Fluctuation relations and their applications in nano-electronic circuits

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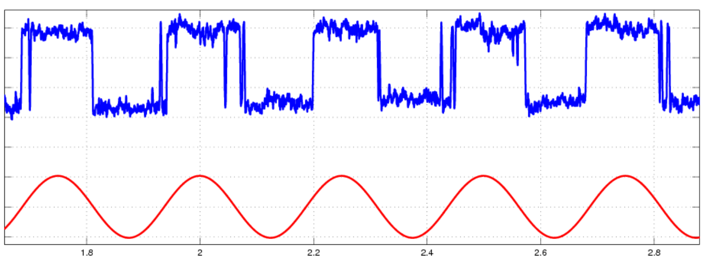
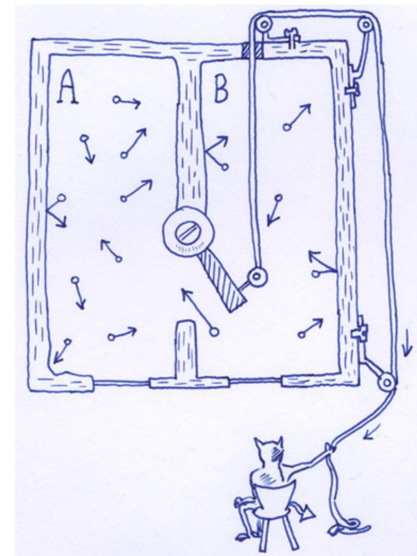
Main topics

LECTURE 1

Stochastic thermodynamics ... and information

$$\frac{P_\tau(\Delta S)}{P_\tau(-\Delta S)} = e^{\Delta S/k_\mathrm{B}} \qquad \langle e^{-\Delta S/k_B} \rangle = 1$$





LECTURES 2 and 3

Energy relaxation, measurement of temperature, autonomous Maxwell's demon, thermodynamics and fluctuation relations in quantum systems

Fluctuation relations

$$\frac{P_\tau(\Delta S)}{P_\tau(-\Delta S)} = e^{\Delta S/k_{\mathrm{B}}}$$

U. Seifert, Rep. Prog. Phys. **75**, 126001 (2012)

$$\langle e^{-\Delta S/k_B} \rangle = 1$$

General theory of thermal fluctuations in nonlinear systems

G. N. Bochkov and Yu. E. Kuzovlev

Gor'kiĭ State University

(Submitted June 17, 1976)

Zh. Eksp. Teor. Fiz. **72**, 238–247 (January 1977)

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Probability of Second Law Violations in Shearing Steady States

Denis J. Evans

Research School of Chemistry, Australian National University, Canberra, Australian Capital Territory 2600, Australia

E. G. D. Cohen

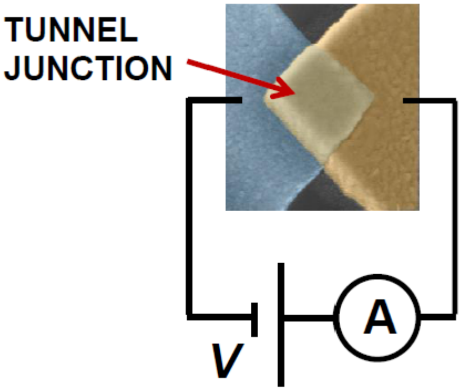
The Rockefeller University, 1230 York Avenue, New York, New York 10021

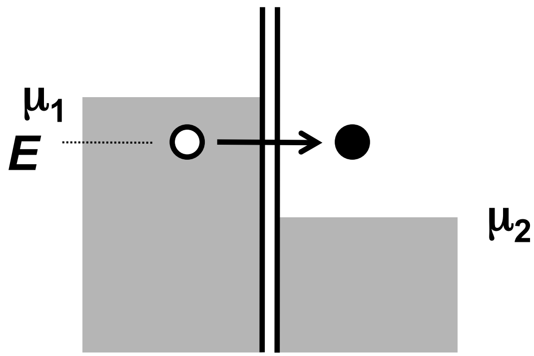
G. P. Morriss

School of Physics, University of South Wales, Kensington, New South Wales, Australia

(Received 26 March 1993)

Dissipation in transport





Dissipation generated by a tunneling event in a junction biased at voltage V

$$\Delta Q = (\mu_1 - E) + (E - \mu_2) = \mu_1 - \mu_2 = eV$$

$\Delta Q = T\Delta S$ is first distributed to the electron system, then typically to the lattice by electron-phonon scattering

