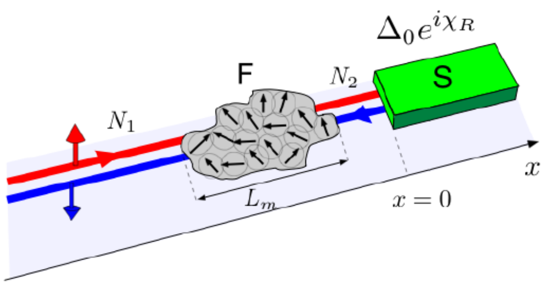
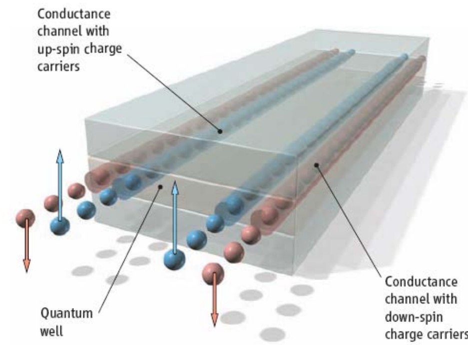
Superconducting hybrids based on QSH systems

Lecture 1



11th Capri Spring School
Transport in Nanostructures

April 13-17, 2015
Capri, Italy



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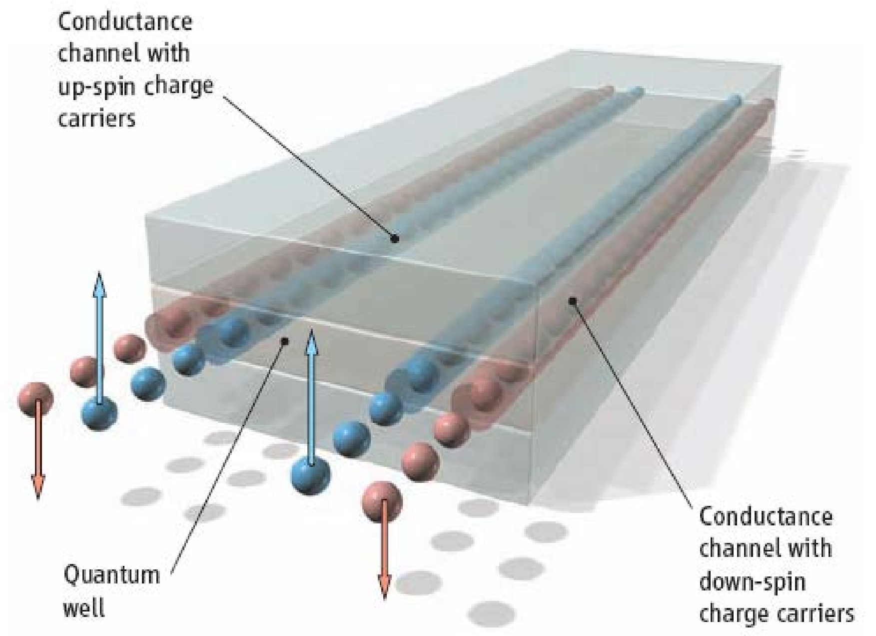
DFG Deutsche Forschungsgemeinschaft
Alexander von Humboldt Stiftung/Foundation
Helmholtz Association

Outline

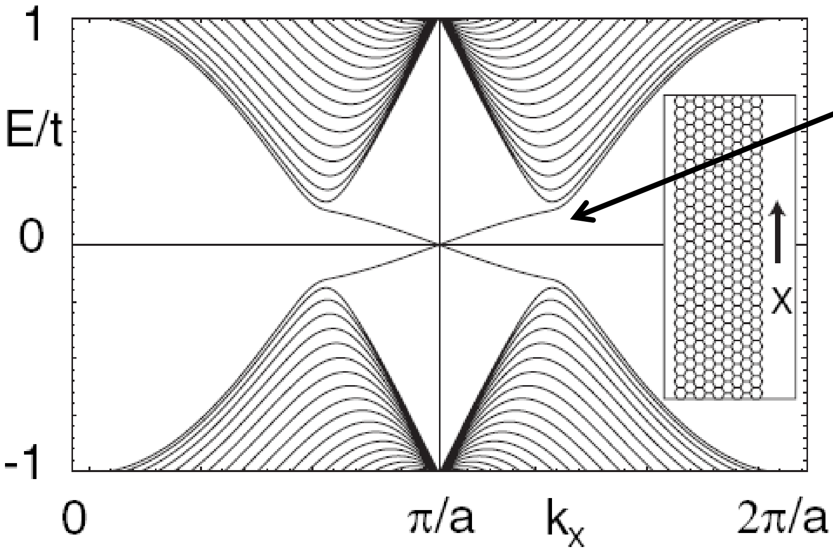
- **Recap**: Quantum spin Hall (QSH) systems
- **Introduction**: Hybrid structures based on QSH systems – Bogoliubov-De Gennes Formalism
- **Application**: Transport signatures in NS junctions

What are QSH systems?



first symmetry-protected topological state of matter

Prediction in graphene



spin filtered edge states:
topologically non-trivial
w/ respect to TRS

$$H = v\left(p_x\sigma_x\tau_x + p_y\sigma_y\right) + \Delta_{SO}\sigma_z\tau_z s_z$$

