Nano- and optomechanics

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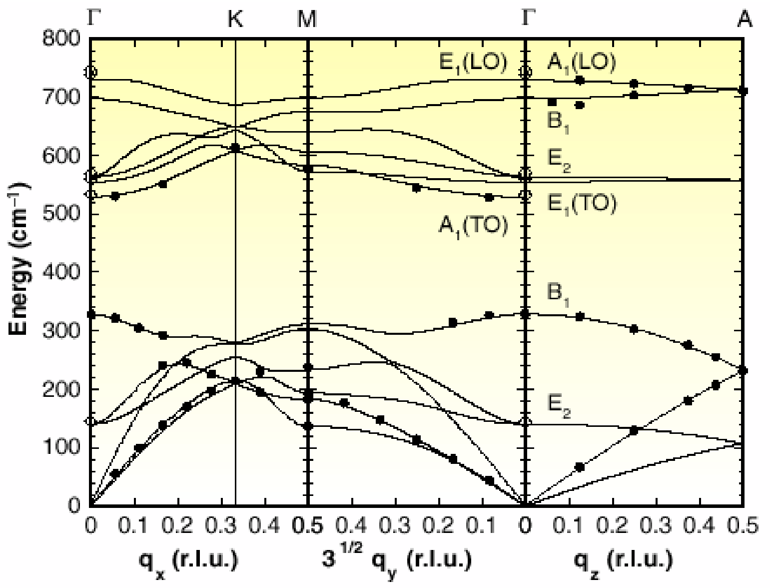
- General optomechanics
- Quantum and non-linear optomechanics
- Nanomechanics

Nano- and optomechanics

As seen by a condensed matter physicist



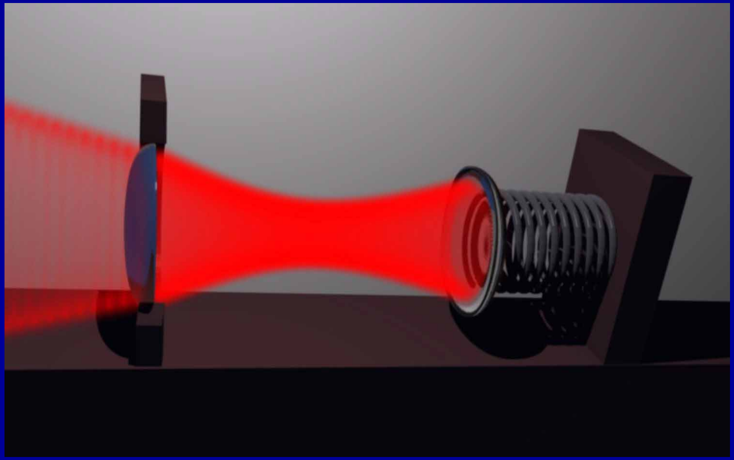
Wurtzite phonon spectrum, Ruf et al, PRL 2001

- Discrete vibration modes
- (only select one or few)
- Interact with electrons

Magnetostriction: interaction between magnetization and mechanical modes

Nano- and optomechanics

As seen by an AMO physicist

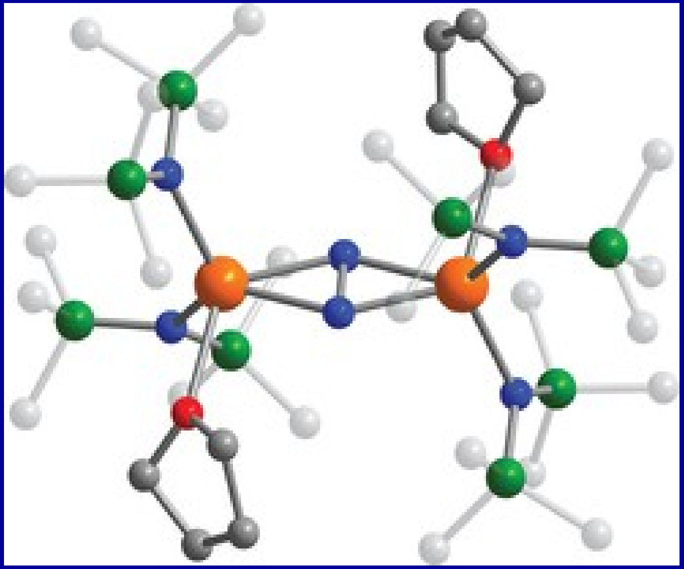


Kippenberg's group website

- One light mode
- One or two vibration modes
- Radiation pressure interaction

Nano- and optomechanics

As seen by a nanophysicist or a nanochemist



– spin-phonon interaction

(different configurations lead to different total spin)

