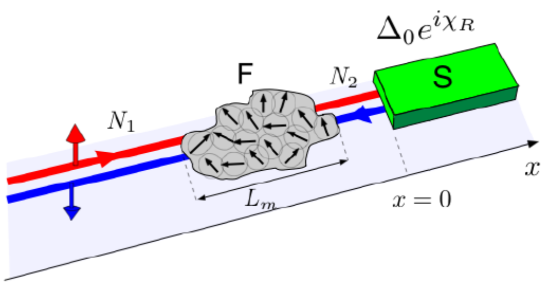
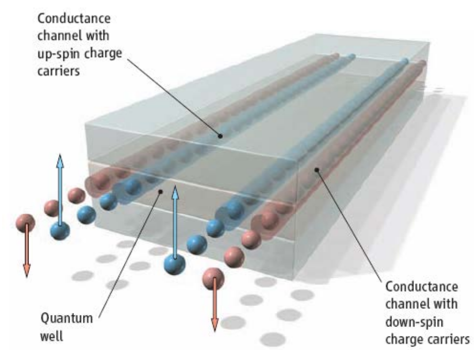
Superconducting hybrids based on QSH systems

Lecture 2



11th Capri Spring School
Transport in Nanostructures

April 13-17, 2015
Capri, Italy



Björn Trauzettel

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Florian Geissler (Uni Würzburg)
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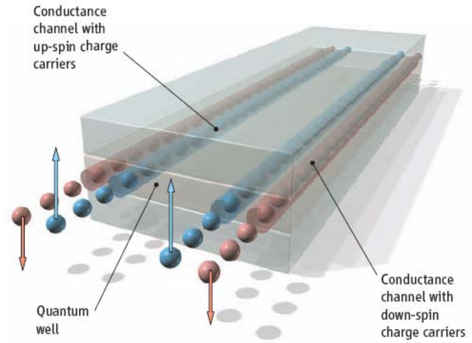
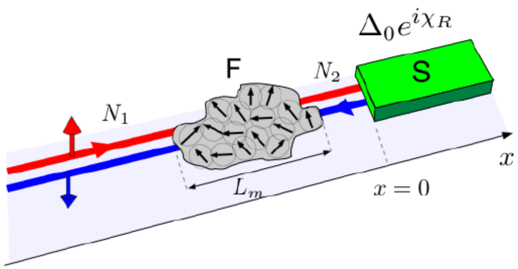
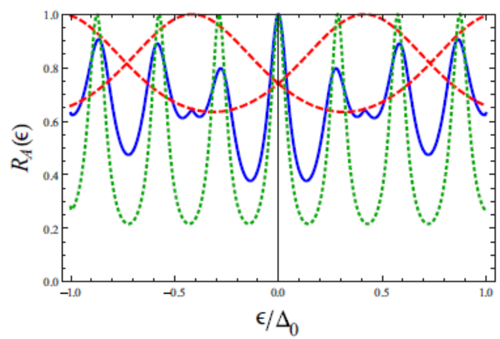
Julius-Maximilians-Universität Würzburg
Elitenetzwerk Bayern
DFG Deutsche Forschungsgemeinschaft
Alexander von Humboldt Stiftung/Foundation
Helmholtz Association

Summary Lecture 1





$$H_{BdG} = \begin{pmatrix} H^{e}_{0+FM} & \Delta(x)\,\sigma_0 \\ \Delta^{*}(x)\,\sigma_0 & H^{h}_{0+FM} \end{pmatrix}$$



(c) $\mu_0 = 0.35, \mu = 0.3, L_m = 2.15, x_0 = 6$

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Outline

- **Classification**: Symmetry of pairing amplitude
- **Transport signature**: Crossed Andreev reflection in NSN junctions
- **Doppler shift:** Topological SQUID based on helical edge states

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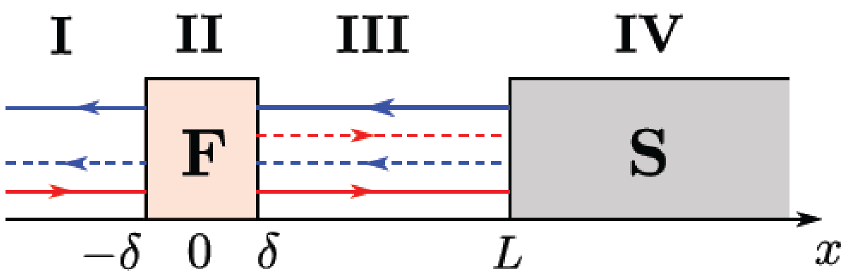
Model and setup



BdG Hamiltonian

$$H = \frac{1}{2}\int dx \Psi^{\dagger} H_{BdG} \Psi$$

$$\Psi^{\dagger} = \left(\psi^{\dagger}_{R\uparrow}, \psi^{\dagger}_{L\downarrow}, \psi_{L\downarrow}, -\psi_{R\uparrow}\right)$$

spin space

$$H_{BdG} = \left(-iv_F \partial_x \sigma_z - \mu\right)\tau_z + \vec{m}\left(x\right)\cdot\vec{\sigma} + \Delta_1\left(x\right)\tau_x + \Delta_2\left(x\right)\tau_y$$

particle-hole space

Crépin, Burset & BT arXiv 2015

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Green's functions

$$r = (x,t)$$

$$G^R(r,r') = -i\theta(t-t')\left\langle\left\{\Psi(r),\Psi^\dagger(r')\right\}\right\rangle$$
$$G^A(r,r') = i\theta(t-t')\left\langle\left\{\Psi(r),\Psi^\dagger(r')\right\}\right\rangle$$
$$G^M(x,\tau,x',\tau') = -\left\langle T_\tau \Psi(x,\tau)\Psi^\dagger(x',\tau')\right\rangle$$

$$G^X = \begin{pmatrix} G^X_{ee} & G^X_{eh} \\ G^X_{he} & G^X_{hh} \end{pmatrix}$$

4x4 matrix

$$G^X_{eh} = \begin{pmatrix} \left[G^X_{eh}\right]_{\uparrow\downarrow} & \left[G^X_{eh}\right]_{\uparrow\uparrow} \\ \left[G^X_{eh}\right]_{\downarrow\downarrow} & \left[G^X_{eh}\right]_{\downarrow\uparrow} \end{pmatrix}$$

Scattering states: N side

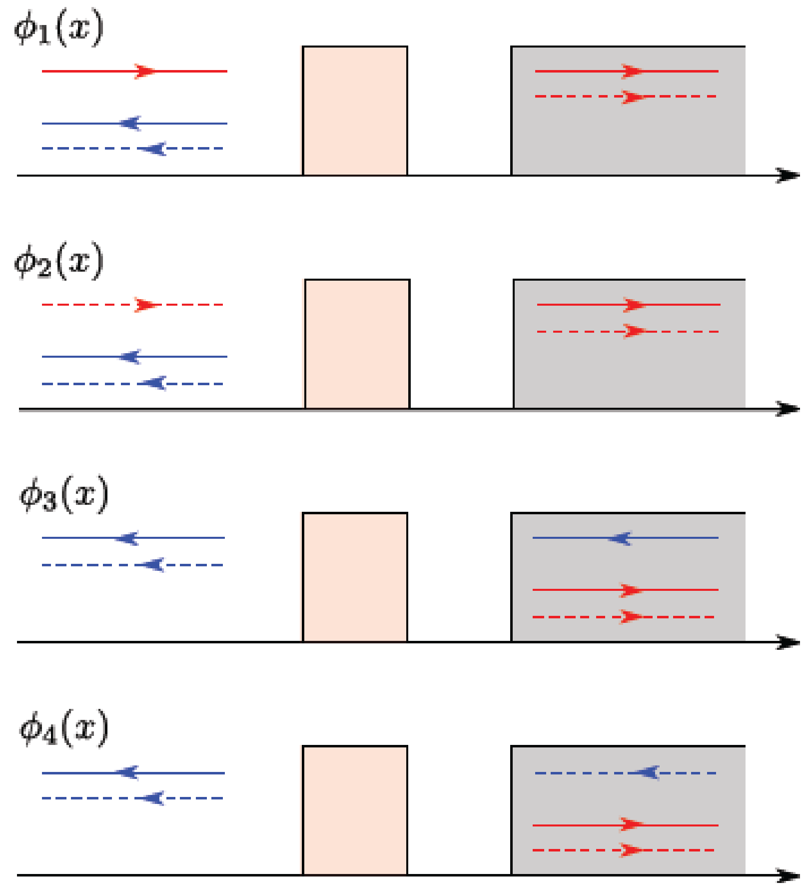
$$\phi_1(x) = \phi_e^{(+)} e^{ik_e x} + a_1 \phi_h^{(-)} e^{ik_h x} + b_1 \phi_e^{(-)} e^{-ik_e x}$$