Kane & Mele PRL 2005

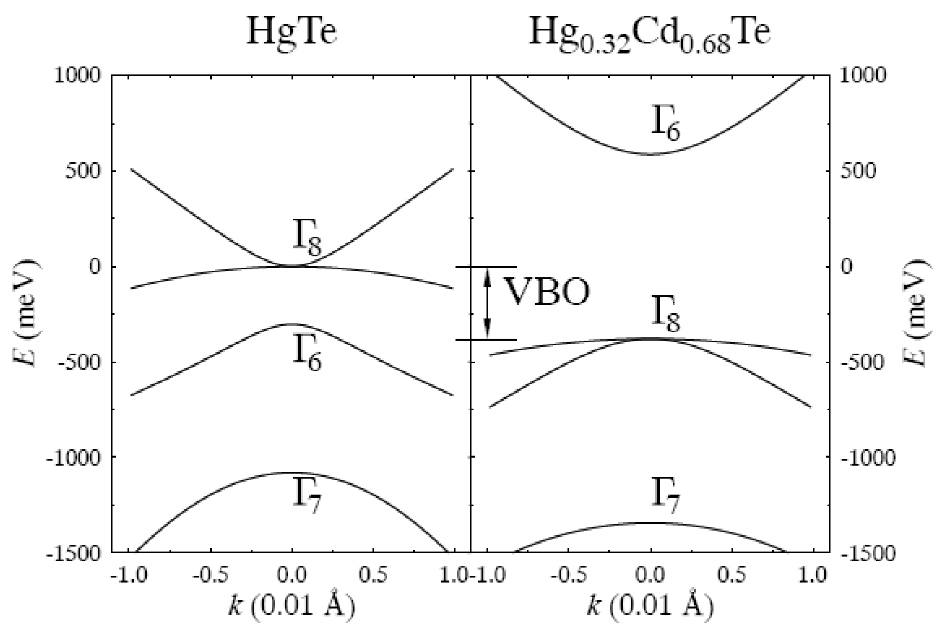
$$\Delta_{SO} \approx 24\mu\text{eV}$$

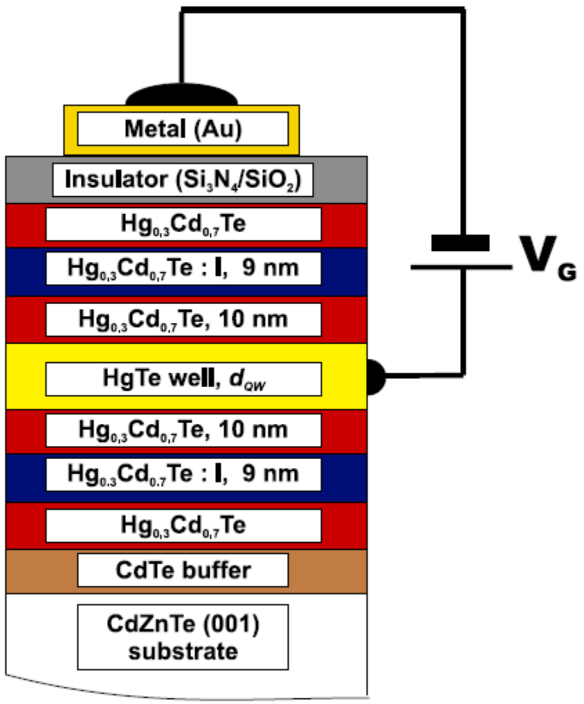
Gmitra, Konschuh, Ertler, Ambrosch-Draxl & Fabian PRB 2009

SOI too small …

How about a better system?

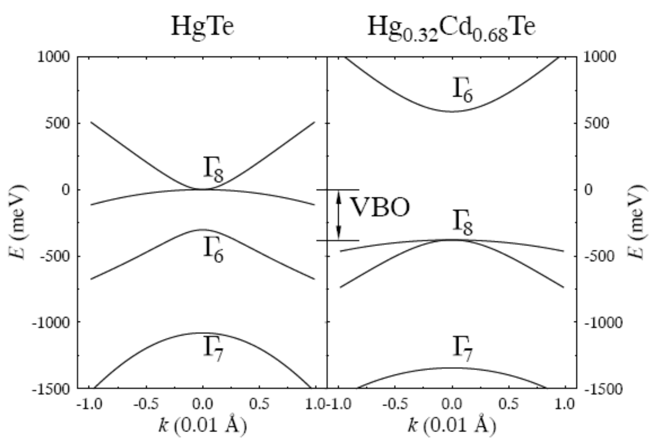
HgTe/CdTe QW I

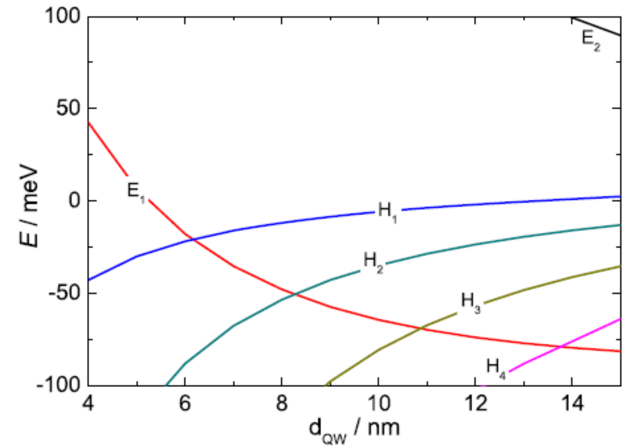


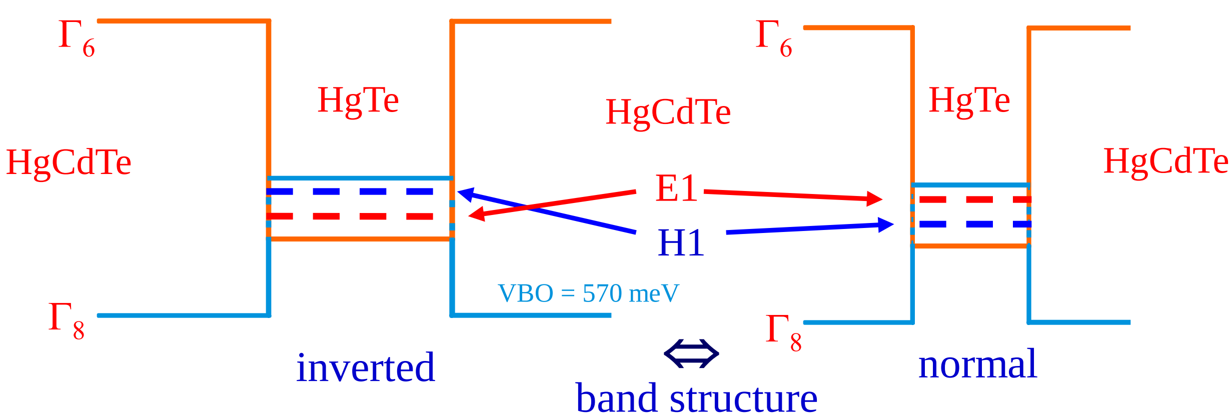


König et al. JPSJ 2008

HgTe/CdTe QW II







Effective Hamiltonian

$$H = \begin{pmatrix} h(\vec{k}) & 0 \\ 0 & h^*(-\vec{k}) \end{pmatrix} \quad \text{with} \quad h(\vec{k}) = \varepsilon(\vec{k}) + d_a(\vec{k})\sigma^a$$

Bernevig, Hughes, Zhang Science 2006

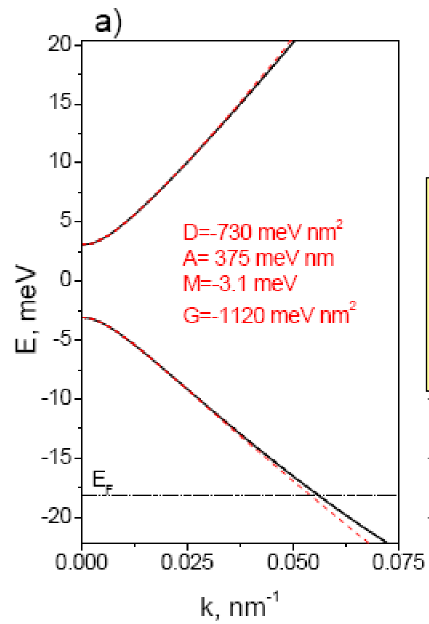
$$\varepsilon(\vec{k}) = C - Dk^2$$

$$\vec{d}(\vec{k}) = (Ak_x, -Ak_y, M - Gk^2)$$

with basis states:

$$\{|E+\rangle, |H+\rangle, |E-\rangle, |H-\rangle\}$$

±: degenerate Kramers partners



comparison to 8-band Kane model

Novik et al. PRB 2005

Schmidt, Novik, Kindermann & BT PRB 2009

Topology in the QSHE I

$$H = \begin{pmatrix} h(\vec{k}) & 0 \\ 0 & h^*(-\vec{k}) \end{pmatrix} \quad \text{with} \quad h(\vec{k}) = \varepsilon(\vec{k}) + d_a(\vec{k})\sigma^a$$