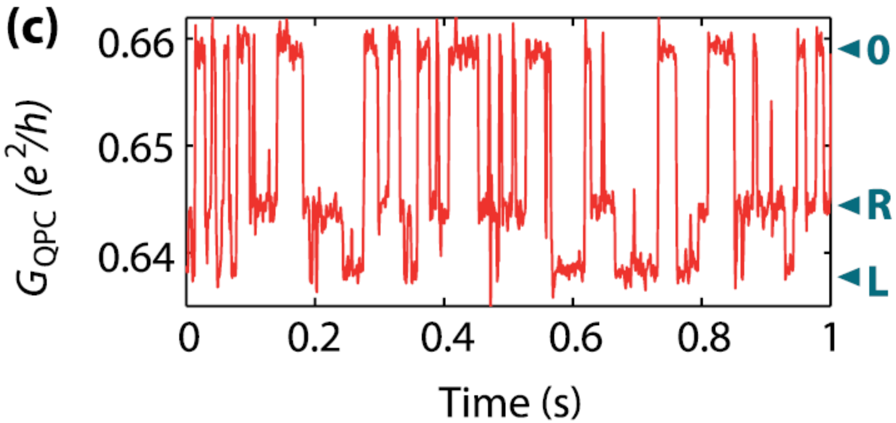
The total average power dissipated is naturally

