University of Basel

Classical and quantum synchronization

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Basel Center for Quantum Computing & Quantum Coherence

QSIT Quantum Science and Technology

National Centre of Competence in Research

global outline

- Lecture I: classical synchronization
- Lecture II: quantum synchronization
- Lecture III: topics in quantum synchronization

lecture III: topics in quantum synchronization

- measures of quantum synchronization
- examples of theoretical proposals
- signatures of energy quantization in synchronization
- quantum chimera states
- conclusion

measures of synchronization

e.g., Hush et al., PRA 91, 061401 (2015); T. Weiss et al., NJP 18, 013043 (2016)

start from the phase distribution $|\phi_j\rangle = \frac{1}{\sqrt{2\pi}} \sum_{n=0}^{\infty} e^{i\phi_j n} |n\rangle$

$$P(\phi) = \int_0^{2\pi} d\phi_1 d\phi_2 \delta(\phi_1 - \phi_2 - \phi) \langle \phi_1, \phi_2 | \rho_{ss} | \phi_1, \phi_2 \rangle$$

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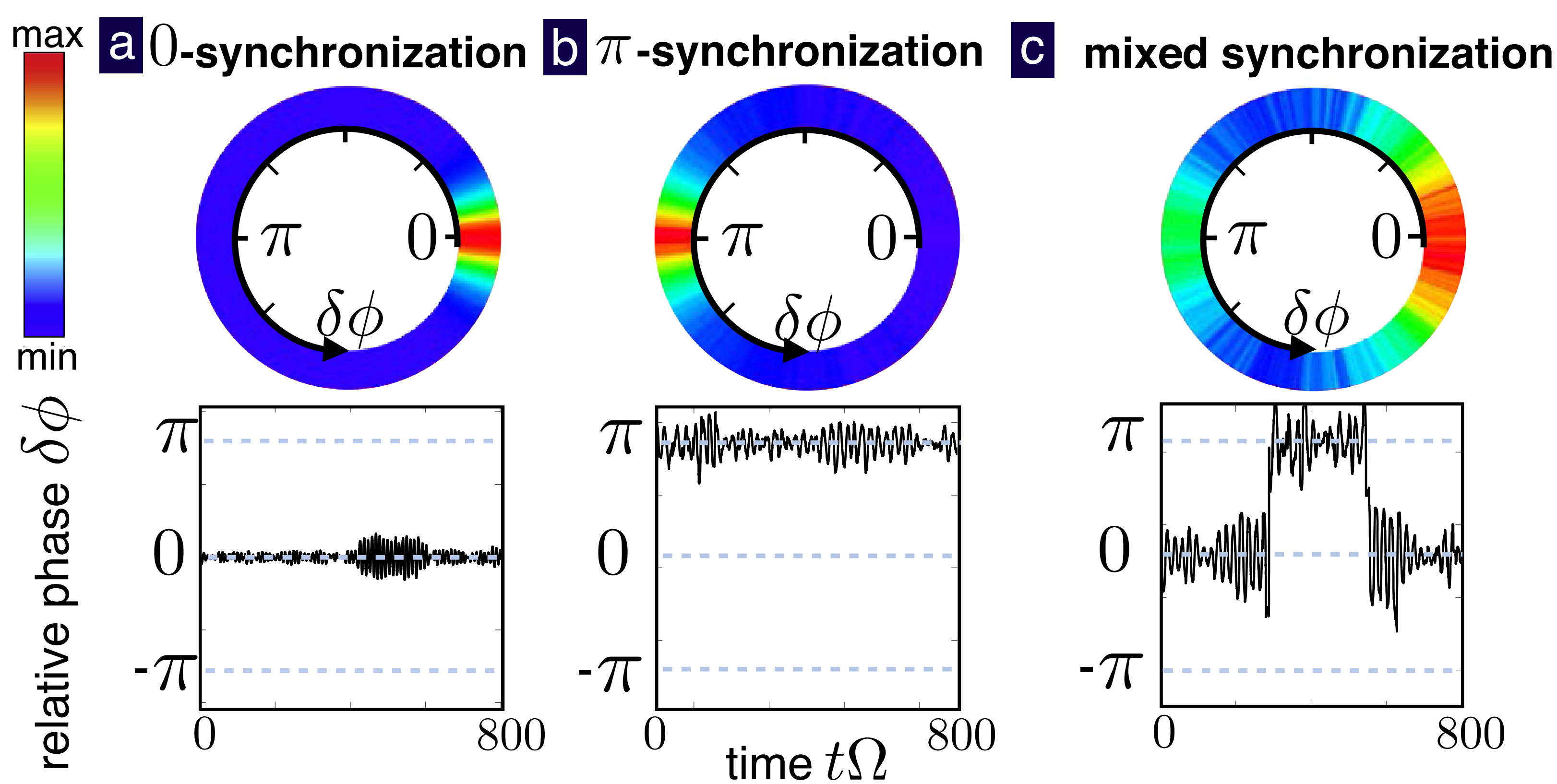

phase distribution and typical quantum trajectories for two coupled (optomechanical) limit-cycle oscillators

measures of synchronization

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proposed measure: $S = 2\pi \max[P(\phi)] - 1$

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what if there are broad peaks / several peaks?!

or use correlators like $$C = \frac{\langle b_1^\dagger b_2 \rangle}{\sqrt{\langle b_1^\dagger b_1 \rangle \langle b_2^\dagger b_2 \rangle}} \approx \langle e^{-i\delta\phi} \rangle$$

measures of synchronization

A. Mari et al., PRL 111, 103605 (2013)

synchronization and quantum uncertainty:

consider two oscillators

$$q_-(t) := (q_1 - q_2)/\sqrt{2} \qquad p_-(t) := (p_1 - p_2)/\sqrt{2}$$

measures of synchronization

A. Mari et al., PRL 111, 103605 (2013)

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complete classical synchronization means $q_-(t), p_-(t) \to 0$

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idea: quantum synchronization measure

$$S_c(t) := \langle q_-(t)^2 + p_-(t)^2 \rangle^{-1}$$

measures of synchronization

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Heisenberg uncertainty relation $\langle q_-(t)^2 \rangle \langle p_-(t)^2 \rangle \geq \frac{1}{4}$

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hence (geometric < arithmetic mean)

$$S_c(t) \leq \frac{1}{2\sqrt{\langle q_-(t)^2 \rangle \langle p_-(t)^2 \rangle}} \leq 1$$

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universal limit to the complete synchronization of two continuous-variable systems

measures of synchronization

A. Mari et al., PRL 111, 103605 (2013)

measure of phase synchronization:

$$\langle b_j(t) \rangle = r_j(t) e^{i\phi_j(t)}$$

$$b_j(t) - \langle b_j(t) \rangle =: b'_j(t) e^{i\phi_j(t)}$$

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fluctuations of amplitude and phase

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fluctuations of amplitude and phase

$$S_p(t) = \frac{1}{\langle p'_-(t)^2\rangle}$$

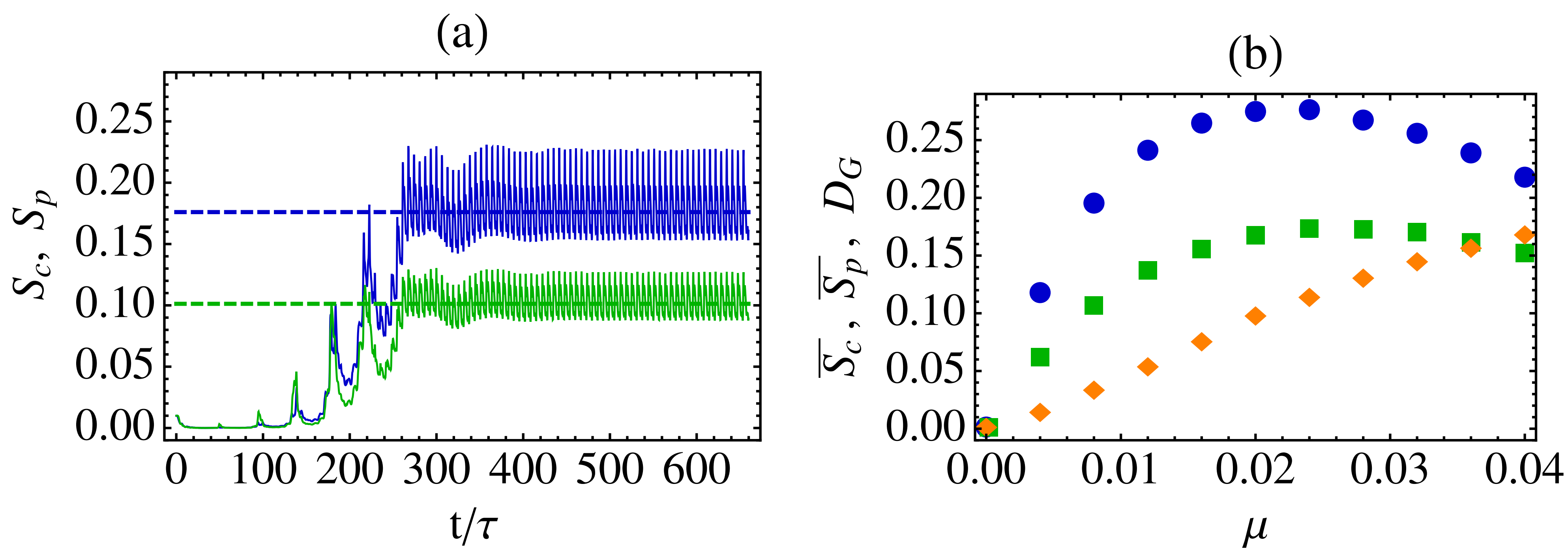
measures phase synchronization

measures of synchronization

A. Mari et al., PRL 111, 103605 (2013)

two optomechanical cells; reactive coupling $\sim \mu$

$$H = \sum_{j=1,2} [-\Delta_j a_j^\dagger a_j + \omega_j b_j^\dagger b_j - g a_j^\dagger a_j (b_j + b_j^\dagger) + iE(a_j - a_j^\dagger)] - \mu(b_1 b_2^\dagger + b_1^\dagger b_2)$$



measures of synchronization

V. Ameri et al., PRA 91, 012301 (2015)

mutual information as an order parameter for quantum synchronization between two subsystems ρ_1, ρ_2

$$
\begin{aligned}
I &= S(\rho_1) + S(\rho_2) - S(\rho) \\
&= -\mathrm{Tr}[\rho_1 \ln \rho_1] - \mathrm{Tr}[\rho_2 \ln \rho_2] + \mathrm{Tr}[\rho \ln \rho]
\end{aligned}
$$

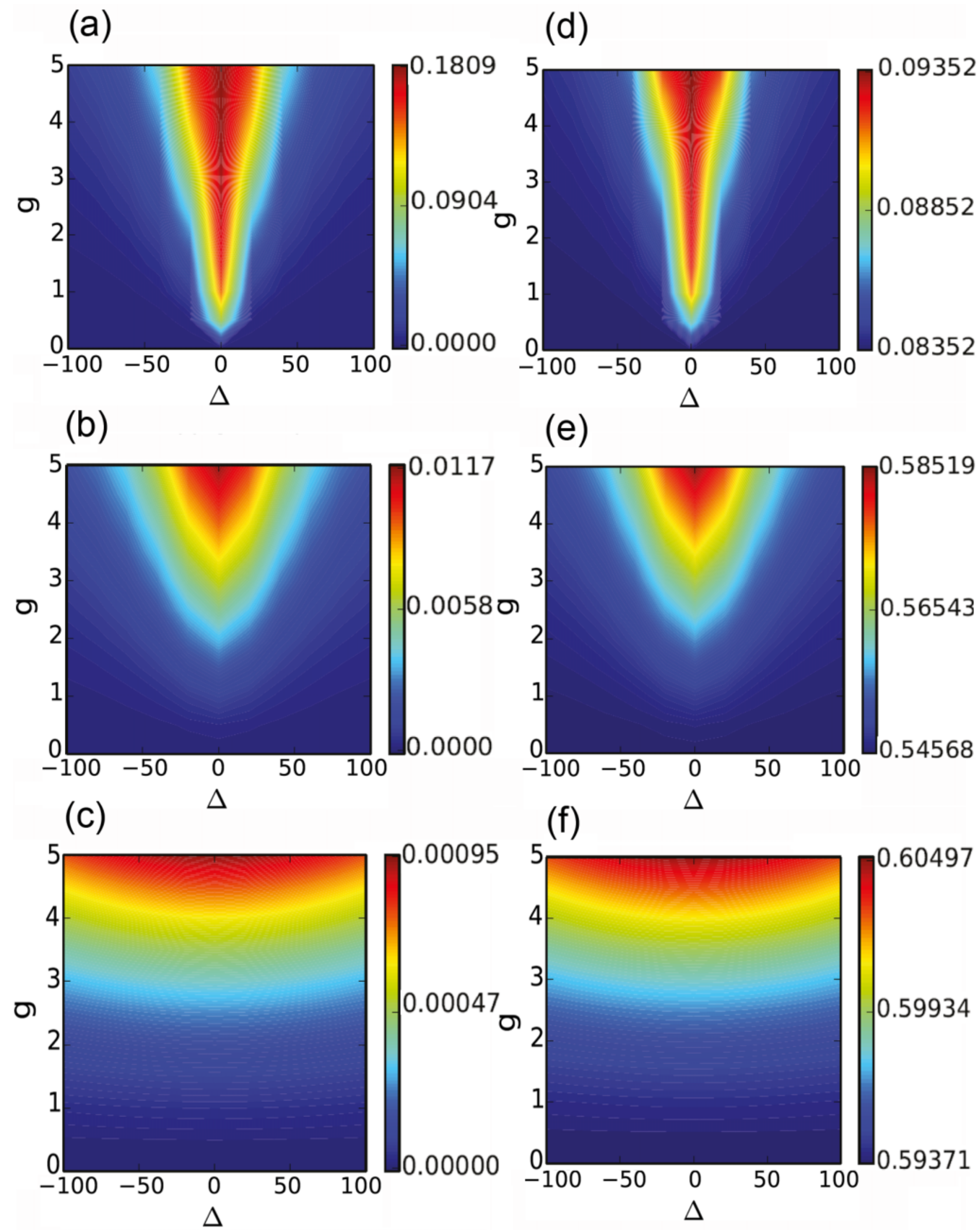
can be applied to both continuous-variable and finite-dimensional systems