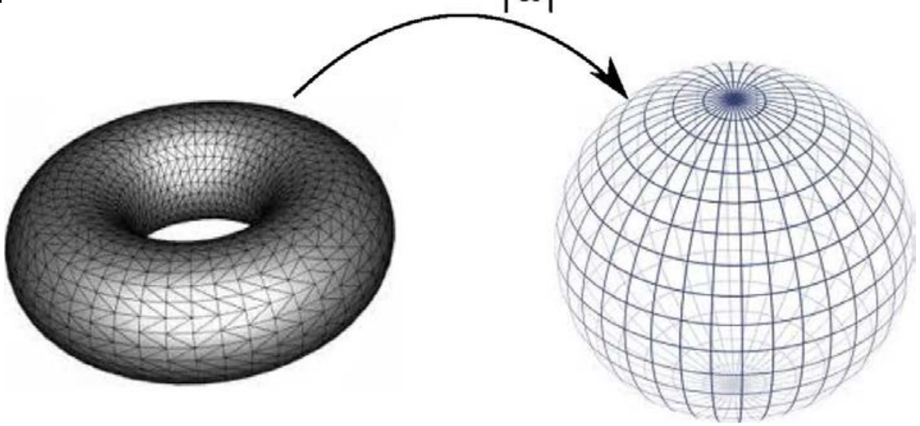
TRS preserved

Thouless, Kohmoto, Nightingale & Den Nijs

TKNN invariant for **one** Kramers partner (quantum anomalous Hall effect):

$$n_\uparrow = \frac{1}{4\pi}\int d\mathbf{k}\left(\partial_{k_x}\hat{d} \times \partial_{k_y}\hat{d}\right)\cdot\hat{d}$$

$$\hat{d} \equiv \frac{\vec{d}}{|\vec{d}|}$$



Avron et al PRL 1988; *Kane & Mele* PRL 2005; *Sheng et al* PRL 2006; *Hasan & Kane* RMP 2010

Topology in the QSHE I

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TKNN invariant for **one** Kramers partner:

$$n_\uparrow = \frac{1}{4\pi}\int d\mathbf{k}\left(\partial_{k_x}\hat{d}\times\partial_{k_y}\hat{d}\right)\cdot\hat{d}$$

zero Hall conductivity:

$$n = n_\uparrow + n_\downarrow = 0$$

quantized spin Hall conductivity:

$$n_\sigma = \frac{n_\uparrow - n_\downarrow}{2}$$

Avron et al PRL 1988; *Kane & Mele* PRL 2005; *Sheng et al* PRL 2006; *Hasan & Kane* RMP 2010

Topology in the QSHE II

quantized spin Hall conductivity:

$$n_\sigma = \frac{n_\uparrow - n_\downarrow}{2}$$

-> Z_2 invariant:

$$\nu = n_\sigma \bmod 2$$

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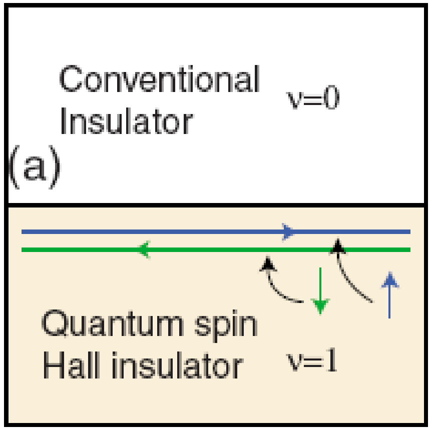
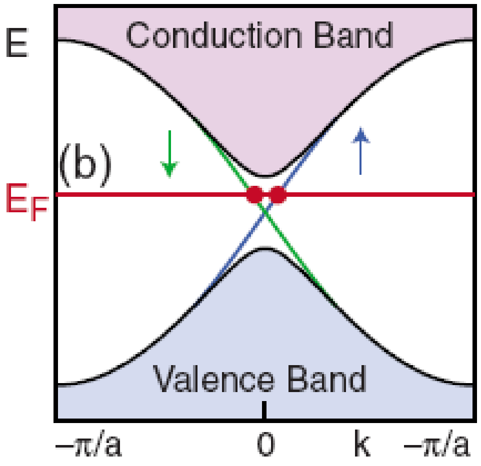




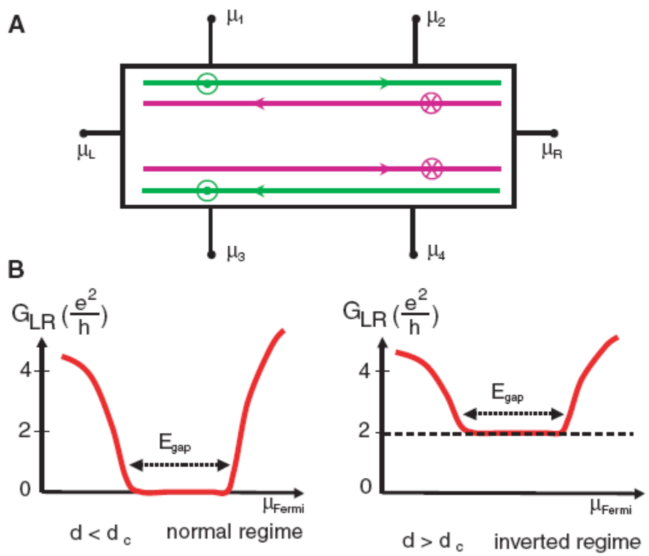
\# of Kramers pairs at edge:

$$N_K = \Delta\nu \bmod 2$$

Avron et al PRL 1988; *Kane & Mele* PRL 2005; *Sheng et al* PRL 2006; *Hasan & Kane* RMP 2010

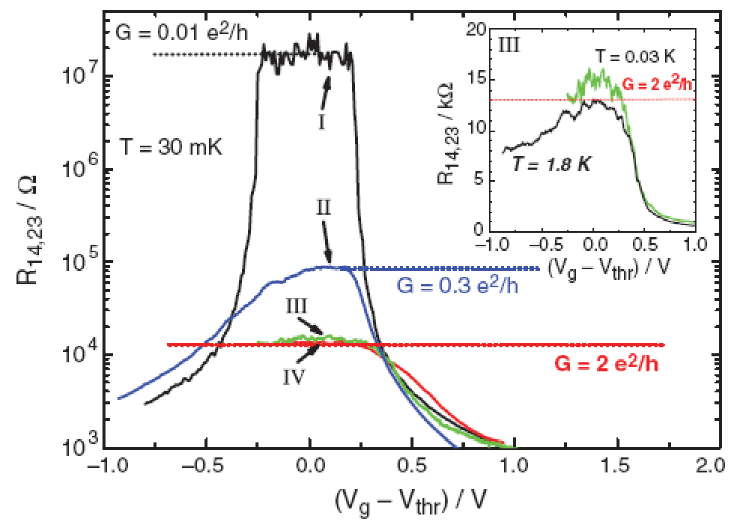
Experimental evidence of edge states

Prediction:



Bernevig, Hughes & Zhang Science 2006

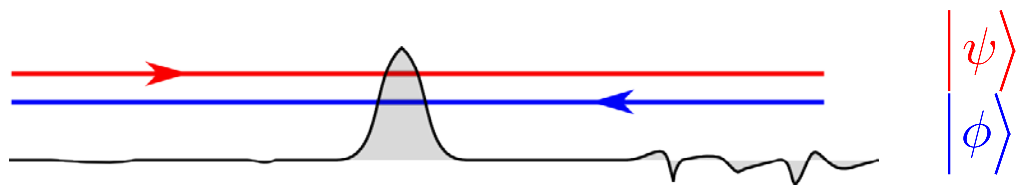
Observation:



König et al. Science 2007

Protection against elastic backscattering

$$\langle \psi | H | \phi \rangle = \langle \phi | H | \psi \rangle^* = \langle T\phi | TH | \psi \rangle$$
$$= \langle \psi | HT | \psi \rangle = \langle \psi | HT^2 | \phi \rangle = -\langle \psi | H | \phi \rangle$$



