Spin Qubits in Semiconducting Nanostructures

Daniel Loss
Department of Physics
University of Basel

$$: \text{Swiss NSF, Nano Basel, Quantum ETH/Basel, EU}$$
I. & II: Quantum dots, spin qubits, quantum gates, decoherence, hole spins in Si/Ge nanowires, scalable systems, surface code, long-distance coupler...

III: Topological quantum computing in nanowires with Majorana fermions, parafermions,..., hybrid spin-Majorana qubits;

Prospects for Spin-Based Quantum Computing in Quantum Dots

Quantum Memories at Finite Temperature
Brown, Loss, Pachos, Self, and Wootton, Rev. Mod. Phys. 88, 045005 (2016)
Front-Runners for Quantum Computers

- spin qubits in semiconductors  ‘small & fast’
- superconducting devices
- trapped ions

\[ \text{more advanced but not so} \quad \text{‘small & fast’} \]

- topological quantum computing?  ‘exotic’
  ‘semi-superconductor hybrids’
  Majorana
  Para- or
  Fibonacci
  fermions?
Front-Runners for Quantum Computers

- spin qubits in semiconductors ‘small & fast’
- semiconducting nanostructures
- topological quantum computing? ‘semi-superconductor hybrids’
- ‘exotic’ Majorana Para- or Fibonacci fermions?
A bit of the action

In the race to build a quantum computer, companies are pursuimg many types of quantum bits, or qubits, each with its own strengths and weaknesses.

### Superconducting loops
A resistance-free current oscillates back and forth around a circuit loop. An injected microwave signal excites the current into superposition states.

<table>
<thead>
<tr>
<th>Longevity (seconds)</th>
<th>0.00005</th>
<th>30-60 s</th>
<th>N/A</th>
</tr>
</thead>
</table>

### Trapped ions
Electrically charged atoms, or ions, have quantum energies that depend on the location of electrons. Tuned lasers cool and trap the ions, and put them in superposition states.

<table>
<thead>
<tr>
<th>Logic success rate</th>
<th>99.9%</th>
<th>N/A</th>
<th>99.2%</th>
</tr>
</thead>
</table>

### Silicon quantum dots
These “artificial atoms” are made by adding an electron to a small piece of pure silicon. Microwaves control the electron’s quantum state.

### Topological qubits
Quasiparticles can be seen in the behavior of electrons channeled through semiconductor structures. Their braided paths can encode quantum information.

### Diamond vacancies
A nitrogen atom and a vacancy add an electron to a diamond lattice. Its quantum spin state, along with those of nearby carbon nuclei, can be controlled with light.

<table>
<thead>
<tr>
<th>Number entangled</th>
<th>9</th>
<th>14</th>
<th>2</th>
<th>N/A</th>
<th>6</th>
</tr>
</thead>
</table>

### Company support

| Google, IBM, Quantum Circuits | ionQ | Intel | Microsoft, Bell Labs | Quantum Diamond Technologies |

### Pros
- Fast working. Build on existing semiconductor industry.
- Very stable. Highest achieved gate fidelities.
- Stable. Build on existing semiconductor industry.
- Greatly reduce errors.
- Can operate at room temperature.

### Cons
- Collapse easily and must be kept cold.
- Slow operation. Many lasers are needed.
- Only a few entangled.
- Must be kept cold.
- Existence not yet confirmed.
- Difficult to entangle.

**Note:** Longevity is the record coherence time for a single qubit superposition state, logic success rate is the highest reported gate fidelity for logic operations on two qubits, and number entangled is the maximum number of qubits entangled and capable of performing two-qubit operations.
Quantum Information

Classical digital computer
network of ‘Boolean logic gates‘, e.g. XOR (CNOT)

- **bits**: \( a, b = 0,1 \)
- **physical implementation**: e.g. 2 voltage levels
- ‘gate‘: electronic circuit

![Diagrams of classical logic gates](image)

Quantum computer

- **qubits** \( |a\rangle, |b\rangle \), \( |a\rangle = \alpha |0\rangle + \beta |1\rangle \), \( |\alpha|^2 + |\beta|^2 = 1 \)
- **physical implementation**: quantum 2-level-system: \( |\uparrow\rangle \equiv |0\rangle, \ |\downarrow\rangle \equiv |1\rangle \)
- ‘quantum gate‘: unitary transformation (is reversible!)
Quantum Computing (basics)

• basic unit: **qubit** → any state of a quantum two-level system
  \[ |\Psi\rangle = a|1\rangle + b|0\rangle \]

"natural" candidate: **electron spin**

• quantum computation:
  1) prepare N qubits (input)
  2) apply unitary transformation in \(2^N\)-dim. Hilbert space → computation
  3) measure result (output)

• quantum computation faster than classical:
  - factoring algorithm (**Shor 1994**): \(\exp N \rightarrow N^2\)
  - database search (**Grover 1996**): \(N \rightarrow N^{1/2}\)
  - quantum simulations
    ...

What a quantum computer could do (faster):

...search large database (→ biology, climate, physics...)
...break `RSA-Encryption’ (banking, industry, military,...)
...simulate physical und chemical processes (or models*)
  (→ energy, catalysts, C-capture, material science, drug design,...)
...machine learning & cloud computing
...play quantum games
...and many unforseen applications (hopefully)

Intense search for new quantum algorithms !

