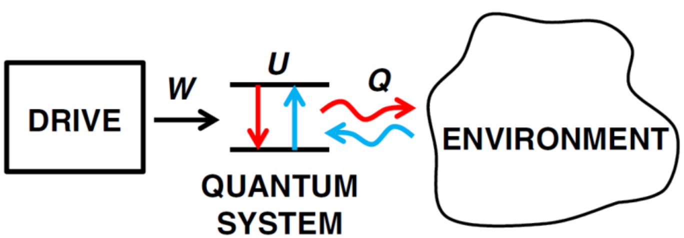
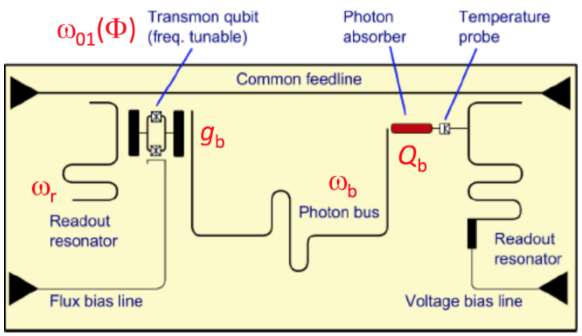
LECTURE 3, Jukka Pekola

Quantum fluctuation relations, calorimetry, quantum trajectories

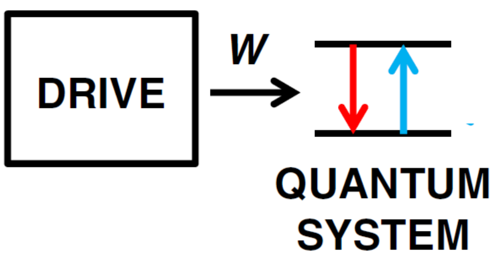


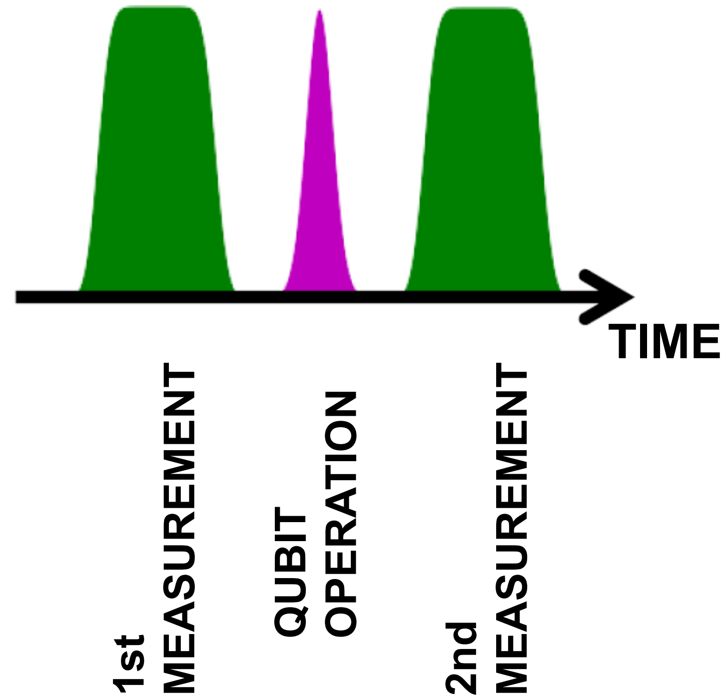


Work measurement in a quantum system

Two-measurement protocol (TMP):

$W = E_f - E_i$





J. Kurchan 2000

Work on a driven quantum system

The simple case of a closed system:

Specifically:

1. System is in thermal equilibrium initially
2. **During the drive**, it obeys unitary evolution, i.e. **no coupling to environment**

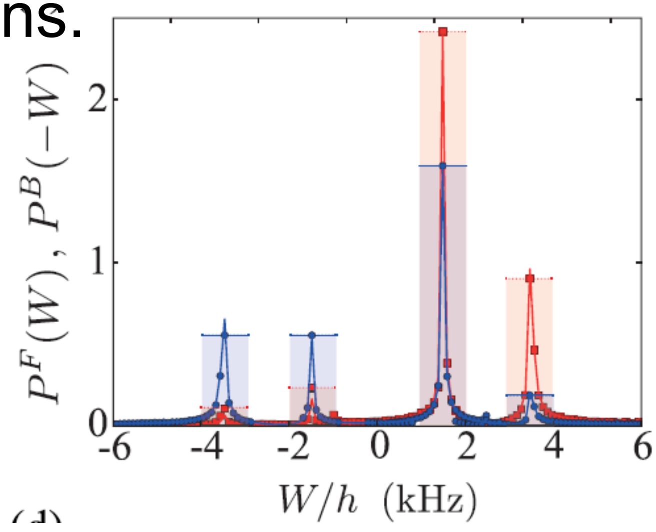
Then the ”two measurement protocol” yields

$$p(w) = \sum_{n,m} \delta(w - [e_m(t_f) - e_n(0)]) p(m, t_f | n) p_n$$

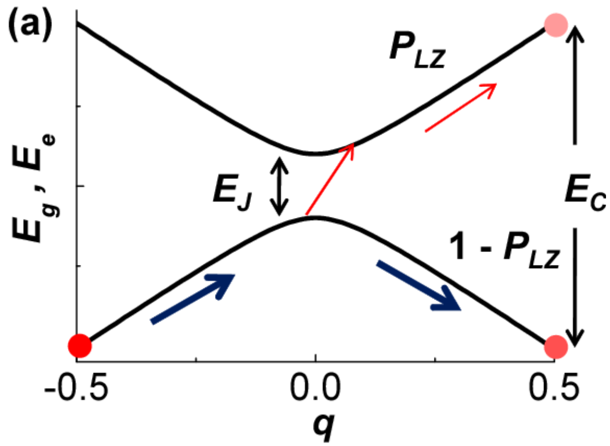
Kurchan 2000
Campisi, Hänggi, Talkner 2011

which satisfies the work fluctuation relations.

NMR measurement:
T. Batalhão et al., PRL 113, 140601 (2014).
Trapped ions:
S. An et al., Nature Phys. 11, 193 (2014).



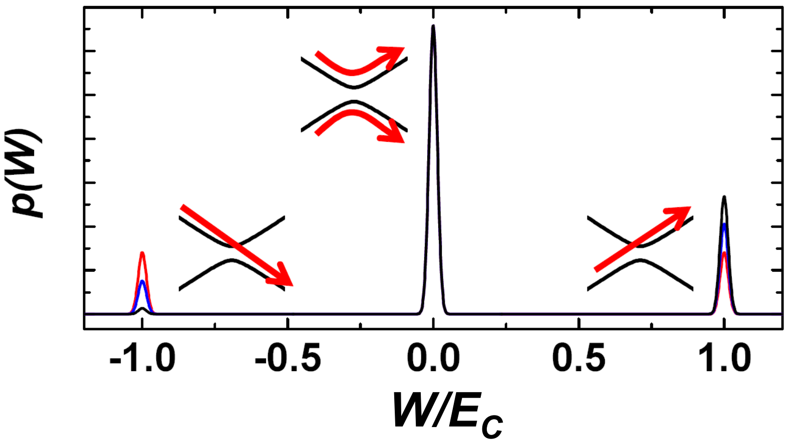
Example: Landau-Zener problem



P_{LZ} = probability of Landau-Zener transition

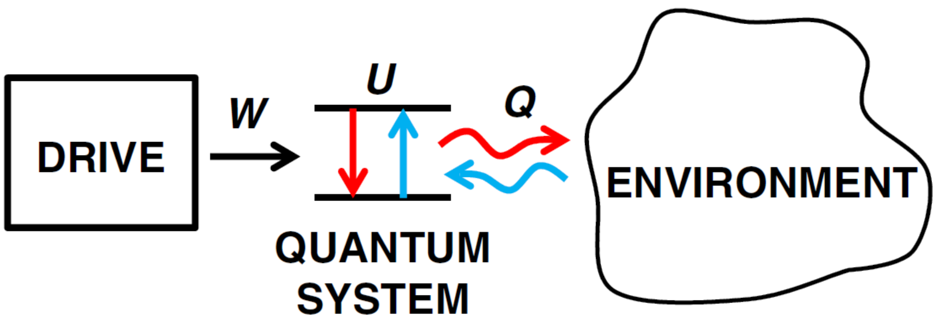
$$\rho^0_{gg} = (1 + e^{-\beta E_C})^{-1}$$