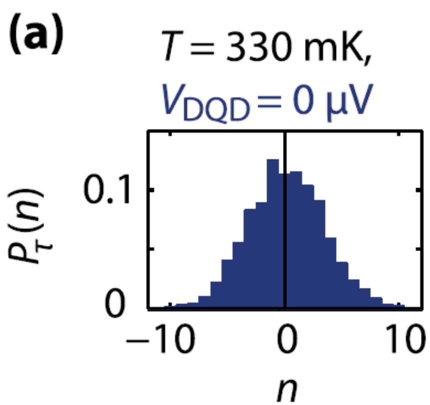
$$P = (I/e)\Delta Q = IV$$

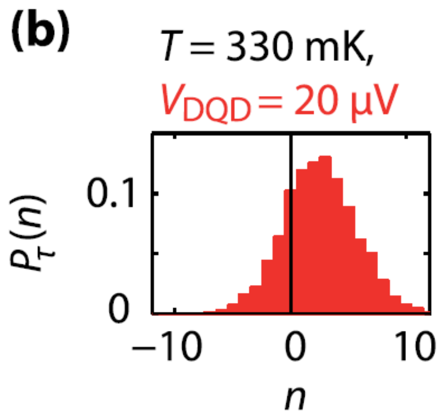
Fluctuation relations in a circuit

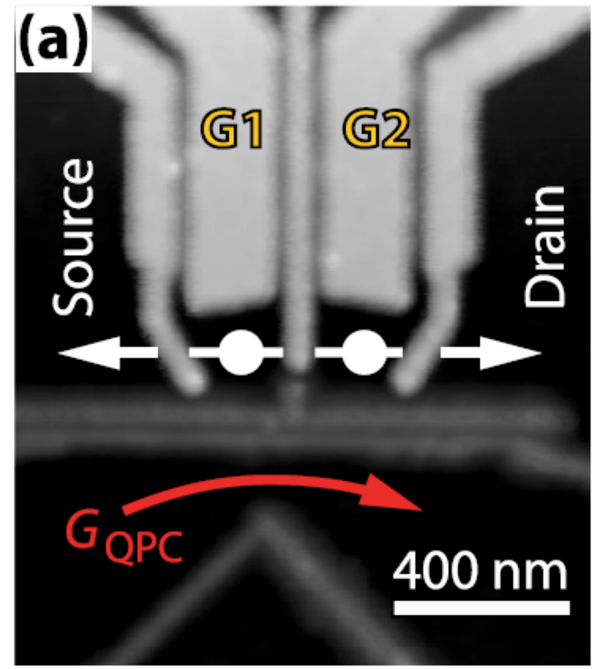
Experiment on a double quantum dot

Y. Utsumi et al. PRB 81, 125331 (2010), B. Kung et al. PRX 2, 011001 (2012)





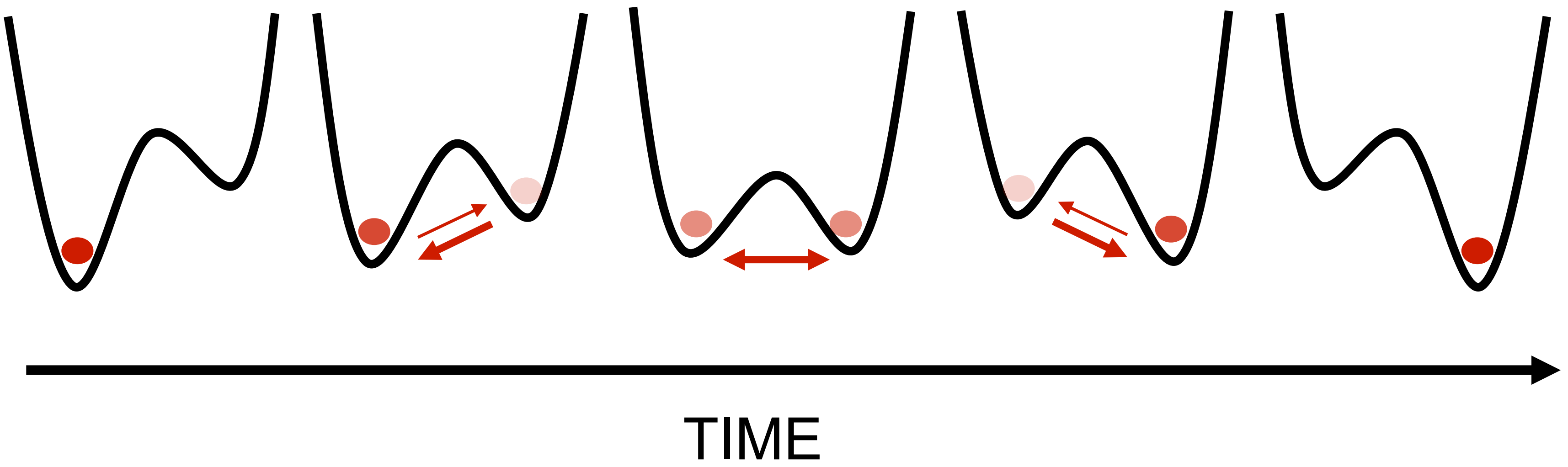




$$\frac{P_\tau(n)}{P_\tau(-n)} = e^{neV_{\rm DQD}/k_{\rm B}T}$$

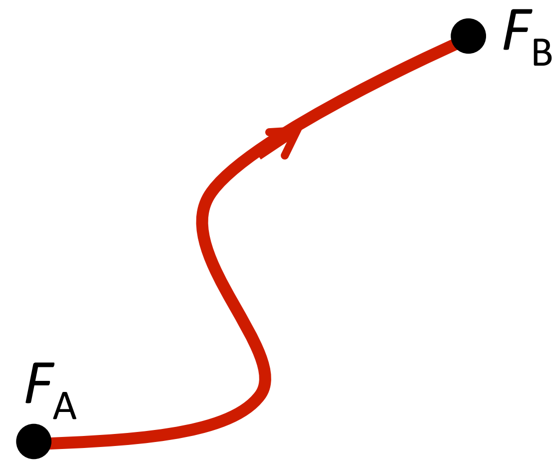
Driven classical systems

Work and dissipation in a driven process?



Crooks and Jarzynski fluctuation relations

Systems **driven** by control parameter(s), starting in equilibrium

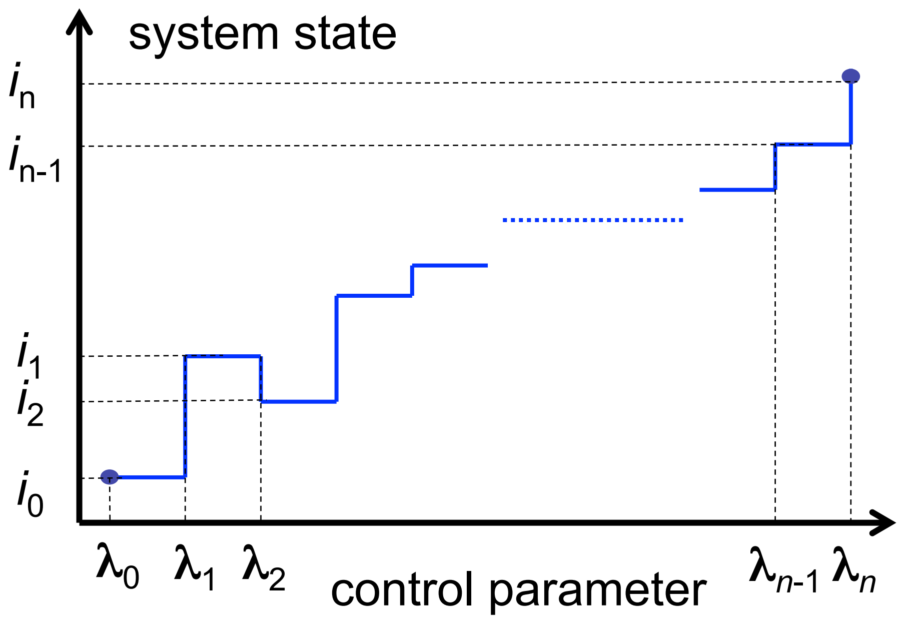


$$W_d = W - \Delta F$$ "dissipated work"