$$\phi_2(x) = \phi_h^{(+)} e^{-ik_h x} + b_2 \phi_h^{(-)} e^{ik_h x} + a_2 \phi_e^{(-)} e^{-ik_e x}$$

$$\phi_3(x) = c_3 \phi_e^{(-)} e^{-ik_e x} + d_3 \phi_h^{(-)} e^{ik_h x}$$

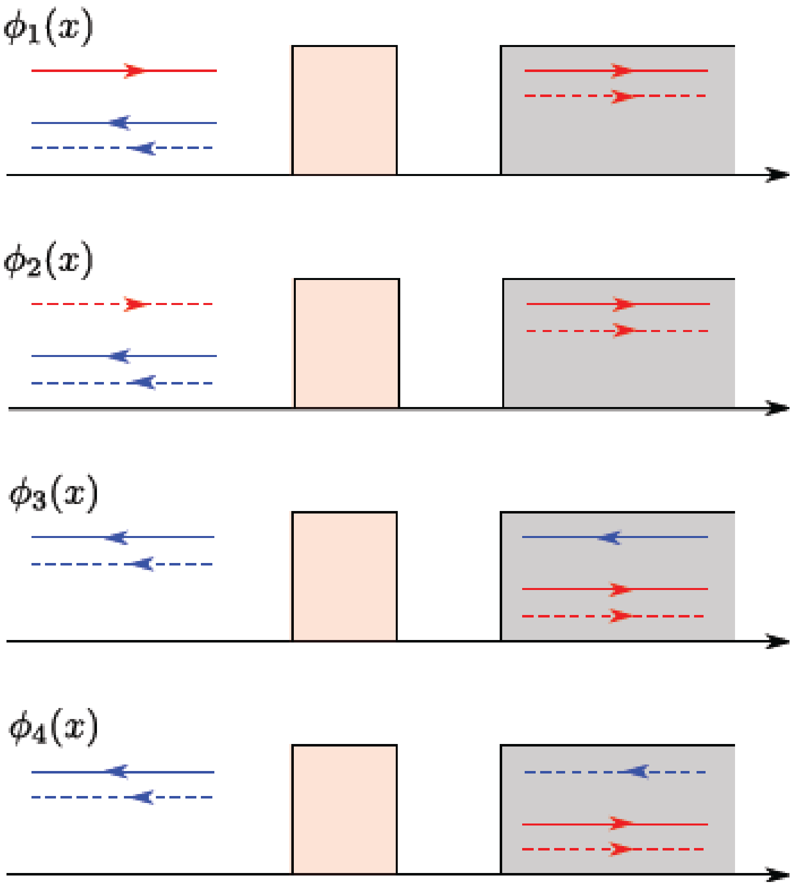
$$\phi_4(x) = d_4 \phi_e^{(-)} e^{-ik_e x} + c_4 \phi_h^{(-)} e^{ik_h x}$$

$$k_e = \mu + \varepsilon \qquad k_h = \mu - \varepsilon$$



McMillan Phys. Rev. 1968

Scattering states: S side



$$\phi_1(x) = c_1 \chi_e^{(+)} e^{ik_e^S x} + d_1 \chi_h^{(+)} e^{-ik_h^S x}$$

$$\phi_2(x) = d_2 \chi_e^{(+)} e^{ik_e^S x} + c_2 \chi_h^{(+)} e^{-ik_h^S x}$$

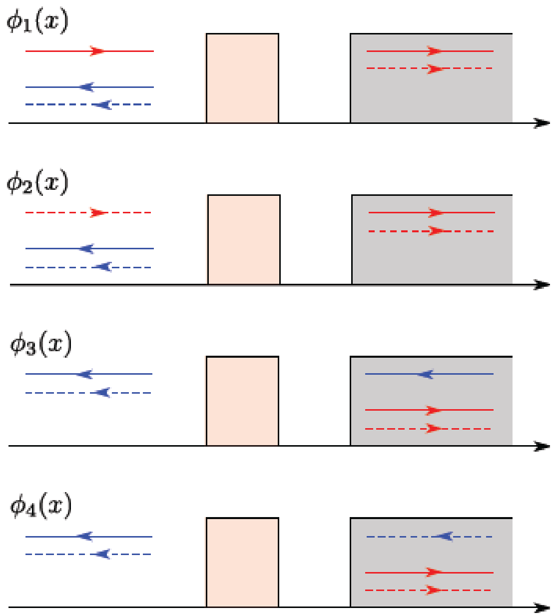
$$\phi_3(x) = \chi_e^{(-)} e^{-ik_e^S x} + a_3 \chi_h^{(+)} e^{-ik_e^S x} + b_3 \chi_h^{(+)} e^{ik_e^S x}$$

$$\phi_4(x) = \chi_h^{(-)} e^{ik_h^S x} + b_4 \chi_h^{(+)} e^{-ik_h^S x} + a_4 \chi_e^{(+)} e^{ik_e^S x}$$

$$k_e^S = \mu + \sqrt{\varepsilon^2 - \Delta^2} \qquad k_h^S = \mu - \sqrt{\varepsilon^2 - \Delta^2}$$

McMillan Phys. Rev. 1968

Green's functions from scattering states



$$G^R_\omega\left(x,x'\right) = \int dt e^{i(\omega+i\eta)(t-t')} G^R\left(x,t,x',t'\right)$$

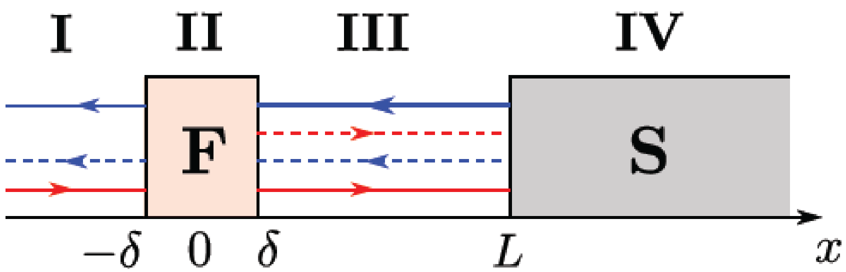
$$\left[w - H_{BdG}\left(x\right)\right] G^R_\omega\left(x,x'\right) = \delta\left(x-x'\right)$$

$$\lim_{\varepsilon\to 0}\left[G^R_\omega\left(x+\varepsilon,x\right) - G^R_\omega\left(x-\varepsilon,x\right)\right] = \frac{1}{iv_F}\sigma_z\tau_z$$

$$G^R_\omega\left(x,x'\right) = \begin{cases} \phi_3\left(x\right) A_3\left(x'\right)^T + \phi_4\left(x\right) A_4\left(x'\right)^T & \text{if } x < x' \\ \phi_1\left(x\right) A_1\left(x'\right)^T + \phi_2\left(x\right) A_2\left(x'\right)^T & \text{if } x > x' \end{cases}$$

McMillan Phys. Rev. 1968

Pairing amplitude



Green's function

$$G^R = \begin{pmatrix} G^R_{ee} & G^R_{eh} \\ G^R_{he} & G^R_{hh} \end{pmatrix}$$

Pairing amplitude

$$F^R = G^R_{eh} i\sigma_2 = \begin{pmatrix} F^R_{\uparrow\uparrow} & F^R_{\uparrow\downarrow} \\ F^R_{\downarrow\uparrow} & F^R_{\downarrow\downarrow} \end{pmatrix}$$

singlet

triplet

$$F^R(x,x',\omega) = \left[f^R_0(x,x',\omega)\sigma_0 + f^R_i(x,x',\omega)\sigma_i \right] i\sigma_2$$

Crépin, Burset & BT arXiv 2015

Antisymmetry of pairing amplitude

singlet

triplet

$$F^R(x,x',\omega) = \left[f_0^R(x,x',\omega)\sigma_0 + f_i^R(x,x',\omega)\sigma_i \right] i\sigma_2$$

$$f_0^R(x,x',\omega) = f_0^A(x',x,-\omega)$$
$$f_i^R(x,x',\omega) = -f_i^A(x',x,-\omega)$$

Crépin, Burset & BT arXiv 2015

Classification of pairing amplitude

$$F^R(x,x',\omega) = \left[f_0^R(x,x',\omega)\sigma_0 + f_i^R(x,x',\omega)\sigma_i\right] i\sigma_2$$

orbital

$$f_0^R(x,x',\omega) = \pm f_0^R(x',x,\omega)$$
$$f_i^R(x,x',\omega) = \pm f_i^R(x',x,\omega)$$

frequency

$$f_0^R(x,x',\omega) = \pm f_0^A(x,x',-\omega)$$
$$f_i^R(x,x',\omega) = \pm f_i^A(x,x',-\omega)$$

