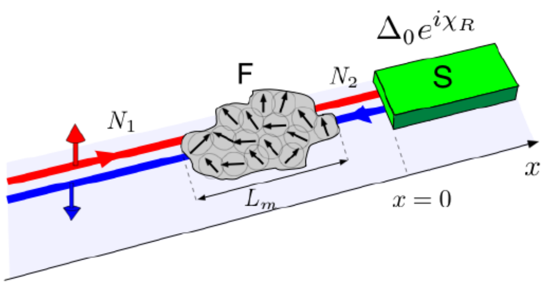
Superconducting hybrids based on QSH systems

Lecture 3

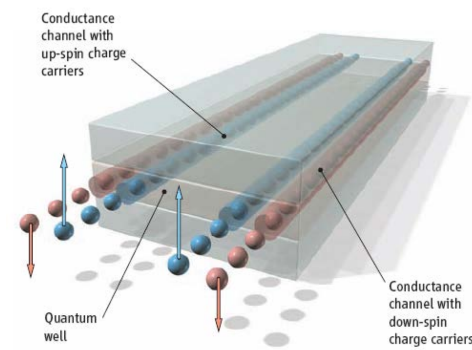


11th Capri Spring School

Transport in Nanostructures

April 13-17, 2015

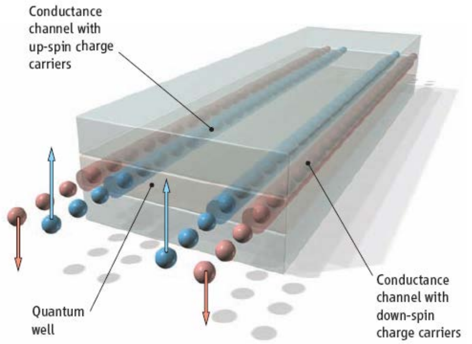
Capri, Italy

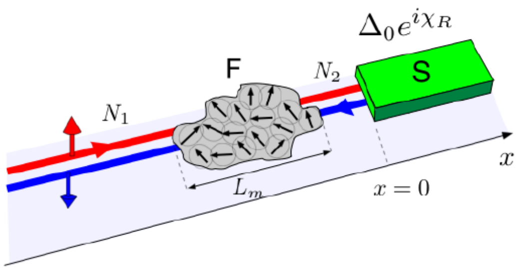


Björn Trauzettel

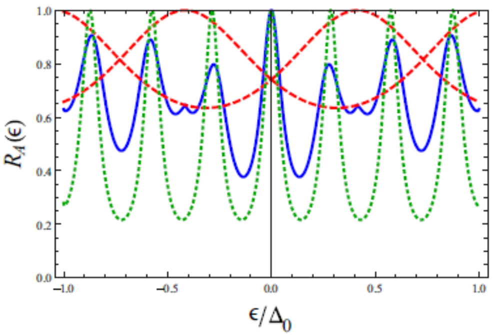
Pablo Burset (Uni Würzburg)
François Crépin (Uni Würzburg)
Fabrizio Dolcini (PolyTech Torino)
Florian Geissler (Uni Würzburg)
Ewelina Hankiewicz (Uni Würzburg)
Naoto Nagaosa (Tokyo University)
Yukio Tanaka (Nagoya University)
Grigory Tkachov (Uni Würzburg)

Summary Lecture 1



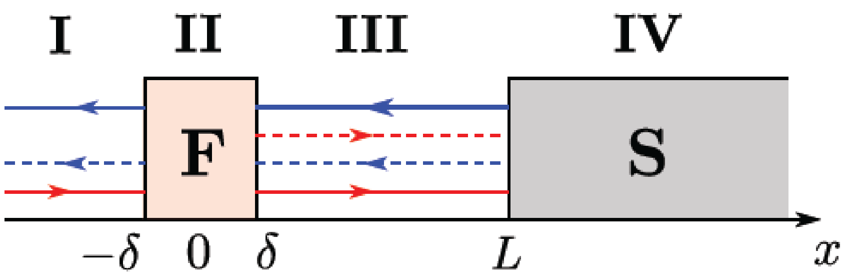


$$H_{BdG} = \begin{pmatrix} H^{e}_{0+FM} & \Delta(x)\sigma_0 \\ \Delta^{*}(x)\sigma_0 & H^{h}_{0+FM} \end{pmatrix}$$

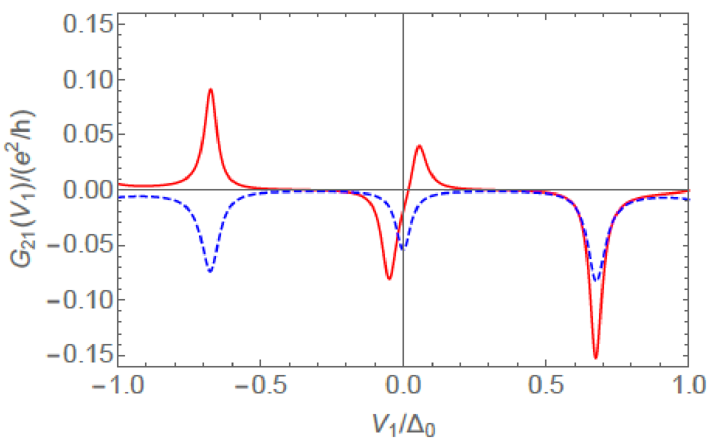


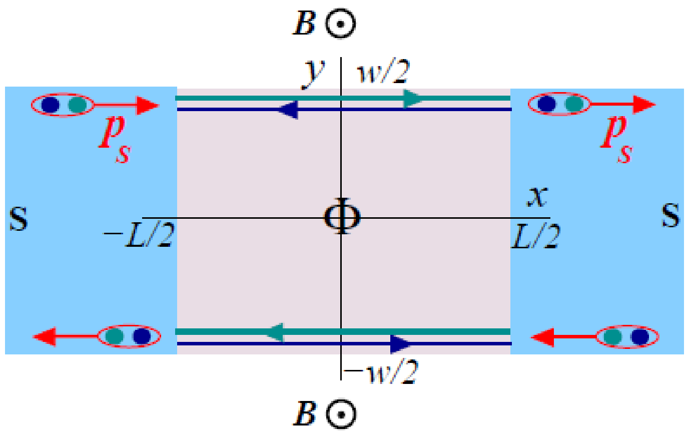
(c) $\mu_0 = 0.35, \mu = 0.3, L_m = 2.15, x_0 = 6$

