

Course: Hybrid Devices for Quantum Information Processing

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Hybrid devices for Quantum Information Processing

Lectures 1 and 2: with Fabio Pedrocchi

Thermal Quasiparticles and Majorana Braiding



I discuss a model calculation of the decoherence of Majorana qubits during braiding in a trijunction, due to thermally generated quasiparticles (bosonic environment). The limitations to coherence are significant.

Lecture 3:

Semiconductor Hall-effect Gyrotrons and Circulators

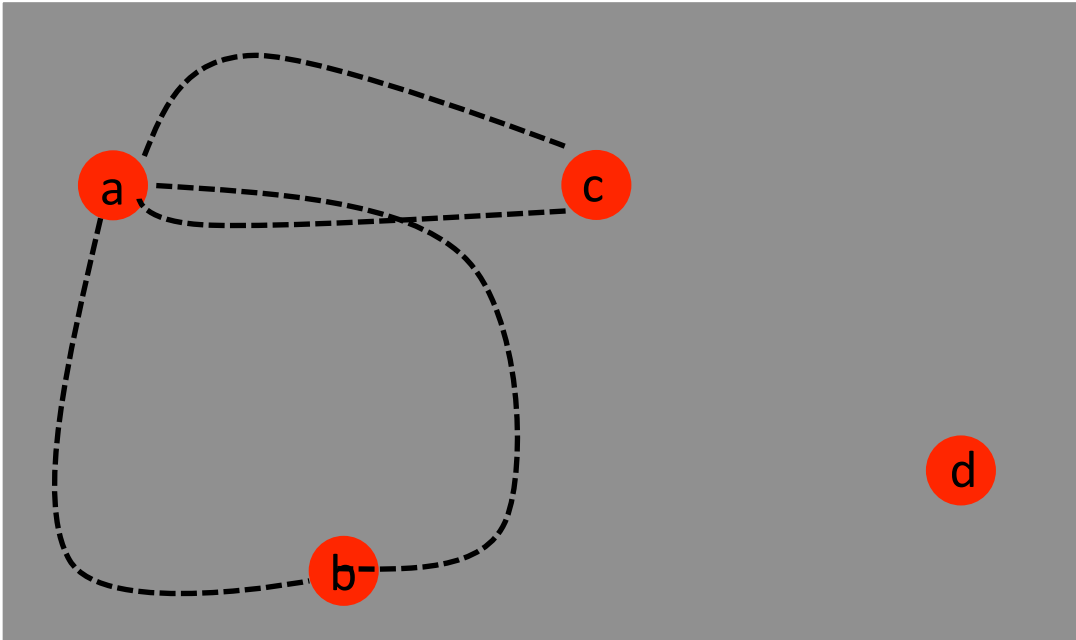
Calculations of driven propagation of chiral edge magnetoplasmons in the integer quantum Hall effect indicate a promising route for these devices in current experiments. They are very important for the miniaturization of multi-qubit quantum computers.

For today:

- What are anyons in general? Compute by braiding!
- Our anyons: Majorana modes
- Canonical model: “Kitaev” wire
- Diagonalize using Majorana-operator representation
- Our first qubit – a ground-state degeneracy
- Moving and braiding Majoranas – the T junction
- Why are Majoranas non-abelian?
- The problem for lecture 2:
 - *Does “topological” really make Majorana qubits fault tolerant?*

Topological Quantum Computing

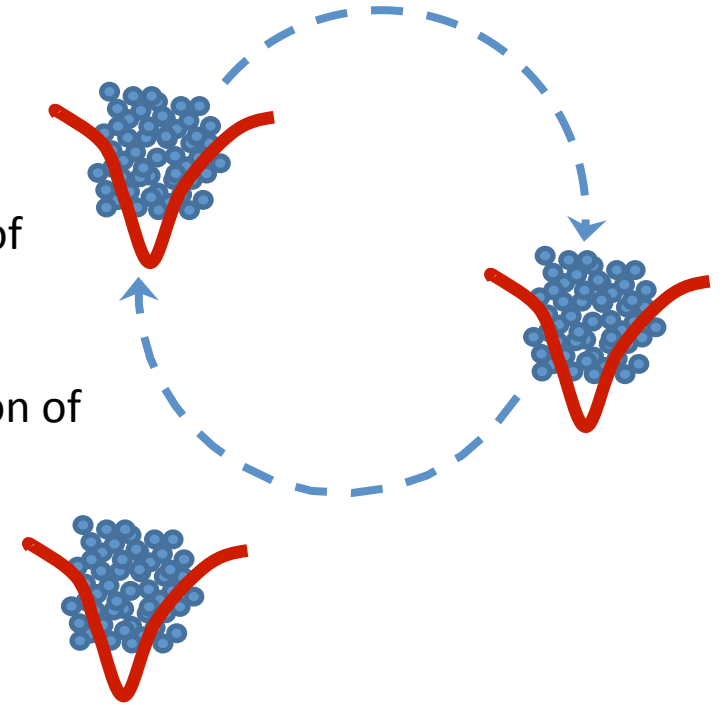
Topological Quantum Computation A. Kitaev, 1997



Degeneracy $|\psi_1\rangle, |\psi_2\rangle, \dots$

$$\begin{pmatrix} |\psi_1\rangle \\ |\psi_2\rangle \\ \dots \end{pmatrix} \xrightarrow{\text{depends only on topology}} U_{ab} \cdot \begin{pmatrix} |\psi_1\rangle \\ |\psi_2\rangle \\ \dots \end{pmatrix}$$

Another interpretation of anyons as “particles”:
Adiabatic motion of one-particle potentials



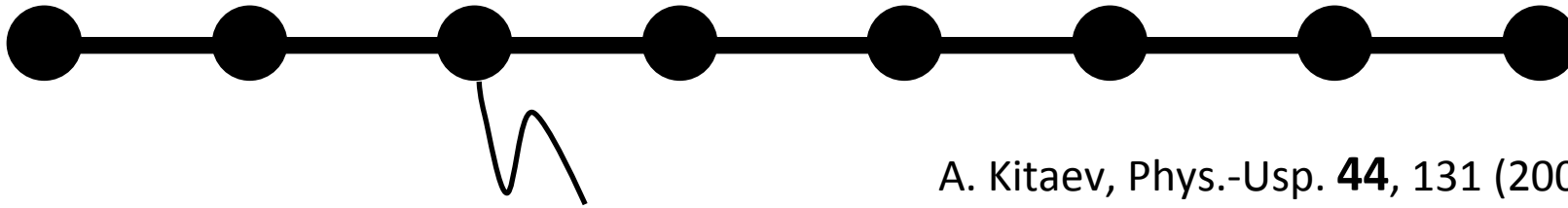
Topological Order

$$[U_{ab}, U_{bc}] \neq 0$$

→ non-abelian anyons

“Kitaev” Wire

Archetypical (1D) model with anyons
(simple model for a nanowire)



A. Kitaev, Phys.-Usp. **44**, 131 (2001)

Lieb, Mattis, Schultz, Ann. Phys. **16** (1961)

$$H = - \sum_{j=1}^L \mu_j a_j^\dagger a_j - \sum_{j=1}^{L-1} (t a_j^\dagger a_{j+1} - \Delta a_j a_{j+1} + \text{h.c.})$$

chemical potential

hopping

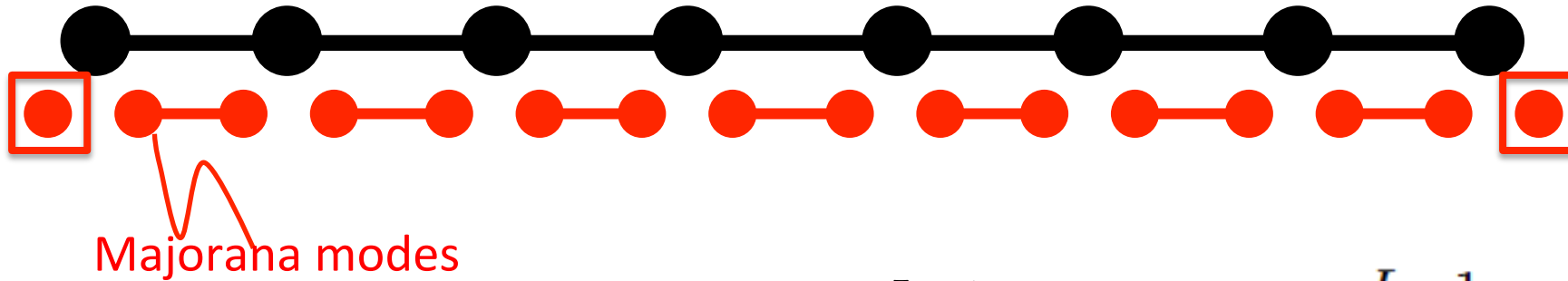
superconducting pairing
(proximity effect, Mourik et al. Science **336**, 1003 (2012))

Majorana operators $\{\gamma_j, \gamma_k\} = 2\delta_{jk}$
 $\gamma^\dagger = \gamma$

$$a_j = \frac{(\gamma_{2j-1} + i\gamma_{2j})}{2}$$

Kitaev Wire

Archetypical (1D) model with anyons
(simple model for a nanowire)



$$\mu_j = 0 \quad t = \Delta \quad H = -\Delta \sum_{j=1}^{L-1} i\gamma_{2j+1}\gamma_{2j} = \sum_{j=0}^{L-1} \epsilon_j (2d_j^\dagger d_j - 1)$$

