Time-dependent response of interacting quantum capacitors in the Coulomb Blockade regime

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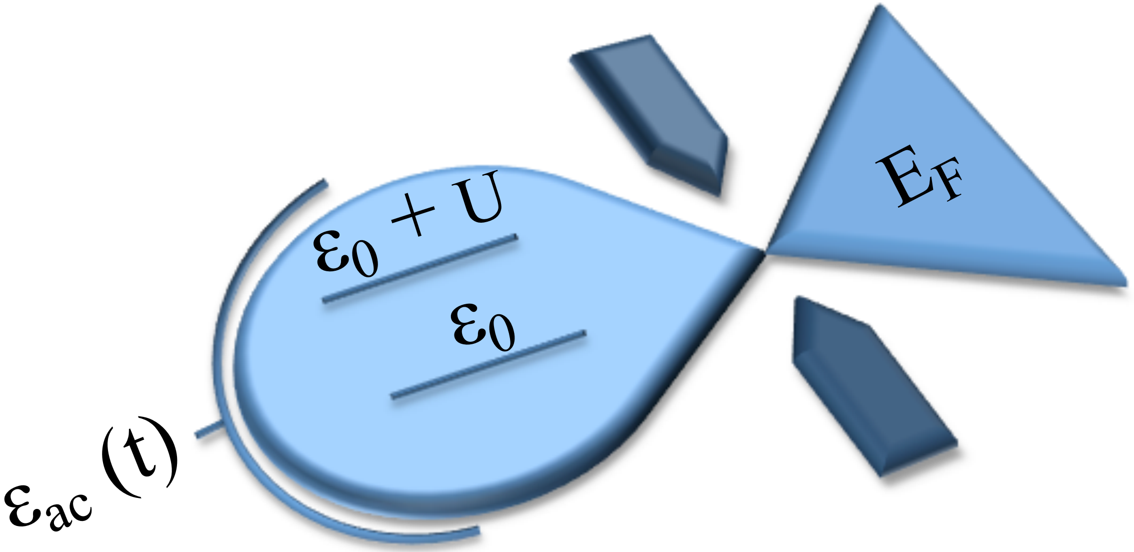
➢ *What is a mesoscopic capacitor?*



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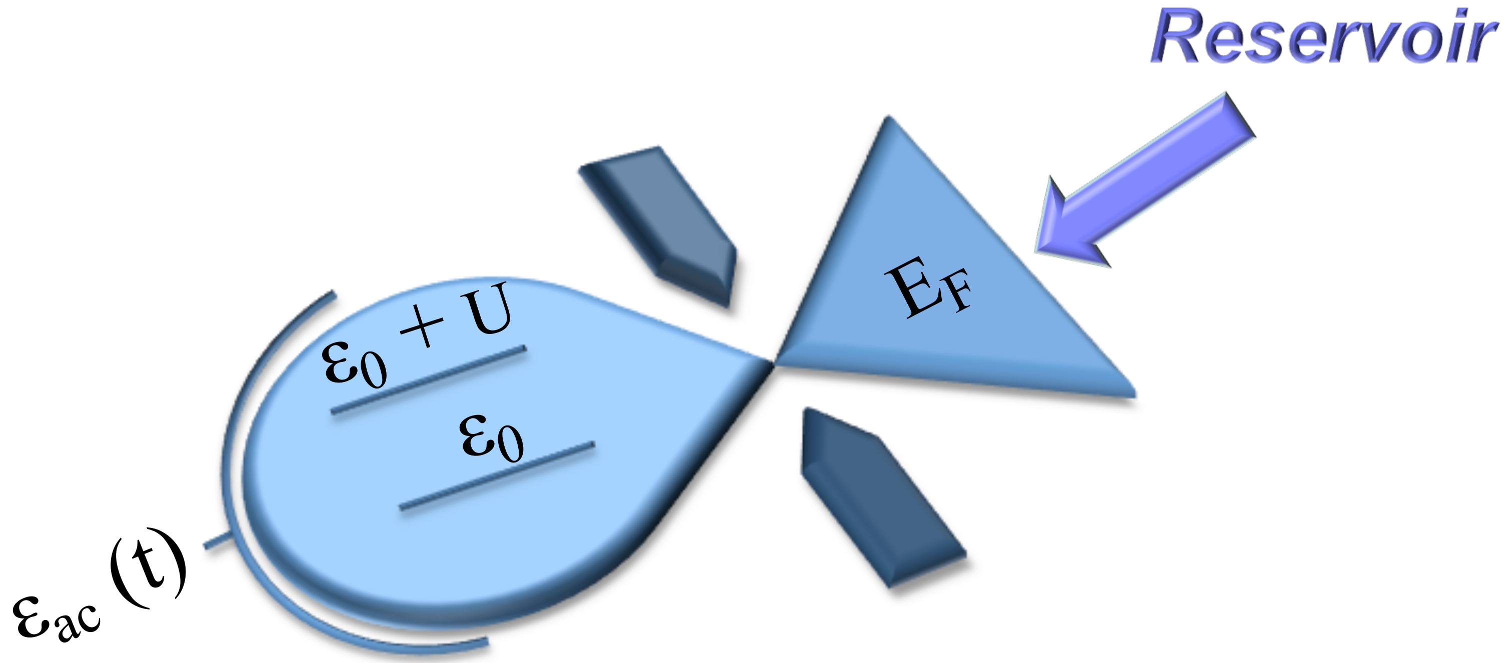

