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Shot noise in magnetic tunnel junctions: effect of the geometric phase

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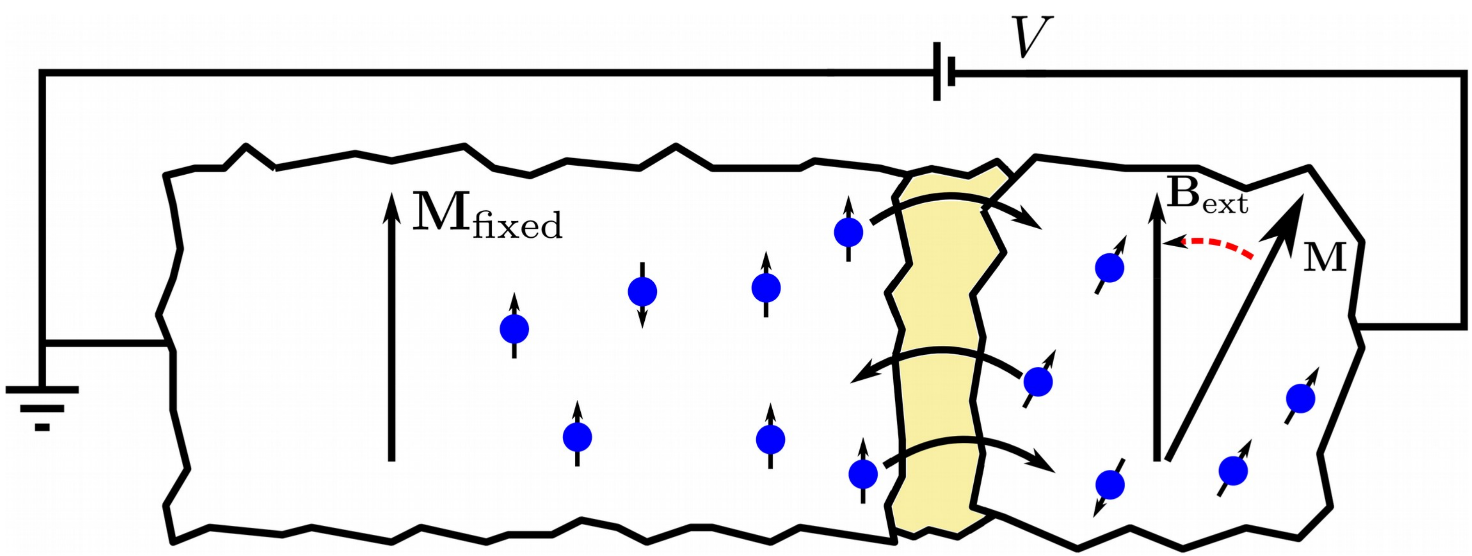
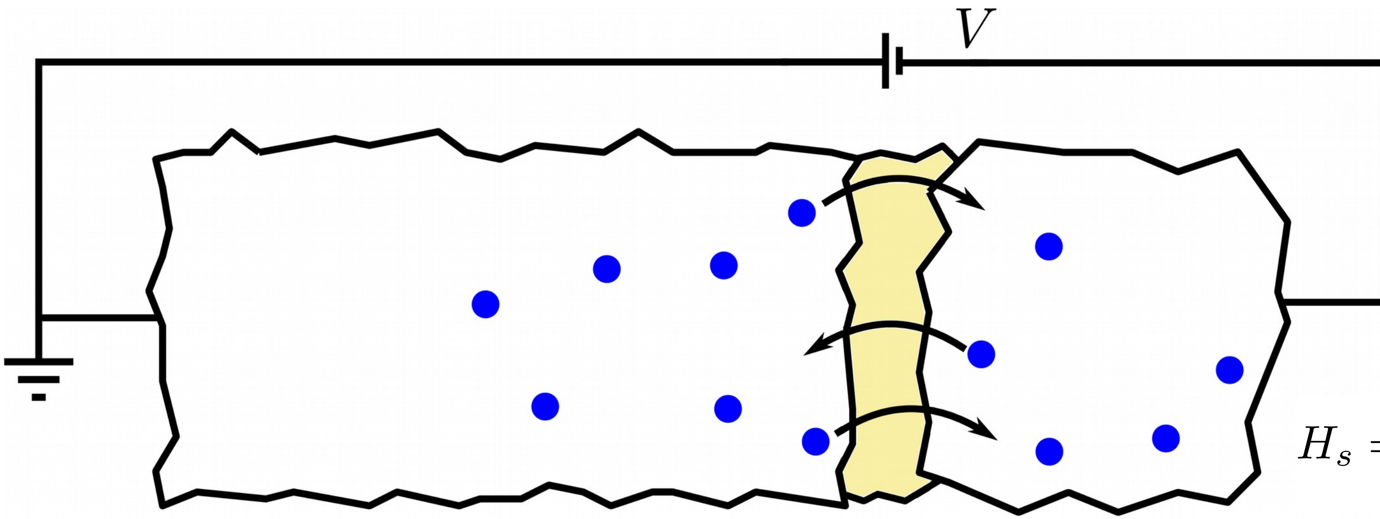




