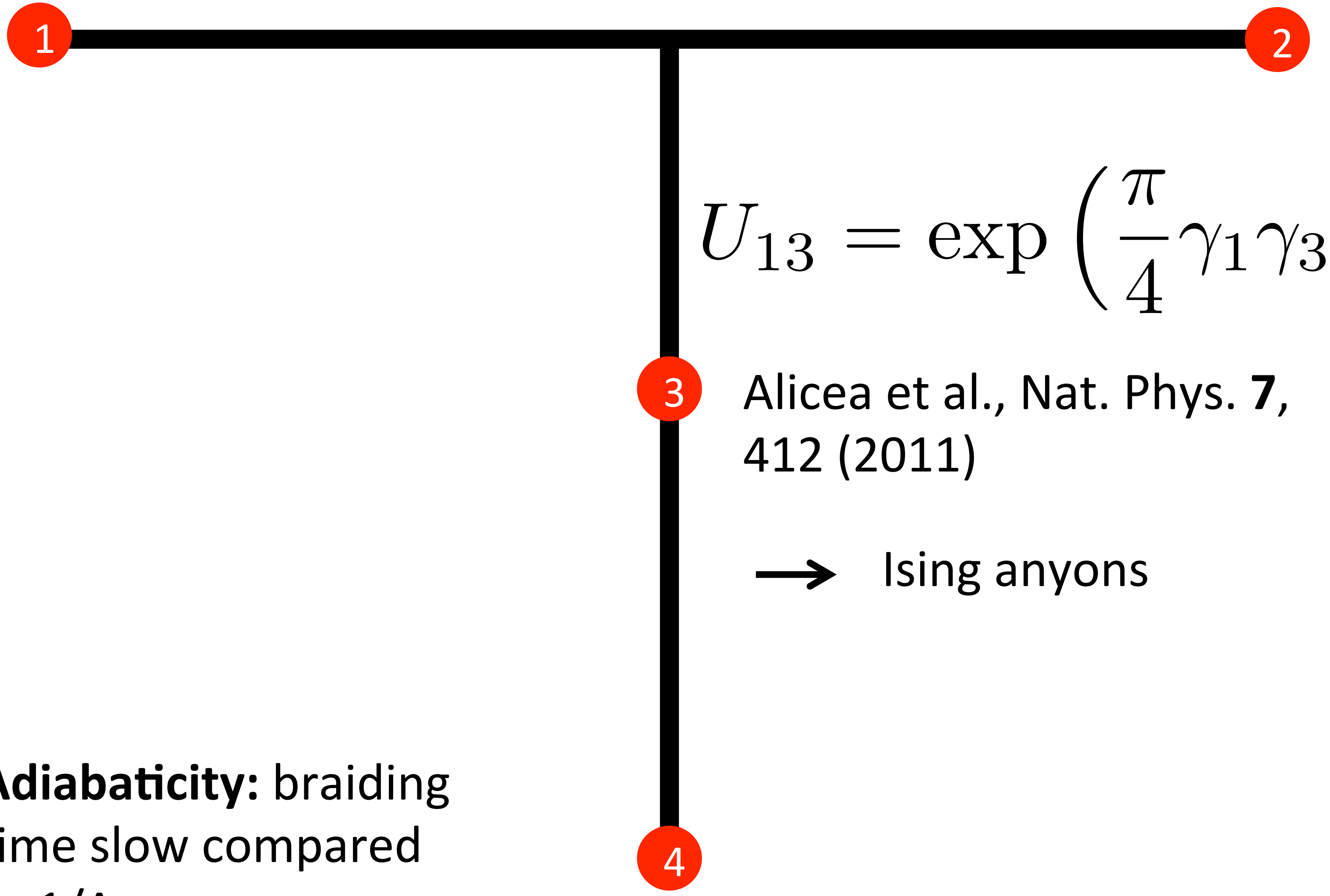
Braiding



$$U_{13} = \exp\left(\frac{\pi}{4}\gamma_1\gamma_3\right)$$

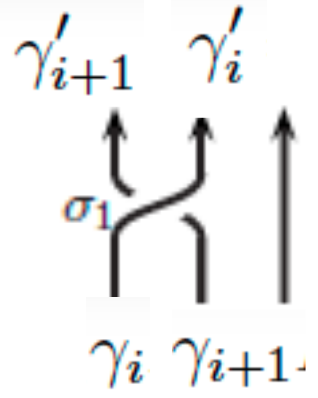
Alicea et al., Nat. Phys. **7**, 412 (2011)

→ Ising anyons

Adiabaticity: braiding time slow compared to $1/\Delta$

Fabian Hassler, “Majorana Qubits”, arXiv:1404.0897

denote the Majorana operators after the exchange by γ_i' and γ_{i+1}'.



$$\gamma_i' = B_{i,i+1}\gamma_i B_{i,i+1}^\dagger, \; \gamma_{i+1}' = B_{i,i+1}\gamma_{i+1} B_{i,i+1}^\dagger.$$

Since the position of the two Majoranas are interchanged by this operation,

Erroneous prime!!

$$\gamma_i' = \alpha_i\gamma_{i+1}, \; \gamma_{i+1}' = \alpha_{i+1}\gamma_i, \qquad \alpha_i, \alpha_{i+1} \in \Re$$

(Hermiticity)

$$-i\gamma_i\gamma_{i+1} = -i\gamma_i'\gamma_{i+1}'.$$

(physical electron number -> Bogoliubov fermion parity)

$$n_j = f_j^\dagger f_j = \frac{1}{2}(1 + i\gamma_{2j-1}\gamma_{2j}), j = 1, \ldots, n.$$

$$P = (-1)^{c^\dagger c} = 1 - 2c^\dagger c$$
$$= -i\gamma_a\gamma_b = \pm 1$$

$$\alpha_i\alpha_{i+1} = -1. \rightarrow \alpha_i = 1, \alpha_{i+1} = -1.$$

(convention)

$$\begin{aligned} \gamma_i &\to \gamma_{i+1}, \\ \gamma_{i+1} &\to -\gamma_i, \\ \gamma_j &\to \gamma_j, \; j \notin \{i, i+1\} \end{aligned} \rightarrow B_{i,i+1} = \exp\left(-\frac{\pi}{4}\gamma_i\gamma_{i+1}\right) = \frac{1}{\sqrt{2}}(1 - \gamma_i\gamma_{i+1})$$

Thermal environment

Main focus: how does **a thermal environment** destroy the stored quantum information when braiding is executed?

$$H(\tau) = H_T(\tau) + H_B + H_{SB}$$

Bosonic Bath $H_B = \sum_j B_j$

System-Bath coupling $H_{SB} = -\sum_j B_j \otimes (2a_j^\dagger a_j - 1) = -i \sum_j B_j \otimes \gamma_{2j-1}\gamma_{2j}$

Markovian master equation in adiabatic limit

$$\dot{\rho}_S(\tau) = -i[H_T(\tau), \rho_S(\tau)] + \mathcal{D}(\rho_S(\tau))$$

Unitary evolution

Dissipation ↔ Γ (Ohmic bath)

Approximately end of lecture 1

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Calculations of driven propagation of chiral edge magnetoplasmons in the integer quantum Hall effect indicate a promising route for these devices in current experiments. They are very important for the miniaturization of multi-qubit quantum computers.

