

# Nanomechanics

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- Detection
- Shuttling
- Doubly-clamped beams
- Graphene
- Strain engineering

# Persistent currents

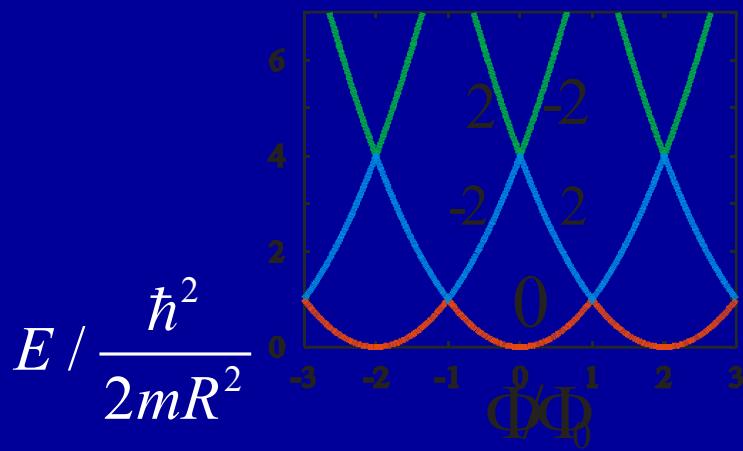
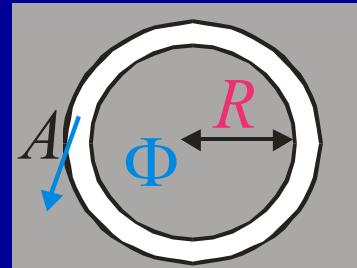
Interference is affected by  
Aharomov-Bohm flux

$$\hat{H} = \frac{\hbar^2}{2m} \left( \vec{p} - \frac{e}{c} \vec{A} \right)^2 \Rightarrow$$

$$\hat{H} = \frac{\hbar^2}{2mR^2} \left( -i \frac{\partial}{\partial \phi} - \frac{\Phi}{\Phi_0} \right)^2;$$

$$\Psi(\phi) \propto e^{iN\phi} \Rightarrow$$

$$E = \frac{\hbar^2}{2mR^2} \left( N - \frac{\Phi}{\Phi_0} \right)^2$$



Energy levels vs. flux

# Measurements of persistent currents

$$I = \frac{\partial E}{\partial \Phi} = \sum_{\text{filled levels}} \frac{\partial E_i}{\partial \Phi}$$

Amplitude clean, single ring:  $\frac{e\hbar}{mR^2}$

Amplitude disordered:

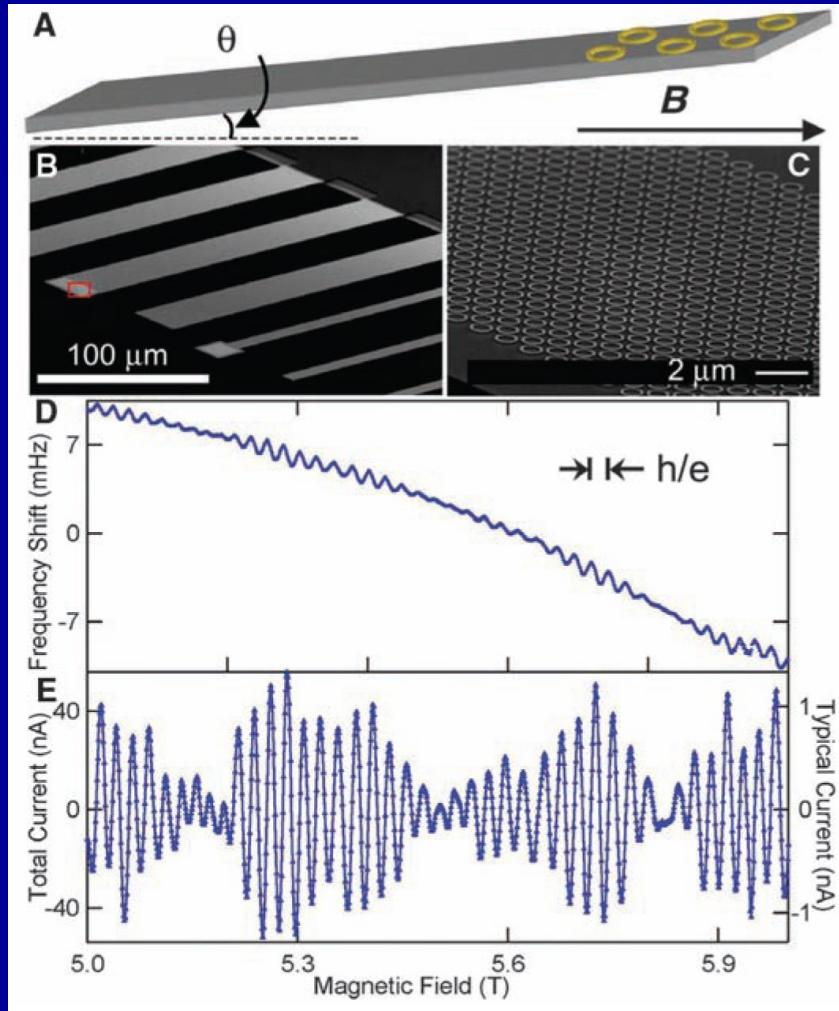
$$I = \sum_l I_l \sin \frac{2\pi l \Phi}{\Phi_0}, I_l = -\frac{4e\delta}{\pi^2 \hbar}$$

Difficult to measure!

A. C. Bleszynski-Jayich et al,  
Science **326**, 272 (2009)

Experiments to date have produced a number of confusing results in apparent contradiction with theory and even among the experiments themselves (2, 3). These conflicts have remained without a clear resolution for nearly 20 years, suggesting that our understanding of how to measure and/or calculate the ground-state properties of as simple a system as an isolated metal ring may be incomplete.