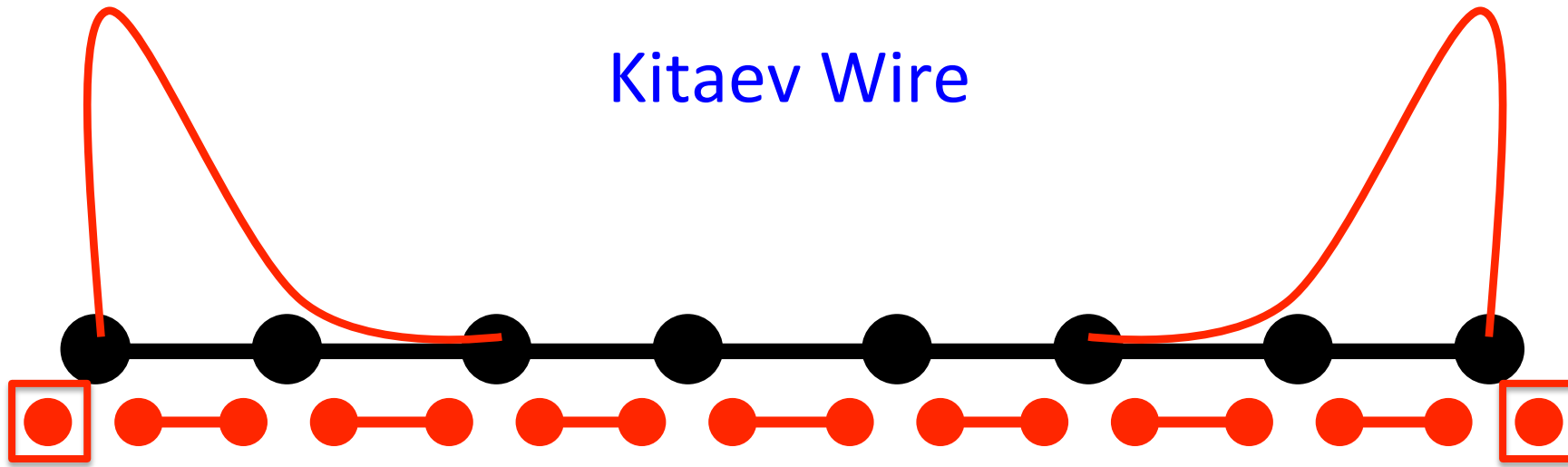
→ γ_1 and γ_{2L} are decoupled from H $\epsilon_0 = 0$ and $\epsilon_j = |\Delta|$

→ Non-local mode

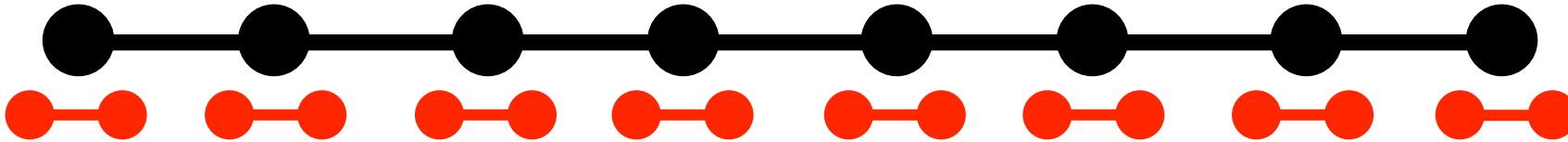
$$d_0 = \frac{\gamma_1 + i\gamma_{2L}}{2} \quad d_j = \frac{1}{2}(\gamma_{2j} + i\gamma_{2j+1})$$

$$d_0|0\rangle = 0 \quad d_0^\dagger|0\rangle = |1\rangle$$

Kitaev Wire



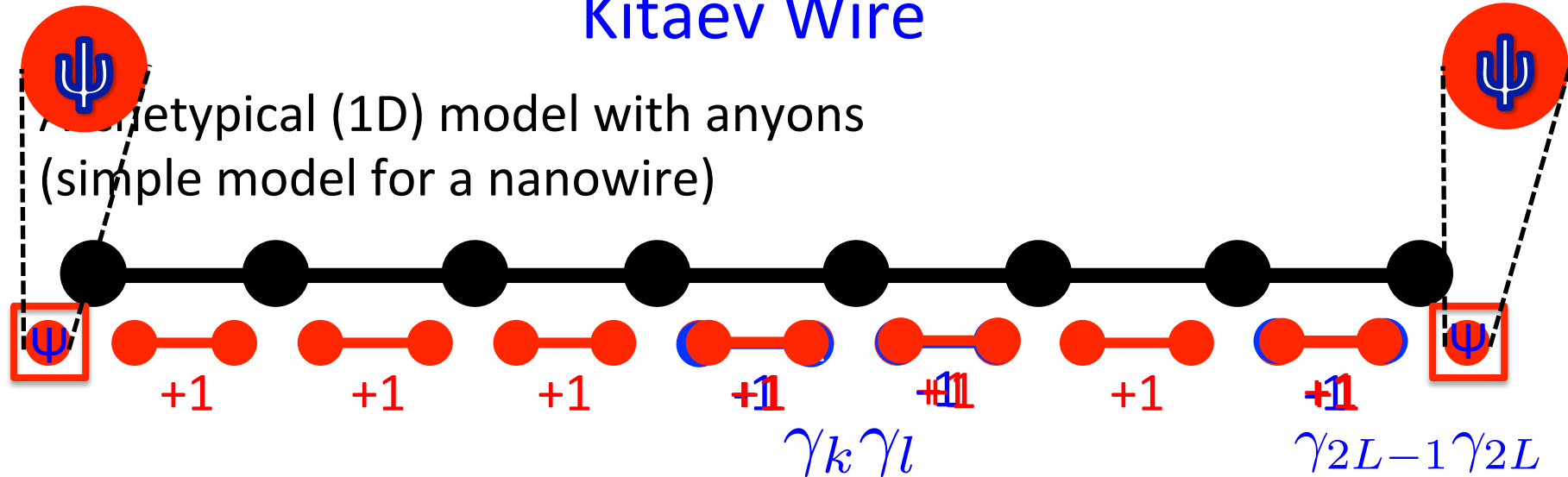
$|\mu_j| \leq 2t$ **Topological phase (Majorana bound states)**



$|\mu_j| \gg 2t$
$$H = -i \frac{\mu}{2} \sum_{j=1}^L \gamma_{2j-1} \gamma_{2j}$$

Non Topological phase (No Majorana bound states)

Kitaev Wire



$$\mu_j = 0 \quad t = \Delta \quad H = -\Delta \sum_{j=1}^{L-1} i \gamma_{2j+1} \gamma_{2j}$$

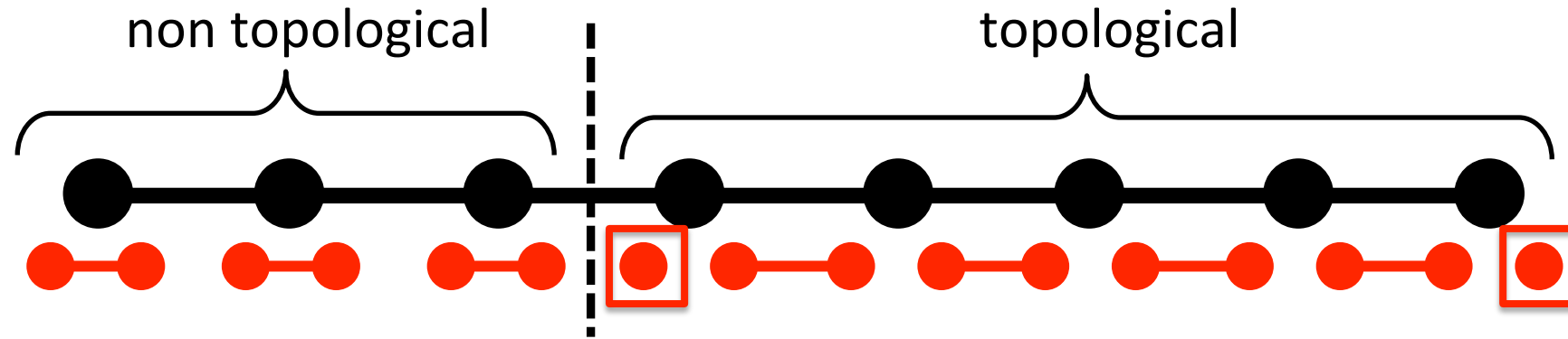
In the ground-state subspace $\longrightarrow i \gamma_{2j+1} \gamma_{2j} = +1$

Excitations $\longrightarrow i \gamma_{2j+1} \gamma_{2j} = -1$ Ψ quasi-particle

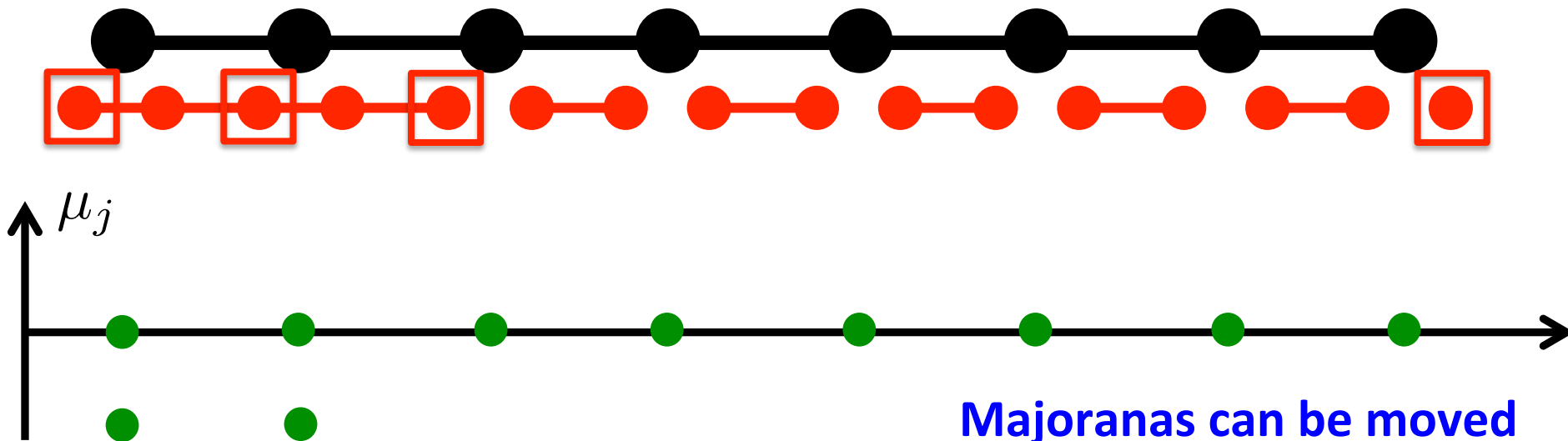
Parity flip $\longrightarrow i \gamma_1 \gamma_{2L} \rightarrow -i \gamma_1 \gamma_{2L}$

