Josephson junction between topological and conventional superconductors

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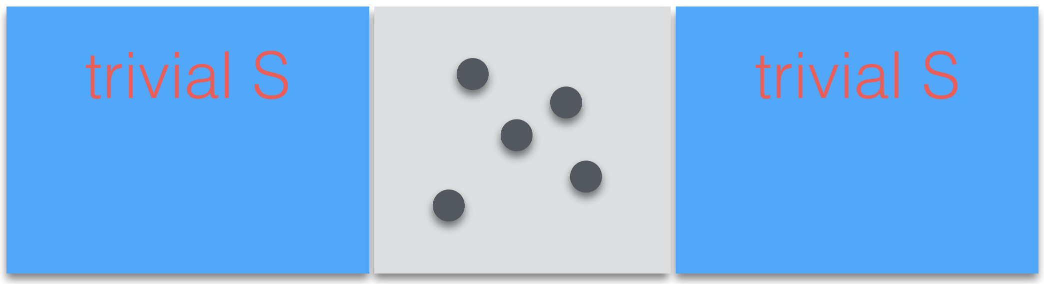
Motivation

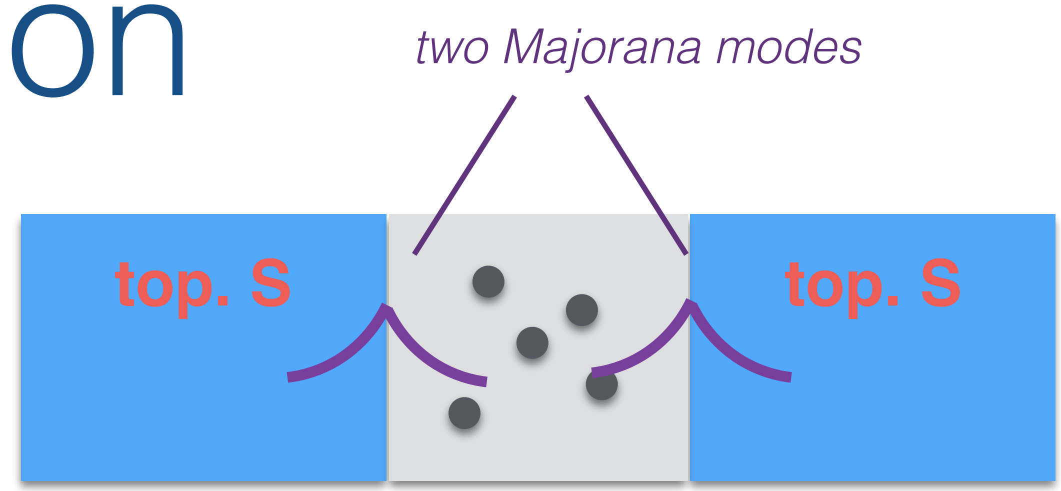


studied for decades; for short junction

$$E_i = \Delta\sqrt{1 - T_i \sin^2\frac{\varphi}{2}} \qquad (1)$$

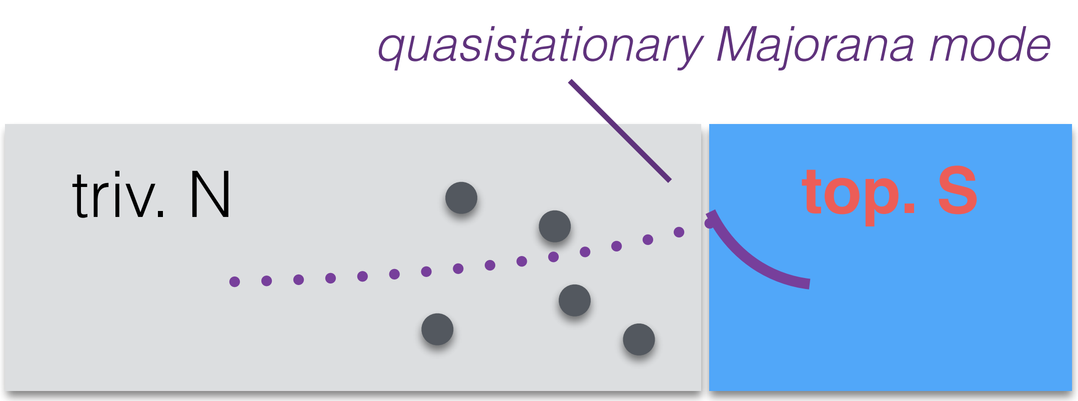
with transmission eigenvalues T_i

Beenakker, 1991



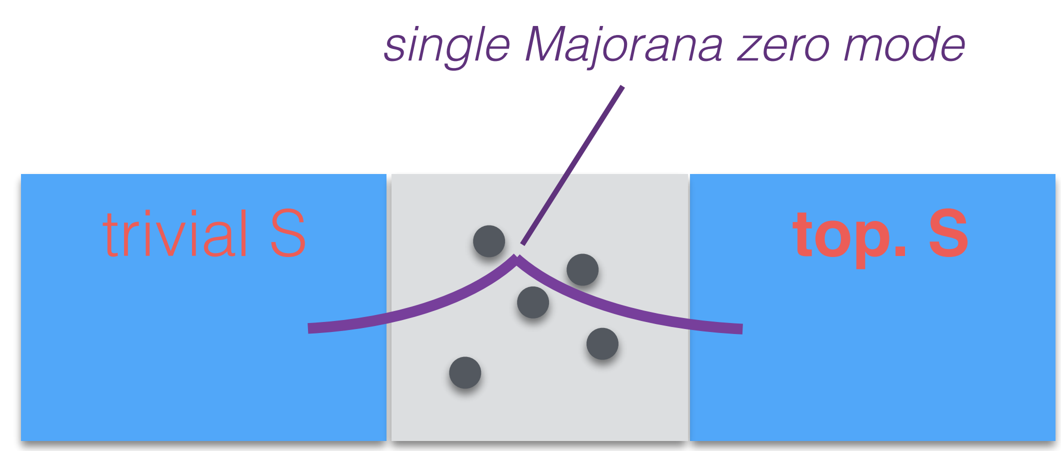
coupled Majorana pair; parity switch; fractional Josephson effect

Kwon, Sengupta, Yakovenko 2003