  ‘Solving the 2D Hubbard model on a quantum computer’
Quantum Computing with Quantum Gates

Barenco et al., PRA 52, 3457 (1995)

Single-qubit operations and a two-qubit gate that generates entanglement are sufficient for universal quantum computation:

**Single-qubit gates**

- **Not-gate**
  \[ \alpha |0\rangle + \beta |1\rangle \rightarrow \beta |0\rangle + \alpha |1\rangle \]

- **Z-gate**
  \[ \alpha |0\rangle + \beta |1\rangle \rightarrow \alpha |0\rangle - \beta |1\rangle \]

- **Hadamard-gate**
  \[ \alpha |0\rangle + \beta |1\rangle \rightarrow \frac{\alpha |0\rangle + |1\rangle}{\sqrt{2}} + \frac{\beta |0\rangle - |1\rangle}{\sqrt{2}} \]

**2-qubit gate:**

CNOT (XOR) gate

\[ |A\rangle \rightarrow |A\rangle \]

\[ |B\rangle \rightarrow |B \oplus A\rangle \]

\[ \Leftrightarrow \text{entanglement} \]
Quantum Gates

- quantum gate: unitary transformation acting on a few qubits at a time (universal set of quantum gates: all unitary operations on n qubits [U(2^n)] can be expressed as a composition of these gates)

- XOR together with one-qubit gates is a universal set for quantum computation (Barenco et al. 1995)

- action of the quantum XOR gate:
  two-particle state $|\psi\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$

\[
U_{XOR} = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0
\end{pmatrix} = \exp \left(-\frac{i}{\hbar} \int dt H(t) \right)
Spin Qubits under Study (Among Others)

Quantum dots (spin and charge states)

Molecular magnets

Donor atoms

Defects in diamond

Optically trapped atoms

ion traps

...
Spin Qubits under Study (Among Others)

Quantum dots (spin and charge states)

- Molecular magnets
- Donor atoms
- Defects in diamond
- Optically trapped atoms
- Ion traps
...

In this lecture, we will mostly discuss spin qubits in quantum dots.
Historical remarks:
Electron qubit: `spin better than charge`
due to longer relaxation/decoherence* times

\[ \tau_{spin}^{\phi} \gg \tau_{charge}^{\phi} \]

10ns - 1min

1-10ns

GaAs

Awschalom et al., '97
Tarucha et al., '02
Kouwenhoven/Vandersypen et al., '03-'09
Abstreiter et al., '04/08; Warburton et al., '09
Zumbuhl et al., '08-'17
Marcus, Yacoby et al.,'05- '15 (T_2 \sim 270...850 \mu s)

\[ \rightarrow \text{natural choice for qubit: spin } \frac{1}{2} \text{ of electron} \]

*) theory: T_2 \sim T_1 for single spin in GaAs dot (‘everything optimized’

‘mesoscopics’
Fujisawa et al. ‘03
Marcus et al. ‘01
Magnetic moment of single spin (Bohr magneton) is very weak:

**Advantage:** spin couples weakly to environment

- spin has long decoherence time (0.001-1000 μs)

**Disadvantage:** spin couples weakly to “observer”

- spin is difficult to control

Instead: control spin via charge, made possible by Pauli exclusion principle which “locks spin to charge”

- manipulation & detection of spin-dynamics via charge (orbital) degrees of freedom of electron

D. Loss & D. DiVincenzo, PRA 57 (1998) 120
Quantum computation with quantum dots

Daniel Loss and David P. DiVincenzo

Institute for Theoretical Physics, University of California, Santa Barbara, Santa Barbara, California 93106-4030
Department of Physics and Astronomy, University of Basel, Klingelbergstrasse 82, 4056 Basel, Switzerland
IBM Research Division, T.J. Watson Research Center, P.O. Box 218, Yorktown Heights, New York 10598

(Received 9 January 1997; revised manuscript received 22 July 1997)

\[ U_{XOR} = e^{i \frac{\pi}{2} S_1^z} e^{-i \frac{\pi}{2} S_2^z} U_{SW}^{1/2} e^{i \pi S_1^z} U_{SW}^{1/2} \]

'Spintronics' = spin-electronics = all-electrical spin control

Spin rotation by exchange
Spin-charge conversion for read-out
Spintronics scheme
Exchange coupling
Quantum computation with quantum dots

Daniel Loss and David P. DiVincenzo

Physics, University of California, Santa Barbara, Santa Barbara, California 93106-4030
Department of Physics and Astronomy, University of Basel, Klingelbergstrasse 82, 4056 Basel, Switzerland
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'Spintronics' = spin-electronics = all-electrical spin control
Electric fields vs. Magnetic fields

- Strong electric fields easy to produce (gates, STM-tips, etc)
- Fast switching of electric fields (picoseconds)
- Easy to apply electric fields locally and on nanoscale

- Strong magnetic (ac) fields hard to produce
- Slow switching of magnetic fields (nanoseconds)
- Hard to apply magnetic fields locally and on nanoscale

spinstronics
Quantum Processor for Spin-Qubits

DL & DiVincenzo, PRA 57 (1998)