$$H_{lead} = \sum_{\gamma} \epsilon_\gamma \, c_\gamma^\dagger c_\gamma$$

$$H_{tunn} = \sum_{\alpha,\gamma} t_{\alpha,\gamma} \, a_\alpha^\dagger c_\gamma + h.c.$$

$$H_s = \sum_{\alpha} \epsilon_\alpha \, a_\alpha^\dagger a_\alpha + \frac{1}{2} E_C \, (N - N_0)^2$$

$$i\mathcal{S}_{tunn} = \mathrm{tr}\,\ln\left[i \left(G_s^{-1} + V(t) - \Sigma \right) \right]$$

U(1) gauge-trafo: $U = \mathrm{e}^{i\psi}$ with $\dot{\psi} = V(t)$

$$i\mathcal{S}_{tunn} = \mathrm{tr}\,\ln\left[i \left(G_s^{-1} - U^\dagger \Sigma U \right) \right]$$

Expansion in $\Sigma = T G_{lead} T^\dagger$

$$i\mathcal{S}_{\mathrm{AES}} = - \oint_K dt \oint_K dt' \, \mathrm{tr}\left[G_s(t - t')\, U^\dagger(t')\, \Sigma(t' - t)\, U(t) \right]$$

AES-action for U(1)-phase [1]

$$i\mathcal{S}_{\text{AES}} = -\oint_K dt \oint_K dt' \,\text{tr}\left[G_s(t-t')\,U^\dagger(t')\,\Sigma(t'-t)\,U(t)\right]$$

$$= -g \oint_K dt \oint_K dt' \,\mathrm{e}^{-i\psi}\,\alpha(t-t')\,\mathrm{e}^{i\psi} = i\mathcal{S}^R_{\text{AES}} + i\mathcal{S}^K_{\text{AES}}$$

