Influence of the electrodynamic environment on ac driven tunnel junctions

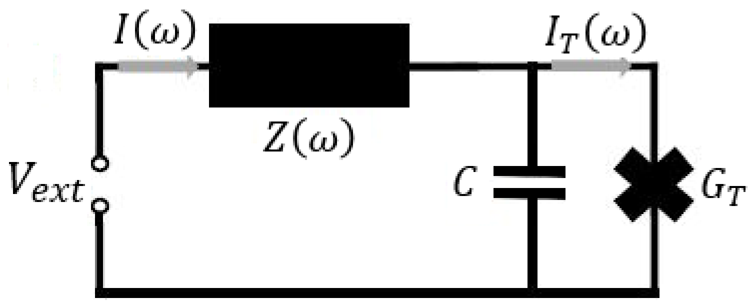
Albert-Ludwigs-Universität Freiburg

Moritz Frey & Hermann Grabert

Standard Model of Dynamical Coulomb Blockade

Tunnel element is coupled to an external voltage source via an environmental impedance.

→ Environmental modes influence the tunneling current



Circuit diagram of a voltage biased tunneling element

Junction charge:

$$\dot{Q} = I - I_T$$

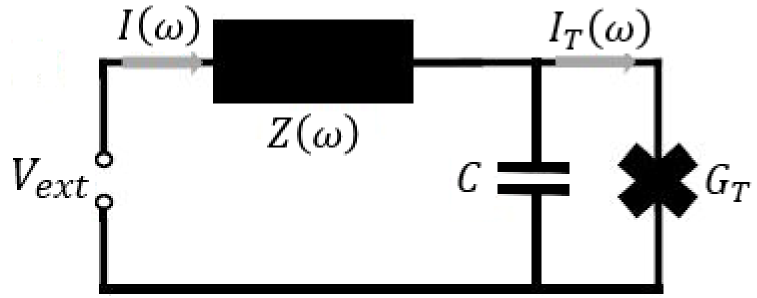
Average currents $\langle I \rangle$ and $\langle I_T \rangle$ coincide only for dc driving → $P(E)$-Theory

Ac driving: driven environmental modes!

Hamiltonian

Standard Hamiltonian of DCB-Theory:

$$H = H_{el} + H_T + H_{env}$$



Circuit diagram of a voltage biased tunneling element

$$H_{el} = \sum_{k,\sigma} \epsilon_{k\sigma} a^\dagger_{k\sigma} a_{k\sigma} + \sum_{q,\sigma} \epsilon_{q\sigma} a^\dagger_{q\sigma} a_{q\sigma}$$

$$H_{env}(t) = \frac{Q^2}{2C} + \sum_n \left\{ \frac{Q_n^2}{2C_n} + \frac{1}{2L_n} \left(\frac{\hbar}{e}\right)^2 [\varphi - \varphi_n - \varphi_{ext}(t)]^2 \right\}$$

contains the phase: $V_{ext}(t) = \frac{\hbar}{e} \dot{\varphi}_{ext}(t)$

$H_T = \Theta\, e^{-i\varphi} + \Theta^\dagger e^{i\varphi}$ with the quasiparticle tunnel operator $\Theta = \sum_{k,q,\sigma} t_{kq\sigma} a^\dagger_{k\sigma} a_{q\sigma}$

Method

Time dependent Hamiltonian:

Unitary Transformation for dc driving (standard theory):

$$\mathcal{U}(t) = e^{-iQ\varphi(t)}$$

A corresponding unitary transformation for ac driving must now involve the environmental modes:

$$\Lambda(t) = \exp\left\{\frac{i}{e}\left[\bar{\varphi}(t)Q + \sum_n \bar{\varphi}_n(t)Q_n\right]\right\} \times \exp\left\{-\frac{i\hbar}{e^2}\left[C\dot{\bar{\varphi}}(t)\varphi + \sum_n C_n\dot{\bar{\varphi}}_n(t)\varphi_n\right]\right\}$$