Key idea: 
all-electrical control of spins ➔ scalable nanotechnology

Simple effective Hamiltonian:

\[ H(t) = J(t)S_1 \cdot S_2 + b_1(t) \cdot S_1 + b_2(t) \cdot S_2 \]

Exchange coupling ➔ Zeeman couplings

2 quantum dots, each with
1 electron-spin (= qubit)
Quantum Processor for Spin-Qubits

DL & DiVincenzo, PRA 57 (1998)

Key idea: all-electrical control of spins ➔ scalable nanotechnology

2 quantum dots, each with 1 electron-spin (= qubit)

artificial hydrogen molecule ➔ exchange splitting $J \sim t^2/U$

➔ ‘CNOT quantum gate’
Quantum Dot Molecular Physics

- two coupled dots = artificial ‘‘H₂ – molecule’’

- use approximative methods from molecular physics:
  → Hund-Mullikan (molecular orbits) Burkard ea, PRB 59, ’99
  → large scale numerics: Das Sarma & Hu ’01, Leburton ea ’01, Landman ea ’01

<table>
<thead>
<tr>
<th></th>
<th>artificial atom</th>
<th>real atom</th>
</tr>
</thead>
<tbody>
<tr>
<td>energy</td>
<td>≈ 1 meV</td>
<td>1 Ry = 13.6 eV</td>
</tr>
<tr>
<td>length a_B</td>
<td>≈ 100 – 500 Å</td>
<td>0.5 Å</td>
</tr>
</tbody>
</table>

- scale:

- magnetic length l_B ≈ 100 Å at B ≈ 1 T → molecular properties of quantum dots are very sensitive to magnetic fields B

- time dependent B field: → J(t) = J(B(t))
  electrical gate V(t): → J(t) = J(V(t))
quantum gate = two coupled dots

• idea: Hubbard physics: \( J(t) \approx 4 \frac{t_0(t)^2}{U} \) exchange

  \( t_0 = t_0(t) \): tunable tunneling barrier

• e.g. swap and square-root-of-swap \( U_{SW}^{1/2} \):

  \[
  \int_0^t J(t') dt' / \hbar \approx J_0 \tau_S / \hbar = \pi / 2 (\text{mod} \pi)
  \]

note: \( \tau_s = 50 \text{ ps} \ll \tau_2 = 1 \text{ ms} \) (GaAs)
Electronic Model

- how do we find the exchange coupling $J$?

\[ H_{el} \rightarrow \begin{array}{c} \overline{J} \\ \end{array} \rightarrow H_S = J(t) \, \vec{S}_1 \cdot \vec{S}_2 \]

- 2D potential for electrons:

- Hamiltonian for one electron per dot:

\[ H = \sum_{i=1,2} \left( \frac{\varepsilon^2}{\epsilon |\vec{r}_1 - \vec{r}_2|} \right) + \sum_{i=1,2} \left( \frac{1}{2m} \left( \vec{p}_i - \frac{e}{c} \vec{A}(\vec{r}_i) \right)^2 + e \varepsilon_i E + \frac{m \omega^2}{2} \left( \frac{1}{4a^2} (x_i^2 - a^2)^2 + y_i^2 \right) \right) \]

\[ \vec{A}(\vec{r}) = \frac{B}{2} (-y, x, 0); \quad \vec{r'} = (x, y, 0) \]
I. Heitler-London Method

- single-dot problem in a magnetic field has exact solution
  \((\text{Fock '28, Darwin '30}) \rightarrow \varphi(\vec{r})\)

  two-particle trial wavefunction (Heitler-London)

\[
\psi_{\pm} = N \left[ \varphi_{-a}(\vec{r}_1)\varphi_{+a}(\vec{r}_2) \pm \varphi_{-a}(\vec{r}_2)\varphi_{+a}(\vec{r}_1) \right]
\]

\[
J = \langle \psi_{-} | H_{el} | \psi_{-} \rangle - \langle \psi_{+} | H_{el} | \psi_{+} \rangle
\]

- results: \(d = a/a_B, \ b^2 = 1 + \omega_L^2/\omega_0^2, \ c \sim (e^2/\varepsilon a_B)/\hbar \omega_0, \ \omega_L = eB/2m\)
  (Burkard, Loss, DiVincenzo '99)

\[
J = \frac{\hbar \omega_0}{\sinh(2d^2(2b-1/b))} \left[ c \sqrt{b} \left( e^{-bd^2} I_0(bd^2) \right. \right.
\]

\[
- \left. e^{d^2(b-1/b)} I_0(d^2(b-1/b)) \right] + \frac{3}{4b} (1 + bd^2)
\]

- Theorem: \(J > 0\) for 2 electrons and \(B = 0\).