C. Jarzynski 1997 $$\langle e^{-\beta W_d} \rangle = 1$$ $$\langle W \rangle \geq \Delta F$$

G. Crooks 1999 $$p_F(W_d)/p_R(-W_d) = e^{\beta W_d}$$

Crooks and Jarzynski relations



$$W = \sum_{k=0}^{n-1} [E(i_k, \lambda_{k+1}) - E(i_k, \lambda_k)]$$

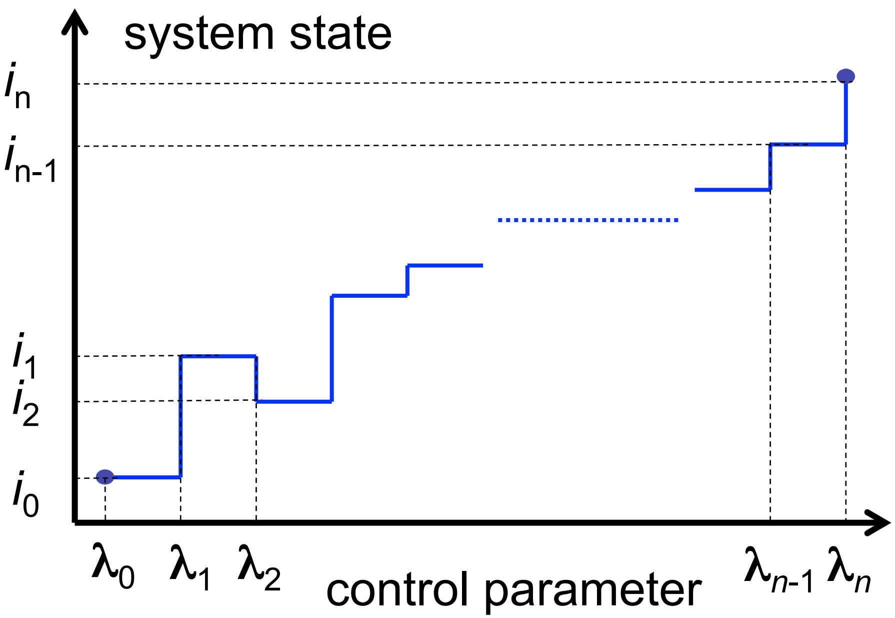
$$Q = \sum_{k=1}^{n} [E(i_{k-1}, \lambda_k) - E(i_k, \lambda_k)]$$

$$\Delta E = E(i_n, \lambda_n) - E(i_0, \lambda_0) = W - Q$$

W = work done on the system
Q = heat to the bath
Δ*E* = change of the internal energy

G. Crooks, J. Stat. Phys. 90, 1481 (1998).

Crooks and Jarzynski relations



We make the following assumptions:

1. Equilibrium in the beginning

$$P(i_0|\lambda_0) = \frac{e^{-\beta E(i_0,\lambda_0)}}{Z(\lambda_0)} = e^{\beta[F(\lambda_0)-E(i_0,\lambda_0)]}$$

2. Markovianity

$$P(i_0 \to^{\lambda_1} i_1 \to^{\lambda_2} i_2 ... \to^{\lambda_n} i_n) = P(i_0 \to^{\lambda_1} i_1) P(i_1 \to^{\lambda_2} i_2) ... P(i_{n-1} \to^{\lambda_n} i_n)$$

3. Detailed balance for the transition rates

$$\frac{P(i_k \to^{\lambda_{k+1}} i_{k+1})}{P(i_k \;{}^{\lambda_{k+1}}\!\leftarrow i_{k+1})} = e^{-\beta[E(i_{k+1},\lambda_{k+1})-E(i_k,\lambda_{k+1})]}$$

G. Crooks, J. Stat. Phys. 90, 1481 (1998).

Crooks and Jarzynski relations

Comparing the probabilities of the forward (F) and reverse (R) trajectories, we obtain

$$\frac{P(i_0|\lambda_0)P(i_0 \to^{\lambda_1} i_1 \to^{\lambda_2} i_2 ... \to^{\lambda_n} i_n)}{P(i_n|\lambda_n)P(i_0 \;^{\lambda_1}\!\leftarrow i_1 \;^{\lambda_2}\!\leftarrow i_2 ... ^{\lambda_n}\!\leftarrow i_n)} = \frac{P(i_0|\lambda_0)}{P(i_n|\lambda_n)} \frac{P(i_0 \to^{\lambda_1} i_1)}{P(i_0 \;^{\lambda_1}\!\leftarrow i_1)} \cdots \frac{P(i_{n-1} \to^{\lambda_n} i_n)}{P(i_{n-1} \;^{\lambda_n}\!\leftarrow i_n)} =$$

$$e^{-\beta[E(i_0,\lambda_0)-F(\lambda_0)]} e^{\beta[E(i_n,\lambda_n)-F(\lambda_n)]} e^{-\beta[E(i_1,\lambda_1)-E(i_0,\lambda_1)]} ... e^{-\beta[E(i_n,\lambda_n)-E(i_{n-1},\lambda_n)]} =$$

$$e^{\beta[\Delta E + F(\lambda_0) - F(\lambda_n) + Q]} = e^{\beta(W-\Delta F)}$$