measures of synchronization

V. Ameri et al., PRA 91, 012301 (2015)

I (left), S_c (right)

for two vdP oscillators



measures of synchronization

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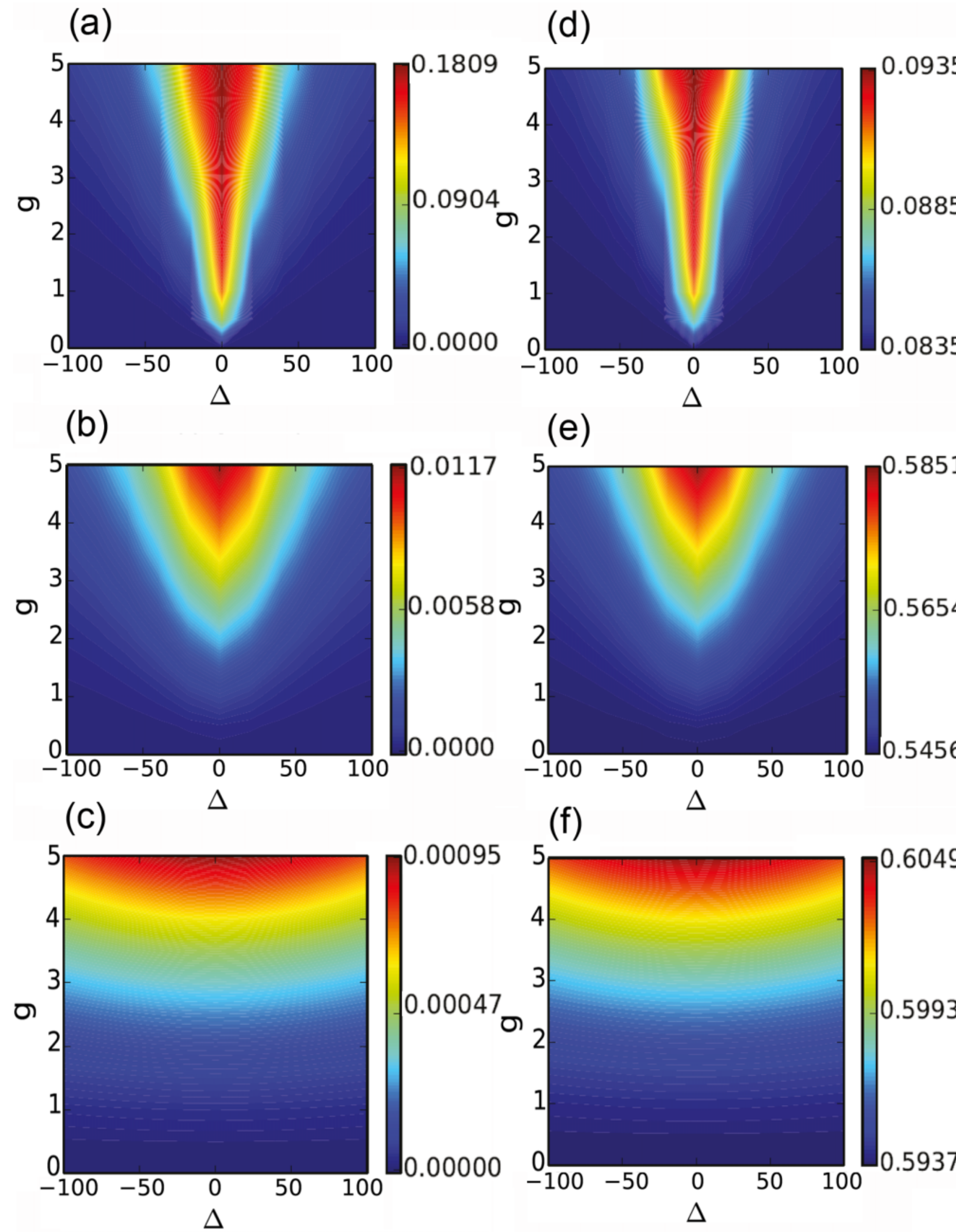
I (left), S_c (right)

for two vdP oscillators

detuning Δ,
coupling $g(b_1^\dagger b_2 + b_2^\dagger b_1)$



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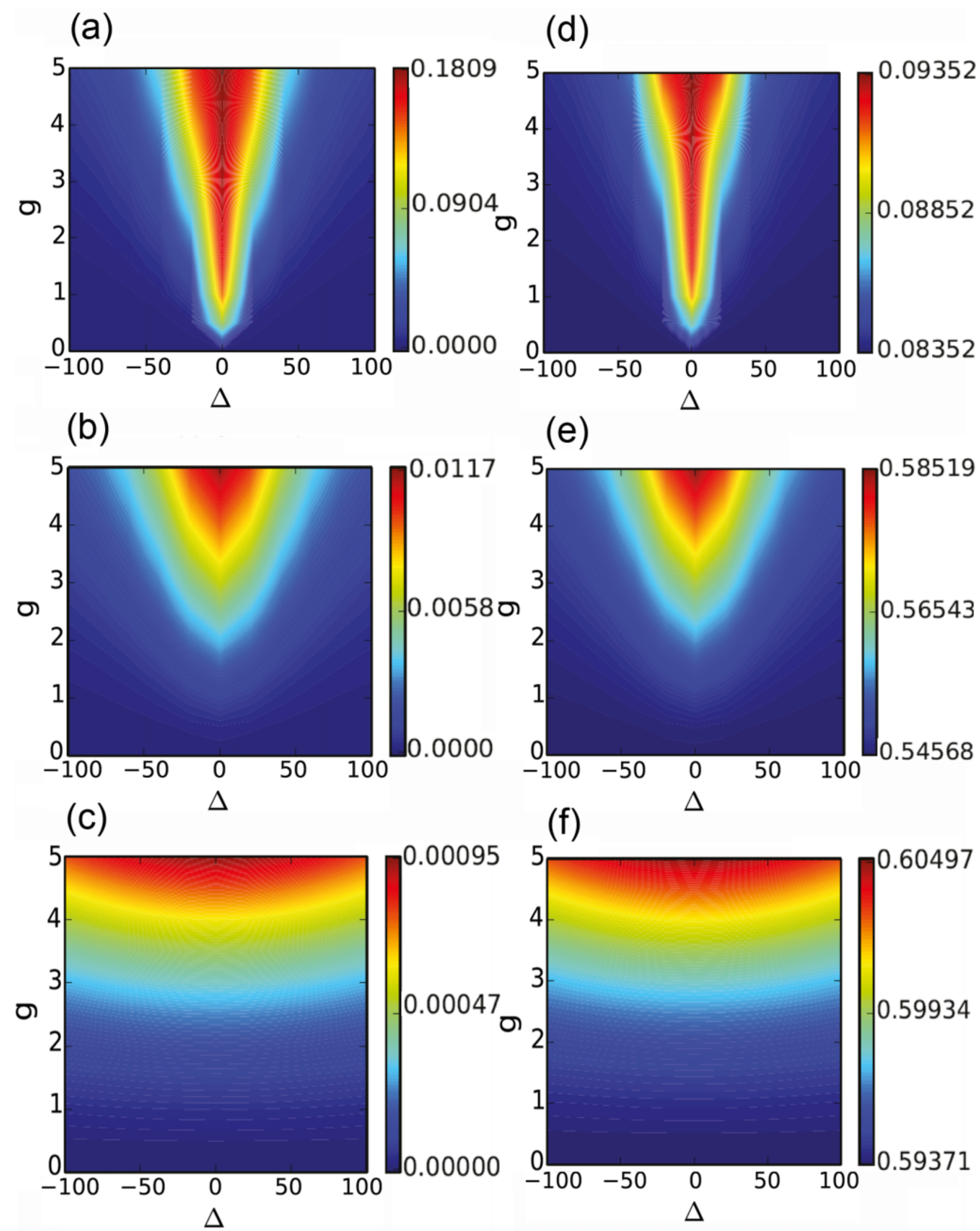
$\gamma_2/\gamma_1 =$ 0.1 (top),

10 (middle), 100 (bottom)



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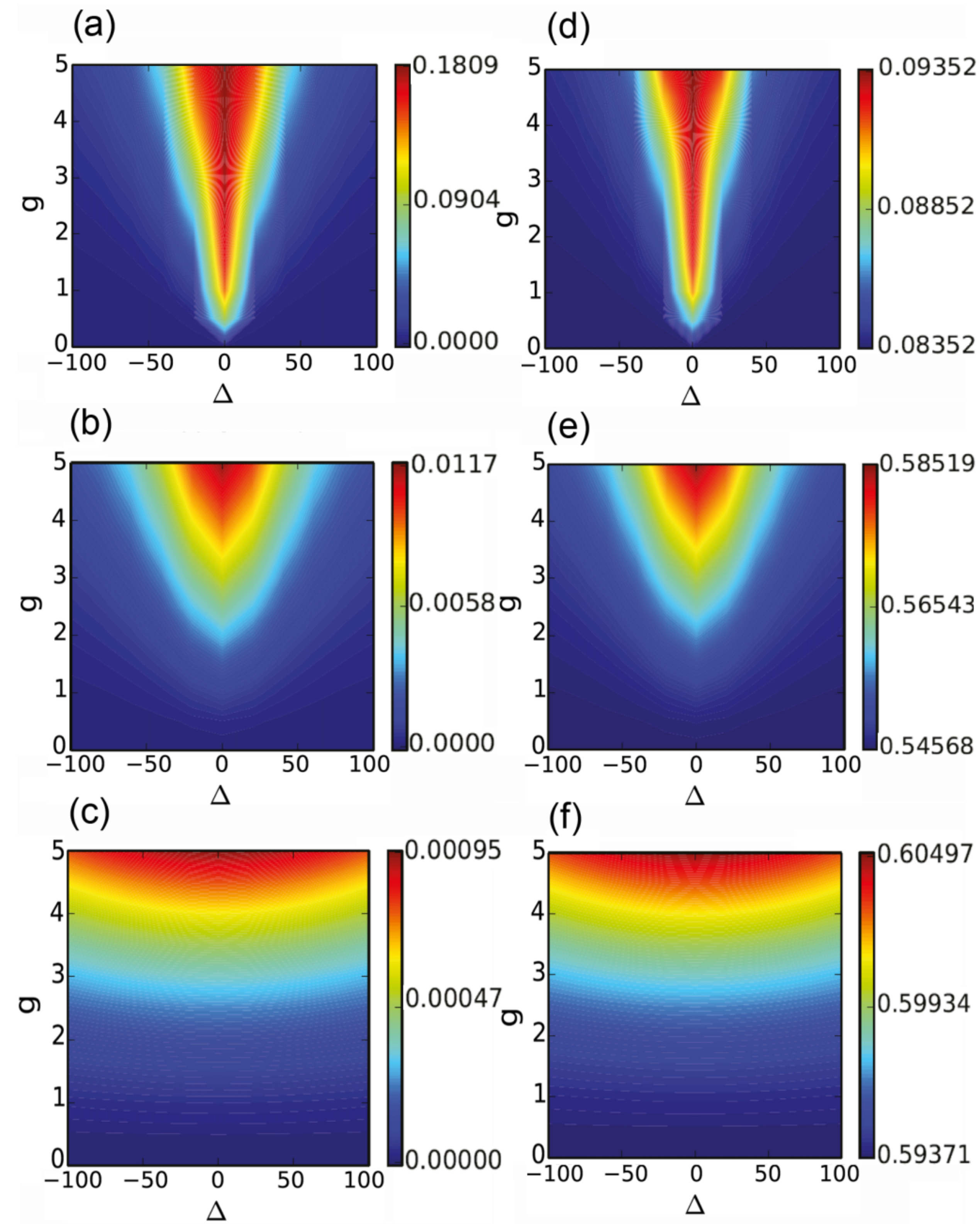
qualitative agreement!



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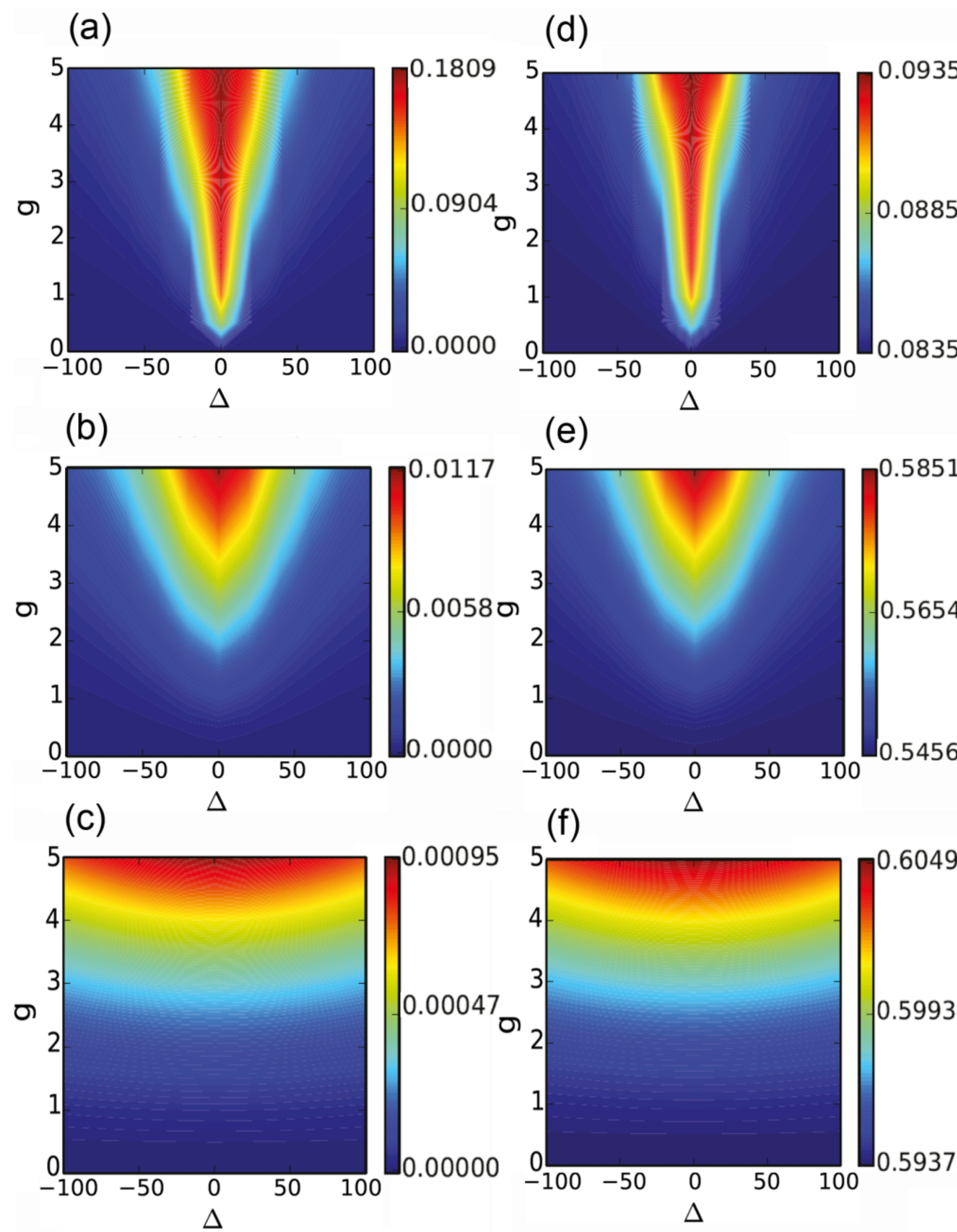
qualitative agreement!