Kane & Mele PRL 2005; *Xu & Moore* PRB 2006; *Wu, Bernevig & Zhang* PRL 2006

Protection against elastic backscattering

$$\langle \lambda | \mu \rangle = \langle \mu | \lambda \rangle^*$$

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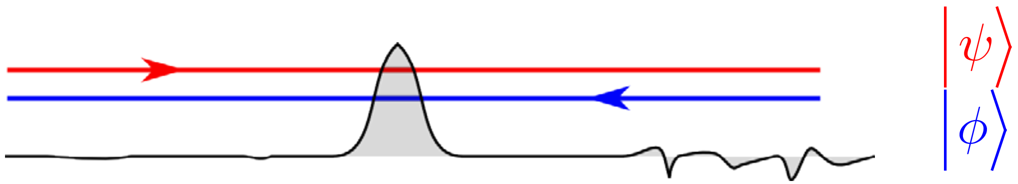

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T: anti-unitary operator

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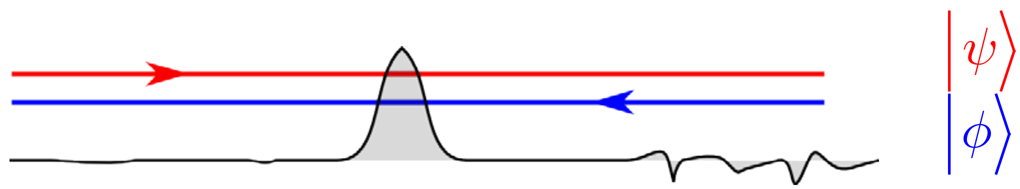

Kane & Mele PRL 2005; *Xu & Moore* PRB 2006; *Wu, Bernevig & Zhang* PRL 2006

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$[H,T] = 0$ and $|\psi\rangle = T|\phi\rangle$

Kane & Mele PRL 2005; *Xu & Moore* PRB 2006; *Wu, Bernevig & Zhang* PRL 2006

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$[H, T] = 0 \text{ and } |\psi\rangle = T|\phi\rangle$

$T^2 = -1$

Kane & Mele PRL 2005; *Xu & Moore* PRB 2006; *Wu, Bernevig & Zhang* PRL 2006

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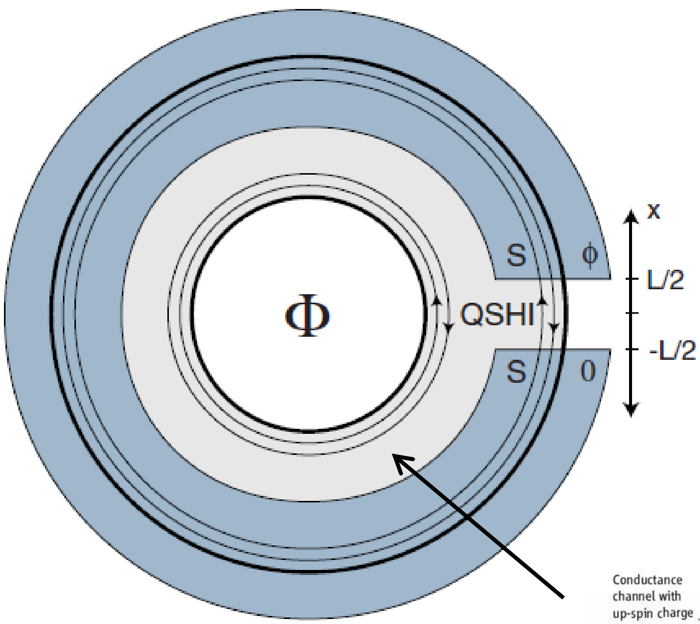
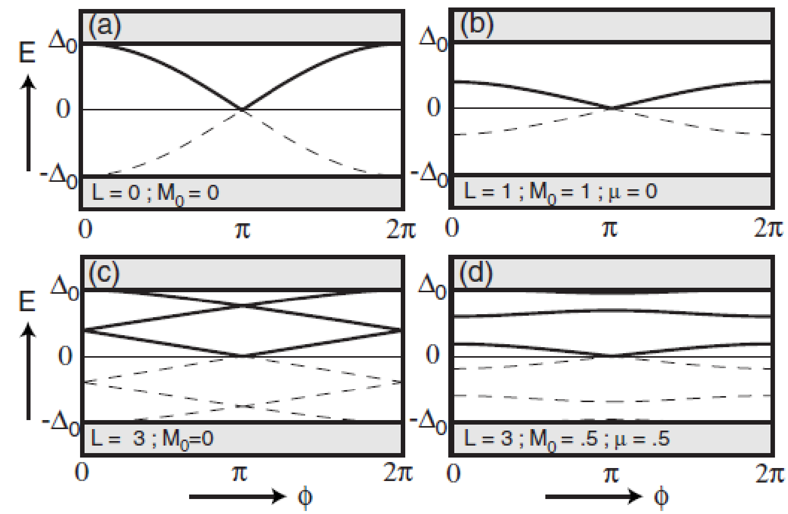
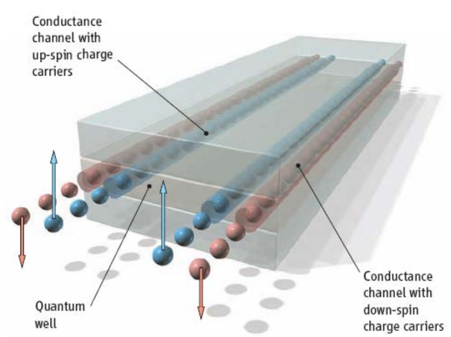
Pioneering prediction





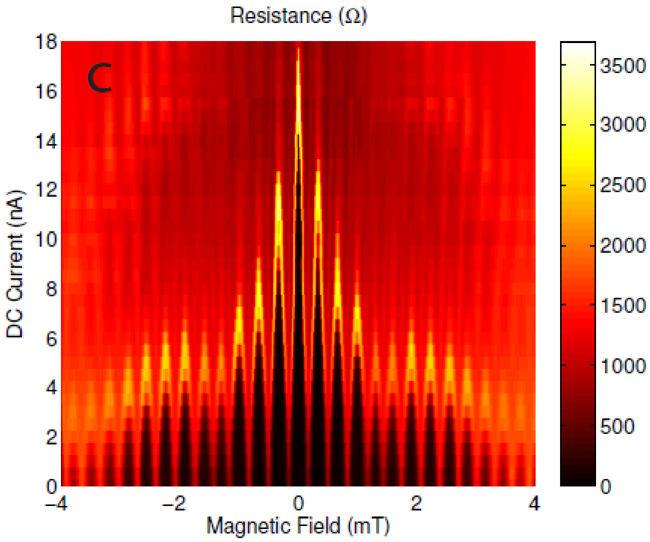


signatures of **p-wave** superconductivity?