Gd-based single molecular magnet
Rinehart et al, Nat. Chem. **22**, 538 (2011)

Nano- and optomechanics

Mechanical elements can be coupled to:

- charge
- light (photons)
- spin (magnetization)

What do we need to understand?

- coupling strength
- dissipation
- non-linearity

Applications?

- actuators
- detectors
- in quantum information

Optomechanics

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- Hamiltionian
- Coupling
- Static effects
- Optomechanically induced transparency
- Mechanical squeezing of light

Topical review: Aspelmeyer, Kippenberg, and Marquardt Rev. Mod. Phys. **86**, 1391 (2014)

LIGO

The Laser Interferometer Gravitational-Wave Observatory

Michelson-Morley interferometer

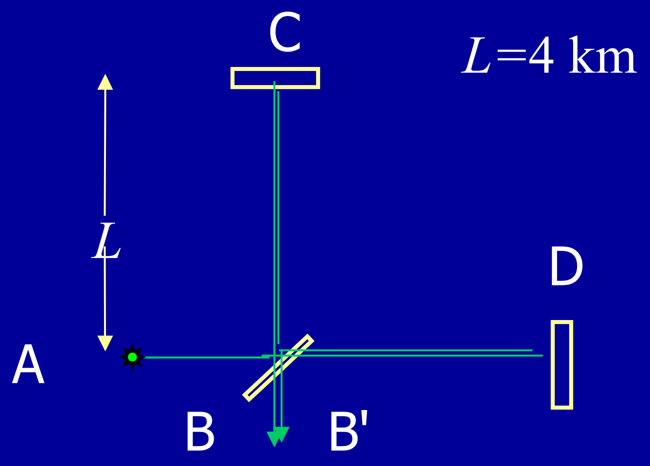


Photo credit: Cfoellmi

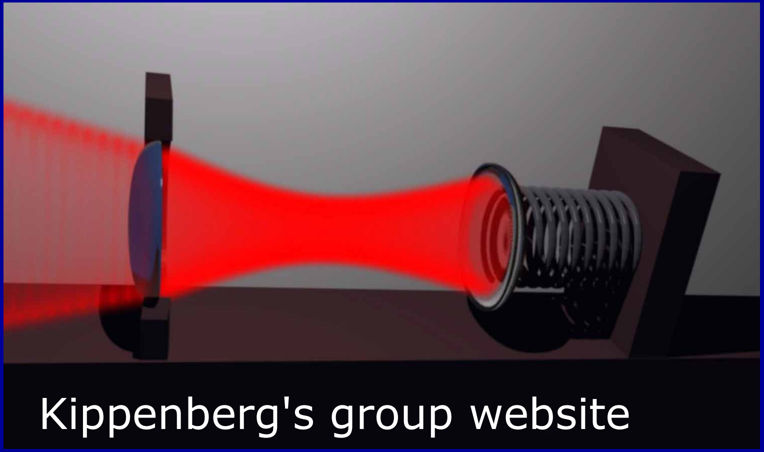
Principle: gravitational waves distort space-time