Summary Lecture 2

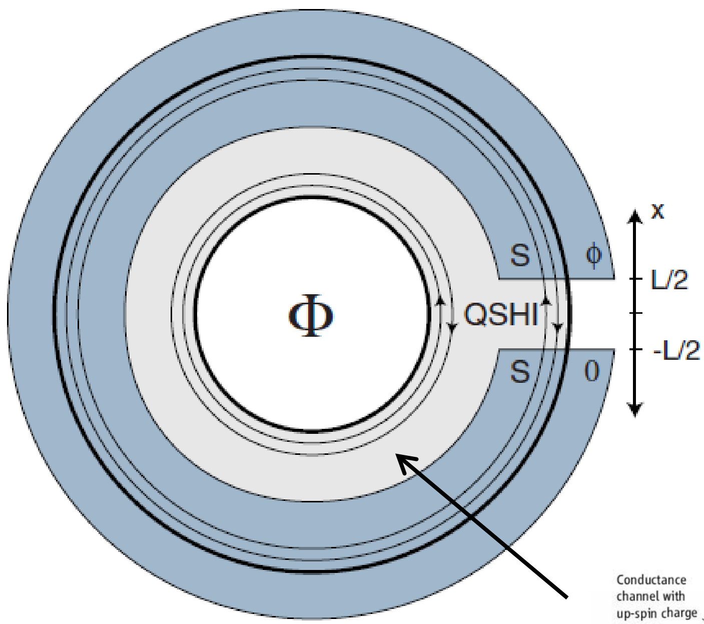


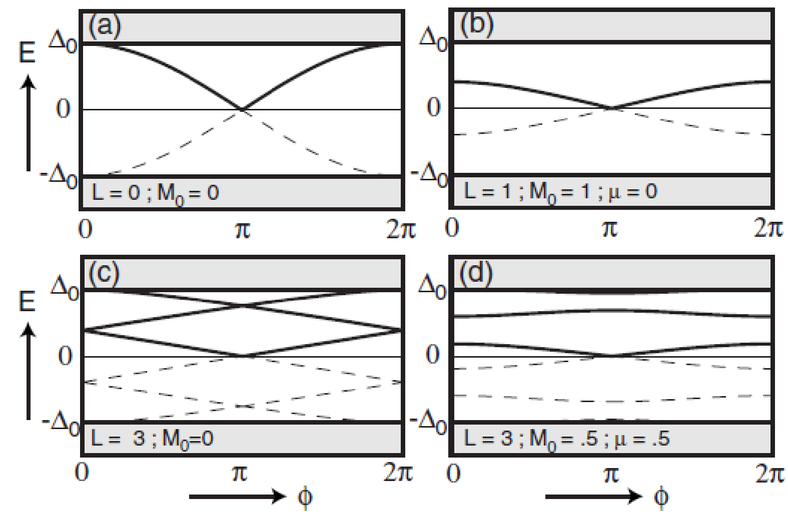
	Pairing	Interface	Bulk
f_0	$\uparrow\downarrow - \downarrow\uparrow$	ESE+OSO	ESE
f_3	$\uparrow\downarrow + \downarrow\uparrow$	ETO+OTE	ETO
$f_\pm$	$\uparrow\uparrow$, $\downarrow\downarrow$	OTE	X

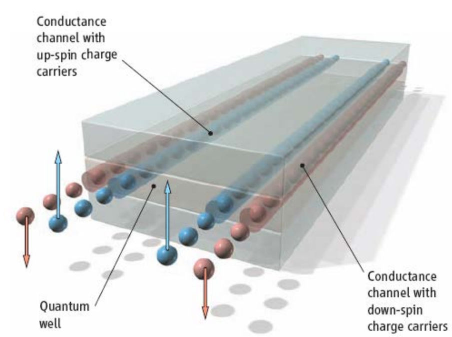




Pioneering prediction







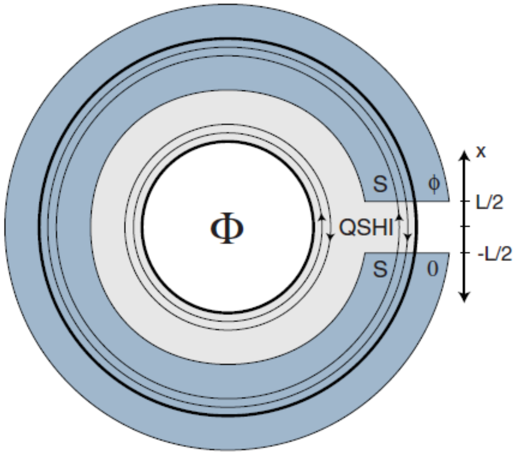
signatures of **p-wave** superconductivity?

Fu & Kane PRB 2009

Outline

- **Parity constraints** in topological JJs
- **Short junction limit:** simple current-phase relations -> parity measurement
- **Long junction limit:** bosonization -> influence of Coulomb interactions

S/QSHI/S: rf SQUID geometry



$$H = \frac{1}{2}\int dx \Psi^\dagger H_{BdG} \Psi$$

$$\Psi^\dagger = \left(\psi^\dagger_{R\uparrow}, \psi^\dagger_{L\downarrow}, \psi_{L\downarrow}, -\psi_{R\uparrow}\right)$$

$$H_{BdG} = \left(-iv_F \partial_x \sigma_z - \mu\right)\tau_z + \frac{1}{2} M(x) \cdot \sigma_x + \Delta_1(x) \cdot \tau_x + \Delta_2(x) \cdot \tau_y$$

Fu & Kane PRB 2009

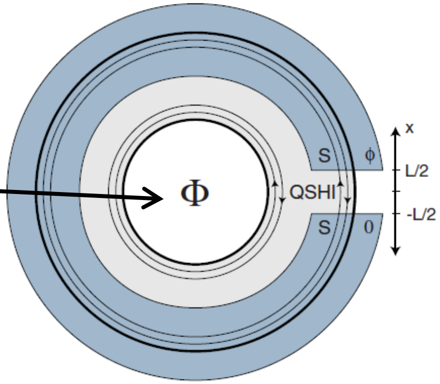
Spectrum of Andreev BS

$$\Delta_1(x) = \Delta_0\left[\theta\left(-x - L/2\right) + \theta\left(x - L/2\right)\cos\phi\right]$$

$$\Delta_2(x) = \Delta_0\theta\left(x - L/2\right)\sin\phi$$

$$M(x) = M_0\theta\left(x + L/2\right)\theta\left(-x + L/2\right)$$

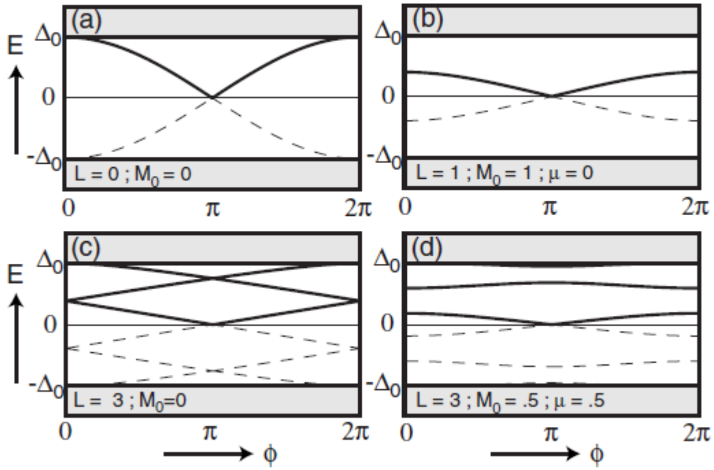
$$\phi = \frac{2e}{\hbar}\Phi$$



short junction limit:

$$\varepsilon_0(\phi) = \sqrt{D}\Delta_0\cos\left(\phi/2\right)$$

normal-state transmission prob.