Berezinskii JETP Lett 1974

Tanaka, Sato & Nagaosa JPSJ 2012

Classification of pairing amplitude

$$F^R(x,x',\omega) = \left[f_0^R(x,x',\omega)\sigma_0 + f_i^R(x,x',\omega)\sigma_i \right] i\sigma_2$$

frequency, spin, orbital

even, singlet, even -> ESE
odd, singlet, odd -> OSO
even, triplet, odd -> ETO
odd, triplet, even -> OTE

orbital

$$f_0^R(x,x',\omega) = \pm f_0^R(x',x,\omega)$$
$$f_i^R(x,x',\omega) = \pm f_i^R(x',x,\omega)$$

frequency

$$f_0^R(x,x',\omega) = \pm f_0^A(x,x',-\omega)$$
$$f_i^R(x,x',\omega) = \pm f_i^A(x,x',-\omega)$$

Berezinskii JETP Lett 1974

Tanaka, Sato & Nagaosa JPSJ 2012

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Classification of pairing amplitude

$$F^R(x,x',\omega)=\left[f_0^R(x,x',\omega)\sigma_0+f_i^R(x,x',\omega)\sigma_i\right]i\sigma_2$$

frequency, spin, orbital

even, singlet, even -> ESE
odd, singlet, odd -> OSO
even, triplet, odd -> ETO
odd, triplet, even -> OTE

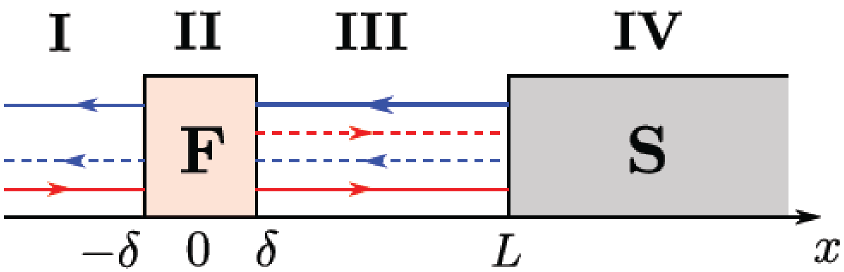
S and T mix if spin rotational symmetry is broken

(orbital) E and O mix if inversion is broken -> NS junctions

Berezinskii JETP Lett 1974

Tanaka, Sato & Nagaosa JPSJ 2012

Classification of pairing amplitude: results



frequency, spin, orbital

-> ESE, OSO, ETO, OTE

	Pairing	Interface	Bulk
f_0	$\uparrow\downarrow - \downarrow\uparrow$	ESE+OSO	ESE
f_3	$\uparrow\downarrow + \downarrow\uparrow$	ETO+OTE	ETO
$f_\pm$	$\uparrow\uparrow, \downarrow\downarrow$	OTE	X

$$f_\alpha^R\left(x,x',\omega\right) = f_{\alpha,bulk}^R\left(x,x',\omega\right) + f_{\alpha,edge}^R\left(x,x',\omega\right)$$

$$f_{\alpha,bulk}^R\left(x,x',\omega\right) \propto e^{-\kappa\left(\omega\right)\left|x-x'\right|}$$

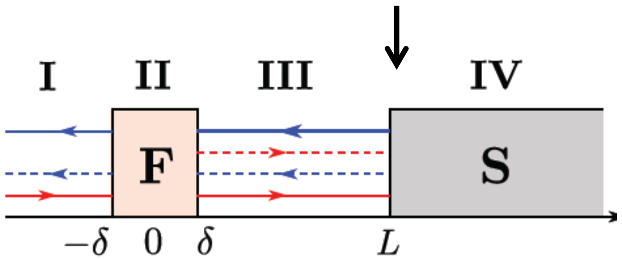
$$f_{\alpha,edge}^R\left(x,x',\omega\right) \propto e^{-\kappa\left(\omega\right)\left(x+x'\right)}$$

(in region IV)

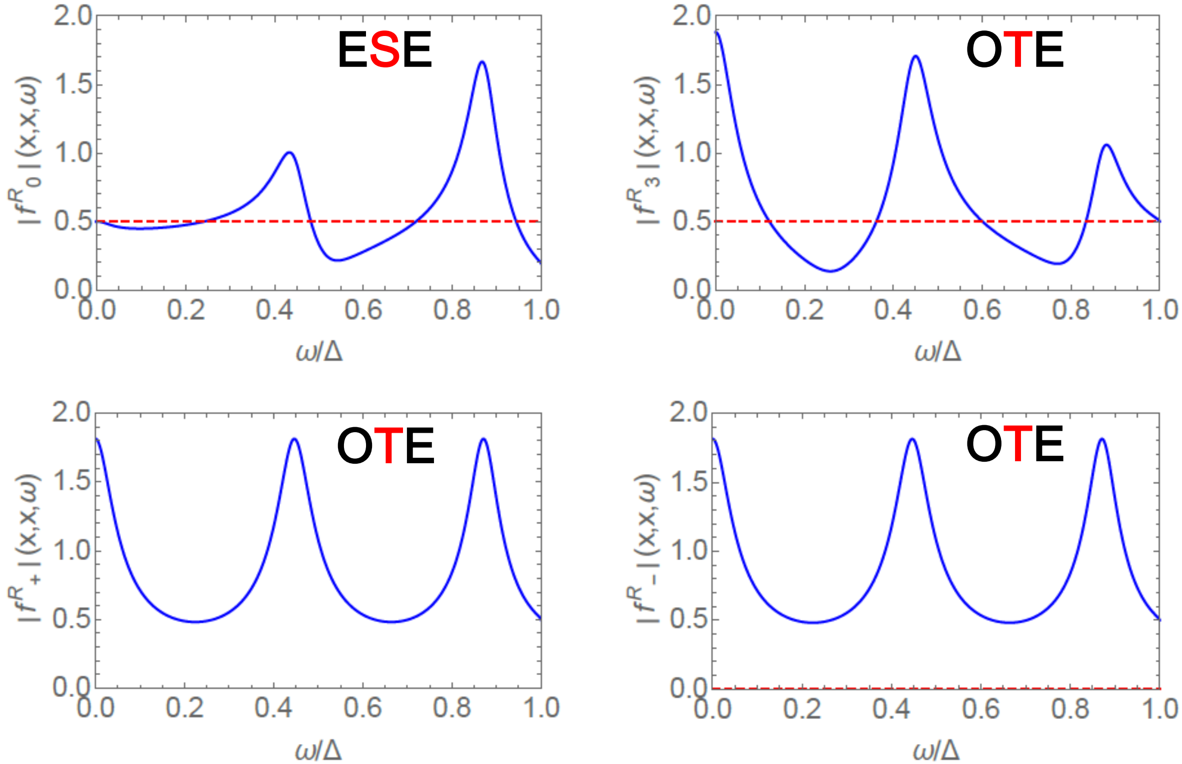
Crépin, Burset & BT arXiv 2015

Classification of pairing amplitude: results

$$x = L = 3\xi$$



$$m_0 = 0.5\Delta$$



$$f^R_\alpha(x,x,\omega) = f^R_{\alpha,bulk}(x,x,\omega) + f^R_{\alpha,edge}(x,x,\omega)$$