Relative distortion needed to measure: 10^{-21}

Cavity/circuit optomechanics

Movable mirror

Static mirror



Kippenberg's group website

Radiation pressure coupling

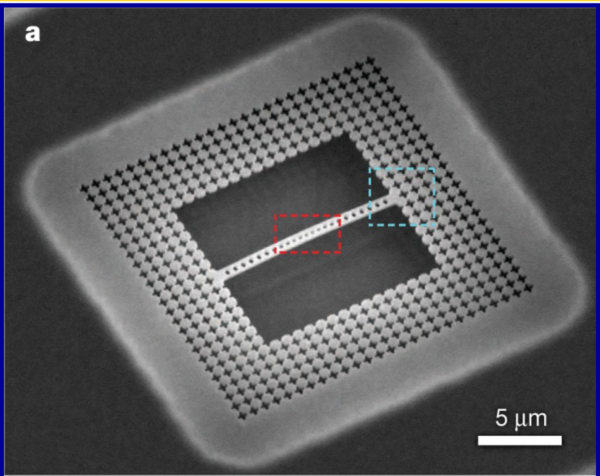
$$H = \hbar\omega_{cav}\hat{a}^{\dagger}\hat{a} + \hbar\omega_{m}\hat{b}^{\dagger}\hat{b} - \hbar g_{0}\hat{a}^{\dagger}\hat{a}(b^{\dagger} + b) \quad \omega_{cav}(x)$$

Cavity

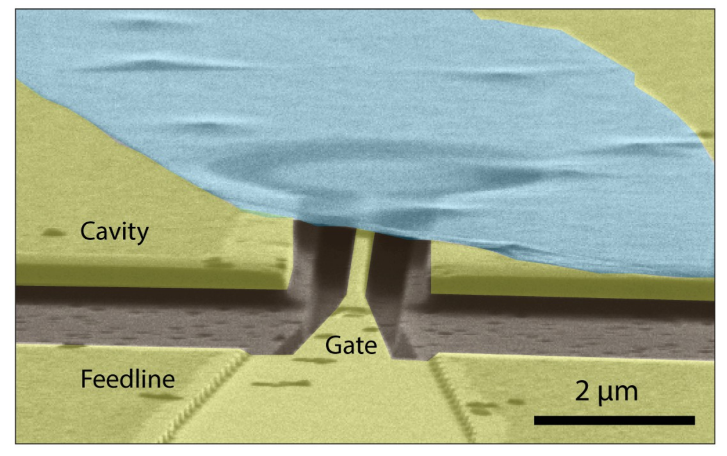
Mechanical resonator

$$\hat{x} = x_{ZPM}\left(\hat{b} + \hat{b}^{\dagger}\right)$$

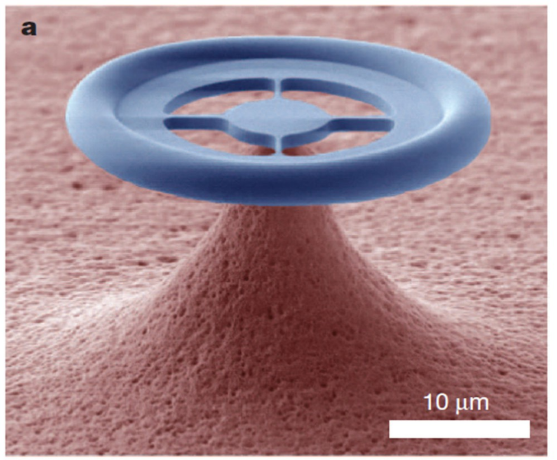
Cavity/circuit optomechanics



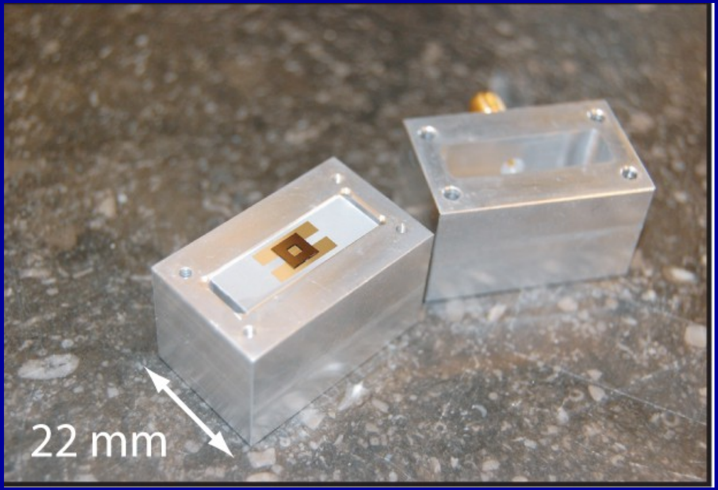
Chan et al, Nature **478**, 89 (2011)