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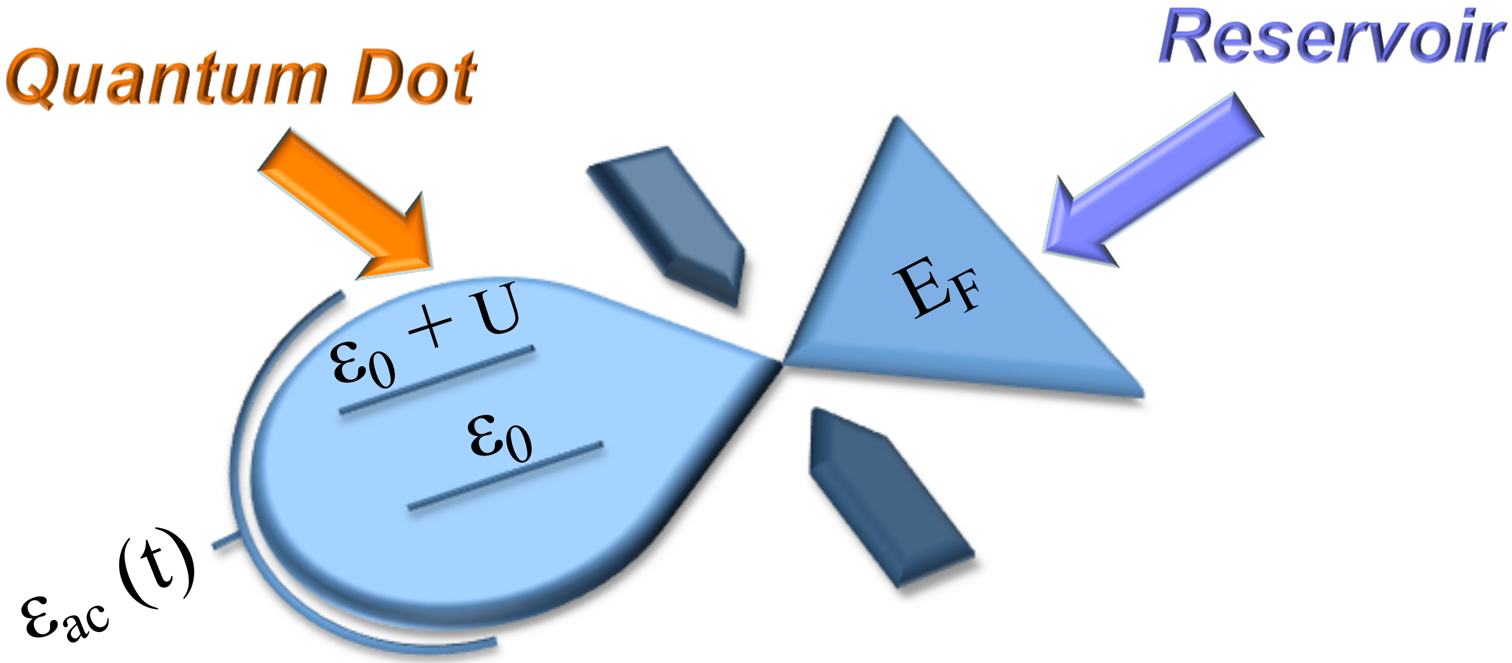