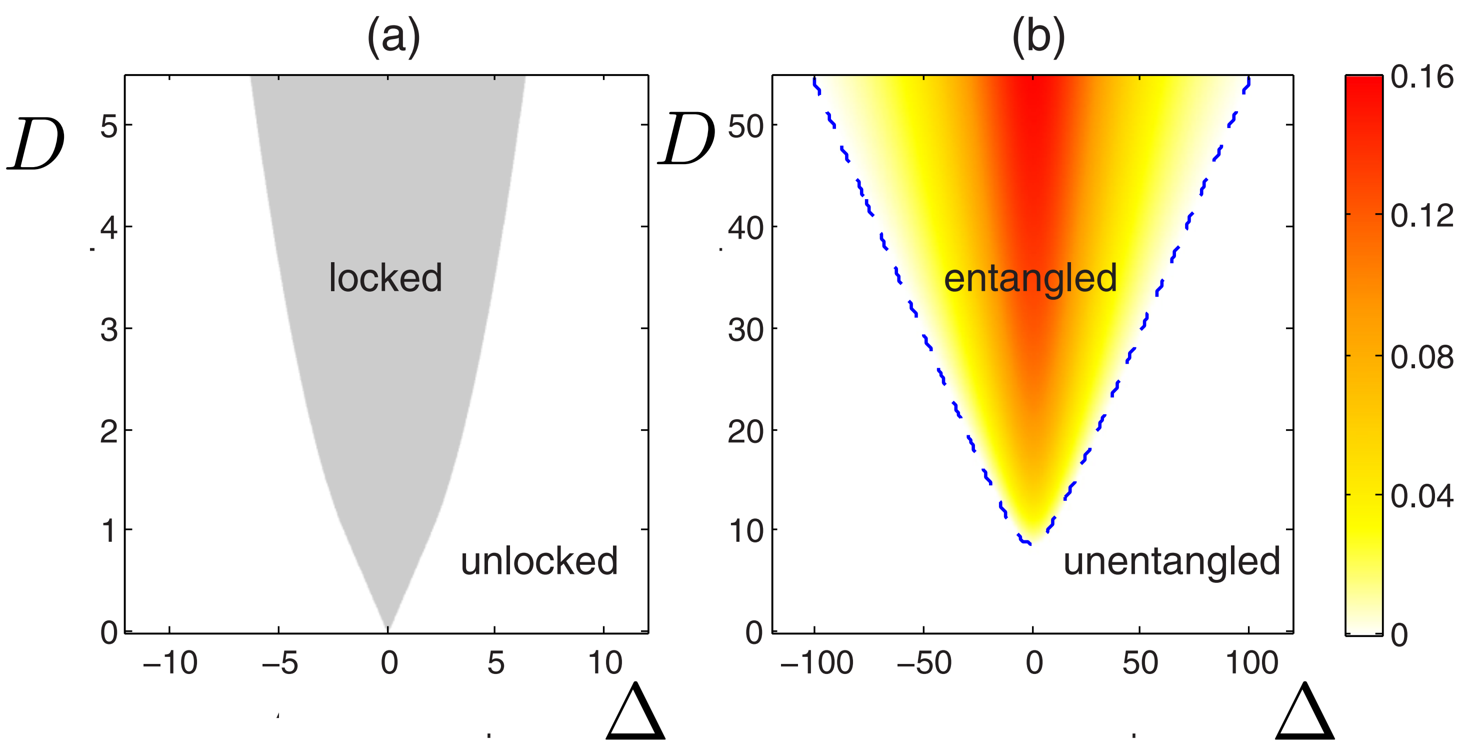
but: different scales; $S_c \neq 0$
in unsynchronized region



measures of synchronization

T.E. Lee, C.-K. Chan, and S. Wang, PRE 89, 022913 (2014)

relation to entanglement for two dissipatively coupled vdP oscillators:
left classical, right extreme quantum limit $\gamma_1 \ll \gamma_2$

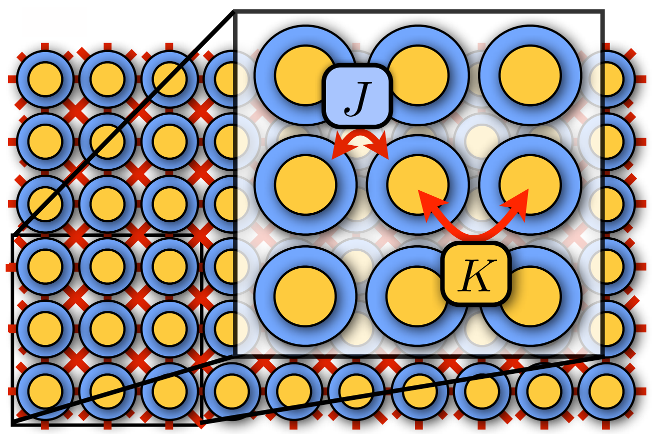


note: different scales

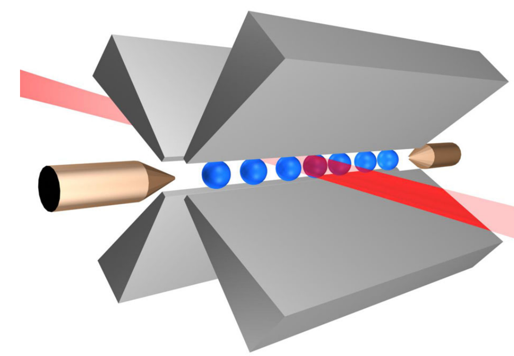
"entanglement tongue"; appears to depend on the coupling/parameters chosen - open question

theoretical proposals

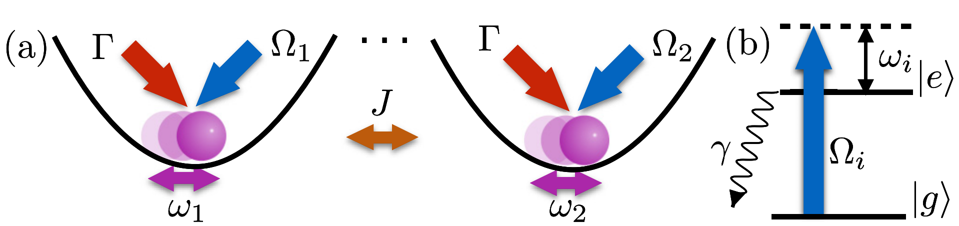
O.V. Zhirov and D.L. Shepelyansky, EPJD 2006
A.M. Hriscu and Yu.V. Nazarov, PRL 2013



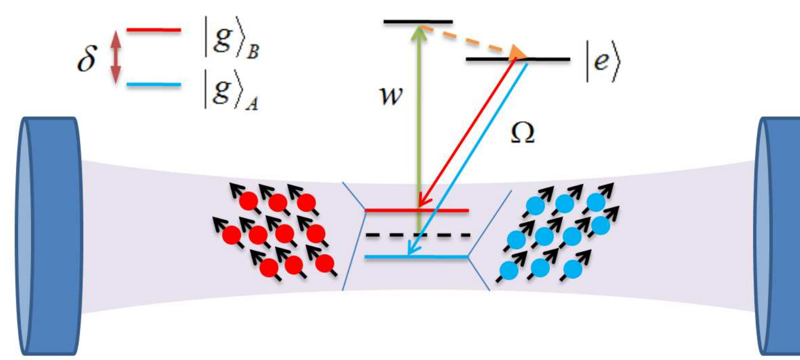
M. Ludwig and F. Marquardt
PRL 111, 073603 (2013)



T.E. Lee and H.R. Sadeghpour
PRL 111, 234101 (2013)



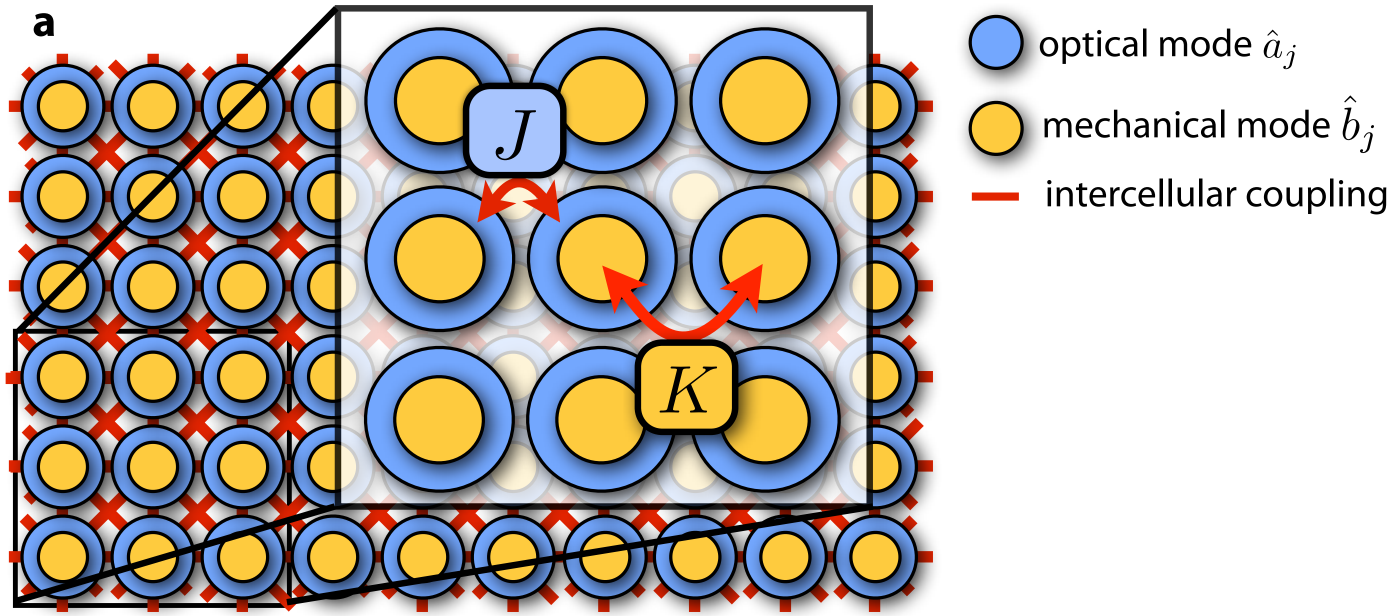
M.R. Hush, W. Li, S. Genway, I. Lesanovsky,
A.D. Armour, PRA 91, 061401 (2015)



M. Xu, D.A. Tieri, E.C. Fine, J.K. Thompson, M.J. Holland
PRL 113, 154101 (2014)

optomechanical array

M. Ludwig and F. Marquardt, arXiv:1208.0327v1; PRL 111, 073602 (2013)



$a, a^\dagger$: photons
$b, b^\dagger$: phonons

$$\hat{H} = \sum_j \hat{H}_{\mathrm{om},j} + \hat{H}_{\mathrm{int}}$$

$$\hat{H}_{\mathrm{om},j} = -\Delta \hat{a}_j^\dagger \hat{a}_j + \Omega \hat{b}_j^\dagger \hat{b}_j - g_0(\hat{b}_j^\dagger + \hat{b}_j)\hat{a}_j^\dagger \hat{a}_j + \alpha_L(\hat{a}_j^\dagger + \hat{a}_j)$$

$$\hat{H}_{\mathrm{int}} = -\frac{J}{z}\sum_{\langle i,j\rangle}(\hat{a}_i^\dagger \hat{a}_j + \hat{a}_i \hat{a}_j^\dagger) - \frac{K}{z}\sum_{\langle i,j\rangle}(\hat{b}_i^\dagger \hat{b}_j + \hat{b}_i \hat{b}_j^\dagger)$$

optomechanical array

driven many-body model...very hard!

optomechanical array

driven many-body model...very hard!

$J = 0$; Gutzwiller approximation ⇒ single-site problem

$$\hat{H}_{\rm mf} = \hat{H}_{\rm om} - K(\hat{b}^\dagger \langle \hat{b} \rangle + \hat{b} \langle \hat{b}^\dagger \rangle)$$

optomechanical array

driven many-body model...very hard!