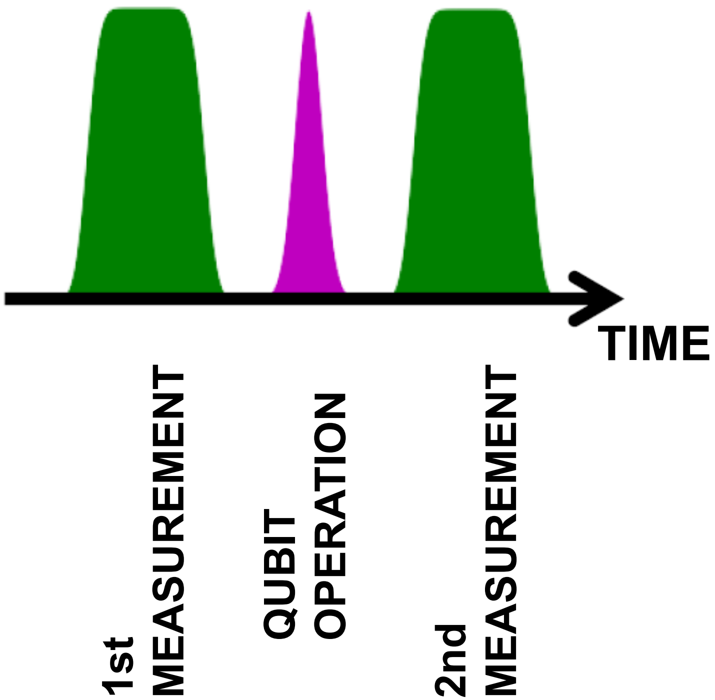
$$p(W) = (1 - \rho^0_{gg}) P_{LZ}\, \delta(W + E_C) + (1 - P_{LZ})\, \delta(W) + \rho^0_{gg} P_{LZ}\, \delta(W - E_C)$$



$$\langle e^{-\beta W} \rangle = \int_{-\infty}^{\infty} dW\, p(W) e^{-\beta W} = 1$$

TMP in a qubit coupled to environment





If the interval between the qubit operation and the second measurement is long, one has for any driving protocol

$$\langle e^{-\beta U} \rangle = 1 + \tanh^2(\beta \hbar \omega_0 / 2)$$

Work and heat in an open quantum system

Power operator $P(t) = \dot{H}(t)$ and work $W = \int_0^\tau P(t)dt$

$$\langle W \rangle = \langle H(\tau) \rangle - \langle H(0) \rangle - \int_0^\tau \mathrm{Tr}\big[\dot{\rho}(t)H(t)\big]dt$$

$$\equiv \langle U \rangle + \langle Q \rangle$$

P. Solinas et al., PRB 87, 060508 (2013)

<Work> = <Change of internal energy> + <Dissipated heat>

Heat to the bath expressed in the eigenbasis of a two-level system

$$\langle Q \rangle = \int_0^\tau dt \dot{\rho}_{gg}(t) \Delta E(t)$$

Difficult to proceed beyond the average work in general

M. Esposito et al., RMP 81, 1665 (2009), M. Campisi et al., RMP 83, 711 (2011),
S. Suomela et al., PRB 90, 094304 (2014)

Quantum jump method

Objective: **unravel** the system evolution **into single realizations** (”single experiments”) instead of averages (the latter ones come naturally from the density matrix)

Construct the Monte Carlo wave function (MCWF) for the system Dalibard, Castin and Mölmer, PRL 1992

$$|\psi(t)\rangle = a(t)|g\rangle + b(t)|e\rangle$$

At $t = t + \Delta t$, we have three possibilities:

1. Relaxation $|\psi(t+\Delta t)\rangle_\downarrow = |g\rangle$ with probability $\Gamma_\downarrow |b(t)|^2 \Delta t$

2. Excitation $|\psi(t+\Delta t)\rangle_\uparrow = |e\rangle$ with probability $\Gamma_\uparrow |a(t)|^2 \Delta t$

3. Evolution without photon absorption/emission

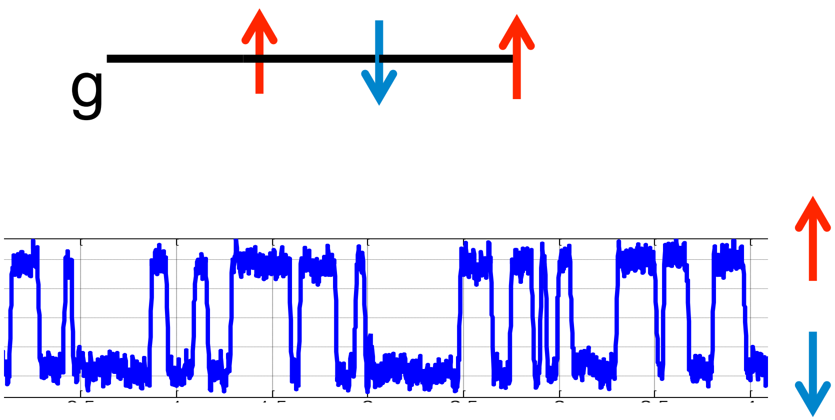
$$|\psi^{(0)}(t+\Delta t)\rangle = [1 - \frac{i}{\hbar} H \Delta t]|\psi(t)\rangle$$

Here the hamiltonian is non-hermitian (to preserve the norm)

$$H = H_S - i\hbar\Gamma_\downarrow |e\rangle\langle e|/2 - i\hbar\Gamma_\uparrow |g\rangle\langle g|/2$$

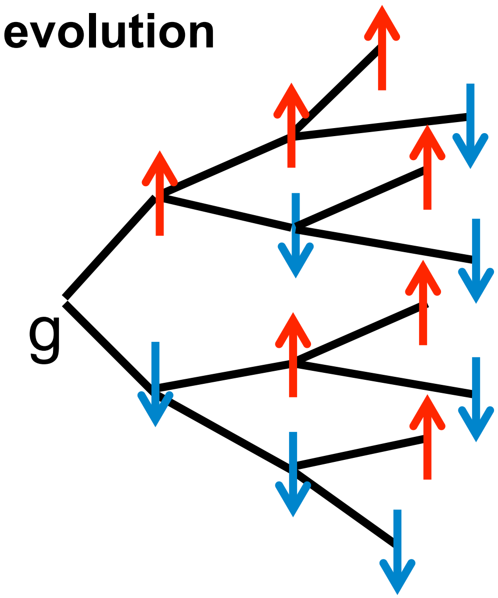
Evolution of a classical vs quantum two-level system

Classical evolution



TIME

Quantum evolution



Between relaxation/excitation events the MCWF evolves as

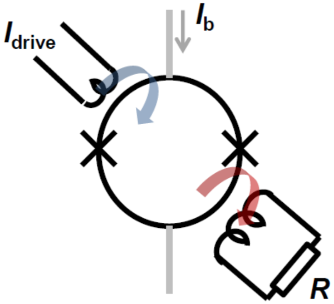
$$i\hbar\dot{a} = \lambda(t)b + i\hbar\Delta\Gamma|b(t)|^2 a(t)/2,$$

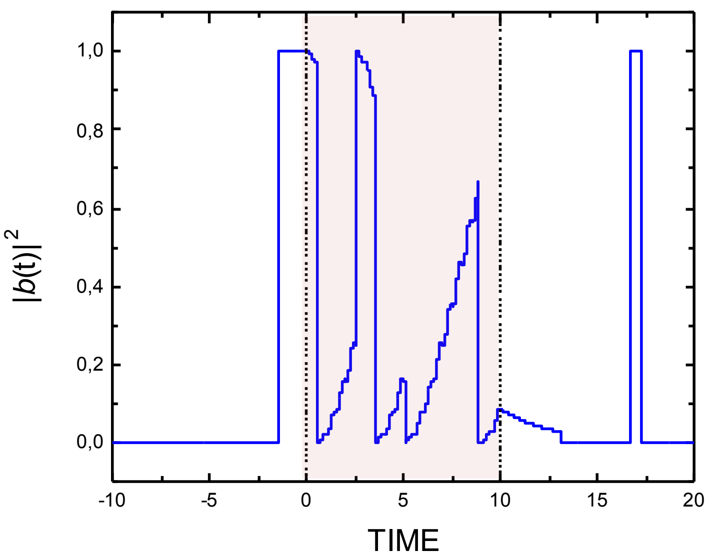
$$i\hbar\dot{b} = \lambda(t)a - i\hbar\Delta\Gamma|a(t)|^2 b(t)/2$$

$$\Delta\Gamma = \Gamma_\downarrow - \Gamma_\uparrow$$

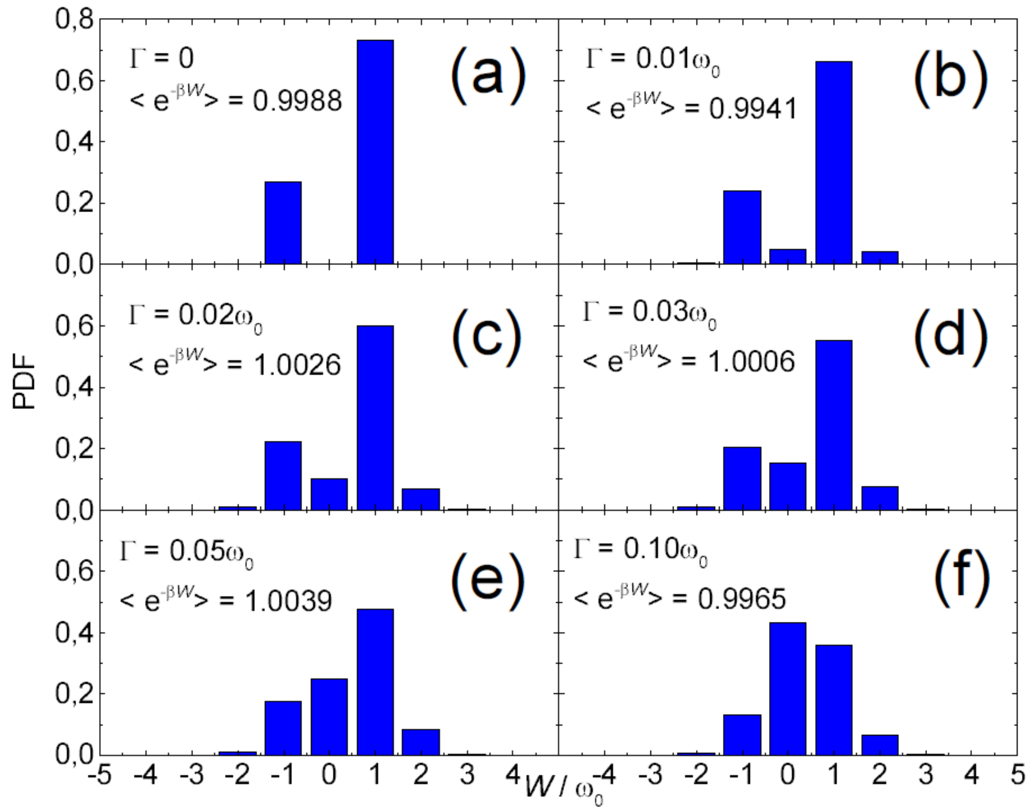
Quantum jump approach for analyzing distribution of dissipation