Fu, Kane, 2009
…



resonant Andreev reflection, zero-bias conductance peak

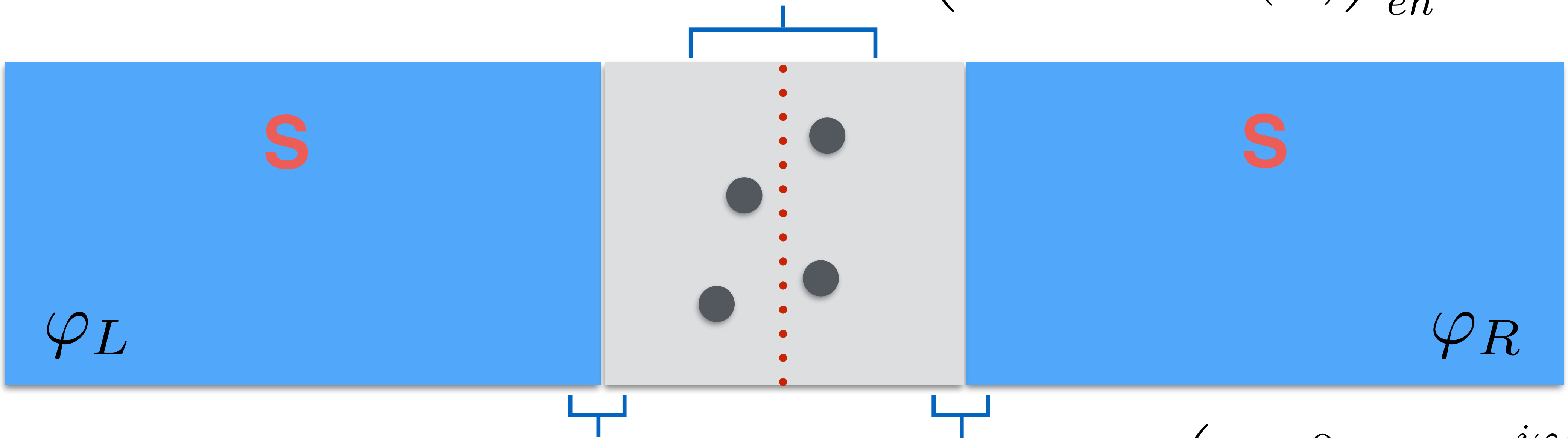
Law, Lee, Ng, 2010
Beenakker *et al*, 2011
PI, Ostrovsky, Feigel'man, 2012
…



Protected zero mode, does not carry supercurrent.

broken time-reversal symmetry in normal state, formula (1) inapplicable.

normal-state scattering matrix: $S_N = \begin{pmatrix} S(E) & 0 \\ 0 & -S^*(E) \end{pmatrix}_{eh}$



S_{AL}

$$S_{AR} = -i \begin{pmatrix} 0 & e^{i\varphi_R - \theta_R} \\ e^{-i\varphi_R - \theta_R} & 0 \end{pmatrix}_{eh}$$