$$\gamma_1 \gamma_2 \cdots \gamma_{2L} \propto Z - \text{error}$$

Kitaev Wire



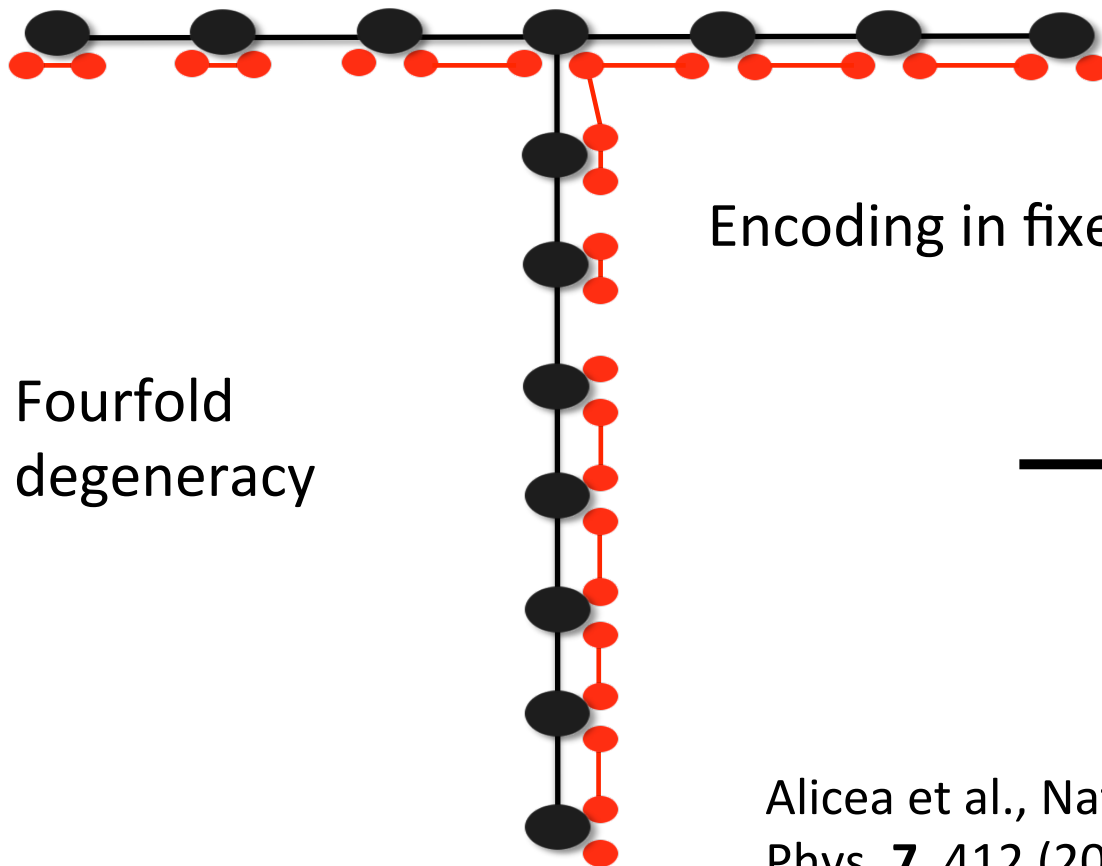
→ Majorana bound states appear at the **junction** between topological and non topological segments



Trijunction

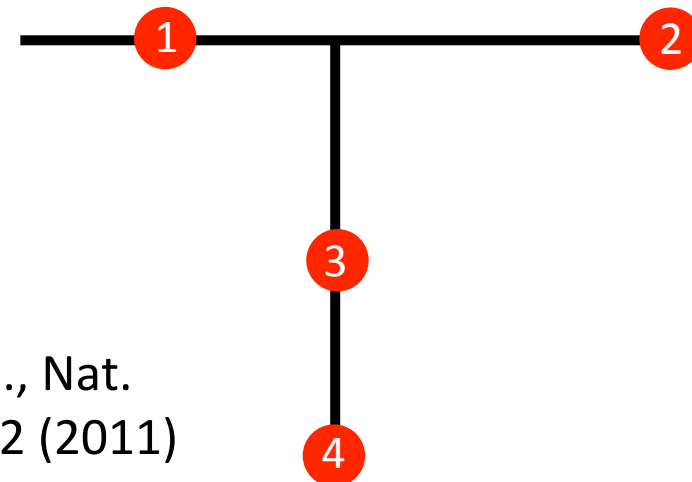
Not enough space to exchange Majoranas

→ Trijunction $H_T(\tau) = H_H(\tau) + H_V(\tau) - (ta_{L/2}^\dagger a_{L+1} - \Delta a_{L/2} a_{L+1} + \text{h.c.})$



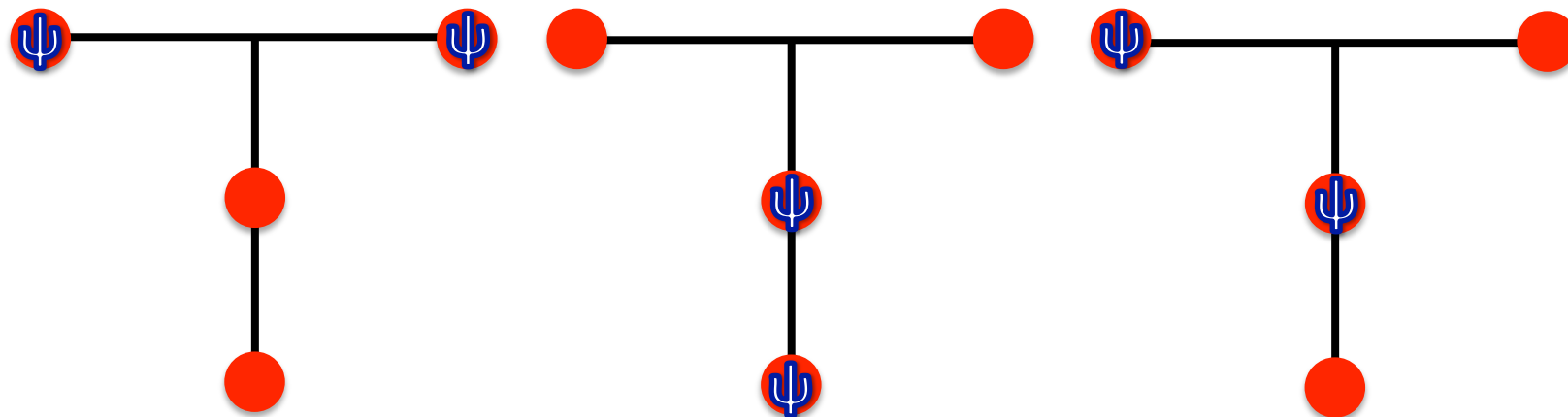
Encoding in fixed parity sector

$$i\gamma_1\gamma_2 i\gamma_3\gamma_4 = +1$$



Alicea et al., Nat.
Phys. **7**, 412 (2011)

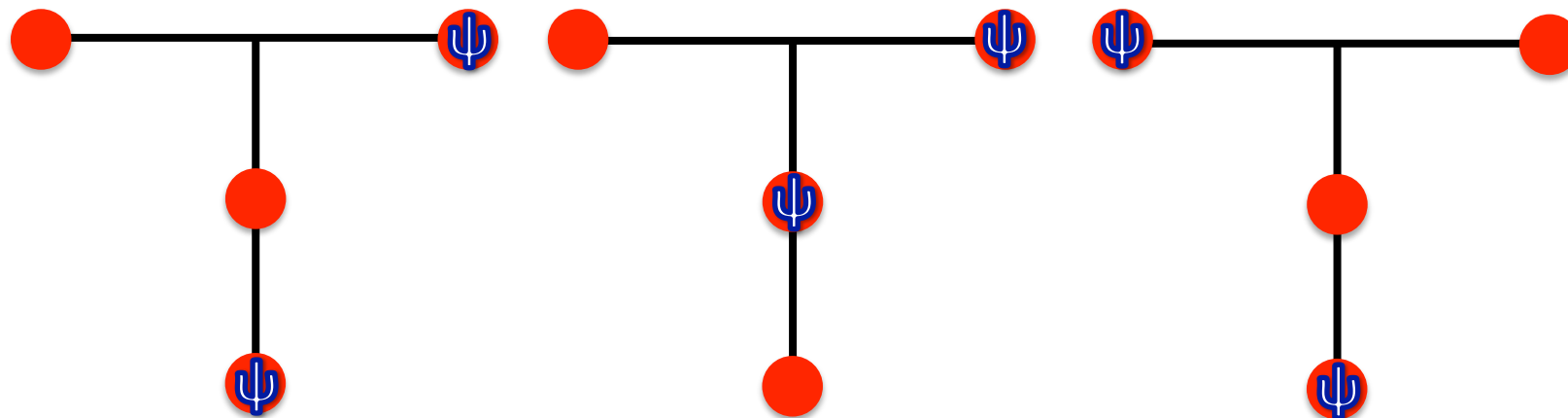
Logical errors



Z-error

Z-error

X-error



X-error

Y-error

Y-error

Majorana Braiding with Thermal Noise

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(Received 7 July 2015; published 15 September 2015)