Fu & Kane PRB 2009

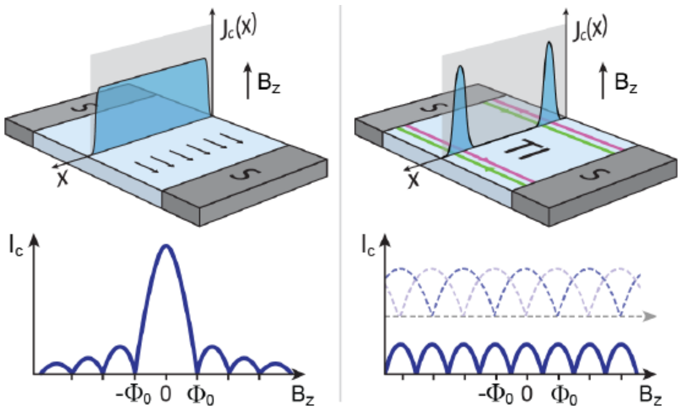
Inspiring experiments

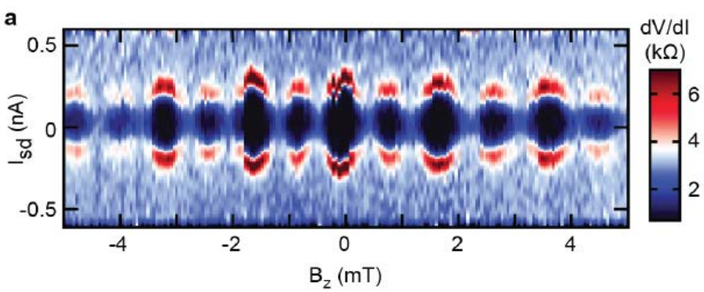
Hg(Cd)Te QWs



Hart et al. Nature Phys 2014

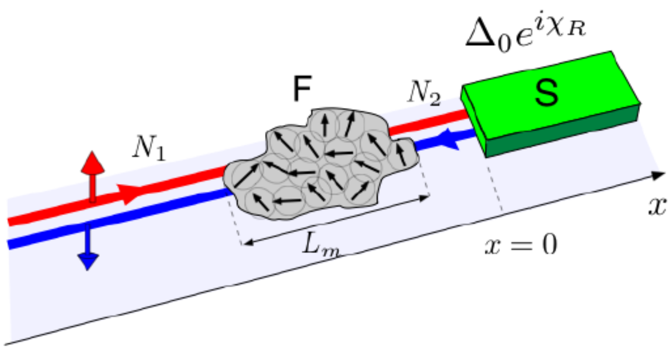
InAs/GaSb QWs





Pribiag et al. arXiv 2014

Setup & Hamiltonian

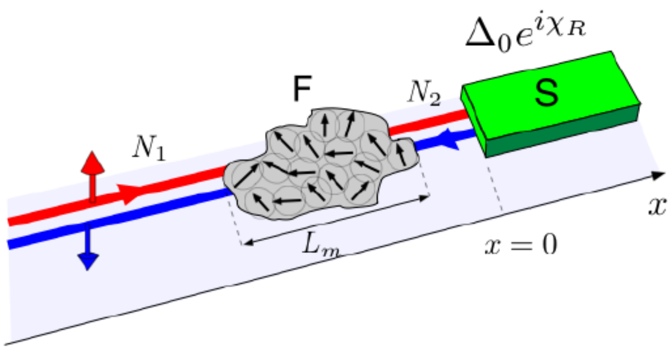


$$H = H_0 + H_{FM} + H_{SC}$$

$$H_0 = \int dx \left(\psi^\dagger_{R\uparrow}, \psi^\dagger_{L\downarrow}\right)\left[v_F p_x \sigma_z - \mu\right]\begin{pmatrix}\psi_{R\uparrow} \\ \psi_{L\downarrow}\end{pmatrix}$$

Crépin, BT & Dolcini PRB 2014

Setup & Hamiltonian



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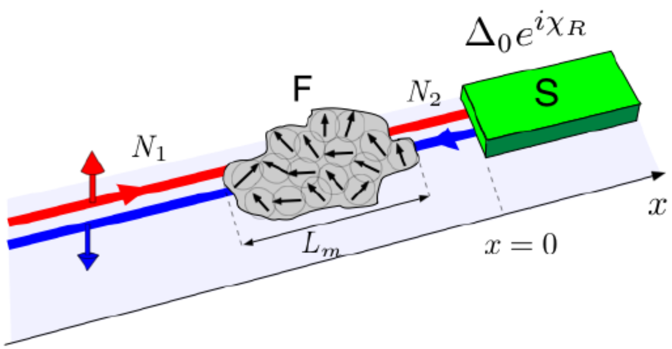
$$H_{FM} = \int dx \left(\psi^\dagger_{R\uparrow}, \psi^\dagger_{L\downarrow}\right)\vec{m}(x)\cdot\vec{\sigma}\begin{pmatrix}\psi_{R\uparrow} \\ \psi_{L\downarrow}\end{pmatrix}$$

Crépin, BT & Dolcini PRB 2014

Setup & Hamiltonian



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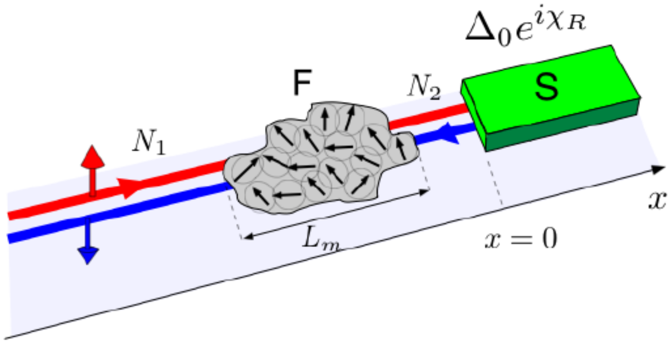
$$H_{SC} = \int dx \left[\Delta(x)\psi^\dagger_{R\uparrow}\psi^\dagger_{L\downarrow} + \Delta^*(x)\psi_{L\downarrow}\psi_{R\uparrow}\right]$$

Crépin, BT & Dolcini PRB 2014

BdG Hamiltonian



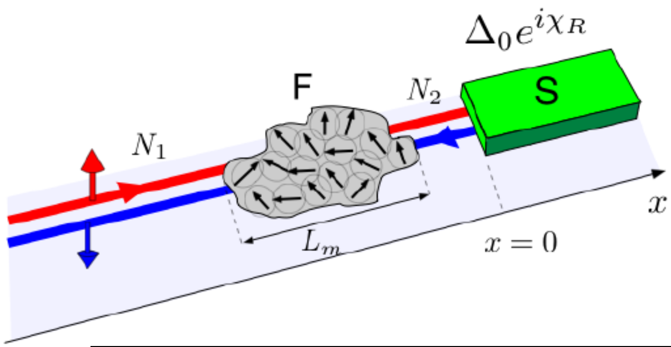
$$H = \frac{1}{2}\int dx \Psi^\dagger H_{BdG} \Psi$$

$$\Psi^\dagger = \left(\psi^\dagger_{R\uparrow}, \psi^\dagger_{L\downarrow}, \psi_{L\downarrow}, -\psi_{R\uparrow}\right)$$

$$H_{BdG} = \begin{pmatrix} H^e_{0+FM} & \Delta(x)\sigma_0 \\ \Delta^*(x)\sigma_0 & H^h_{0+FM} \end{pmatrix}$$