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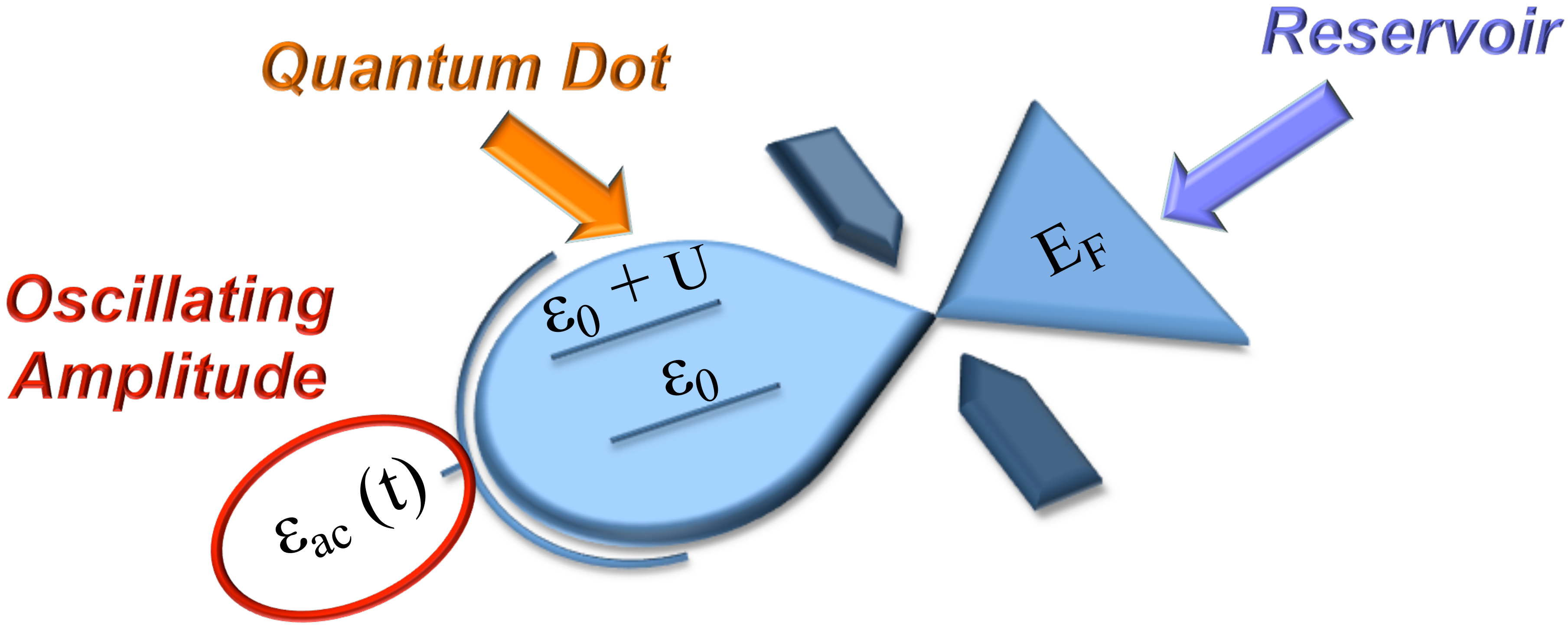
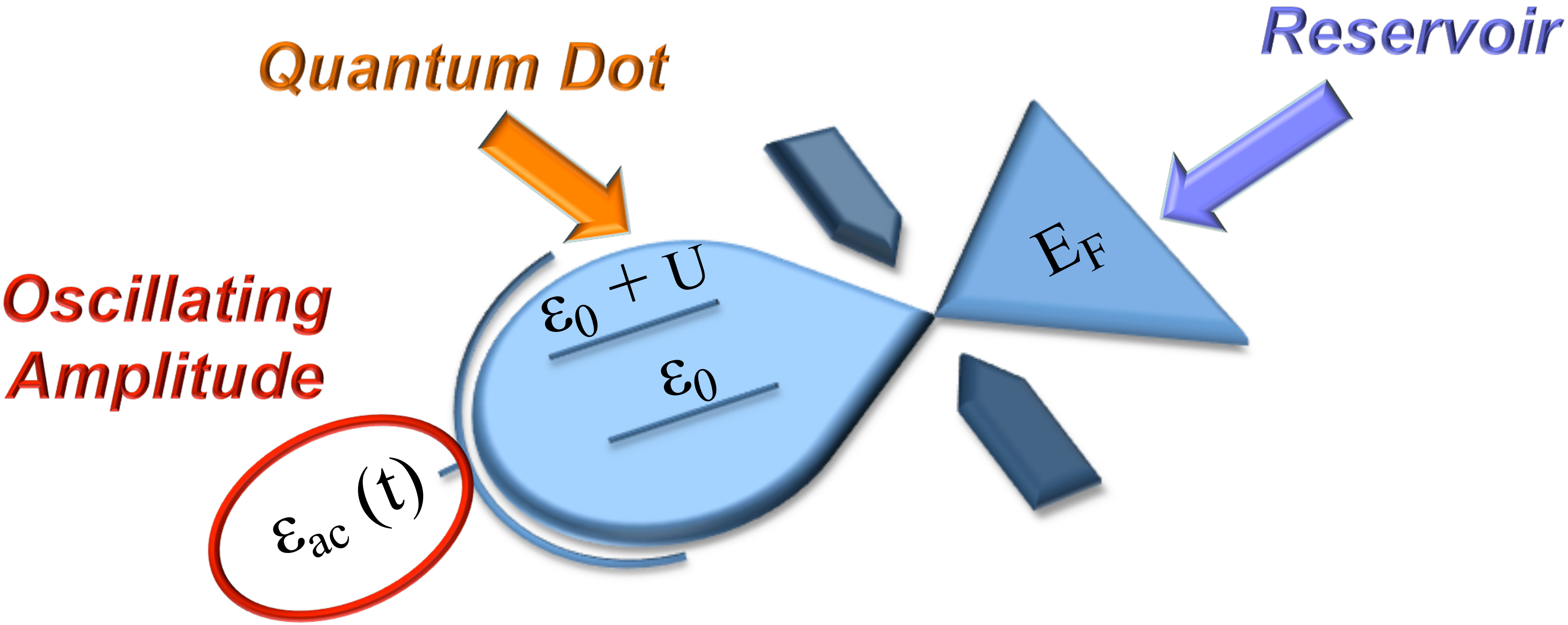