Singh et al, Nature Nanotech. **9**, 820 (2014)



Verhagen et al, Nature **482**, 63 (2012)



Yuan et al, Nature Comms. 6, 8491 (2015)

Coupling

$$H = \hbar\omega_{cav}\hat{a}^{\dagger}\hat{a} + \hbar\omega_{m}\hat{b}^{\dagger}\hat{b} - \hbar g_{0}\hat{a}^{\dagger}\hat{a}(b^{\dagger} + b)$$

Dissipation rate in the cavity

Sideband-resolved regime

$$\Gamma, \kappa \ll \omega_{m} \ll \omega_{cav}$$

Where is g_0?

Weak coupling ← → Strong coupling

Driving and linearization: $g = g_0\sqrt{n_{cav}}$ $\hbar g\left(\hat{a}^{\dagger}b + \hat{a}b^{\dagger}\right)$

Coupling

$$H_{\text{int}} = -\hbar g_0 \hat{a}^\dagger \hat{a}(b^\dagger + b) \rightarrow -\hbar g(\hat{a}^\dagger + \hat{a})(b^\dagger + b)$$

Non-resonant? Depends how we drive. $g = g_0\sqrt{n_{cav}}$

In the rotating frame: $\sqrt{n_{cav}} \propto e^{i\omega_d t}; a \propto e^{i\omega_{cav} t}; b \propto e^{i\omega_m t}$