We apply the jump method to a driven qubit



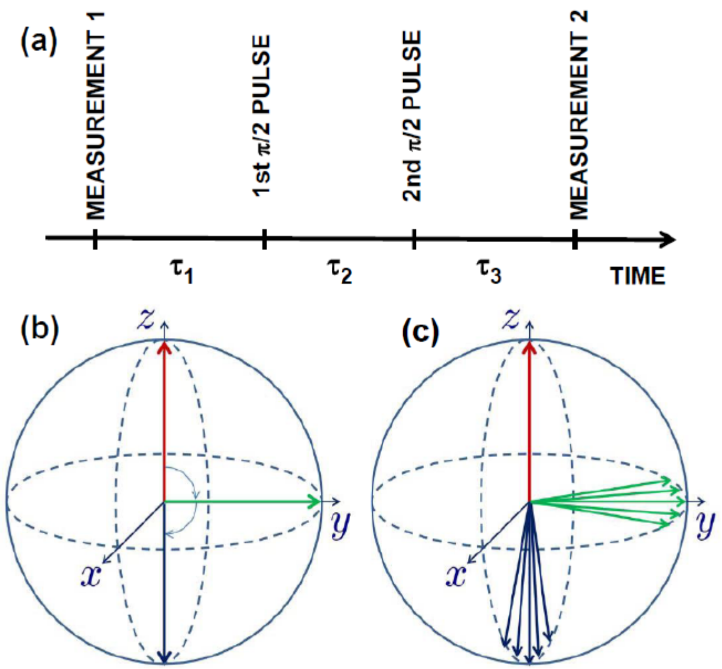


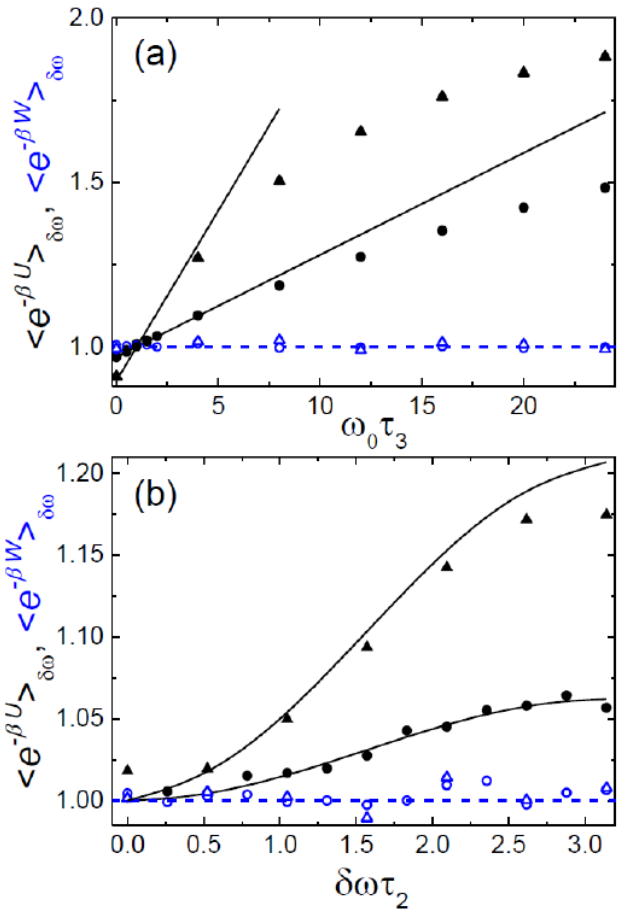
π pulse with dissipation



F. Hekking and JP, PRL 111, 093602 (2013).
Horowitz and Parrondo, NJP 15, 085028 (2013).

TMP in a qubit coupled to environment





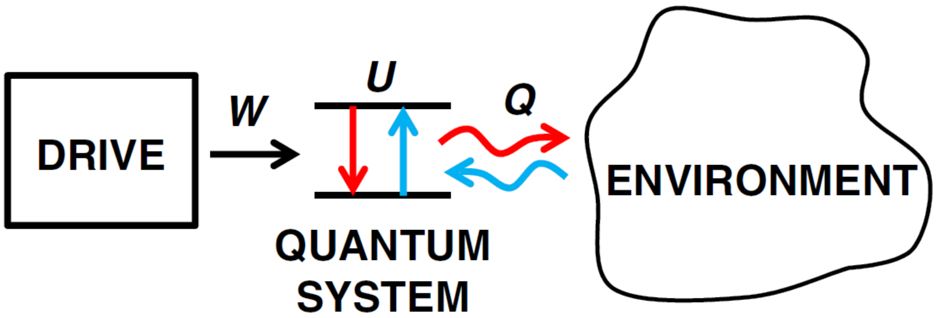
$$\langle e^{-\beta W} \rangle = 1$$

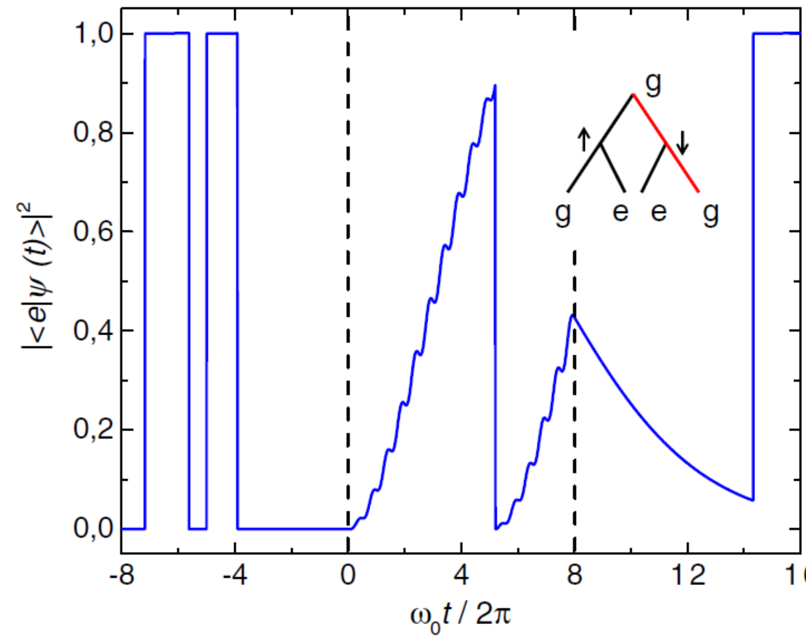
In weak dissipation regime

$$\langle e^{-\beta U} \rangle - 1 = [(\tau_3 - e^{-(\Gamma_\varphi \tau_2)^2} \tau_1] \, \Gamma_\Sigma \tanh^2(\beta \hbar \omega_0/2).$$

J. P. Pekola, Y. Masuyama, Y. Nakamura, J. Bergli, and Y. M. Galperin, PRE 91, 062109 (2015).
Measurement of *U* by Y. Masuyama et al., Tokyo Univ (unpublished) seems to confirm these predictions.