→ Average current can then be calculated along lines similar to standard $P(E)$ -theory

Average Current

Results for: $V_{ext}(t) = V_{dc} + V_{ac}\cos(\Omega t)$

Fourier expansion: $\langle I_{env}(t)\rangle = \sum_{n=-\infty}^{\infty} I_n\, e^{-in(\Omega t-\eta)}$, with fourier components:

$$I_n = \frac{1}{2}\frac{Y(n\Omega)}{Y(n\Omega)-in\Omega C}\sum_{k=-\infty}^{\infty} J_k(a)$$

$(n>0)$

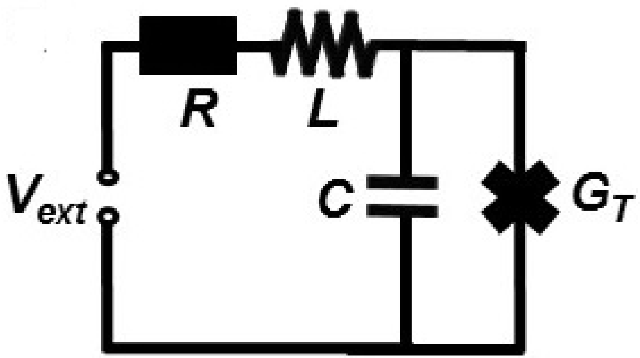
$$\times\{[J_{k+n}(a)+J_{k-n}(a)]\,I_{dc}(V_{dc}+k\hbar\Omega/e)$$
$$+i\,[J_{k+n}(a)-J_{k-n}(a)]\,I_{KK}(V_{dc}+k\hbar\Omega/e)\}$$
$$-\frac{i}{2}\delta_{n,1}\Omega C\Xi V_{ac}$$

$$I_0 = \sum_{k=-\infty}^{\infty} J_k^2\left(\frac{e\Xi V_{ac}}{\hbar\Omega}\right) I_{dc}(V_{dc}+k\hbar\Omega/e)$$

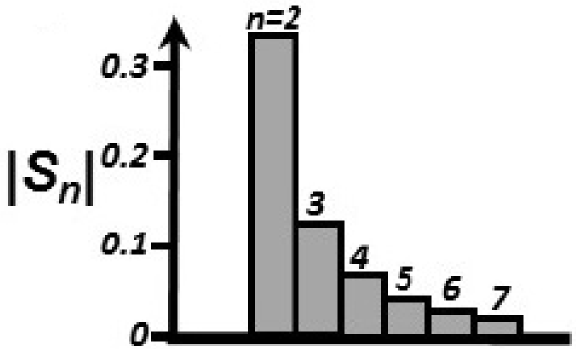
→ Renormalized amplitude of the ac voltage across the junction and suppression of the n^{th} harmonic of the current

Suppression of higher harmonics

Results for: $V_{ext}(t) = V_{dc} + V_{ac}\cos(\Omega t)$



Model circuit: LC-resonator with lead resistance R, inductance L and quality factor $Q_f = \frac{\sqrt{L}}{R\sqrt{C}}$



Modulus of the suppression factor of the n^{th} harmonic amplitude for driving at the resonance frequency and quality Factor $Q_f = 10$

Current Noise

Spectral function of current fluctuations:

$$S(\omega) = \langle \delta I(\omega) \delta I(-\omega) \rangle$$

Inserting the current fluctuations

$$\delta I(\omega) = Y(\omega) Z_t(\omega) [\delta I_T(\omega) + i\omega C \delta v_N(\omega)]$$

we find, that the spectral function consists of three contributions

$$S(\omega) = S_N(\omega) + S_T(\omega) + S_{NT}(\omega)$$

1.Johnson-Nyquist noise:

$$S_N(\omega) = |\omega C Z_t(\omega)|^2 \frac{2\hbar\omega}{1-e^{-\beta\hbar\omega}} Y'(\omega)$$