Red-detuned drive: $\omega_d = \omega_{cav} - \omega_m$

$$H_{\text{int}} = -\hbar g(\hat{a}^\dagger b + \hat{a} b^\dagger)$$

Blue-detuned drive: $\omega_d = \omega_{cav} + \omega_m$

$$H_{\text{int}} = -\hbar g(\hat{a}^\dagger b^\dagger + \hat{a} b)$$

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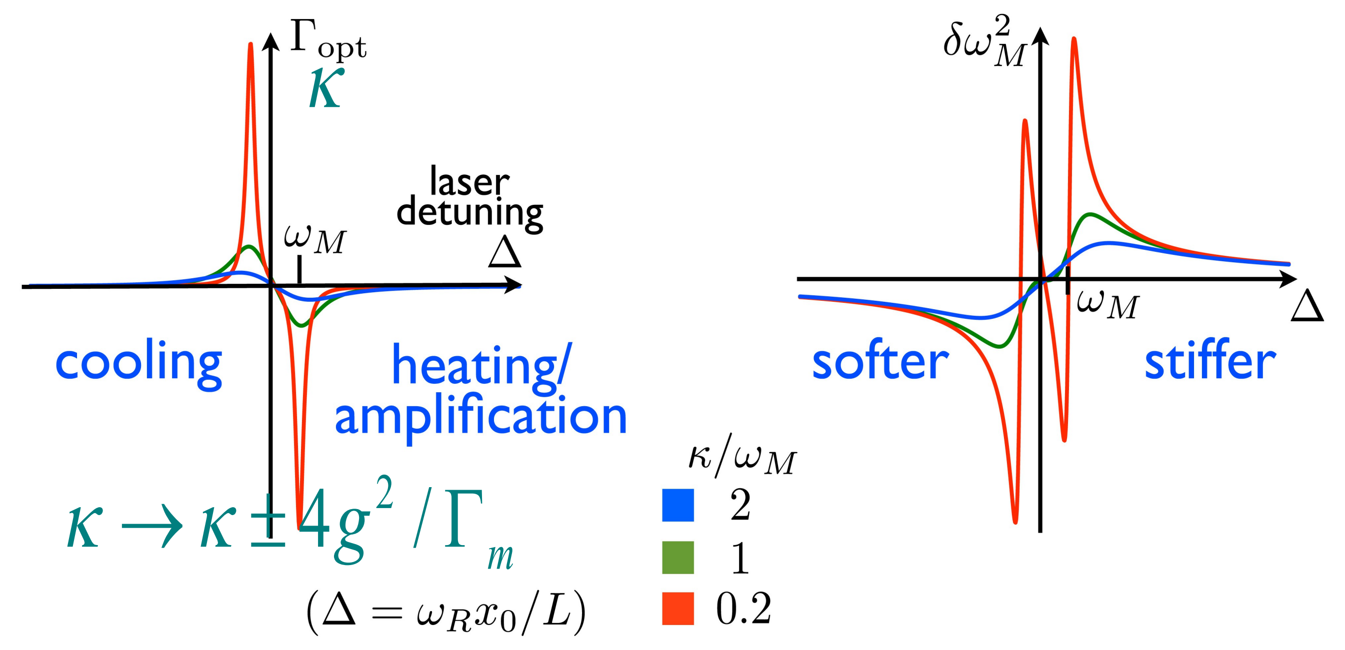
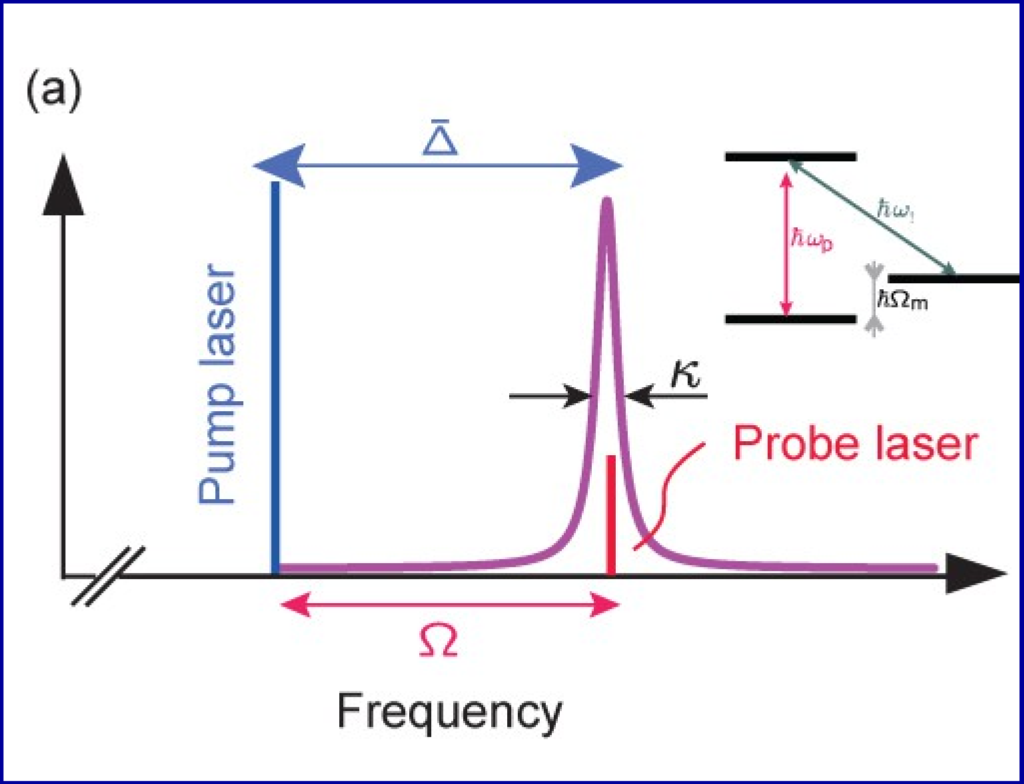
Static effects

If we only look at the mechanical resonator:
- Equilibrium position is shifted
- Frequency is renormalized (optical spring)
- Damping coefficient is renormalized
- Non-linearity appears and can lead to instabilities