# Measurements of persistent currents



A. C. Bleszynski-Jayich et al,  
 Science **326**, 272 (2009)

Currents produce torque and shift  
 the cantilever frequency

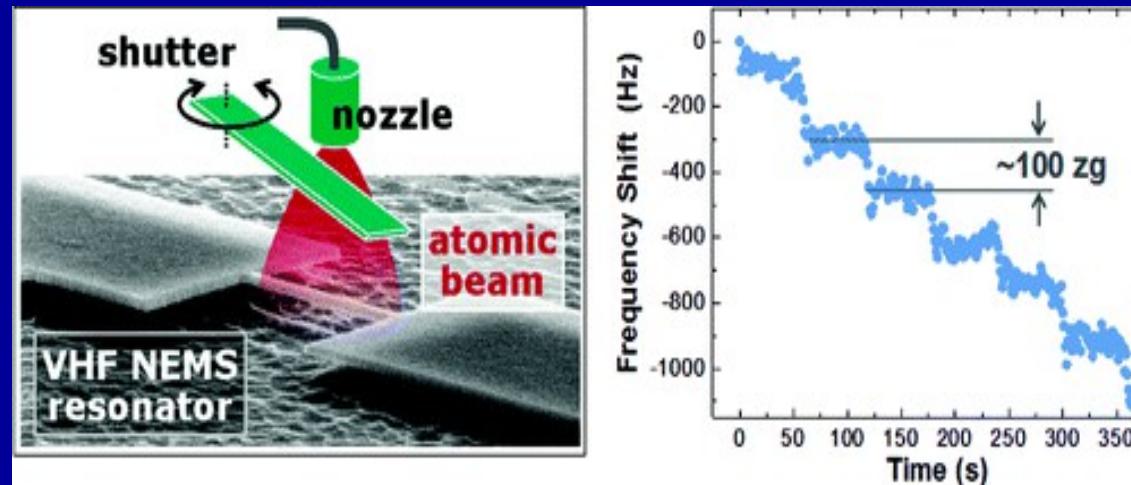


$$\tau = \mu \times B$$

Good agreement with the theory predictions

# Mass detection

Y. T. Yang et al (Roukes group, Caltech) Nano Letters **6**, 583 (2006)



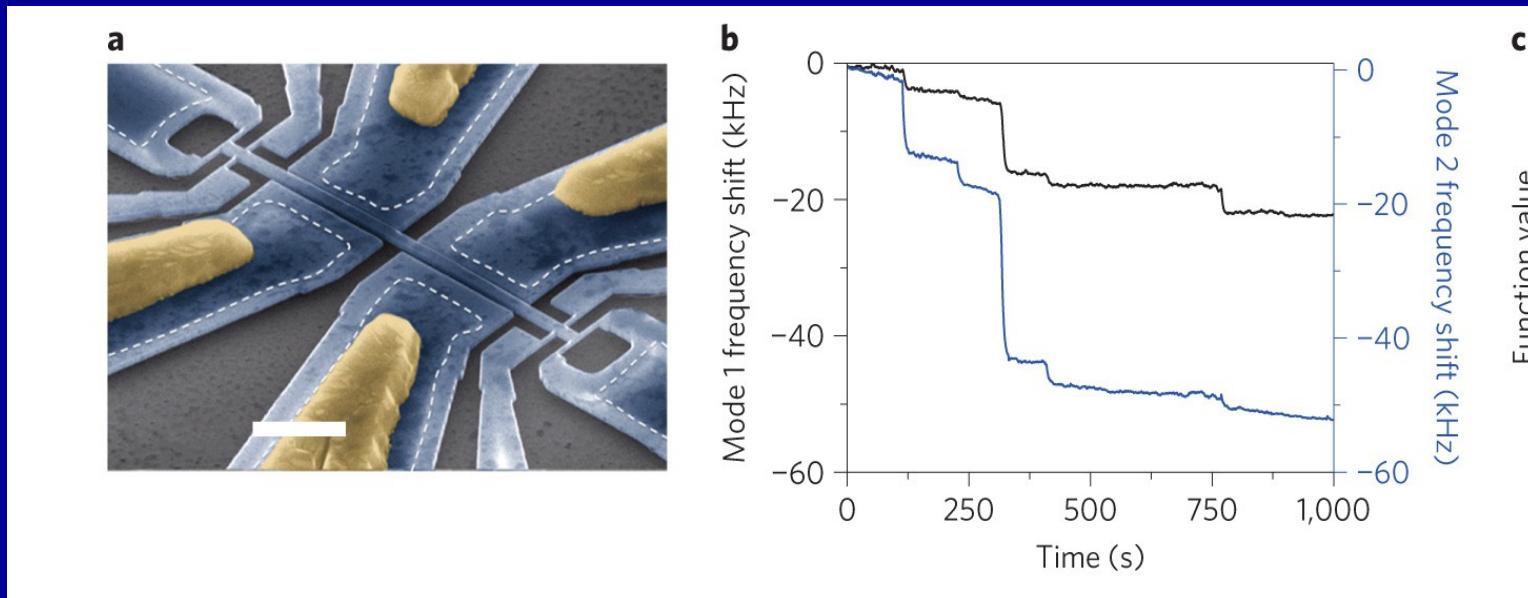
Resonant frequency: 133 MHz

Size: 2300 x 150 x 70 nm

Mass sensitivity: 100 zg

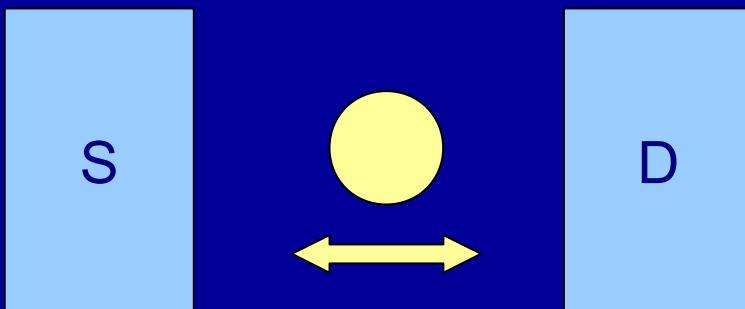
# Single-molecule detection

M. S. Hanay et al, Nature nanotech. 7, 602 (2012)



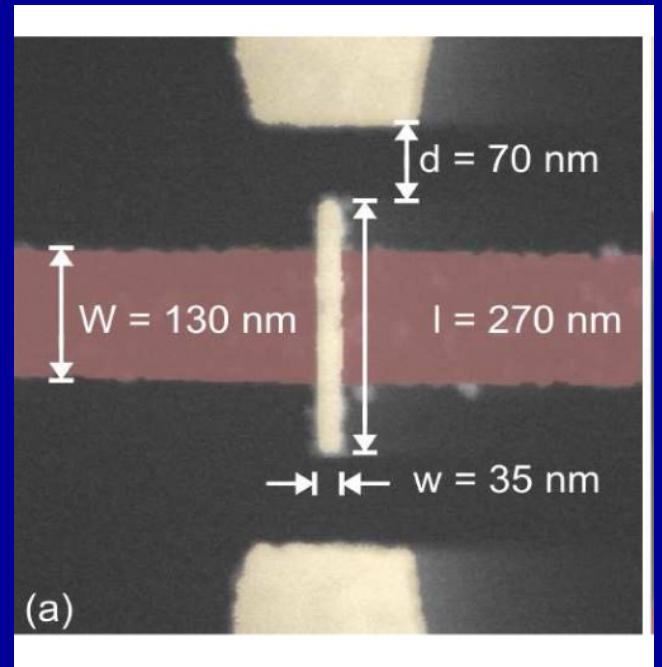
# Shuttling

L. Y. Gorelik et al  
PRL **80**, 4526 (1998)



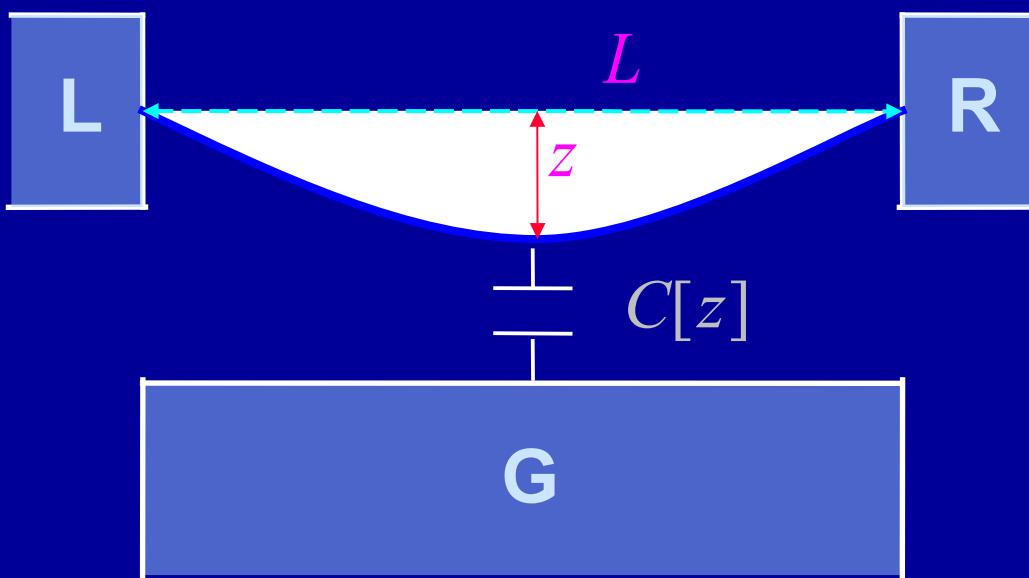