$$\langle \delta v_N(\omega) \delta v_N(-\omega) \rangle$$

transmission factor

Johnson Nyquist noise of admittance

Current Noise

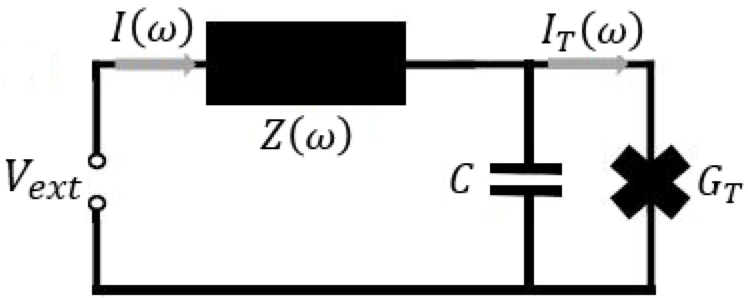
2.Shot noise: $S_T(\omega) = |Y(\omega)Z_T(\omega)|^2 e\left[\frac{I(V+\hbar\omega/e)}{1-e^{-\beta(eV+\hbar\omega)}} + \frac{I(V-\hbar\omega/e)}{e^{\beta(eV-\hbar\omega)-1}}\right]$

$\langle \delta I_T(\omega)\delta I_T(-\omega)\rangle$

transmission factor Shot noise of tunneling current

3.Cross-correlation noise:

$\langle \delta I_T(\omega)\delta v_N(-\omega) + \delta v_N(\omega)\delta I_T(-\omega)\rangle$

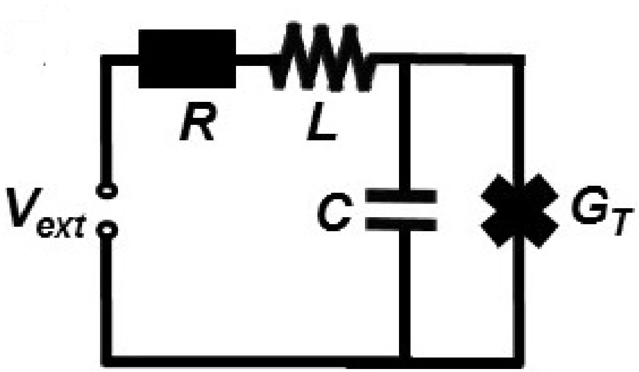


H. Lee and L. S. Levitov, Phys. Rev. B 53, 7383 (1996)

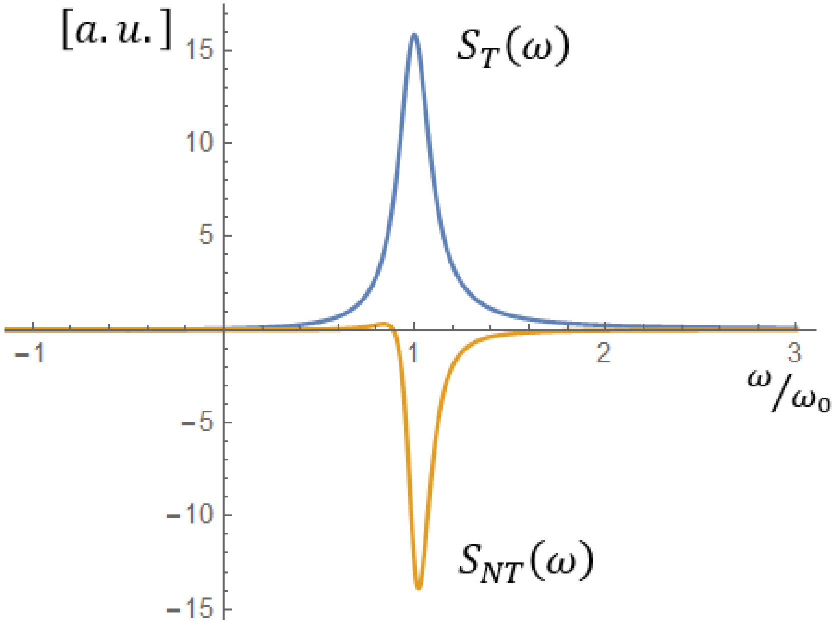
$$S_{NT}(\omega) = \frac{2e}{1-e^{-\beta\hbar\omega}}\Xi(\omega)^2 \times \left\{\sin^2[\eta(\omega)][I(V-\hbar\omega/e) - I(V+\hbar\omega/e)] + \right.$$