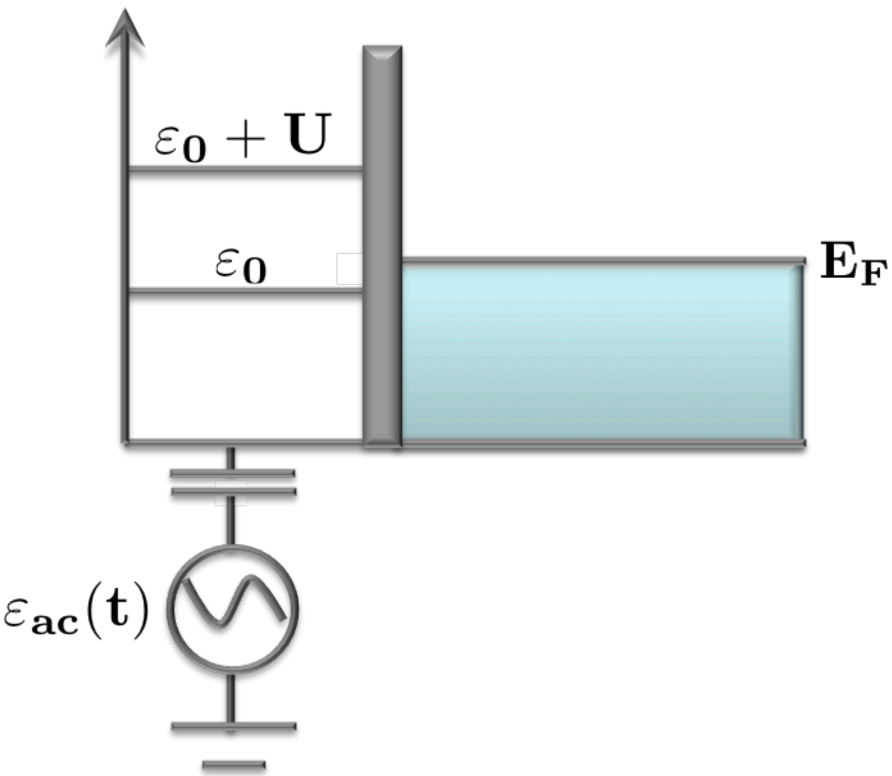
- ***What is a mesoscopic capacitor?***



Purely ac response !!!

J. Gabelli *et al.*, Science 313, 499 (2006)

Single-level Anderson Model



- $H_R = \sum_{k\sigma} \varepsilon_k C^\dagger_{k\sigma} C_{k\sigma}$
- $H_T = \sum_{k\sigma} \left(V_k^* d^\dagger_\sigma C_{k\sigma} + V_k C^\dagger_{k\sigma} d_\sigma \right)$
- $H_C = \sum_{\sigma} \varepsilon_\sigma(t) d^\dagger_\sigma d_\sigma + U n_\sigma n_{\bar{\sigma}}$

- $\varepsilon_\sigma(t) = \varepsilon_{0\sigma} + \varepsilon_{ac}(t) = \varepsilon_{0\sigma} + eV_{ac}(t)$
- $n_\sigma = d^\dagger_\sigma d_\sigma$
- $\sigma = -\bar{\sigma}$

Keldysh Green function formalism

- $$I(t) = e\partial_t \sum_\sigma \mathcal{N}_\sigma(t) = e\partial_t \sum_\sigma \int \frac{d\varepsilon}{2\pi i} G^<_{\sigma,\sigma}(t,\varepsilon)$$

  Where $\mathcal{N}_\sigma(t) = \langle n_\sigma \rangle (t)$

  $$G(t,\varepsilon) = \sum_n e^{-in\Omega t} G(n,\varepsilon)$$ Mixed representation

- Series expansion in powers of $\hbar\Omega$

  $$G^<_{\sigma,\sigma}(t,\varepsilon) = G^{<,f}_{\sigma,\sigma}(t,\varepsilon) + (\hbar\Omega) G^{<,(1)}_{\sigma,\sigma}(t,\varepsilon) + \dots$$

  Then,

  $$I(t) = e\partial_t \sum_\sigma (\mathcal{N}^f_\sigma(t) + \mathcal{N}^{(1)}_\sigma(t) + \dots)$$
  $$\simeq -C_\partial(t)(\partial_t V_{ac}(t)) + R_\partial(t) C_\partial(t) \partial_t (C_\partial(t)(\partial_t V_{ac}(t)))$$