Couples phonons to charge due to the position dependence of tunnel rates and of Coulomb-induced force

D. R. König and E. M. Weig  
APL **101**, 213111 (2012)



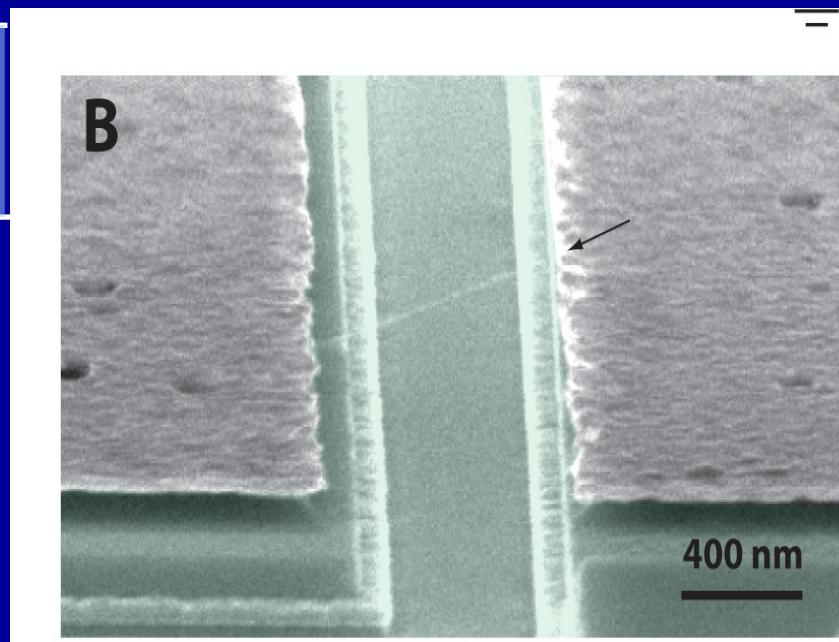
# Double-clamped beam

S. Sapmaz et al  
PRB **67**, 235414 (2003)



Couples phonons to charge due to  
the Coulomb-induced force

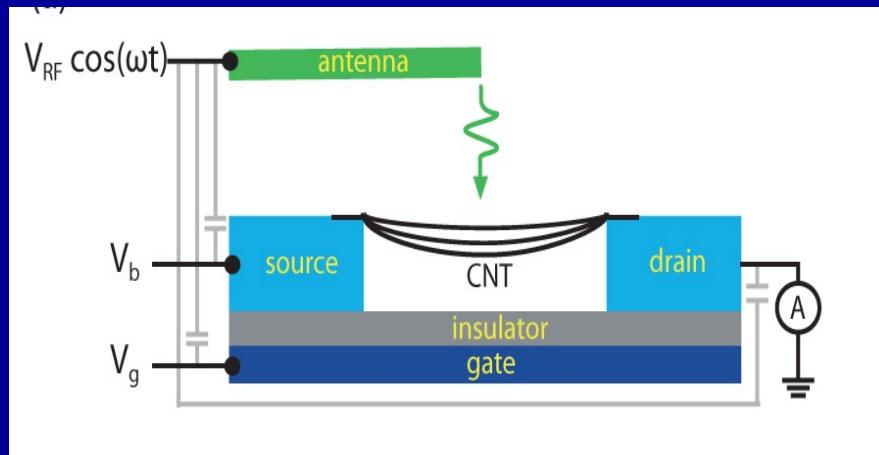
G. Steele et al  
Science **325**, 1103 (2009)



Size: 500 nm  
Frequency: 140 MHz  
Q-factor: over  $10^5$

# Backaction in a double-clamped beam

H. B. Meerwaldt, G. Labadze, B. H. Schneider, A. Taspinar, YMB,  
 H. S. J. van der Zant, and G. A. Steele, PRB **86**, 115454 (2012)



$$M\ddot{x} + \frac{M\omega_0}{Q}\dot{x} + M\omega_0^2 x = F_{RF} \cos \omega t + F[x]$$