containing: $Y(\omega)Z_T(\omega) = \Xi(\omega)e^{i\eta(\omega)}$

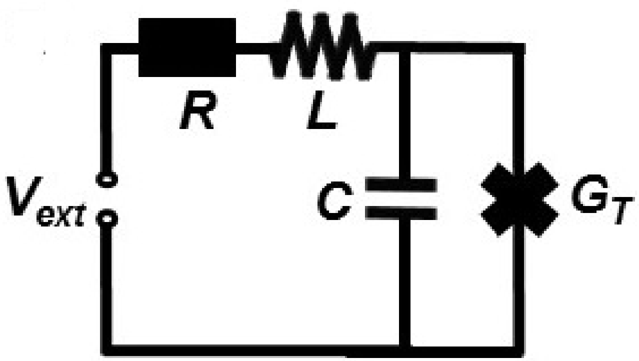
Current Noise LC-Resonator



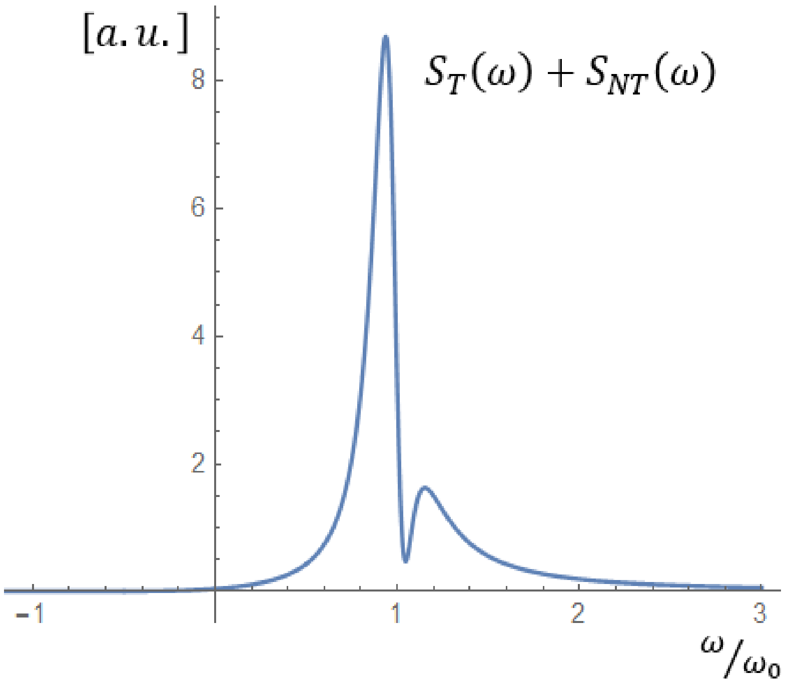
Contributions of S_T and S_{NT} to the total noise spectrum for an LC-resonator with Q_f=5



Current Noise LC-Resonator



Contributions of S_T and S_{NT} to the total noise spectrum for an *LC*-resonator with Q_f=5



Conclusion

Influence of the electromagnetic environment on ac driven tunnel junctions:

- Suppression of higher harmonics of the average current
- Current noise spectrum is not a sum of shot-noise and Johnson-Nyquist noise: Third component due to cross correlation
- Contribution of Cross correlation is not negligible
- All contributions to the noise spectrum are already present for pure dc driving! In case of an ac driving, the spectral function can be expressed as a weighted sum of the spectral functions in presence of dc driving (Tien-Gordon relation)