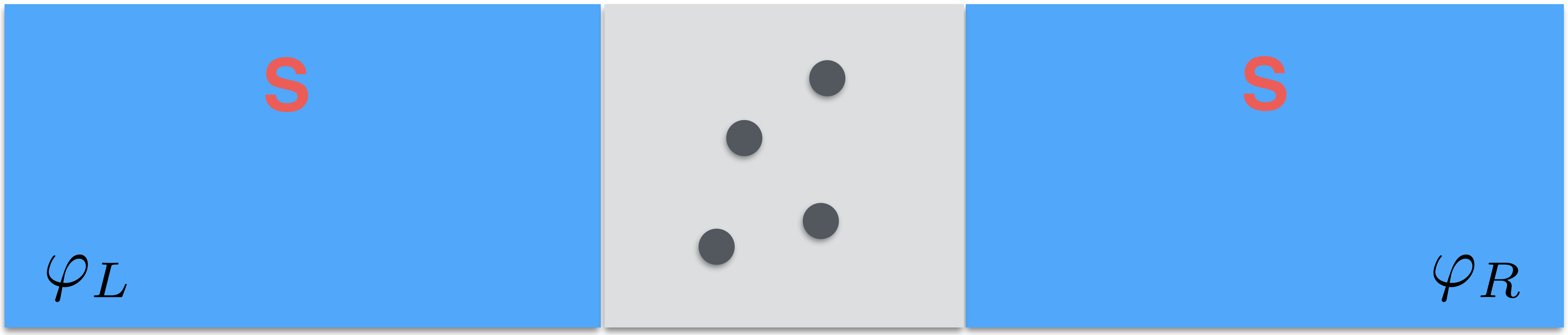
Andreev reflection on NS boundary,

$$\tanh \theta_R = \frac{\omega}{\Delta_R}$$

$$I \equiv \frac{2e}{h}\frac{\partial F}{\partial \varphi} = -\frac{e}{h}\frac{\partial}{\partial \varphi}\int \frac{i\omega d\omega dx}{2\pi}\mathrm{Tr}[G_\omega(x,x)]$$

Green function can be expressed through scattering matrices

$$I = -\frac{e}{h}\frac{\partial}{\partial \varphi}\int \frac{d\omega}{2\pi}\ln\det(1 - S_A S_N)$$



$$I = -\frac{e}{h}\frac{\partial}{\partial\varphi}\int\frac{d\omega}{2\pi}\ln\det(1 - S_A S_N)$$

eliminate Nambu space

$$\hat{\theta} = \begin{pmatrix}\theta_L & 0\\ 0 & \theta_R\end{pmatrix}_{LR}$$

$$\hat{\varphi} = \begin{pmatrix}\varphi_L & 0\\ 0 & \varphi_R\end{pmatrix}_{LR}$$

$$\det\left(1 - e^{-\hat{\theta}-i\hat{\varphi}}S_E e^{-\hat{\theta}+i\hat{\varphi}}S^*_{-E}\right)$$

At zero energy

$$\sim \det(S^T e^{-i\hat{\varphi}} - e^{-i\hat{\varphi}}S)$$

equals zero for odd-sized S

in a time-reversal invariant junction

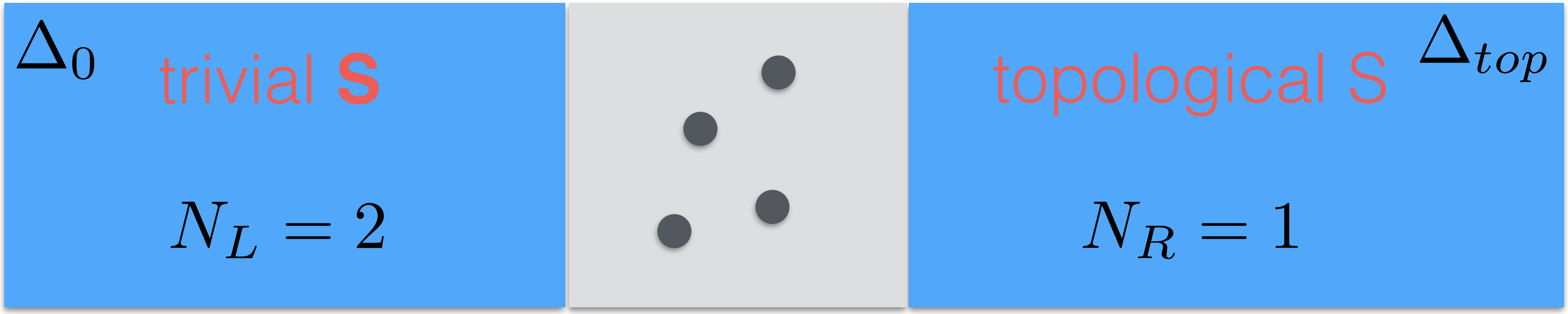
$$S = -S^T$$

which is incompatible with odd size

protected Majorana zero mode appears if and only if $N_L - N_R$ is odd

class D, $\mathcal{T}^2 = -1$

Minimal topological junction



normal scattering matrix is 3x3, determinant can be calculated

$$\det = T\cosh\theta_R\cosh\theta_L + T_A\cos(\varphi - \chi) + \frac{\Delta_0}{\Delta_{top}}\left[(2-T)\sinh^2\theta_L + \rho\right]$$