$J = 0$; Gutzwiller approximation ⇒ single-site problem

$$\hat{H}_{\mathrm{mf}} = \hat{H}_{\mathrm{om}} - K(\hat{b}^{\dagger}\langle\hat{b}\rangle + \hat{b}\langle\hat{b}^{\dagger}\rangle)$$

to check for collective oscillations, calculate

$$\langle\hat{b}\rangle(t) = \bar{b} + re^{-i\Omega_{\mathrm{eff}}t}$$

optomechanical array

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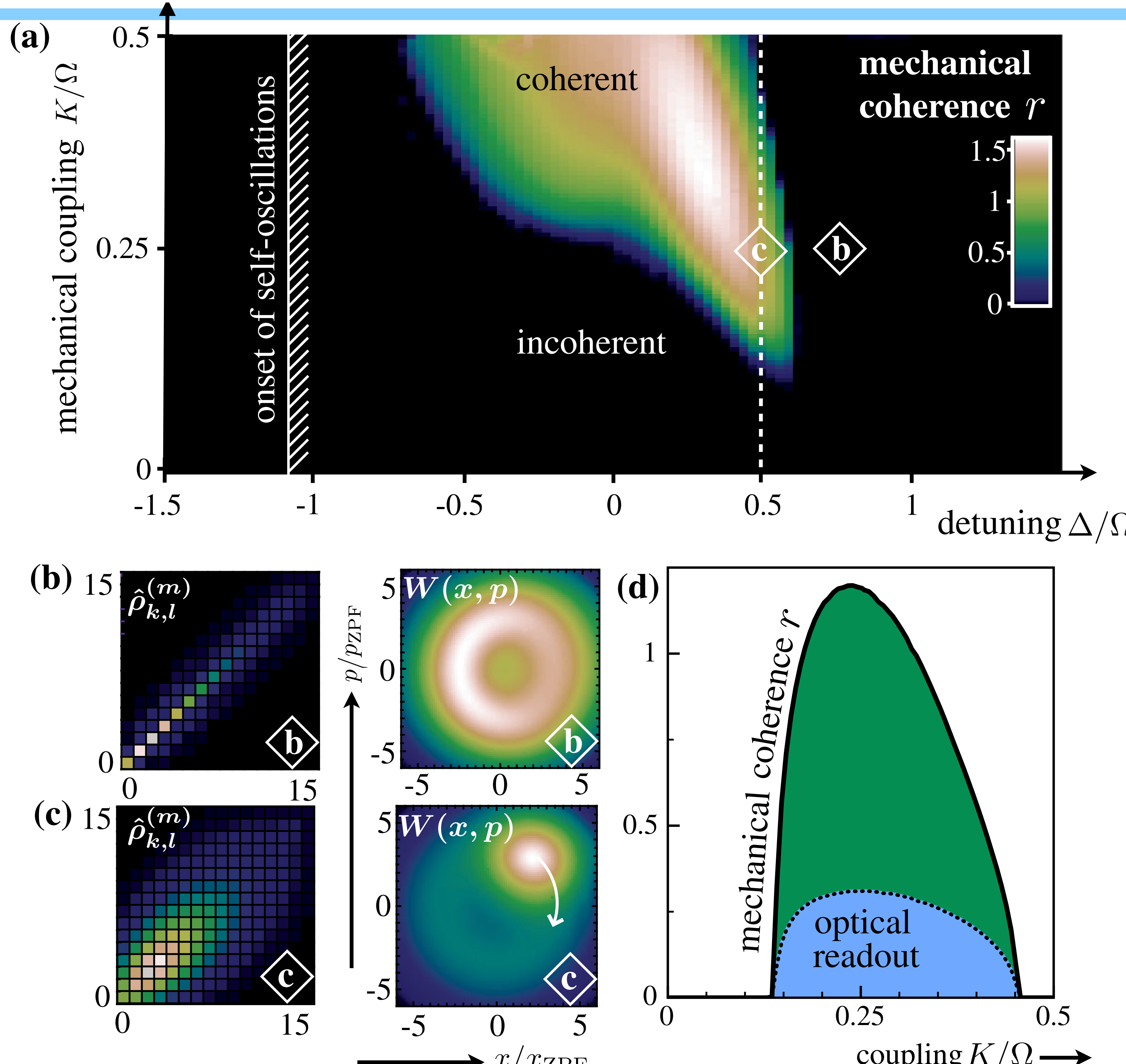
- at weak coupling K, self-oscillations are incoherent, $r = 0$
  although the bare frequencies of the oscillators agree!
  quantum effect - due to quantum fluctuations

- collective mechanical motion of the array for $r > 0$

optomechanical array



trapped ions = realization of vdP oscillators

T.E. Lee and H.R. Sadeghpour PRL 111, 234101 (2013)

trapped ion with ground state, excited state $|g\rangle,\ |e\rangle$

motional mode with frequency ω_0 = harmonic oscillator

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T.E. Lee and H.R. Sadeghpour PRL 111, 234101 (2013)

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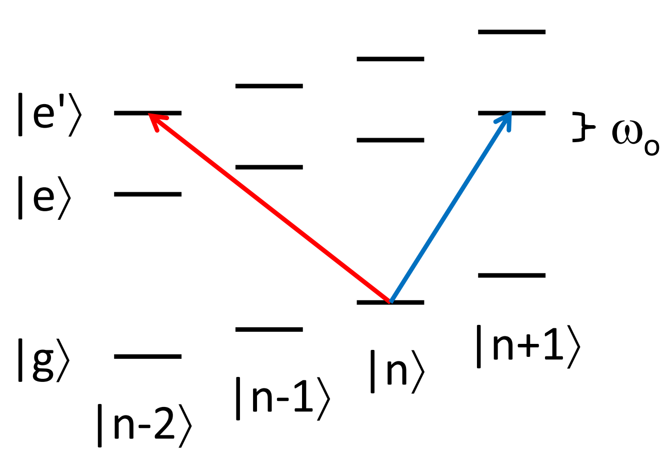
motional mode with frequency ω_0 = harmonic oscillator

to realize

$$\frac{d\rho}{dt} = -i\left[\Delta b^\dagger b + \Omega(b^\dagger + b), \rho\right] + \gamma_1 \mathcal{D}[b^\dagger]\rho + \gamma_2 \mathcal{D}[b^2]\rho$$

laser-excite to $|e\rangle$ (detuning $+\omega_0$), and simultaneously to another state $|e'\rangle$ (detuning $-2\omega_0$)

(a)



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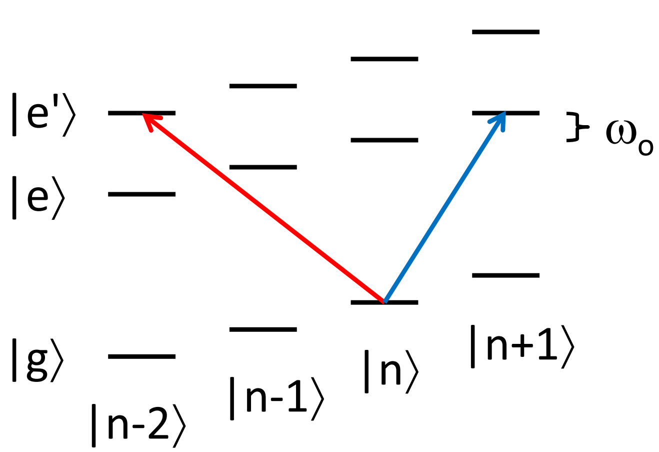


external drive implemented by RF signal

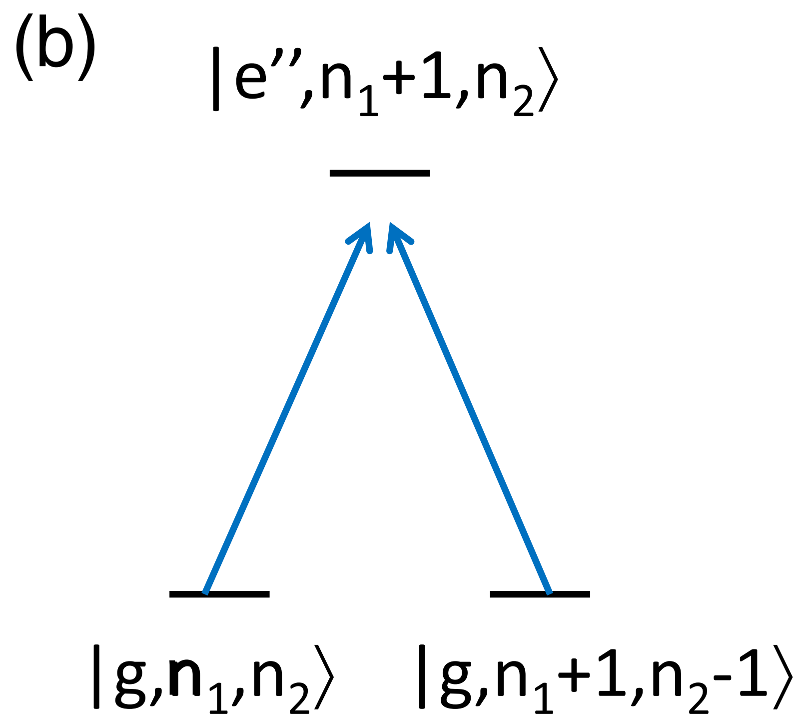
trapped ions = realization of vdP oscillators

T.E. Lee and H.R. Sadeghpour PRL 111, 234101 (2013)

coupling of two “ionic” vdP oscillators with similar frequencies:

driving the blue sideband transition of both modes ⇒ reactive coupling

$$g(b_1^\dagger b_2 + b_2^\dagger b_1)$$



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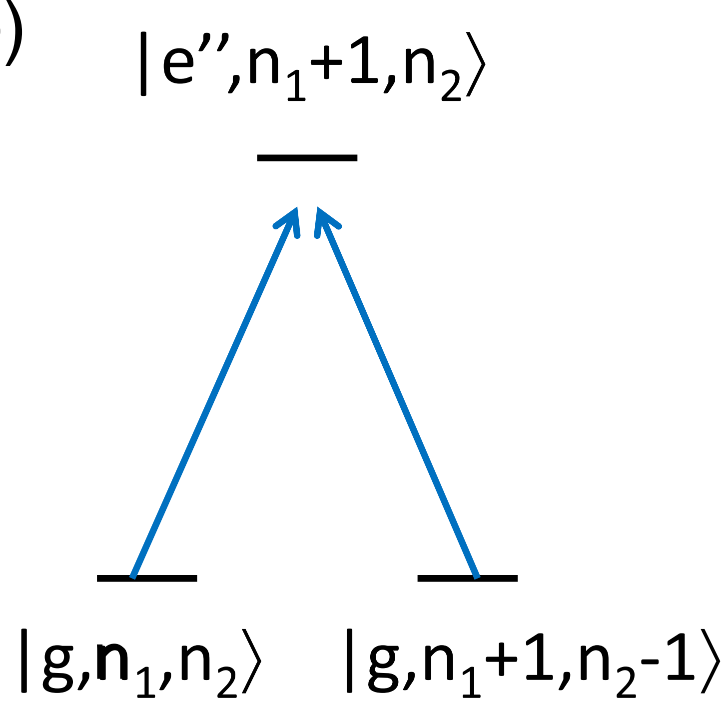
(b)



Wigner function can be directly measured

D. Leibfried et al., PRL 77, 4281 (1996)

quantum kicked rotator

O.V. Zhirov and D.L. Shepelyansky, EPJD 2006

$$H = p^2/2 - fx + K\cos(x)\sum_m \delta(t-m)$$

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additional frictional term $\quad F = -\gamma p$

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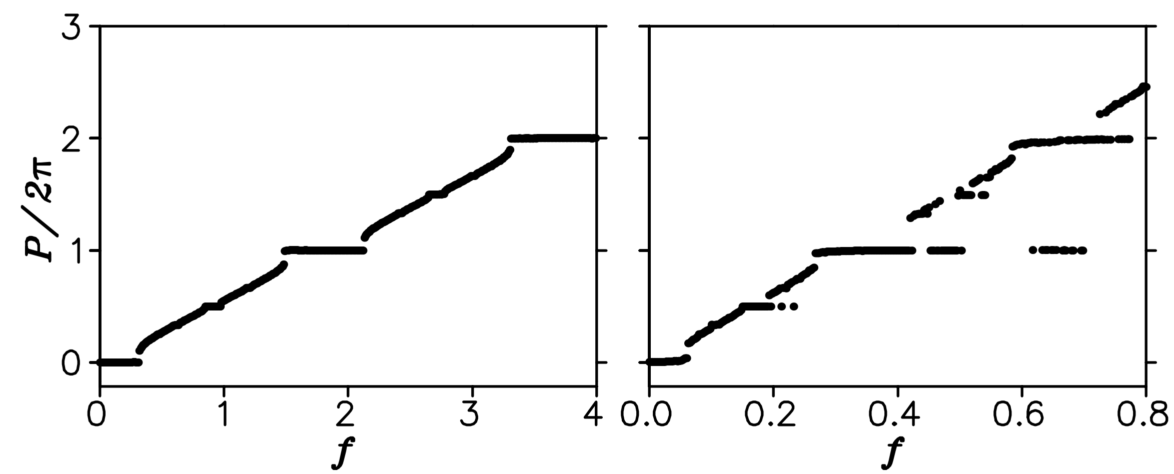
average momentum without kicks $P = \dfrac{f}{\gamma}$

with kicks, synchronization plateaus
(devil's staircase)