PHYSICAL REVIEW B **92**, 115441 (2015)



Monte Carlo studies of the self-correcting properties of the Majorana quantum error correction code under braiding

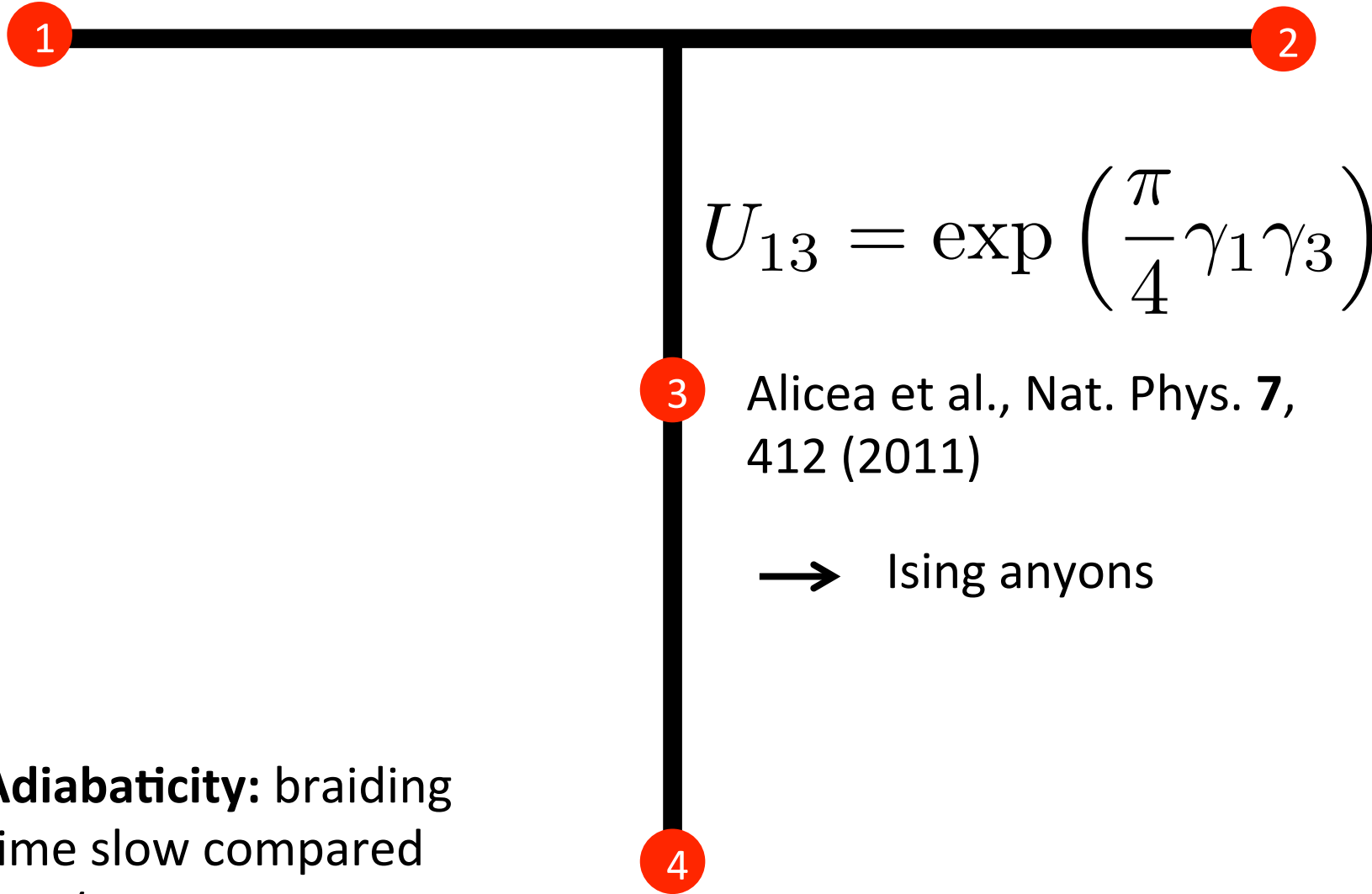
Fabio L. Pedrocchi,¹ N. E. Bonesteel,² and David P. DiVincenzo¹

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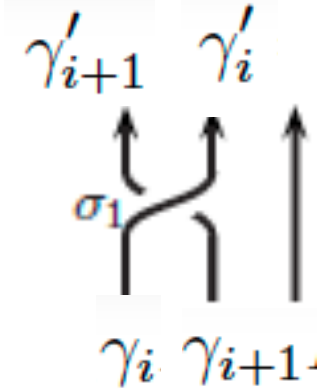
Braiding



Adiabaticity: braiding
time slow compared
to $1/\Delta$

denote the Majorana operators after the exchange by γ'_i and γ'_{i+1} .

Fabian Hassler, "Majorana Qubits",
[arXiv:1404.0897](https://arxiv.org/abs/1404.0897)



$$\gamma'_i = B_{i,i+1} \gamma_i B_{i,i+1}^\dagger, \quad \gamma'_{i+1} = B_{i,i+1} \gamma_{i+1} B_{i,i+1}^\dagger.$$

Since the position of the two Majoranas are interchanged by this operation,

$$\gamma'_i = \alpha_i \gamma_{i+1}, \quad \gamma'_{i+1} = \alpha_{i+1} \gamma_i, \quad \alpha_i, \alpha_{i+1} \in \mathbb{R} \quad (\text{Hermiticity})$$

$$-i\gamma_i \gamma_{i+1} = -i\gamma'_i \gamma'_{i+1} \longrightarrow$$

$$\alpha_i \alpha_{i+1} = -1. \longrightarrow \alpha_i = 1, \alpha_{i+1} = -1. \quad (\text{convention})$$

(physical electron number $\rightarrow$
 Bogoliubov fermion parity)

$$n_j = f_j^\dagger f_j \equiv \frac{1}{2}(1 + i\gamma_{2j-1}\gamma_{2j}), j = 1, \dots, n.$$

$$P = (-1)^{c^\dagger c} = 1 - 2c^\dagger c \\ = -i\gamma_a \gamma_b = \pm 1$$

$$\begin{aligned} \gamma_i &\rightarrow \gamma_{i+1}, \\ \gamma_{i+1} &\rightarrow -\gamma_i, \\ \gamma_j &\rightarrow \gamma_j, \quad j \notin \{i, i+1\}. \end{aligned} \longrightarrow B_{i,i+1} = \exp\left(-\frac{\pi}{4}\gamma_i \gamma_{i+1}\right) = \frac{1}{\sqrt{2}}(1 - \gamma_i \gamma_{i+1})$$

Thermal environment

Main focus: how does a **thermal environment** destroy the stored quantum information when braiding is executed?

$$H(\tau) = H_T(\tau) + H_B + H_{SB}$$

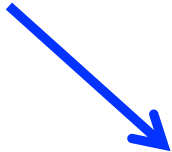
Bosonic Bath $H_B = \sum_j B_j$

System-Bath coupling $H_{SB} = - \sum_j B_j \otimes (2a_j^\dagger a_j - 1) = -i \sum_j B_j \otimes \gamma_{2j-1} \gamma_{2j}$

Markovian master equation in adiabatic limit

$$\dot{\rho}_S(\tau) = -i[H_T(\tau), \rho_S(\tau)] + \mathcal{D}(\rho_S(\tau))$$


Unitary evolution


Dissipation $\leftrightarrow \Gamma$ (Ohmic bath)

Approximately end of lecture 1



Normal-metal quasiparticle traps for superconducting qubits

R.-P. Riwar,^{1,2} A. Hosseinkhani,^{1,3} L. D. Burkhardt,² Y. Y. Gao,² R. J. Schoelkopf,² L. I. Glazman,² and G. Catelani¹

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The presence of quasiparticles in superconducting qubits emerges as an intrinsic constraint on their coherence. While it is difficult to prevent the generation of quasiparticles, keeping them away from active elements of the qubit provides a viable way of improving the device performance. Here we develop theoretically and validate



Normal-metal quasiparticle traps for superconducting qubits

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The presence of quasiparticles in superconducting qubits emerges as an intrinsic constraint on their coherence. While it is difficult to prevent the generation of quasiparticles, keeping them away from active elements of the qubit provides a means to extend the coherence time and validate the qubit's performance.

it is difficult to prevent the generation of quasiparticles

Basic error processes

Excitations are always created in pairs

Creation bulk, energy cost is -4Δ

Annihilation bulk, energy cost is 4Δ

Hopping bulk, energy cost is 0

Creation boundary, energy cost is -2Δ

Annihilation boundary, energy cost is 2Δ

Hopping onto (out from)

Majorana, energy cost 2Δ (-2Δ)

Non topological

$$H_{SB} = -i \sum_j B_j \otimes \gamma_{2j-1} \gamma_{2j}$$

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Main focus: how does **a thermal environment** destroy the stored quantum information when braiding is executed?

$$H(\tau) = H_T(\tau) + H_B + H_{SB}$$

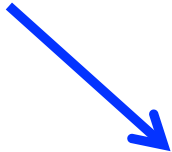
Bosonic Bath $H_B = \sum_j B_j$

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Markovian master equation in adiabatic limit

$$\dot{\rho}_S(\tau) = -i[H_T(\tau), \rho_S(\tau)] + \mathcal{D}(\rho_S(\tau))$$


Unitary evolution


Dissipation $\leftrightarrow \Gamma$ (Ohmic bath)