$i\mathcal{S}^R_{AES} \longrightarrow$ **dissipation**

EOM: Ohm's law

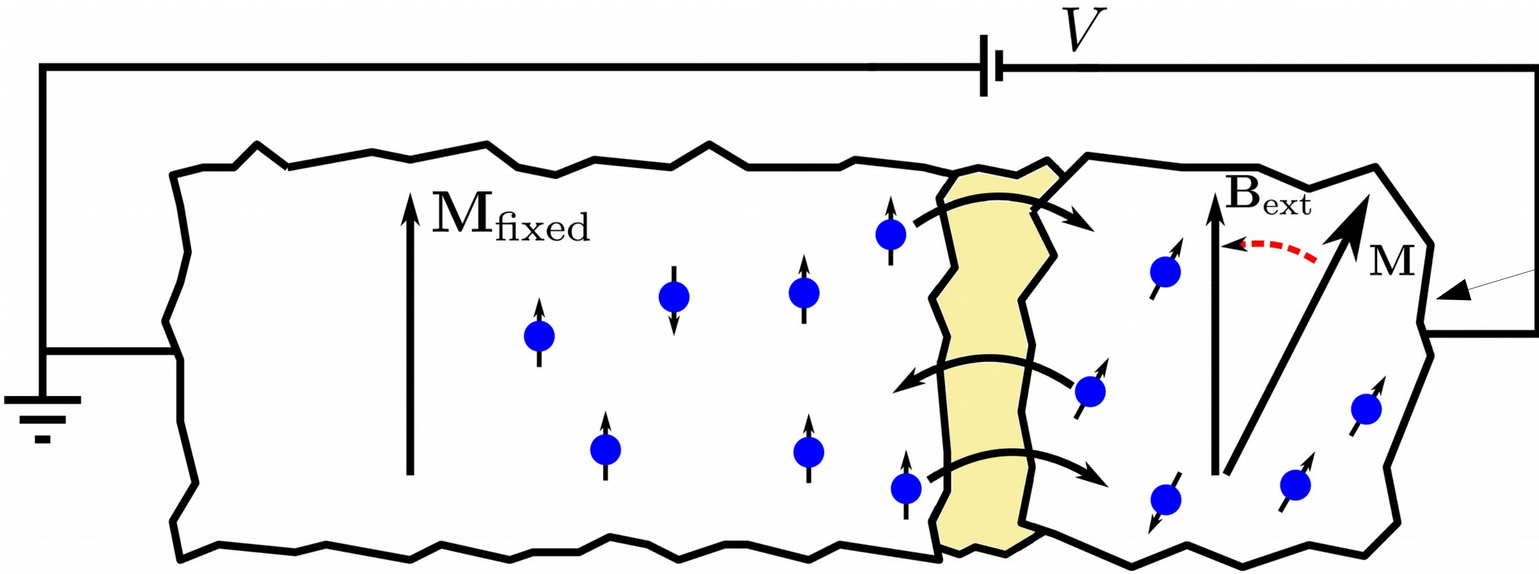
$$g\,\dot{\psi}_c = I \quad \longrightarrow \quad \psi_c = \frac{I}{g}\,t = V_0\,t$$

$i\mathcal{S}^K_{AES} \longrightarrow$ **fluctuation**

stochastic EOM

$$g\,\dot{\psi}_c = I + \delta I \quad \longrightarrow \quad \langle \delta I \delta I \rangle_{\omega=0} = \frac{g}{2}\,\alpha_K(V_0) \xrightarrow{T \ll V_0} g|V_0|$$

[1] V. Ambegaokar et al. , Phys. Rev. Lett. 48, 1745 (1982)



single domain Stoner-magnet

$J\rho_s > 1$

$|\mathbf{M}| \approx M_0$

$$i\mathcal{S}_M = \text{tr}\ln\left[i\left(G_s^{-1} + V + \mathbf{M}\frac{\boldsymbol{\sigma}}{2} - \Sigma\right)\right] - i\oint_K dt\,\frac{|\vec{\mathbf{M}} - \vec{\mathbf{B}}|^2}{4J}$$

U(1) gauge-trafo: electrical potential

SU(2) gauge-trafo: direction of the magnetization

$\mathrm{U} = \mathrm{e}^{i\psi} R$

$\mathrm{Q} = \mathrm{R}^\dagger(-i\partial_t)R$

$$i\mathcal{S}_M = \text{tr}\ln\left[i\left(G_s^{-1} + M_0\frac{\sigma_z}{2} - Q - U^\dagger\Sigma U\right)\right] - i\oint_K dt\,\frac{|\vec{\mathbf{M}} - \vec{\mathbf{B}}|^2}{4J}$$

SU(2) gauge transformation $\vec{n}\cdot\vec{\sigma} = R\sigma_z R^\dagger$

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Euler angle representation:

$$R = \mathrm{e}^{-i\frac{\phi}{2}\sigma_z}\,\mathrm{e}^{-i\frac{\theta}{2}\sigma_y}\mathrm{e}^{i\frac{(\phi-\chi)}{2}\sigma_z} \longrightarrow \textbf{gauge freedom } \chi$$