$\gamma = 0.25$ $\gamma = 0.05$



$K = 0.8; \quad \hbar = 0$

quantum kicked rotator

O.V. Zhirov and D.L. Shepelyansky, EPJD 2006

quantum case $\dot{\rho} = -i\,[H,\rho] + \gamma \sum_j \left[L_j \rho L_j^\dagger - \frac{1}{2}\{L_j^\dagger L_j, \rho\} \right]$

$$L_1 = \sum_{n=0} \sqrt{n+1}|n\rangle\langle n+1|$$

$$L_2 = \sum_{n=0} \sqrt{n+1}|-n\rangle\langle -n-1|$$

$|n\rangle$ momentum eigenstates

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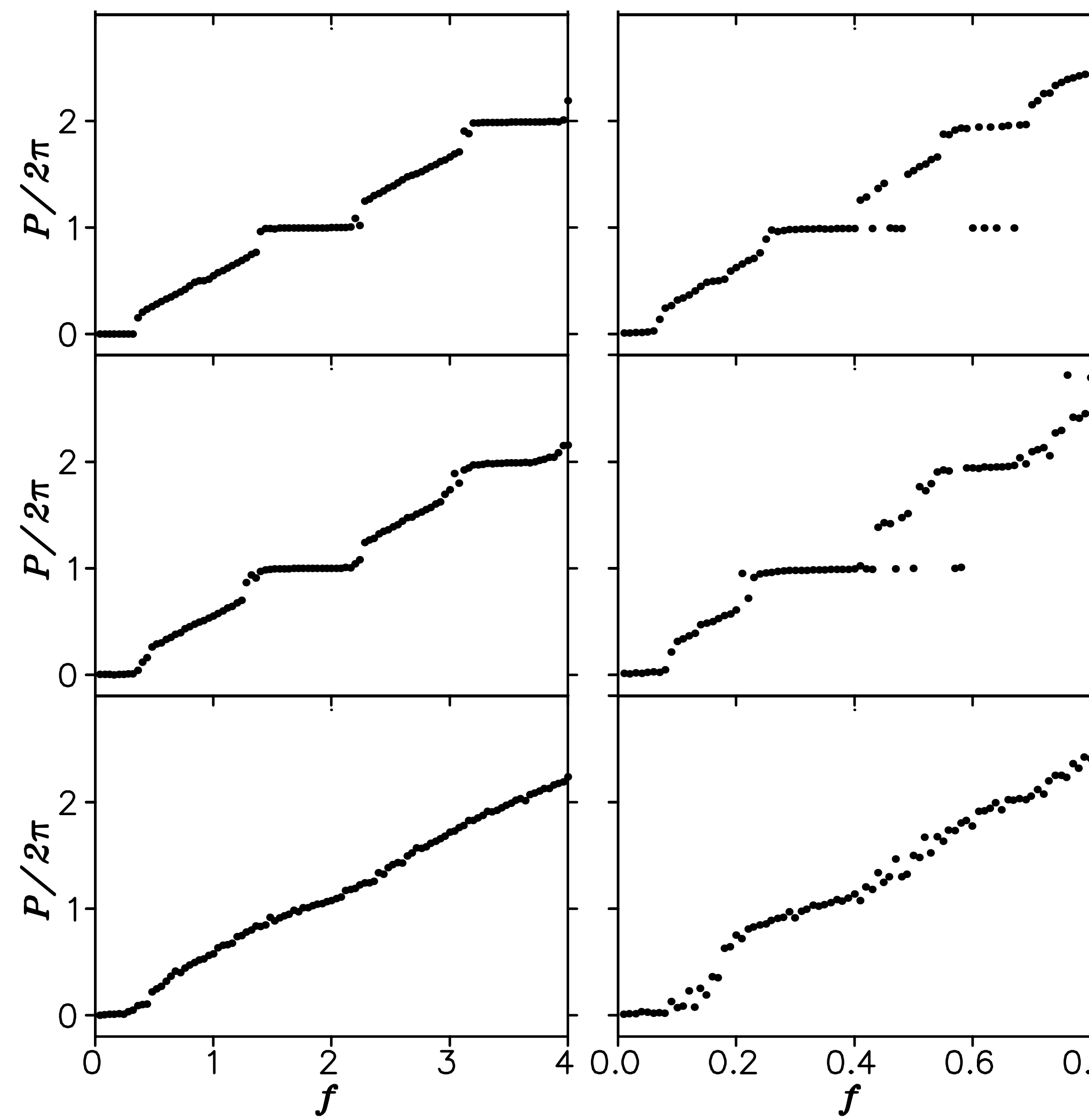

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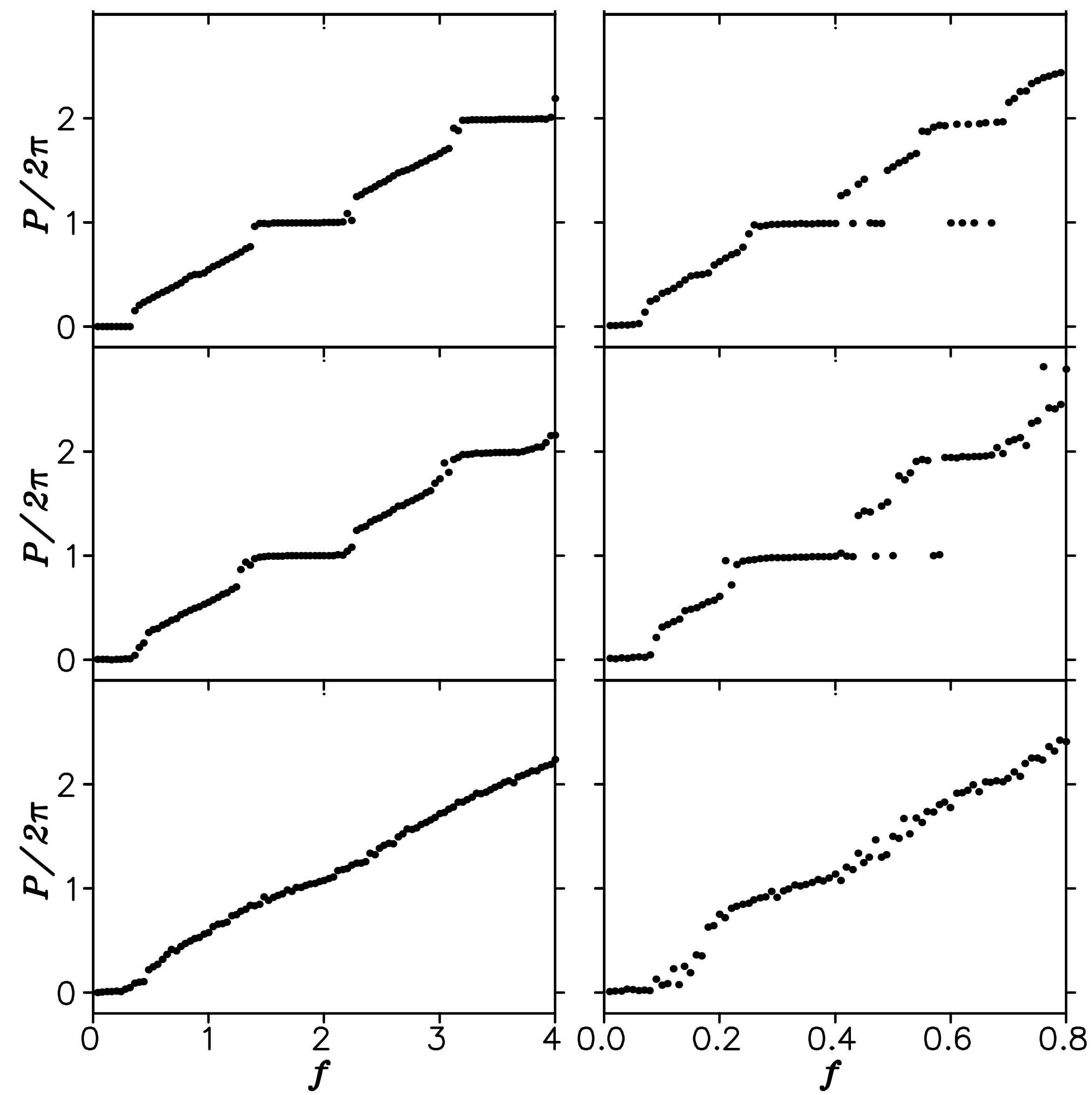
$|n\rangle$ momentum eigenstates

steps persist at small values of $\hbar$
⇒ quantum synchronization

$\gamma = 0.25$ $\gamma = 0.05$



$K = 0.8; \quad \hbar = 0.012, 0.05, 0.5$

quantum phase slip junction

J.E. Mooij and Yu.V. Nazarov, Nature Phys. 2, 169 (2006)

phase slip junction: thin superconducting wire + coherent quantum phase slips ⇒ finite voltage

duality of phase slip junction and Josephson junction:

$$(\hat{q}, \hat{\phi}) \to (-\hat{\phi}/2\pi, 2\pi\hat{q})$$

$$E_S \to E_J; \quad E_L \to E_C; \quad I \leftrightarrow R_q^{-1} V$$

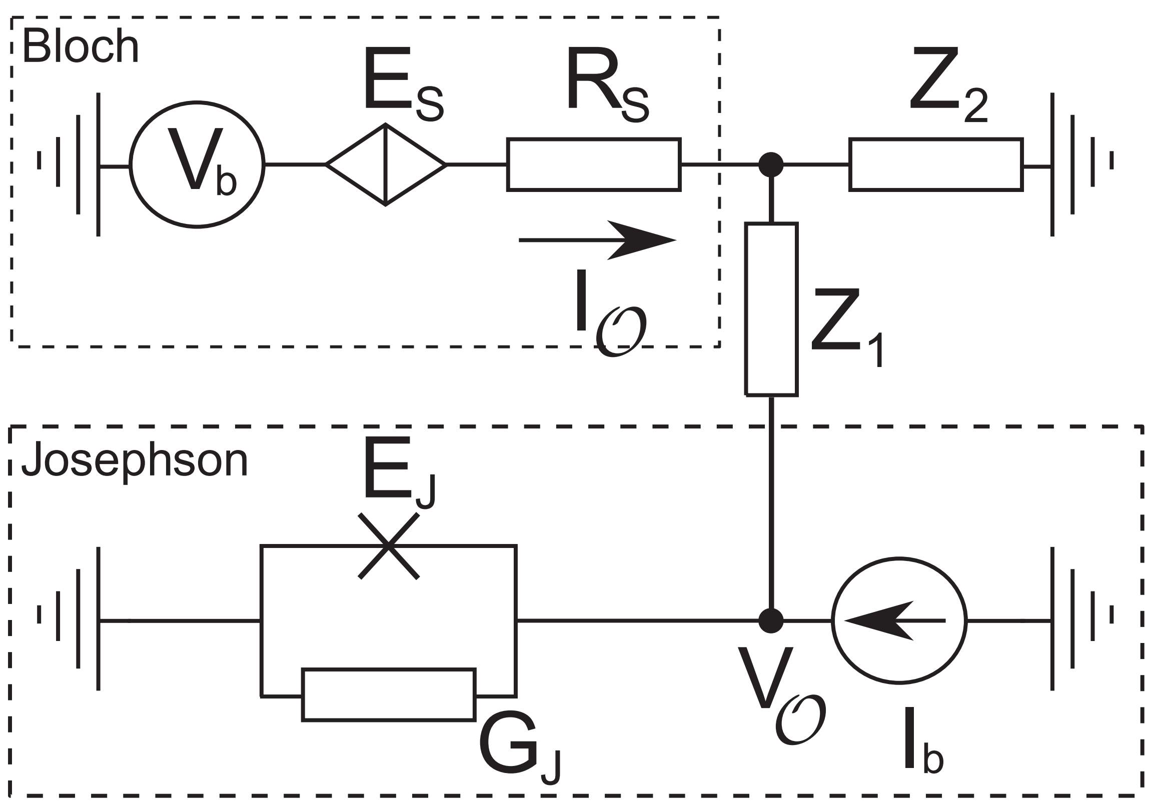
transforms $H = \frac{E_L}{(2\pi)^2}\hat{\phi}^2 - E_S \cos(2\pi\hat{q})$

into $H = E_C \hat{q}^2 - E_J \cos(\hat{\phi})$

Q synchronization of conjugated variables

A.M. Hriscu and Yu.V. Nazarov, PRL 2013

Josephson oscillator = current-biased Josephson junction
Bloch oscillator = voltage-biased phase-slip junction



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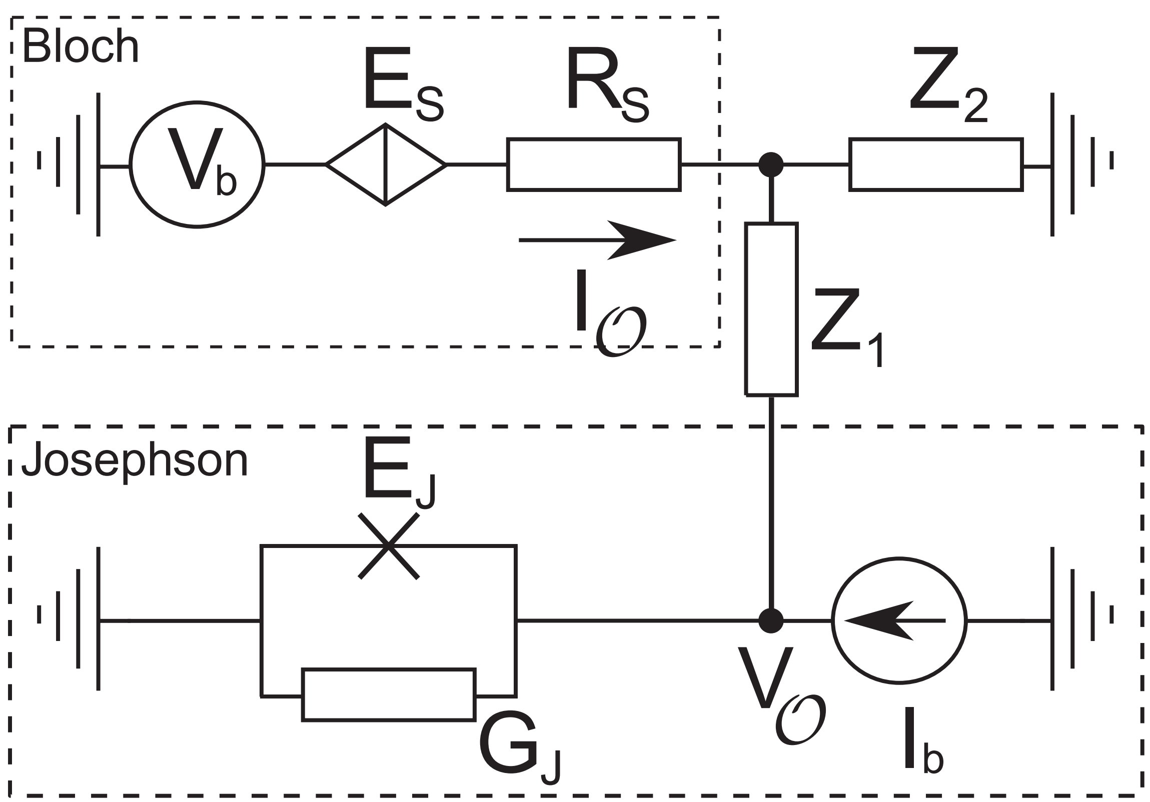