Davies Prescription

Markovian master equation in the weak-coupling limit

$$\mathcal{D}(\rho_S(t)) = \sum_{i,j} \sum_{\omega} \gamma^{ij}(\omega) \left(A^i(\omega) \rho_S(t) (A^j(\omega))^{\dagger} - \frac{1}{2} \{ (A^j(\omega))^{\dagger} A^i(\omega), \rho_S(t) \} \right)$$

 Jump operator

Spectral function

$$\gamma^{ij}(\omega) = \int_{-\infty}^{+\infty} ds e^{i\omega s} \langle B_i^{\dagger}(s) B_j(0) \rangle$$

Davies Prescription

System-Bath interaction

$$H_{SB} = -2 \sum_j B_j \otimes a_j^\dagger a_j = -i \sum_j B_j \otimes \gamma_{2j-1} \gamma_{2j}$$

Rewrite in terms of **eigenoperators**

$$\begin{aligned} H_{SB} = & -iB_1 \otimes (d_0 + d_0^\dagger)(d_1 + d_1^\dagger) \\ & - \sum_{j=1}^{\lceil L/2 \rceil - 1} B_{2j} \otimes (d_{2j-1} - d_{2j-1}^\dagger)(d_{2j} + d_{2j}^\dagger) \\ & - \sum_{j=1}^{\lceil L/2 \rceil - 1} B_{2j+1} \otimes (d_{2j} - d_{2j}^\dagger)(d_{2j+1} + d_{2j+1}^\dagger) \\ & + iB_L \otimes (d_{L-1} - d_{L-1}^\dagger)(d_0 - d_0^\dagger). \end{aligned}$$

(corrects a few factors of 2 on p. 17 of Pedrocchi et al. PRB. Our apologies! No change of the physics.

Davies Prescription

Terms have a clear physical meaning:

$$A_{\text{hopping}} := -iB_1 \otimes d_0 d_1^\dagger - \sum_{j=1}^{\lceil L/2 \rceil - 1} B_{2j} \otimes d_{2j-1} d_{2j}^\dagger$$

$$- \sum_{j=1}^{\lceil L/2 \rceil - 1} B_{2j+1} \otimes d_{2j} d_{2j+1}^\dagger - iB_L \otimes d_{L-1} d_0^\dagger + \text{h.c.}$$

Energy cost: $\pm 2\Delta$

Energy cost: 0

$$A_{\text{creation}} :=$$

$$-iB_1 \otimes d_0^\dagger d_1^\dagger + \sum_{j=1}^{\lceil L/2 \rceil - 1} (B_{2j} \otimes d_{2j-1}^\dagger d_{2j}^\dagger + B_{2j+1} \otimes d_{2j}^\dagger d_{2j+1}^\dagger)$$

$$+iB_L \otimes d_{L-1}^\dagger d_0^\dagger$$

Energy cost: -2Δ

Energy cost: -4Δ

Davies Prescription

Time evolution under H_S

$$\begin{aligned} e^{iH_S t} A_\zeta e^{-iH_S t} \\ &= \sum_{m,n,k,\ell} |m\rangle \langle m| e^{iH_S t} |k\rangle \langle k| A_\zeta |\ell\rangle \langle \ell| e^{-iH_S t} |n\rangle \langle n| \\ &= \sum_{m,n} e^{it(\epsilon_m - \epsilon_n)} |m\rangle \langle m| A_\zeta |n\rangle \langle n| \end{aligned}$$

→ Fourier transform gives the **jump operators**

$$A_\zeta(\omega) = \sum_{\epsilon_m - \epsilon_n = \omega} |m\rangle \langle m| A_\zeta |n\rangle \langle n|$$

Davies Prescription

→ Fourier transform gives the **jump operators**

$$A_{\zeta}(\omega) = \sum_{\epsilon_m - \epsilon_n = \omega} |m\rangle \langle m| A_{\zeta} |n\rangle \langle n|$$

Example of jump operators:

$$A_{\text{hopping}}^1(-2|\Delta|) = \sum_{\epsilon_m - \epsilon_n = -2|\Delta|} |m\rangle \langle m| d_0 d_1^{\dagger} |n\rangle \langle n|$$

$$A_{\text{hopping}}^{2j}(0) = \sum_{\epsilon_m - \epsilon_n = 0} |m\rangle \langle m| d_{2j-1} d_{2j}^{\dagger} + d_{2j} d_{2j-1}^{\dagger} |n\rangle \langle n|$$

$$A_{\text{creation}}^1(-2|\Delta|) = \sum_{\epsilon_m - \epsilon_n = -2|\Delta|} |m\rangle \langle m| d_0^{\dagger} d_1^{\dagger} |n\rangle \langle n|$$

Davies Prescription

Pauli Master equation

Technical Condition: (satisfied in our model)

$$\langle m_\alpha | A_\eta^i(\omega) | n_k \rangle \neq 0 \longrightarrow \begin{array}{l} \text{No other jump} \\ \text{operators cause} \\ \text{transitions} \\ |n_k\rangle \leftrightarrow |m_\beta\rangle \end{array}$$

 Diagonal elements decouple from off-diagonal elements

Davies Prescription



Diagonal elements decouple from off-diagonal elements

$$\frac{dP(n, \tau)}{d\tau} = \sum_m [W(n|m)P(m, \tau) - W(m|n)P(n, \tau)]$$

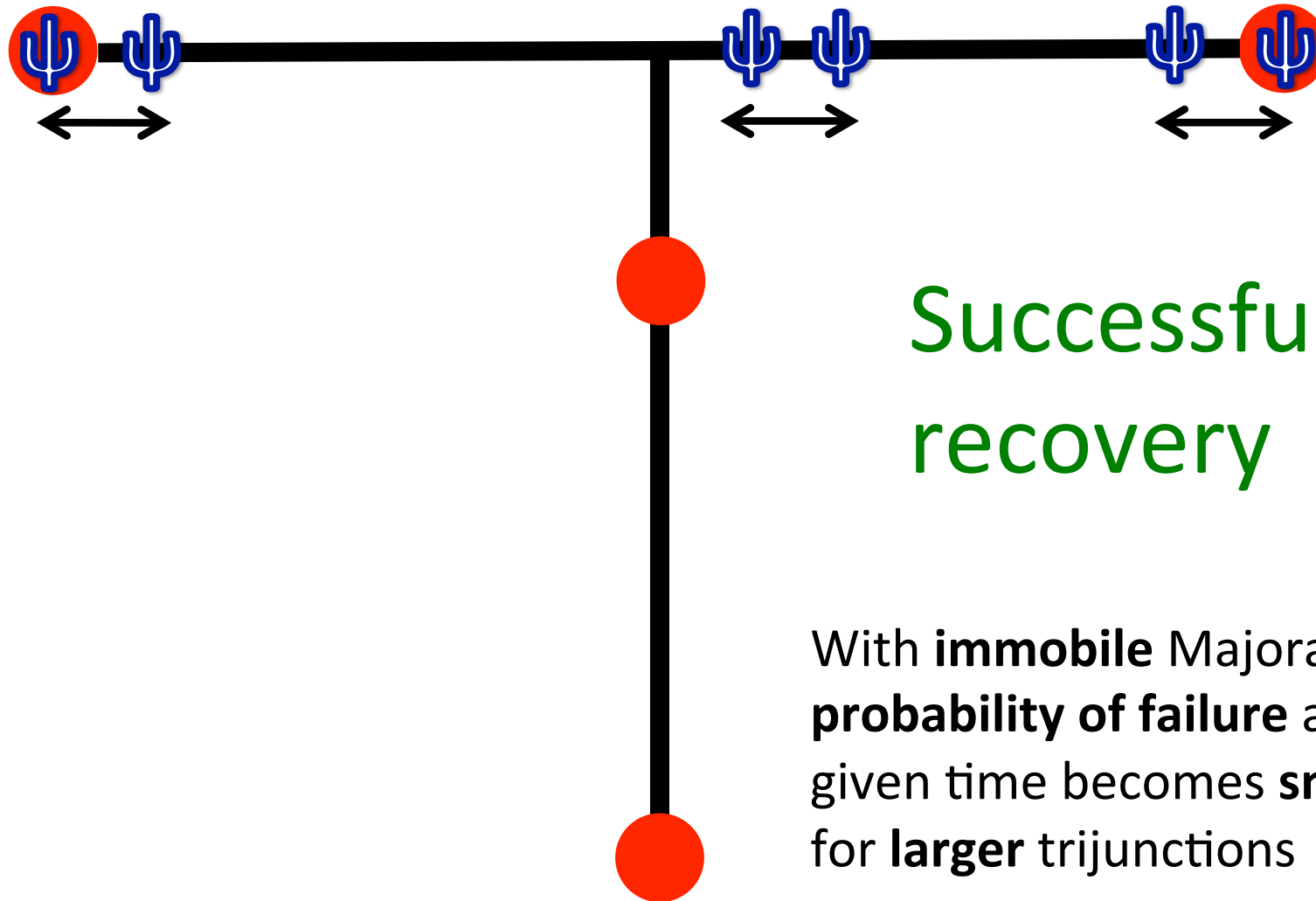
With transition rates

$$W(n|m) = \gamma(\omega_{mn}) |\langle m | A^{i_{mn}}(\omega_{mn}) | n \rangle|^2$$