From:Florian Marquardt Windsor School 2010

Optomechanically induced transparency

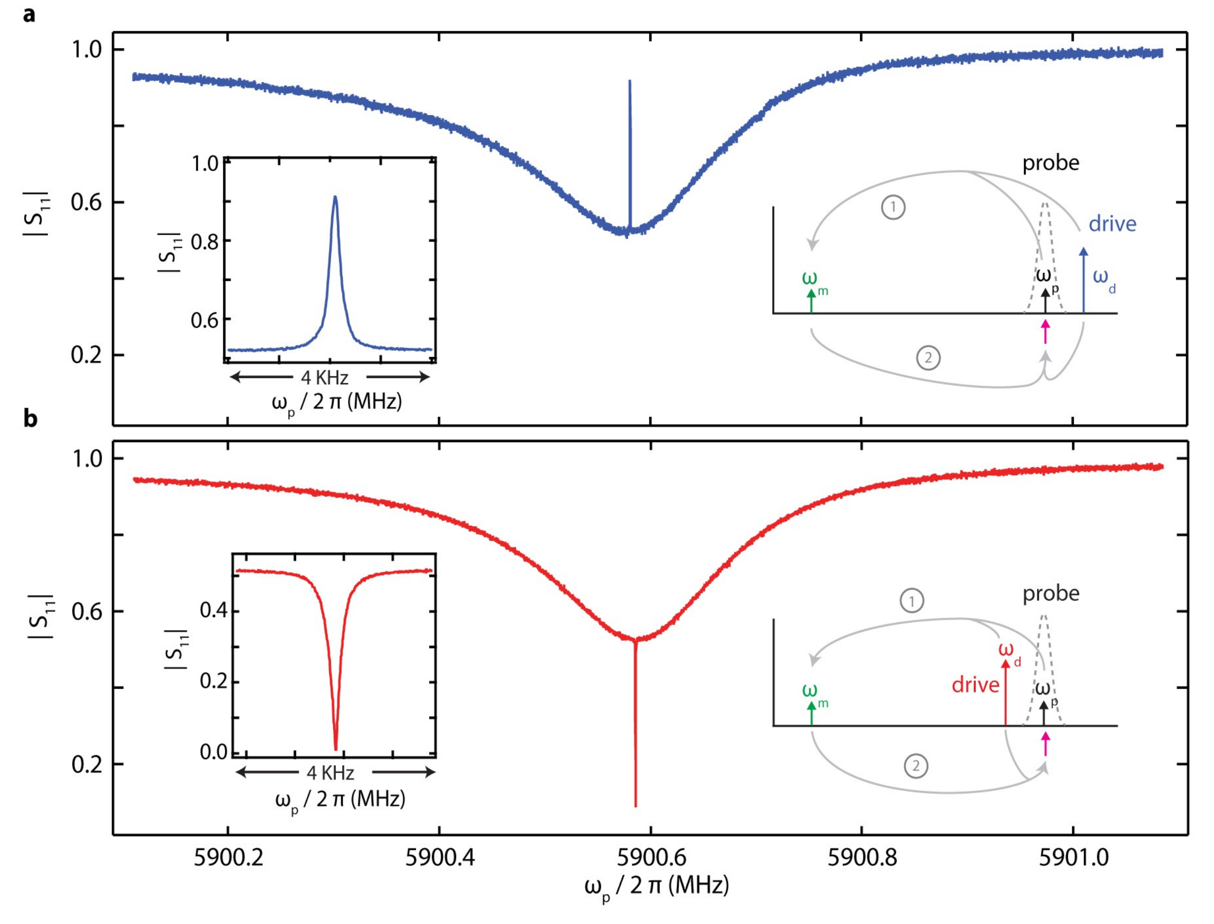


From: Aspelmeyer, Kippenberg, and Marquardt Rev. Mod. Phys. **86**, 1391 (2014)

Cavity is strongly red-driven at $\omega_{cav} - \omega_m$ (red-detuned)

Probe laser measures the transmission around the cavity resonance

Optomechanically induced transparency



Singh et al, Nature Nanotech. **9**, 820 (2014)

First observation:
S. Weis et al, Science 330, 1520 (2010)