uncoupled: $Z_1 = \infty; \quad Z_2 = 0$

$$\omega_B = \frac{\pi}{e} I_{\mathcal{O}} = \frac{\pi}{e R_S} \sqrt{V_b^2 - V_C^2}$$

$$\omega_J = \frac{2e}{\hbar} V_{\mathcal{O}} = \frac{2e}{\hbar G_J} \sqrt{I_b^2 - I_C^2}$$

$$V_C = \frac{\pi E_S}{e}; \quad I_C = \frac{2e E_J}{\hbar}$$

Q synchronization of conjugated variables

A.M. Hriscu and Yu.V. Nazarov, PRL 2013

LC coupling $Z_1 = C; \quad Z_2 = L$

⇒ synchronization of the two oscillations for $n\omega_J = m\omega_B$

Q synchronization of conjugated variables

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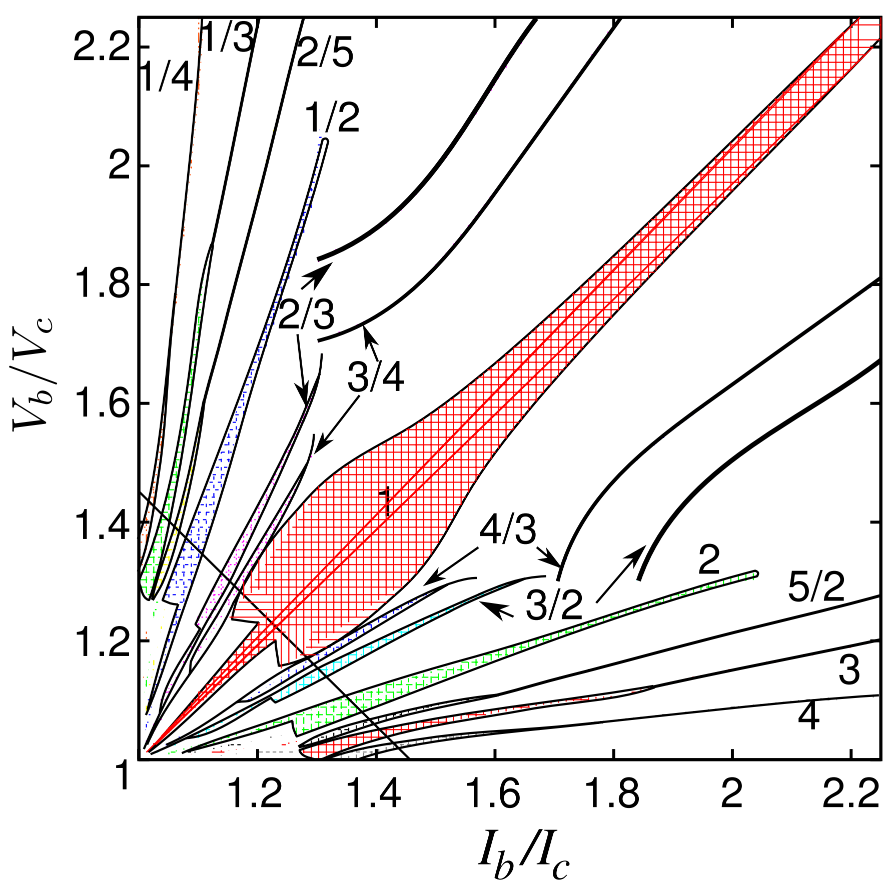

synchronization domains (n/m)

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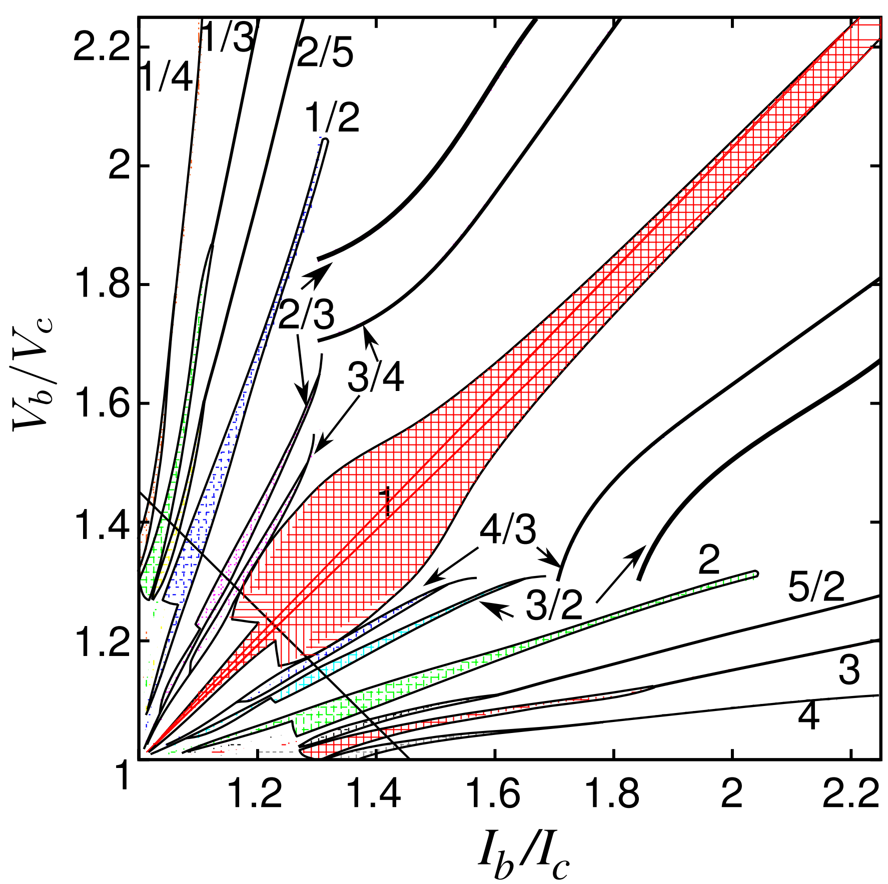
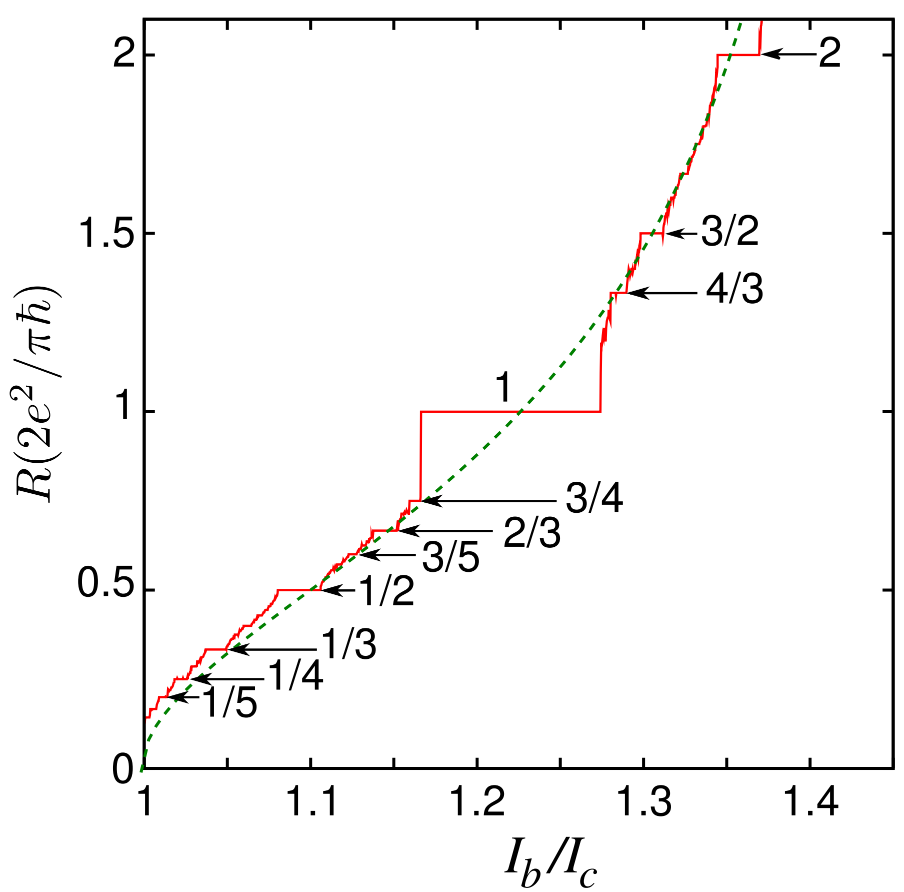
$\Rightarrow$ synchronization of the two oscillations for $n\omega_J = m\omega_B$





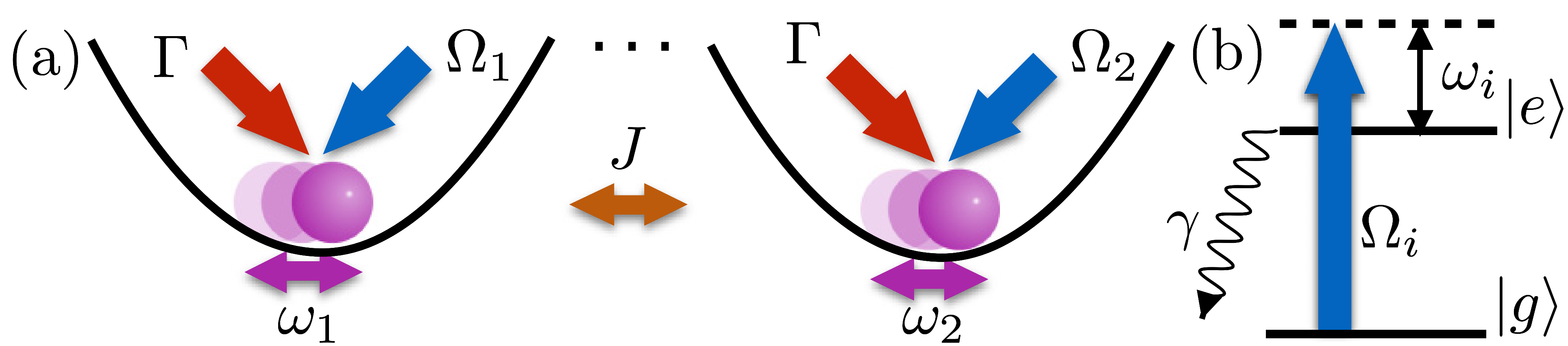
$$R = \frac{V_{\mathcal{O}}}{I_{\mathcal{O}}} = \frac{\pi\hbar}{2e^2}\frac{m}{n}$$

synchronization domains (n/m)

probe of quantum synchronization

Hush et al., PRA 91, 061401 (2015)

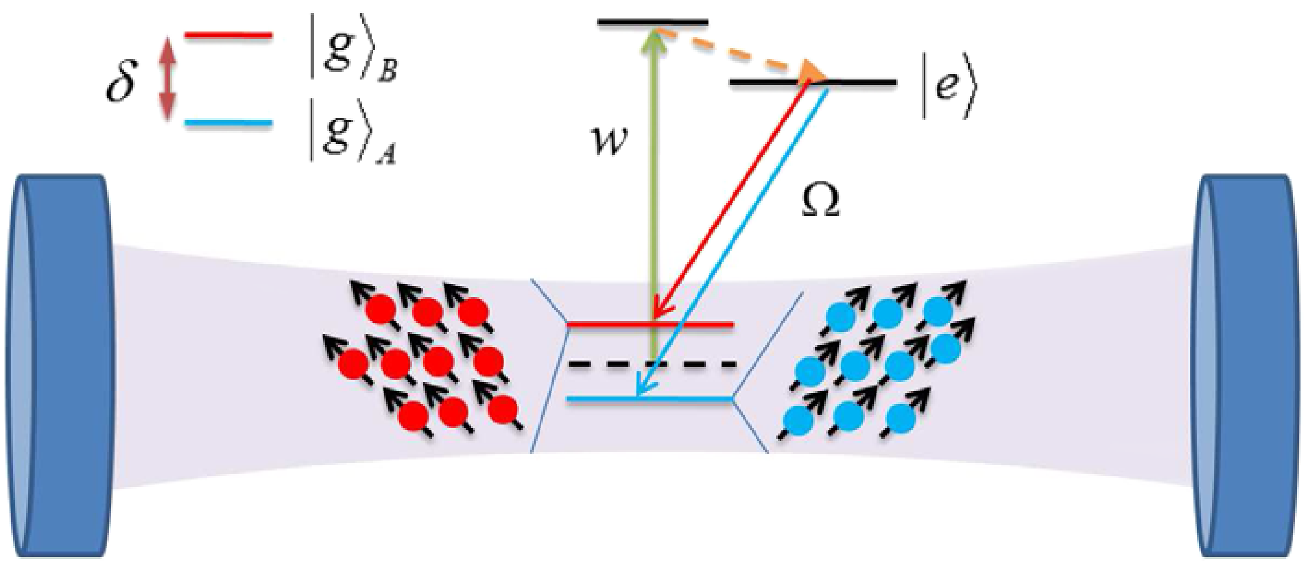
- two cold ions in microtraps
- probe synchronization by measuring correlation functions of the ions' internal states



synchronization of ensembles of atoms

M. Xu, D.A. Tieri, E.C. Fine, J.K. Thompson, M.J. Holland, PRL 113, 154101 (2014)

synchronization of two active atomic clocks to a common single-mode optical cavity