For today (Tuesday):

- The problem for lecture 2:
  - *Does "topological" really make Majorana qubits fault tolerant?*
- Bosonic bath − no parity problem?
- Bath causes creation, hopping, and destruction of thermal quasiparticles
- Derivation (Davies) of how all these terms emerge from one deformation potential
- Failure of error correction when the Majoranas are braided

PRL **115,** 120402 (2015) PHYSICAL REVIEW LETTERS week ending 18 SEPTEMBER 2015

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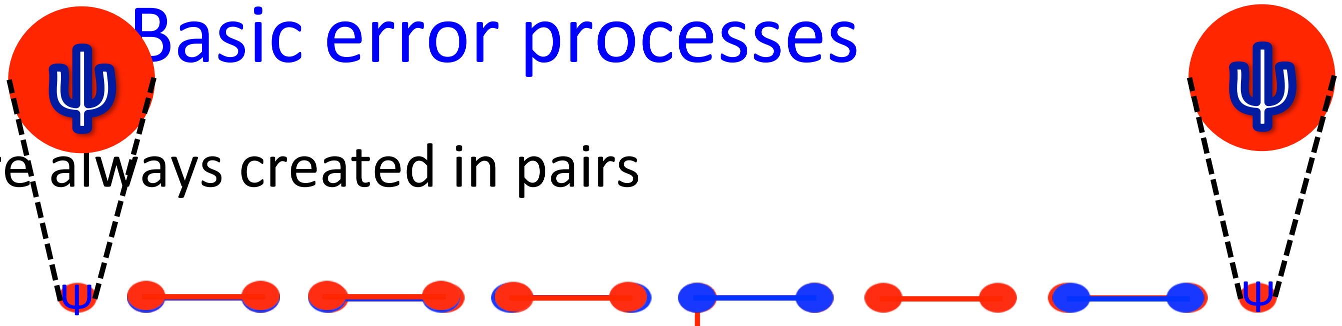
Basic error processes

Excitations are always created in pairs



Creation bulk, energy cost is -4Δ

Annihilation bulk, energy cost is 4Δ

Hopping bulk, energy cost is 0

Non topological

Creation boundary, energy cost is -2Δ

Annihilation boundary, energy cost is 2Δ

Hopping onto (out from) Majorana, energy cost 2Δ (-2Δ)

$$H_{SB} = -i \sum_j B_j \otimes \gamma_{2j-1}\gamma_{2j}$$

PHYSICAL REVIEW B **94**, 104516 (2016)

Normal-metal quasiparticle traps for superconducting qubits

R.-P. Riwar,[1,2] A. Hosseinkhani,[1,3] L. D. Burkhart,[2] Y. Y. Gao,[2] R. J. Schoelkopf,[2] L. I. Glazman,[2] and G. Catelani[1]

[1]*Peter Grünberg Institut (PGI-2) and JARA Institute for Quantum Information, Forschungszentrum Jülich, 52425 Jülich, Germany*

[2]*Departments of Physics and Applied Physics, Yale University, New Haven, Connecticut 06520, USA*

[3]*JARA-Institute for Quantum Information, RWTH Aachen University, D-52056 Aachen, Germany*

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The presence of quasiparticles in superconducting qubits emerges as an intrinsic constraint on their coherence. While it is difficult to prevent the generation of quasiparticles, keeping them away from active elements of the qubit provides a viable way of improving the device performance. Here we develop theoretically and validate

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it is difficult to prevent the generation of quasiparticles

Thermal environment

Main focus: how does **a thermal environment** destroy the stored quantum information when braiding is executed?

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Bosonic Bath $\quad H_B = \sum_j B_j$

System-Bath coupling $\quad H_{SB} = -\sum_j B_j \otimes (2a_j^\dagger a_j - 1) = -i \sum_j B_j \otimes \gamma_{2j-1}\gamma_{2j}$

Markovian master equation in adiabatic limit

$$\dot{\rho}_S(\tau) = -i[H_T(\tau), \rho_S(\tau)] + \mathcal{D}(\rho_S(\tau))$$

Unitary evolution

Dissipation ↔ Γ (Ohmic bath)

Davies Prescription

Markovian master equation in the weak-coupling limit

$$\mathcal{D}(\rho_S(t)) = \sum_{i,j}\sum_{\omega}\gamma^{ij}(\omega)\left(A^i(\omega)\rho_S(t)(A^j(\omega))^\dagger - \frac{1}{2}\{(A^j(\omega))^\dagger A^i(\omega), \rho_S(t)\}\right)$$

Jump operator

Spectral function

$$\gamma^{ij}(\omega) = \int_{-\infty}^{+\infty} ds\, e^{i\omega s}\langle B_i^\dagger(s) B_j(0)\rangle$$

Davies Prescription

System-Bath interaction

$$H_{SB} = -2 \sum_j B_j \otimes a_j^\dagger a_j = -i \sum_j B_j \otimes \gamma_{2j-1}\gamma_{2j}$$

Rewrite in terms of **eigenoperators**

(corrects a few factors of 2 on p. 17 of Pedrocchi et al. PRB. Our apologies! No change of the physics.

$$\begin{aligned} H_{SB} = & -iB_1 \otimes (d_0 + d_0^\dagger)(d_1 + d_1^\dagger) \\ & - \sum_{j=1}^{\lceil L/2 \rceil - 1} B_{2j} \otimes (d_{2j-1} - d_{2j-1}^\dagger)(d_{2j} + d_{2j}^\dagger) \\ & - \sum_{j=1}^{\lceil L/2 \rceil - 1} B_{2j+1} \otimes (d_{2j} - d_{2j}^\dagger)(d_{2j+1} + d_{2j+1}^\dagger) \\ & + iB_L \otimes (d_{L-1} - d_{L-1}^\dagger)(d_0 - d_0^\dagger) \,. \end{aligned}$$