conductance: $$T = \mathrm{tr}(t_{RL}t^\dagger_{RL})$$

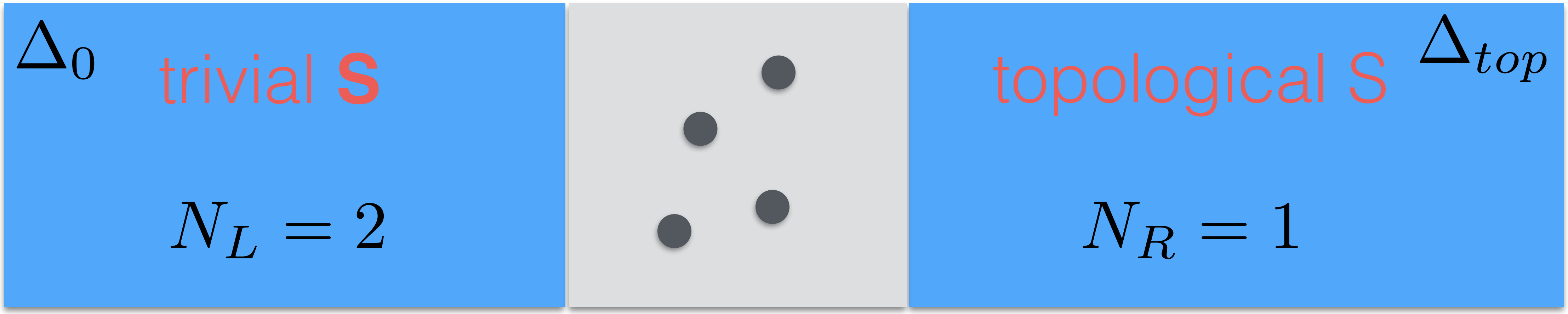
three further invariants: $$T_A e^{i\chi} = -\mathrm{tr}(t_{RL}t^*_{LR})$$

$$\rho = \mathrm{tr}\left[r_L\frac{(r^\dagger_L - r^*_L)}{2}\right]$$

together the 4 scalars fully define S up to basis rotation $S \to V^T S V$

in time-reversal invariant systems $S = -S^T$ and T alone defines transport

Minimal topological junction

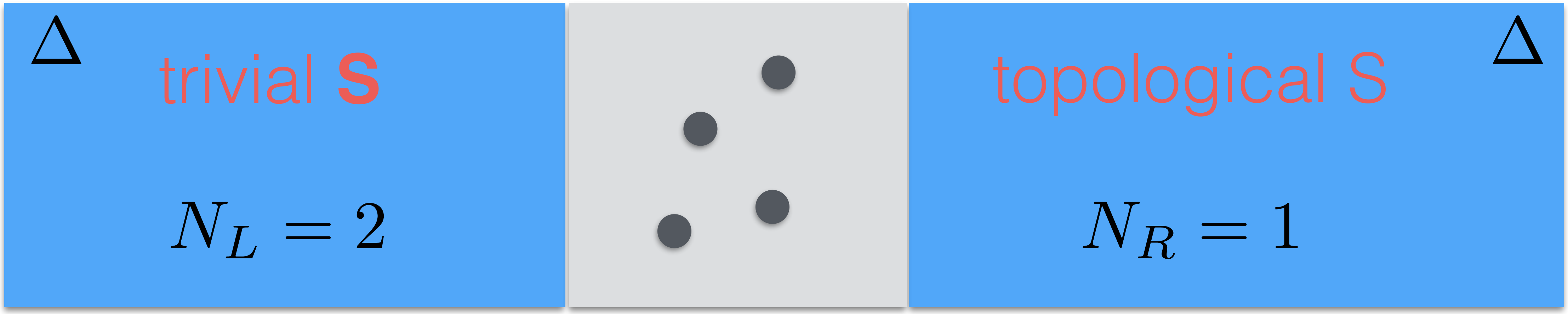


$\varphi = \varphi_R - \varphi_L - \chi$

$$I = \frac{e\Delta_0\Delta_{top}}{h} T_A \sin\varphi \int_{-\infty}^{\infty} \frac{d\omega/2\pi}{(2-T)\omega^2 + \Delta_0^2\rho + T\sqrt{(\omega^2+\Delta_0^2)(\omega^2+\Delta_{top}^2)} + T_A\Delta_0\Delta_{top}\cos\varphi}$$

asymmetry with respect to gaps!