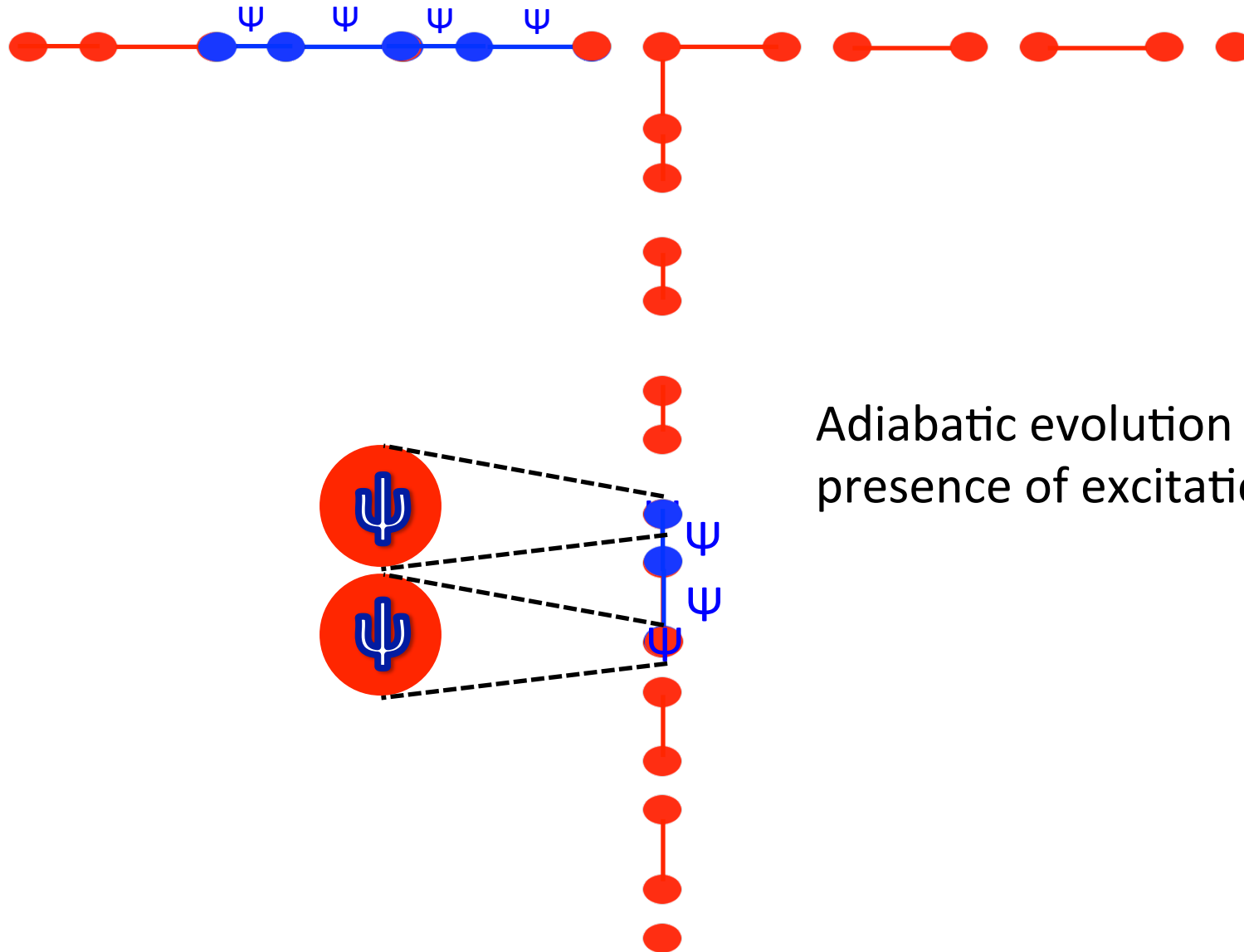
We take in our model an Ohmic spectral function