Davies Prescription

Terms have a clear physical meaning:

$$A_{\text{hopping}} := -iB_1 \otimes d_0 d_1^\dagger - \sum_{j=1}^{\lceil L/2 \rceil - 1} B_{2j} \otimes d_{2j-1} d_{2j}^\dagger$$

$$- \sum_{j=1}^{\lceil L/2 \rceil - 1} B_{2j+1} \otimes d_{2j} d_{2j+1}^\dagger - iB_L \otimes d_{L-1} d_0^\dagger + \text{h.c.}$$

Energy cost: ±2Δ

Energy cost: 0

$$A_{\text{creation}} :=$$

$$-iB_1 \otimes d_0^\dagger d_1^\dagger + \sum_{j=1}^{\lceil L/2 \rceil - 1} (B_{2j} \otimes d_{2j-1}^\dagger d_{2j}^\dagger + B_{2j+1} \otimes d_{2j}^\dagger d_{2j+1}^\dagger)$$

$$+iB_L \otimes d_{L-1}^\dagger d_0^\dagger$$

Energy cost: -2Δ

Energy cost: -4Δ

Davies Prescription

Time evolution under H_S

$$e^{iH_S t} A_\zeta \, e^{-iH_S t}$$

$$= \sum_{m,n,k,\ell} |m\rangle\langle m|e^{iH_S t}|k\rangle\langle k|A_\zeta|\ell\rangle\langle \ell|e^{-iH_S t}|n\rangle\langle n|$$

$$= \sum_{m,n} e^{it(\epsilon_m - \epsilon_n)} |m\rangle\langle m|A_\zeta|n\rangle\langle n|$$

⟶ Fourier transform gives the jump operators

$$A_\zeta(\omega) = \sum_{\epsilon_m - \epsilon_n = \omega} |m\rangle\langle m|A_\zeta|n\rangle\langle n|$$

Davies Prescription

→ Fourier transform gives the jump operators

$$A_\zeta(\omega) = \sum_{\epsilon_m - \epsilon_n = \omega} |m\rangle\langle m|A_\zeta|n\rangle\langle n|$$

Example of jump operators:

$$A^1_{\text{hopping}}(-2|\Delta|) = \sum_{\epsilon_m - \epsilon_n = -2|\Delta|} |m\rangle\langle m|d_0 d_1^\dagger|n\rangle\langle n|$$

$$A^{2j}_{\text{hopping}}(0) = \sum_{\epsilon_m - \epsilon_n = 0} |m\rangle\langle m|d_{2j-1} d_{2j}^\dagger + d_{2j} d_{2j-1}^\dagger|n\rangle\langle n|$$

$$A^1_{\text{creation}}(-2|\Delta|) = \sum_{\epsilon_m - \epsilon_n = -2|\Delta|} |m\rangle\langle m|d_0^\dagger d_1^\dagger|n\rangle\langle n|$$

Davies Prescription

Pauli Master equation

Technical Condition: (satisfied in our model)