$$F[x] = -\Delta kx - \beta x^2 - \alpha x^3$$

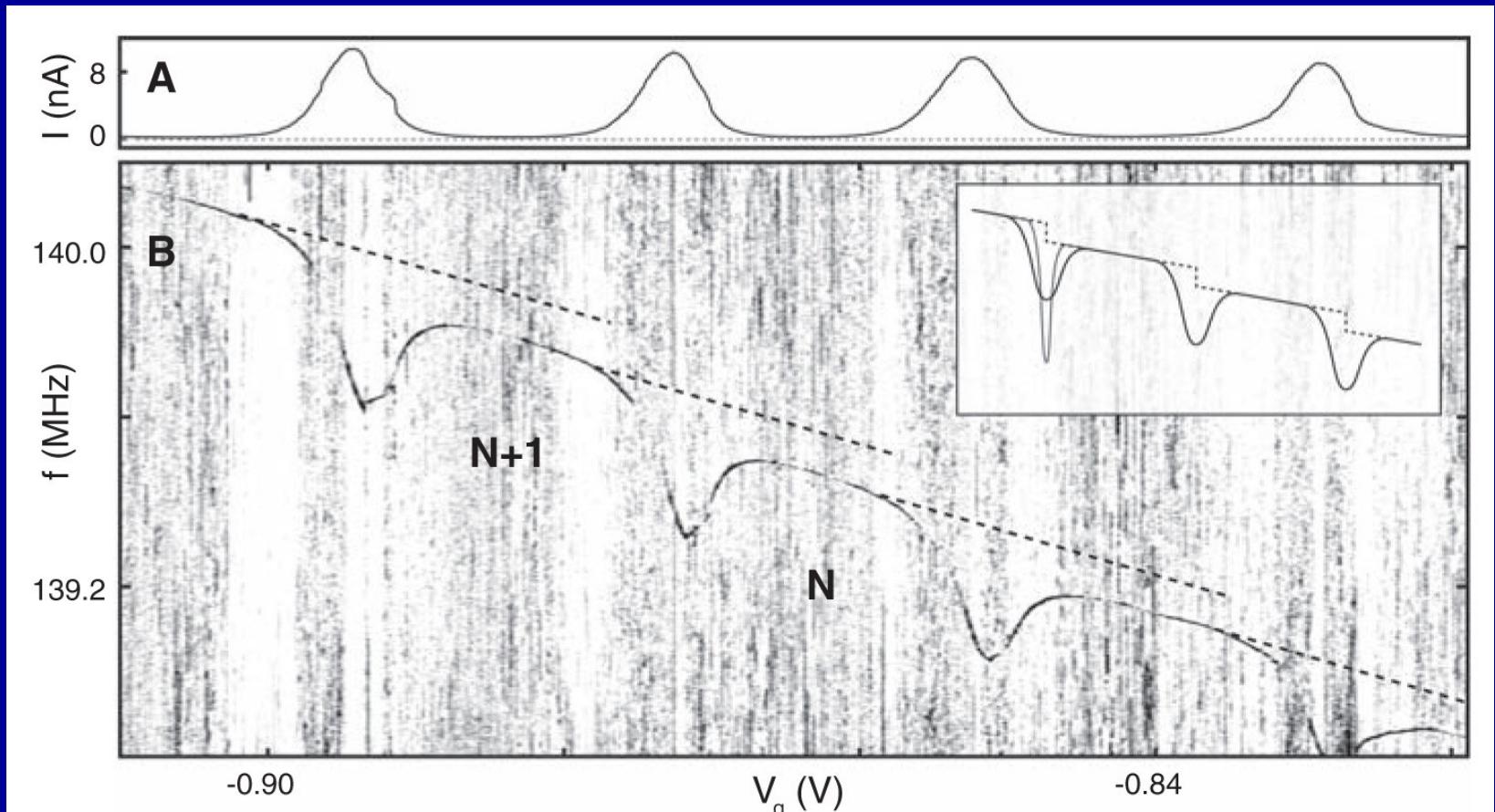
$$F = \frac{1}{2} \frac{dC_g}{dx} (V_g - V_{CNT})^2$$

Renormalization of frequency and introducing of Duffing parameter

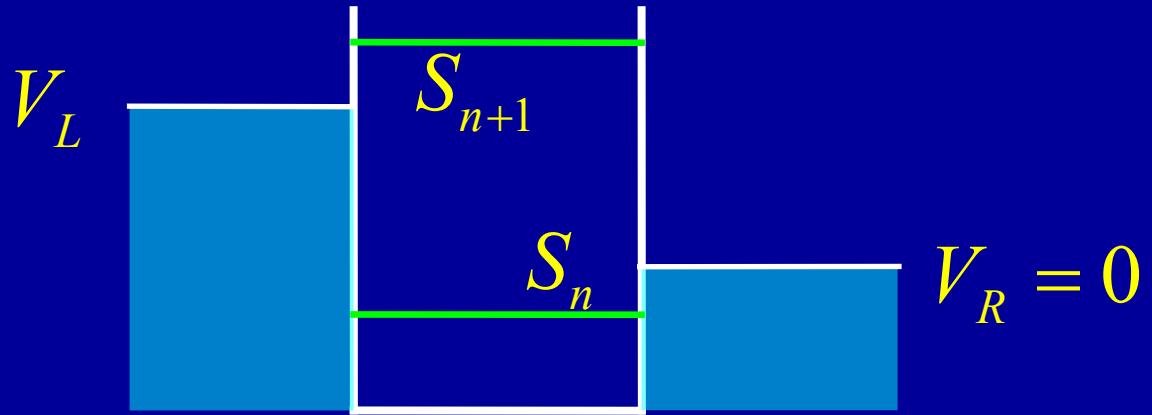
# Frequency stiffening by Coulomb effects

$$\Delta\omega_0 \propto \sim 1 - \frac{C_{tot}}{C_g} - \frac{e}{C_g} \frac{\partial \langle N \rangle}{\partial V_g}$$

G. Steele et al  
Science **325**, 1103 (2009)

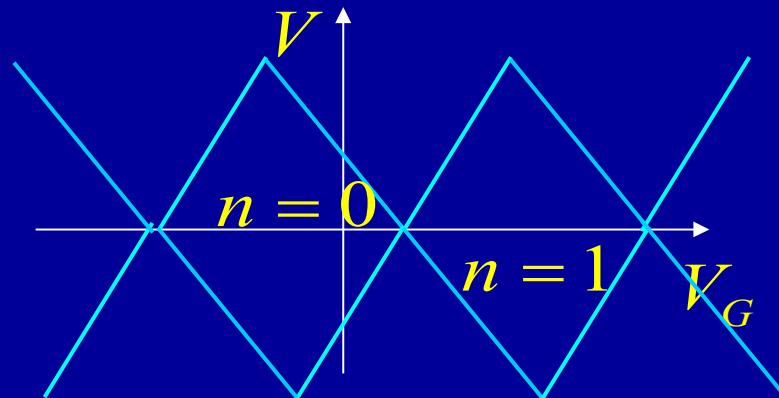


# Coulomb blockade



Conditions that current is not flowing:

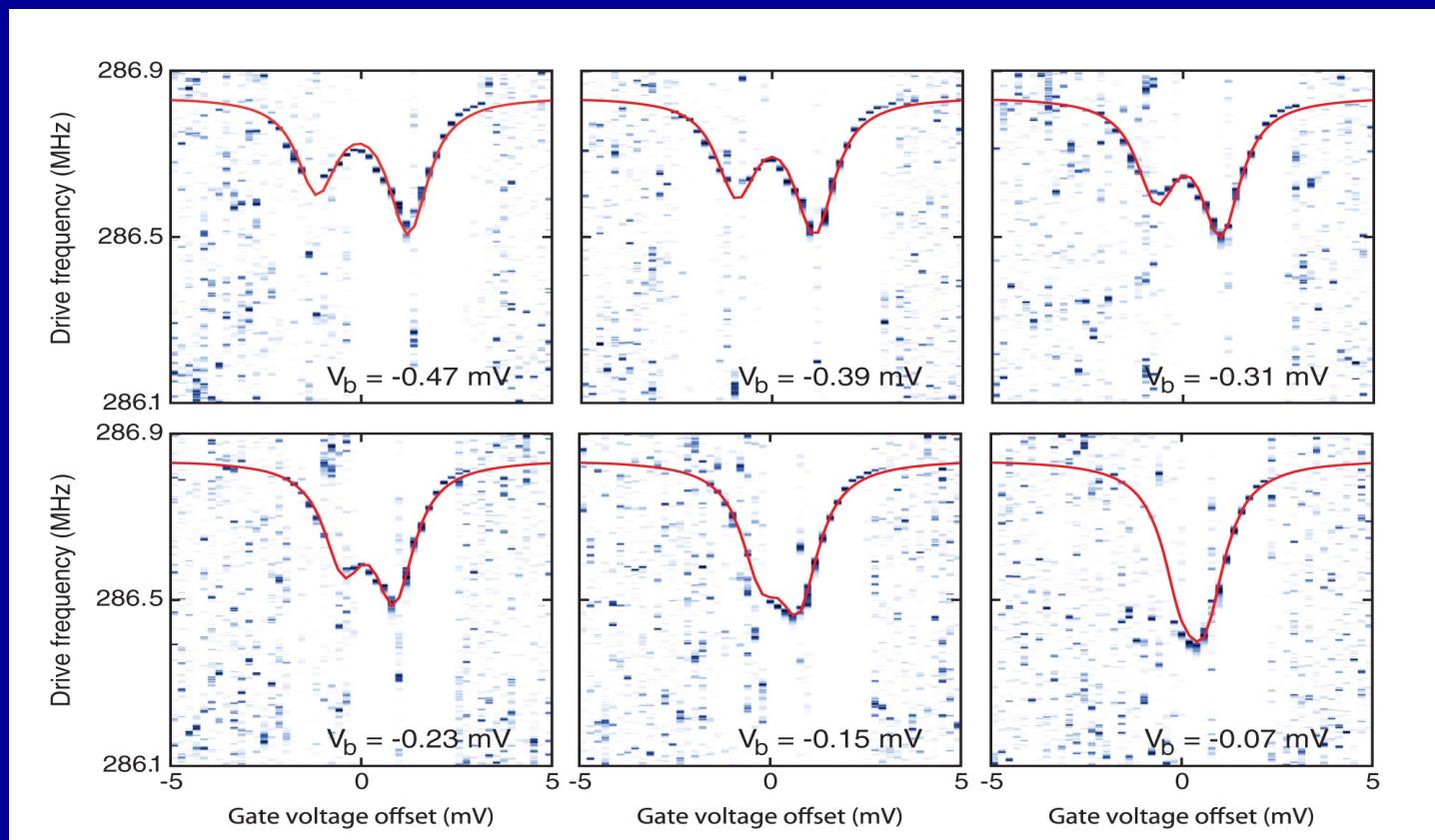
- (a)  $S_{n+1} > eV_L$
- (b)  $S_n < eV_L$
- (c)  $S_{n+1} > 0$
- (d)  $S_n < 0$



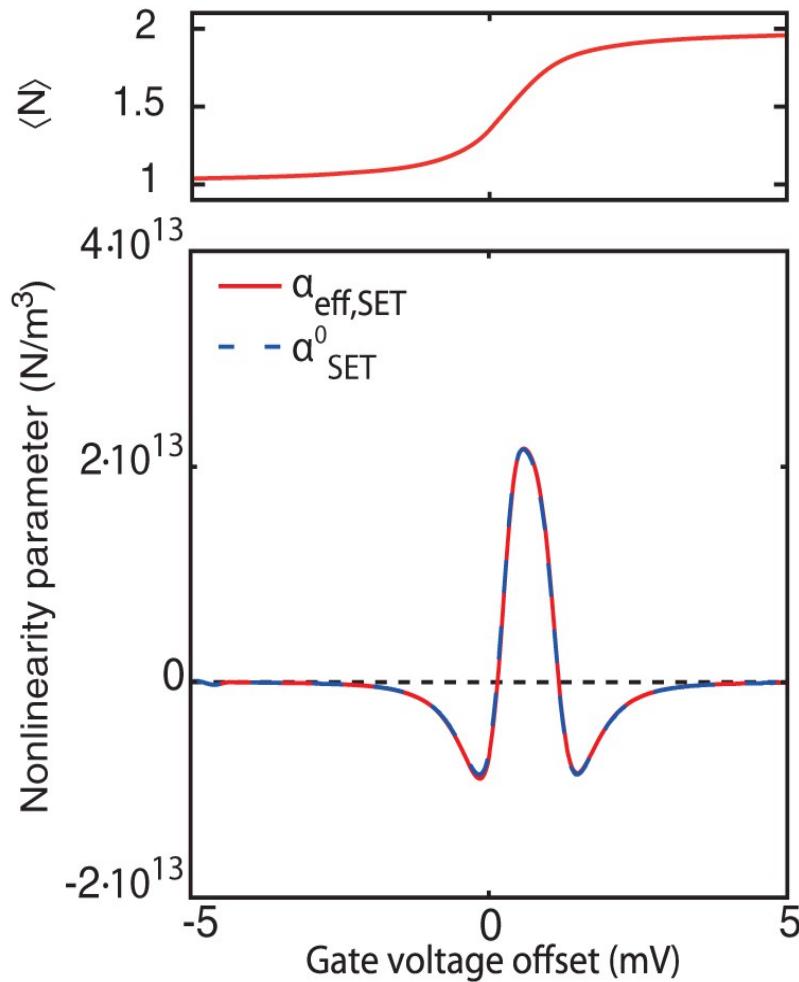
# Frequency softening by Coulomb effects

H. B. Meerwaldt et al, PRB **86**, 115454 (2012)

$$\Delta\omega_0 \propto \sim 1 - \frac{C_{tot}}{C_g} - \frac{e}{C_g} \frac{\partial \langle N \rangle}{\partial V_g}$$