Equal gaps



$\varphi = \varphi_R - \varphi_L - \chi$

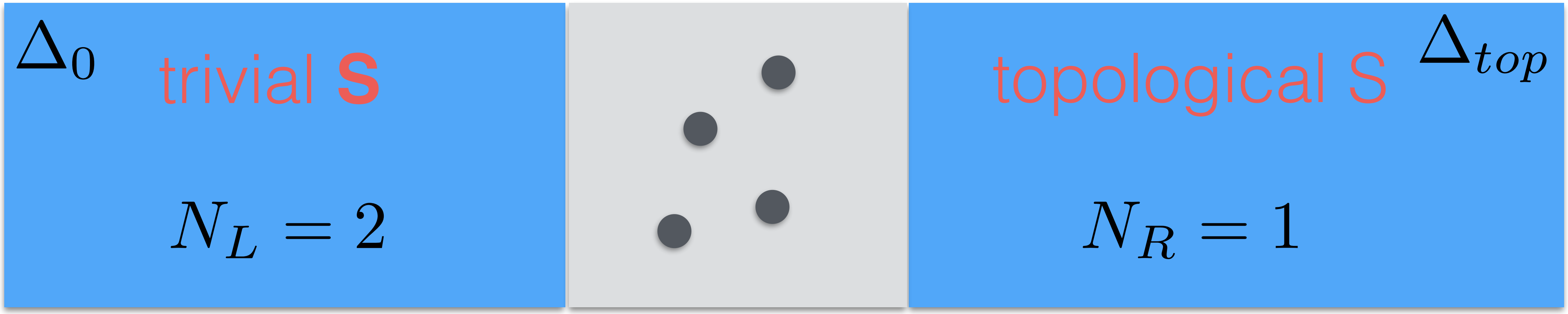
Current is carried by Andreev bound state

$$I = -\frac{e}{h} \tanh\left(\frac{E_0}{2T}\right) \frac{\partial E_0}{\partial \varphi}$$

$$E_0 = \Delta \sqrt{\frac{\rho + T + T_A}{2} - T_A \sin^2 \frac{\varphi}{2}}$$

Somewhat similar to time-reversal invariant junction, where $E = \Delta \sqrt{1 - T \sin^2 \frac{\varphi}{2}}$

Tunneling junction



$\varphi = \varphi_R - \varphi_L - \chi$

$$I = \frac{e\Delta_0\Delta_{top}}{h} T_A \sin\varphi \int_{-\infty}^{\infty} \frac{d\omega/2\pi}{(2-T)\omega^2 + \Delta_0^2\rho + T\sqrt{(\omega^2+\Delta_0^2)(\omega^2+\Delta_{top}^2)} + T_A\Delta_0\Delta_{top}\cos\varphi}$$

in the tunneling limit $T \to 0$

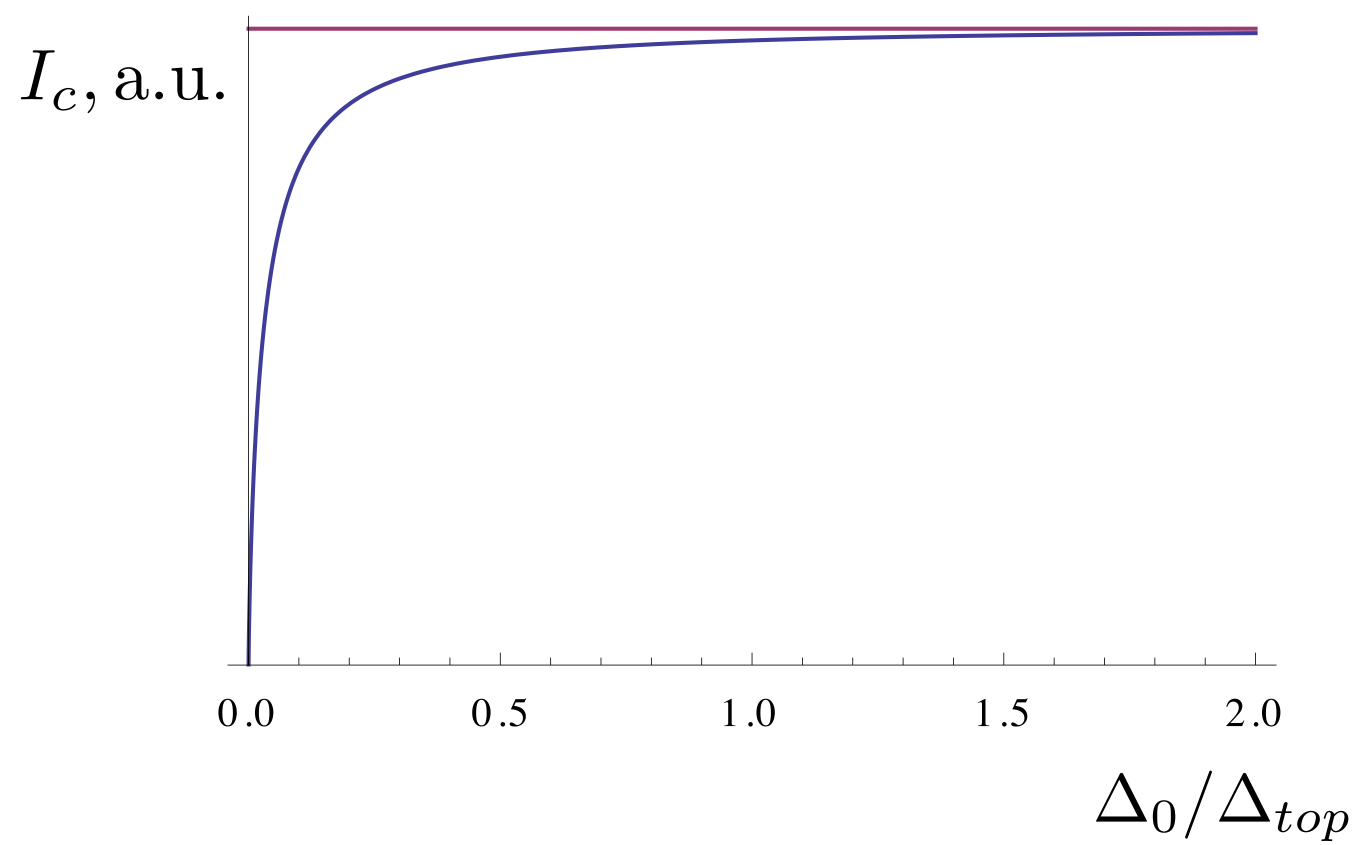
$$I = \frac{e\Delta_{top}}{h\sqrt{2\rho}} T_A \sin\varphi$$