$$\langle m_\alpha | A^i_\eta(\omega) | n_k \rangle \neq 0 \longrightarrow$$

No other jump operators cause transitions

$$|n_k\rangle \leftrightarrow |m_\beta\rangle$$

➡ Diagonal elements decouple from off-diagonal elements

Davies Prescription

➡ Diagonal elements decouple from off-diagonal elements

$$\frac{dP(n,\tau)}{d\tau} = \sum_m [W(n|m)P(m,\tau) - W(m|n)P(n,\tau)]$$

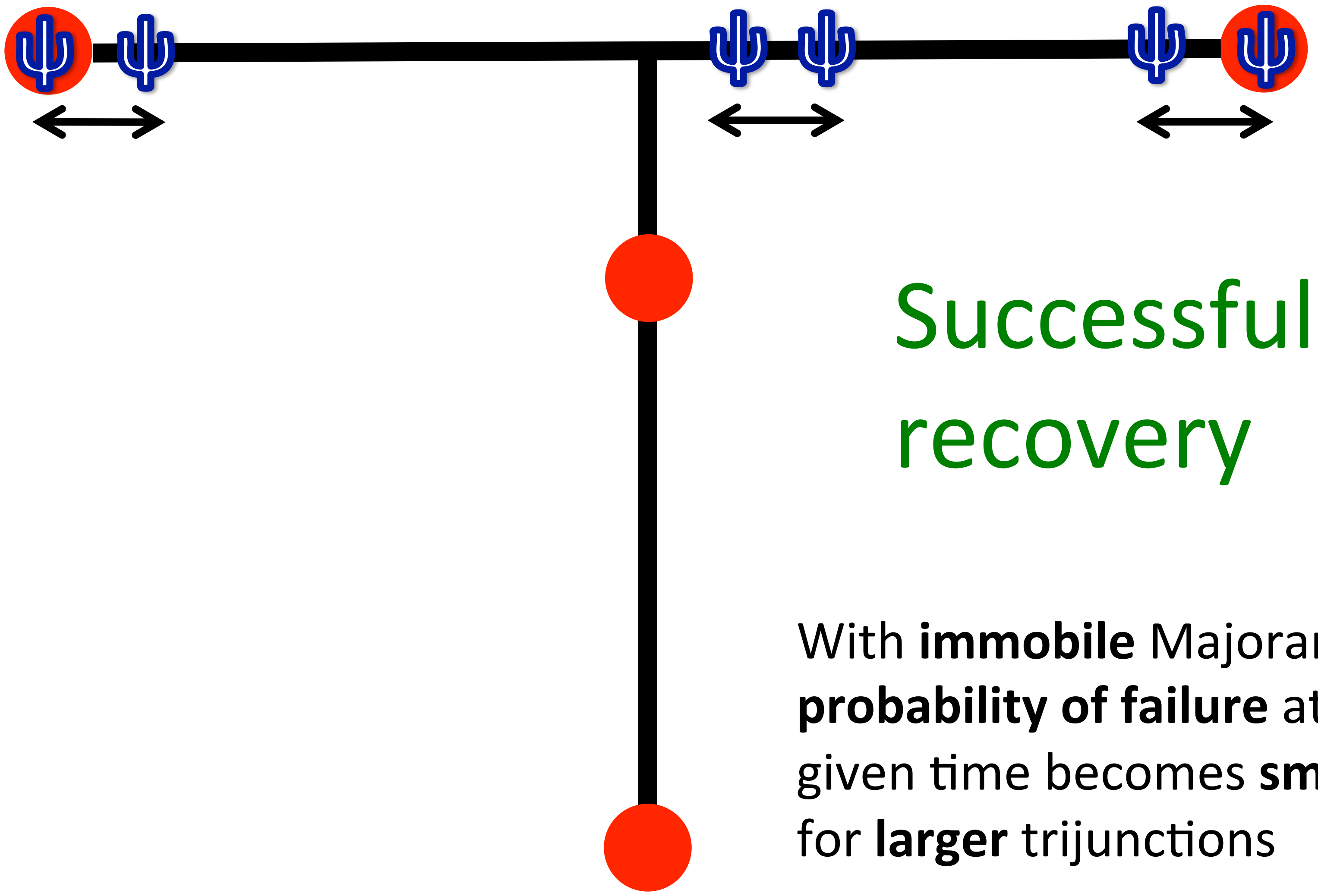
With transition rates