  (see also numerics by X. Hu et al., PRB '00, include higher orbitals)
II. Hund-Mullikan calculation

Burkard, Loss, DiVincenzo, PRB 59, 2070 (1999)

- confinement is approximated by a quartic potential, with typically \( \hbar \omega_0 = 3 \) meV

\[
W(x, y) = \frac{m \omega_0^2}{2} \left( \frac{(x^2 - a^2)^2}{4a^2} + y^2 \right)
\]

- separates into two harmonic wells, if

\[
a >> a_B \equiv \sqrt{\hbar / m \omega_0}
\]

- Hamiltonian (neglecting the Zeeman splitting for GaAs):

\[
H_d = \sum_{i=1,2} \left[ \frac{1}{2m} \left( p_i - \frac{e}{c} A(r_i) \right)^2 + W(r_i) \right] + \frac{e^2}{\kappa |r_1 - r_2|}
\]

- gauge:

\[
A = (-yB/2, xB/2, 0) \quad \Rightarrow \quad B \parallel z
\]
Fock-Darwin states (Fock ‘28; Darwin ‘30), translated by $\pm a$ in presence of magnetic field $B$ (Burkard, Loss, DiVincenzo ‘99):

$$
\phi_{\pm a}(x, y) = \frac{1}{\lambda \sqrt{\pi}} \exp \left[ - \frac{(x \mp a)^2 + y^2}{2\lambda^2} \mp \frac{iy}{2l^2} \right]
$$

$$
l = \sqrt{\frac{\hbar c}{e|B|}} \quad \lambda = \sqrt{\frac{\hbar}{m\omega}} \quad \omega = \sqrt{\omega_0^2 + \omega_L^2} \quad \omega_L = \frac{|e|B}{2mc}
$$

$$
d_{\pm,\sigma} : \quad \psi_{\pm,\sigma} = \chi_\sigma \left( \phi_{-a} \pm \phi_{+a} \right) / \sqrt{2(1 \pm S)}
$$

Two-particle states: 6 possible configurations

Lowest energy eigenstates of DD:

\[ |S1\rangle = \frac{1}{\sqrt{2}} \left( d_{-\uparrow}^\dagger d_{+\downarrow}^\dagger - d_{-\downarrow}^\dagger d_{+\uparrow}^\dagger \right) |0\rangle \]

\[ |S2\rangle = \frac{1}{\sqrt{1+\phi^2}} \left( \phi d_{+\uparrow}^\dagger d_{+\downarrow}^\dagger + d_{-\downarrow}^\dagger d_{-\uparrow}^\dagger \right) |0\rangle \]

\[ |T_+\rangle = d_{-\uparrow}^\dagger d_{+\uparrow}^\dagger |0\rangle, \quad |T_-\rangle = d_{-\downarrow}^\dagger d_{+\downarrow}^\dagger |0\rangle \]

\[ |T_0\rangle = \frac{1}{\sqrt{2}} \left( d_{-\uparrow}^\dagger d_{+\downarrow}^\dagger + d_{-\downarrow}^\dagger d_{+\uparrow}^\dagger \right) |0\rangle \]

\[ |S\rangle = \frac{1}{\sqrt{1+\phi^2}} \left( d_{+\uparrow}^\dagger d_{+\downarrow}^\dagger - \phi d_{-\uparrow}^\dagger d_{-\downarrow}^\dagger \right) |0\rangle \]

\[ \phi = \sqrt{1 + \left( \frac{4t_H}{U_H} \right)^2 - \frac{4t_H}{U_H}} \]

Exchange:

\[ J = v - \frac{U_H}{2} + \frac{1}{2} \sqrt{U_H^2 + 16t_H^2} \]

Burkard et al. '99; Schliemann et al. '00, Golovach et al. '03

Concurrence:

\[ c[|S\rangle] = \frac{2\phi}{1+\phi^2} \]

Double occupancy:

\[ D[|S\rangle] = \frac{(1-\phi)^2}{2(1+\phi^2)^2} \]
Lateral Coupling (GaAs dots)

- **extended** Hubbard physics:

\[ J = \frac{4t^2}{U} + V(B) \]

- note: HM $\rightarrow$ HL for increasing on-site Coulomb repulsion, i.e.

\[ U(B, c) \sim c\sqrt{B} \]
GaAs/AlGaAs Heterostruktur
2DEG 90 nm depth, $n_s = 2.9 \times 10^{11}$ cm$^{-2}$

Temp.: 100 mK

C. Marcus et al., PRL 2004
Many sources cause **decoherence** of spin qubit:

Fischer and DL, Science 324, 1277 (2009)

The goal is to reach long decoherence times $T_2$ and short gate times $\tau$ such that $T_2/\tau > 10^3$!
Sources of spin decoherence in GaAs quantum dots:

• **spin-orbit interaction** (band structure effects): couples lattice vibrations with spin ➞ **spin-phonon** interaction, but weak in quantum dots due to 1. low momentum, 2. no 1\textsuperscript{st} order s-o terms due to confinement (Khaetskii&Nazarov, ’00; Golovach et al., ‘04-’10)

• **spin-orbit interaction ➞ gate errors** (XOR); but they can be minimized (Bonesteel et al., Burkard et al., ’02, ’03)

• **dipole-dipole interaction**: weak

• **hyperfine interaction with nuclear spins**: dominant decoherence source (Burkard, DL, DiVincenzo, PRB ’99; Coish &DL, 2004-10, Das Sarma 2006..., Erlingson&Nazarov 2002,...), but absent e.g. in Si/Ge based dots!
Swichting Rate