M. Moskalets, P. Samuelsson, and M. Büttiker, Phys. Rev. Lett. **100**, 086601 (2008).

- ***Equation of motion:*** $i\hbar\partial_t A(t) = [A(t), H]$

  - Dyson equation for the Keldysh matrices: $\widehat{G} = \begin{pmatrix} G^t & G^< \\ G^> & G^{\bar{t}} \end{pmatrix}$

$$\widehat{G}_{\sigma,\sigma}(t,t') = \widehat{g}_\sigma(t,t') + \int \frac{ds}{\hbar} \widehat{G}_{\sigma,\sigma}(t,s)\varepsilon_{ac}(s)\widehat{g}_\sigma(s,t') + \int \frac{ds}{\hbar} \int \frac{ds'}{\hbar} \widehat{G}_{\sigma,\sigma}(t,s')\widehat{\Sigma}_0(s',s)\widehat{g}_\sigma(s,t') + U \int \frac{ds}{\hbar} \ll d_\sigma, d_\sigma^\dagger n_{\bar{\sigma}} \gg (t,s)\widehat{g}_\sigma(s,t')$$

- $\widehat{\Sigma}_0(t,t') = \sum_k |V_k|^2 \widehat{g}_k(t,t')$
- $\widehat{g}_{\sigma/k}(t,t')$ Non-interacting equilibrium Green function.

Non-Interacting Solution U = 0:

$$G^{<,f}_{\sigma,\sigma}(t,\varepsilon) = \frac{\Sigma_0^<(\varepsilon)}{(\varepsilon - \varepsilon_{0\sigma} - \varepsilon_{ac}(t) - \Sigma_0^r)(\varepsilon - \varepsilon_{0\sigma} - \varepsilon_{ac}(t) - \Sigma_0^a)}$$
$$= G^{r,f}_{\sigma,\sigma}(t,\varepsilon)\Sigma_0^<(\varepsilon)G^{a,f}_{\sigma,\sigma}(t,\varepsilon)$$

$$G^{<,(1)}_{\sigma,\sigma}(t,\varepsilon) = \frac{i}{\Omega}\partial_t\varepsilon_{ac}(t)\left(G^{a,f}_{\sigma,\sigma}(t,\varepsilon)\partial_\varepsilon G^{<,f}_{\sigma,\sigma}(t,\varepsilon) + G^{<,f}_{\sigma,\sigma}(t,\varepsilon)\partial_\varepsilon G^{r,f}_{\sigma,\sigma}(t,\varepsilon)\right)$$

- Wide band limit $\Gamma = 2\pi|V_k|^2\rho$
- $\Sigma_0^<(\varepsilon) = 2i\Gamma f(\varepsilon)$
- $\Sigma_0^{r/a} = \mp i\Gamma$

★ In the **Non-Magnetic** case $\sigma = \bar{\sigma}$ $\begin{cases} \varepsilon_{0\uparrow} = \varepsilon_{0\downarrow} \equiv \varepsilon_0 \\ D_{\uparrow 0}(t,\varepsilon) = D_{\downarrow 0}(t,\varepsilon) \equiv D_0(t,\varepsilon) \\ \mathcal{N}_{\uparrow 0}(t) = \mathcal{N}_{\downarrow 0}(t) \equiv \mathcal{N}_0(t)/2 \end{cases}$

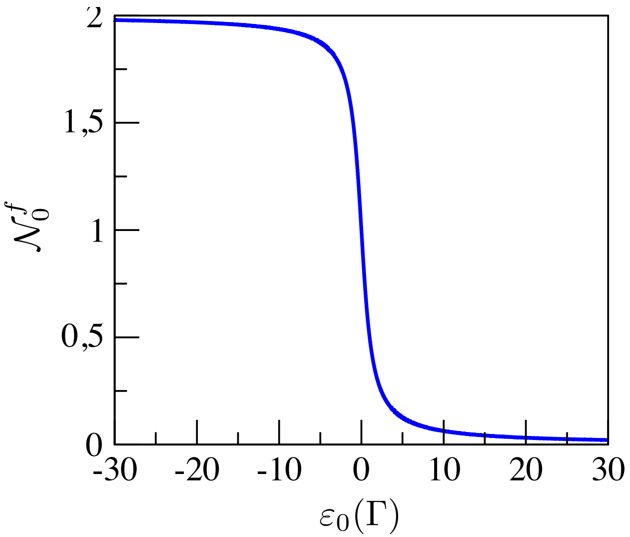
$$D_0(t,\varepsilon) = \frac{1}{\pi}\frac{\Gamma}{(\varepsilon - \varepsilon_0 - \varepsilon_{ac}(t))^2 + \Gamma^2}$$