$$W(n|m) = \gamma(\omega_{mn})|\langle m|A^{i_{mn}}(\omega_{mn})|n\rangle|^2$$

We take in our model an Ohmic spectral function

$$\gamma(\omega) = \kappa\left|\frac{\omega}{1-\exp(-\beta\omega)}\right|$$

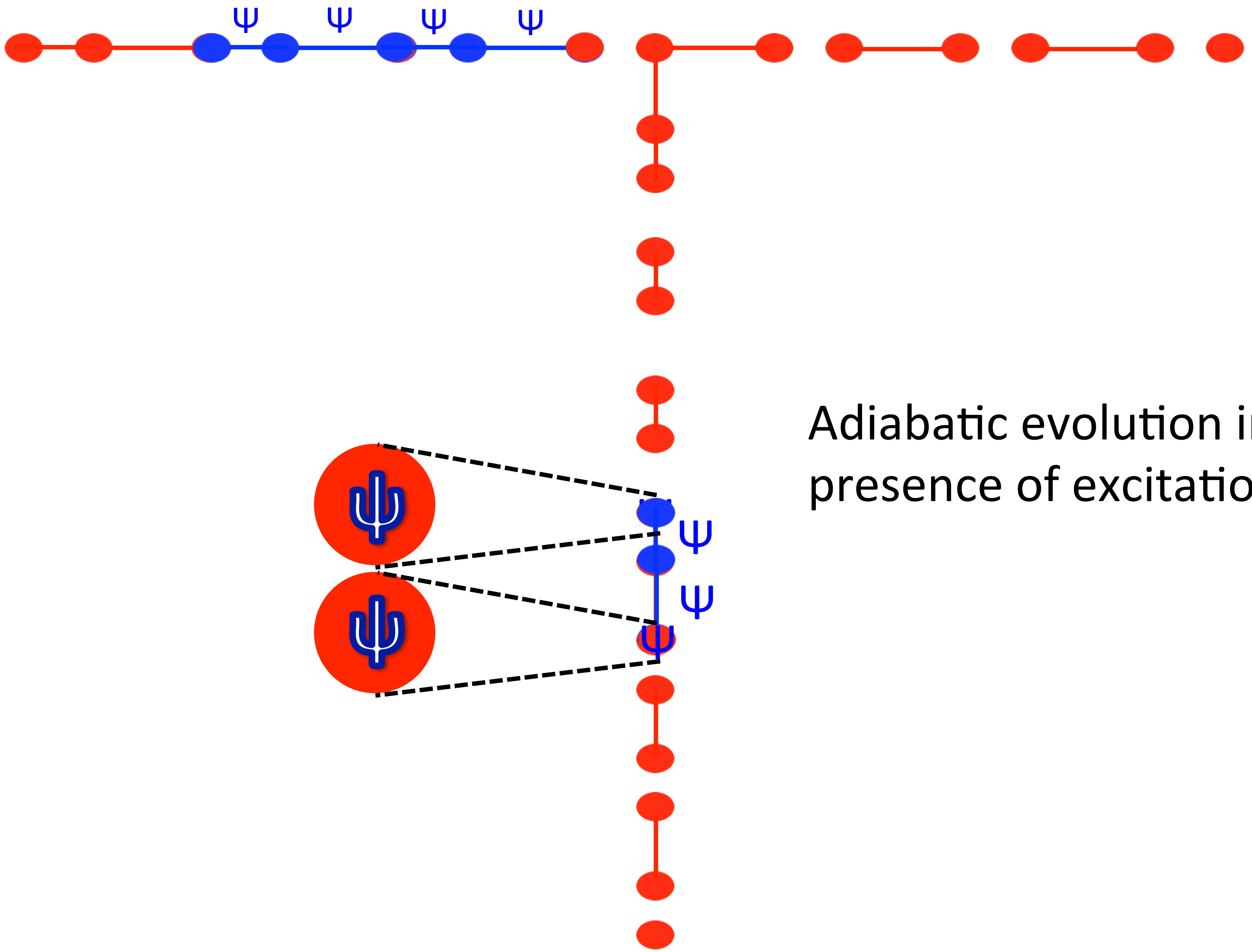
Error Correction



Successful recovery

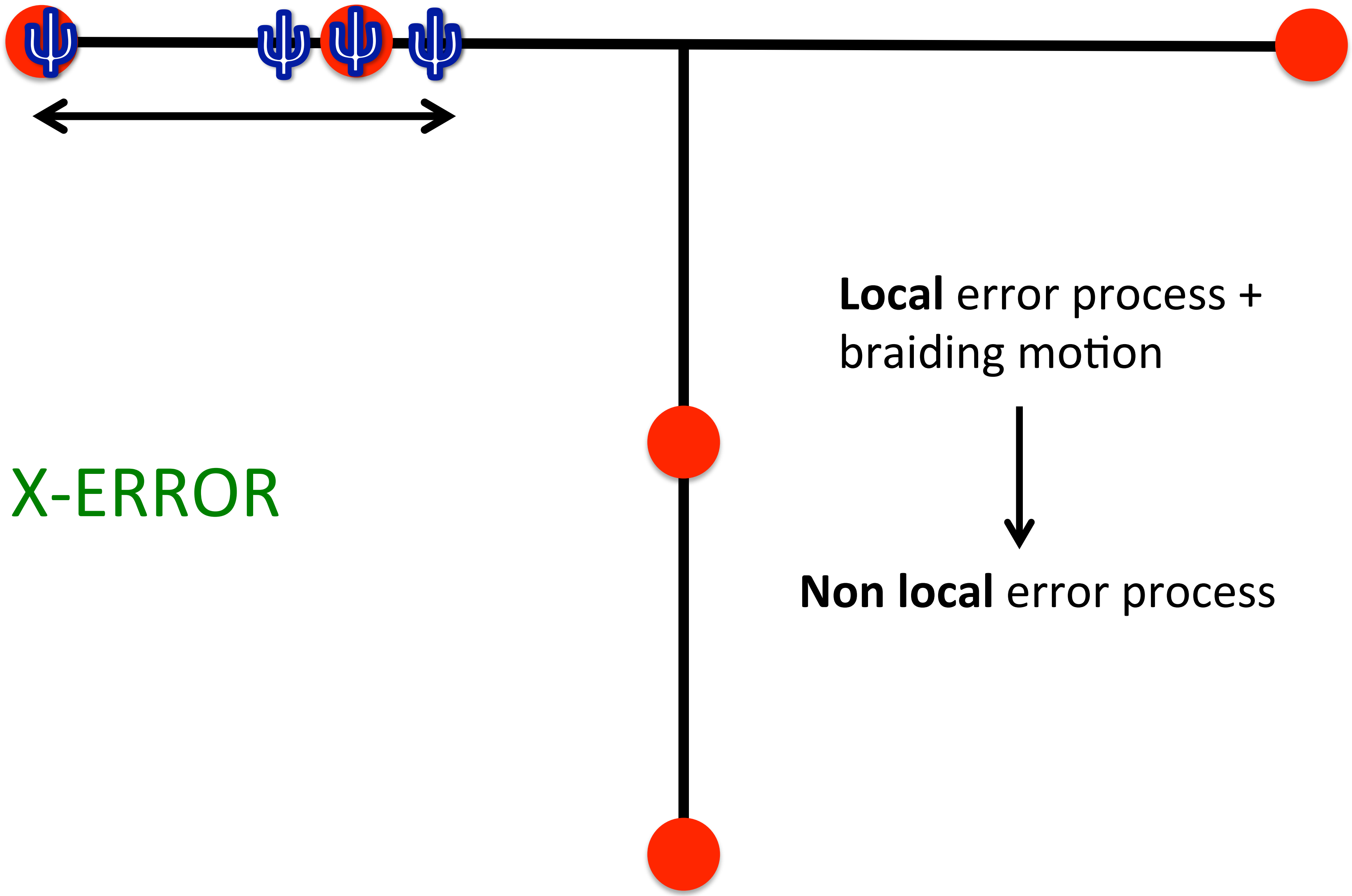
With **immobile** Majoranas, the **probability of failure** at a given time becomes **smaller** for **larger** trijunctions