Crépin, Burset & BT arXiv 2015

Outline

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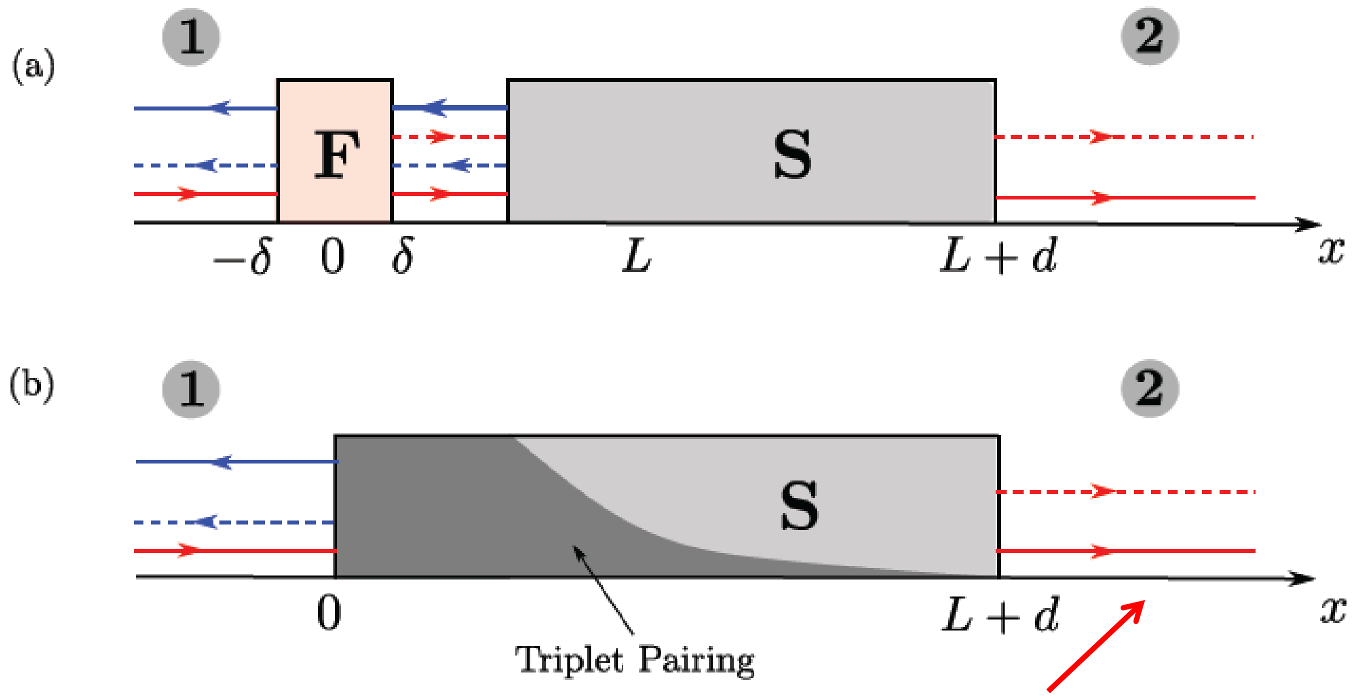
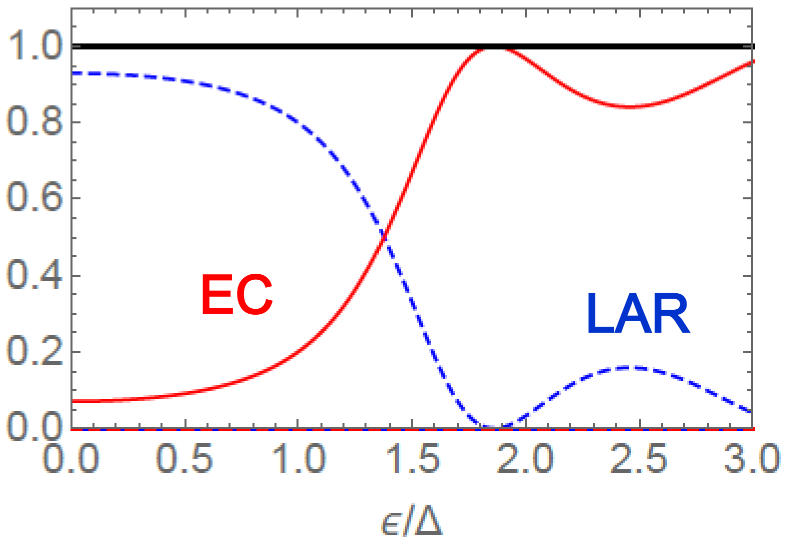
Detection of OTE: idea



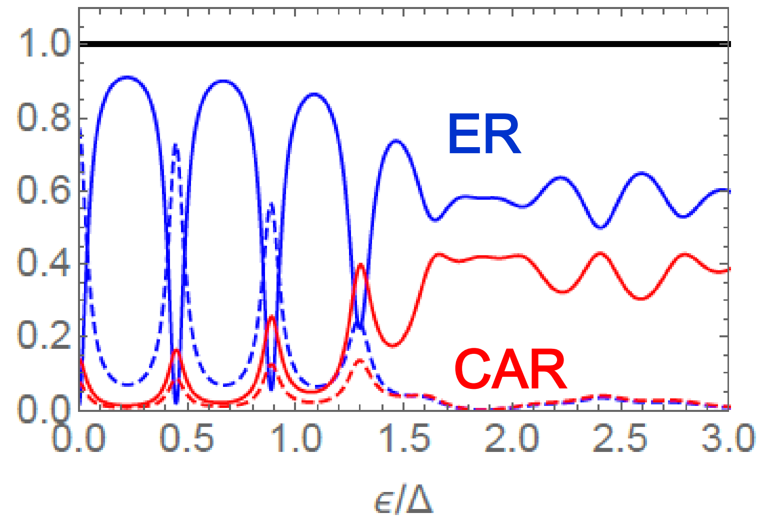
use crossed Andreev reflection

Crépin, Burset & BT arXiv 2015

Detection of OTE: results



(a) $d = 2\xi,\ m_0 = 0$



(c) $d = 2\xi,\ m_0 = 0.5,\ L = 3\xi$

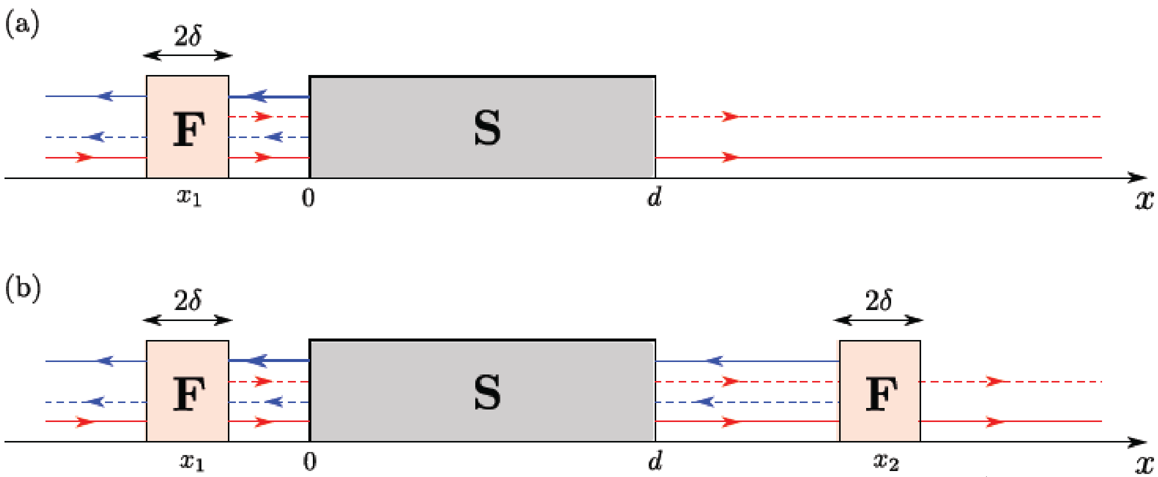
Adroguer et al. PRB 2010

-> dilemma

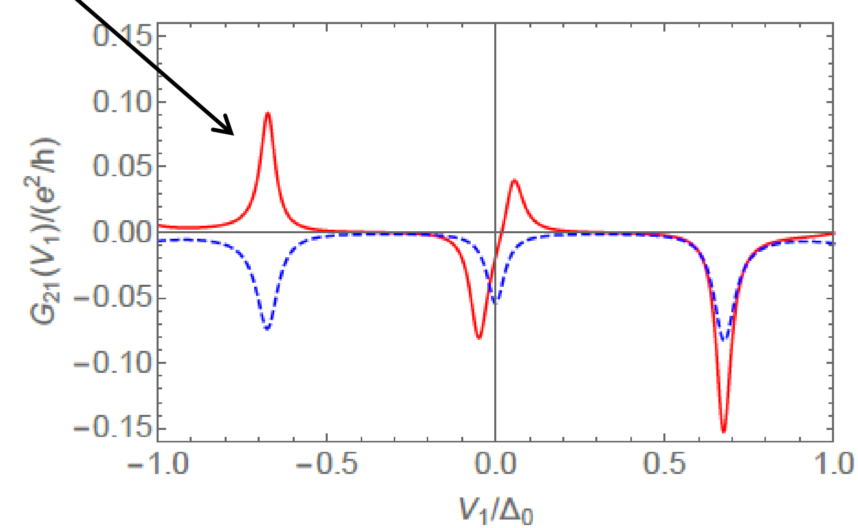
$$\frac{T_{CAR}}{T_{EC}} = \tanh^2\left(2m_0\right)\tanh^2\left(d/\xi\right) \le 1$$