Determine $N_{Op} \approx \tau_\phi / \tau_s$ for GaAs

- calculate $J(\nu)$ **statically** and then take $J(t) = J(\nu(t))$ for time-dependent $\nu(t)$, where $\nu = V, B, a, E$ is control parameter

- sufficient criterion for this to work

\[ \bar{J} = \left( \frac{1}{\tau_s} \right) \int_0^{T_s} dt \, J(t) \]

\[ \frac{1}{\tau_s} \approx \left| \frac{\dot{\nu}}{\nu} \right| \ll \frac{\bar{J}}{\hbar} \quad \text{adiabaticity condition} \]

- compatible with

\[ J \tau_s = n \pi, \quad n = 1, 3, 5, \ldots \quad \text{(needed for XOR)} \]

- self-consistency of calculation of $J$

\[ J \ll \Delta \epsilon \]

- thus:

\[ \frac{1}{\tau_s} \ll \frac{\bar{J}}{\hbar} \ll \frac{\Delta \epsilon}{\hbar} \quad \text{(no double occupancy)} \]

- numbers: $J \approx 0.2 \text{ meV} \implies \tau_s \gtrsim 50 \text{ ps}$

- decoherence of spin ca. $100 \mu$s

\[ N_{Op} \approx \frac{\tau_\phi}{\tau_s} \approx 10^6 \quad \text{sufficient for upscaling} \]

\[ \rightarrow \]

\[ \bar{J} \]
Quantum XOR (CNOT) via Hamiltonian

\[ U(t) = T \exp \left\{ -\frac{i}{\hbar} \int_0^t H(t') \, dt' \right\}, \quad H \neq 0 \text{ during } \tau_s \]

can show that: \((\text{Loss+DiVincenzo, PRA } 57 (120), 1998)\)

\[ U_{\text{XOR}} \leftrightarrow \int^t H_{\text{XOR}} = \pi S_1^z S_2^z - \frac{\pi}{2} (S_1^z + S_2^z) \quad s = \frac{1}{2} \]

\(H_{\text{XOR}}\) is pure Ising: not very physical (for real spin)!
instead use Heisenberg \(H = JS_1 \cdot S_2\) for \(U_{\text{XOR}}\)
and Zeeman \(H_B = B_1 \cdot S_1 + B_2 \cdot S_2\) for single-qubit operations

\(\text{e.g. swap gate: qubit } 1 \leftrightarrow \text{qubit } 2, \text{ choose } \int^t J(t)/\hbar \approx J_0\tau_s/\hbar = \pi \text{ (mod } 2\pi)\)

\[ U(t) = e^{i\pi/4} U_{\text{sw}} = e^{i\pi/4} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \]

basis: \(\{ |\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle \}\)

\[ U_{\text{XOR}} = e^{i\frac{\pi}{2}S_1^z} e^{-i\frac{\pi}{2}S_2^z} U_{\text{sw}}^2 e^{i\pi S_1^z} U_{\text{sw}}^2 \quad (\text{DL+DDV '97}) \]
Entanglement with 'sqrt-of-swap'

$$U_{XOR} = e^{i(\pi/2)S_1^z} e^{-i(\pi/2)S_2^z} U_{SW}^{1/2} e^{i\pi S_1^z} U_{SW}^{1/2} ,$$

Square-root-of-swap: $$|\uparrow\downarrow\rangle \Rightarrow |\uparrow\downarrow\rangle + i |\downarrow\uparrow\rangle$$

= entangler: product state $$|\uparrow\rangle_1 \times |\downarrow\rangle_2 \Rightarrow$$ entangled state

→ Entanglement is crucial for quantum computing!
Dynamics of Entanglement for square-root-of-swap

The square-root of a swap is obtained by halfing the duration of the tunneling pulse.

The result is a fully entangled two-qubit state having only a vanishingly small amplitude for double-occupancies of one of the dots.

During the process the indistinguishability of electrons and their fermionic statistics are essential.

Quantum XOR gate (DL & DDV `97)

\[
U_{XOR} = e^{i\frac{\pi}{2}S^z_1} e^{-i\frac{\pi}{2}S^z_2} U_{SW}^{1/2} e^{i\pi S^z_1} U_{SW}^{1/2},
\]

| \[ | \uparrow \uparrow \rangle \] | \[ | \uparrow \downarrow \rangle \] | \[ | \downarrow \uparrow \rangle \] | \[ | \downarrow \downarrow \rangle \] |
| --- | --- | --- | --- |
| \[ U_{SW}^{1/2} \] | \[ \downarrow \] | \[ \downarrow \] | \[ \downarrow \] | \[ \downarrow \] |
| \[ e^{i\pi S^z_1} \] | \[ i | \uparrow \uparrow \rangle \] | \[ \frac{ie^{-i\pi/4}}{\sqrt{2}} (| \uparrow \downarrow \rangle + i | \downarrow \uparrow \rangle) \] | \[ \frac{e^{-i\pi/4}}{\sqrt{2}} (i | \uparrow \downarrow \rangle + | \downarrow \uparrow \rangle) \] | \[ \downarrow \] | \[ \downarrow \] | \[ \downarrow \] | \[ \downarrow \] |
| \[ U_{SW}^{1/2} \] | \[ \downarrow \] | \[ \downarrow \] | \[ \downarrow \] | \[ \downarrow \] |
| \[ e^{-i\frac{\pi}{2}S^z_2} \] | \[ i | \uparrow \uparrow \rangle \] | \[ | \uparrow \downarrow \rangle \] | \[ \frac{ie^{-i\pi/4}}{\sqrt{2}} (| \uparrow \downarrow \rangle - i | \downarrow \uparrow \rangle) \] | \[ \frac{ie^{-i\pi/4}}{\sqrt{2}} (i | \uparrow \downarrow \rangle - | \downarrow \uparrow \rangle) \] | \[ -i | \downarrow \downarrow \rangle \] |
| \[ \times e^{i\pi S^z_1} \] | \[ i | \uparrow \uparrow \rangle \] | \[ i | \uparrow \downarrow \rangle \] | \[ i | \downarrow \uparrow \rangle \] | \[ -i | \downarrow \downarrow \rangle \] |
How to make entanglement ‘visible’