Fu & Kane PRB 2009

Low energy Hamiltonian

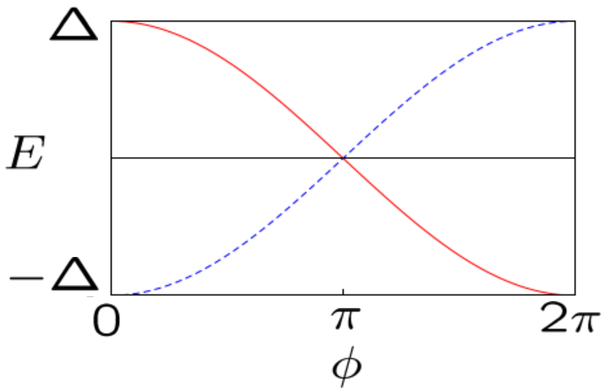
$$E \ll \Delta_0$$

$$H = \varepsilon_0(\phi)\left(\gamma_0^\dagger \gamma_0 - 1/2\right)$$

describes 2 states:

$$N_0 = \gamma_0^\dagger \gamma_0 = 0,1$$

$$\varepsilon_0(\phi) = \sqrt{D}\Delta_0 \cos\left(\phi/2\right)$$



mixing requires a change in N_0 <->
parity constraint

-> crossing at E=0 protected

What happens if phi is advanced by 2pi?

Fu & Kane PRB 2009

What happens if ϕ is advanced by 2π?

$$E \ll \Delta_0$$

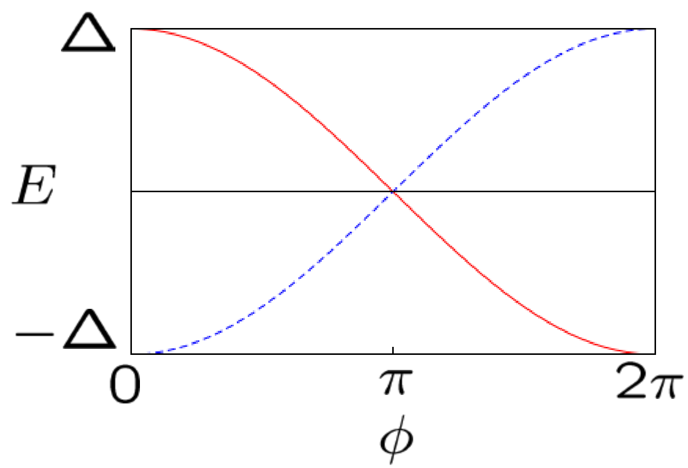
$$H = \varepsilon_0(\phi)\left(\gamma_0^\dagger \gamma_0 - \tfrac{1}{2}\right)$$

describes 2 states:

$$N_0 = \gamma_0^\dagger \gamma_0 = 0,1$$

$$\varepsilon_0(\phi) = \sqrt{D}\,\Delta_0 \cos\left(\phi/2\right)$$

Fermion parity changes
<-> fermion parity anomaly



$$\phi \to \phi + 2\pi$$

$$\phi/2\pi = \Phi/\Phi_0 \quad \text{with } \Phi_0 = h/2e$$

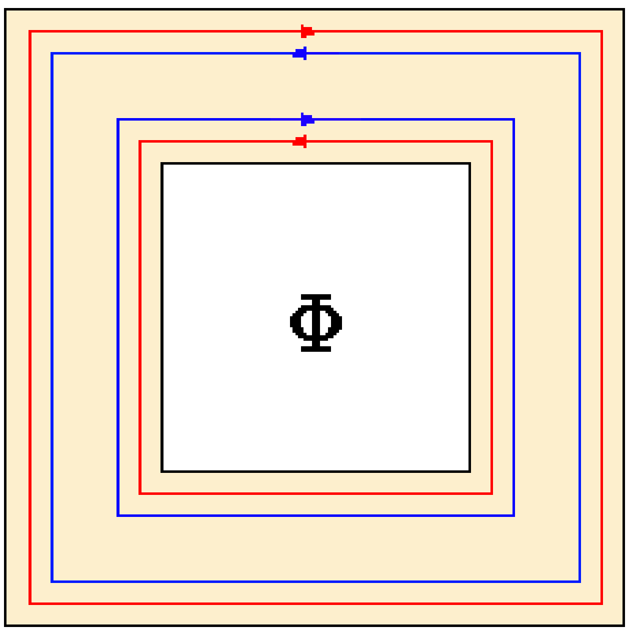
Fu & Kane PRB 2009

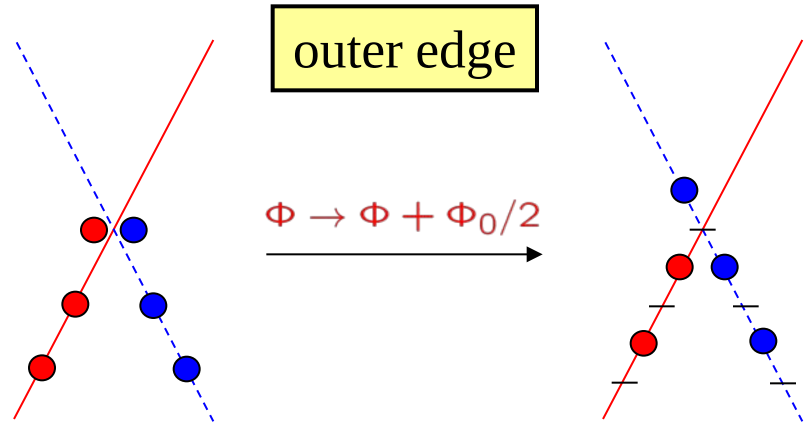
Parity pumping

quantization condition (outer edge):

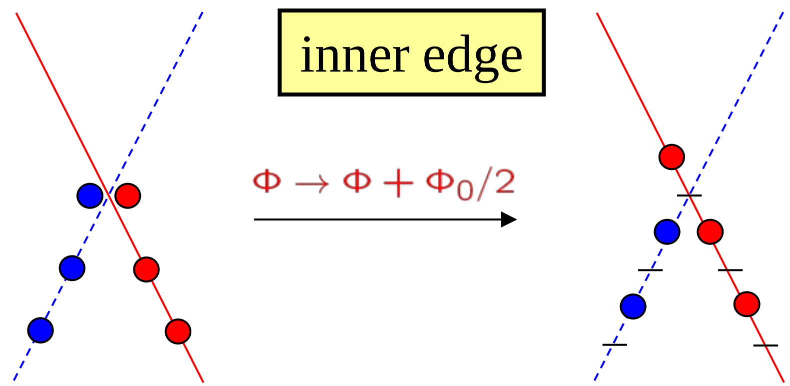
$$k_n = \frac{2\pi}{L_{out}}\left(n \pm \frac{\Phi}{\Phi_0}\right)$$