Crépin, Burset & BT arXiv 2015

Way out …

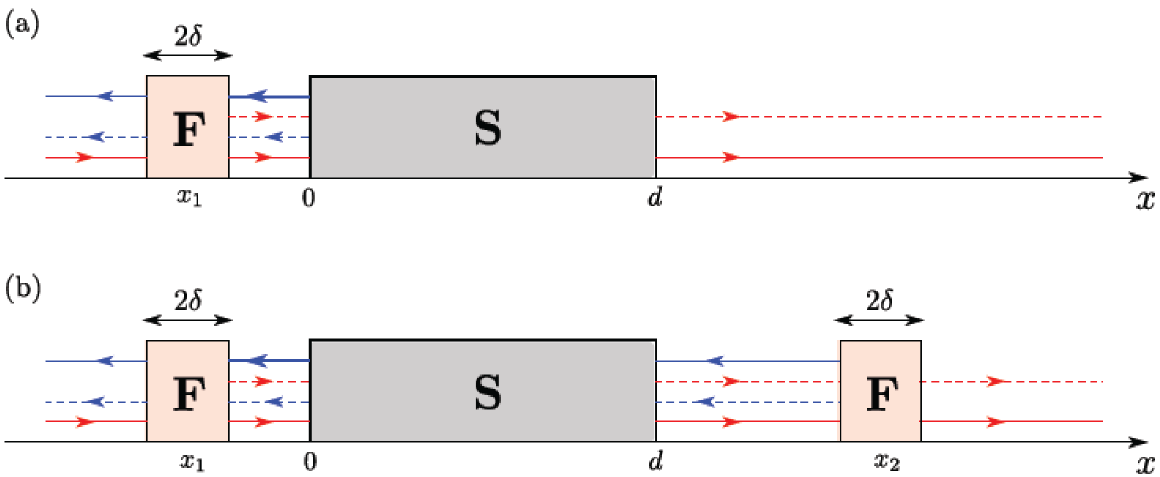


… add complexity



$$G_{21} = -\partial I_2 / \partial V_1 = T_{CAR} - T_{EC}$$

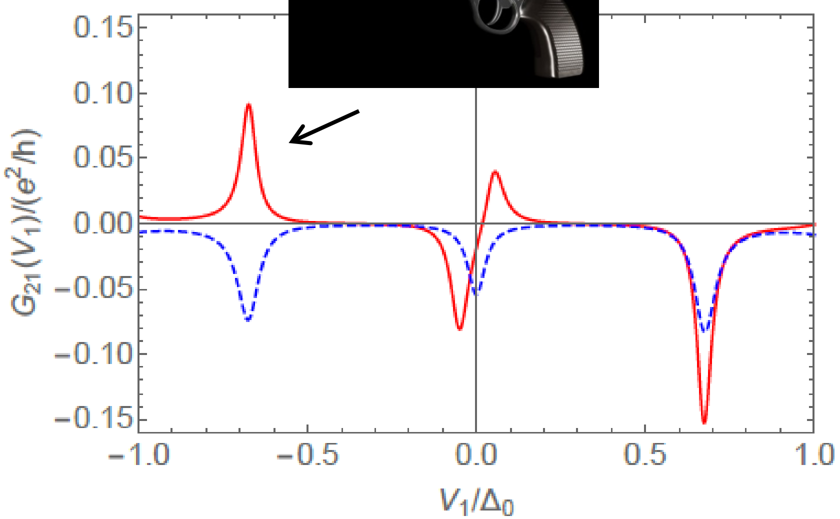
Way out …



… add complexity

OTE

$$G_{21} = -\partial I_2 \Big/ \partial V_1 = T_{CAR} - T_{EC}$$



Outline

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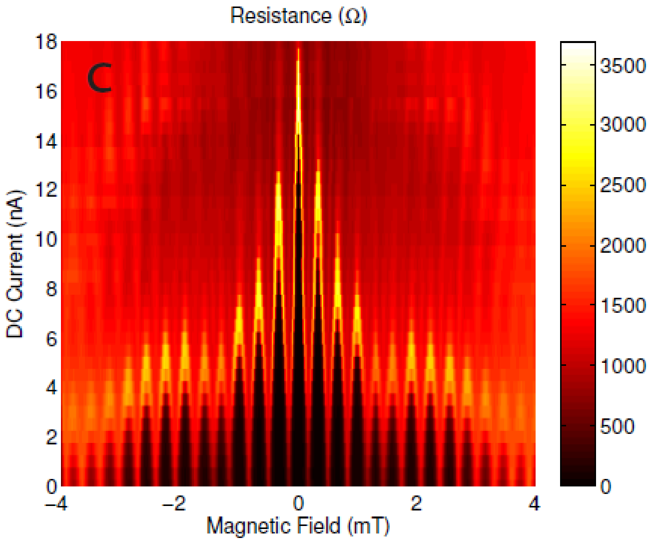
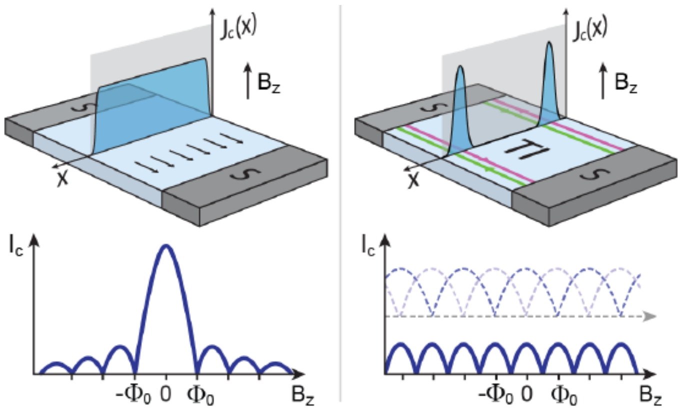
Inspiring experiments

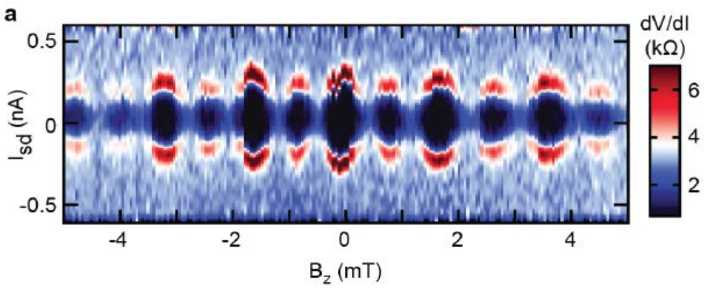
Hg(Cd)Te QWs



Hart et al. Nature Phys 2014

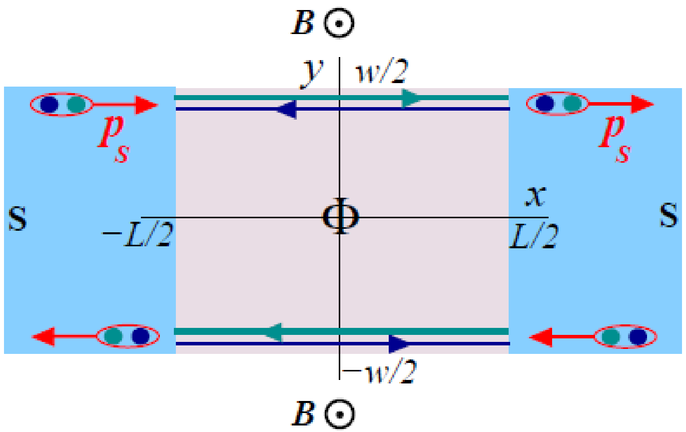
InAs/GaSb QWs





Pribiag et al. arXiv 2014

Setup & Hamiltonian



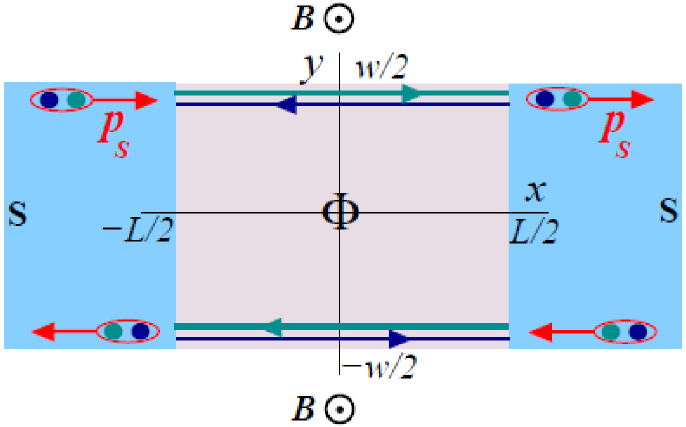
$$H_{BdG} = \begin{pmatrix} h(x) & i\sigma_y \Delta(x) e^{i\varphi_0(x)} \\ -i\sigma_y \Delta(x) e^{-i\varphi_0(x)} & -h^*(x) \end{pmatrix}$$

$$\Delta(x) = \begin{cases} 0, & |x| < L/2 \\ \Delta, & |x| \geq L/2 \end{cases}$$

$$\varphi_0(x) = \begin{cases} -\phi_0/2, & \mathrm{x} \leq -L/2 \\ +\phi_0/2, & \mathrm{x} \geq +L/2 \end{cases}$$