Loss & DiVincenzo, 1998

$|\downarrow\downarrow\rangle - i|\uparrow\uparrow\rangle$

$|\downarrow\uparrow\rangle$

$|\uparrow\downarrow\rangle$

$|\uparrow\uparrow\rangle + i|\downarrow\downarrow\rangle$
How to make entanglement ‘visible’

Loss & DiVincenzo, 1998

\[ | \uparrow \downarrow \rangle - i | \downarrow \uparrow \rangle \]

\[ | \downarrow \uparrow \rangle + i | \uparrow \downarrow \rangle \]

\[ \rightarrow \text{entanglement oscillates!} \]
Entanglement oscillations
Petta, Yacoby, Marcus et al., Science 2005

ultra-fast ‘clock speed’: entanglement generated in 180 ps!
Switching of exchange $J$

1. **Asymmetric via bias $\varepsilon$**

2. **Symmetric via barrier height**

Burkard, Loss, and DiVincenzo, PRB (1999)
Noise Suppression Using Symmetric Exchange Gates in Spin Qubits

Frederico Martins, Filip K. Malinowski, Peter D. Nissen, Edwin Barnes, Saeed Fallahi, Geoffrey C. Gardner, Michael J. Manfra, Charles M. Marcus, and Ferdinand Kuemmeth

Center for Quantum Devices, Niels Bohr Institute, University of Copenhagen, 2100 Copenhagen, Denmark

Sqrt-of-Swap gate via J

FIG. 3. (a) Tilt-induced exchange oscillations (i.e., $\gamma_x = 0$ mV) for $\epsilon_x = 79.5$ and 82 mV, generating oscillation frequencies indicated by $J$. (b) Same as (a) but for the symmetric mode of operation ($\epsilon_x = 13.5$ mV), with $\gamma_x = 100, 120$, and 140 mV.
Reduced Sensitivity to Charge Noise in Semiconductor Spin Qubits via Symmetric Operation


HRL Laboratories, LLC, 3011 Malibu Canyon Road, Malibu, California 90265, USA
(Received 5 August 2015; published 16 March 2016)

Si/Ge quantum dots
Serial vs. Parallel gate

I. Serial gate: [LD, PRA 57, 120 (1998)]

\[ H(t) = J(t)S_1 \cdot S_2 + b_1(t) \cdot S_1 + b_2(t) \cdot S_2 \]

controlled such that

\[ H(t) = J(t)S_1 \cdot S_2 \quad \text{or} \quad H(t) = b_1(t) \cdot S_1 + b_2(t) \cdot S_2 \]

\[ U_{SW}^{1/2} = \exp(i \int_0^{\tau_s} dt J(t)S_1 \cdot S_2), \quad \text{if} \quad \int_0^{\tau_s} dt J(t) = \pi / 2 + 2\pi n \]

\[ U_{XOR} = e^{-i(\pi/2)S_2^y} \left[ e^{i(\pi/2)S_1^x} e^{-i(\pi/2)S_2^z} U_{SW}^{1/2} e^{i\pi S_1^z} U_{SW}^{1/2} e^{i(\pi/2)S_2^y} \right] \]

\[ \Rightarrow \quad \text{need 7 pulses (5 for CPF)} \]
II. Parallel gate: Burkard et al., PRB 60, 11404 (1999)

\[
H(t) = J(t)S_1 \cdot S_2 + b_1(t) \cdot S_1 + b_2(t) \cdot S_2
\]

\[
U_{CPF} = \exp(i \int_0^{\tau_s} dt H(t))
\]

only 1 pulse for CPF!

if \[\int J = \pi / 2, \text{ and } \int b_{1/2}^z = \pi(1 \pm \sqrt{3}) / 4\]

\[
U_{XOR} = e^{-i(\pi / 2)S_2^y} U_{CPF} e^{i(\pi / 2)S_2^y}
\]

\[\Rightarrow \text{ need only 3 pulses}\]

Implementation scheme: Meunier et al., PRB 83, 121403 (2011)
Single-Qubit Operations or *How to Flip a Spin?*

1. Electron Spin Resonance (ESR)
   An ac magnetic field is applied perpendicular to a static magnetic field, with a frequency that matches the Zeeman splitting.