Measurement of the environment



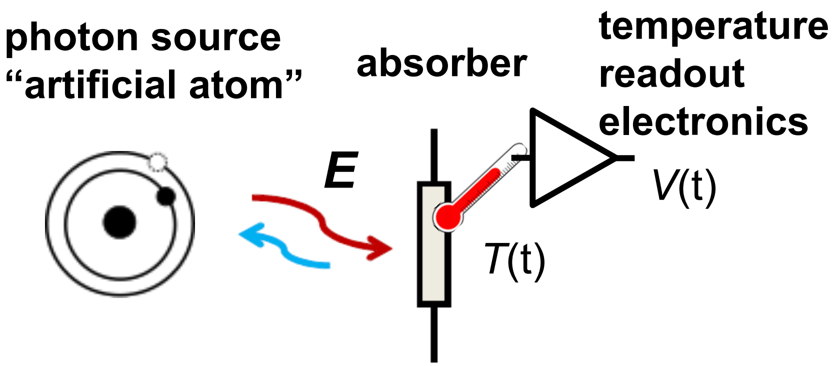


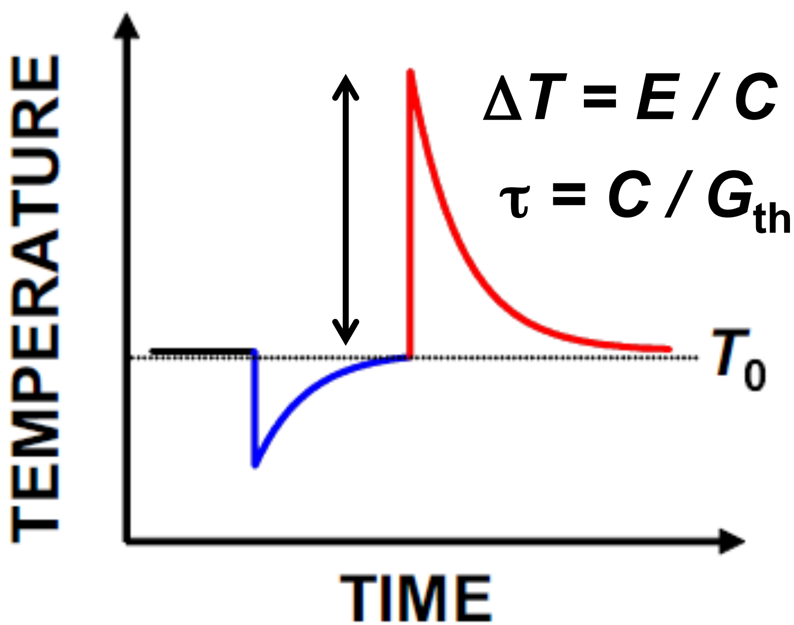
$$W = Q + U$$

In a two-level system the measurement of the environment is in principle perfect since it yields *Q* and ALSO *U* via the measurement of the "guardian photons".

Calorimetry for measuring the photons

Requirements for calorimetry on single microwave quantum level. Photons from relaxation of a superconducting qubit.





Typical parameters:

Operating temperature
$T = 0.1$ K

$E/k_B = 1$ K, $C = 300...1000 k_B$

$\Delta T \sim 1 - 3$ mK, $\tau \sim 0.01 - 1$ ms

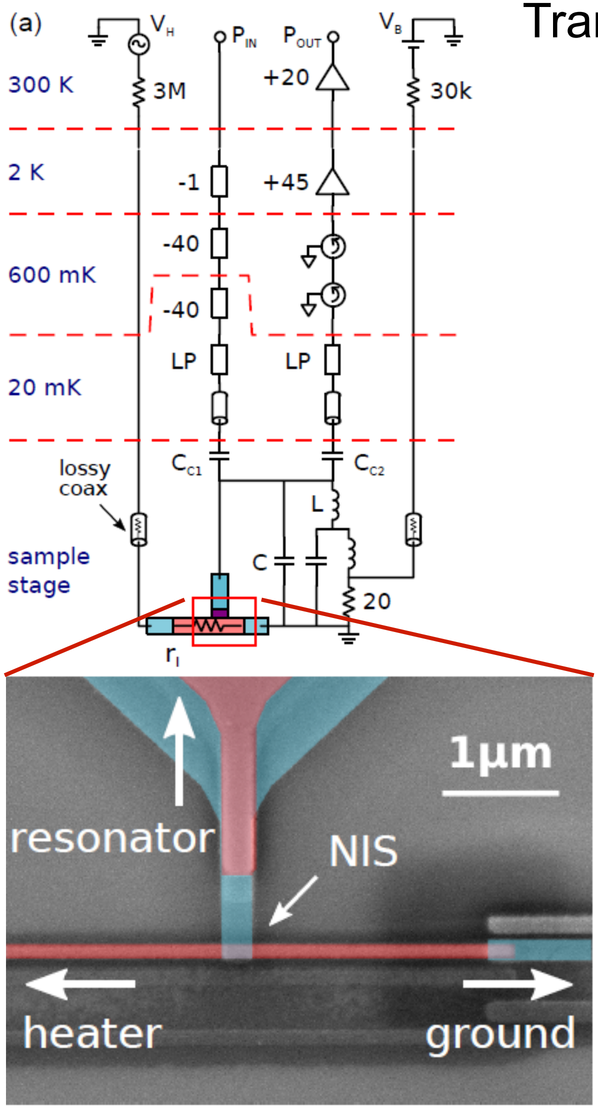
NET = 10 μK/(Hz)$^{1/2}$ is sufficient for single photon detection

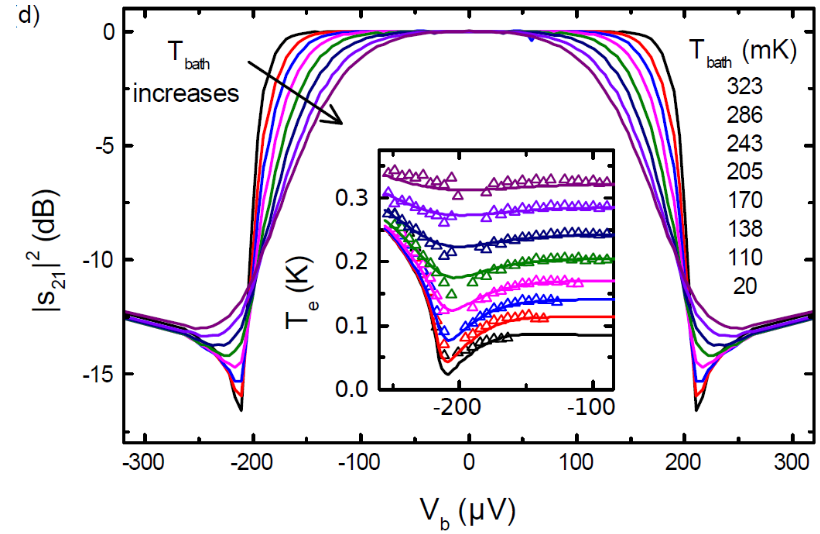
$\delta E = \mathrm{NET}\,(C\,G_{th})^{1/2}$

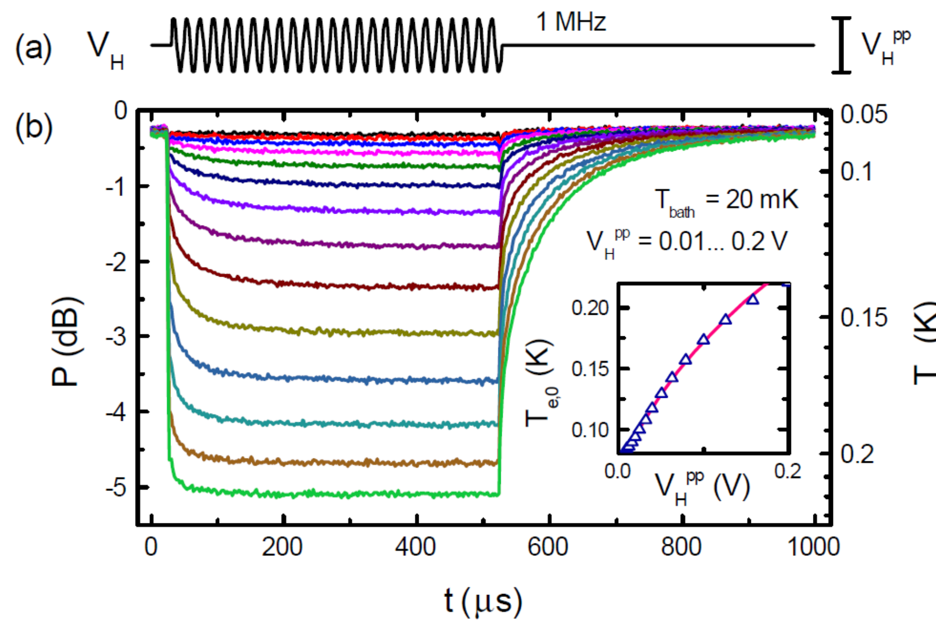
JP et al., NJP 15, 115006 (2013)