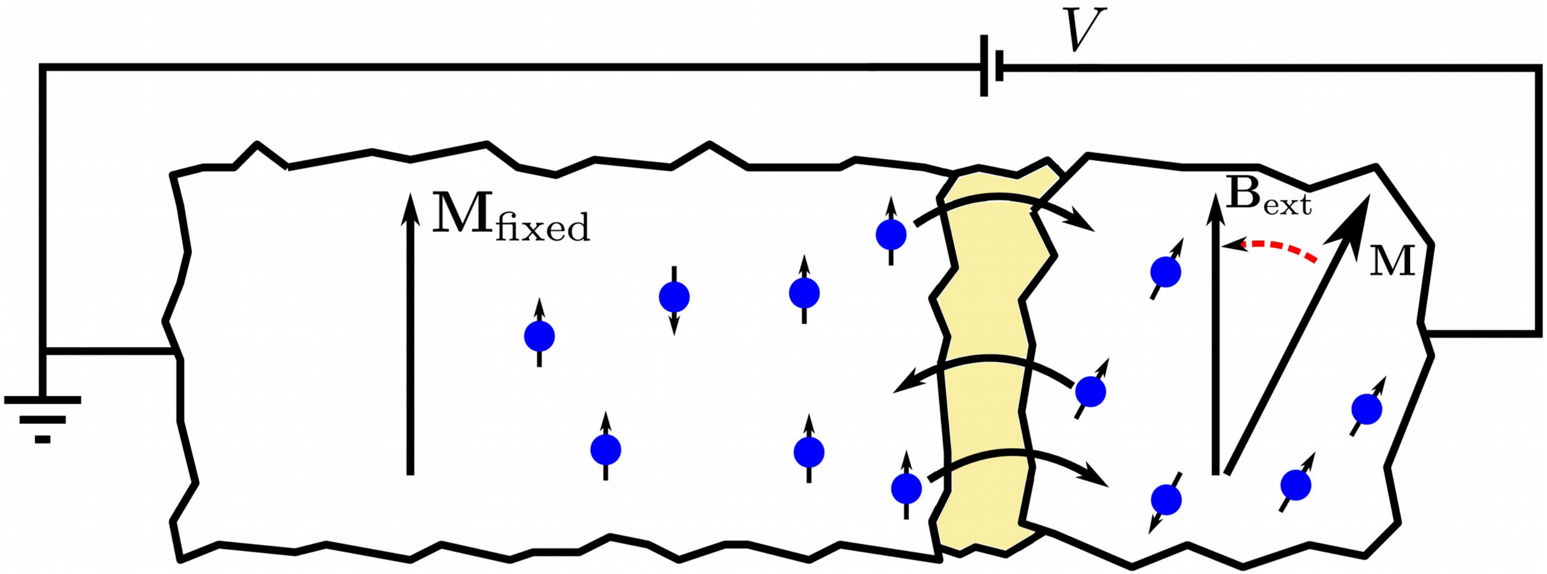
$$Q \equiv R^\dagger(-i\partial_t)R$$

$$= \underbrace{\frac{1}{2}\left(\dot{\phi}(1-\cos\theta)-\dot{\chi}\right)\sigma_z}_{\text{Berry phase } Q_{||}} \underbrace{-\frac{1}{2}\mathrm{e}^{i\chi\sigma_z}\left[\dot{\theta}\,\sigma_y - \dot{\phi}\sin\theta\,\sigma_x\right]\mathrm{e}^{i\phi\sigma_z}}_{\text{Landau–Zener transition } Q_\perp}$$

adiabatic approx. ⟶ only keep $Q_{||}$

gauge fixing [2]: $\dot{\chi}_c = \dot{\phi}_c(1-\cos\theta_c)$ $\chi_q = \phi_q(1-\cos\theta_c)$