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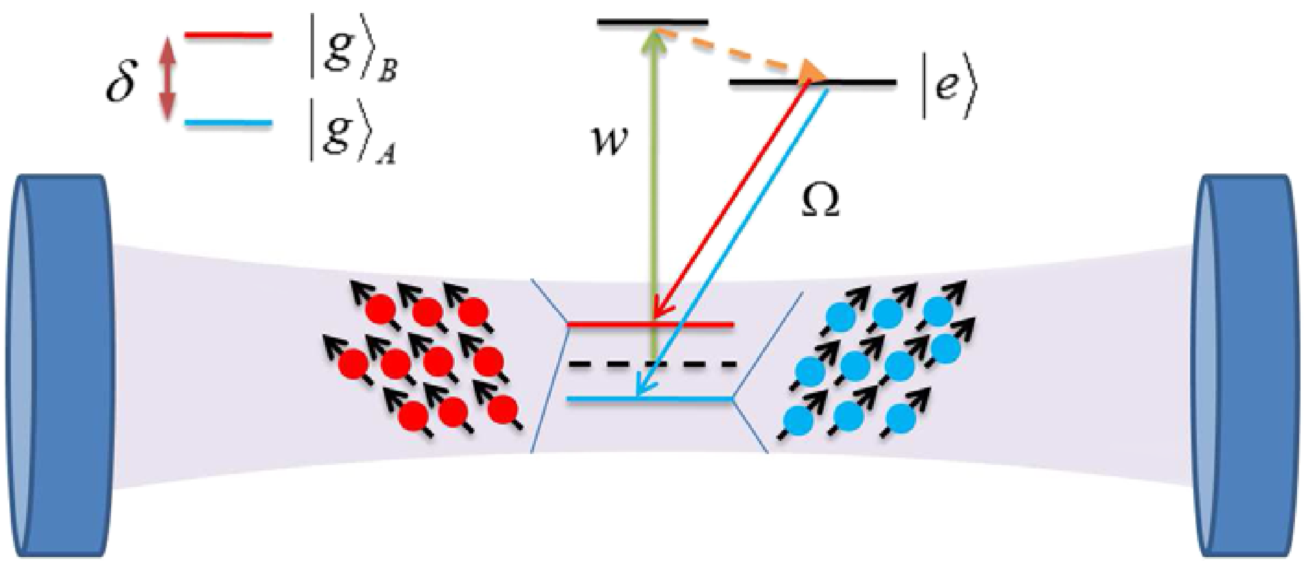
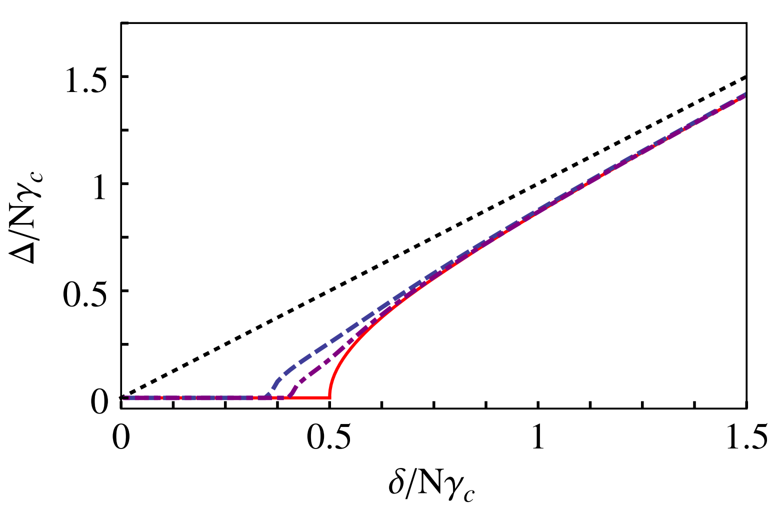




steady-state relative phase precession as a function of detuning

$$N = 100, 500, 10^6$$

energy quantization and Q sync

N. Lörch, E. Amitai, A. Nunnenkamp, CB arXiv:1603.01409

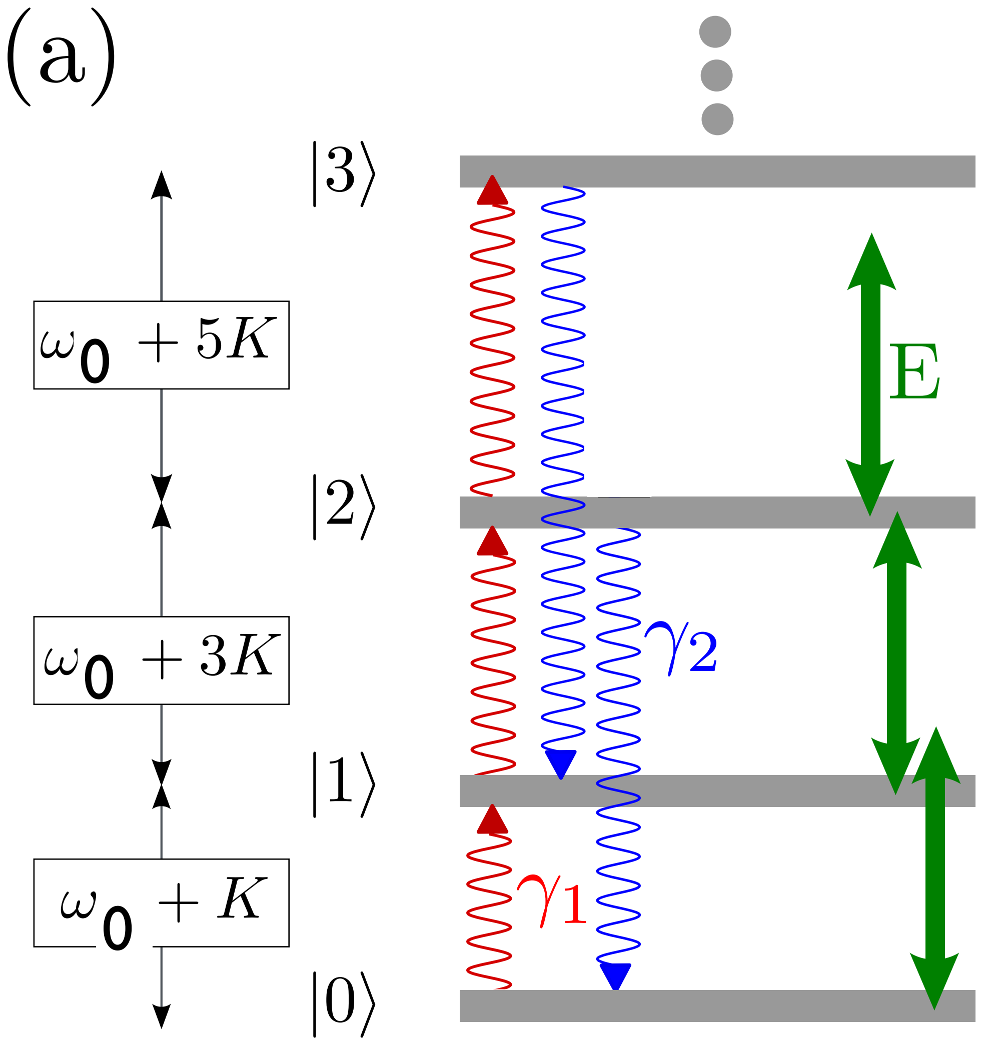
energy quantization $\hbar\omega_0$ not visible in the synchronization behavior of the quantum vdP oscillator

add a Kerr nonlinearity:

$$\hbar\omega_0 b^\dagger b \rightarrow \hbar\omega_0 b^\dagger b + K(b^\dagger b)^2$$

non-harmonic level spacing

$$\hbar\omega_0 + (2m+1)K$$



energy quantization and Q sync

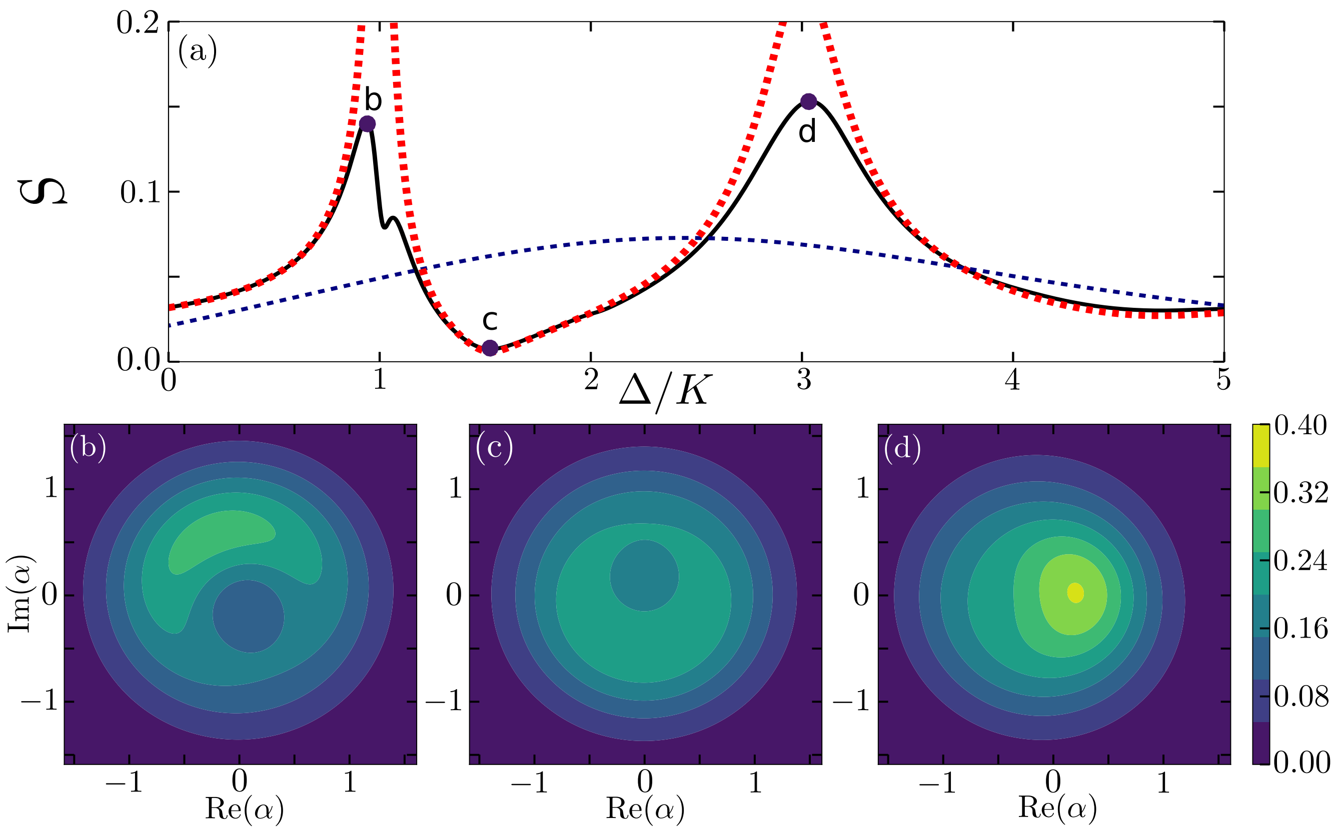
N. Lörch, E. Amitai, A. Nunnenkamp, CB arXiv:1603.01409

to study the tendency to phase locking, define $S = \frac{|\langle b \rangle|}{\sqrt{\langle b^\dagger b \rangle}}$

non-harmonic level spacing ⇒

several synchronization resonances

$$\gamma_2/\gamma_1 = 7,\ \Omega/\gamma_1 = 2.25,\ K/\gamma_1 = 50$$



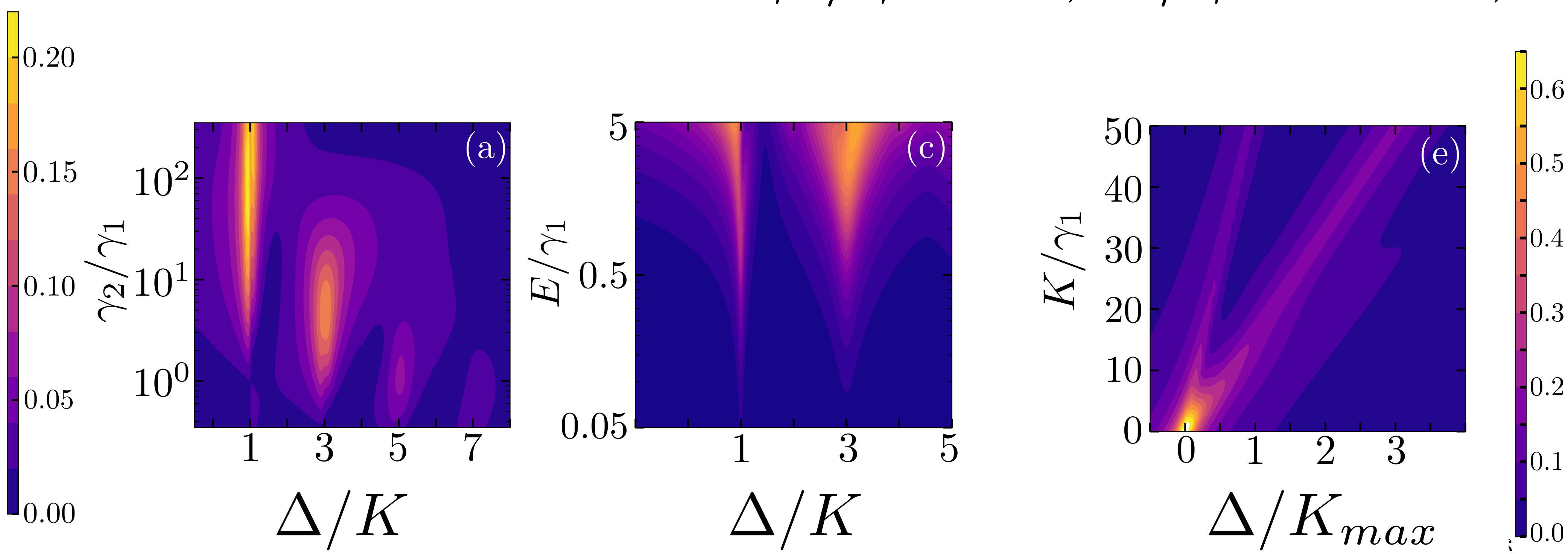
Wigner distribution

energy quantization and Q sync

N. Lörch, E. Amitai, A. Nunnenkamp, CB arXiv:1603.01409

global behavior of S

for $\gamma_2/\gamma_1 = 5,\ \Omega/\gamma_1 = 2.25,\ K/\gamma_1 = 50$



quantum chimera states

M.J. Panaggio and D.M. Abrams, Nonlinearity 28, R67 (2015)

chimera: "fire-breathing hybrid of a lion, a goat, and a snake"

quantum chimera states

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classical chimera states:

- complex spatio-temporal pattern in networks of identical oscillators
- coexistence of synchronized and unsynchronized dynamics

quantum chimera states

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chimera: "fire-breathing hybrid of a lion, a goat, and a snake"

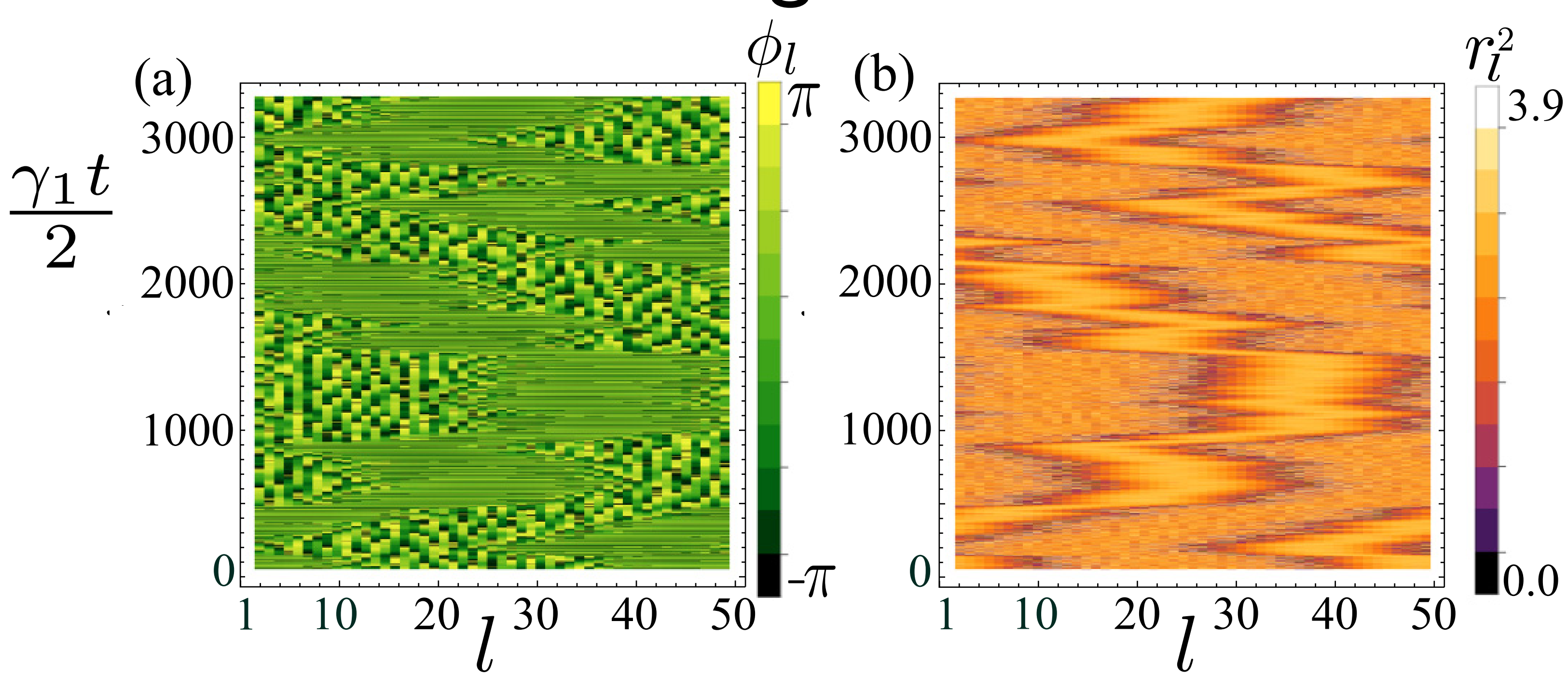
classical chimera states:

- complex spatio-temporal pattern in networks of identical oscillators
- coexistence of synchronized and unsynchronized dynamics

50 classical vdP oscillators, periodic boundary conditions, interaction range 10

$$\alpha_l(t) = r_l(t) e^{i\phi_l(t)}$$

$$\gamma_2/\gamma_1 = 0.2$$

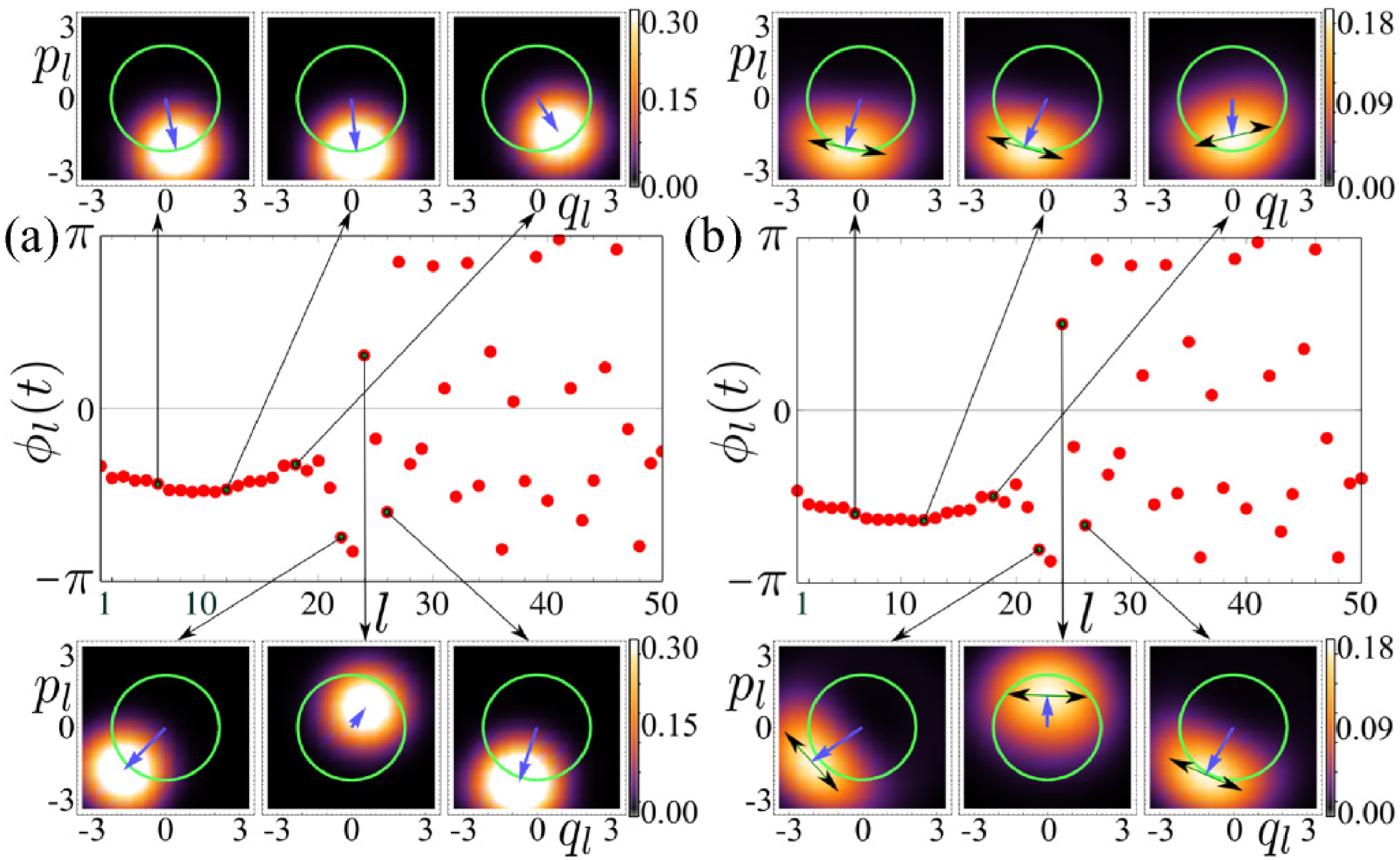


Bastidas et al., PRE 92, 062924 (2015)

quantum chimera states

quantum chimera state?

Bastidas et al., PRE 92, 062924 (2015)



(a) $\rho(t_0)$ "classical"; tensor product of coherent states

(b) after short time, quantum correlations develop (squeezing)

quantum chimera states

quantum chimera state? Bastidas et al., PRE 92, 062924 (2015)

covariance matrix $\mathcal{C}_{ij} = \langle \frac{1}{2}(R_i R_j + R_j R_i) \rangle - \langle R_i \rangle \langle R_j \rangle$

measure for quantum fluctuations in $R_{2l-1} = q_l; \quad R_{2l} = p_l$

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quantum chimera state? Bastidas et al., PRE 92, 062924 (2015)

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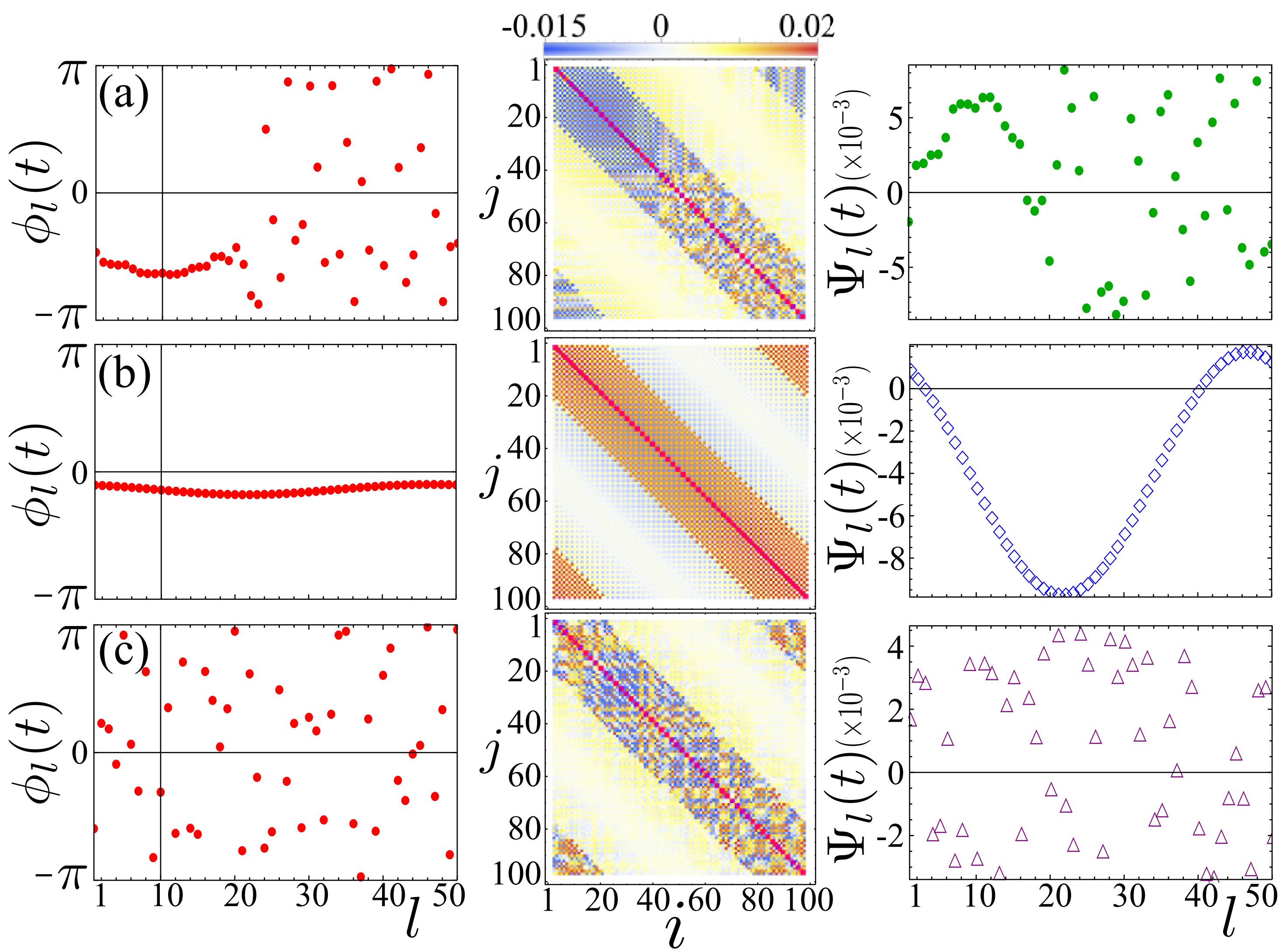

$\mathcal{C}_{ij}$ reflects (un-)synchronized regions

$$\Psi_l(t) \sim \sum_{m=l-d}^{l+d} \mathcal{C}_{2l,2m}(t)$$

conclusion

quantum synchronization can have a variety of meanings:

- (approximate) locking of a quantum self oscillator to an external harmonic drive
- (approximate) spontaneous phase synchronization of two or more quantum self oscillators
- collective coherent motion of an array of equal oscillators
- synchronization of two conjugate variables (like charge and phase)
- phase locking of two atomic ensembles coupled to a single-mode cavity

outlook / open questions

- quantum synchronization measures
- finitely many oscillators / infinite arrays
- pattern formation in arrays; chimera states
- impact on quantum metrology

collaborators

Ehud Amitai

Niels Lörch

Andreas Nunnenkamp, Basel → Cambridge

Stefan Walter, Basel → Erlangen

S. Walter, A. Nunnenkamp, and C. Bruder, Phys. Rev. Lett. 112, 094102 (2014)
S. Walter, A. Nunnenkamp, and C. Bruder, Ann. Phys. (Berlin) 527, 131 (2015)
N. Lörch, E. Amitai, A. Nunnenkamp, and C. Bruder, arXiv:1603.01409

appendix

quantum phase slip junction

J.E. Mooij and Yu.V. Nazarov, Nature Phys. 2, 169 (2006)

duality of phase slip junction and Josephson junction:

$$(\hat{q}, \hat{\phi}) \to (-\hat{\phi}/2\pi, 2\pi\hat{q})$$

$$E_S \to E_J; \quad E_L \to E_C; \quad I \leftrightarrow R_q^{-1} V$$

transforms $H = \frac{E_L}{(2\pi)^2}\hat{\phi}^2 - E_S \cos(2\pi\hat{q})$

into $H = E_C\hat{q}^2 - E_J \cos(\hat{\phi})$

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even more general