Fast NIS thermometry

Transmission read-out at 600 MHz of a NIS junction



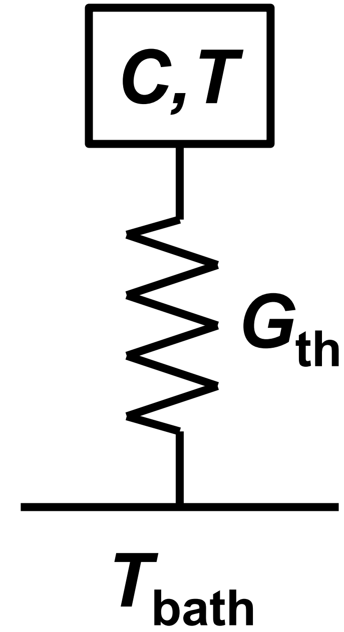




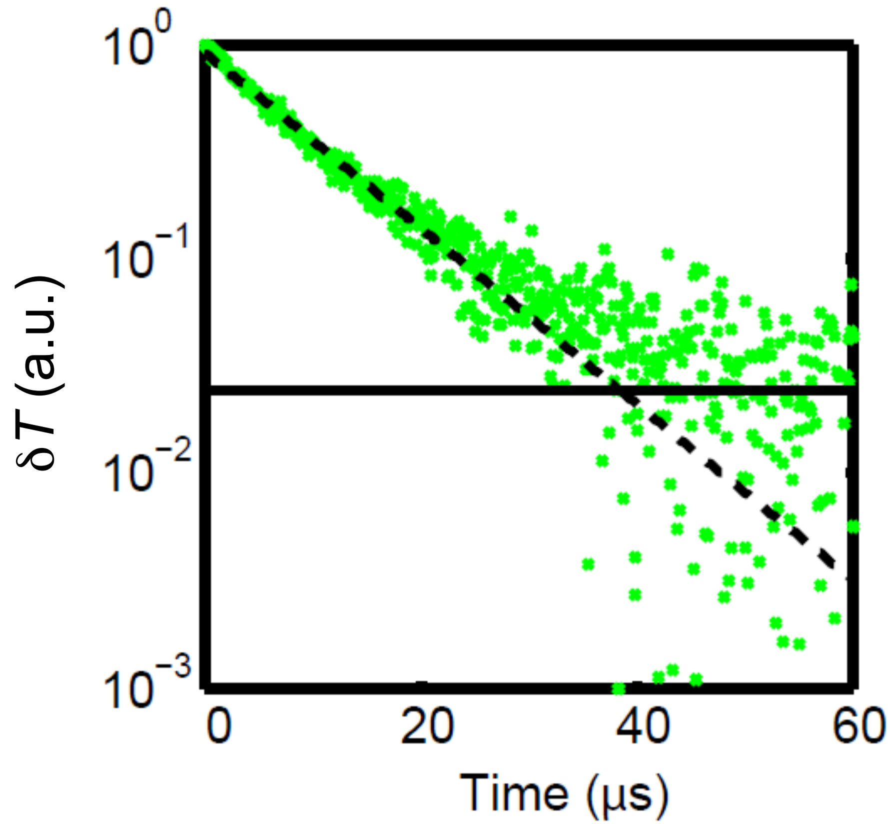
S. Gasparinetti et al., Phys. Rev. Applied 3, 014007 (2015)
(proof of the concept by Schmidt et al., 2003)

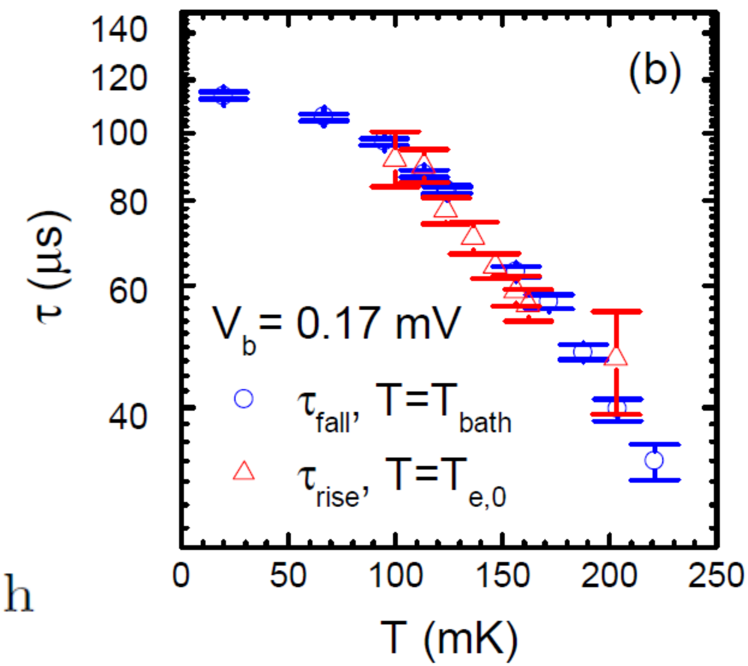
100 µK/(Hz)$^{1/2}$ noise level here

Fast thermometry



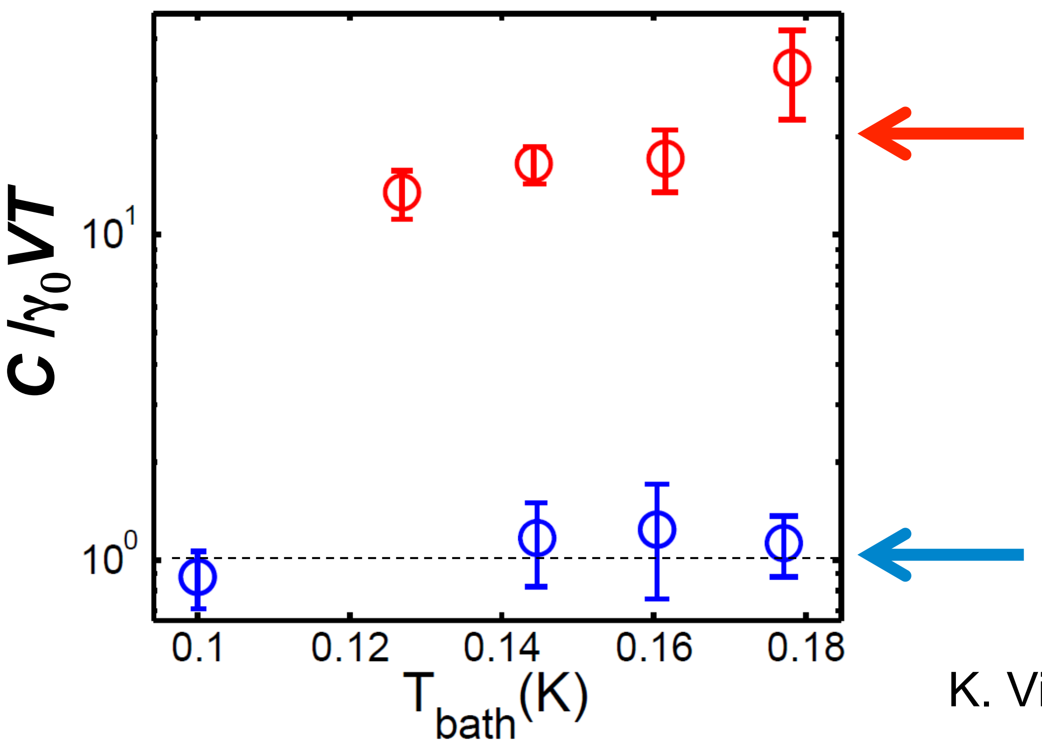
Time-resolved measurements





$$\mathcal{C}\frac{d\delta T}{dt} = -G_{\rm th}\delta T \qquad \tau = \mathcal{C}/G_{\rm th}$$

Heat capacities of metallic thin film absorbers



copper

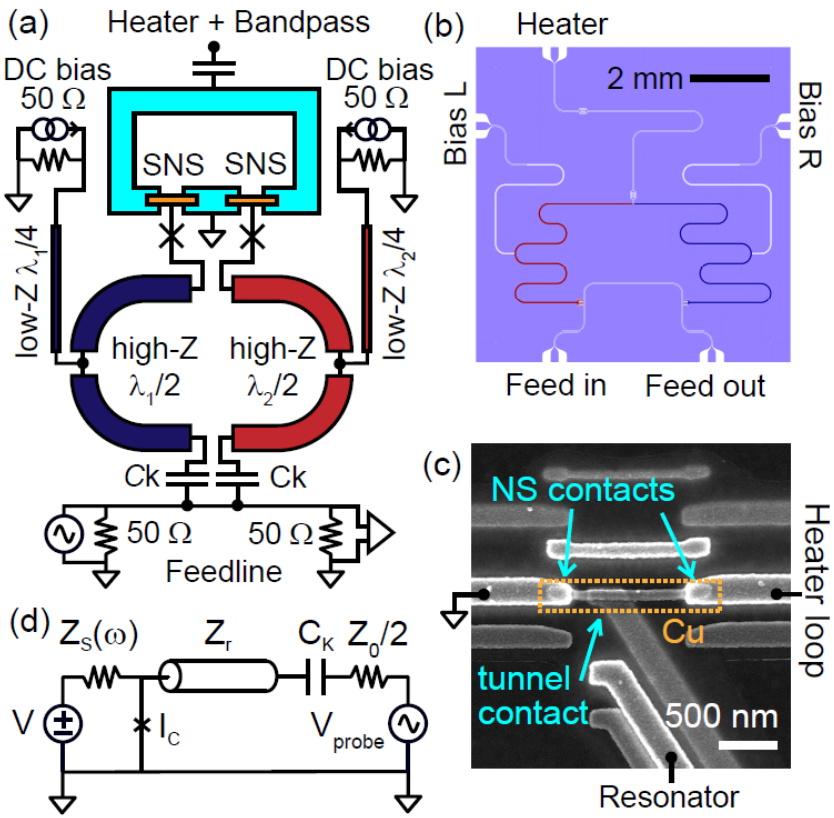
silver

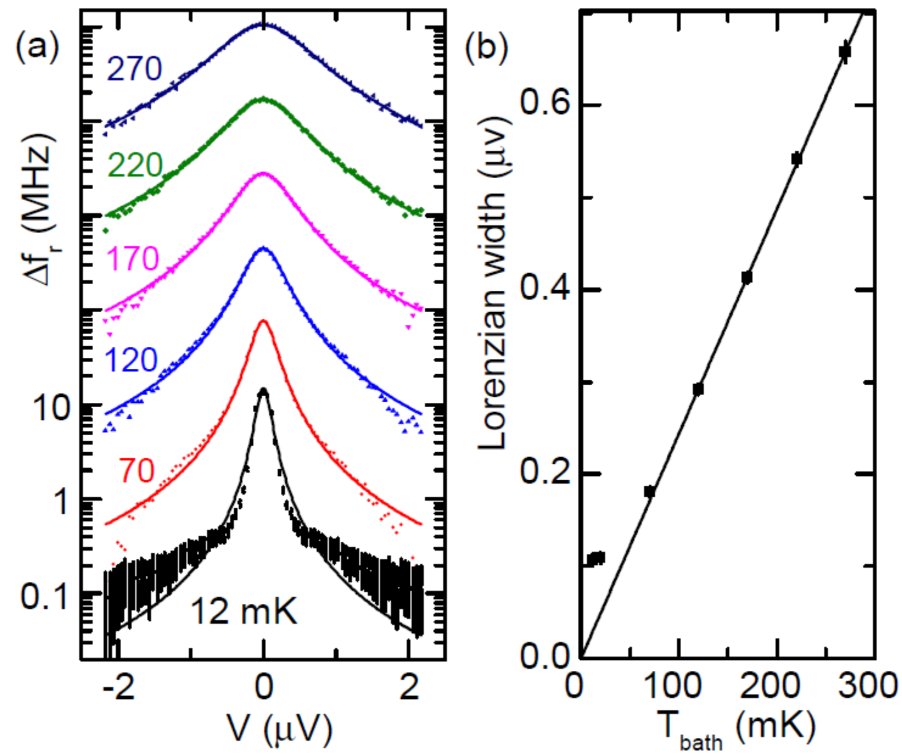
$\gamma_0 = (\pi^2/3)\, N(0) k_B^2$ is the free-electron value

K. Viisanen et al., in preparation (2016)

For V = 1 μm x 0.1 μm x 0.01 μm one obtains $C/k_B = 10^3$ for silver.