[2] A. Shnirman et al. , Phys. Rev. Lett. 114, 176806 (2015)



$$i\mathcal{S}_M = \text{tr}\ln\left[-i\left(G^{-1}_{s,\sigma_z} - Q - U^\dagger \Sigma U\right)\right] \underbrace{-i\oint dt\,\frac{|\vec{\mathbf{M}}-\vec{\mathbf{B}}|^2}{4J}}_{i\mathcal{S}_B}$$

$$\text{first order in } Q \longrightarrow i\mathcal{S}_{WZNW} = -\text{tr}\left[G_{s,\sigma_z} Q\right] = iS\oint \dot{\phi}\,(1-\cos\theta)$$

$$\text{first order in } \Sigma \longrightarrow i\mathcal{S}_{AES} = -\text{tr}\left[G_{s,\sigma_z} U^\dagger \Sigma U\right] = i\mathcal{S}^R_{AES} + i\mathcal{S}^K_{AES}$$

$i\mathcal{S}^R_{AES} \longrightarrow$ **dissipation** $\qquad$ $i\mathcal{S}^K_{AES} \longrightarrow$ **fluctuation**

$U(1)\otimes SU(2)$ – AES – like action :

$$i\mathcal{S}_{AES} = -\oint \mathrm{d}t \oint \mathrm{d}t' \ \mathrm{tr}\left[G_{s,\sigma_z}(t-t')U^\dagger(t')\,\Sigma\,(t'-t)U(t)\right]$$

$$U = A_{\uparrow\uparrow}\sigma_\uparrow + A_{\downarrow\downarrow}\sigma_\downarrow + A_{\downarrow\uparrow}\sigma_+ + A_{\uparrow\downarrow}\sigma_-$$

resembles 4 times the U(1)-AES-problem

$$A_{\uparrow\uparrow} = \cos\frac{\theta}{2}\, e^{i(-\frac{\chi}{2}+\psi)} \qquad A_{\downarrow\uparrow} = -\sin\frac{\theta}{2}\, e^{i(-\phi+\frac{\chi}{2}+\psi)}$$

$$A_{\downarrow\downarrow} = \cos\frac{\theta}{2}\, e^{i(\frac{\chi}{2}+\psi)} \qquad A_{\uparrow\downarrow} = \sin\frac{\theta}{2}\, e^{i(\phi-\frac{\chi}{2}+\psi)}$$