Majoranas and Interactions

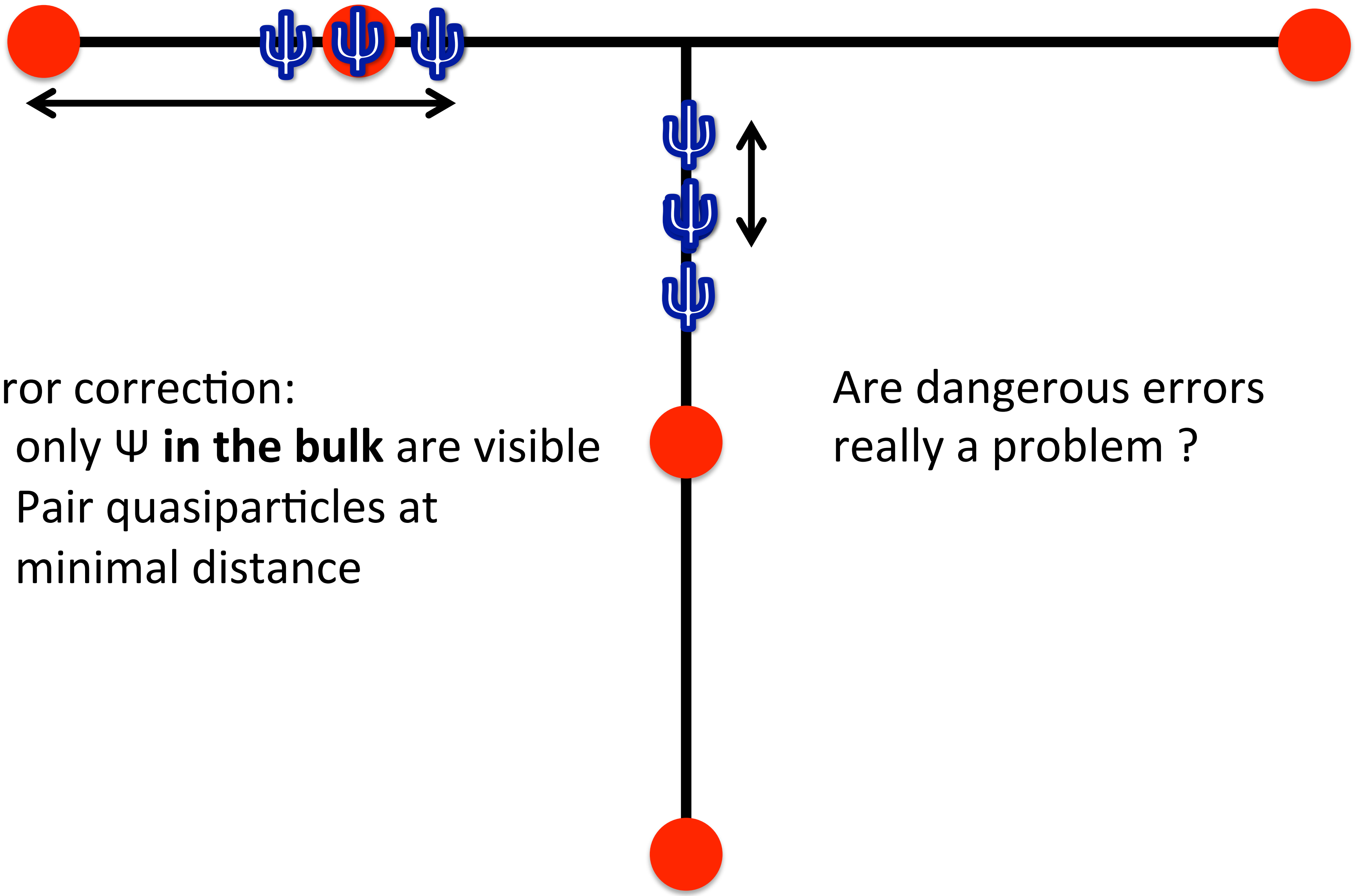


Adiabatic evolution in the presence of excitations

Dangerous errors



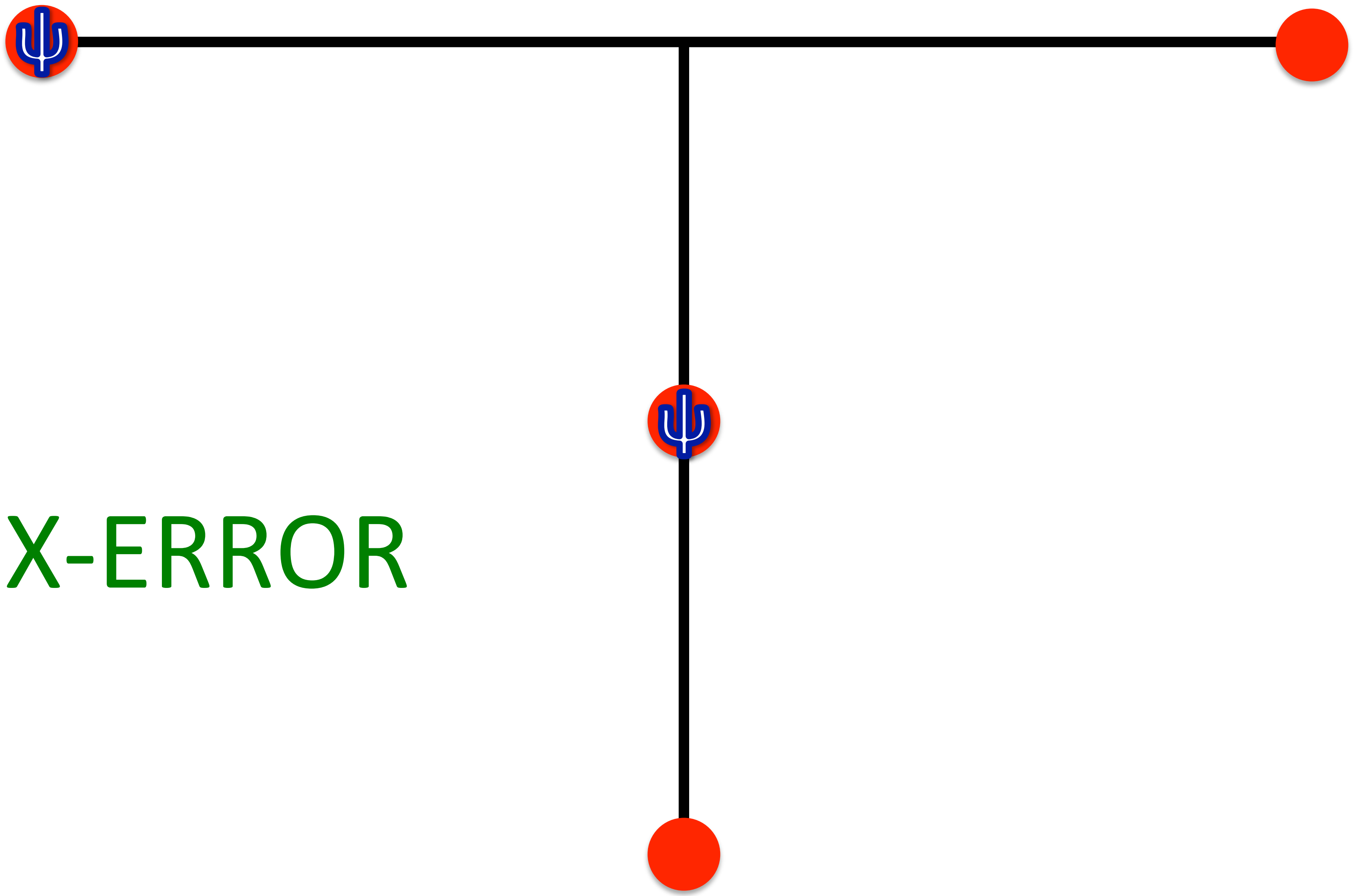