Josephson thermometer (at 5 GHz)





P(E) theory:

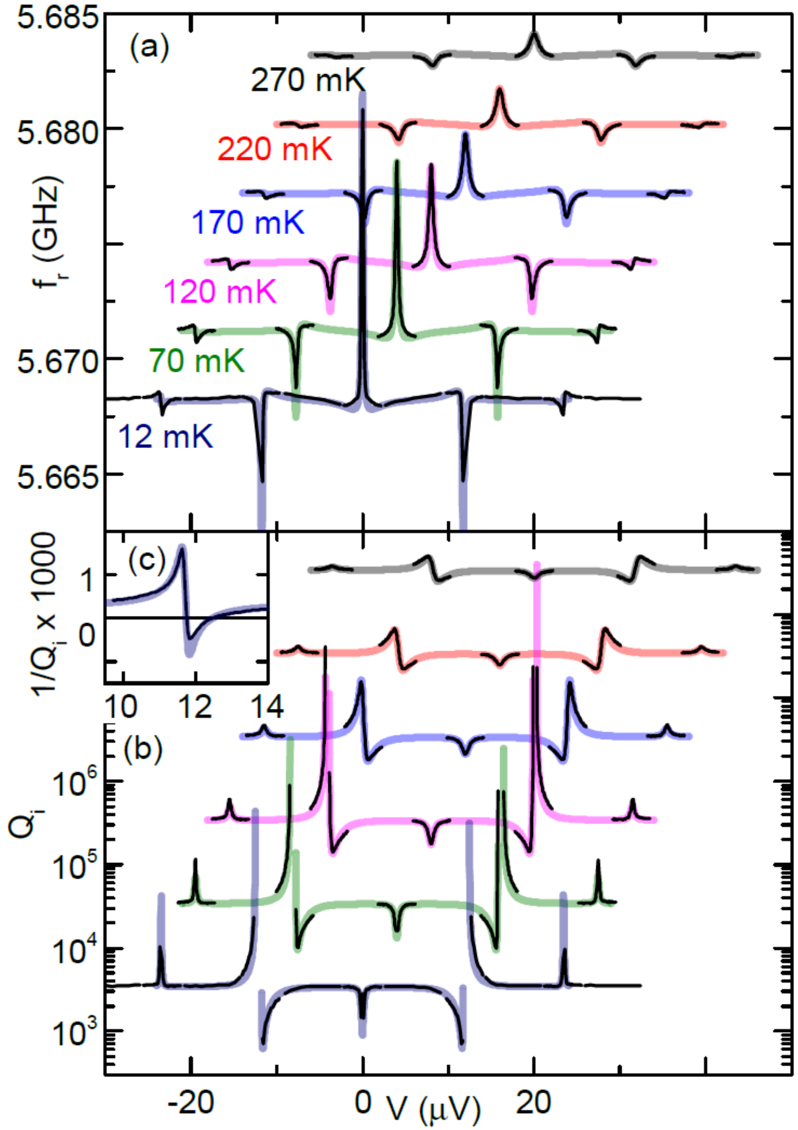
$$f = f_r + \Delta f_T \left(\frac{1}{1 + V^2/V_T^2} - \frac{1}{1 + \hbar\gamma_0/2eV_T} \right)$$

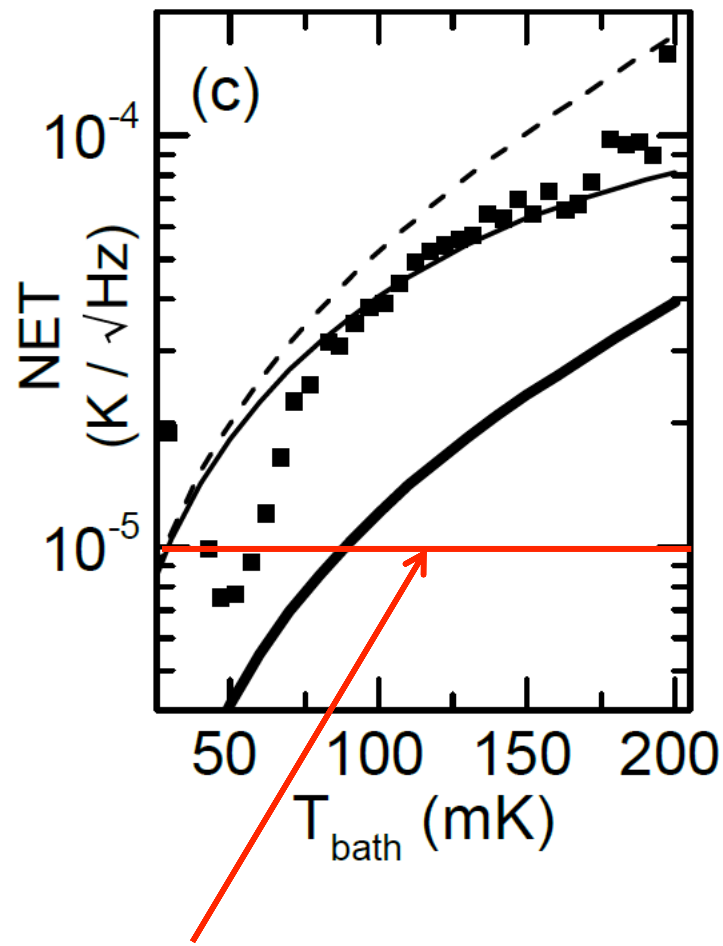
$$\Delta f_T = \frac{I_C^2 Z_r}{4\pi^2 k_B T} \left(2 \sinh \frac{\pi k_B T}{\hbar \gamma_0} \right)^\alpha, \quad \boxed{V_T = \frac{4\pi R_S k_B T}{e R_q}}$$

Only one fit parameter:
R_S = 57.4 Ω.

O.-P. Saira, M. Zgirski, D. Golubev, K. Viisanen and JP, in preparation (2016)

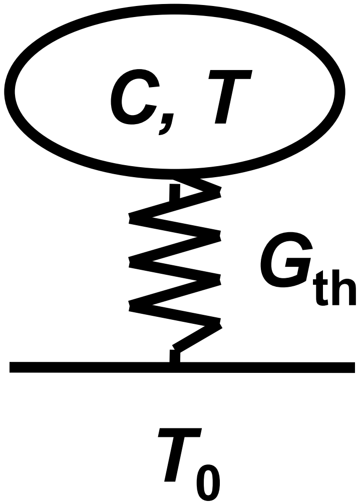
Josephson thermometer (at 5 GHz)