does not depend on Δ_0
in wide parameter range

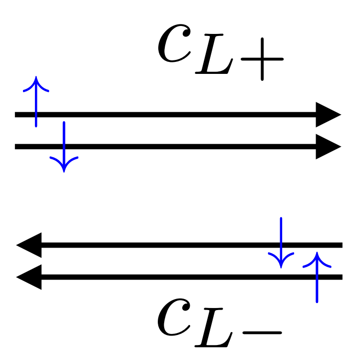
at very small $\Delta_0 \lesssim T\Delta_{top}/\rho$

$$I_c \sim \Delta_0 \ln\frac{\Delta_{top}}{\Delta_0}$$

tunneling limit:



Trivial spin-degenerate wire

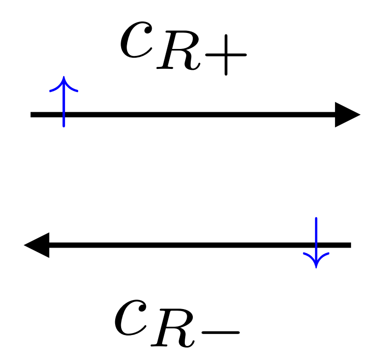


$$H_L = \frac{p^2}{2m} - E_f$$

S_e

Nanowire with SOI, Zeeman field

Lutchyn *et al* 2010
Oreg *et al* 2010



$$H_R = \frac{p^2}{2m} + \sigma_z u p + B\sigma_x - E_f$$

$$H_{eff} = -\sigma_z u p$$

One spin-deg. channel N_L=2

Fermi level inside the Zeeman gap, so that N_R=1

$$S_e = \begin{pmatrix} \frac{4i\gamma^2}{(1+\gamma^2)^2} & \frac{1-\gamma^2}{1+\gamma^2} & \frac{2i\gamma(1-\gamma^2)}{(1+\gamma^2)^2} \\ \frac{-1+\gamma^2}{1+\gamma^2} & 0 & \frac{2\gamma}{1+\gamma^2} \\ \frac{2i\gamma(1-\gamma^2)}{(1+\gamma^2)^2} & -\frac{2\gamma}{1+\gamma^2} & +\frac{i(1-\gamma^2)^2}{(1+\gamma^2)^2} \end{pmatrix}$$

depends on one parameter $\gamma = \sqrt{v_f/u}$

$\gamma \ll 1$ is the tunnelling limit,

$\gamma = 1$ is a transparent contact.