Equations of motion

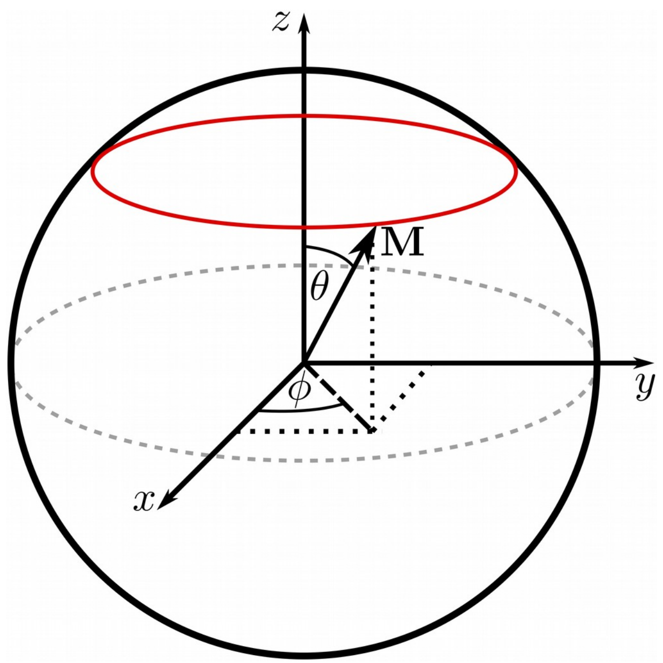
LLG equation + spin torque

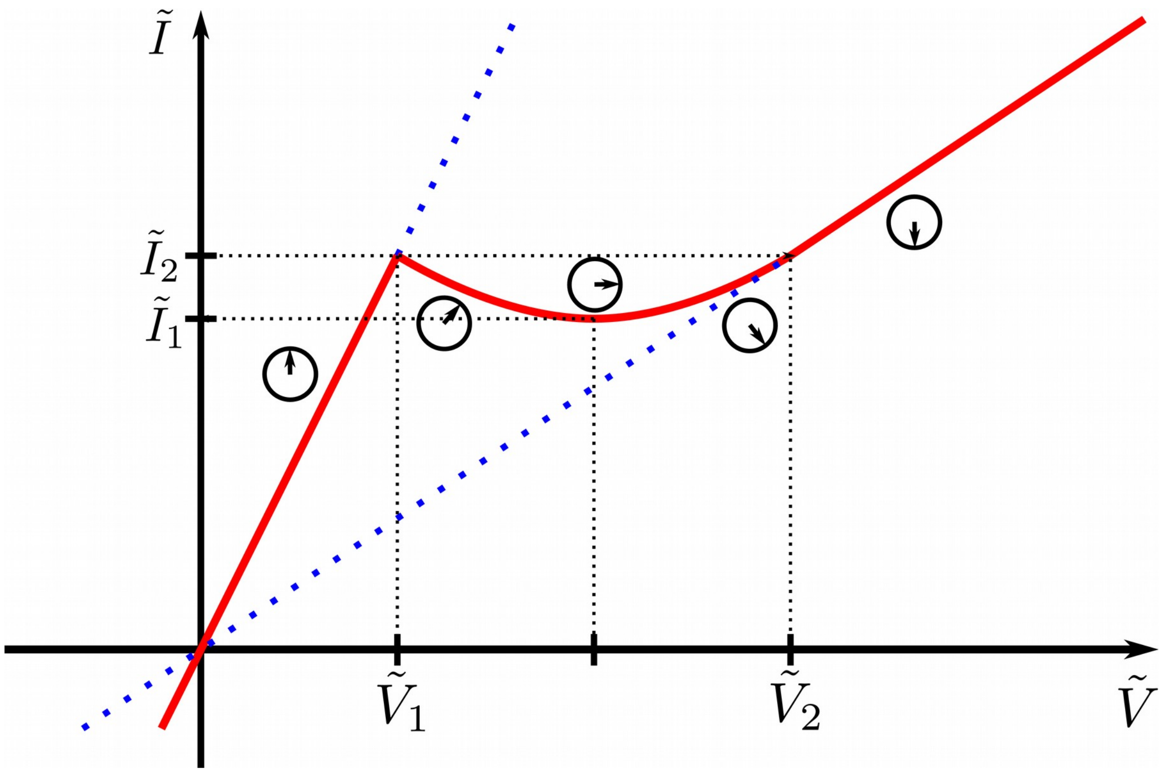
$$\dot\theta = -\frac{\sin\theta}{S}\left[(\tilde g_\theta + g_0)\dot\phi - g_s V\right]$$

$$\dot\phi = B + \frac{\tilde g_\theta + g_0}{S}\,\frac{\dot\theta}{\sin\theta}$$

Ohm's law + pumping current

$$g_\theta V = I + g_s \sin^2\theta\,\dot\phi$$





Ohm's law with pumping current and noise

$$g_\theta V - g_s \sin^2\theta\dot{\phi} = I + \delta I$$

noise correlation close to the **stationary solutions** at **zero frequency**

$$\langle \delta I \delta I \rangle = \frac{1}{2}\cos^2\frac{\theta_0}{2}\left\{g_{\uparrow\uparrow}\,\alpha^{\mathrm{K}}_{\mathrm{AES}}(V_0 - B_-) + g_{\downarrow\downarrow}\,\alpha^{\mathrm{K}}_{\mathrm{AES}}(V_0 + B_-)\right\} + \frac{1}{2}\sin^2\frac{\theta_0}{2}\left\{g_{\downarrow\uparrow}\,\alpha^{\mathrm{K}}_{\mathrm{AES}}(V_0 - B_+) + g_{\uparrow\downarrow}\,\alpha^{\mathrm{K}}_{\mathrm{AES}}(V_0 + B_+)\right\}$$