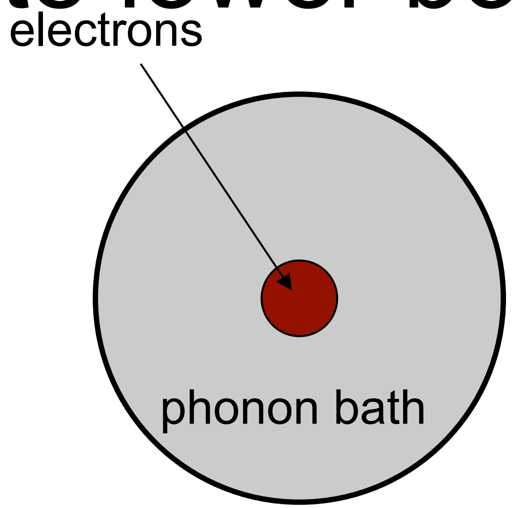


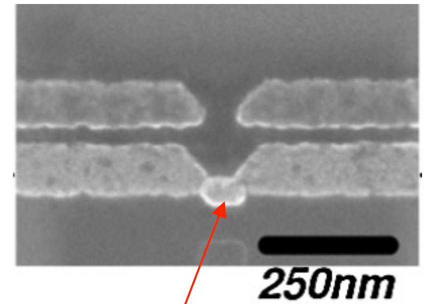


Expected 1 K photon resolution

Classical temperature fluctuations – ultimate lower bound for NET



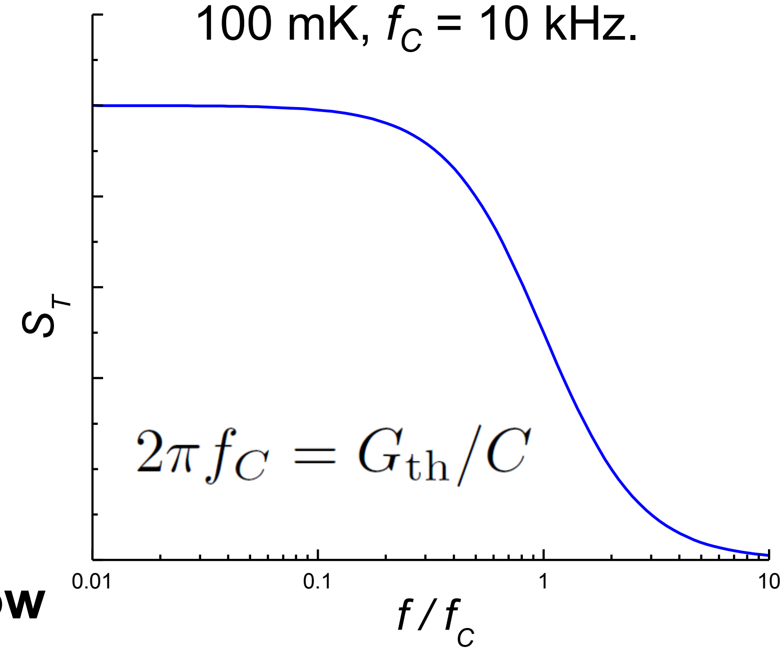




In this grain, $\langle \Delta T^2 \rangle$ is expected to be several mK at 100 mK, f_C = 10 kHz.

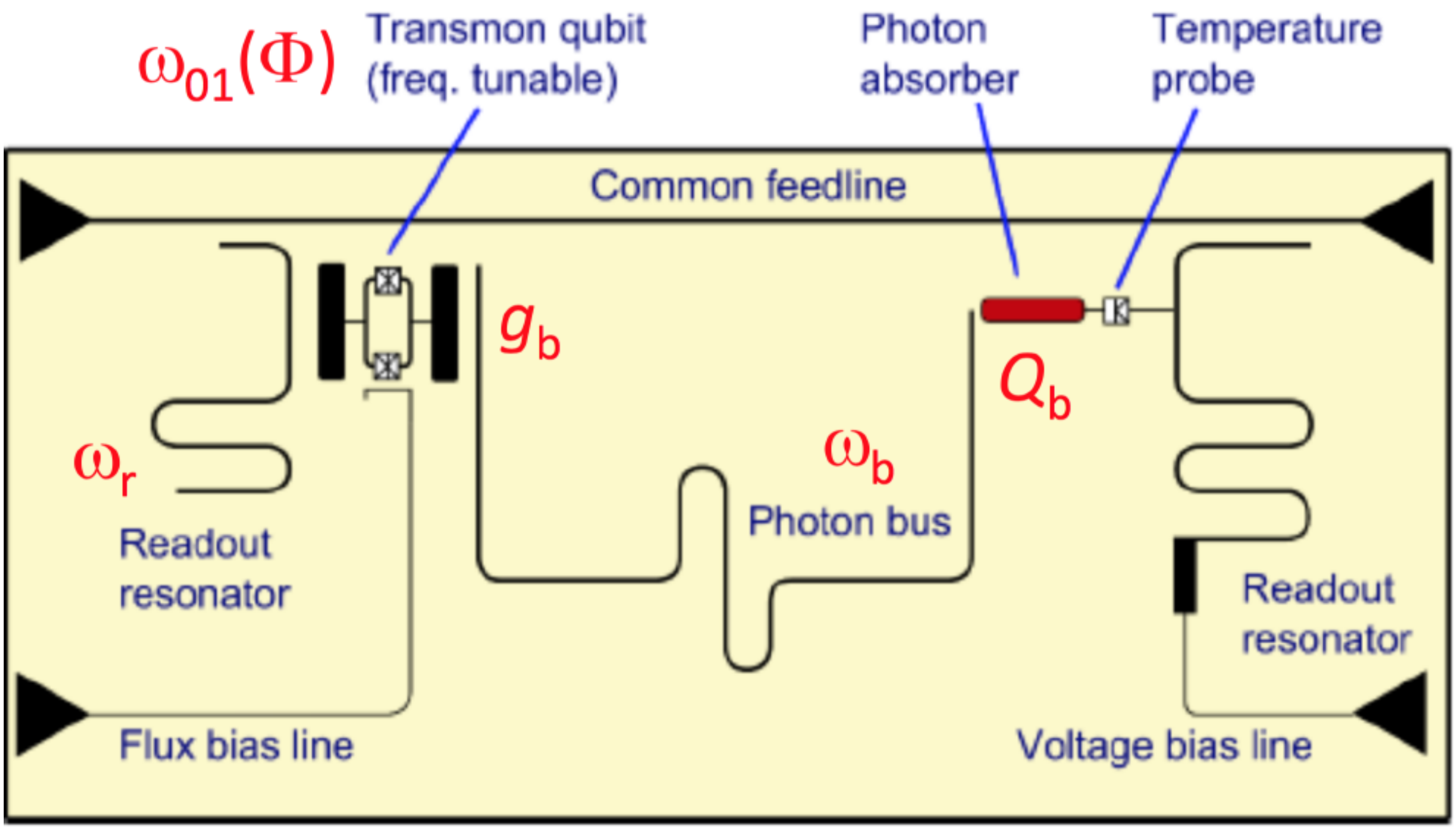
$$\langle \delta T^2 \rangle = k_B T^2 / C$$

$$S_T(\omega) = \frac{2 k_B T^2}{G_{\rm th}} \frac{1}{1 + \omega^2 C^2 / G_{\rm th}^2}$$

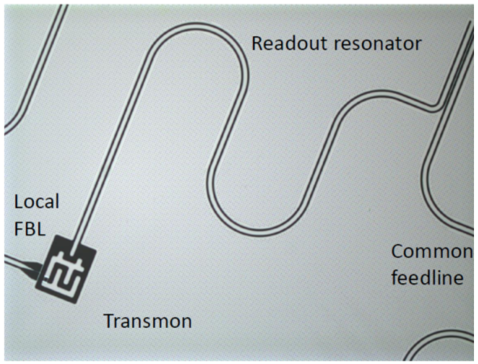


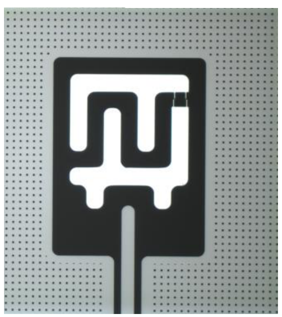
Leads to 1 μK/(Hz)$^{1/2}$ or somewhat below

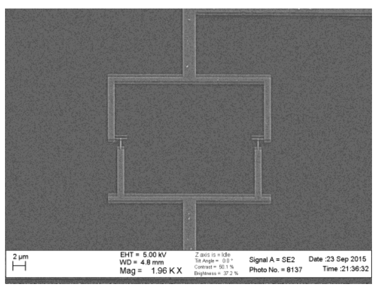
The planned set-up for calorimetric measurement of quantum trajectories