# Coulomb-induced nonlinearity

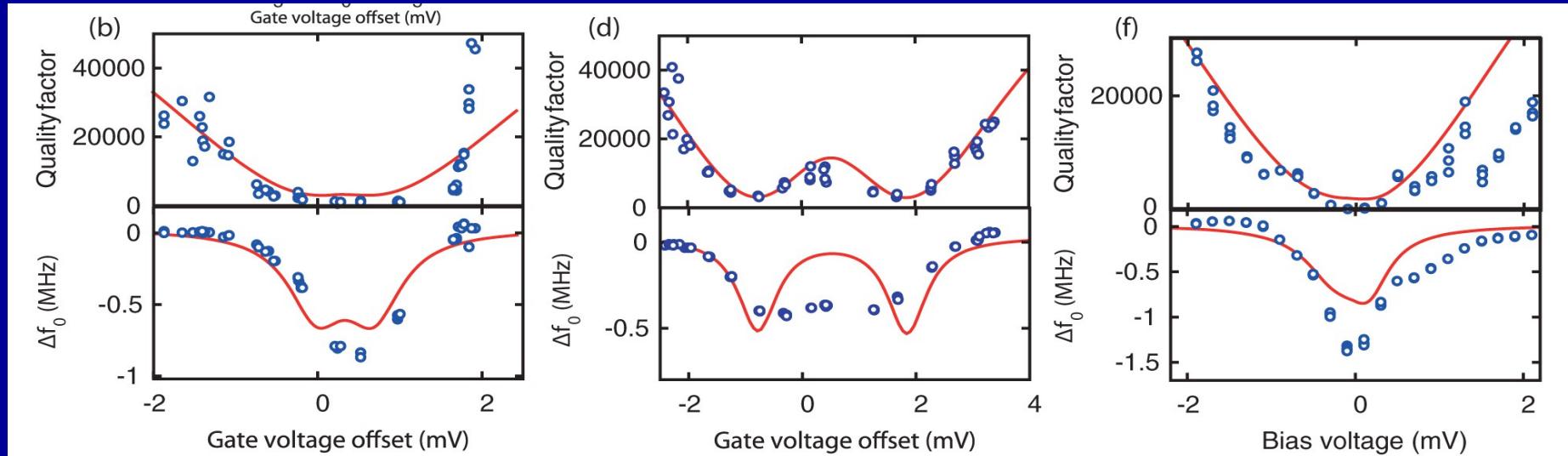


$$F[x] = -\Delta kx - \beta x^2 - \alpha x^3$$

# Coulomb-induced damping

$$\frac{\omega_0}{Q} = \frac{\omega_0}{Q_0} + \frac{F_{stoch} V_g}{m C_g} \frac{1}{\Gamma_{tot}} \frac{dC_g}{dx} \frac{\partial \langle N \rangle}{\partial V_g}$$

O. Usmani, YMB, and Yu. V.Nazarov, PRB **75**, 195312 (2007)



# Backaction in SET coupled to a resonator

$P_n(x, v, t)$  - obeys master equation

$x$  – position  
 $v$  - velocity

O. Usmani, YMB,  
and Yu. V. Nazarov, '04, '07

$$\frac{\partial P_n}{\partial t} + \left( v \frac{\partial}{\partial x} + \frac{\partial}{\partial v} \frac{F}{M} \right) P_n = St[P]$$

$$F(x, v) = -M\omega^2 x - M\gamma v + Fn$$

Adiabaticity: reduce to Fokker-Planck equation

$$\frac{\partial P}{\partial t} + v \frac{\partial P}{\partial x} - \omega^2 x \frac{\partial P}{\partial v} - \left[ \frac{\omega}{Q} + \Psi(x) \right] \frac{\partial}{\partial v} (vP) = D(x) \frac{\partial^2 P}{\partial v^2}$$

Built-in dissipation

Dissipation due to  
tunneling

Diffusion in  
velocity space

$$\Psi(x) = \frac{F^2}{M^2} \frac{\Gamma^-(x)\partial_x\Gamma^+(x) - \Gamma^+(x)\partial_x\Gamma^-(x)}{\Gamma_t^3}$$

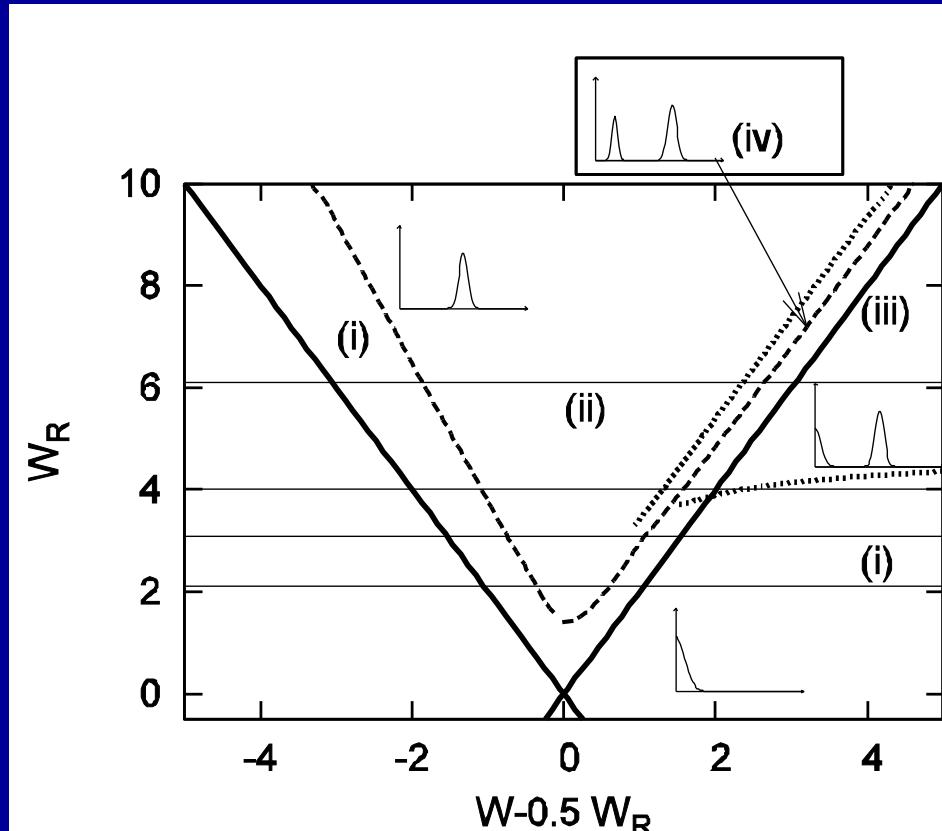
Yaroslav M. Blanter

Capri School, April 2016

# Coulomb-induced instabilities

$$\frac{\omega_0}{Q} = \frac{\omega_0}{Q_0} + \frac{F_{stoch} V_g}{m C_g} \frac{1}{\Gamma_{tot}} \frac{dC_g}{dx} \frac{\partial \langle N \rangle}{\partial V_g}$$

O. Usmani, Ya. M. Blanter, and Yu. V. Nazarov,  
 PRB **75**, 195312 (2007)



$$\Gamma_{L,R}^{\pm} = 2e^{a_{L,R}(\Delta E_{L,R} + Fx)} \times f_F(\pm(\Delta E_{L,R} + Fx))$$

# Strain in graphene

Deformation of a graphene sheet acts at electrons as pseudomagnetic field in the Dirac equation

$$A_x = t\beta(u_{xx} - u_{yy}); A_y = -2t\beta u_{xy}; \beta \approx 3$$

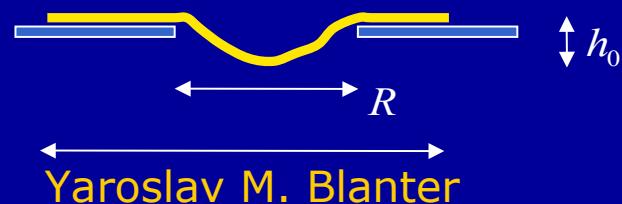
Suzuura, Ando '02  
 Guinea, Katsnelson, Vozmediano '08