the **geometric phase** enters through

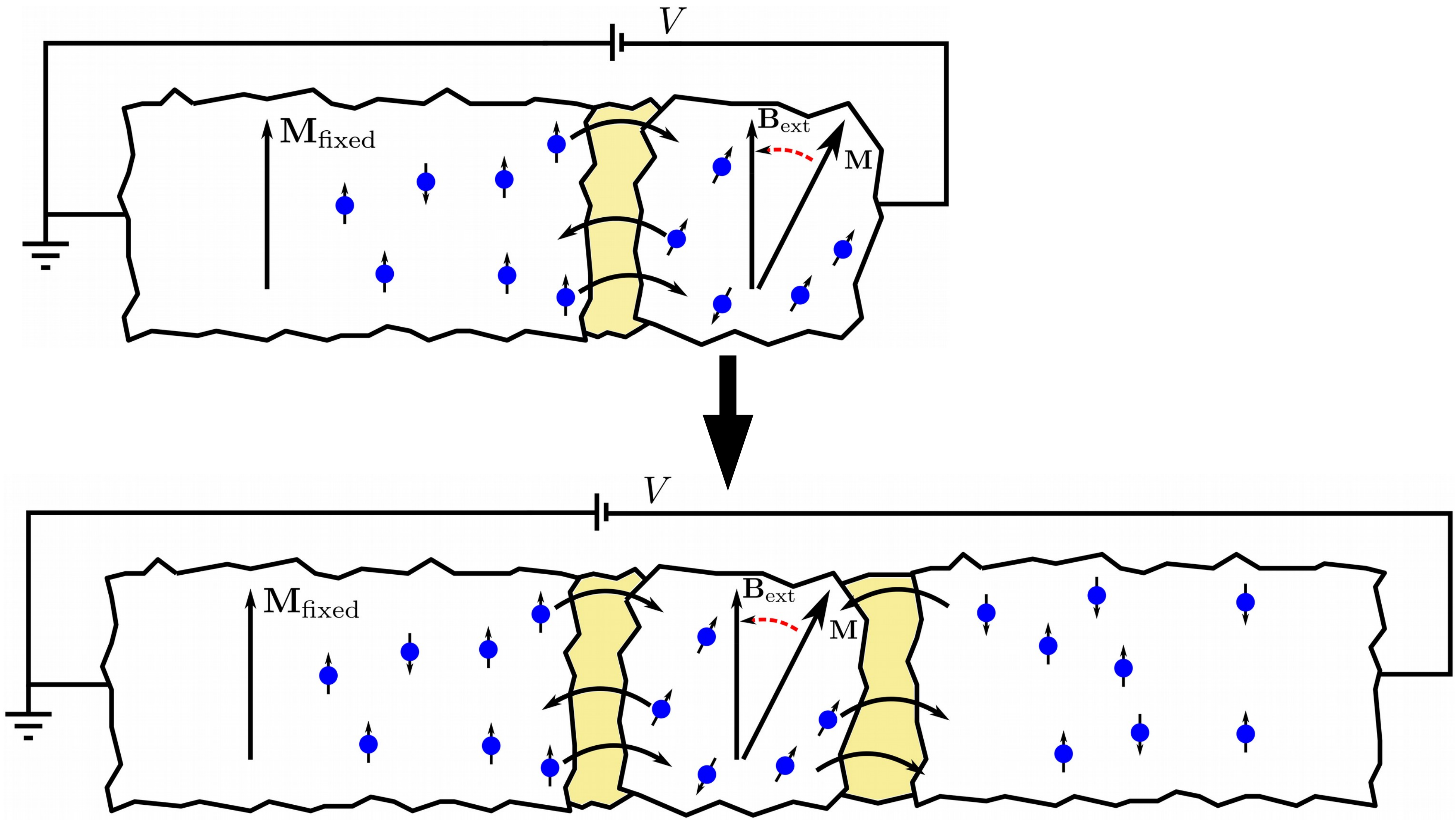
$$B_+ = \dot{\phi}_c - \dot{\phi}_c(1-\cos\theta_c)/2 \qquad B_- = \dot{\phi}_c(1-\cos\theta_c)/2$$

thermal noise (high T): $\langle \delta I \delta I \rangle = 2 g_\theta T$

shot noise (low T): $\langle \delta I \delta I \rangle = I$ → **Fano factor = 1**

geometric noise terms add up to the **pumping current** contribution

Outlook



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