Demonstration of qubit operation at Aalto

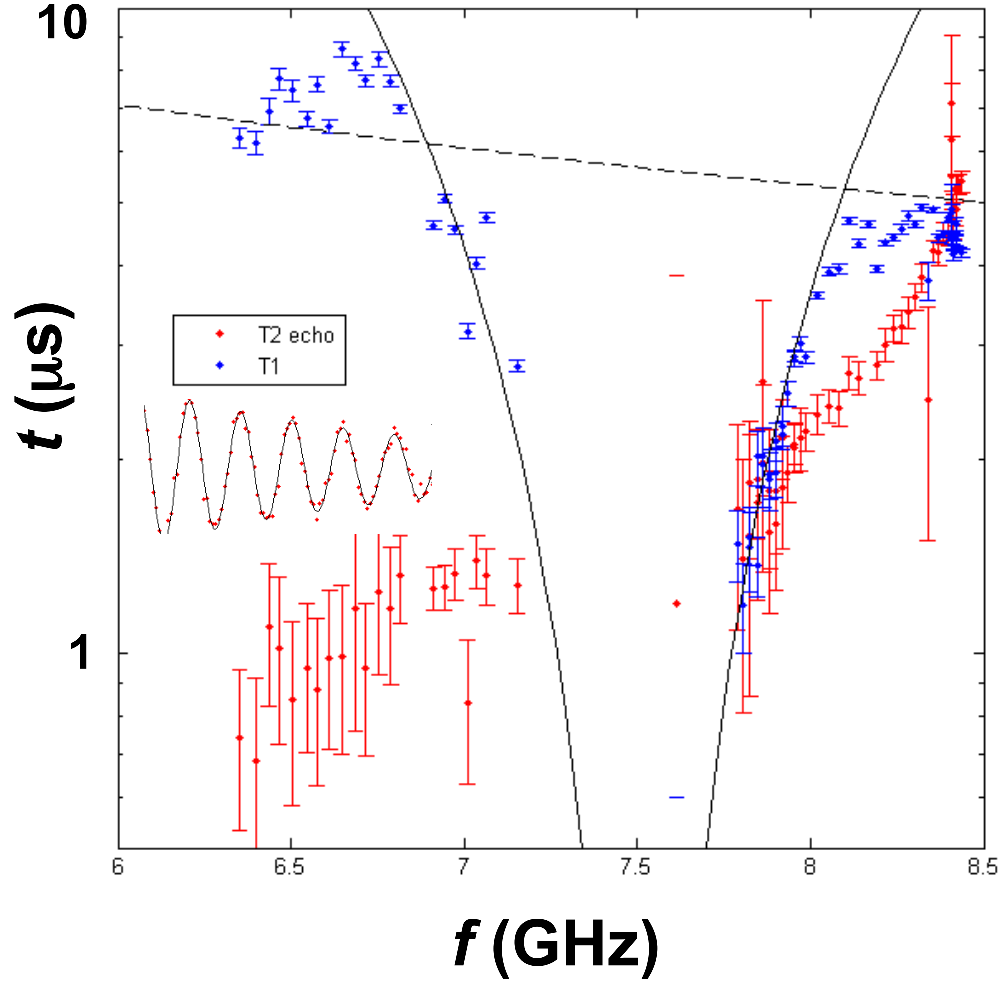






Standard transmon superconducting qubit

J. Koch et al., Phys. Rev. A 76, 04319 (2007)

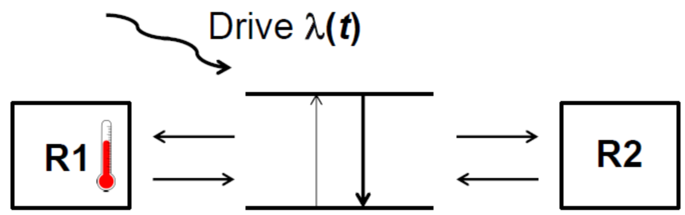


”Purcell effect”, preliminary data

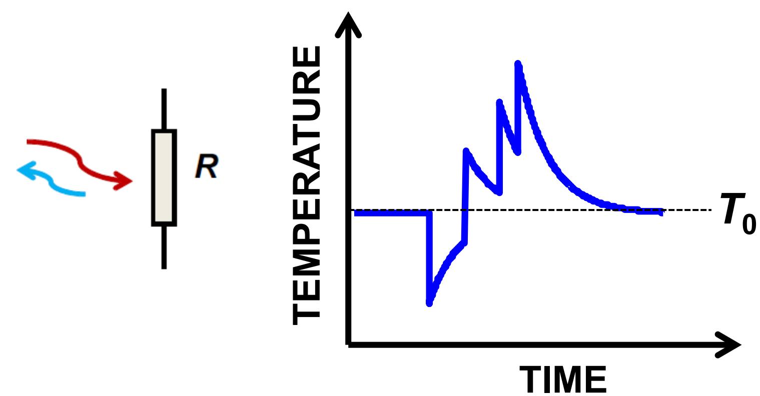
O.-P. Saira, J. Senior, R. George, Y. Pashkin, JP, unpublished data (2016)

Calorimetry on quantum two-level systems: ”errors”

1. Hidden environments/noise sources



2. Finite heat capacity of the absorber (non-Markovian)



Experiments in Helsinki by

Jonne Koski

Olli-Pentti Saira

Ville Maisi

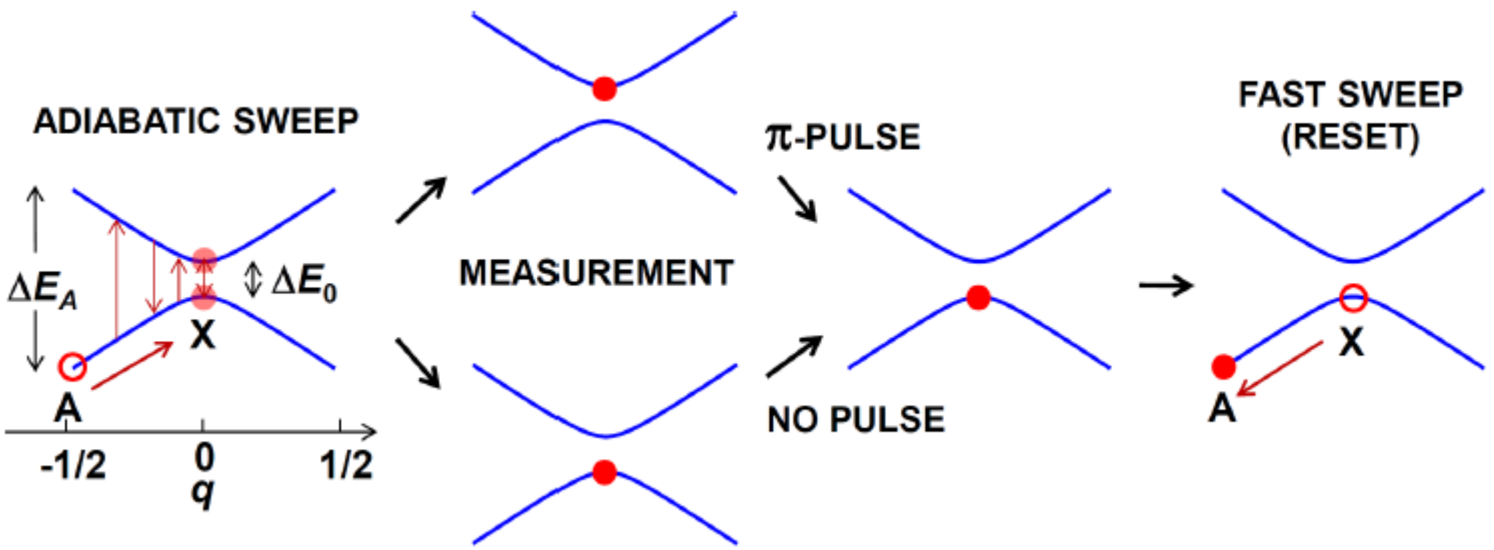
Klaara Viisanen

Simone Gasparinetti

Future experiments

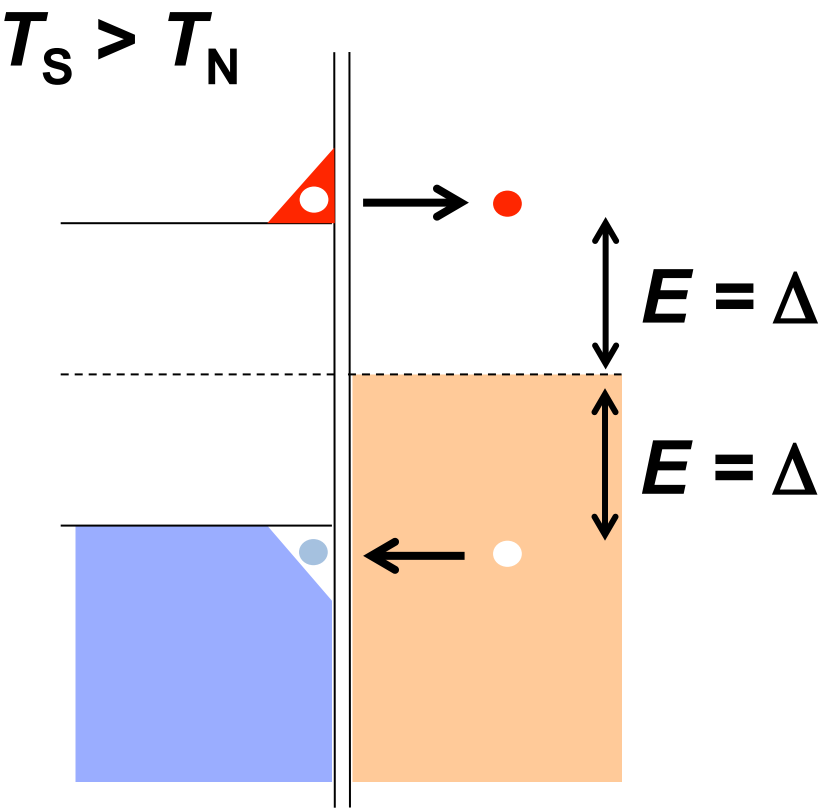
Quantum heat engines

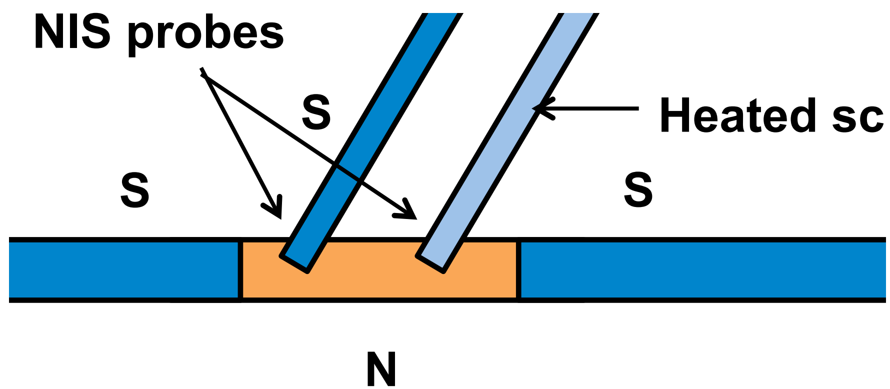
Maxwell's Demon based on a qubit



JP, D. Golubev, D. Averin, PRB 93, 024501 (2016).

Counting tunneling (heat) by calorimetry





For Al Δ/k_B = 2.5 K