Crépin, BT & Dolcini PRB 2014

BdG Hamiltonian



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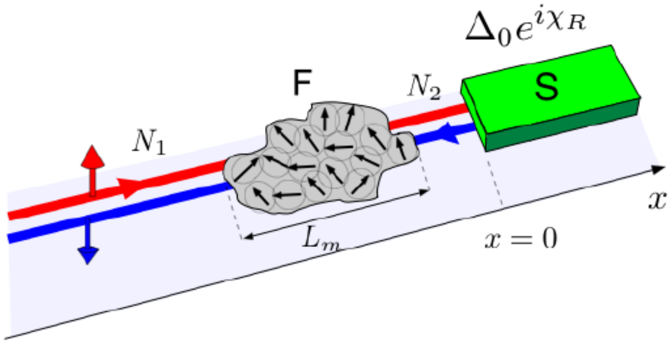
$$H^e_{0+FM} = v_F \sigma_z p_x - \mu\sigma_0 + \vec{m}(x)\cdot\vec{\sigma}$$

$$H_{BdG} = \begin{pmatrix} H^e_{0+FM} & \Delta(x)\sigma_0 \\ \Delta^*(x)\sigma_0 & H^h_{0+FM} \end{pmatrix}$$

$$H^h_{0+FM} = -TH^e_{0+FM}T^{-1} = -v_F \sigma_z p_x + \mu\sigma_0 + \vec{m}(x)\cdot\vec{\sigma}$$

Crépin, BT & Dolcini PRB 2014

BdG Hamiltonian



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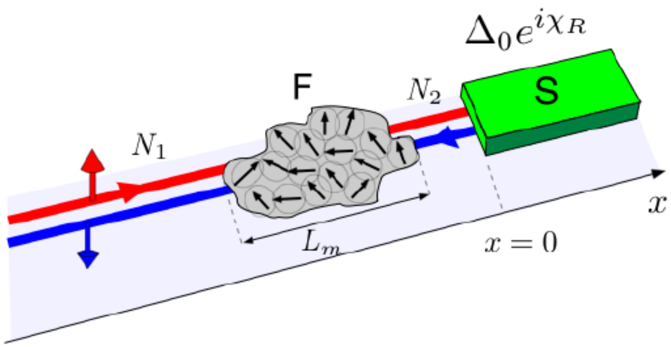
$$H = \sum_{\varepsilon_n \geq 0, j} \varepsilon_n \gamma^{\dagger}_{\varepsilon_n, j} \gamma_{\varepsilon_n, j}$$

$$\Psi(x) = \sum_{\varepsilon_n \geq 0, j} \left\{ \varphi_{\varepsilon_n, j}(x) \gamma_{\varepsilon_n, j} + \left[C\varphi_{\varepsilon_n, j}\right](x) \gamma^{\dagger}_{\varepsilon_n, j} \right\}$$

charge conjugated wave function

Crépin, BT & Dolcini PRB 2014

Symmetries & Majoranas



Particle-hole symmetry

$$C H_{BdG} C^{-1} = -H_{BdG}$$

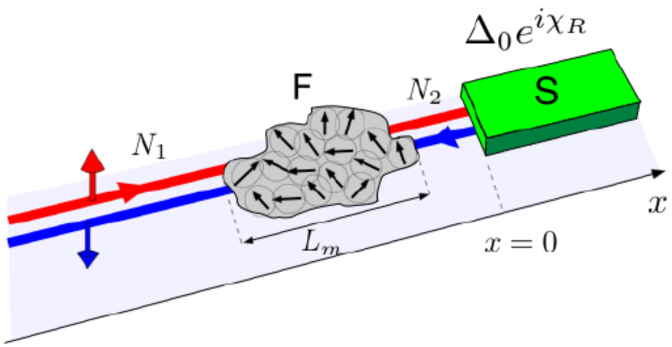
$$C = K\tau_y \otimes \sigma_y = K U_C$$

$$\gamma^{\dagger}_{\varepsilon_n, j} = \gamma_{-\varepsilon_n, j}$$

Chamon, Jackiw, Nishida, Pi & Santos PRB 2010

Crépin, BT & Dolcini PRB 2014

Symmetries & Majoranas



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Majorana fermion

Chamon, Jackiw, Nishida, Pi & Santos PRB 2010

Crépin, BT & Dolcini PRB 2014

Majorana fermions *vs.* anyons

Majorana fermions

$$\Psi(x) = \sum_{\varepsilon_n \geq 0, j} \left\{ \varphi_{\varepsilon_n, j}(x)\, \gamma_{\varepsilon_n, j} + \left[C\varphi_{\varepsilon_n, j} \right](x)\, \gamma^\dagger_{\varepsilon_n, j} \right\}$$

fermions

$$\Psi^\dagger(x) = U_C \Psi(x)$$

$$\gamma^\dagger_{\varepsilon_n, j} = \gamma_{-\varepsilon_n, j}$$

Chamon, Jackiw, Nishida, Pi & Santos PRB 2010

Majorana fermions *vs.* anyons

Majorana fermions

$$\Psi(x) = \sum_{\varepsilon_n \geq 0, j} \left\{ \varphi_{\varepsilon_n, j}(x)\, \gamma_{\varepsilon_n, j} + \left[C \varphi_{\varepsilon_n, j} \right](x)\, \gamma^{\dagger}_{\varepsilon_n, j} \right\}$$

fermions

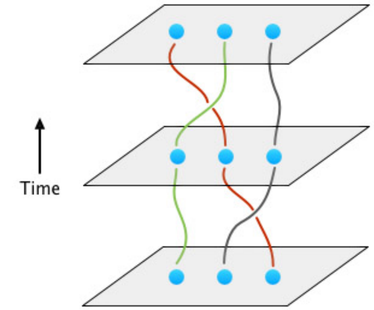
$$\Psi^{\dagger}(x) = U_C \Psi(x)$$

$$\gamma^{\dagger}_{\varepsilon_n, j} = \gamma_{-\varepsilon_n, j}$$

Majorana bound states

$$\gamma^{\dagger}_{0,j} = \gamma_{0,j}$$

anyons



Chamon, Jackiw, Nishida, Pi & Santos PRB 2010

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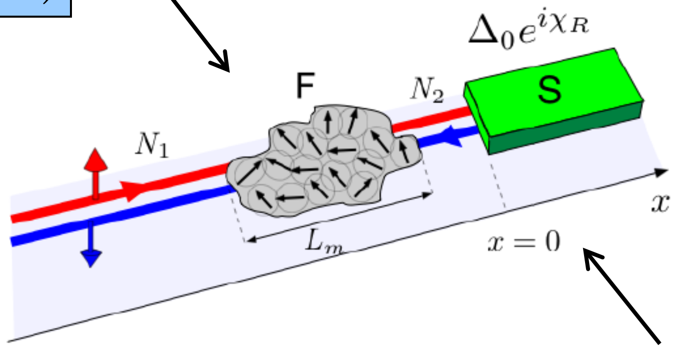
S-matrix construction

$$\begin{pmatrix} b_{e,1} \\ b_{e,2} \\ b_{h,1} \\ b_{h,2} \end{pmatrix} = \begin{pmatrix} S_0^e(\varepsilon) & 0 \\ 0 & S_0^h(\varepsilon) \end{pmatrix} \begin{pmatrix} a_{e,1} \\ a_{e,2} \\ a_{h,1} \\ a_{h,2} \end{pmatrix}$$



perfect AR

$$\begin{pmatrix} a_{e,2} \\ a_{h,2} \end{pmatrix} = \exp\left(-i \arccos\left(\frac{\varepsilon}{\Delta_0}\right)\right) \begin{pmatrix} 0 & e^{i\chi_R} \\ e^{-i\chi_R} & 0 \end{pmatrix} \begin{pmatrix} b_{e,2} \\ b_{h,2} \end{pmatrix}$$