- ***Frozen Approximation:*** $\mathcal{N}_0^f(t) = 2\int \frac{d\varepsilon}{2\pi i} G^{<,f}_{\sigma,\sigma}(t,\varepsilon) = 2\int d\varepsilon f(\varepsilon) D_0(t,\varepsilon)$
- ***Quantum Capacitance:*** $C^0_\partial(t) = 2e^2 \int d\varepsilon (-\partial_\varepsilon f(\varepsilon)) D_0(t,\varepsilon)$
- ***Charge Relaxatin Resistance:***

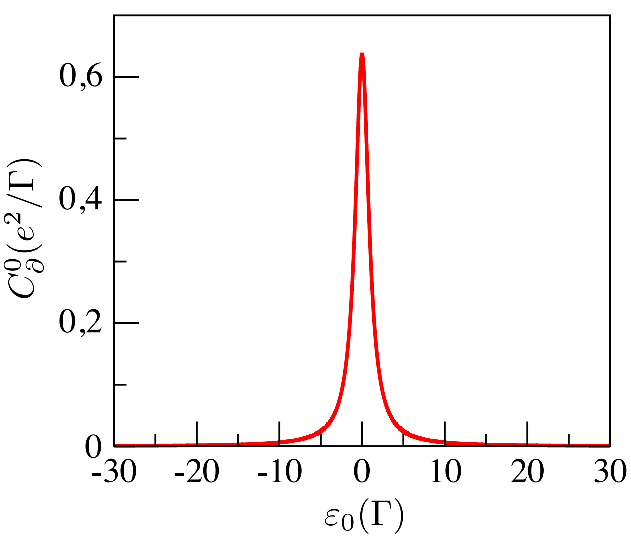
$$R^0_\partial(t) = \frac{h}{4e^2}\frac{\int d\varepsilon(-\partial_\varepsilon f(\varepsilon))\partial_t((\partial_t V_{ac}(t))D_0^2(t,\varepsilon))}{\int d\varepsilon(-\partial_\varepsilon f(\varepsilon))D_0(t,\varepsilon)\int d\varepsilon(-\partial_\varepsilon f(\varepsilon))\partial_t((\partial_t V_{ac}(t))D_0(t,\varepsilon))}$$

★ In the **Linear Regime** $V_{ac} \to 0$ and $k_B T = 0$: $D_0(\varepsilon_0) = \frac{1}{\pi} \frac{\Gamma}{\varepsilon_0^2 + \Gamma^2}$

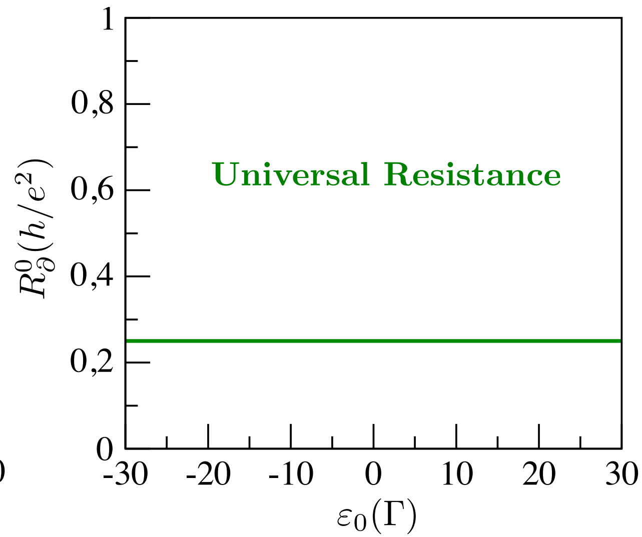
$\mathcal{N}_0^f(\varepsilon_0) = 1 - \frac{2}{\pi} \tan^{-1}\left(\frac{\varepsilon_0}{\Gamma}\right)$



$C_\partial^0(\varepsilon_0) = 2e^2 D_0(\varepsilon_0)$



$R_\partial^0(\varepsilon_0) = \frac{h}{4e^2}$



Prediction of the Universal Charge Relaxation Resistance:

M. Büttiker, H. Thomas, and A. Prêtre, Phys. Lett. A **180**, 346 (1993).

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  - Equation of motion for $\ll d_\sigma, d_\sigma^\dagger n_{\bar{\sigma}} \gg (t,t')$

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    - $\ll d_\sigma, d_\sigma^\dagger C_{k\bar{\sigma}} d_{\bar{\sigma}} \gg (t,t') \simeq 0$ Dot spin excitation
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$$\begin{aligned}\ll d_\sigma, d_\sigma^\dagger n_{\bar\sigma} \gg (t,t') = \mathcal{N}_{\bar\sigma}(t)\widehat{g}_\sigma(t,t') &+ \int \frac{ds}{\hbar} \ll d_\sigma, d_\sigma^\dagger n_{\bar\sigma} \gg (t,s)\varepsilon_{ac}(s)\widehat{g}_\sigma(s,t') \\ &+ U\int \frac{ds}{\hbar} \ll d_\sigma, d_\sigma^\dagger n_{\bar\sigma} \gg (t,s)\widehat{g}_\sigma(s,t') \\ &+ \int \frac{ds}{\hbar}\int \frac{ds'}{\hbar} \ll d_\sigma, d_\sigma^\dagger n_{\bar\sigma} \gg (t,s')\widehat{\Sigma}_0(s',s)\widehat{g}_\sigma(s,t')\end{aligned}$$