Tkachov, Burset, BT & Hankiewicz arXiv 2014

Setup & Hamiltonian

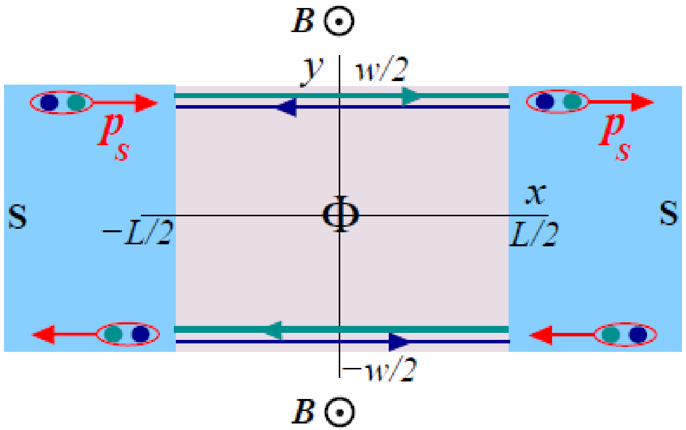


$$H_{BdG} = \begin{pmatrix} h(x) & i\sigma_y \Delta(x) e^{i\varphi_0(x)} \\ -i\sigma_y \Delta(x) e^{-i\varphi_0(x)} & -h^*(x) \end{pmatrix}$$

$$h(x) = v_F \sigma_x \left(-i\partial_x + \frac{p_S}{2} \right) + U(x) - \mu$$

Tkachov, Burset, BT & Hankiewicz arXiv 2014

Setup & Hamiltonian



$$H_{BdG} = \begin{pmatrix} h(x) & i\sigma_y \Delta(x) e^{i\varphi_0(x)} \\ -i\sigma_y \Delta(x) e^{-i\varphi_0(x)} & -h^*(x) \end{pmatrix}$$

$$h(x) = v_F \sigma_x \left(-i\partial_x + \frac{p_S}{2} \right) + U(x) - \mu$$

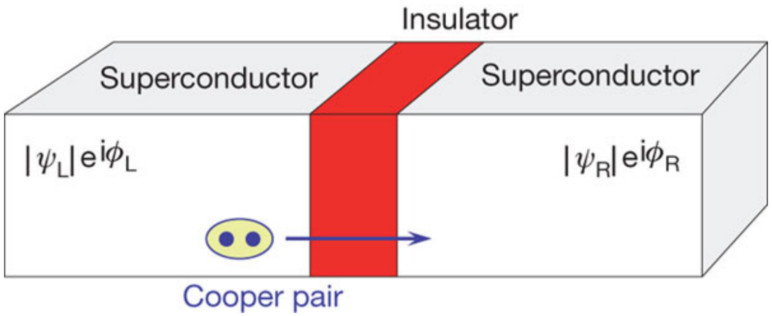
shielding response of SC
-> Doppler shift

$$p_S(B) = -\frac{2e}{c} A_x\left(\pm\frac{w}{2}\right) = \pm\pi\frac{Bw}{\Phi_0}$$

Tkachov & Falko PRB 2004
Tkachov & Richter PRB 2005
Rohlfing et al. PRB 2009

Tkachov, Burset, BT & Hankiewicz arXiv 2014

Josephson current: basics



$$J = \frac{2e}{\hbar}\frac{dF}{d\phi}$$

free energy

phase difference

DOS

$$F = -2k_B T \int_0^\infty d\varepsilon\, \rho(\varepsilon) \ln\left[2\cosh\left(\varepsilon / 2k_B T\right)\right] + [\phi\text{-independent}]$$

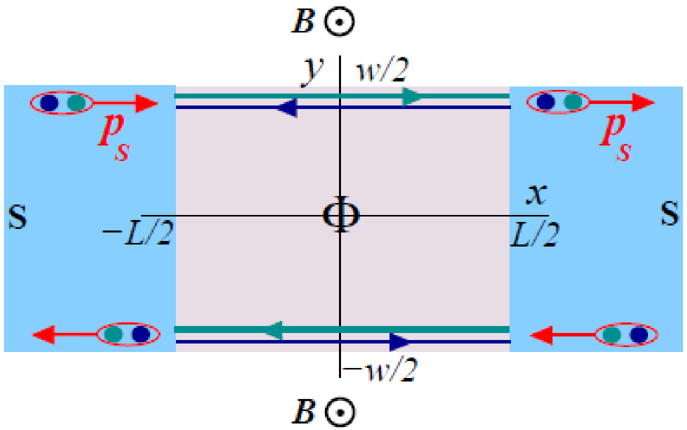
$$J = -\frac{2e}{\hbar} 2k_B T \frac{d}{d\phi} \sum_{n=0}^{\infty} \ln\det\left[1 - S_A(i\omega_n) S_N(i\omega_n)\right]$$

$$\omega_n = (2n+1)\pi k_B T$$

Beenakker & Brouwer Chaos, Solitons & Fractals 1997

Josephson current

$$J_u(\phi_0, B) = -\frac{2e}{\hbar} k_B T \sum_{n=0}^{\infty} \frac{A_n^2(-B)e^{i\phi} - A_n^2(B)e^{-i\phi}}{\left[1 + A_n(-B)A_n(B)\right]^2 + \left[A_n(-B)e^{i\phi/2} - A_n(B)e^{-i\phi/2}\right]^2}$$

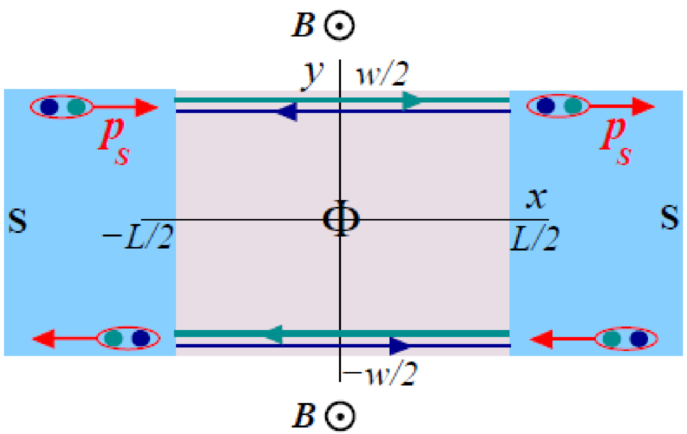


$$\phi = \phi_0 + k_S L = \phi_0 + \pi \, \Phi/\Phi_0$$

Tkachov, Burset, BT & Hankiewicz arXiv 2014

Josephson current

$$J_u\left(\phi_0, B\right) = -\frac{2e}{\hbar} k_B T \sum_{n=0}^{\infty} \frac{A_n^2\left(-B\right)e^{i\phi} - A_n^2\left(B\right)e^{-i\phi}}{\left[1 + A_n\left(-B\right)A_n\left(B\right)\right]^2 + \left[A_n\left(-B\right)e^{i\phi/2} - A_n\left(B\right)e^{-i\phi/2}\right]^2}$$



$$\phi = \phi_0 + k_S L = \phi_0 + \pi \, \Phi / \Phi_0$$

$$J_l\left(\phi_0, B\right) = J_u\left(\phi_0, -B\right)$$

B-dependent Andreev reflection

$$A_n\left(B\right) = \frac{\Delta e^{-\omega_n L/\hbar v_F}}{\omega_n + \frac{i}{2} v_F p_S\left(B\right) + \sqrt{\left[\omega_n + \frac{i}{2} v_F p_S\left(B\right)\right]^2 + \Delta^2}}$$

Tkachov, Burset, BT & Hankiewicz arXiv 2014

When is it relevant?

$$\frac{v_F |p_S|}{2} \geq \max(\Delta, \pi k_B T)$$

$$B \geq B_{AR} = \frac{2\Phi_0}{\pi w \xi_*}; \xi_* = \min\left(\frac{\hbar v_F}{\Delta}, \frac{\hbar v_F}{\pi k_B T}\right)$$

quite small: a few mT

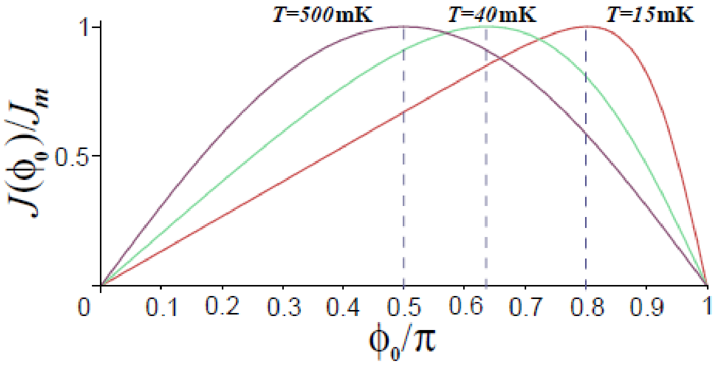
Tkachov, Burset, BT & Hankiewicz arXiv 2014

When is it relevant?

$$\frac{v_F |p_S|}{2} \geq \max(\Delta, \pi k_B T)$$

$$B \geq B_{AR} = \frac{2\Phi_0}{\pi w \xi_*}; \xi_* = \min\left(\frac{\hbar v_F}{\Delta}, \frac{\hbar v_F}{\pi k_B T}\right)$$

How does the maximum current at zero field change as a function of B?