$$\Phi_0 = h/e$$



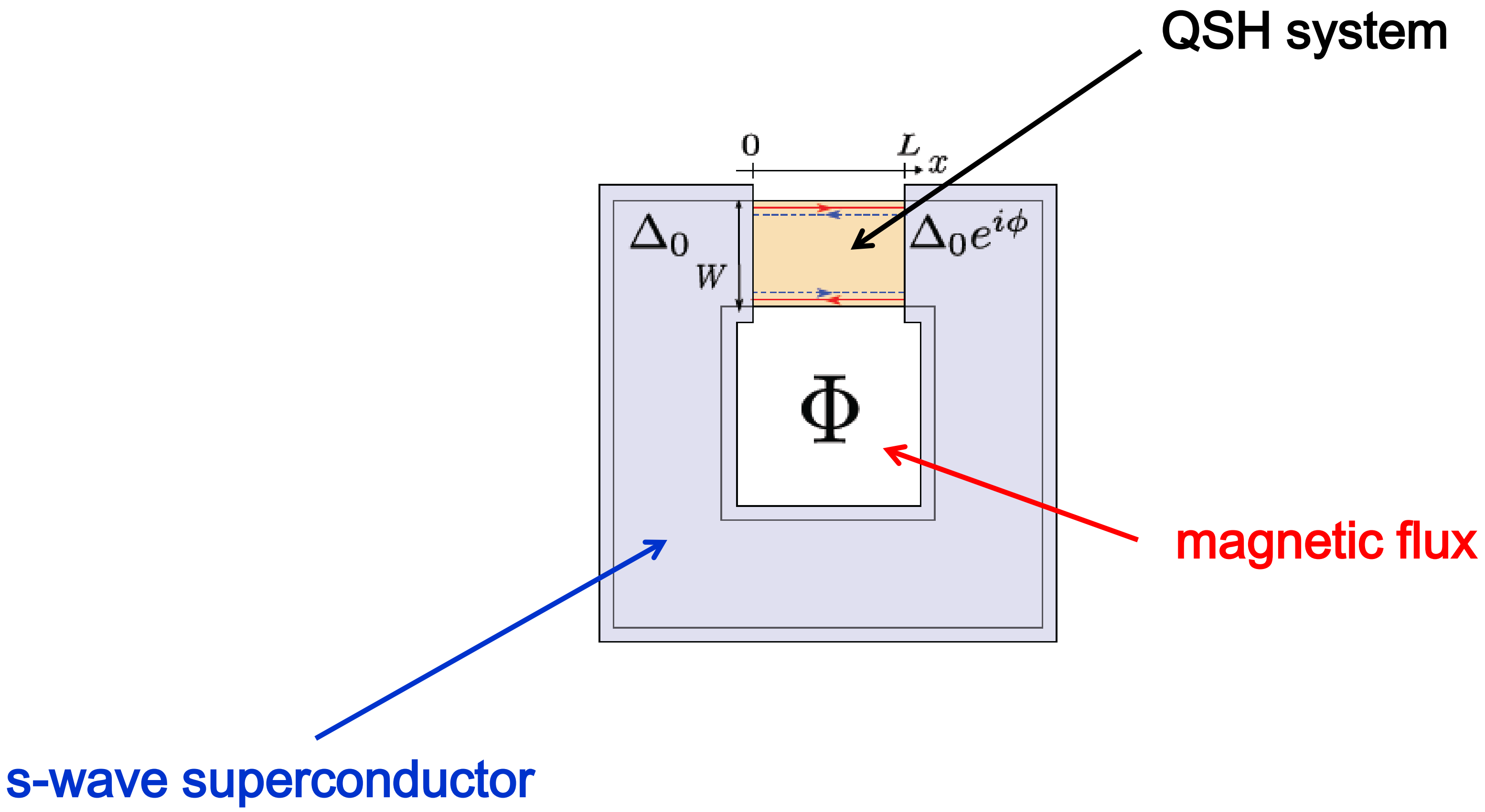


Fermion parity = 0 Fermion parity = 1



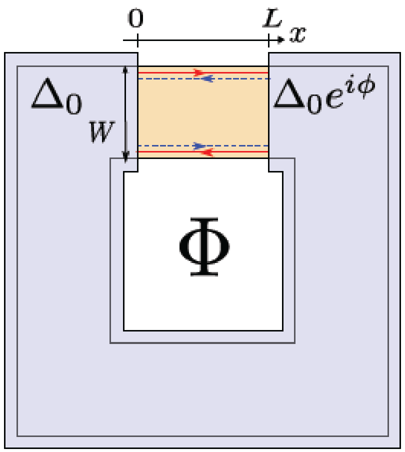
Fermion parity = 0 Fermion parity = 1

Modified setup



Crépin & BT PRL 2014

JJ current vs. parity pumping

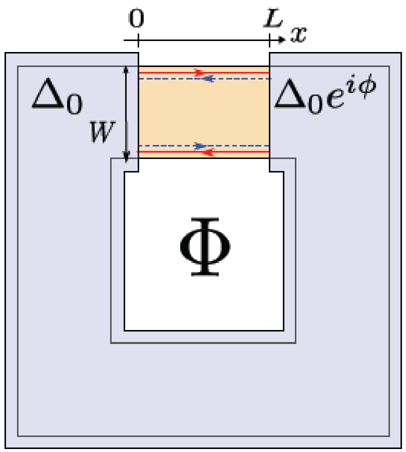


$$J_{\sigma,\sigma'}[\phi] = I_{up,\sigma}[\phi] + I_{down,\sigma'}[\phi]$$

total fermion parity number: $\Sigma = \sigma \cdot \sigma'$

Crépin & BT PRL 2014

JJ current vs. parity pumping

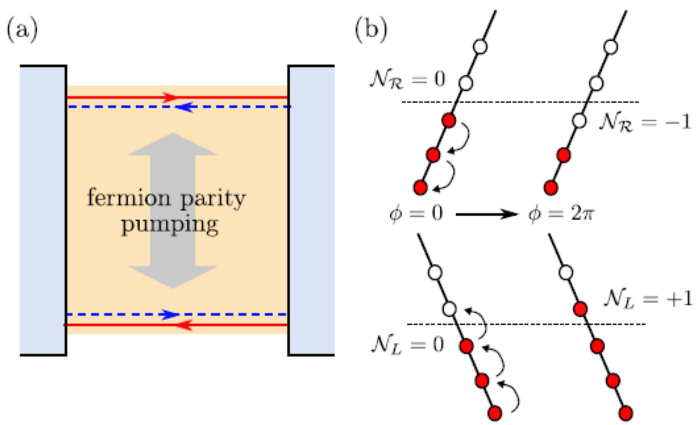


$$J_{\sigma,\sigma'}[\phi] = I_{up,\sigma}[\phi] + I_{down,\sigma'}[\phi]$$

total fermion parity number: $\Sigma = \sigma \cdot \sigma'$

$$\phi \to \phi + 2\pi$$

$$\phi/2\pi = \Phi/\Phi_0 \quad \text{with } \Phi_0 = h/2e$$



Fu & Kane PRB 2006

Crépin & BT PRL 2014

Outline

- **Parity constraints** in topological JJs
- **Short junction limit:** simple current-phase relations -> parity measurement
- **Long junction limit:** bosonization -> influence of Coulomb interactions