Dangerous errors



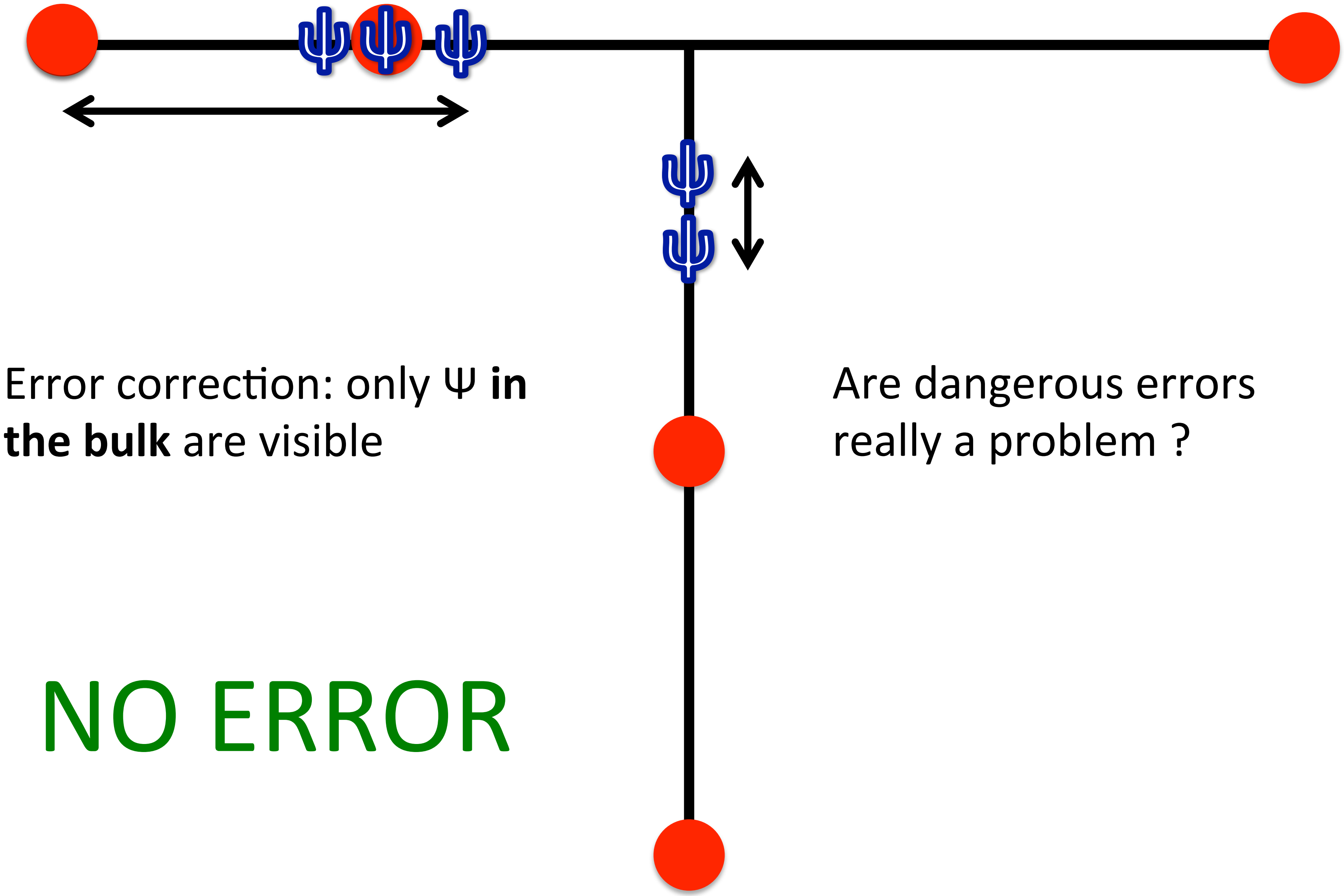
Error correction:
- only Ψ **in the bulk** are visible
- Pair quasiparticles at minimal distance

Are dangerous errors really a problem ?