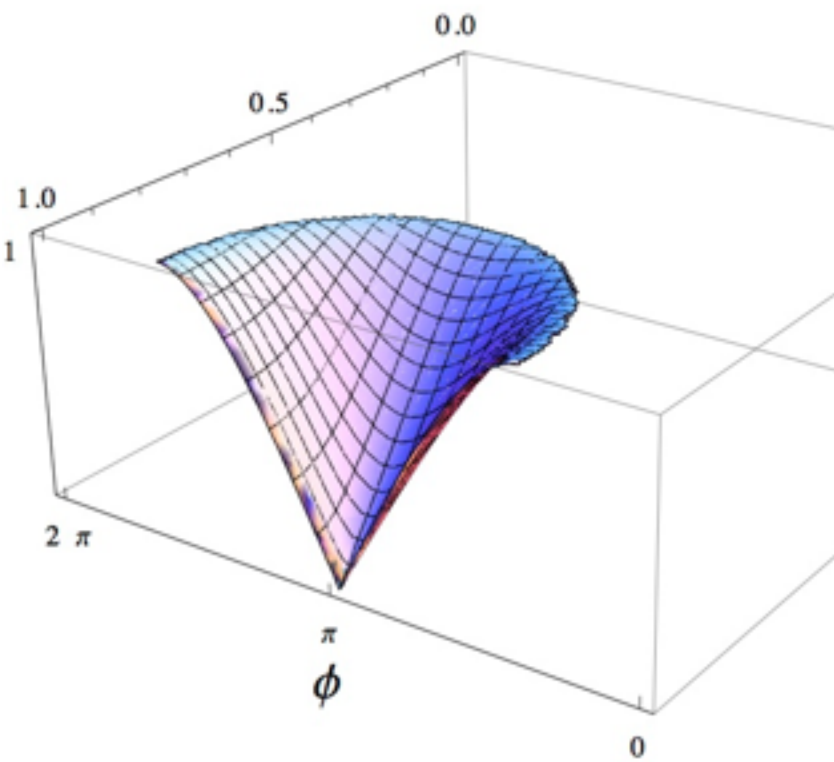
Conductance (transmission eigenvalue):

$$T = 8\gamma^2 \frac{(1+\gamma^4)}{(1+\gamma^2)^4}$$

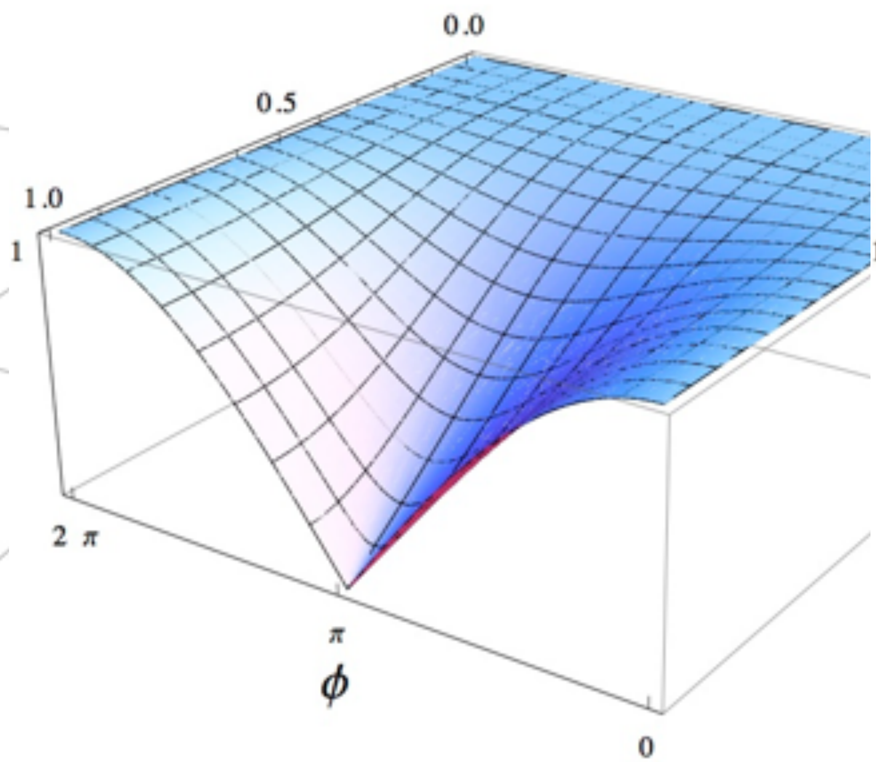
$$T \simeq 8\gamma^2$$
$$T_A \simeq 16\gamma^4$$

in the tunnelling limit.