i.e.,

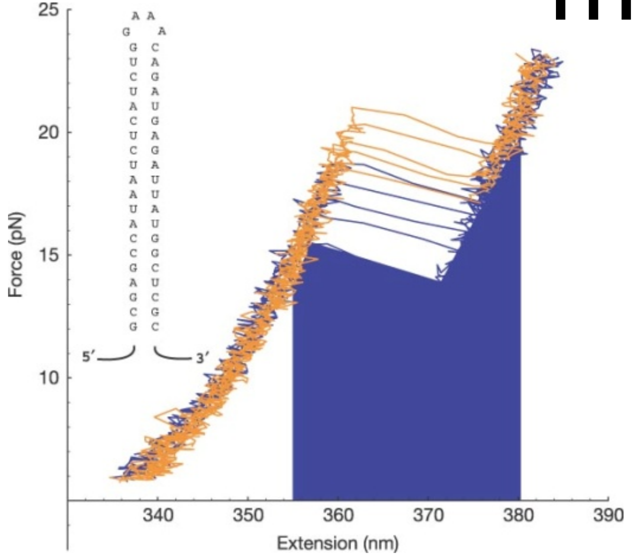
$$\frac{P_F(W_d)}{P_R(-W_d)} = e^{\beta W_d}$$

Jarzynski relation follows again by averaging

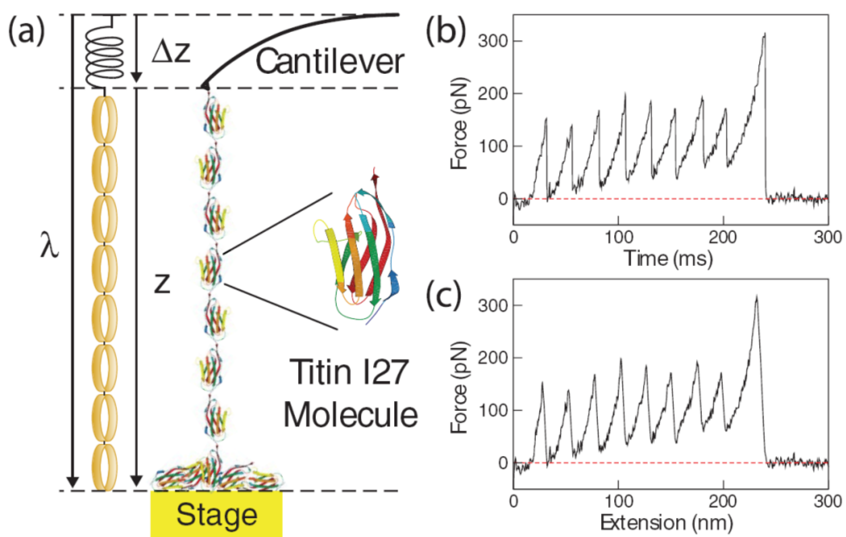
$$\langle e^{-\beta W_d} \rangle = \int dW_d \, P_F(W_d) e^{-\beta W_d} = \int dW_d \, P_R(-W_d) = \int d\tilde{W} \, P_R(\tilde{W}) = 1$$

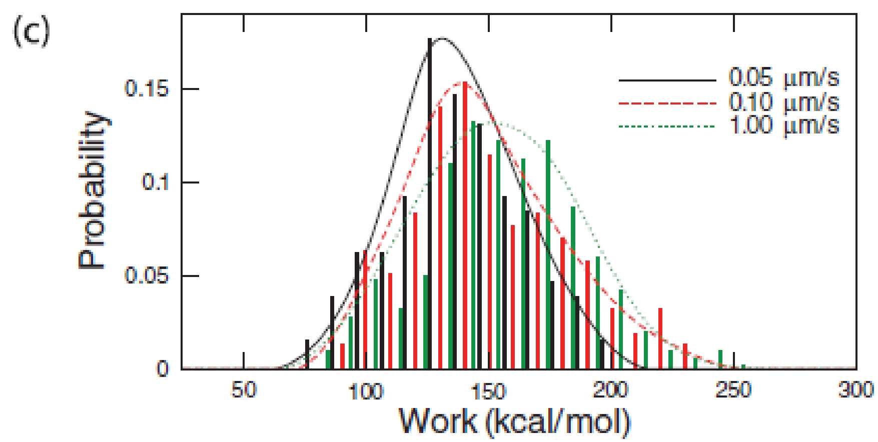
G. Crooks, J. Stat. Phys. 90, 1481 (1998).

Experiments on fluctuation relations: molecules

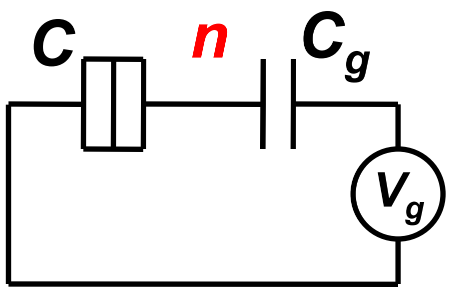


Liphardt et al., Science **292,** 733 (2002)
Collin et al., Nature **437**, 231 (2005)
Harris et al, PRL **99**, 068101 (2007)