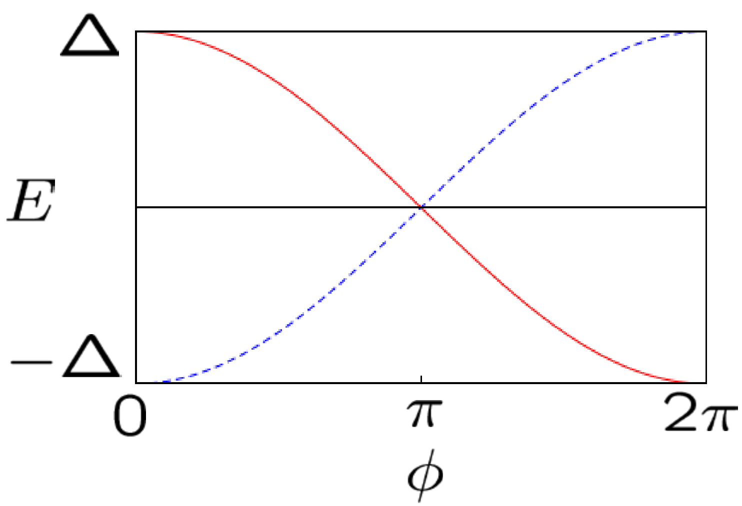
Short junction limit

$$L \ll \xi = {v_F}/{\Delta_0}$$

$$I_{up,\pm}[\phi] = I_{down,\pm}[\phi] = \pm I_c \sin\left({\phi}/{2}\right)$$

$$I_c = {\Delta_0}/{2}$$

<u>Due to Andreev bound states:</u>



Fu & Kane PRB 2009

Crépin & BT PRL 2014

Short junction limit

$$L \ll \xi = v_F / \Delta_0$$

$$I_{up,\pm}[\phi] = I_{down,\pm}[\phi] = \pm I_c \sin\left(\phi/2\right)$$

$$I_c = \Delta_0 / 2$$

odd parity:

$$J_{+,-}[\phi] = J_{-,+}[\phi] = 0$$

even parity:

$$J_{+,+}[\phi] = -J_{-,-}[\phi] = 2I_c \sin\left(\phi/2\right)$$

-> parity detector

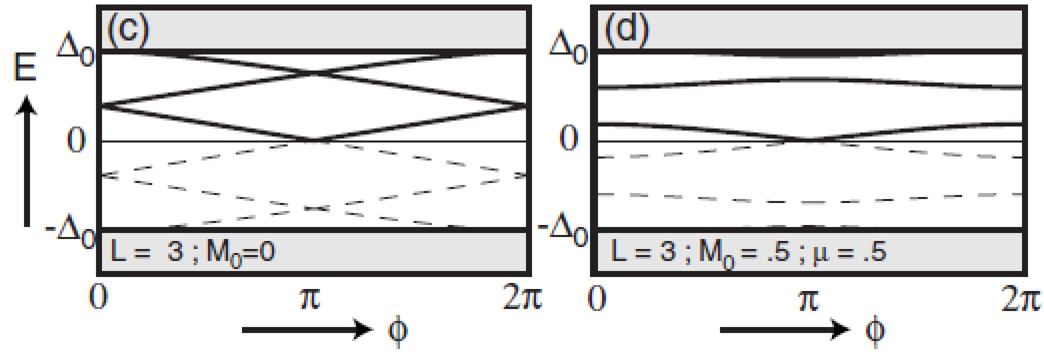
Crépin & BT PRL 2014

Outline

- **Parity constraints** in topological JJs
- **Short junction limit**: simple current-phase relations -> parity measurement
- **Long junction limit:** bosonization -> influence of Coulomb interactions

Long junction: ABS spectrum

$$L \gg \xi = {v_F}/{\Delta_0}$$



ABS spectrum:

$$\varepsilon_n = \frac{\pi v_F}{L}\left(n + \frac{1}{2} \pm \frac{\phi}{2\pi}\right) \quad \text{with } n \in \mathbb{Z}$$

valid for $\varepsilon \ll \Delta_0$

Kulik Sov. Phys. JETP 1970

Long junction: twisted BC

perfect AR (upper edge):

$$\psi_{R,\uparrow}(0) = i\psi^{\dagger}_{L\downarrow}(0)$$
$$\psi_{R,\uparrow}(L) = -ie^{i\phi}\psi^{\dagger}_{L\downarrow}(L)$$

twisted boundary conditions:

$$\psi_{R,\uparrow}(x+2L,t) = -e^{-i\phi}\psi_{R,\uparrow}(x,t)$$
$$\psi_{L,\downarrow}(x+2L,t) = -e^{-i\phi}\psi_{L,\downarrow}(x,t)$$
$$\psi_{R,\uparrow}(x,t) = -i\psi^{\dagger}_{L,\downarrow}(-x,t)$$

$x \in [-L, L]$

Maslov, Stone, Goldbart & Loss PRB 1996

Julius-Maximilians-UNIVERSITÄT WÜRZBURG

Long junction: twisted BC

perfect AR (upper edge):

$$\psi_{R,\uparrow}(0) = i\psi^{\dagger}_{L\downarrow}(0)$$
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$$\psi_{R,\uparrow}(x,t) = -i\psi^{\dagger}_{L,\downarrow}(-x,t)$$

$x \in [-L, L]$

Maslov, Stone, Goldbart & Loss PRB 1996

ABS spectrum:

$$\varepsilon_n = \frac{\pi v_F}{L}\left(n + \frac{1}{2} \pm \frac{\phi}{2\pi}\right) \quad \text{with } n \in \mathbb{Z}$$

Long junction: effective model

BdG Hamiltonian (upper edge):

$$H_{\mathrm{up}} = \frac{1}{2}\int_0^L dx \Psi^\dagger H_{BdG} \Psi$$

$$\Psi^\dagger = \left(\psi^\dagger_{R\uparrow}, \psi^\dagger_{L\downarrow}, \psi_{L\downarrow}, -\psi_{R\uparrow}\right)$$

$$H_{BdG} = -i v_F \partial_x \sigma_z \tau_z$$

twisted boundary conditions:

$$\psi_{R,\uparrow}\left(x+2L,t\right) = -e^{-i\phi}\psi_{R,\uparrow}\left(x,t\right)$$

$$\psi_{L,\downarrow}\left(x+2L,t\right) = -e^{-i\phi}\psi_{L,\downarrow}\left(x,t\right)$$

$$\psi_{R,\uparrow}\left(x,t\right) = -i\psi^\dagger_{L,\downarrow}\left(-x,t\right)$$

Maslov, Stone, Goldbart & Loss PRB 1996

$$H_{\mathrm{up}} = i\frac{v_F}{2}\int_{-L}^{L} dx\left[-\psi^\dagger_{R,\uparrow}\partial_x\psi_{R,\uparrow} + \psi^\dagger_{L,\downarrow}\partial_x\psi_{L,\downarrow}\right]$$

Long junction: effective model

$$H_{\mathrm{up}} = i\frac{v_F}{2}\int_{-L}^{L} dx\left[-\psi^{\dagger}_{R,\uparrow}\partial_x\psi_{R,\uparrow} + \psi^{\dagger}_{L,\downarrow}\partial_x\psi_{L,\downarrow}\right]$$