B=0 current-phase relation

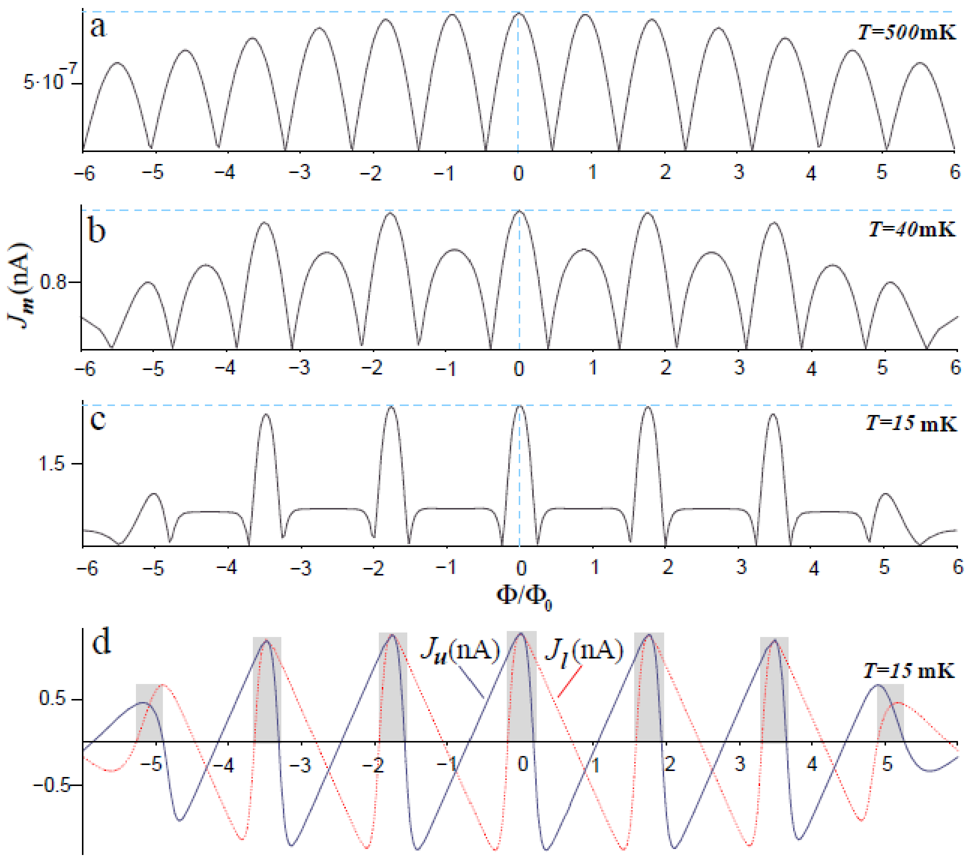
$$J_m(B) = |J(\phi_{0,\max}, B)|$$

$$J(\phi_0, B) = J_u(\phi_0, B) + J_l(\phi_0, B)$$

Tkachov, Burset, BT & Hankiewicz arXiv 2014

Results: long junction

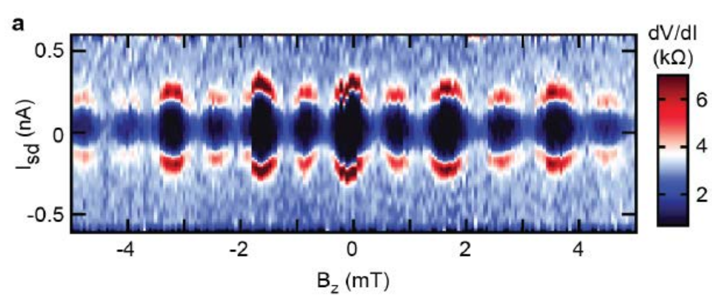
$L > \xi$



$$J_m(B) \approx \left| \frac{8ek_BT}{\hbar} \mathrm{Re}\left(A_0^2(B)\, e^{-i\pi \Phi/\Phi_0} \right) \right|$$

$$B_{AR} > B_{osc} = \frac{\Phi_0}{\pi w L}$$

even-odd effect

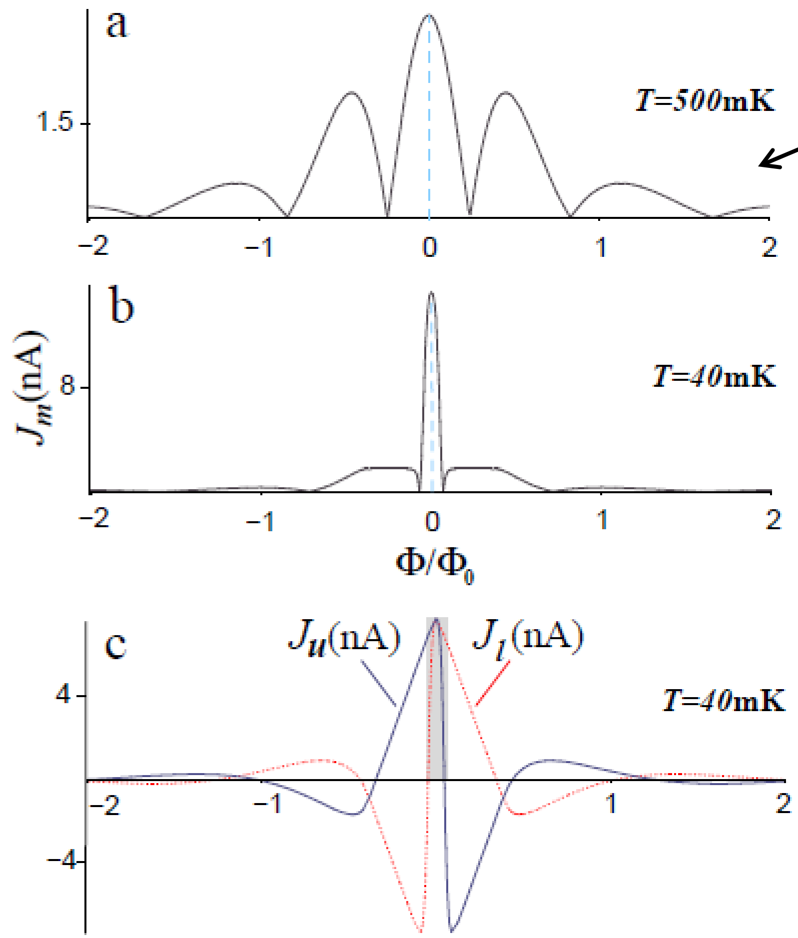


Pribiag et al. arXiv 2014

Tkachov, Burset, BT & Hankiewicz arXiv 2014

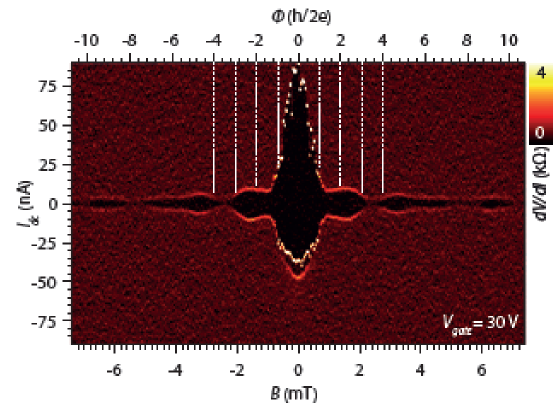
Results: short junction

$L < \xi$



strong suppression

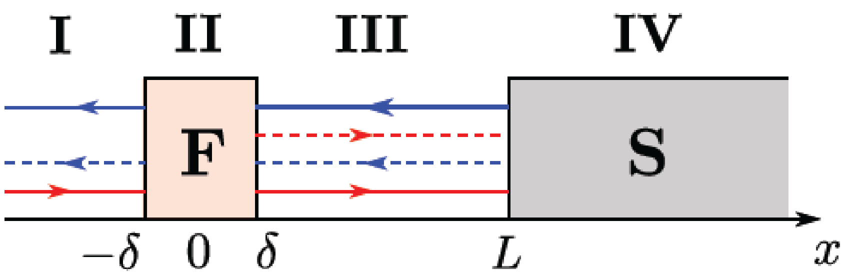
$$B_{AR} < B_{osc} = \frac{\Phi_0}{\pi w L}$$