$$\gamma(\omega) = \kappa \left| \frac{\omega}{1 - \exp(-\beta\omega)} \right|$$

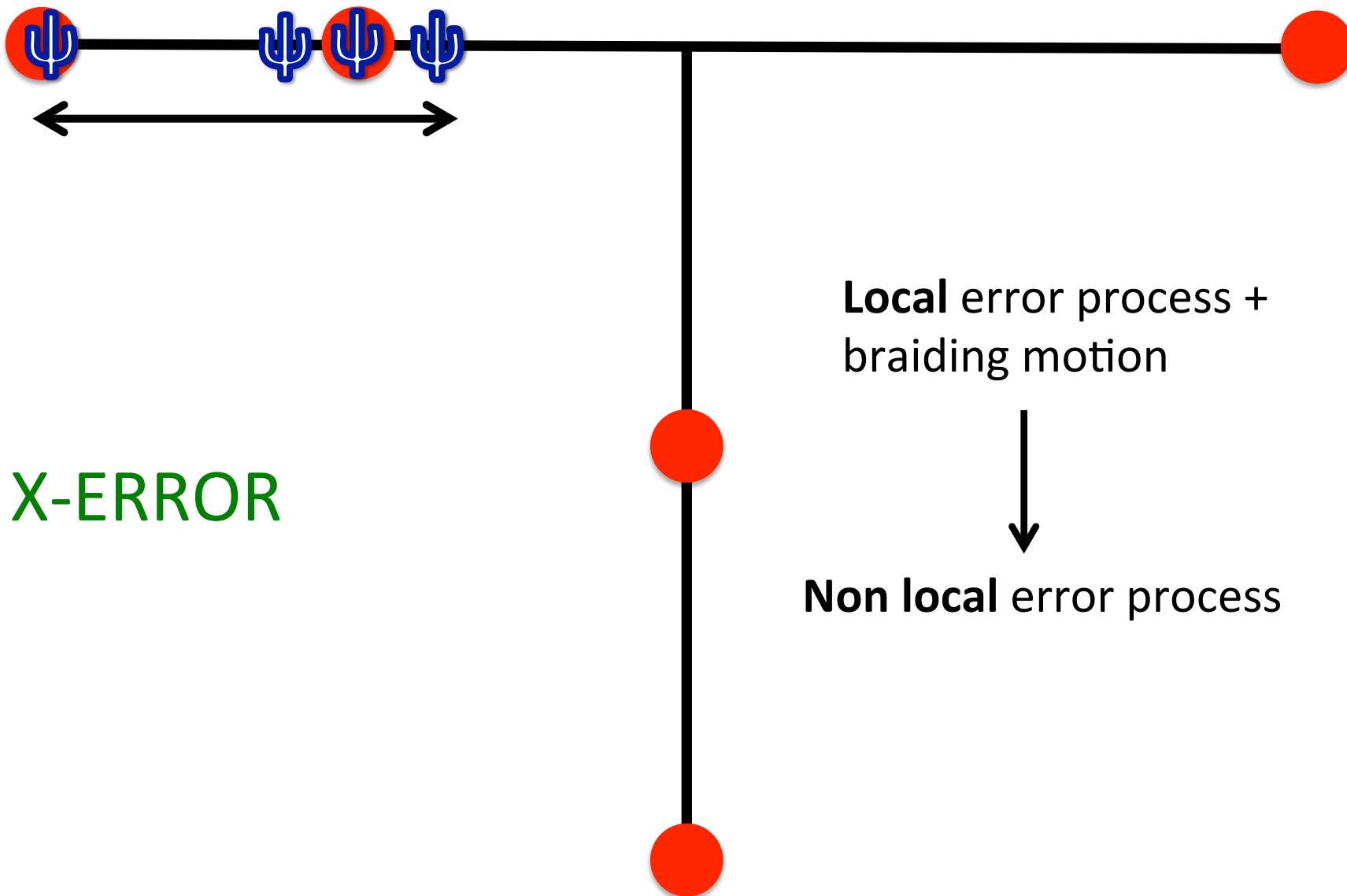
Error Correction



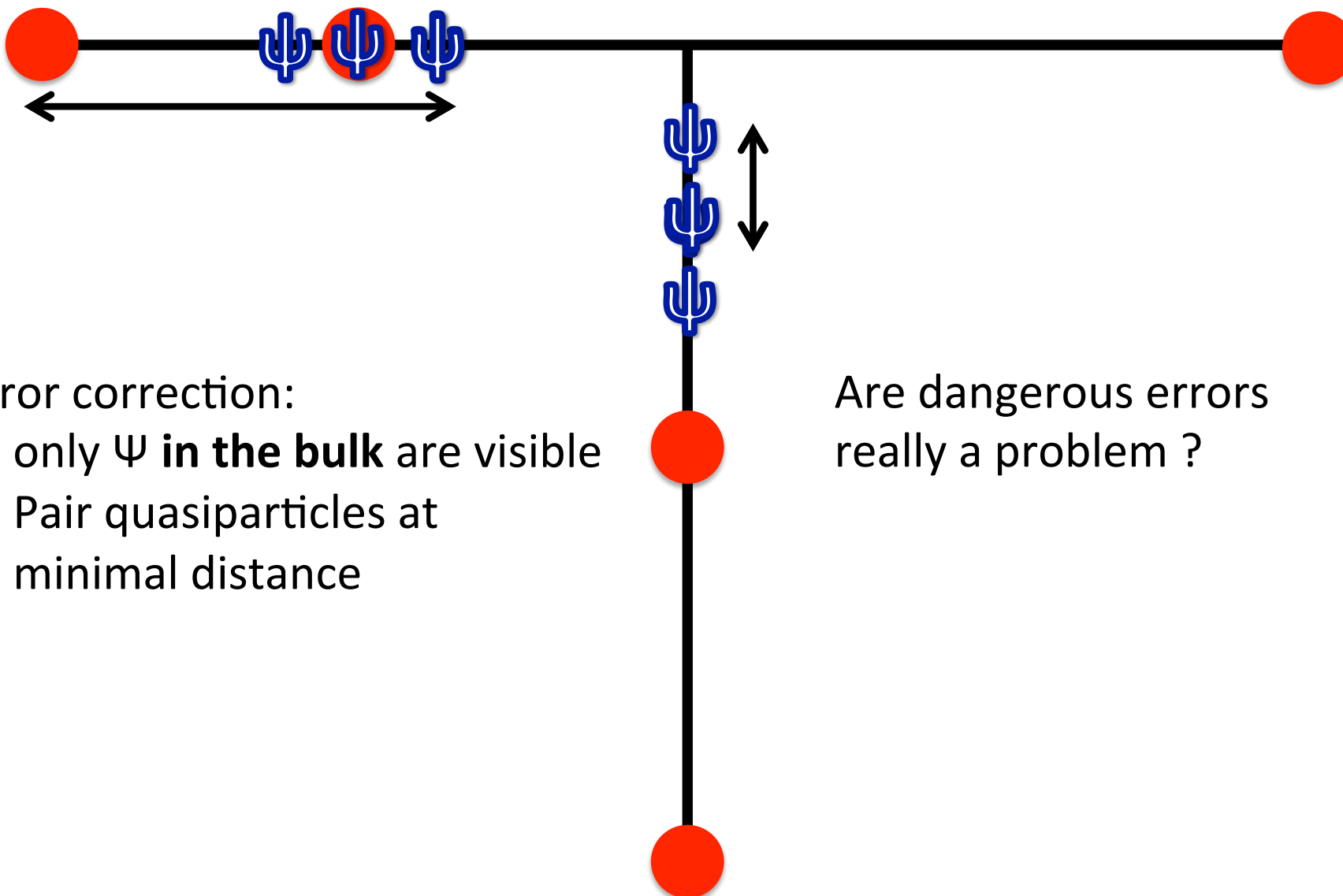
Majoranas and Interactions



Dangerous errors



Dangerous errors

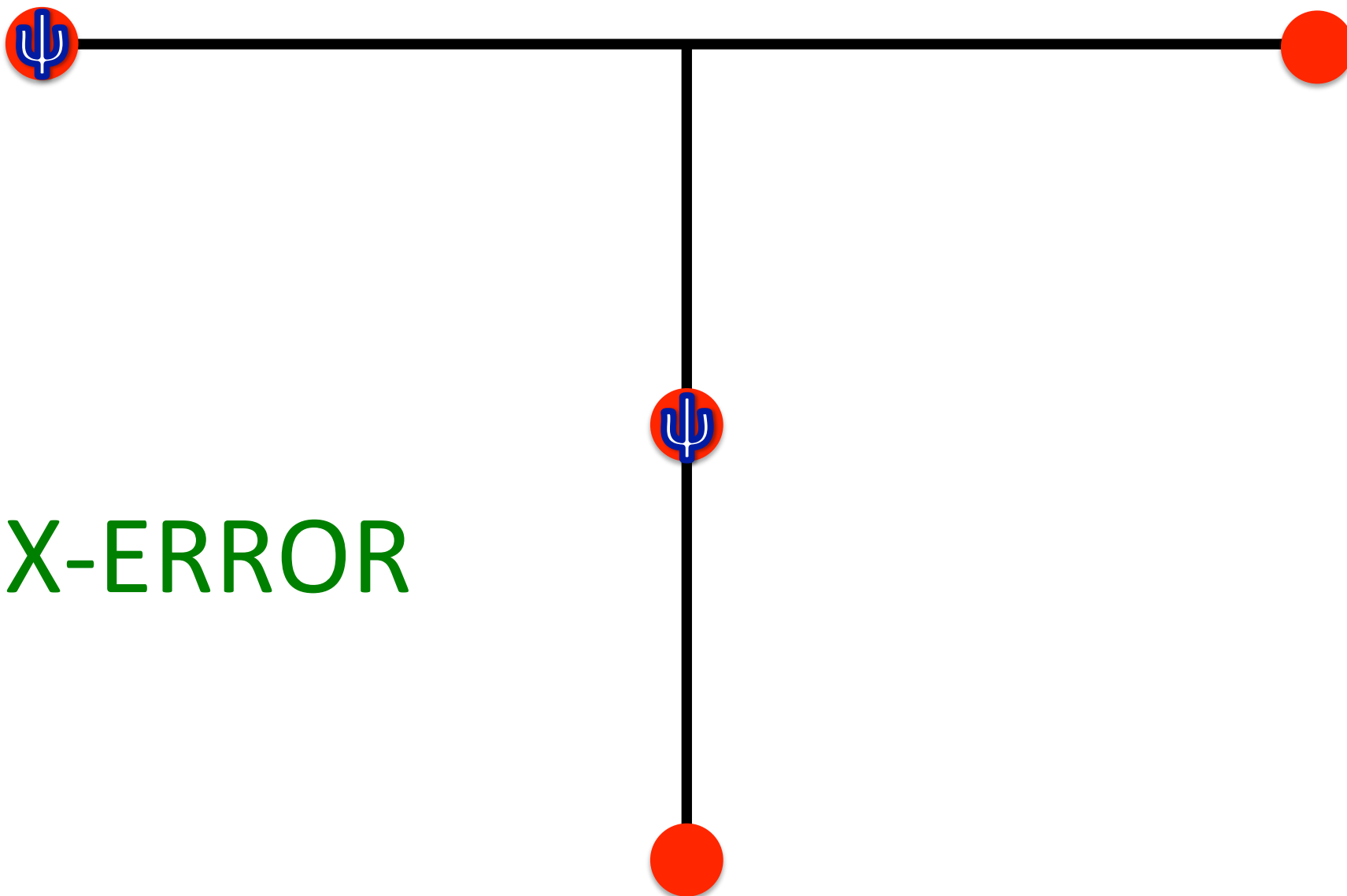


Error correction:

- only Ψ **in the bulk** are visible
- Pair quasiparticles at minimal distance

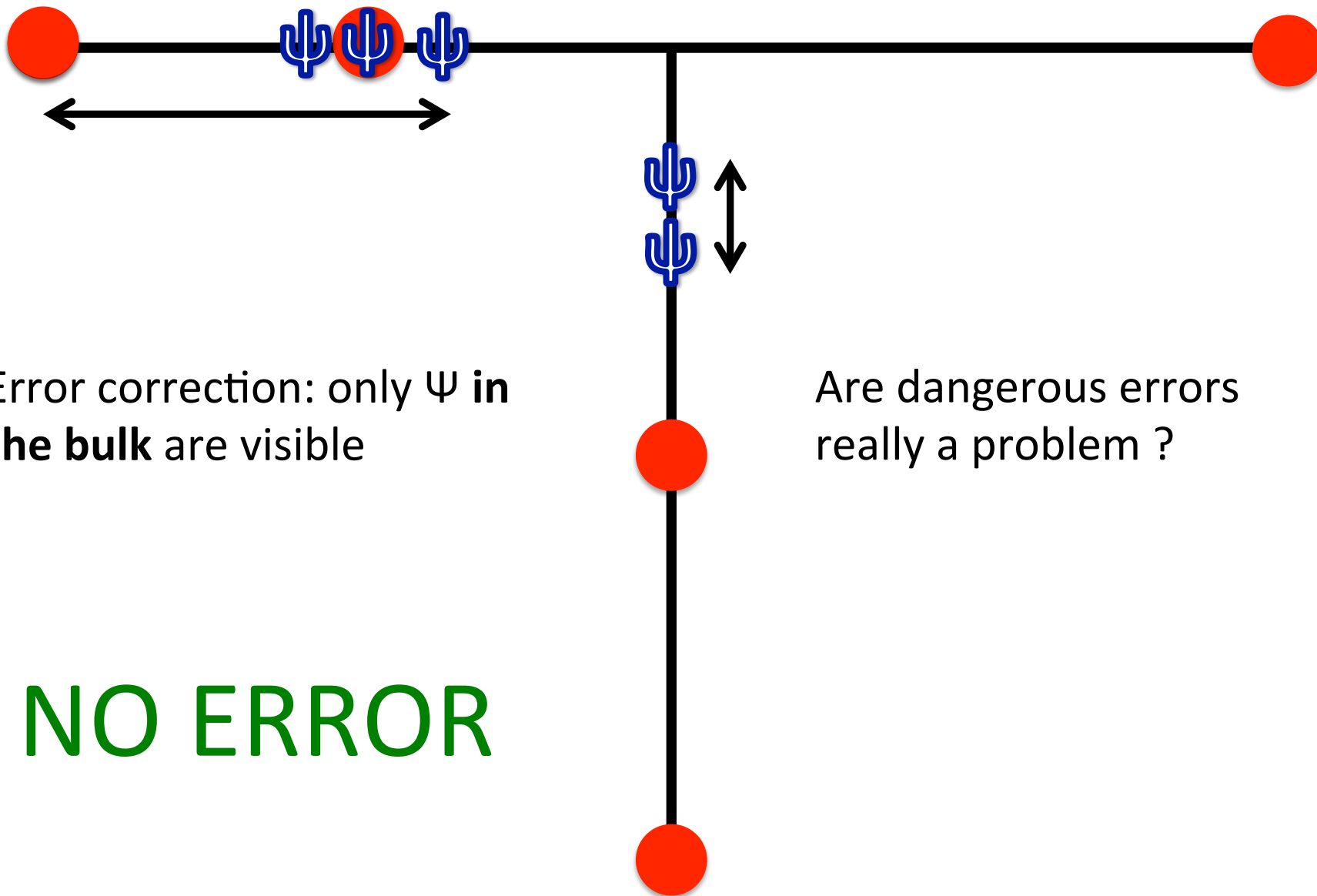
Are dangerous errors really a problem ?