Coulomb Blockade Solution $\mathbf{U} > \pi\mathbf{\Gamma}$:

$$G^{<,f}_{\sigma,\sigma}(t,\varepsilon) = \left(1-\mathcal{N}^{f}_{\bar\sigma}(t)\right) G^{r,f}_{0;\sigma,\sigma}(t,\varepsilon)\Sigma^{<}_{0}(\varepsilon)G^{a,f}_{0;\sigma,\sigma}(t,\varepsilon) + \mathcal{N}^{f}_{\bar\sigma}(t)G^{r,f}_{0;\sigma,\sigma}(t,\varepsilon-U)\Sigma^{<}_{0}(\varepsilon)G^{a,f}_{0;\sigma,\sigma}(t,\varepsilon-U)$$

$$\begin{aligned}G^{<,(1)}_{\sigma,\sigma}(t,\varepsilon) = {} & \mathcal{N}^{(1)}_{\bar\sigma}(t)\Sigma^{<}_{0}(\varepsilon)\left(G^{r,f}_{0;\sigma,\sigma}(t,\varepsilon-U)G^{a,f}_{0;\sigma,\sigma}(t,\varepsilon-U) - G^{r,f}_{0;\sigma,\sigma}(t,\varepsilon)G^{a,f}_{0;\sigma,\sigma}(t,\varepsilon)\right) \\ & + \frac{i}{\Omega}\partial_t\varepsilon_{ac}(t)\Big\{\left(1-\mathcal{N}^{f}_{\bar\sigma}(t)\right)\Big[\partial_\varepsilon\left(\Sigma^{<}_{0}(\varepsilon)G^{r,f}_{0;\sigma,\sigma}(t,\varepsilon)G^{a,f}_{0;\sigma,\sigma}(t,\varepsilon)\right)G^{a,f}_{0;\sigma,\sigma}(t,\varepsilon) \\ & \qquad + \Sigma^{<}_{0}(\varepsilon)G^{r,f}_{0;\sigma,\sigma}(t,\varepsilon)G^{a,f}_{0;\sigma,\sigma}(t,\varepsilon)\partial_\varepsilon G^{r,f}_{0;\sigma,\sigma}(t,\varepsilon)\Big] \\ & + \mathcal{N}^{f}_{\bar\sigma}(t)\Big[\partial_\varepsilon\left(\Sigma^{<}_{0}(\varepsilon)G^{r,f}_{0;\sigma,\sigma}(t,\varepsilon-U)G^{a,f}_{0;\sigma,\sigma}(t,\varepsilon-U)\right)G^{a,f}_{0;\sigma,\sigma}(t,\varepsilon-U) \\ & \qquad + \Sigma^{<}_{0}(\varepsilon)G^{r,f}_{0;\sigma,\sigma}(t,\varepsilon-U)G^{a,f}_{0;\sigma,\sigma}(t,\varepsilon-U)\partial_\varepsilon G^{r,f}_{0;\sigma,\sigma}(t,\varepsilon-U)\Big]\Big\}\end{aligned}$$

$$G^{r/a,f}_{0;\sigma,\sigma}(t,\varepsilon) = (\varepsilon-\varepsilon_\sigma-\varepsilon_{ac}(t)-\Sigma^{r/a}_{0})^{-1}$$

- In the **Non-Magnetic** case: $\sigma = \bar{\sigma}$
- In the **Linear Regime**: $V_{ac} \to 0$
- At **Zero Temperature**: $k_B T = 0$

$$D_0(\varepsilon_0) = \frac{1}{\pi} \frac{\Gamma}{\varepsilon_0^2 + \Gamma^2}$$

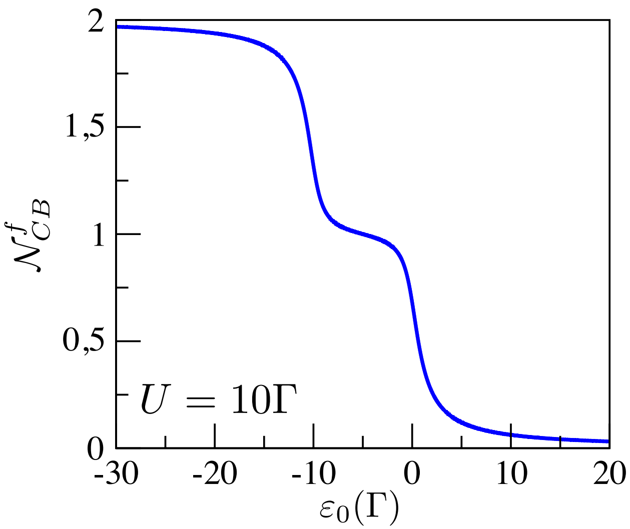
$$\mathcal{N}_0^f(\varepsilon_0) = 1 - \frac{2}{\pi} \tan^{-1}\left(\frac{\varepsilon_0}{\Gamma}\right)$$

$$\mathcal{N}_{CB}^f(\varepsilon_0) = \frac{2\,\mathcal{N}_0^f(\varepsilon_0)}{2 + \mathcal{N}_0^f(\varepsilon_0) - \mathcal{N}_0^f(\varepsilon_0 + U)}$$