Constructive interference results in OMIT; the width is Γ_m

Input-output relations

Langevin equations for the creation/annihilation operators:

$$\frac{d\hat{a}}{dt} = \left(i\Delta - \frac{\kappa}{2} \right)\hat{a} - ig\hat{x}\hat{a} + \sqrt{\kappa_{ext}}\, s_{in} + \sqrt{(1-\eta_c)\kappa}\,\delta\hat{s}_{vac}(t)$$

Detuning and dissipation in the cavity

Input signal

Quantum noise

$$\frac{d\hat{x}}{dt} = \frac{\hat{p}}{m}$$

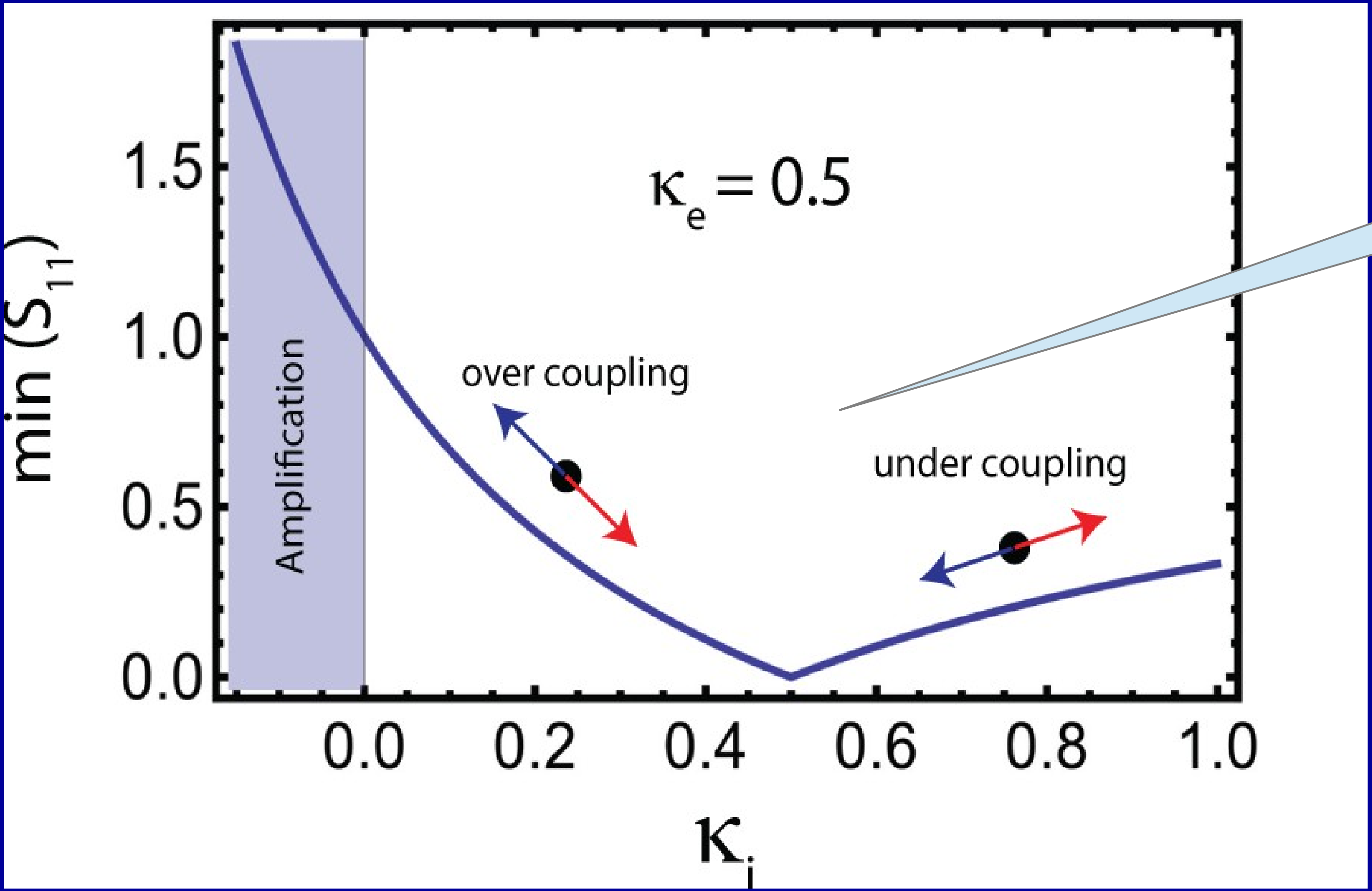
$$\frac{d\hat{p}}{dt} = -m\omega_m{}^2\hat{x} - \alpha\hat{x}^3 + \hbar g\hat{a}^\dagger\hat{a} - \Gamma_m\hat{p} + \delta F_{th}(t)$$