Deformation caused by uniform load:

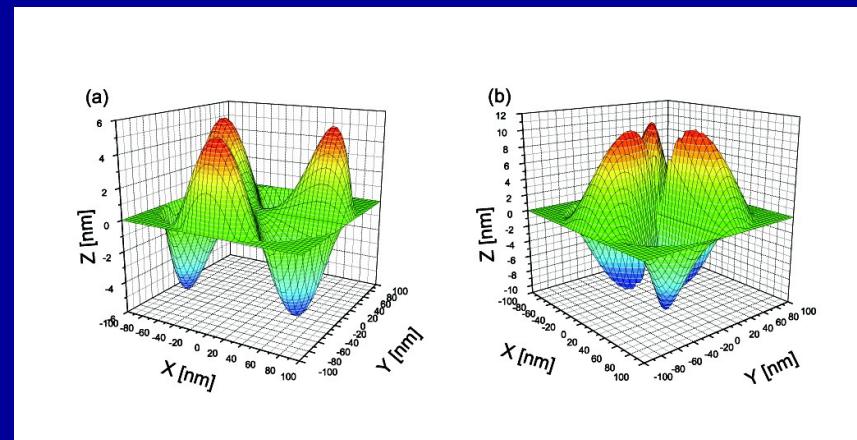
$$h(r) = \frac{h_0}{R^4} (R^2 - r^2)^2$$

Deformation caused by local load:

$$h(r) = \frac{h_0}{R^2} \left( \frac{1}{2} (R^2 - r^2) - r^2 \ln \frac{R}{r} \right)$$



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Uniform load

Local load (center)

Dirac equation: with added gauge fields

$$\vec{\sigma}(v_F \vec{p} + \vec{A}) \Psi(\vec{r}) = E \Psi(\vec{r})$$

K.-J.Kim, YMB, K.-H.Ahn

Phys. Rev. B **84**, 081401 (2011)

Capri School, April 2016

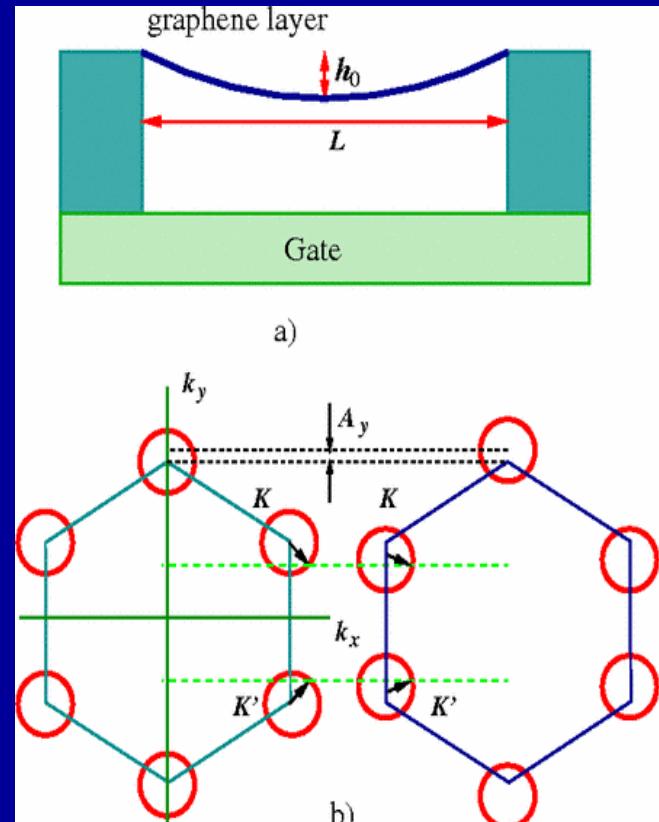
# Piezoeconductivity in graphene

Deformation:

- Creates strain: pseudomagnetic gauge fields
- Creates density redistribution; the profile needs in principle to be calculated self-consistently

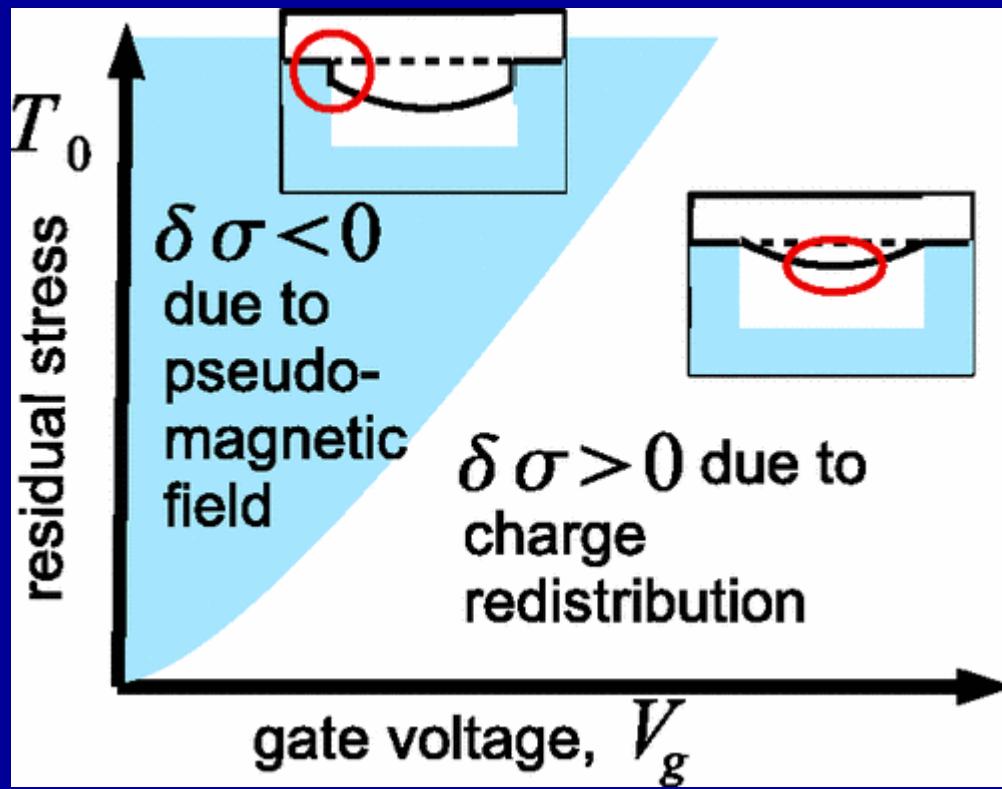
Fogler, Guinea, and Katsnelson  
 Phys. Rev. Lett. **101**, 226804 (2008)

Local shift of the Dirac cones:  
 Predicted metal-insulator transition at certain deformation