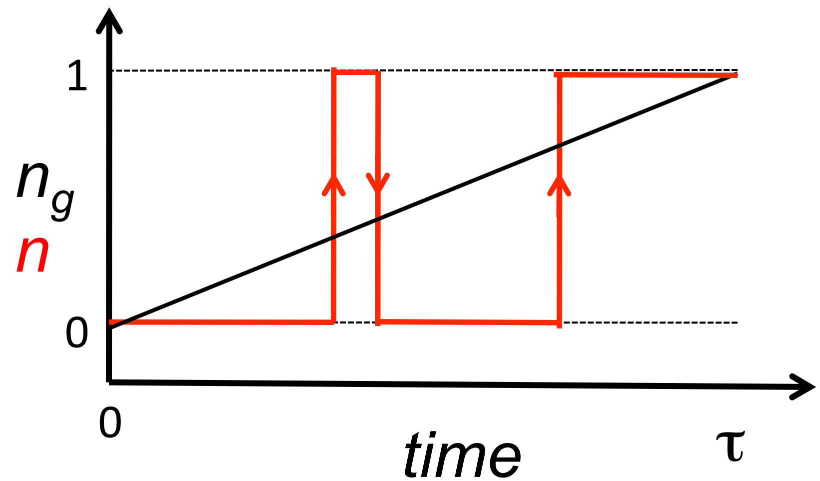




Dissipation in driven single-electron transitions

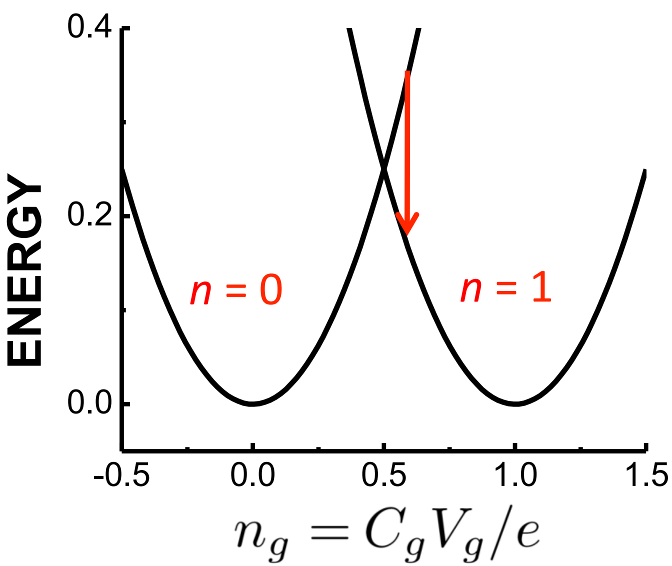


Single-electron box



$$Q_i \quad = E_0(n_{g,i}) - E_1(n_{g,i}) = \; 2E_C\delta n_{g,i}$$

$$E_C = \frac{e^2}{2(C+C_g)} \qquad \delta n_{g,i} = n_{g,i} - 1/2$$



The total dissipated heat in a ramp:

$$Q = 2E_C \sum_i \pm\delta n_{g,i}$$

D. Averin and JP, EPL 96, 67004 (2011)

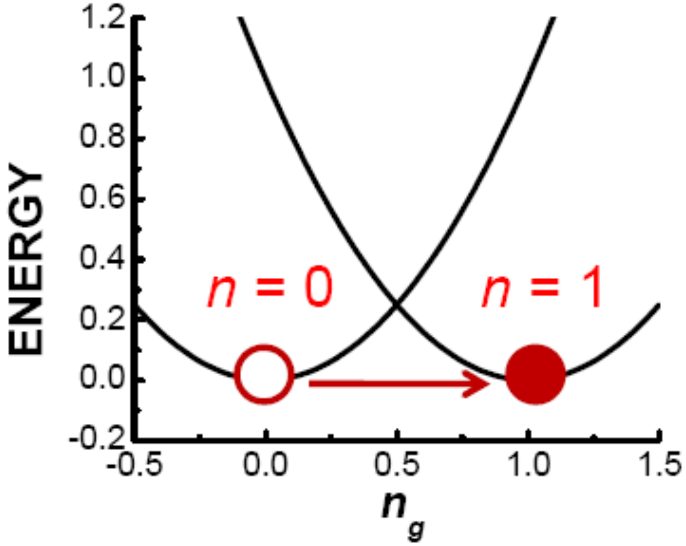
Work done by the gate

In general:

$$W = \int dt \frac{\partial H}{\partial t} = \int dt \dot{\lambda} \frac{\partial H}{\partial \lambda}$$

For a SEB box: (for the gate sweep 0 -> 1)

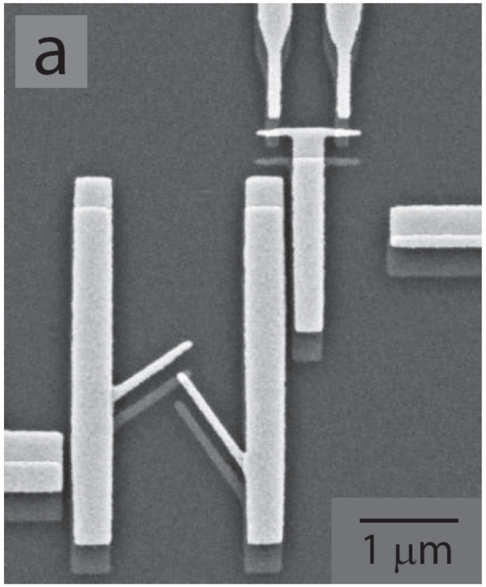
$$W - \Delta F = E_C \left(1 - n_i - n_f\right) + Q$$