Coupling

Mechanical dissipation

Thermal noise

Strong coupling



Impedance matching

Singh et al, Nature Nanotech. **9**, 820 (2014)

$$\kappa \to \kappa \pm 4g^2 / \Gamma_m$$

What happens to the cavity?

It becomes non-linear:

can be used for squeezing

T. P. Purdy et al, Phys. Rev. X 3, 031012 (2013)

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Coherent states

Formal definition: $|\alpha\rangle = \exp\left[\alpha \hat{a}^{\dagger} - \alpha^{*} \hat{a}\right]|0\rangle$

Eigenstate of the annihilation operator: $\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$

In terms of the Fock states: (Poisson distribution)

$$|\alpha\rangle = \exp\left(-|\alpha|^2/2\right)\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle$$

The coherent state is a minimum uncertainty state $\Delta p \Delta q = \hbar/2$

$$q \propto \hat{a} + \hat{a}^{\dagger};\ p \propto \hat{a} - \hat{a}^{\dagger}$$

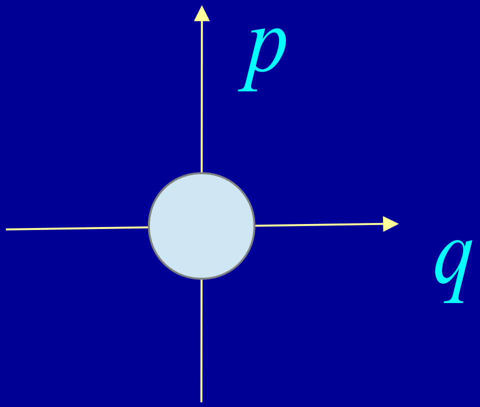
Wigner function

Formal definition:

$$W(q,p) = \frac{1}{4\pi}\int_{-\infty}^{\infty} \langle q+q' | \rho | q-q' \rangle \exp\left(\frac{iq'p}{2}\right) dq'$$

Quasi-probability distribution

For a coherent state: $W(q,p) = \frac{2}{\pi}\exp\left(-\frac{1}{2}\left(p^2+q^2\right)\right)$



$\alpha = 0$

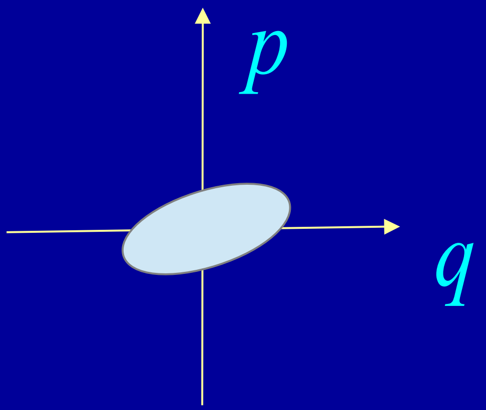
Squeezed states

Also states with the minimum uncertainty but with : $\Delta p \neq \Delta q$

$$|\alpha, \xi\rangle = S(\xi) \exp\left[\alpha \hat{a}^\dagger - \alpha^* \hat{a}\right] |0\rangle$$

$$S(\xi) = \exp\left[\frac{1}{2}\xi^* \hat{a}^2 - \frac{1}{2}\xi \hat{a}^{\dagger 2}\right]$$

Wigner function:



Mechanical squeezing of light

How to detect squeezed states: Measure noise

$$S_q(\omega) = \int d(t-t')q(t)q(t')\exp(i\omega(t-t'))$$