# Piezoeconductivity in graphene

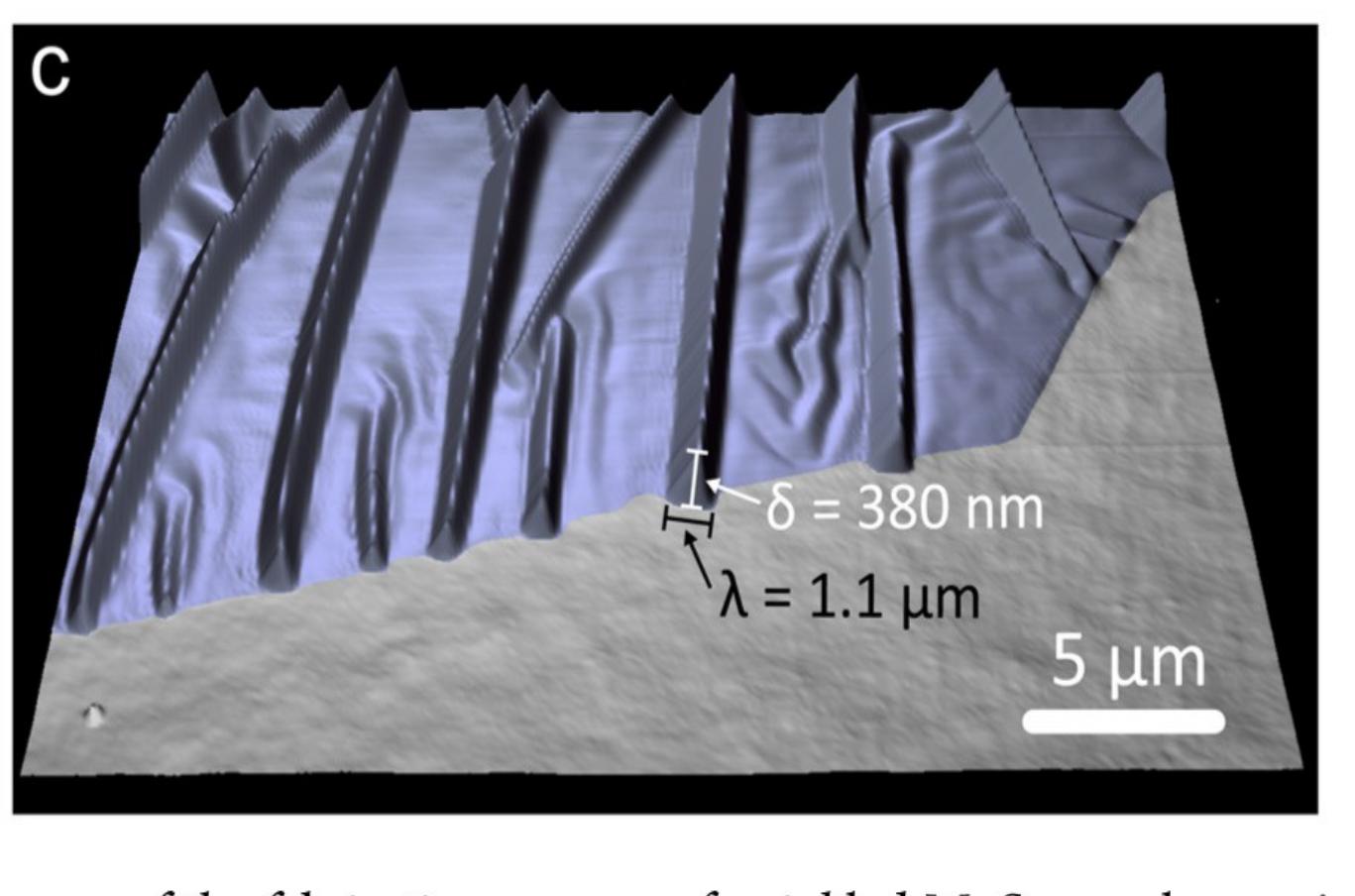
M. V. Medvedyeva and YMB  
Phys. Rev. B **83**, 045426 (2011)



# Strain engineering in MoS<sub>2</sub>

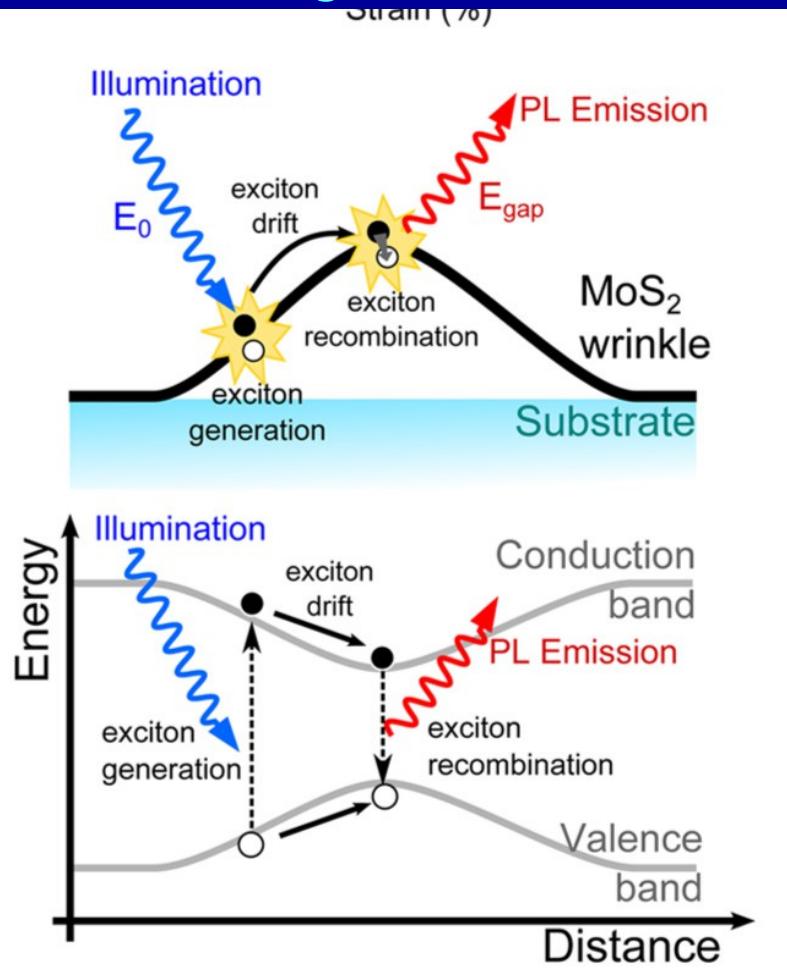
Wrinkles: large strain difference

A. Castellanos-Gomez et al  
Nano Lett. **13**, 5361 (2013)



# Strain engineering in MoS<sub>2</sub>

## Funneling of excitons



A. Castellanos-Gomez et al  
Nano Lett. **13**, 5361 (2013)