2. Electric-Dipole-Induced Spin Resonance (EDSR)
   Exploits spin-orbit interaction and ac electric field.

3. Electrically Driven ESR in a Slanting Magnetic Field
   Exploits a magnetic field gradient and ac electric field.

4. Electrically Driven ESR in an exchange field of auxiliary spin
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Single-Qubit Operations or *How to Flip a Spin*?

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   Exploits the *exchange field*, *magnetic field gradient*, and *ac electric field*.
Two-Qubit Gate of Combined Single-Spin Rotation and Interdot Spin Exchange in a Double Quantum Dot

R. Brunner,¹,²* Y.-S. Shin,¹ T. Obata,¹,³ M. Pioro-Ladrière,⁴ T. Kubo,⁵ K. Yoshida,¹ T. Taniyama,⁶,⁷ Y. Tokura,¹,⁵ and S. Tarucha¹,³

double dot in B field gradient

EDSR ~ MHz

entanglement ~ GHz
Ultra-fast single-qubit gates via exchange

Alternative to ESR/EDSR: double dot with pulsed J-gate!

DL and DiVincenzo, PRA 57 (1998)
Coish and DL, PRB 75, 161302 (2007)
Chesi, Wang, Yoneda, Otsuka, Tarucha, and DL, PRB 90, 235311 (2014)

Single-qubit gates with high fidelity and ultrafast ~ 1 ns
Single-spin rotation via exchange: Two regimes

I. \( b_1 \gg b_2 \)

Advantage: hybridization of logical states and (1,1) charge configuration can be made very small; but difficult to reach

II. \( b_1 \simeq b_2 \simeq B \)

This is a more typical situation in exp.:

\[
B \gtrsim 1 \text{ T} \quad \text{(due to saturation field of micromagnet)}
\]

\[
\Delta b = b_1 - b_2 \simeq 10 - 50 \text{ mT}
\]

Ultra-short gate times: 1 ns, with very high fidelity for GaAs double dots

Coish & DL, PRB 2007

Chesi et al., PRB 2014
Single-spin manipulation in double dots with micromagnet

Chesi et al., PRB 90, 235311 (2014)

• Single-qubit gates implemented via exchange (as for two-qubit gates)

• Ultra-short gate times: 1 ns, with very high fidelity for GaAs double dots

• Noise sources: Nuclear and charge noise present but not a problem
Most Advanced: Spin qubits in \textbf{GaAs} quantum dots


Westervelt, Gossard 1995
Kouwenhoven, Tarucha 1996
Sachrajda 2000
Kouwenhoven, Tarucha 2003-13
Vandersypen, Koppens, 2003
Marcus 2004

Petta, Marcus, Yacoby 2005
Ensslin, Ihn 2006
Zumbuhl, Kastner 2008
Petta 2010
Bluhm, Dial, Yacoby 2010-13
\( T_2 \sim 300 \, \mu\text{s} \)
Brunner, Pioro-Ladriere, Tarucha 2011

... and many more ...
Spin-Qubits from Electrons

simplest spin-qubit: spin-1/2 of 1 electron $|0\rangle = \uparrow$, $|1\rangle = \downarrow$

Many more choices for spin qubits:

- 'exchange-only qubits' DiVincenzo, Burkard et al. `00; Sachrajda `12; Marcus `13;
  3 electrons: $|0\rangle = S \uparrow$, $|1\rangle = T_+ \downarrow - T_0 \uparrow$ Doherty `15; Taylor `16; Rashba/Halperin `13
- 'singlet-triplet' qubits Levy `02, Taylor et al. `05, Klinovaja et al. `12
  2 electrons: $|0\rangle = S$, $|1\rangle = T_0$

- 'spin-cluster qubits' Meier, Levy & DL, `03
  N electrons: AF spin chains, ladders, clusters,...

- 'spin-orbit qubits' Golovach, Borhani &DL, `07; Kouwenhoven et al., `11;

  hole spins: Bulaev & DL, `05; Marcus et al., `11; Kloeffel, Trif & DL, `11-`16 (Si/Ge NW)

- molecular magnets Leuenberger & DL, `01; Affronte et al., `06,
  Lehmann et al., `07; Trif et al., `08, `10, `16
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- molecular magnets Leuenberger & DL, `01; Affronte et al., `06, Lehmann et al., `07; Trif et al., `08, `10, `16
Most popular spin qubits (in GaAs)

LD spin qubit:
spin-1/2 of 1 electron

\[ |0\rangle = \uparrow, \quad |1\rangle = \downarrow \]

'singlet-triplet' qubits:
2 electrons:
Levy (2002), Taylor et al. (2005)

\[ |0\rangle = S, \quad |1\rangle = T_0 \]

Prospects for Spin-Based Quantum Computing in Quantum Dots
C. Kloeffel and D. Loss, Annu. Rev. Condens. Matter Phys. 4, 51 (2013);
Singlet-Triplet (ST) Qubit

four dots = two ST-qubits

Computational basis:

\[ S^z = 0 \]

\[
\begin{align*}
[10] &= (+ - - +) \\
[01] &= (- + + -) \\
[00] &= (- + - +)
\end{align*}
\]