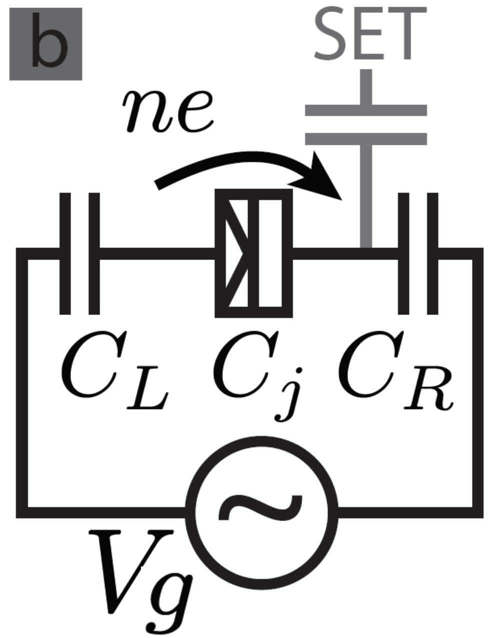


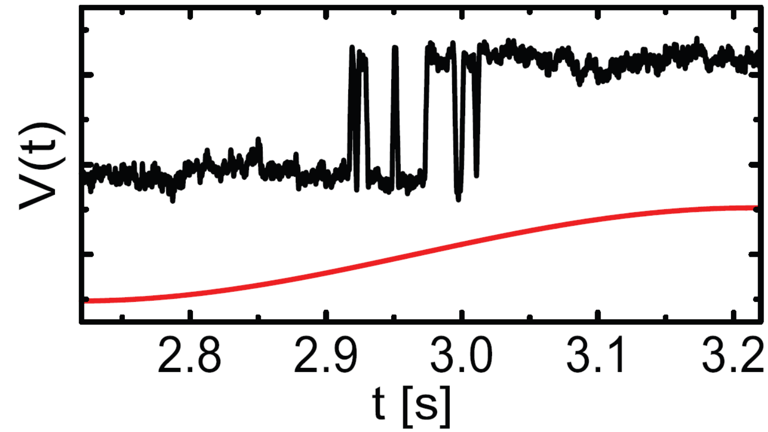
When the system is in the preferred charge state at the ends of the sweep,