Crépin, BT & Dolcini PRB 2014

S-matrix of FM domain

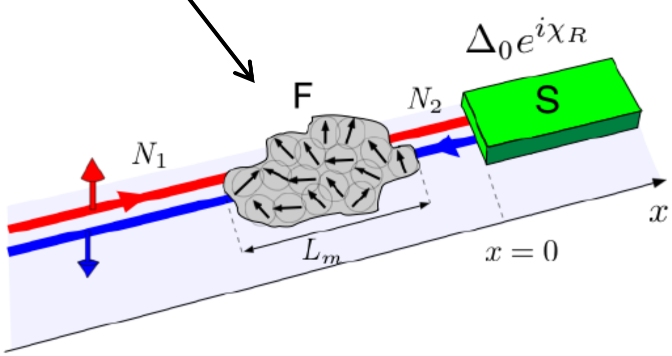
$\Gamma_m(\varepsilon) \sim k_F L_m$

$\Phi_m(\varepsilon) \sim k_F x_0$

$$S_0^e(\varepsilon) = e^{i\Gamma_m(\varepsilon)} \begin{pmatrix} -ie^{i\Phi_m(\varepsilon)}\sqrt{1-T_\varepsilon} & e^{i\chi_m(\varepsilon)}\sqrt{T_\varepsilon} \\ e^{-i\chi_m(\varepsilon)}\sqrt{T_\varepsilon} & -ie^{-i\Phi_m(\varepsilon)}\sqrt{1-T_\varepsilon} \end{pmatrix}$$

$\chi_m(\varepsilon) \sim m_z L_m$

all symmetries broken



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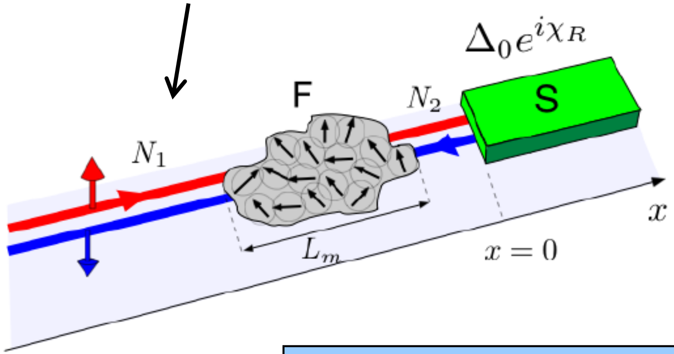
Andreev reflection

$$\begin{pmatrix} b_{e,1} \\ b_{h,1} \end{pmatrix} = \begin{pmatrix} r_{ee} & r_{eh} \\ r_{he} & r_{hh} \end{pmatrix} \begin{pmatrix} a_{e,1} \\ a_{h,1} \end{pmatrix}$$

$$R_A = |r_{eh}|^2 = |r_{he}|^2$$



$$\Phi_m^A(\varepsilon) = \frac{1}{2}\left(\Phi_m(\varepsilon) - \Phi_m(-\varepsilon)\right)$$

$$R_A = \frac{T_\varepsilon T_{-\varepsilon}}{\left(1 - \sqrt{R_\varepsilon R_{-\varepsilon}}\right)^2 + 4\cos^2\left[\arccos\left(\frac{\varepsilon}{\Delta_0}\right) + \Phi_m^A(\varepsilon)\right]\sqrt{R_\varepsilon R_{-\varepsilon}}}$$

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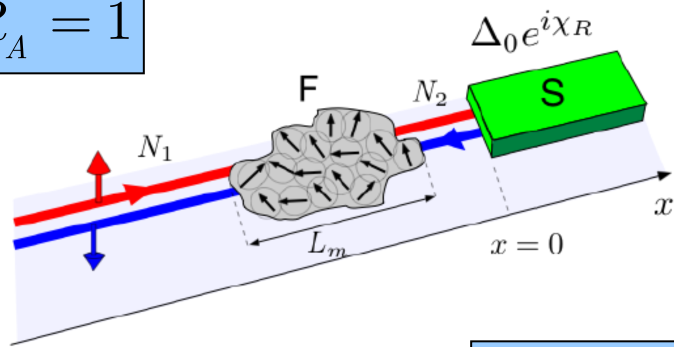
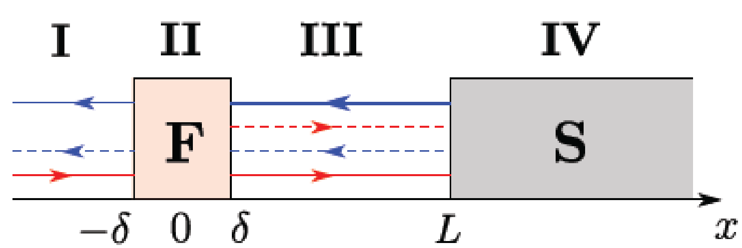
Resonance condition

$$R_A = 1$$



$$\left(\sqrt{R_\varepsilon}-\sqrt{R_{-\varepsilon}}\right)^2+4\cos^2\left[\arccos\left(\frac{\varepsilon}{\Delta_0}\right)+\Phi_m^A(\varepsilon)\right]\sqrt{R_\varepsilon R_{-\varepsilon}}=0$$

Fabry-Perot resonator
for electrons/holes



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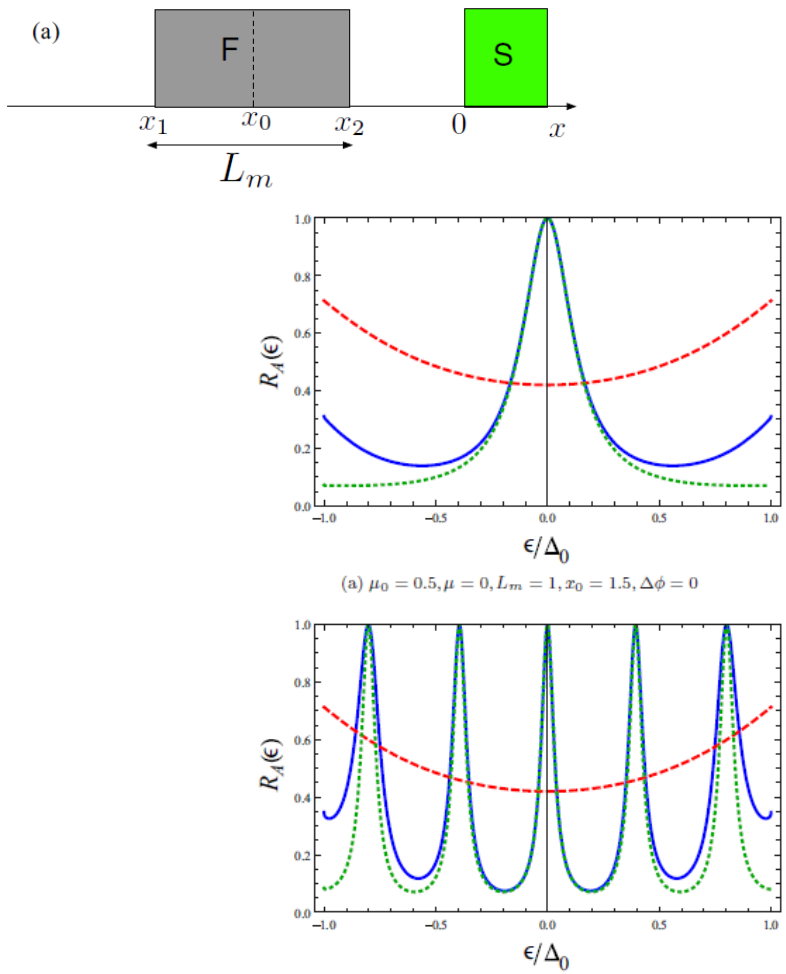
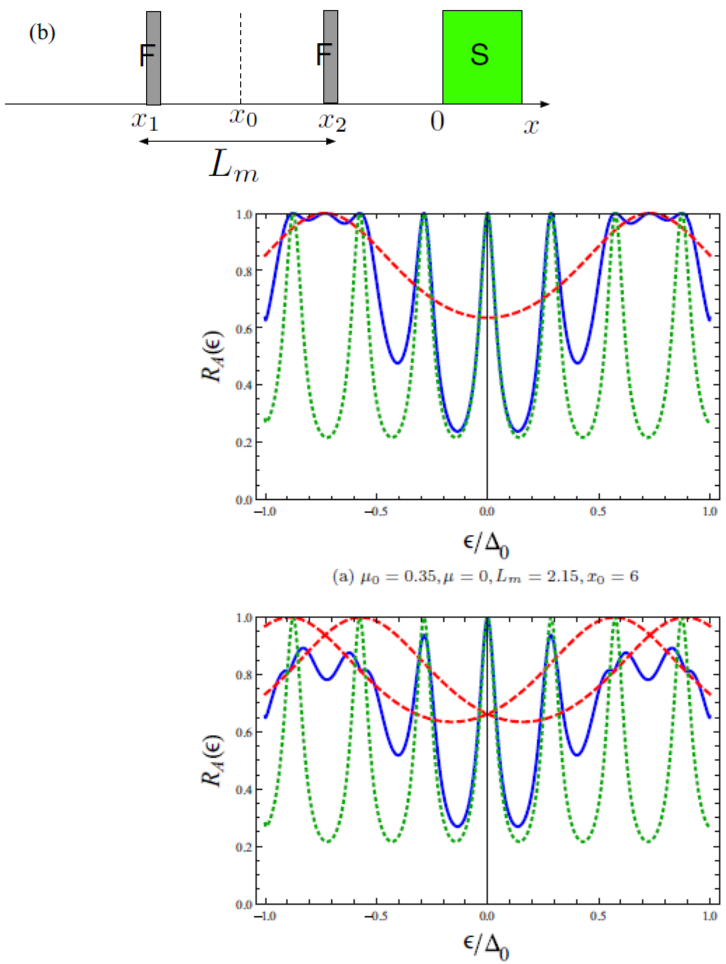
FM domain vs. double barrier