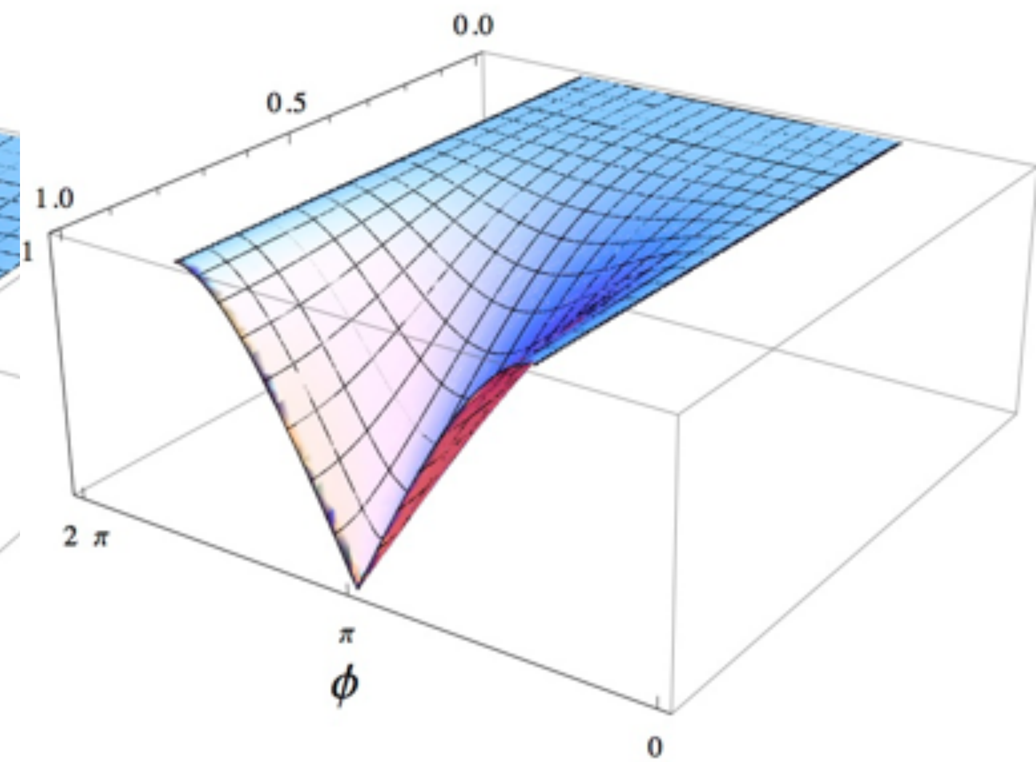
Bound state at different gaps



$$\Delta_0 = 3\Delta_{\text{top}}$$



$$\Delta_0 = \Delta_{\text{top}}$$



$$3\Delta_0 = \Delta_{\text{top}}$$

bound state exists if

$$\cos\varphi < \frac{\Delta_{\text{top}}}{\Delta_0} - \frac{\Delta_0^2 - \Delta_{\text{top}}^2}{8\Delta_0\Delta_{\text{top}}}\left(\gamma^2 - \gamma^{-2}\right)^2$$

bound state exists if

$$\cos\varphi < \frac{\Delta_0}{\Delta_{\text{top}}}$$

In the tunneling limit we find that

$$I_c = \frac{e\Delta_{top}}{2h} T_A = \frac{e\Delta_{top}}{h} \frac{T^2}{16}$$

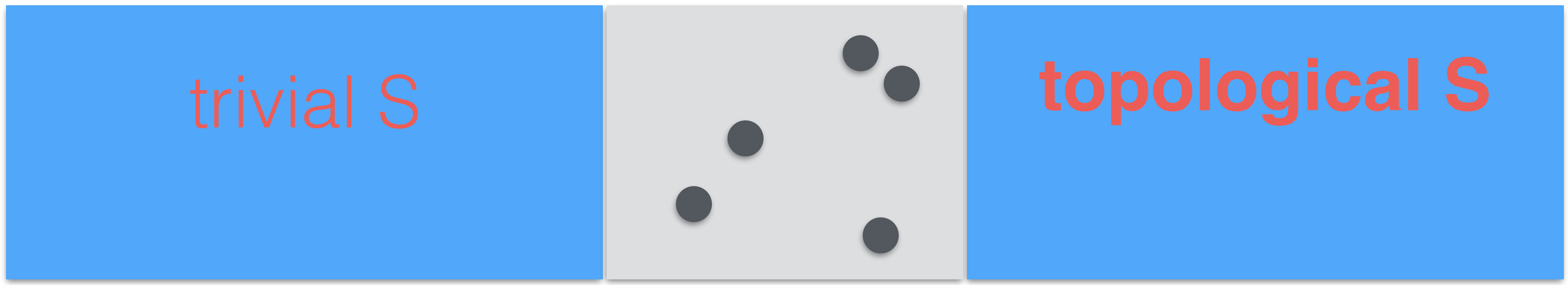
so that

$$eI_cR_N = \Delta_{top} \frac{h}{16e^2 R_N}$$

This is in strong contrast with the Ambegaokar-Baratoff relation:

1. The supercurrent is strongly suppressed and scales as conductance squared.
2. It depends on the gap in the topological superconductor, but not on the gap in the trivial superconductor.

Summary



Topological Josephson junction necessarily breaks time-reversal symmetry, leading to unconventional Josephson current behavior:

1. Asymmetric dependence on the two gaps.
2. Current can be strongly suppressed in the tunneling limit.
3. There may be no sub-gap levels apart from the Majorana zero mode.