Note: Decoherence time is very long \( T_2 \sim 250 \, \mu s \)

Bluhm et al., Nat. Phys. 7, 109 (2011)
CNOT Gate via Exchange: fast and noise-free


\[ H^B = \sum_{i=1}^{4} (b + b_i) \hat{S}_i^z \]

Zeeman interaction

\[ b = g \mu_B B \quad \text{and} \quad b_i = g \mu_B B_i, \]

global magnetic field \( B \) \( \rightarrow \) quantization axis
local magnetic field \( B_i \) \( \rightarrow \) single qubit operations

\[ H_{\text{ex}}(t) = J_{12}(t) \mathbf{S}_1 \cdot \mathbf{S}_2 + J_{23}(t) \mathbf{S}_2 \cdot \mathbf{S}_3 + J_{34}(t) \mathbf{S}_3 \cdot \mathbf{S}_4 \]

exchange interaction

Rashba SOI leads to*) \[ H_{\text{SO}} = \beta(t) \cdot (\mathbf{S}_i \times \mathbf{S}_j) \]

Dzyaloshinskii-Moriya (SOI) term

CNOT Gate via Exchange: fast and noise-free


\[ H^B = \sum_{i=1}^{4} (b + b_i) \hat{S}_i^z \]
\[ b = g \mu_B B \quad \text{and} \quad b_i = g \mu_B B_i, \]

Zeeman interaction

same time-dependence!

\[ H_{\text{ex}}(t) = J_{12}(t) \mathbf{S}_1 \cdot \mathbf{S}_2 + J_{23}(t) \mathbf{S}_2 \cdot \mathbf{S}_3 + \]
\[ H_{SO} = \beta(t) \cdot (\mathbf{S}_i \times \mathbf{S}_j) \]

Rashba SOI leads to* \)


Dzyaloshinskii-Moriya (SOI) term

global magnetic field $B \Rightarrow$ quantization axis

local magnetic field $B_i \Rightarrow$ single qubit operations
Computational scheme

Klinovaja et al., PRB 86, 085423 (2012)

- phase gate $C_{23}$ for spins on dots 2 and 3
- $\pi_{12}$ pulses ($J_{12}$ swaps spins 1 and 2) and $\pi_{34}$ ($J_{34}$ swaps spins 3 and 4)
- phase gate $C_{23}$ for spins on dots 2 and 3
- $\pi_{12}$ ($J_{12}$ swaps spins 1 and 2) and $\pi_{34}$ ($J_{34}$ swaps spins 3 and 4)

rotation axis: $J_{23}e_x - \beta_{23}e_y + \Delta b_{23}e_z$

rotation axis: $J_{12}e_x - \beta_{12}e_y + \Delta b_{12}e_z$
...from one to many quantum dots...
12 quantum dots - 4 RX spin qubits

Marcus & Kuemmeth et al., 2015/16

<table>
<thead>
<tr>
<th>Qubit 1</th>
<th>Qubit 2</th>
<th>Qubit 3</th>
<th>Qubit 4</th>
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<tbody>
<tr>
<td>$J_{O1}^{12}$</td>
<td>$J_{O1}^{23}$</td>
<td>$J_{O1-O2}^{eff}$</td>
<td>$J_{O2}^{12}$</td>
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<td>$J_{O2}^{12}$</td>
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<td>$J_{O4}^{12}$</td>
<td>$J_{O4}^{23}$</td>
<td></td>
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</tbody>
</table>
FIG. 1: (a) SEM image of a sample nominally identical to the one used for the measurements. Dotted circles indicate quantum dots, squares indicate Fermi reservoirs in the 2DEG, which are connected to ohmic contacts. The gates that are not labeled are grounded. The reflectance of SD2, $V_{RF,SD2}$, is monitored. (b) Charge stability diagram of the quadruple dot. The occupancy of each dot is denoted by $(l, m, n, p)$ corresponding to the number of electrons in dot 1 (leftmost), 2, 3 and 4 (rightmost) respectively. The fading of charge transition lines from dot 2 and 3 can be explained in a similar way as in Ref. 17 (black dotted lines indicate their positions) and becomes less prominent for a slow scan (see Supplementary Information II). The pulse sequence for loading and read-out is indicated in the charge stability diagram via arrows, see also panel b. The black rectangle corresponds to the hot spot in dot 4 where spins relax on a sub-microsecond timescale; this hot spot is only used for the measurements of Fig. 3. (c) Read from left to right and top to bottom. The system is initialized by loading one electron from the left reservoir. Next, we shuttle the electron to dot 2, 3 and 4 sequentially and finally read out the spin state using spin-selective tunneling.
12 (=9+3) quantum dots in Si/SiGe heterostructure
A logical qubit in a linear array of semiconductor quantum dots

Cody Jones,* Mark F. Gyure, and Thaddeus D. Ladd
HRL Laboratories, LLC, 3011 Malibu Canyon Road, Malibu, CA 90265, USA

Michael A. Fogarty, Andrea Morello, and Andrew S. Dzurak
Centre for Quantum Computation and Communication Technology,
School of Electrical Engineering and Telecommunications,
The University of New South Wales, Sydney, New South Wales 2052, Australia

arXiv:1608.06335