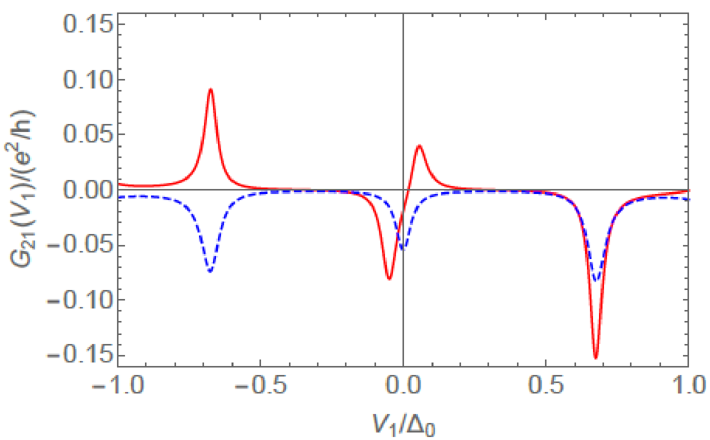
Calado et al. arXiv 2015

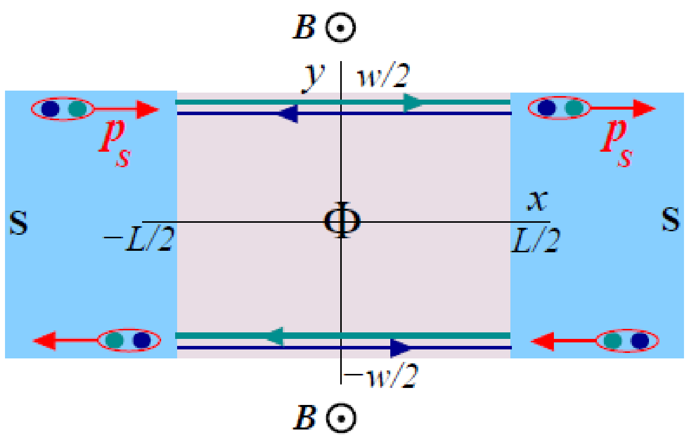
Tkachov, Burset, BT & Hankiewicz arXiv 2014

Summary Lecture 2

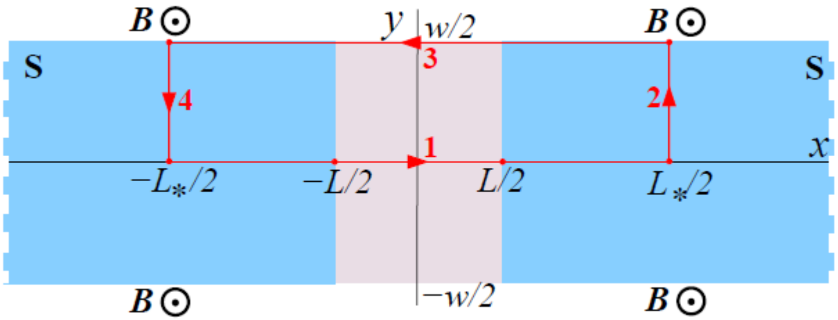


	Pairing	Interface	Bulk
f_0	$\uparrow\downarrow - \downarrow\uparrow$	ESE+OSO	ESE
f_3	$\uparrow\downarrow + \downarrow\uparrow$	ETO+OTE	ETO
$f_\pm$	$\uparrow\uparrow, \downarrow\downarrow$	OTE	X





Gauge-invariant phase



$$\mathbf{B} = \left[0, 0, B\left(x, y\right)\right]$$

$$B\left(x, \pm {}^{w}\!/_{2}\right) = B$$

Ampère‘s law:

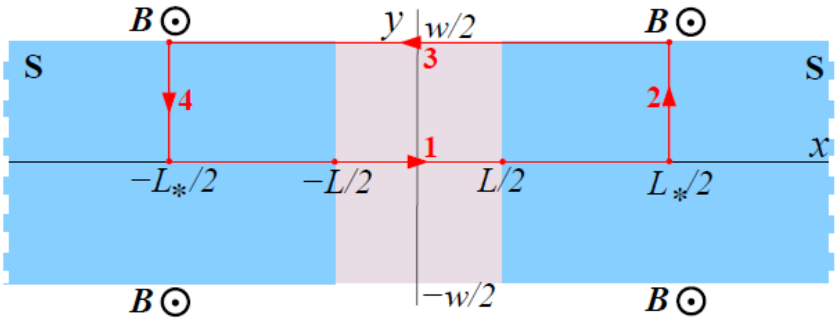
$$\partial_y B\left(x, y\right) = \frac{4\pi}{c} j_x\left(x, y\right), -\partial_x B\left(x, y\right) = \frac{4\pi}{c} j_y\left(x, y\right)$$

bc imply:

$$B\left(x, -y\right) = B\left(x, y\right) \Rightarrow j_x\left(x, -y\right) = -j_x\left(x, y\right)$$

Tkachov, Burset, BT & Hankiewicz arXiv 2014

Gauge-invariant phase



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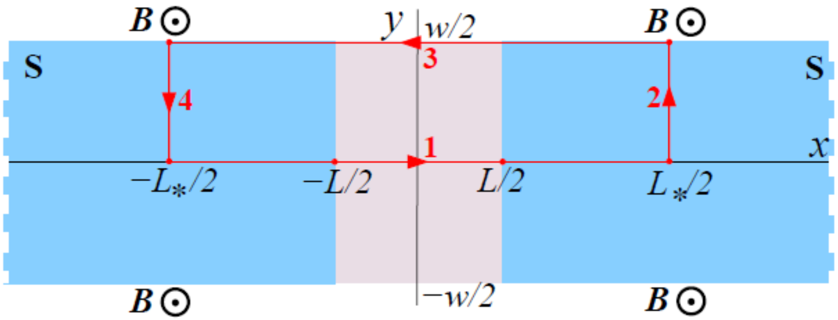
type-II SC -> $\vec{j} \propto \vec{\nabla}\varphi$

gauge-invariant
phase gradient

$$\vec{\nabla}\varphi = \vec{\nabla}\varphi_0 - \frac{2\pi}{\Phi_0}\vec{A}$$

Tkachov, Burset, BT & Hankiewicz arXiv 2014

Gauge-invariant phase



$$\oint \nabla\varphi d\mathbf{l} = 2\pi N - 2\pi \frac{\Phi/2}{\Phi_0}$$

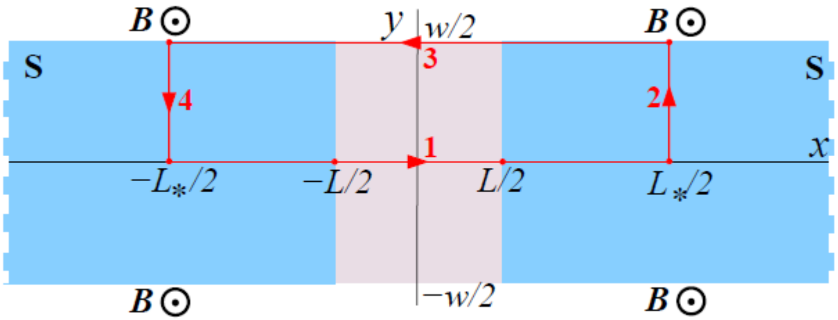
winding number

$$j_x(x,-y) = -j_x(x,y) \Rightarrow \partial_x \varphi(x,0) = 0$$

$$\Rightarrow \varphi(x,0) = \text{piecewise const.}$$

Tkachov, Burset, BT & Hankiewicz arXiv 2014

Gauge-invariant phase



$$\oint \nabla\varphi d\mathbf{l} = 2\pi N - 2\pi \frac{\Phi/2}{\Phi_0}$$

integral over path 1

$$\varphi\left(L/2, 0\right) - \varphi\left(-L/2, 0\right) \equiv \phi_0$$

integral over path 3

$$\varphi\left(-L_*/2, w/2\right) - \varphi\left(L_*/2, w/2\right) \equiv -\phi\left(w/2\right)$$

integrals over path 2&4 vanish

$$\phi\left(w/2\right) = \phi(0) + \pi\, \Phi/\Phi_0 - 2\pi N$$

Tkachov, Burset, BT & Hankiewicz arXiv 2014