Dangerous errors



Dangerous errors

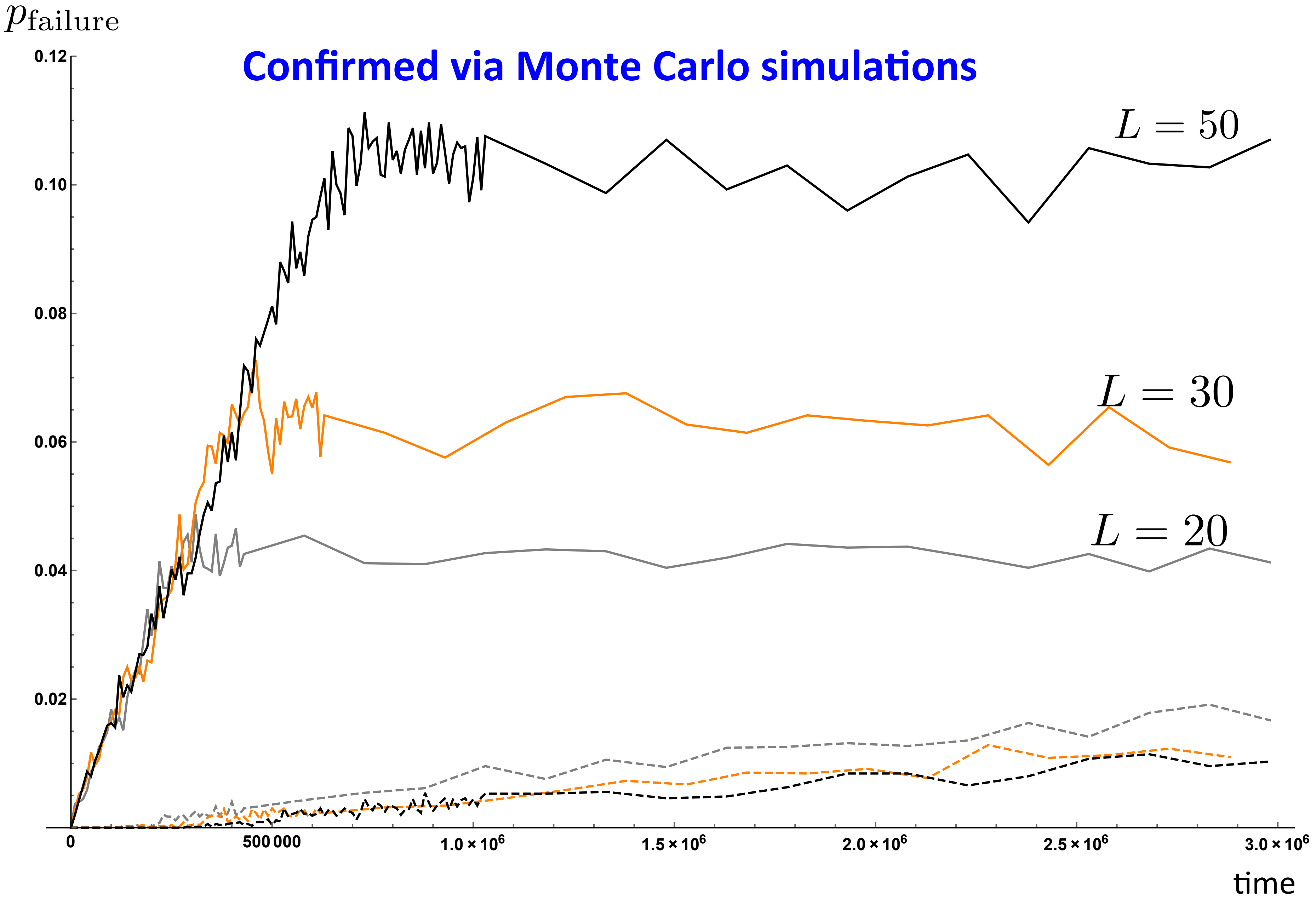


Error correction: only Ψ **in the bulk** are visible

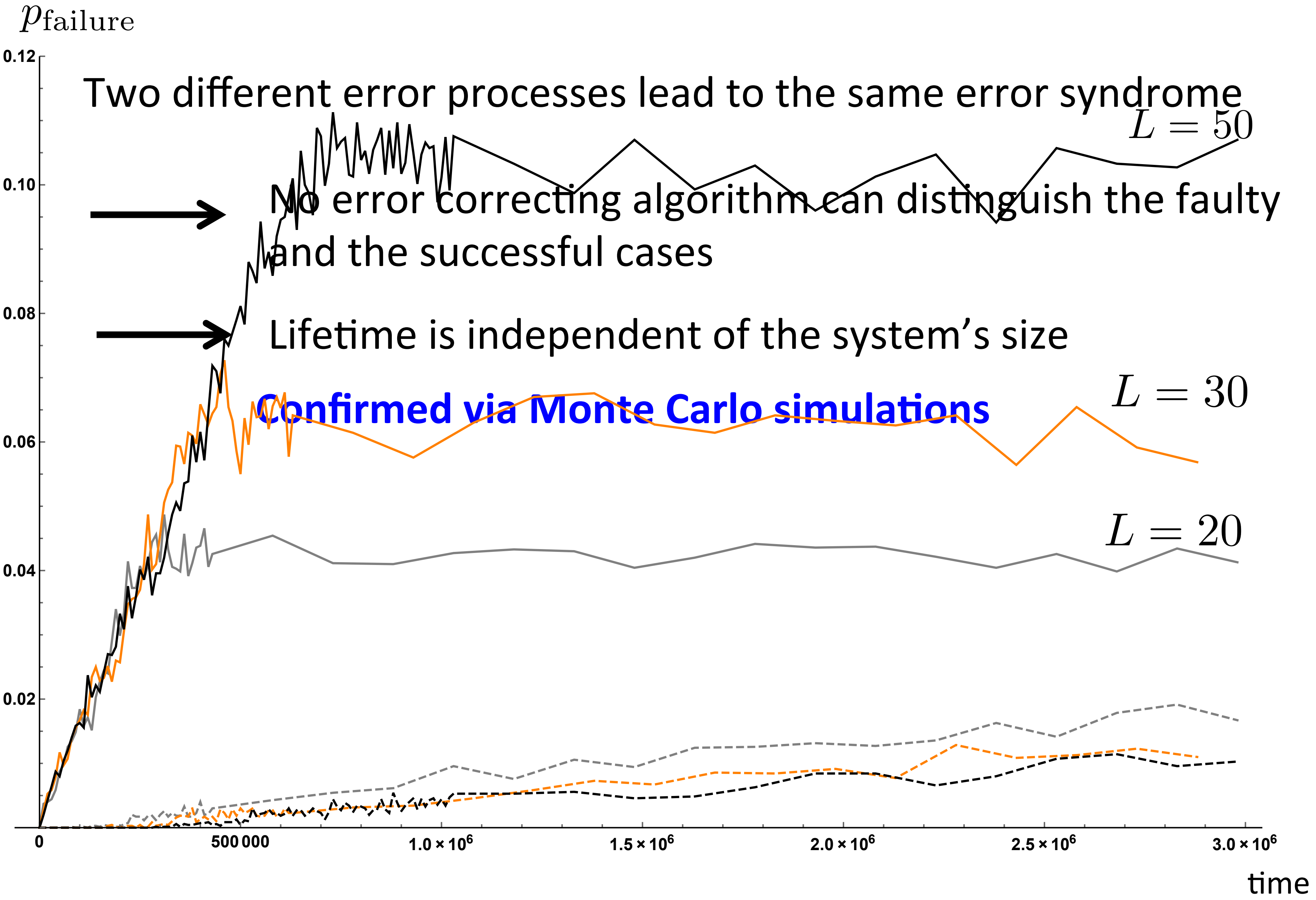
Are dangerous errors really a problem ?

NO ERROR

Dangerous errors



Dangerous errors



Two different error processes lead to the same error syndrome

→ No error correcting algorithm can distinguish the faulty and the successful cases

→ Lifetime is independent of the system's size

Confirmed via Monte Carlo simulations

Dangerous errors

Braiding Majoranas in a trijunction setup is problematic

⟶ Braiding renders errors non local

⟶ Dangerous errors

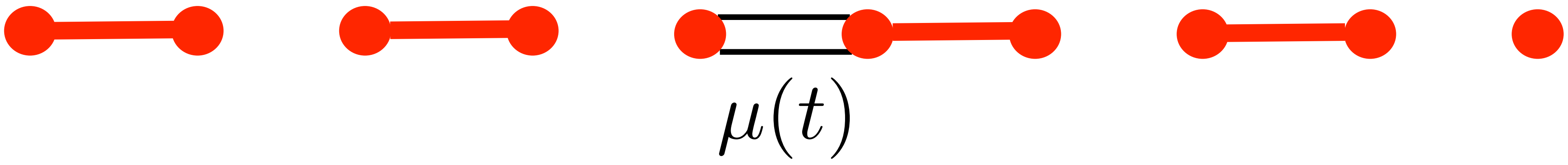
⟶ **Lifetime does not grow with system size**

Other schemes ?

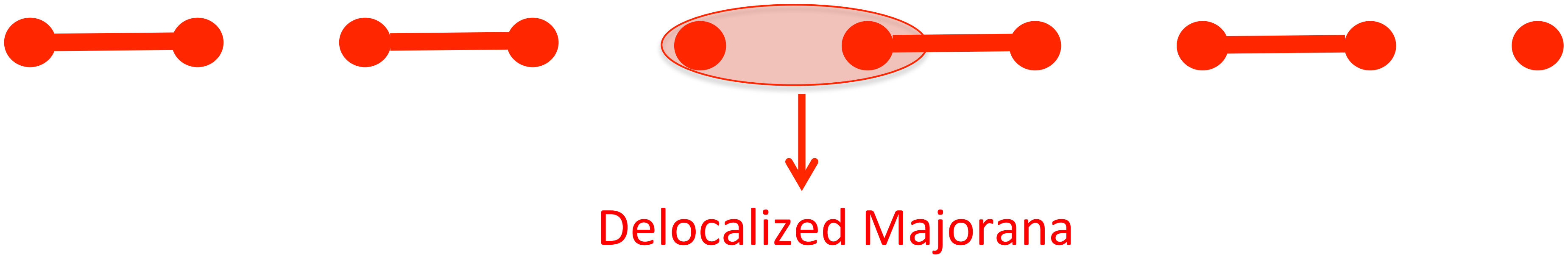
THANK YOU !

Backup

Linear case



One Majorana is delocalized



Trijunction case