$$W - \Delta F = Q$$

Experiment on a single-electron box

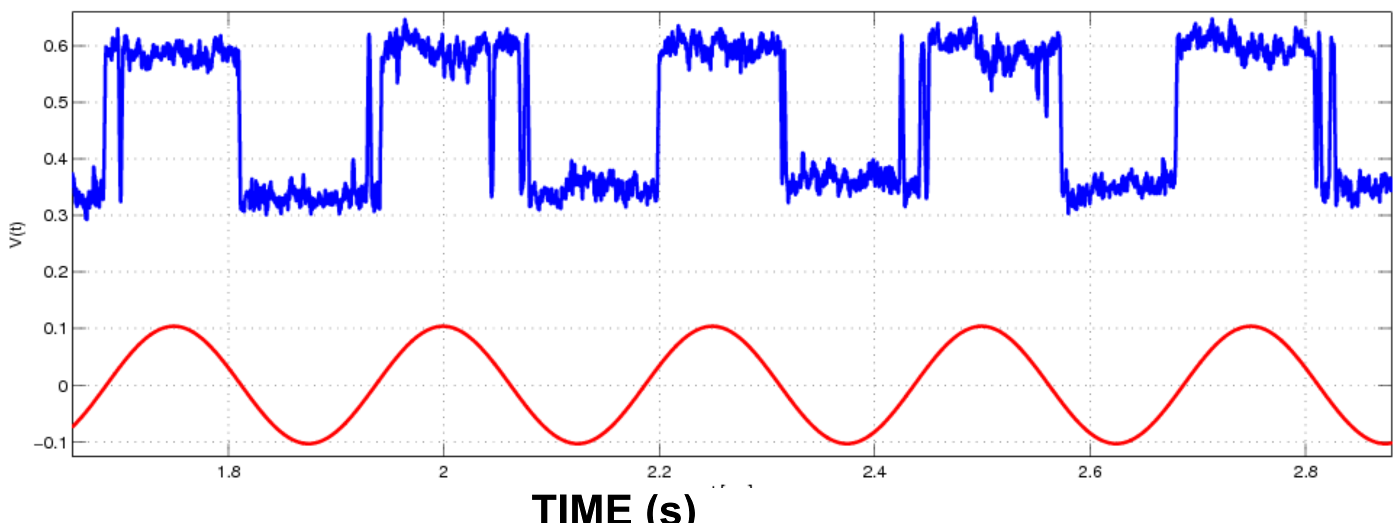






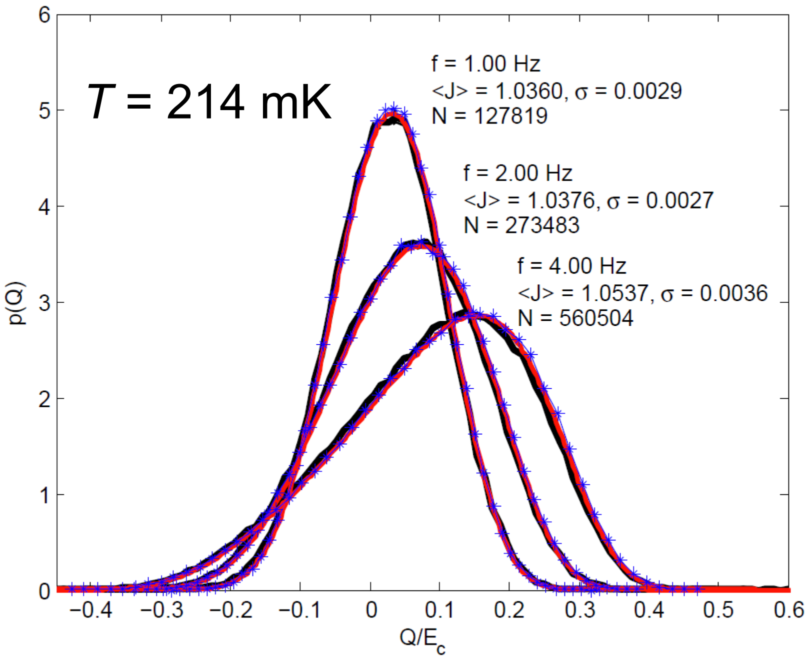
Detector current

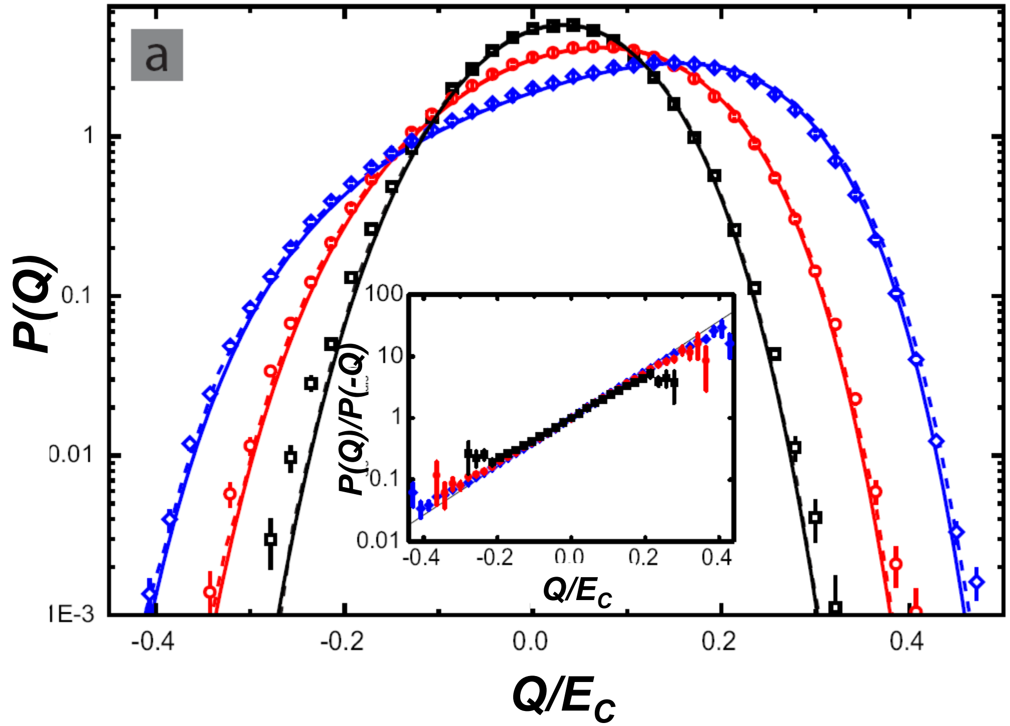
Gate drive



O.-P. Saira et al., PRL 109, 180601 (2012); J.V. Koski et al., Nature Physics 9, 644 (2013).

Experimental distributions





Measured distributions of Q at three different ramp frequencies

Taking the finite bandwidth of the detector into account (about 1% correction) yields

$$\langle e^{-\beta(W-\Delta F)} \rangle = 1.03 \pm 0.03$$

Measurements of the heat distributions at various frequencies and temperatures

symbols: experiment;
full lines: theory;
dashed lines:

$$\sigma_Q^2 = 2k_B T\langle Q\rangle$$