Fourier transformation:

$$\psi_{R,\uparrow}(x,t) = \frac{1}{\sqrt{2L}}\sum_n \exp\left(i\frac{\pi}{L}\left(x+\frac{1}{2}+\frac{\phi}{2\pi}\right)x\right)c_{R,\uparrow,n}$$

$$\psi_{L,\downarrow}(x,t) = \frac{1}{\sqrt{2L}}\sum_n \exp\left(-i\frac{\pi}{L}\left(x+\frac{1}{2}-\frac{\phi}{2\pi}\right)x\right)c_{L,\downarrow,n}$$

chiral constraint:

$$\psi_{R,\uparrow}(x,t) = -i\psi^{\dagger}_{L,\downarrow}(-x,t)$$

$$c_{R,\uparrow,n} = -ic^{\dagger}_{L,\downarrow,-n-1}$$

$$H_{\mathrm{up}} = \frac{v_F}{2}\sum_n\left[\frac{\pi}{L}\left(n+\frac{1}{2}-\frac{\phi}{2\pi}\right)c^{\dagger}_{R,\uparrow,n}c_{R,\uparrow,n} + \frac{\pi}{L}\left(n+\frac{1}{2}+\frac{\phi}{2\pi}\right)c^{\dagger}_{L,\downarrow,n}c_{L,\downarrow,n}\right]$$

Bosonization

number of fermions w/ respect to Fermi sea

chiral bosonic field (periodic on [-L,L])

$$\psi_{R,\uparrow}(x,t) = F_R \frac{1}{\sqrt{2\pi a}} e^{i\frac{\pi}{L}\left(\hat{N}_R + \frac{1}{2} - \frac{\phi}{2\pi}\right)x} e^{-i\tilde{\phi}_R(x)}$$

Klein factors

Crépin & BT PRL 2014

Bosonization

number of fermions w/ respect to Fermi sea

chiral bosonic field (periodic on [-L,L])

$$\psi_{R,\uparrow}(x,t) = F_R \frac{1}{\sqrt{2\pi a}} e^{i\frac{\pi}{L}\left(\hat{N}_R + \frac{1}{2} - \frac{\phi}{2\pi}\right)x} e^{-i\tilde{\phi}_R(x)}$$

Klein factors

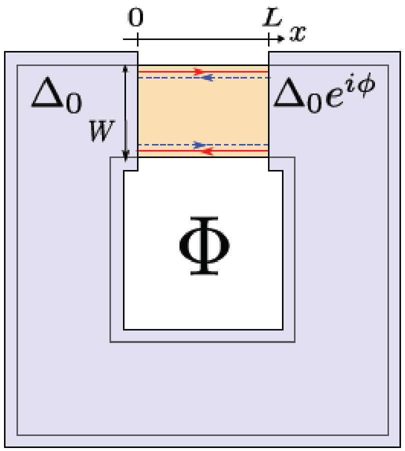
$$\tilde{\phi}_R(x) = -\sum_{q>0} \sqrt{\frac{\pi}{Lq}} \left(e^{iqx} b_{R,q} + e^{-iqx} b^{\dagger}_{R,q}\right) e^{-aq/2}$$

Crépin & BT PRL 2014

Hamiltonian

$$H_{up} = \frac{v_F \pi}{2L}\left(\hat{N}_R - \frac{\phi}{2\pi}\right)^2 + v_F \sum_{q>0} q b^\dagger_{R,q} b_{R,q}$$

$$H_{down} = \frac{v_F \pi}{2L}\left(\hat{N}_L + \frac{\phi}{2\pi}\right)^2 + v_F \sum_{q>0} q b^\dagger_{L,q} b_{L,q}$$



$\phi \rightarrow \phi + 2\pi$ -> parity pumping

Crépin & BT PRL 2014

Josephson current: calculation

$$I_{up,\pm}[\phi] = -\frac{2e}{\hbar}\frac{1}{\beta}\frac{\partial}{\partial\phi}\ln Z^t_{up,\pm}[\phi]$$

Topological part of partition function:

$$Z^t_{up,\pm}[\phi] = \sum_{\substack{N_R\in\mathbb{Z} \\ even/odd}} \exp\left(-\beta\frac{\hbar\pi v_F}{2L}\left(N_R - \frac{\phi}{2\pi}\right)^2\right)$$