(a) $\mu_0 = 0.5, \mu = 0, L_m = 1, x_0 = 1.5, \Delta\phi = 0$

(b) $\mu_0 = 0.5, \mu = 0, L_m = 1, x_0 = 4.5, \Delta\phi = 0$



(a) $\mu_0 = 0.35, \mu = 0, L_m = 2.15, x_0 = 6$

(b) $\mu_0 = 0.35, \mu = 0.16, L_m = 2.15, x_0 = 6$

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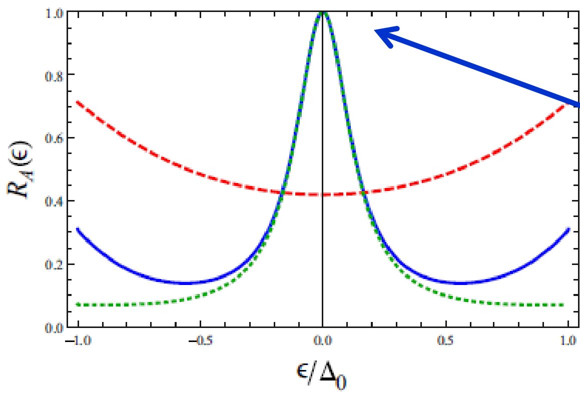
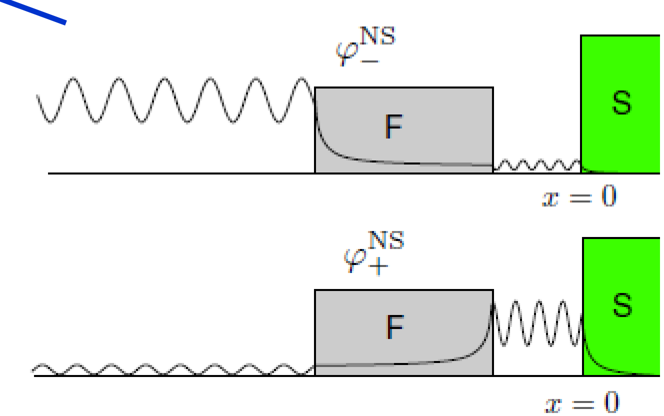
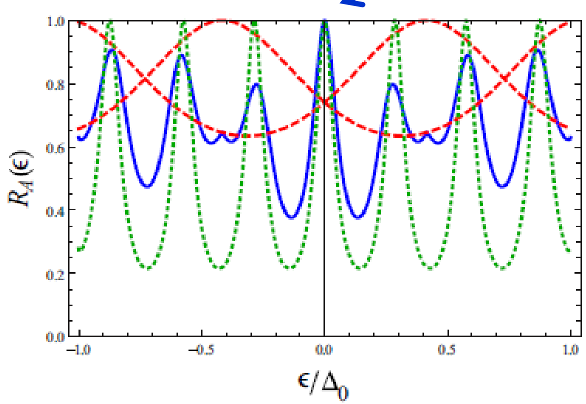
Robust MBS signature



(a) $\mu_0 = 0.5, \mu = 0, L_m = 1, x_0 = 1.5, \Delta\phi = 0$

$$C\varphi_{\pm}^{NS} = \varphi_{\pm}^{NS}$$





(c) $\mu_0 = 0.35, \mu = 0.3, L_m = 2.15, x_0 = 6$

Majorana bound states $\varepsilon = 0$

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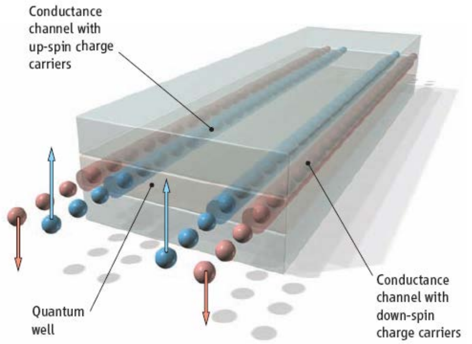
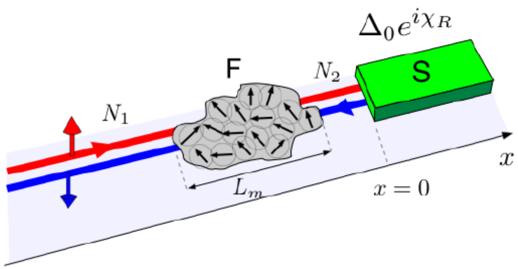
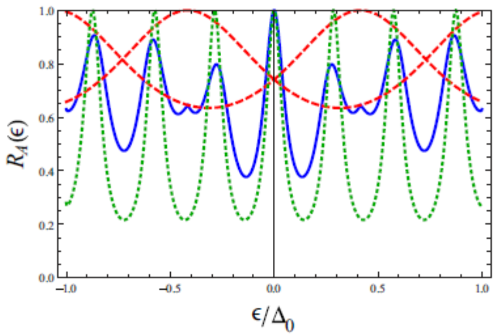
Summary Lecture 1





$$H_{BdG} = \begin{pmatrix} H^{e}_{0+FM} & \Delta(x)\sigma_0 \\ \Delta^{*}(x)\sigma_0 & H^{h}_{0+FM} \end{pmatrix}$$



(c) $\mu_0 = 0.35, \mu = 0.3, L_m = 2.15, x_0 = 6$