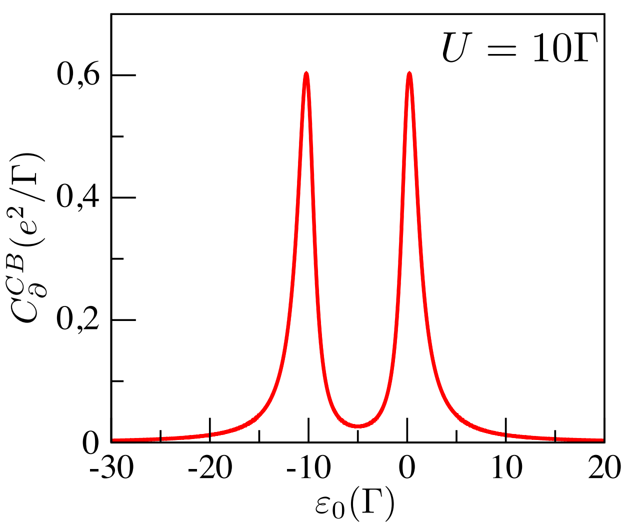
$$C_\partial^{CB}(\varepsilon_0) = 4\,\frac{D_0(\varepsilon_0)(2 - \mathcal{N}_0^f(\varepsilon_0 + U)) + D_0(\varepsilon_0 + U)\mathcal{N}_0^f(\varepsilon_0)}{(2 + \mathcal{N}_0^f(\varepsilon_0) - \mathcal{N}_0^f(\varepsilon_0 + U))^2}$$

$$R_\partial^{CB}(\varepsilon_0) = \frac{h}{8e^2} \frac{D_0^2(\varepsilon_0)(2 - \mathcal{N}_0^f(\varepsilon_0 + U)) + D_0^2(\varepsilon_0 + U)\mathcal{N}_0^f(\varepsilon_0)}{(D_0(\varepsilon_0)(2 - \mathcal{N}_0^f(\varepsilon_0 + U)) + D_0(\varepsilon_0 + U)\mathcal{N}_0^f(\varepsilon_0))^2}(2 + \mathcal{N}_0^f(\varepsilon_0) - \mathcal{N}_0^f(\varepsilon_0 + U))^2$$

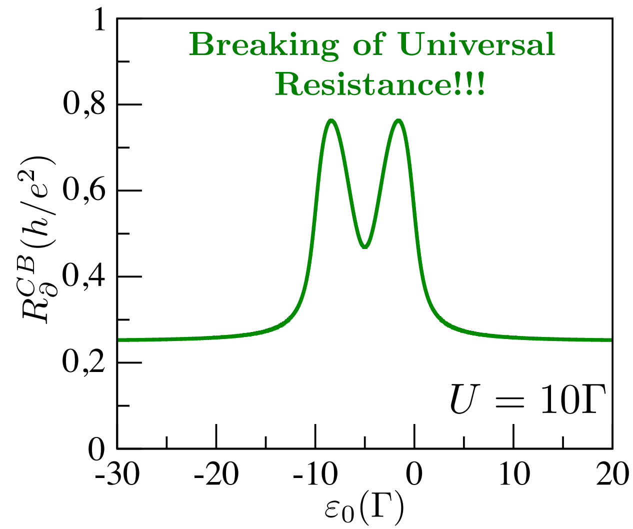
Frozen Occupation



Quantum Capacitance



Charge Relaxation Resistance



Middle plateau between $\varepsilon_0 = 0$ and $\varepsilon_0 = -U$.

Simetric curve around $\varepsilon_0 = -U/2$.
Charge fluctuations induces increased Resistance.
Non-interacting case for $\varepsilon_0 \to \pm\infty$.

Double resonant peak around $\varepsilon_0 = 0$ and $\varepsilon_0 = -U$.

Conclusions

- Purely ac response of an interacting quantum dot for arbitrary ac amplitudes in the limit of low frequencies.
- Breakdown of the charge relaxation resistance universality due to Coulomb charging effects.

Work in progress

- Ac amplitude similar to Coulomb interaction strength $\varepsilon_{ac} \sim U$
- Non-zero temperatures $k_B T \neq 0$

Future work

- Magnetic field.