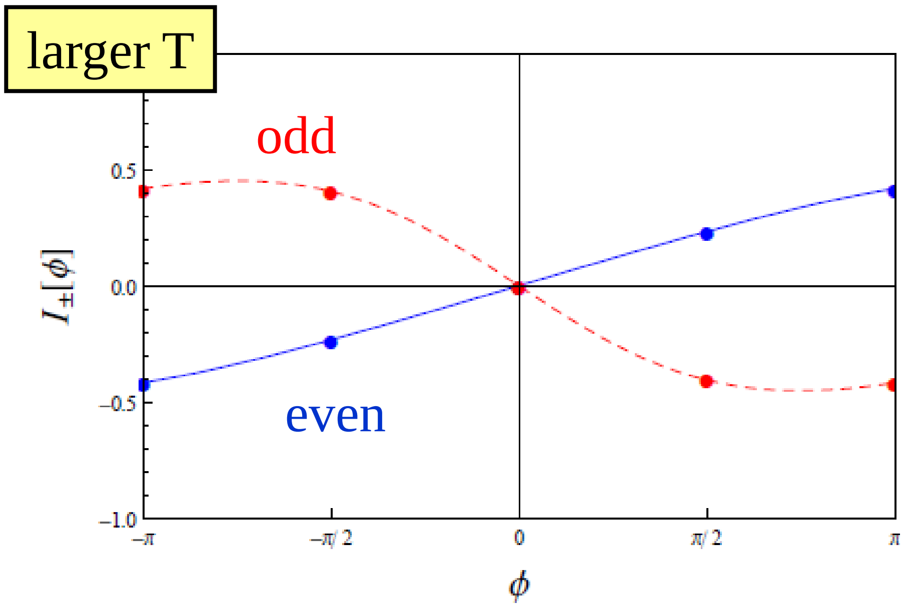
parity constraint

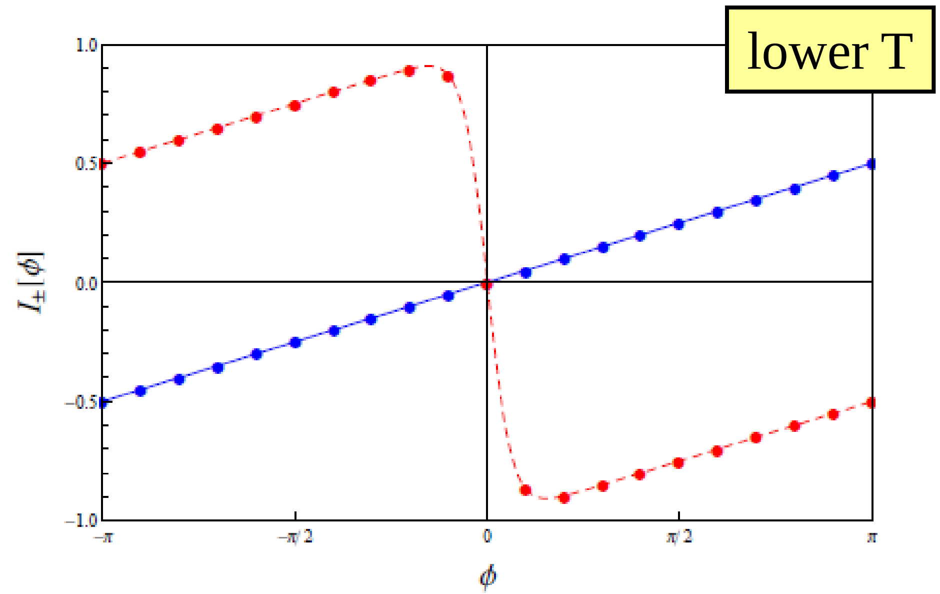
Related work: *Beenakker, Pikulin, Hyart, Schomerus & Dahlhaus* PRL 2013

Crépin & BT PRL 2014

Josephson current: result

$$I_{up,\pm}\left[\phi\right] = e\frac{v_F}{L}\frac{\phi}{2\pi} - \frac{2e}{\hbar\beta}\partial_\phi \ln\theta_{3/2}\left[i\beta\frac{\hbar\pi v_F}{2L}\frac{\phi}{\pi}, e^{-2\beta\frac{\hbar\pi v_F}{L}}\right]$$



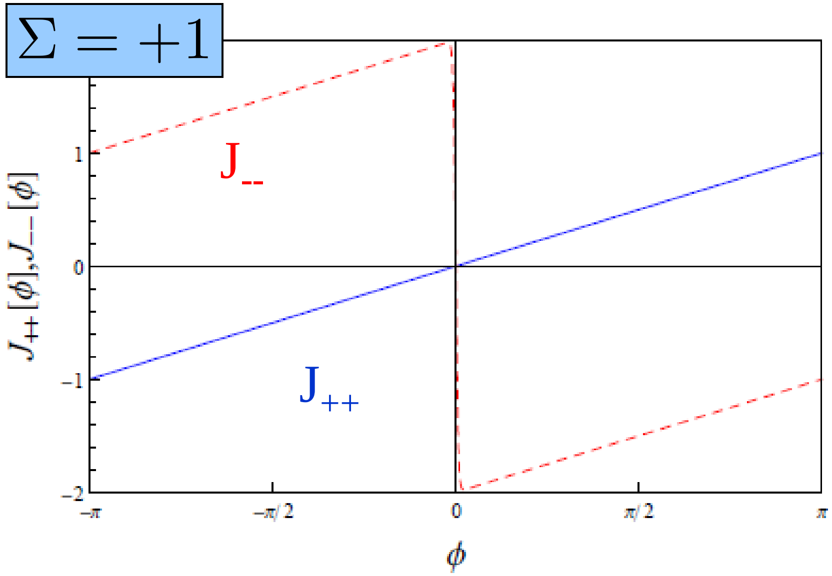


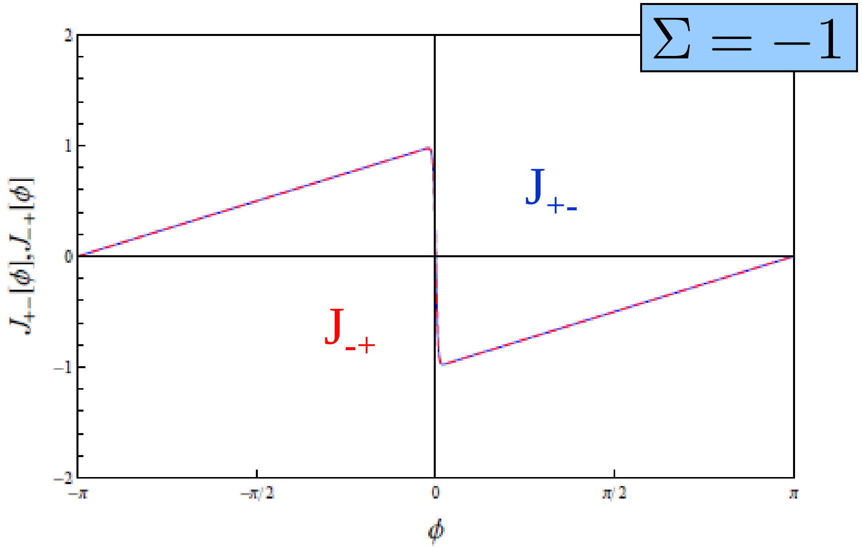
Related work: *Beenakker, Pikulin, Hyart, Schomerus & Dahlhaus* PRL 2013

Crépin & BT PRL 2014

Josephson current: combined

$$J_{\sigma,\sigma'}[\phi] = I_{up,\sigma}[\phi] + I_{down,\sigma'}[\phi]$$





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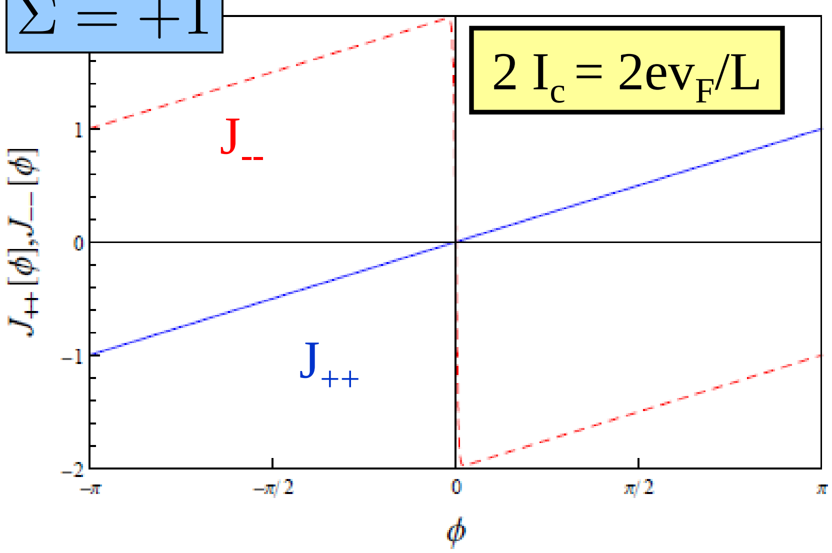
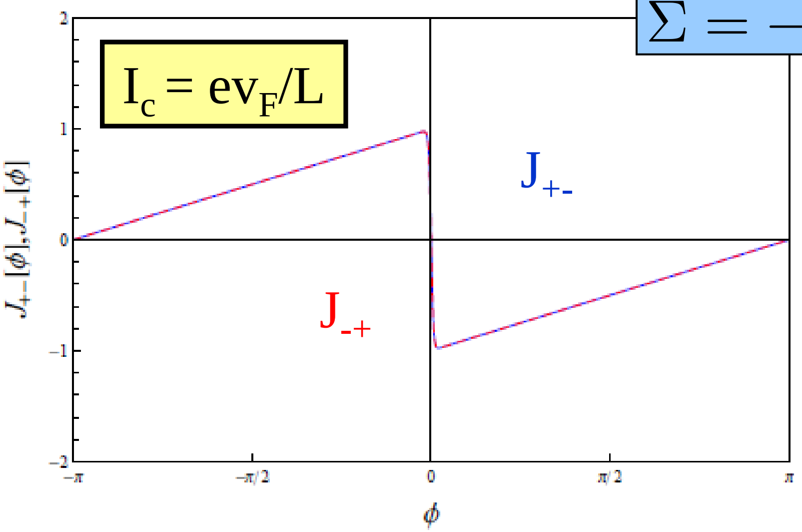
Josephson current 2