Dangerous errors



X-ERROR

Dangerous errors



Error correction: only ψ in
the bulk are visible

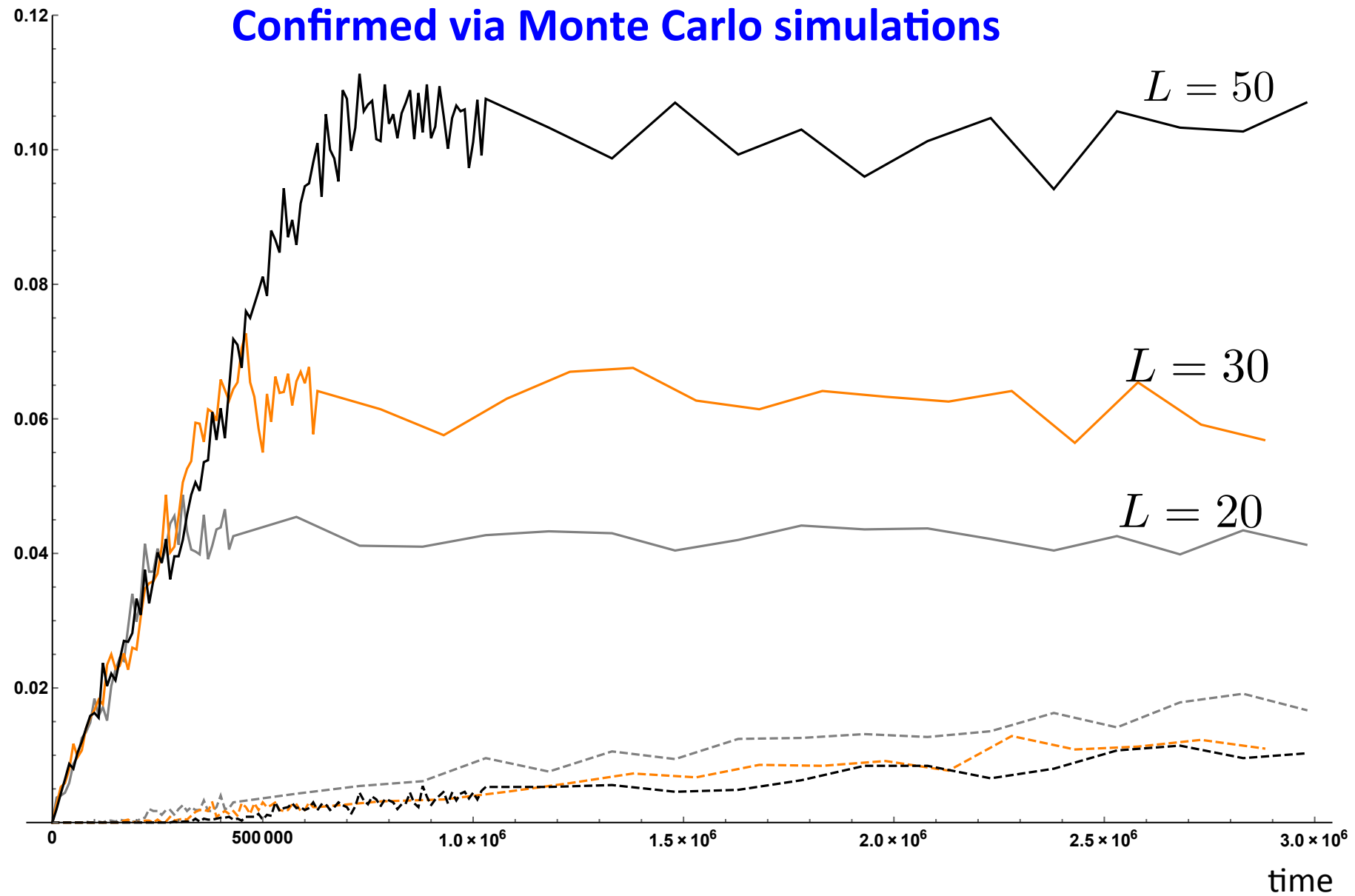
Are dangerous errors
really a problem ?

NO ERROR

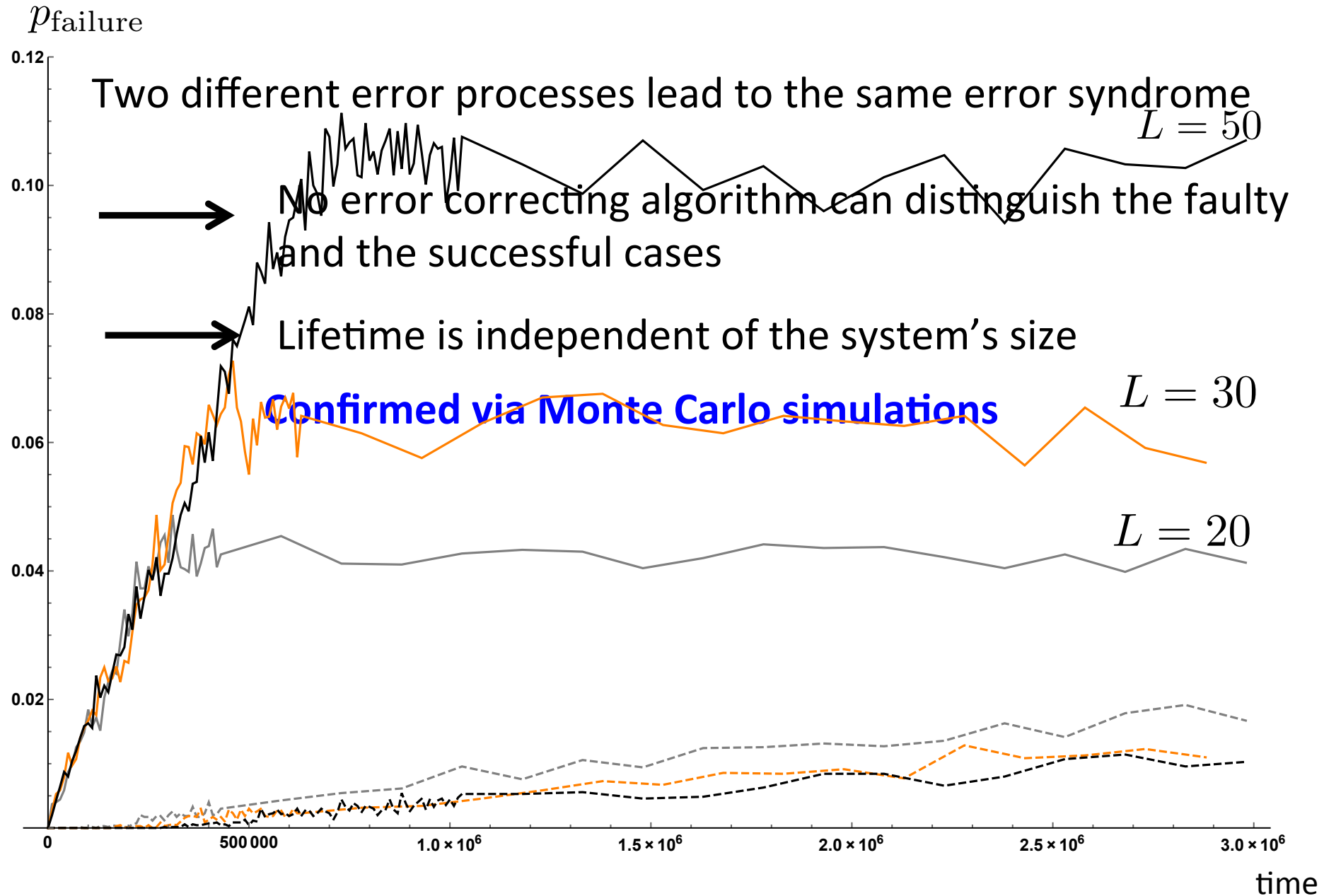
Dangerous errors

p_{failure}

Confirmed via Monte Carlo simulations



Dangerous errors



Dangerous errors

Braiding Majoranas in a trijunction setup is problematic

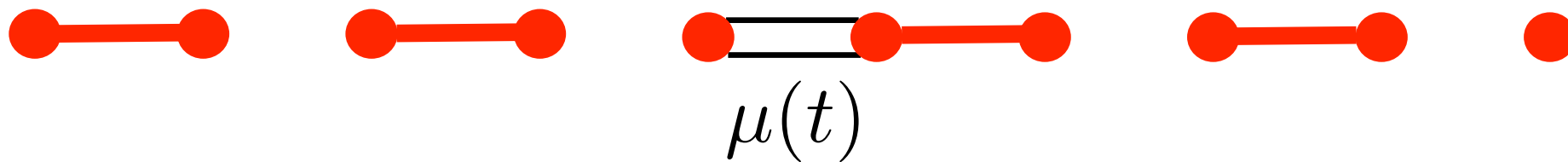
- Braiding renders errors non local
- Dangerous errors
- **Lifetime does not grow with system size**

Other schemes ?

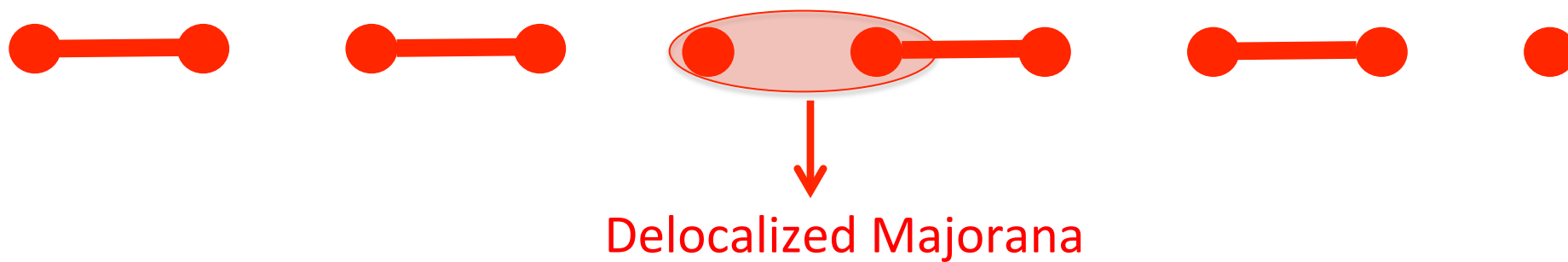
THANK YOU !

Backup

Linear case



One Majorana is delocalized



Trijunction case