$$J_{\sigma,\sigma'}[\phi] = I_{up,\sigma}[\phi] + I_{down,\sigma'}[\phi]$$

$\Sigma = +1$



$\Sigma = -1$



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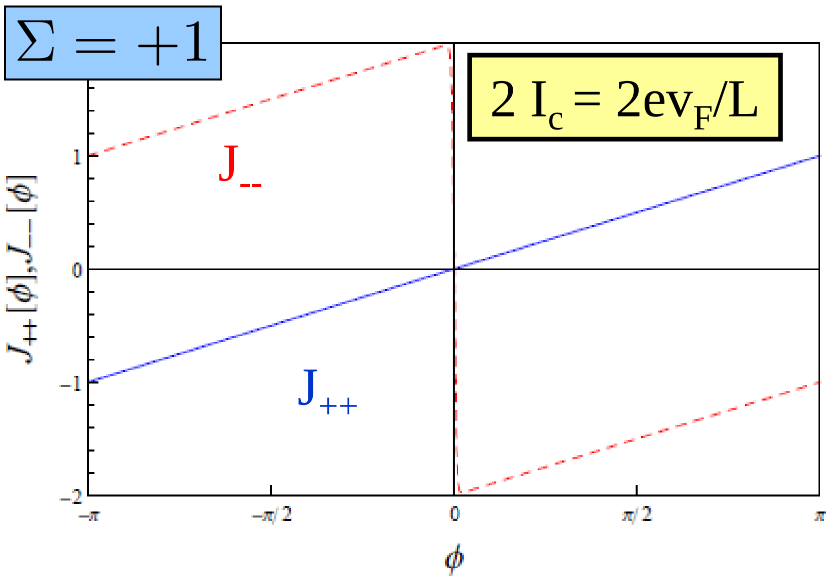
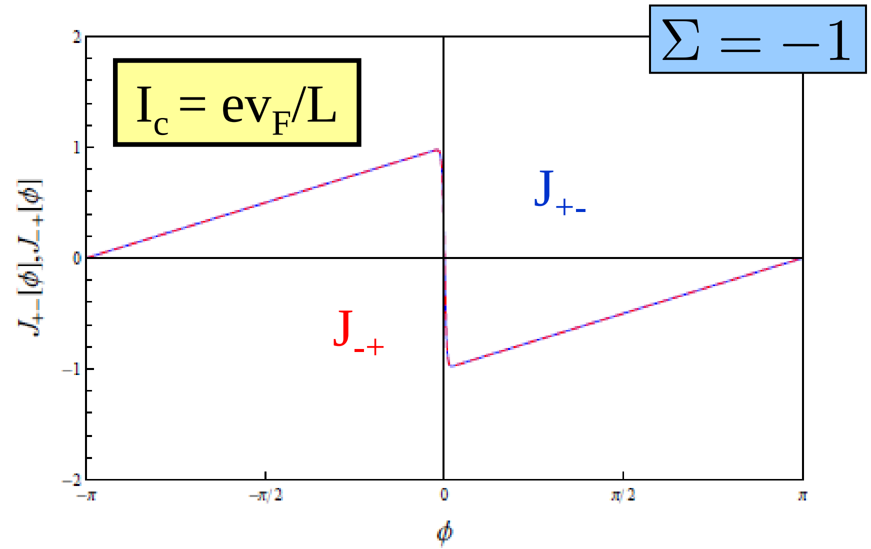
Josephson current 2

$$J_{\sigma,\sigma'}[\phi] = I_{up,\sigma}[\phi] + I_{down,\sigma'}[\phi]$$



4pi-periodic



2pi-periodic

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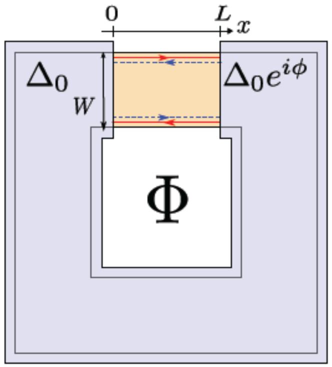
Influence of Coulomb interactions



$$H_{int} = g_2 \int_0^L dx : \rho_{R,\uparrow}(x)\, \rho_{L,\downarrow}(x) :$$

$$H_{up} = \frac{v_N \pi}{2L} \left(\hat{N}_R - \frac{\phi}{2\pi} \right)^2 + v_S \sum_{q>0} q B^\dagger_{R,q} B_{R,q}$$

$$v_N = v_F \left(1 - \frac{g_2}{2\pi v_F} \right)$$

-> rescaling of the critical current $I_c = ev_N/L$

Take home messages

- Critical current:

$$\begin{matrix} 2I_c \text{ even case} \\ I_c \text{ odd case} \end{matrix} \quad \text{with } I_c = \frac{ev_F}{L}$$

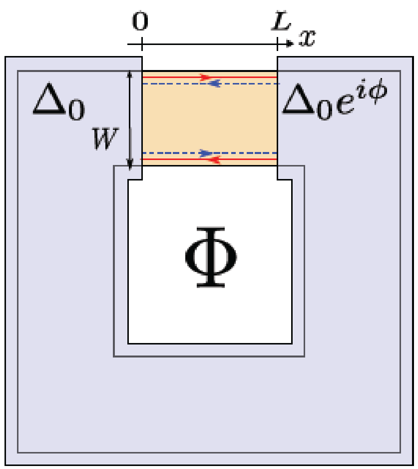
- Phase dependence:

$$J[\phi] = J[\phi + 4\pi] \text{ even case}$$
$$J[\phi] = J[\phi + 2\pi] \text{ odd case}$$

-> parity detector

- Phase dependence could be modified by disorder

Summary Lecture 3



$$J_{\sigma,\sigma'}\left[\phi\right] = I_{up,\sigma}\left[\phi\right] + I_{down,\sigma'}\left[\phi\right]$$

total fermion parity:

$$\Sigma = \sigma \cdot \sigma'$$

$$\psi_{R,\uparrow}\left(x+2L,t\right) = -e^{-i\phi}\psi_{R,\uparrow}\left(x,t\right)$$
$$\psi_{L,\downarrow}\left(x+2L,t\right) = -e^{-i\phi}\psi_{L,\downarrow}\left(x,t\right)$$
$$\psi_{R,\uparrow}\left(x,t\right) = -i\psi^{\dagger}_{L,\downarrow}\left(-x,t\right)$$